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Algorithms by Sedgewick and Wayne (4th edition) [SW11]
September 30th, 2024

1.4: Analysis of Algorithms

Exercise 1. Show that the number of different triples that can be chosen from n items is precisely $n(n-1)(n-2)/6$. *Hint*: Use mathematical induction or a counting argument.

Solution.

Proof. This is the problem of choosing a combination of 3 out of n, which is given by $\binom{n}{3}$, and

$$\binom{n}{3} = \frac{n!}{3!(n-3)!} = \frac{n \cdot (n-1) \cdot (n-2) \cdot (n-3)!}{3!(n-3)!} = \frac{n(n-1)(n-2)}{6}$$

□

Exercise 2. Modify `ThreeSum` to work properly even when the `int` values are so large that adding two of them might cause integer overflow.

Solution. There are two cases when it comes to overflow:

(i) *Positive overflow.* The sum exceeds `Integer.MAX_VALUE`. If two terms sum to `Integer.MAX_VALUE + 1`, overflow occurs, and the value wraps around to `Integer.MIN_VALUE`. Thus, if $a + b =$ `Integer.MAX_VALUE + 1` and $c =$ `Integer.MIN_VALUE`, then we have a valid sum. However, if $a + b$ sums to anything larger, then no value of c will do because c cannot be smaller than `Integer.MIN_VALUE`.

(ii) *Negative overflow.* The sum of two negative numbers a and b, yielding a value below `Integer.MIN_VALUE`. In this case, it's impossible to have $a + b = c$ for any 32-bit two's complement integer c.

References

[SW11] Robert Sedgewick and Kevin Wayne. *Algorithms*. 4th ed. Addison-Wesley, 2011. ISBN: 9780321573513.