Prime Numbers as Superposed Wavefunctions: A Harmonic Approach to Composite Detection and Distribution

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We recast the sieve of Eratosthenes in a harmonic framework. Rigourously, for each prime p we build a *divisibility detector* from a normalized cosine sum

$$\chi_p(n) \coloneqq \frac{1}{p}\sum_{k=0}^{p-1}\cos\Big(\frac{2\pi kn}{p}\Big),$$

which equals 1 iff $p \mid n$ and 0 otherwise. A multiplicative *sieve mask*

$$\mathcal{M}_Y(n) \coloneqq \prod_{p\le Y}\big(1-\chi_p(n)\big)$$

vanishes exactly at integers n divisible by a prime $\le Y$, and equals 1 otherwise. Choosing $Y = \sqrt{x}$ eliminates every composite $\le x$. We analyze the uncovered density via the Mertens product $\prod_{p\le Y}(1-1/p) \sim \mathrm{e}^{-\gamma}/\log Y$ and connect this to the prime number theorem heuristics. As a visualization, we also discuss a *wave superposition* picture (cosine "interference") that matches the rigorous mask while providing intuition and figures. Numerical experiments illustrate the mask, uncovered fraction, and the approach to zero uncovered density as $Y \to \infty$.

I. INTRODUCTION

Prime numbers are the multiplicative atoms of the integers [**?**]. Classical sieves remove multiples of small primes to reveal larger primes. We revisit this process through harmonic constructions: periodic signals that *exactly* detect divisibility and, when combined multiplicatively, implement a sieve.

Contributions. (i) We formalize for each prime p a normalized cosine-sum detector $\chi_p(n)$ with $\chi_p(n) = 1$ iff $p \mid n$; (ii) we define a multiplicative sieve mask $\mathcal{M}_Y$ that removes all composites $\le x$ by taking $Y = \sqrt{x}$; (iii) we quantify the uncovered density with Mertens' product and relate it to prime density heuristics; (iv) we provide a wave-superposition visualization consistent with the rigorous construction, and report numerical illustrations.

II. PRIME WAVEFUNCTIONS AND DIVISIBILITY DETECTORS

A. Rigorous detector via cosine superposition

Define for prime p,

$$\chi_p(n) \coloneqq \frac{1}{p}\sum_{k=0}^{p-1}\cos\Big(\frac{2\pi kn}{p}\Big). \tag{1}$$

Lemma 1 (Divisibility Indicator)**.** *For any integer n and prime p, $\chi_p(n) = 1$ if $p \mid n$ and $\chi_p(n) = 0$ otherwise.*

Proof. □

B. Heuristic "wavefunction" for visualization

Define a phase-shifted cosine

$$\Psi_p^{\mathrm{viz}}(n) \coloneqq -\cos\Big(\frac{2\pi n}{p}\Big), \tag{2}$$

so $\Psi_p^{\mathrm{viz}}(kp) = -1$ at multiples kp.

Remark 1. Ψ_p^{viz} *is* not *an exact detector; it visualizes "troughs at multiples" (helpful for figures), while χ_p in Eq. (**??**) does the exact combinatorial work.*

III. MULTIPLICATIVE SIEVE MASK AND "INTERFERENCE"

A. Exact multiplicative mask

For a cutoff $Y \ge 2$, define

$$\mathcal{M}_Y(n) \coloneqq \prod_{p\le Y}\big(1-\chi_p(n)\big). \tag{3}$$

Proposition 1 (Correctness up to a range)**.** *Let $x \ge 4$ and choose $Y = \sqrt{x}$. Then for all $2 \le n \le x$,*

$$\mathcal{M}_Y(n) = \begin{cases} 0, & \textit{if } n \textit{ is composite,} \\ 1, & \textit{if } n \textit{ is prime.} \end{cases}$$

Proof. □

B. Additive "interference" view (visual story)

Define an additive score

$$S_Y(n) := \sum_{p \le Y} \Psi_p^{\mathrm{viz}}(n). \tag{4}$$

Multiples of small primes accumulate negative contributions (many "troughs"). While S_Y is not a classifier, its contrast aligns qualitatively with the exact mask $\mathcal{M}_Y$ and produces informative images.

IV. COVERAGE RATIOS AND PRIME DENSITY LINKS

A. Novelty of a new prime in the sieve order

If primes are applied in increasing order, the fresh fraction eliminated by p is

$$f(p) = \frac{1}{p} \prod_{q<p} \Big(1 - \frac{1}{q}\Big), \tag{5}$$

by inclusion–exclusion. Summing $f(p)$ over $p \le Y$ gives the total eliminated density, i.e.,

$$1 - U(Y), \qquad U(Y) := \prod_{p \le Y} \Big(1 - \frac{1}{p}\Big).$$

B. Asymptotics via Mertens' product

Theorem 1 (Mertens' product)**.** *As $Y \to \infty$,*

$$\prod_{p \le Y} \Big(1 - \frac{1}{p}\Big) \sim \frac{\mathrm{e}^{-\gamma}}{\log Y}, \tag{6}$$

where γ is the Euler–Mascheroni constant.

Proof. □

Remark 2 (Heuristic bridge to PNT)**.** *If the sieve removes all composites up to x by taking $Y = \sqrt{x}$, then*

$$U(\sqrt{x}) \sim \frac{\mathrm{e}^{-\gamma}}{\frac{1}{2}\log x} = \frac{2\mathrm{e}^{-\gamma}}{\log x},$$

which mirrors the leading $1/\log x$ scale in the prime number theorem. This does not constitute a proof of PNT but is a standard consistency check of sieve heuristics.

V. FIGURES

VI. LIMITING BEHAVIOR AND "RARITY"

As $Y \to \infty$, the uncovered density $U(Y) = \prod_{p \le Y}(1 - 1/p)$ tends to 0 by Eq. (**??**). In sieve terms, primes collectively remove "almost all" integers in the sense that numbers with no small prime factor have vanishing density. This aligns with the intuitive rarity of integers that survive many sieving stages, without implying any maximal prime.

psi2_psi3_plot.png

FIG. 1. Visualization waves Ψ_2^{viz} and Ψ_3^{viz} over $n \in [1, 36]$, showing troughs at multiples. (Heuristic; the exact detector uses Eq. (**??**).)

VII. DISCUSSION

VIII. CONCLUSION

We presented a harmonic reformulation of sieving: exact divisibility detectors χ_p built from cosine superpositions, an associated multiplicative mask that eliminates all composites up to a range, and density estimates via Mertens' product consistent with prime asymptotics. The companion wave visualization aids intuition and supports clear numerical demonstrations.

[1] G. H. Hardy and E. M. Wright, *An Introduction to the Theory of Numbers*, Oxford Univ. Press, 5th ed., 1979.

[2] F. Mertens, Über eine zahlentheoretische Funktion, Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften, Wien (1874).

fft_mask_210.png

FIG. 2. Exact mask $\mathcal{M}_{11}(n)$ on $1 \leq n \leq 210$. White indicates $\mathcal{M}_{11}(n) = 1$ (no factor ≤ 11), black indicates eliminated.

uncovered_fraction_plot.png

FIG. 3. Uncovered fraction $U(Y)$ vs. Y (points: empirical; solid: $\prod_{p \leq Y}(1 - 1/p)$; dashed: $\mathrm{e}^{-\gamma}/\log Y$). Convergence follows Mertens' product.

[3] G. Tenenbaum, *Introduction to Analytic and Probabilistic Number Theory*, 3rd ed., AMS, 2015.

[4] H. L. Montgomery and R. C. Vaughan, *Multiplicative Number Theory I: Classical Theory*, Cambridge, 2006.