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Computer Systems: A Programmer's Perspective, by Bryant and O'Hallaron
Chapter 1: A Tour of Computer Systems
March 5, 2024

Practice Problems

Exercise 1.1. Suppose you work as a truck driver, and you have been hired to carry a load of potatoes from Boise, Idaho, to Minneaopolis, Minnesota, a total distance of 2,500 kilometers. You estimate you can average 100 km/hr driving within the speed limits, requiring a total of 25 hours for the trip.

(a) You hear on the news that Montana has just abolished its speed limit, which constitutes 1,500 km of the trip. Your truck can travel at 150 km/hr. What will be your speedup for the trip?

(b) You can buy a new turbocharger for your truck at www.fasttrucks.com. They stock a variety of models, but the fast you go, the more it will cost. How fast must you travel through Montana to get an overall speedup of your trip of $1.67\times$?

Solution: Recall that Amdhal's Law says if part of a system takes a fraction α of the overall time, and if we speed up by a factor of k, then the overall system speedup is given by

$$S = \frac{1}{(1-\alpha) + \alpha/k},$$

where α is the fraction of a system

(a) The distance through Montana corresponds to a fraction $\alpha = \frac{1500}{2500} = \frac{3}{5}$ of the overall trip. During the Montana part, we travel at 150 km/hr, whereas we travel at 100 km/hr for the rest of the trip, meaning that our performance improve factor through Montana is $k = \frac{150}{100} = \frac{3}{2}$. Therefore, the overall speedup is

$$\begin{aligned} S &= \frac{1}{(1-\frac{3}{5}) + \frac{3}{5}\cdot\frac{2}{3}} \\ &= \frac{1}{\frac{2}{5} + \frac{2}{5}} \\ &= \frac{5}{4} \end{aligned}$$

That is, the speedup for the trip is $1.25\times$.

(b) To find the necessary speed of our track to obtain and overall speedup of $S = 1.67\times$, we use Amdhal's Law to find k:

$$\begin{aligned} S(1-\alpha)+\frac{\alpha S}{k} &= 1 \\ \frac{\alpha S}{k} &= 1+S(\alpha-1) \\ k &= \frac{\alpha S}{1+S(\alpha-1)} \end{aligned}$$

Since $1.67 \approx \frac{5}{3}$, we get

$$\begin{aligned} k &= \frac{\frac{3}{5}\cdot\frac{5}{3}}{1+\frac{5}{3}\left(\frac{3}{5}-1\right)} \\ &= \frac{1}{1-\frac{2}{3}} \\ &= 3 \end{aligned}$$

Hence, we would need to travel 100 km/hr $\times 3 = 300$ km/hr.

Exercise 2. The marketing department at your company has promised your customers that the next software release will show a $2\times$ performance improvement. You have been assigned the task of delivering on that promise. You have determined that only 80% of the system can be improved. How much (i.e, what value of k) would you need to improve this part to meet the overall performance target?

Solution: After re-arranging Amdhal's Law in the previous question, the equation for k became

$$k = \frac{\alpha S}{1+S(\alpha-1)}$$

Given $S = 2$ and $\alpha = 0.8 = \frac{4}{5}$, we find the required performance improvement k:

$$\begin{aligned} k &= \frac{\frac{4}{5}\cdot 2}{1+2\left(\frac{4}{5}-1\right)} \\ &= \frac{\frac{8}{5}}{\frac{3}{5}} \\ &= \frac{8}{3} \end{aligned}$$

Hence, we need a performance improvement of about $2.67\times$.