Sergio E. Garcia Tapia
Algorithms by Sedgewick and Wayne (4th edition) [SW11]
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2.4: Priority Queues

Exercise 1. Suppose that the sequence `P R I O * R * * I * T * Y * * * Q U E * * * U * E` (where a letter means *insert* and an asterisk means *remove the maximum*) is applied to an initially empty priority queue. Give the sequence of letters returned by the *remove the maximum* operations.

Solution.

```
P
P R
P R I
P R I O
P I O // max removed: R
P I O R
P I O // max removed: R
I O // max removed: P
I O I
I I // max removed: O
I I T
I I // max removed: T
I I Y
I I // max removed: Y
I // max removed: I
// max removed: I
Q
Q U
Q U E
Q E // max removed: U
E // max removed: Q
// max removed: E
U
// max removed: U
E
```

At the end, `E` remains on the queue. The sequence letters returned is:

`R R P O T Y I I U Q E U`

Exercise 2. Criticize the following idea: To implement *find the maximum* in constant time, why not use a stack or a queue, but keep track of the maximum value inserted so far, then return that value for *find the maximum*.

Solution. One issue is that this only guarantees that the first *find the maximum* operation can be returned in constant time. Once that items is removed, if the client then asks for the next value, this operation then requires linear time.

Exercise 3. Provide priority-queue implementations that support *insert* and *remove the maximum*, one for each of the following underlying data structures: unordered array, ordered array, unordered linked list, and ordered linked list. Give a table of the worst-case bounds for each operation for each of your four implementations.

Solution. See package `com.segarciat.algs4.ch2.sec4.ex03`. The time complexities for the two main operations are given below for priority queue with n items:

	Insert	Remove the maximum
`UnorderedArrayMaxPQ`	$O(1)$	$O(n)$
`OrderedArrayMaxPQ`	$O(n)$	$O(1)$
`UnorderedListMaxPQ`	$O(1)$	$O(n)$
`OrderedListMaxPQ`	$O(n)$	$O(1)$

References

[SW11] Robert Sedgewick and Kevin Wayne. *Algorithms*. 4th ed. Addison-Wesley, 2011. ISBN: 9780321573513.