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Practice Problems

Exercise 6.1. In the following, let r be the number of rows in a DRAM array, c the number of columns, b_r the number of bits needed to address the rows, and b_c the number of bits needed to address the columns. For each of the following DRAMs, determine the power-of-2 array dimensions that minimize $\max(b_r, b_c)$, the maximum number of bits needed to address the rows or columns of the array.

Organization	r	c	b_r	b_c	$\max(b_r, b_c)$
16×1	____	____	____	____	____
16×4	____	____	____	____	____
128×8	____	____	____	____	____
512×4	____	____	____	____	____
1024×4	____	____	____	____	____

Solution: The notation 16×1 means $d = 16$ supercells, each of width $w = 1$ bit. The number of rows r and columns c must satisfy $rc = d = 16$. Since 16 is a perfect square, the minimum is given when we pick the square root, so $r = c = 4$. Meanwhile, $d = 128$ is not a perfect square, and we must pick either $r = 8$ and $c = 16$, or $r = 16$ and $c = 8$. For $d = 512$, we can pick $r = 32$ and $d = 16$, and for $d = 1024$, we can pick $r = d = 32$.

Organization	r	c	b_r	b_c	$\max(b_r, b_c)$
16×1	4	4	2	2	2
16×4	4	4	2	2	2
128×8	16	8	4	3	4
512×4	32	16	5	4	5
1024×4	32	32	5	5	5

Exercise 6.2. What is the capacity of a disk with 2 platters, $10,000$ cylinders, an average of 400 sectors per track, and 512 bytes per sector?

Solution: According Section 6.12, page 592, the capacity of a rotating disk is given by

$$\text{Capacity} = \frac{\#\text{ bytes}}{\text{sector}} \times \frac{\text{average }\#\text{ sectors}}{\text{track}} \times \frac{\text{average }\#\text{ tracks}}{\text{surface}} \times \frac{\#\text{ surfaces}}{\text{platter}} \times \frac{\#\text{ platters}}{\text{disk}}$$

Recall that a cylinder is the collection of tracks on all the surfaces that are equidistant from the center of the spindle. Since the disk has 10,000 cylinders, this means there's an average of 10,000 tracks per surface. Therefore the capacity is:

$$\begin{aligned}\text{Capacity} &= \frac{512\text{ bytes}}{\text{sector}} \times \frac{400\text{ sectors}}{\text{track}} \times \frac{10,000\text{ tracks}}{\text{surface}} \times \frac{2\text{ surfaces}}{\text{platter}} \times \frac{2\text{ platters}}{\text{disk}} && (1)\\ &= 8.192 \times 10^9\text{ bytes} && (2)\\ &= 8.192\text{ GB} && (3)\end{aligned}$$

Exercise 6.3. Estimate the average time (in ms) to access a sector on the following disk:

Parameter	Value
Rotational Rate	15,000 RPM
$T_{\text{avg seek}}$	8 ms
Average number of sectors/track	500

Solution: The average time to access a sector on a disk is the sum of the seek time, the rotational latency, and the transfer time. The seek time is how long it takes the actuator arm on a hard disk to move to the track that contains the target sector. The rotational latency is how long the drive waits for the first bit of the sector to pass under the head of the actuator arm. The transfer time is how long it takes to read the contents of the sector.

The average rotational latency is given by half of the maximum rotational latency, which in turn is given by:

$$T_{\text{max rotation}} = \frac{1}{\text{RPM}} \times \frac{60 \text{ secs}}{\text{min}}$$

Therefore,

$$T_{\text{avg rotation}} = \frac{1}{2} \times \frac{1 \text{ minute}}{15000 \text{ rotations}} \times \frac{60 \text{ secs}}{\text{min}} \times \frac{1000 \text{ ms}}{\text{second}} = 2 \text{ ms}$$

Meanwhile, the average transfer time is given by

$$\begin{aligned} T_{\text{avg transfer}} &= \frac{1}{\text{RPM}} \times \frac{1}{\text{average \# sectors/track}} \times \frac{60 \text{ secs}}{\text{min}} \\ &= \frac{\text{minute}}{15,000 \text{ rotations}} \times \frac{1}{500 \text{ sectors/track}} \times \frac{60 \text{ secs}}{\text{min}} \times \frac{1000 \text{ ms}}{\text{second}} \\ &= 0.008 \text{ ms} \end{aligned}$$

Therefore we have, the average access time is about $T_{\text{access}} = 8 \text{ ms} + 2 \text{ ms} + 0.008 \text{ ms} = 10.008 \text{ ms}$.

Exercise 6.4. Suppose that a 1 MB file consisting of 512-byte logical blocks is stored on a disk drive with the following characteristics:

Parameter	Value
Rotational rate	10,000 RPM
$T_{\text{avg seek}}$	5 ms
Average number of sectors/track	1,000
Surfaces	4
Sector size	512 bytes

For each case below, suppose that a program reads the logical blocks of the file sequentially, one after the other, and that the time to position the head over the first block is $T_{\text{avg seek}} + T_{\text{avg rotation}}$.

(a) *Best case*: Estimate the optimal time (in ms) required to read the file given the best possible mapping of logical blocks to disk sectors (i.e., sequential).

(b) *Random case*: Estimate the time (in ms) required to read the file if blocks are mapped randomly to disk sectors.

Solution:

(a)

(b) 1 MB is equivalent to 1000 KB, so with a logical block size of 512 bytes, it will take up about 2000 logical blocks on disk. Since the size of a block equals the size of a sector, and there are 1000 tracks per sector on average for the given disk, this means that a sector can hold about 1000 blocks. In the best case, the data for the file is mapped onto 2 tracks on two surfaces on the same platter with the increasing order of the logical blocks matching that of the sector. After the initial delay of $T_{\text{avg seek}} + T_{\text{avg rotation}}$ to place the head on the first block, it will take about $T_{\text{avg transfer}}$ to transfer all bytes in the sector, and about $\frac{1}{1000}T_{\text{max rotation}}$ to get to the next sector (logical block) on the same track to continue. These latter two steps are performed 1000 times per track. Since the two surfaces are on the same platter, the head is already in place to start transferring the data from the second surface. Therefore, the optimal time would be about:

$$\begin{aligned} T_{\text{access, optimal}} &= T_{\text{avg seek}} + T_{\text{avg rotation}} + 2 \cdot 1000 \cdot T_{\text{avg transfer}} \\ &= T_{\text{avg seek}} + T_{\text{max rotation}} + 2000 \cdot T_{\text{avg transfer}} \end{aligned}$$

The maximum rotational latency for the given disk is about

$$T_{\text{maximum rotation}} = \times \frac{1 \text{ minute}}{10000 \text{ rotations}} \times \frac{60 \text{ secs}}{\text{min}} \times \frac{1000 \text{ ms}}{\text{second}} = 6 \text{ ms}$$

The average transfer latency is about

$$\begin{aligned} T_{\text{avg transfer}} &= \frac{\text{minute}}{10,000 \text{ rotations}} \times \frac{1}{1,000 \text{ sectors/track}} \times \frac{60 \text{ secs}}{\text{min}} \times \frac{1000 \text{ ms}}{\text{second}} \\ &= 0.006 \text{ ms} \end{aligned}$$

Altogether, the optimal access time is about

$$\begin{aligned} T_{\text{access, optimal}} &= 5 \text{ ms} + \frac{1}{2} \cdot 6 \text{ ms} + 2000 \cdot 0.006 \text{ ms} \\ &= 20 \text{ ms} \end{aligned}$$

(c) For a random case, we may not have contiguous blocks for the file. The 2000 blocks for the file may be split evenly among all 4 surfaces, meaning there is about 500 blocks per surface. If each block is on a different track, it may take about

$$\begin{aligned} T_{\text{access, avg}} &= 2000 \times (T_{\text{avg seek}} + T_{\text{avg rotation}} + T_{\text{avg transfer}} + T_{\text{access}}) \\ &= 2000 \times (5 + \text{ ms} + 3 \text{ ms} + 0.006 \text{ ms}) \\ &= 16012 \text{ ms} \\ &\approx 16 \text{ seconds} \end{aligned}$$

Exercise 6.5. As we have seen, a potential drawback of SSDs is that the underlying flash memory can wear out. For example, for the SSD in Figure 6.14 (shown below), Intel guarantees about 12 petabytes (128×10^{15} bytes) of writes before the drive wears out:

Reads		Writes	
Sequential read throughput	550 MB/s	Sequential write throughput	470 MB/s
Random read throughput (IOPS)	89,000 IOPS	Random write throughput (IOPS)	74,000 IOPS
Random read throughput (MB/s)	365 MB/s	Random write throughput (MB/s)	303 MB/s
Avg. sequential read access time	50 μs	Avg. sequential write access time	60 μs

Given this assumption, estimate the lifetime (in years) of this SSD for the following workloads:

(a) *Worst case for sequential writes*: The SSD is written to continuously at a rate of 470 MB/s (the average sequential write throughput of the device).

(b) *Worst case for random writes*: The SSD is written continuously at a rate of 303 MB/s (the average random write throughput of the device).

(c) *Average case*: The SSD is written to at a rate of 20 GB/day (the average daily write rate assumed by some computer manufacturers in their mobile computer workload simulations).

Solution:

(a) It would take

$$\begin{aligned} T_{\text{worst seq. writes}} &= 128 \times 10^{15} \text{ bytes} \cdot \frac{1 \text{ MB}}{10^6 \text{ bytes}} \cdot \frac{1 \text{ second}}{470 \text{ MB}} \cdot \frac{1 \text{ day}}{86400 \text{ seconds}} \cdot \frac{1 \text{ year}}{365 \text{ days}} \\ &\approx 8.63 \text{ years} \end{aligned}$$

(b)

$$\begin{aligned} T_{\text{worst ran. writes}} &= 128 \times 10^{15} \text{ bytes} \cdot \frac{1 \text{ MB}}{10^6 \text{ bytes}} \cdot \frac{1 \text{ second}}{303 \text{ MB}} \cdot \frac{1 \text{ day}}{86400 \text{ seconds}} \cdot \frac{1 \text{ year}}{365 \text{ days}} \\ &\approx 13.4 \text{ years} \end{aligned}$$

(c)

$$\begin{aligned} T_{\text{avg}} &= 128 \times 10^{15} \text{ bytes} \cdot \frac{1 \text{ GB}}{10^9 \text{ bytes}} \cdot \frac{1 \text{ day}}{20 \text{ GB}} \cdot \frac{1 \text{ year}}{365 \text{ days}} \\ &\approx 17534 \text{ years} \end{aligned}$$

Exercise 6.6. Using the data from the years 2005 to 2015 in Figure 6.15(c) on page 603, estimate the year when you will be able to buy a petabyte (10^{15} bytes) of rotating disk storage for \$500 bytes. Assume actual dollars (no inflation).

Solution: Expressed in dollars per GB, the cost would be

$$\frac{\$500}{10^{15}\text{ bytes}} \cdot \frac{10^9\text{ byte}}{1\text{ GB}} = \frac{\$0.0005}{\text{GB}}$$

According to the table, the price for rotating disk storage in dollars per GB for the years 2005, 2010, and 2015 were \$5/GB, \$0.3/GB, and \$0.03/GB, indicating a decrease by a factor between 10 and 16.67. It would therefore take at least 5 years but no more than 10 years, so around 2025.