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Digital Design and Computer Architecture: RISC-V
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Exercise 2.1. Write a Boolean Equation in sum-of-products canonical form for each of the truth tables below:

(a)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	1

(b)

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

(c)

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

(d)

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

(e)

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

Solution:

(a) $Y = \bar{A}\bar{B} + A\bar{B} + AB$

(b) $Y = \bar{A}\bar{B}\bar{C} + ABC$

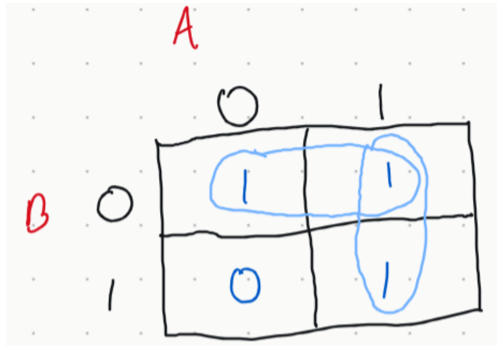

Figure 1: Exercise 2.5 (a): K-Map for Truth Table in Exercise 2-1 (a)

(c) $Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + A\bar{B}C + ABC$

(d) $Y = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}CD + A\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D} + ABCD\bar{D}$

(e) $\bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}B\bar{C}D + \bar{A}BC\bar{D} + A\bar{B}\bar{C}D + A\bar{B}C\bar{D} + AB\bar{C}\bar{D} + ABCD$

Exercise 2-3. Write a Boolean equation in product-of-sums canonical form for the truth tables in Exercise 2-1.

Solution:

(a) $Y = A + \bar{B}$

(b) $Y = (A + B + \bar{C})(A + \bar{B} + C)(A + \bar{B} + \bar{C})(\bar{A} + B + C)(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + C)$

(c) $Y = (A + B + \bar{C})(A + \bar{B} + \bar{C})(\bar{A} + \bar{B} + C)$

(d) $Y = (A + \bar{B} + C + D)(A + \bar{B} + C + \bar{D})(A + \bar{B} + \bar{C} + D)(A + \bar{B} + \bar{C} + \bar{D})(\bar{A} + B + C + \bar{D})(\bar{A} + B + \bar{C} + \bar{D})(\bar{A} + \bar{B} + C + D)(\bar{A} + \bar{B} + C + \bar{D})(\bar{A} + \bar{B} + \bar{C} + \bar{D})$

(e) $Y = (A + B + C + \bar{D})(A + B + \bar{C} + D)(A + \bar{B} + C + D)(A + \bar{B} + \bar{C} + \bar{D})(\bar{A} + B + C + D)(\bar{A} + B + \bar{C} + \bar{D})(\bar{A} + \bar{B} + C + \bar{D})(\bar{A} + \bar{B} + \bar{C} + D)$

Exercise 2-5. Minimize each of the Boolean equations from Exercise 2-1.

Solution:

(a) The K-map is shown in Figure 1. To minimize $Y = \bar{A}\bar{B} + A\bar{B} + AB$, we notice from the K-map that the minterm $A\bar{B}$ is shared, so we use the idempotency to duplicate it and use it in simplification:

$$Y = \bar{A}\bar{B} + A\bar{B} + AB = (\bar{A}\bar{B} + A\bar{B}) + (A\bar{B} + AB) = \bar{B} + A$$

(b) The K-map is shown in Figure 2. It shows that the implicants in $Y = \bar{A}\bar{B}\bar{C} + ABC$ are prime implicants and hence, the equation cannot be reduced further.

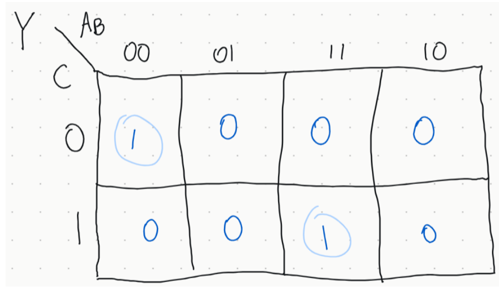

Figure 2: Exercise 2.5 (b): K-Map for Truth Table in Exercise 2-1 (b)

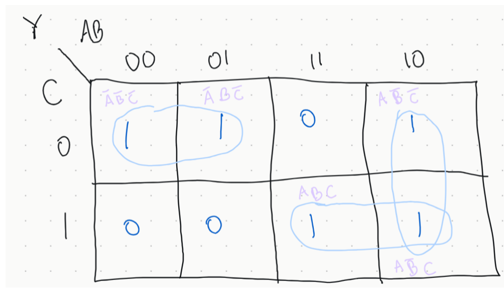

Figure 3: Exercise 2.5 (c): K-Map for Truth Table in Exercise 2-1 (c)

(c) The K-map is shown in Figure 3. It reveals that $Y = \bar{A}\bar{B}\bar{C}+\bar{A}B\bar{C}+A\bar{B}\bar{C}+A\bar{B}C+ABC$ can be reduced:

$$\begin{aligned} Y &= \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + A\bar{B}C + ABC \\ &= (\bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C}) + (A\bar{B}\bar{C} + A\bar{B}C) + (A\bar{B}C + ABC) \\ &= \bar{A}\bar{C} + A\bar{B} + AC \end{aligned}$$

(d) The K-map is shown in Figure 4. It reveals how to reduce the equation $Y = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}CD + A\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D} + ABCD\bar{D}$. First we work with the implicants belonging to the rectangle of size 4 in the first column:

$$((\bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D) + (\bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D})) = (\bar{A}\bar{B}\bar{C}) + (\bar{A}\bar{B}C) = (\bar{A}\bar{B})$$

Then we work with the other 4-by-4 rectangle that wraps around:

$$(\bar{A}\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D}) + (\bar{A}\bar{B}C\bar{D} + A\bar{B}C\bar{D}) = \bar{B}\bar{C}\bar{D} + \bar{B}C\bar{D} = \bar{B}\bar{D}$$

Finally, we work with the 2-by-2 square that shares a minterm $A\bar{B}C\bar{D}$:

$$ABC\bar{D} + A\bar{B}C\bar{D} = AC\bar{D}$$

Altogether, the minimized equation is $Y = AC\bar{D} + \bar{A}\bar{B} + \bar{B}\bar{D}$.

(e) The K-Map is shown in Figure 5. It reveals that it is not possible to reduce sum, so $Y = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}B\bar{C}D + \bar{A}BC\bar{D} + A\bar{B}\bar{C}D + A\bar{B}C\bar{D} + AB\bar{C}\bar{D} + ABCD$.

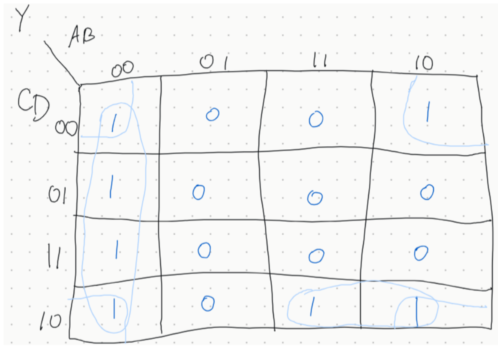

Figure 4: Exercise 2.5 (d): K-Map for Truth Table in Exercise 2-1 (d)

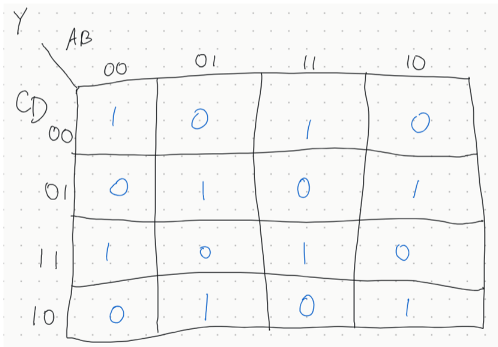

Figure 5: Exercise 2.5 (e): K-Map for Truth Table in Exercise 2-1 (e)

Exercise 2.7. Sketch a reasonably simple combinational circuit implementing each of the functions from Exercise 2.5. *Reasonably simple* means that you are not wasteful of gates, but you don't waste vast amounts of time checking every possible combination of the circuit either.

Solution: (a) See Figure 6.

(b) See Figure 7. See Figure 8 for an alternative.

(c) Note that $\bar{A}\bar{C} + AC$ corresponds to the following 2-input truth table, which shows it is equivalent to a NOT gate followed by an XOR gate, which is a XNOR gate.

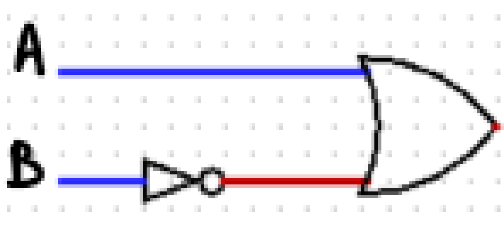

Figure 6: Exercise 2.7 (a): Circuit implementing boolean equation in Exercise 2.5(a)

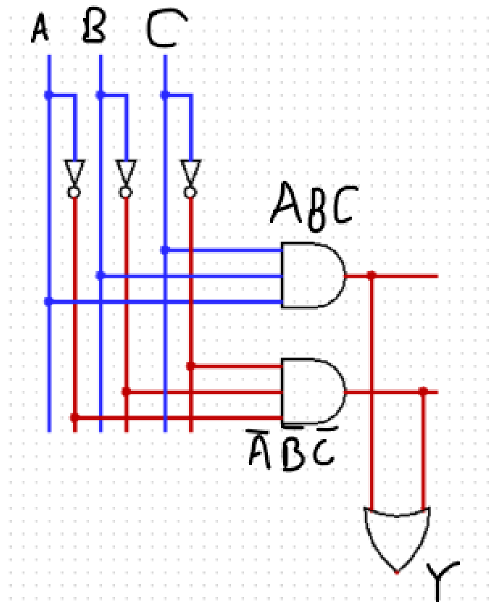

Figure 7: Exercise 2.7 (b): Circuit implementing boolean equation in Exercise 2.5(b)

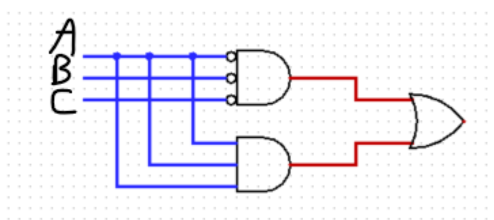

Figure 8: Exercise 2.7 (b) - Alternnative: Circuit implementing boolean equation in Exercise 2.5(b)

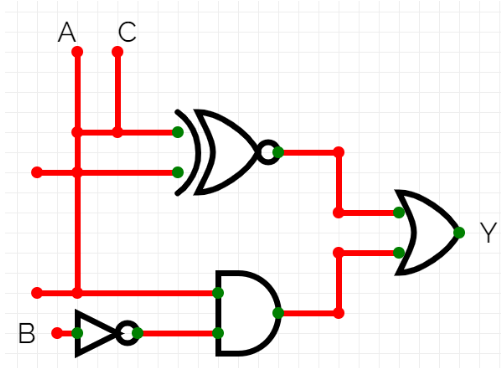

Figure 9: Exercise 2.7 (c): Circuit implementing boolean equation in Exercise 2.5 (c)

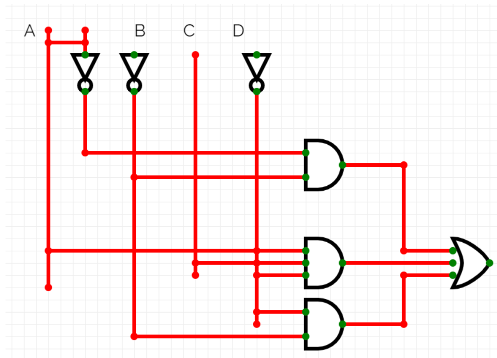

Figure 10: Exercise 2.7 (d): Circuit implementing boolean equation in Exercise 2.5 (d)

A	C	Y
0	0	1
0	1	0
1	0	0
1	1	1

With that in mind, the circuit is given in Figure 9

(d) See Figure 10

(e) From the truth corresponding to Exercise 2.5 (e), and hence Exercise 2.1 (e), the output is 1 when there is an even number of 1s, or when we have all 0s. This is the opposite of an XOR gate, which is 1 when an odd number of inputs are 1. Hence, this is the negation of a 4-input NOR gate. According to section 2.5.1 in the book, we can build a XOR gate from using cascading XOR gates. Therefore, we can use two XOR gates and one XNOR gate. See Figure 11

Exercise 2.9. Repeat Exercise 2.7 using only NOT gates, and AND, and OR gates.

(a) Same as Exercise 2.5 (a), which is in Figure 1.

(b) Same as version 1 of Exercise 2.5(b), which is in Figure 7.

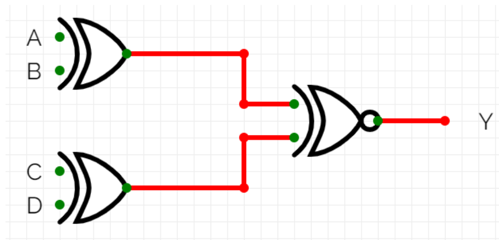

Figure 11: Exercise 2.7 (e): Circuit implementing boolean equation in Exercise 2.5 (e)

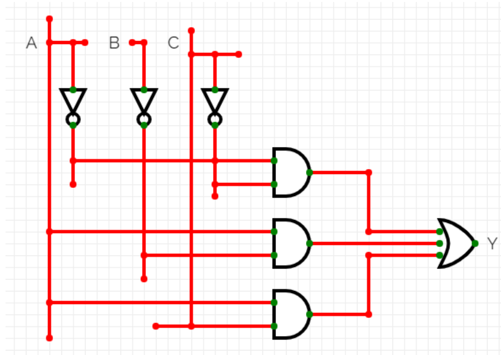

Figure 12: Exercise 2.9 (c): Circuit implementing boolean equation in Exercise 2.5 (c) using only AND, OR, NOT

(c) See Figure 12.

(d) We can write the equation as $Y = \bar{D}(AC + \bar{B}) + \bar{A}\bar{B}$. See Figure 13.

(e) We can use the distributive property to re-write the equation as:

$$\begin{aligned} Y &= \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}B\bar{C}D + \bar{A}BC\bar{D} + A\bar{B}\bar{C}D + A\bar{B}C\bar{D} + AB\bar{C}\bar{D} + ABCD \\ &= (\bar{A}\bar{B})(\bar{C}\bar{D} + CD) + (\bar{A}B)(\bar{C}D + C\bar{D}) + (A\bar{B})(\bar{C}D + C\bar{D}) + (AB)(\bar{C}\bar{D} + CD) \\ &= (\bar{A}\bar{B} + AB)(\bar{C}\bar{D} + CD) + (\bar{A}B + A\bar{B})(\bar{C}D + C\bar{D}) \end{aligned}$$

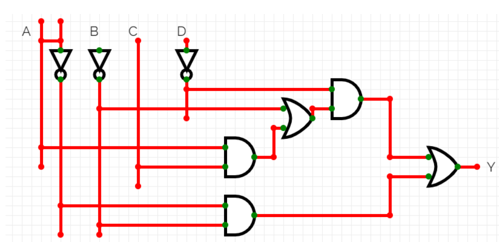

Figure 13: Exercise 2.9 (d): Circuit implementing boolean equation in Exercise 2.5 (d) using only AND, OR, NOT

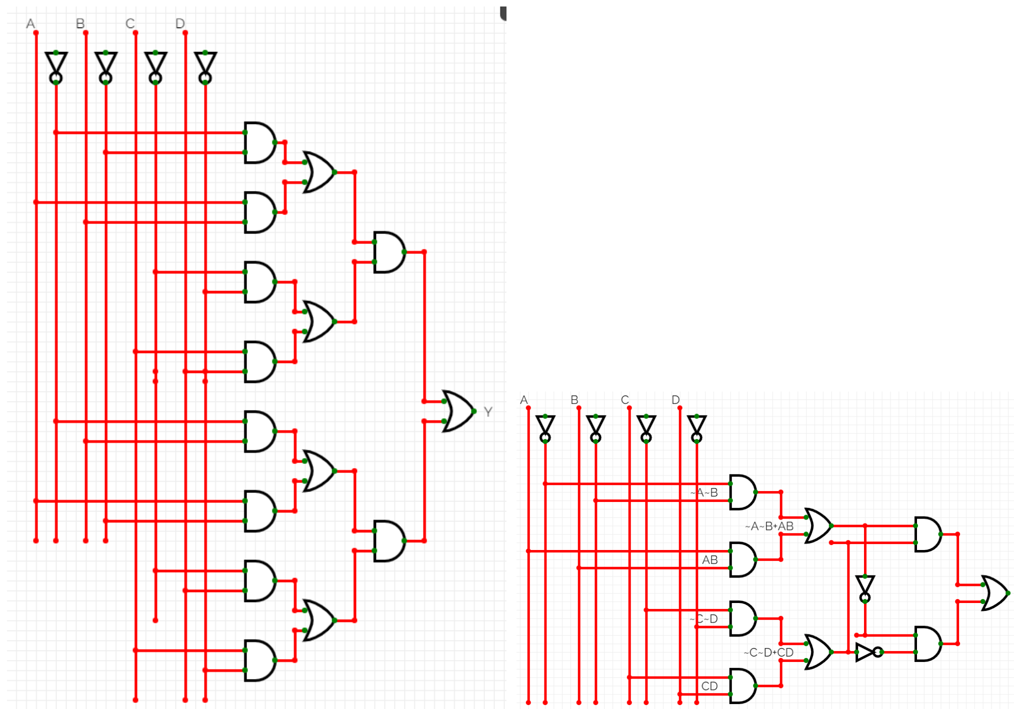

Figure 14: Exercise 2.9 (e): Two Circuits implementing boolean equation in Exercise 2.5 (e) using only AND, OR, NOT

Alternatively, note that:

$$\begin{aligned}\overline{(\bar{A}\bar{B}+AB)} &= (A+B)(\bar{A}+\bar{B}) \qquad \text{(De Morgan's Theorem)}\\ &= A\bar{A}+A\bar{B}+B\bar{;}A+B\bar{B}\\ &= A\bar{B}+\bar{A}B\end{aligned}$$

Using this, we could further simplify our equation. Letting $G=\bar{A}\bar{B}+AB$ and $H=\bar{C}\bar{D}+CD$, we can write Y as:

$$Y = HG + \bar{H}\bar{G}$$

See Figure 14.

Exercise 2.11. Repeat Exercise 2.7 using only NOT and NAND and NOR gates.

Solution: (a) The equation $Y=\bar{B}+A$ is equivalent to $Y=\overline{B\bar{A}}$. This implies the circuit in Figure 15.

(b) We can brute force it by replacing AND gates with NAND and NOT, and OR gates with NOR and NOT. Then we simplify and get Figure 16

(c) The equation is $Y=\bar{A}\bar{C}+A\bar{B}+AC$.

(d) The equation is $Y=\bar{D}(AC+\bar{B})+\bar{A}\bar{B}$.

(e) See Figure 19

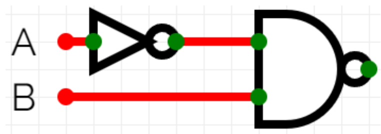

Figure 15: Exercise 2.11 (a): Circuit implementing boolean equation in $Y = \bar{B} + A$ with only NAND and NOT

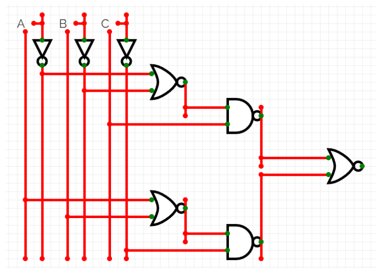

Figure 16: Exercise 2.11 (b): Circuit implementing $Y = \bar{A}\bar{B}\bar{C} + ABC$ using only NOT, NAND, and NOR gates.

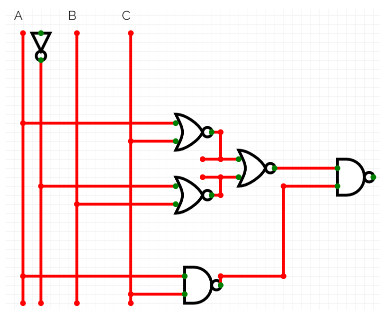

Figure 17: Exercise 2.11 (c): Circuit implementing $Y = \bar{A}\bar{C} + A\bar{B} + AC$. using only NOT, NAND, and NOR gates.

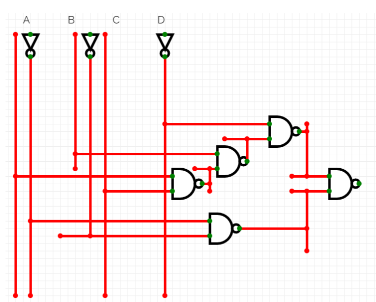

Figure 18: Exercise 2.11 (d): Circuit implementing $Y = \bar{A}\bar{C} + A\bar{B} + AC$. using only NOT, NAND, and NOR gates.

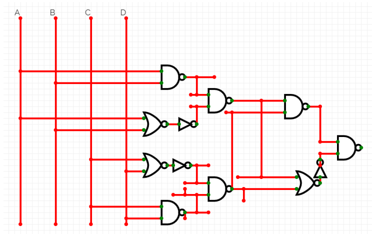

Figure 19: Exercise 2.11 (e): Circuit implementing $Y = \bar{A}\bar{C} + A\bar{B} + AC$. using only NOT, NAND, and NOR gates.

Exercise 2.19. Give an example of a truth table requiring between 3 billion and 5 billion rows that can be constructed using fewer than 40 (but at least 1) two-input gates.

Solution: Consider the 32-input AND boolean function:

$$Y = \prod_{n=1}^{32} A_n$$

There are 2^{32} outputs (around 4.3 billion). It can be constructed with 31 two-input AND gates.

Exercise 2.21. Alyssa P. Hacker says that any Boolean function can be written in minimal sum-of-product form as the sum of all of the prime implicants of the function. Ben Bitdiddle says there are some funnctions whose minimal equation does not involve all of the prime implicants. Explain why Alyssa is right or proved a counterexample demonstrating Ben's point.

Solution: Alyssa is not correct. Consider Figure 20, representing a K-Map for a boolean equation that is reduced in two ways:

$$\begin{aligned} Y &= A\bar{B}\bar{C}D + \bar{A}B\bar{C}D + AB\bar{C}D + \bar{A}BCD + \bar{A}BC\bar{D} \\ Y &= \bar{A}BC\bar{D} + \bar{A}B\bar{C}D + \bar{A}BCD + AB\bar{C}D + A\bar{B}\bar{C}D \end{aligned}$$

We can now reduce them to (respectively):

$$\begin{aligned} Y &= A\bar{B}\bar{C}D + B\bar{C}D + \bar{A}BC \\ Y &= \bar{A}BC\bar{D} + \bar{A}BD + A\bar{C}D \end{aligned}$$

Note that both of this equations are minimal, and both contain distinct prime implicants.

Exercise 2.22. Prove the following theorems are true using perfect induction. You need not prove their duals.

(a) The idempotency theorem (T3)

(b) The distributivity theorem (T8)

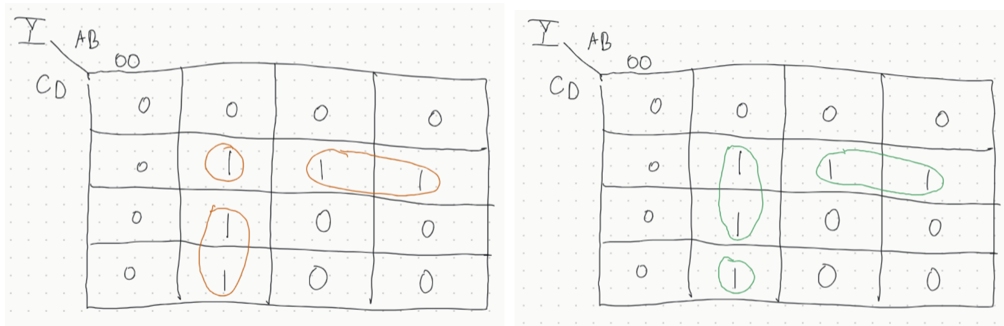

Figure 20: 2-21: K-Map for two equivalent, minimal boolean equations

(c) The combining theorem (T10)

Solution: (a) Idempotency says that $B \cdot B = B$. If $B = 0$, then equation says $0 \cdot 0 = 0$, which is a true equation. If $B = 1$, then it says $1 \cdot 1 = 1$, which is also a true equation.

(b) Distributivity says $(B \cdot C) + (B \cdot D) = B \cdot (C + D)$. Consider the truth tables:

B	C	D	$(B \cdot C) + (B \cdot D)$	$B \cdot (C + D)$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	0	0
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

(c) Combining theorem says: $(B \cdot C) + (B \cdot \bar{C}) = B$. The truth table is as follows:

C	B	$(B \cdot C) + (B \cdot \bar{C})$
0	0	0
0	1	1
1	0	0
1	1	1

Exercise 2.23. Prove De Morgan's Theorem for three variables, A, B, and C, using perfect induction.

Solution: De Morgan's says that

$$\overline{\prod B_n} = \sum \bar{B_n}$$

There are two cases:

1. Suppose that a term B_k is 0. Then the left-hand product is 0, and the inverse operation makes the result 1. On the right-hand side, since $B_k = 0$, it follows that $\bar{B}_k = 1$, and since the right-hand side consists of OR operations only, the entire result becomes 1. Hence, both sides are 1.

2. Suppose that no term is 0. That is, every Bn is 1. The product is 1, and the inverse is 0, meaning the left-hand side is 0. For the right-hand side, since every B_n, it follows that every $\bar{B}_n$ is 0, and the OR of them all is still 0. Hence, both sides are 0.

Exercise 2.24. Write Boolean equations for the circuit in Figure 2.82 (in the book, page 98). You need not minimize the equations.

Solution: The equations are:

$$Y = \bar{A}D + A\bar{C}D + A\bar{B}C + ABCD$$
$$Z = BD + A\bar{C}D$$

Exercise 2.25. Minimize the Boolean Equations from Exercise 2.24 and sketch an improved circuit with the same function.

Solution: The following is the boolean table for this function:

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

The corresponding K-Map is in Figure 21. We have a prime implicant consisting of 8 minterms, and a prime implicant with one minterm:

$$Y = \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD + \bar{A}B\bar{C}D + \bar{A}BCD + AB\bar{C}D + ABCD + A\bar{B}\bar{C}D + A\bar{B}CD + A\bar{B}C\bar{D}$$
$$Y = \bar{A}\bar{B}D + \bar{A}BD + ABD + A\bar{B}D + A\bar{B}C\bar{D}$$
$$Y = \bar{A}D + AD + A\bar{B}C\bar{D}$$
$$Y = D + A\bar{B}C\bar{D}$$

The improved circuit can be seen on Figure 22.

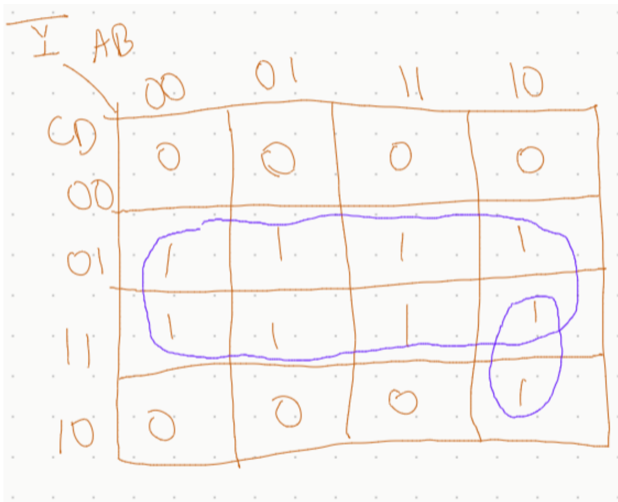

Figure 21: Exercise 2-25: Boolean table for boolean function on Exercise 2-23

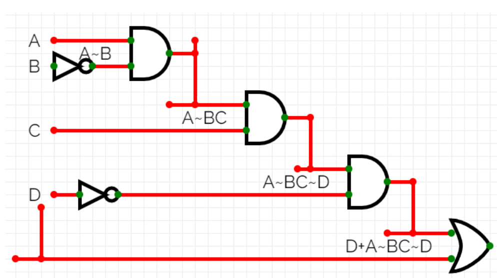

Figure 22: Exercise 2-25: Improved version of the circuit in Exercise 2.24.

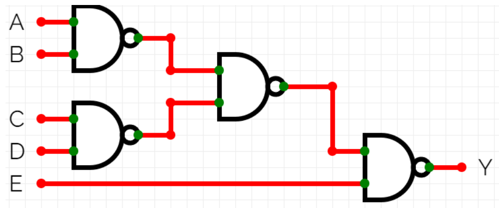

Figure 23: Exercise 2-26: Original circuit to reduce

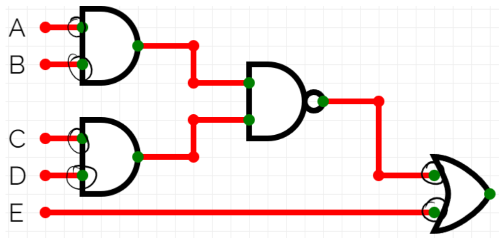

Figure 24: Exercise 2-26: Re-written circuit

Exercise 2.26. Using De Morgan equivalent gates and bubble pushing methods, redraw the circuit in Figure 23 so that you can find the Boolean equation by inspection. Write the Boolean equation.

Solution: The re-written circuit is in Figure 24. The boolean equation can be written as:

$$Y = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{E}$$

Exercise 2.27. Repeat Exercise 2.26 for the circuit in Figure 25.

Solution: The rewritten circuit is in Figure 26.

$$Y = (ABC + \bar{D}) + (\bar{E}(\bar{F} + \bar{G}))$$

Exercise 2.28. Find a minimal Boolean equation for the function in the table below. Remember to take advantage of the don't care entries.

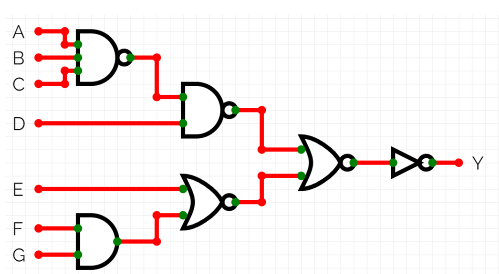

Figure 25: Exercise 2-27: Circuit to rewrite

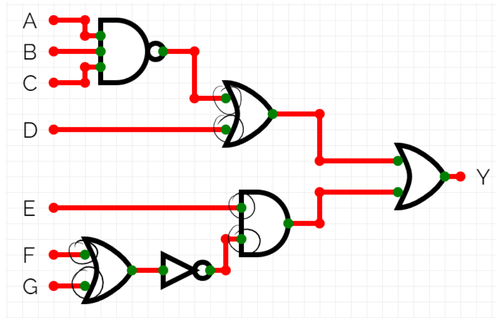

Figure 26: Exercise 2-27: Rewritten version

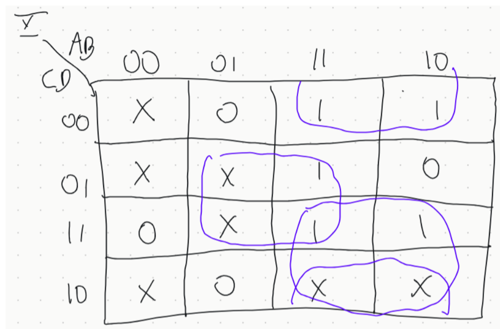

Figure 27: Exercise 2-28: K-Map for truth table with don't cares

A	B	C	D	Y
0	0	0	0	X
0	0	0	1	X
0	0	1	0	X
0	0	1	1	0
0	1	0	0	0
0	1	0	1	X
0	1	1	0	0
0	1	1	1	X
1	0	0	0	1
1	0	0	1	0
1	0	1	0	X
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	X
1	1	1	1	1

Solution: The corresponding K-Map is given in Figure 27. The corresponding equation is:

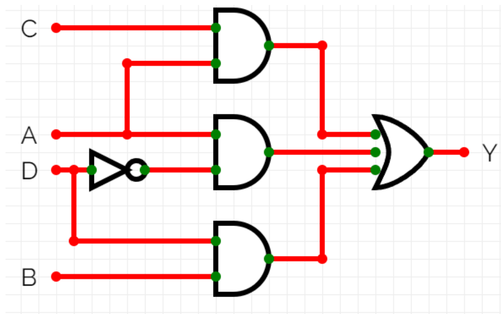

Figure 28: Exercise 02-29: Circuit from Boolean function on Exercise 02-28

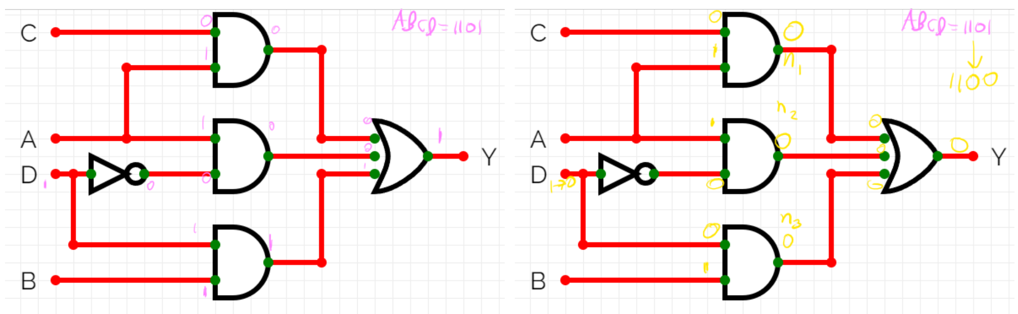

Figure 29: Exercise 02-30: Glitch across prime implicant boundary from ABCD=1101 to ABCD=1100.

$$
\begin{aligned}
Y &= \bar{A}B\bar{C}D + \bar{A}BCD + AB\bar{C}D + ABCD \\
&\quad + ABCD + A\bar{B}CD + ABC\bar{D} + A\bar{B}C\bar{D} \\
&\quad + AB\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D} + ABC\bar{D} + A\bar{B}C\bar{D} \\
&= \bar{A}BD + ABD + ACD + AC\bar{D} + A\bar{C}\bar{D} + AC\bar{D} \\
&= BD + AC + A\bar{D}
\end{aligned}
$$

Exercise 2.29. Sketch a circuit for the function from Exercise 2.28.

Solution: See Figure 28

Exercise 2.30. Does your circuit from Exercise 2.29 have any potential glitches when one of the inputs changes? If not, explain why not. If so, show how to modify the circuit to eliminate the glitches.

Solution: Yes, there is a glitch. Note in Figure 27 that there $AB\bar{C}\bar{D}$ and $AB\bar{C}D$ belong to different prime implicants, but if D changes from 0 to 1, or from 1 to 0, the boundary between the implicants is crossed. In particular, consider the state of the system when $ABCD = 1101$ (corresponding to the prime implicant $AB\bar{C}D$ just mentioned). See Figure 29.

In particular, node n_3 falls from 1 to 0 before n_2 rises from 0 to 1, and thus there is a short time during which Y has a value of 0 and not 1. To fix the issue, we add a circle to cover the prime implicant boundary. In doing so, we stretch it to cover all 4 squares on the

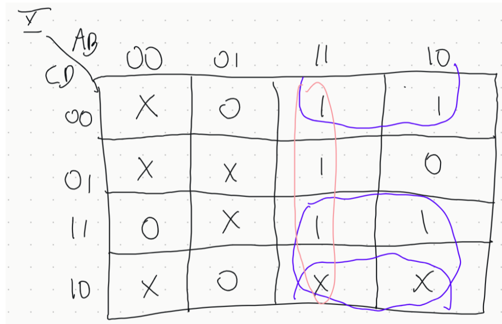

Figure 30: Exercise 02-30: Updated K-Map with no term crossing a prime implicant boundary.

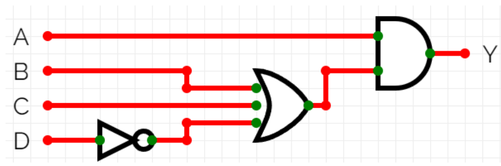

Figure 31: Exercise 02-30: Updated circuit without glitch

third column. This makes the center square superflous so we remove it, and the updated K-Map is shown on Figure 30

Now we can simplify this updated K-Map:

$$
\begin{aligned}
Y &= AB\bar{C}\bar{D} + AB\bar{C}D + ABCD + ABC\bar{D} \\
&+ ABCD + A\bar{B}CD + ABC\bar{D} + A\bar{B}C\bar{D} \\
&+ ABC\bar{D} + A\bar{B}C\bar{D} + AB\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D} \\
&= AB\bar{C} + ABC + ACD + AC\bar{D} + AC\bar{D} + A\bar{C}\bar{D} \\
&= AB + AC + A\bar{D} \\
&= A(B + C + \bar{D})
\end{aligned}
$$

The updated, glitch-less circuit is in Figure 31.

Exercise 2.31. Find a minimal Boolean equation for the function in the Boolean table below.

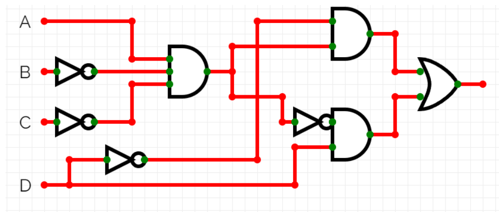

Figure 32: Exercise 02-32: Circuit based on boolean table in Exercise 2-31.

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	1
0	0	1	0	X
0	0	1	1	X
0	1	0	0	0
0	1	0	1	X
0	1	1	0	X
0	1	1	1	X
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	X
1	1	1	1	1

Remember to take advantage of the don't care entries.

Solution:

$$\begin{aligned}
Y &= A\bar{B}\bar{C}\bar{D} \\
&+ \bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}D + \bar{A}\bar{B}CD + \bar{A}BCD \\
&+ \bar{A}B\bar{C}D + \bar{A}BCD + AB\bar{C}D + ABCD \\
&+ \bar{A}\bar{B}CD + \bar{A}BCD + ABCD + A\bar{B}CD \\
&= A\bar{B}\bar{C}\bar{D} + \bar{A}\bar{C}D + \bar{A}CD + \bar{A}BD + ABD + \bar{A}CD + ACD \\
&= A\bar{B}\bar{C}\bar{D} + \bar{A}D + BD + CD
\end{aligned}$$

Exercise 2.32. Sketch a circuit for the function from Exercise 2.31

Solution: We can simplify the function further to

$$Y = A\bar{B}\bar{C}\bar{D} + D(\bar{A} + B + C) = \bar{D}A\bar{B}\bar{C} + D\overline{A\bar{B}\bar{C}}$$

The corresponding circuit is on Figure 32.

Exercise 2.33. Ben Bitdiddle will enjoy his picnic on sunny days that have no ants. He will also enjoy his picnic any day he sees a hummingbird, as well as on days where there are ants and lady bugs. Write a Booelan equation for his enjoyment (E) in terms of sun (S), ants (A), hummingbirds (H), and ladybugs (L).

Solution:

$$E = S\bar{A} + H + AL$$