Chapitre 2 Camera Calibration

2.1 Introduction
2.2 Linear Camera Model
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Chapitre 2 Camera Calibration

2.1 Introduction

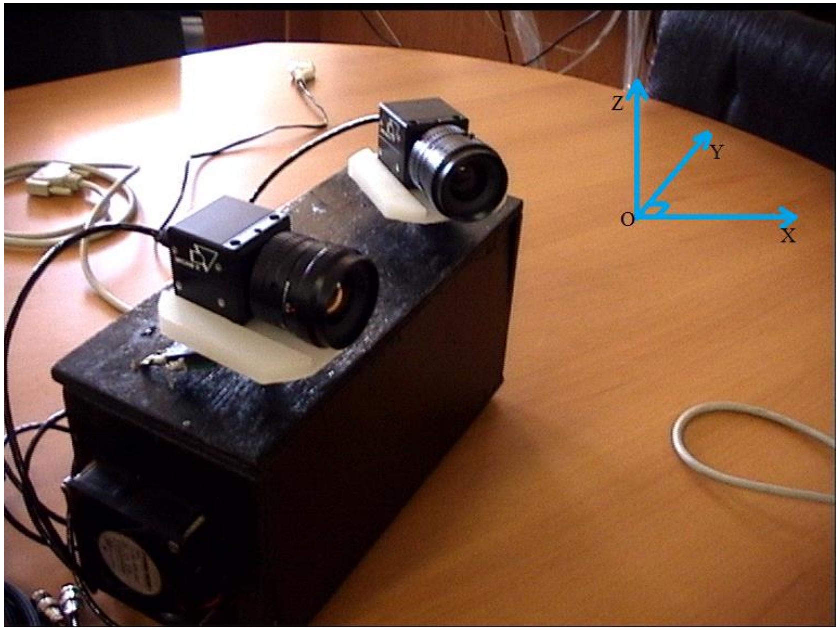
How the camera maps the perspective projection points in the world onto its image plane?
Camera calibration is a method for finding camera’s **internal** and external parameters:

- **Focal length: f**
- **Position of impact of optical axis (Ox, Oy)**
- **Dimensions of the pixel (mm) ex, ey**

Chapitre 2. Camera Calibration

2.1 Introduction

Camera calibration is a method for finding camera's internal and **external** parameters.



- **The position and Orientation of the camera coordinate frame relatively to the world coordinate frame (OXYZ).**

Chapitre 2. Camera Calibration

2.1 Introduction

In this chapter, we study:

- The linear camera model
- The camera calibration
- Extracting intrinsic and extrinsic matrices
- Example Application: Simple stereo

Chapitre 2. Camera Calibration

2.2 Linear Camera Model

Coordinates of the point P may be known with respect to the world coordinate W such as P(0,3,4) in figure 3.

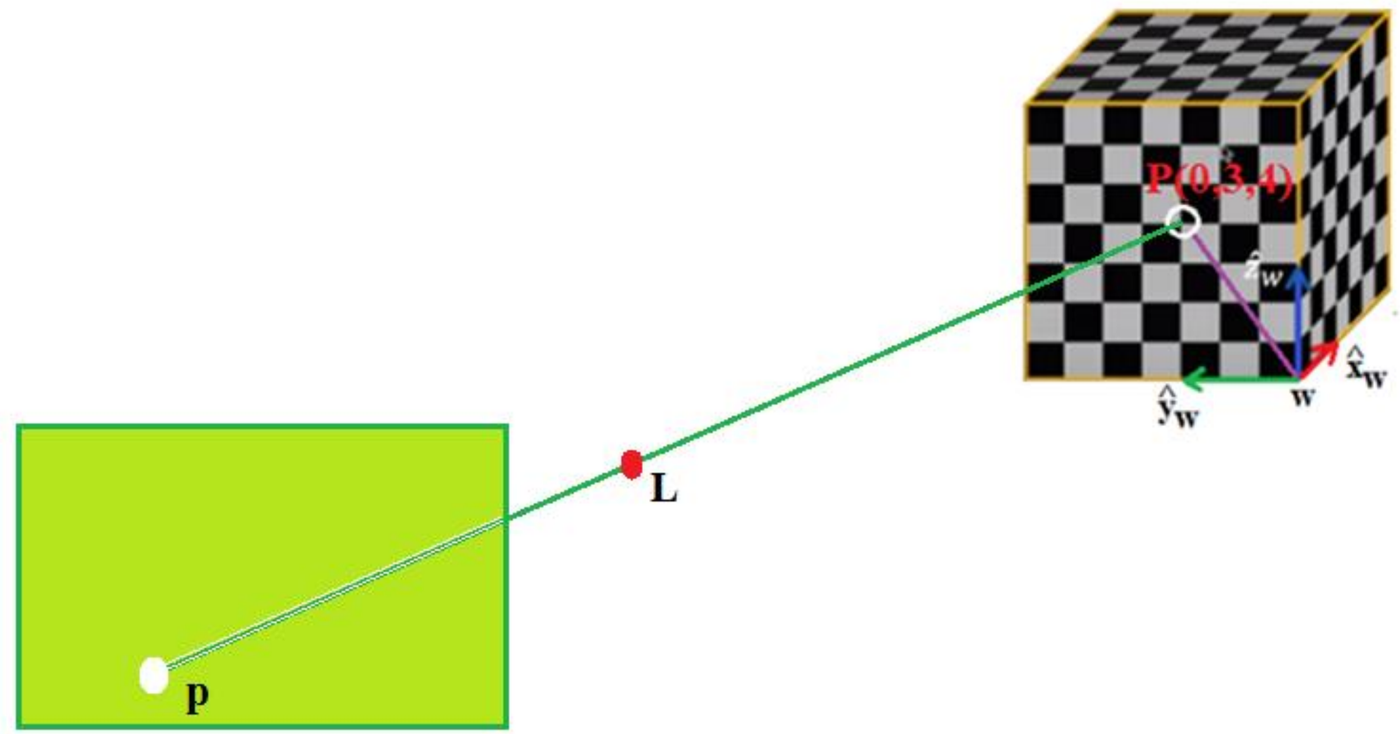


Figure 2. Forward imaging model: 3D to 2D

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2.2 Linear Camera Model

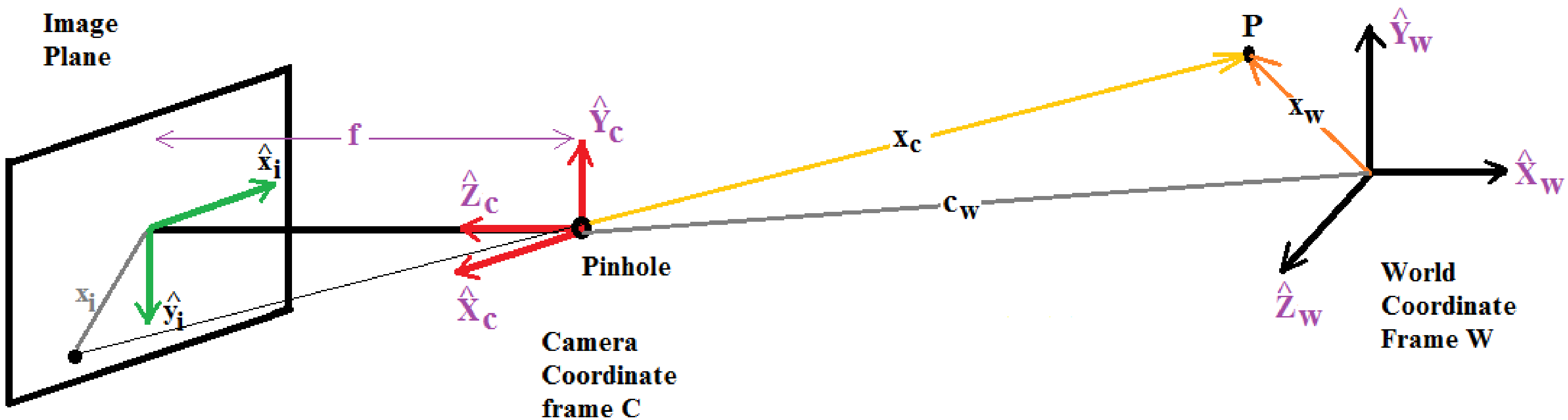

Figure 2. Forward imaging model: 3D to 2D

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2.2 Linear Camera Model

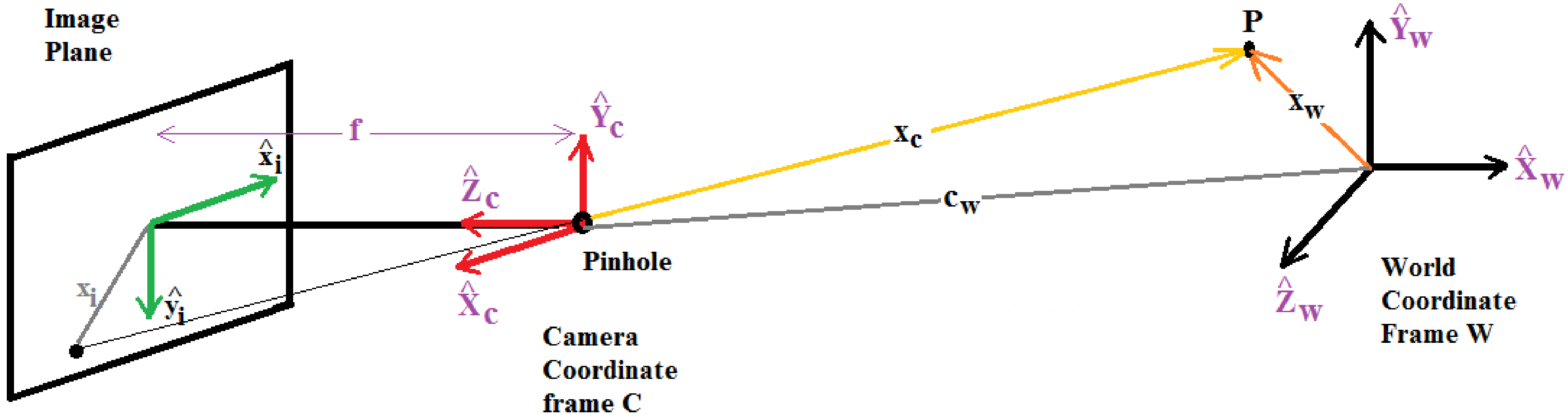

Figure 2. Forward imaging model: 3D to 2D

Coordinates of the point P ($\overrightarrow{CP} = \overrightarrow{X_C}$) can't be known with respect to the camera coordinate C because we don't know the position of the projection center C (Pinhole).

The coordinates of P with respect to camera coordinate C are necessary for the computation of its projection on the image plane

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2.2 Linear Camera Model

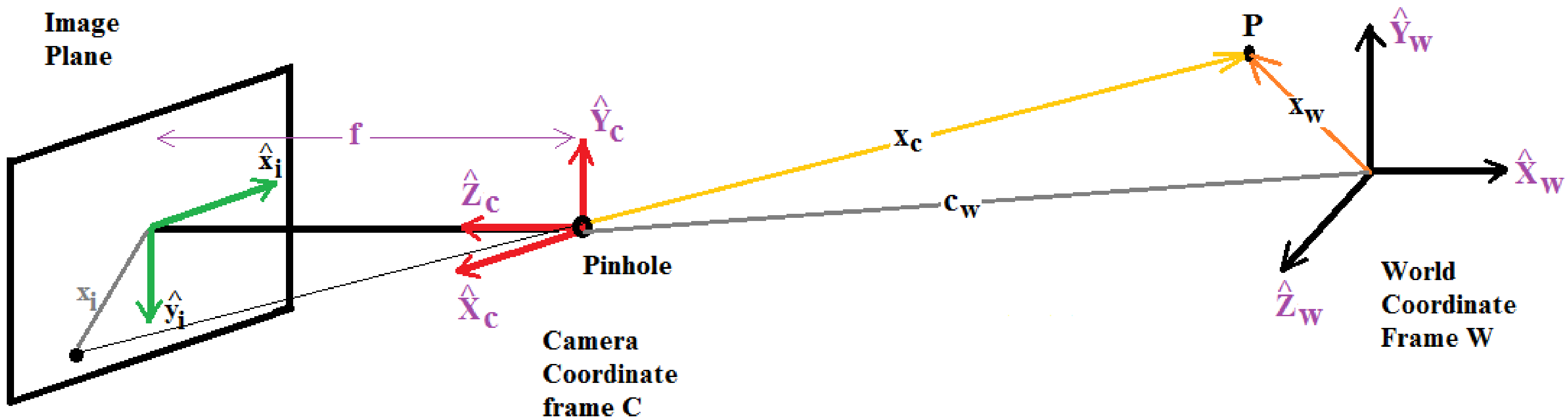

Figure 2. Forward imaging model: 3D to 2D

Consequently, we need to write the coordinate of P (known with respect to World coordinate W) with respect to camera coordinate: make a Transformation

2.2 Linear Camera Model

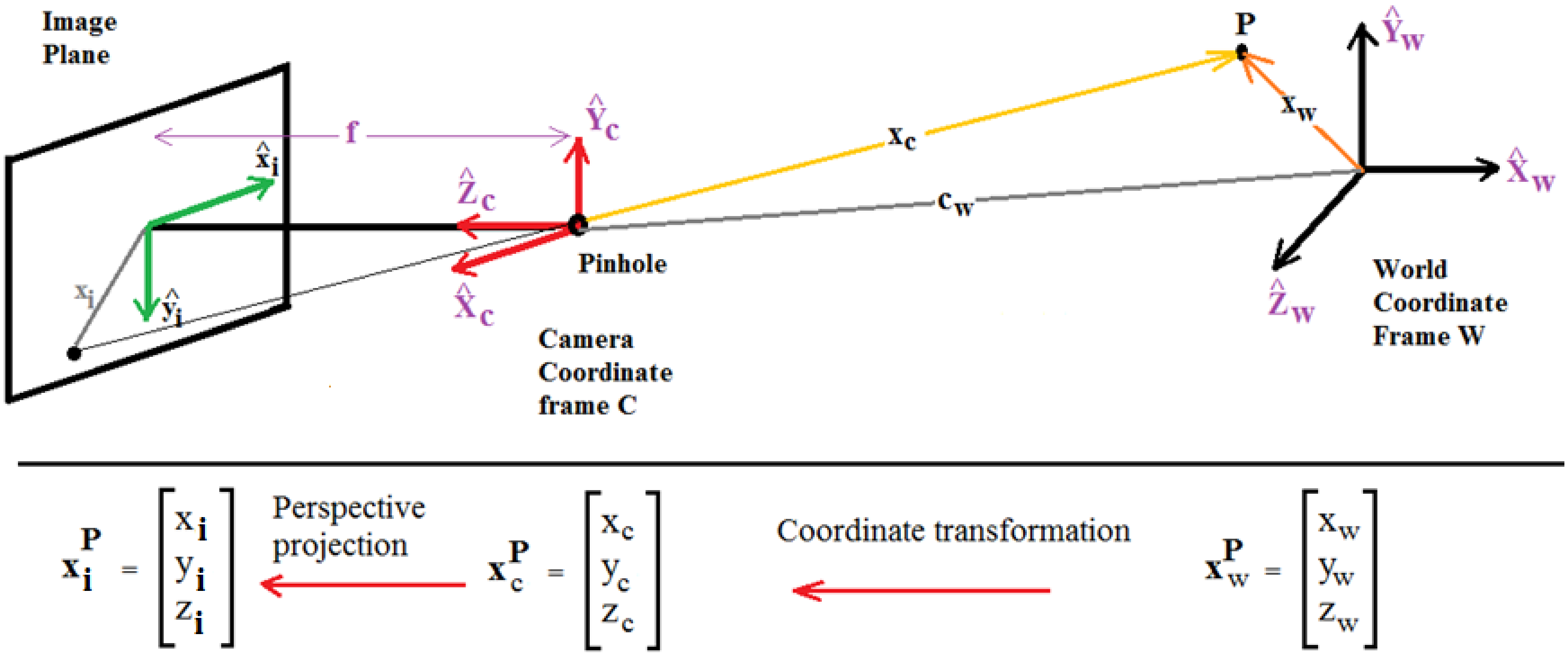

Figure 2. Forward imaging model: 3D to 2D

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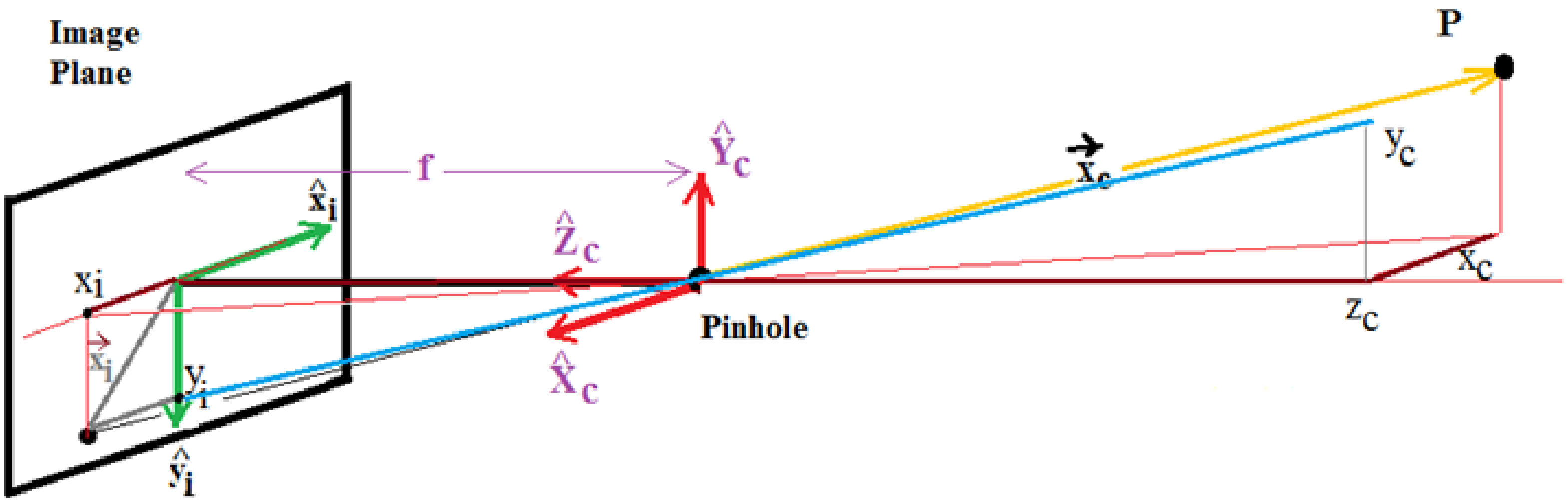
2.2 Linear Camera Model



We apply Thales's Theorem and we obtain:

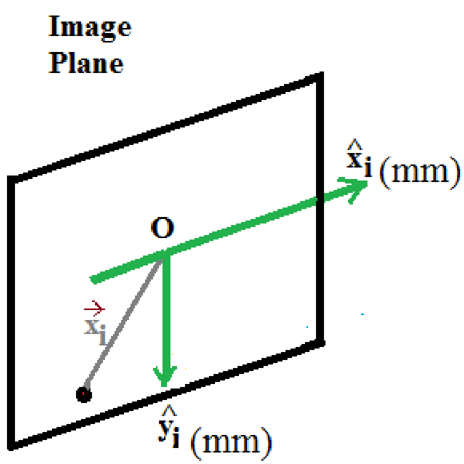
$\frac{x_i}{f} = \frac{x_c}{z_c}$ and $\frac{y_i}{f} = \frac{y_c}{z_c}$

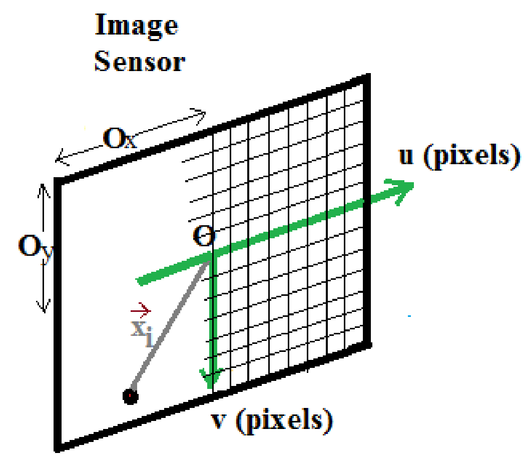
Therefore: $x_i = f\,\frac{x_c}{z_c}$, $y_i = f\,\frac{y_c}{z_c}$

2.2 Linear Camera Model

$$x_i = f\frac{x_c}{z_c}, y_i = f\frac{y_c}{z_c}$$

If we assume that m_x, m_y are the pixel densities (pixels/mm) in x and y directions, the coordinates of the pixel are (u, v) where:





Chapitre 2. Camera Calibration

2.2 Linear Camera Model

If we assume that m_x, m_y are the pixel densities (pixels/mm) in x and y directions, the coordinates of the pixel are (u, v) where:

$$u = m_x x_i = m_x f \frac{x_c}{z_c} \qquad v = m_y y_i = m_y f \frac{y_c}{z_c}$$

Let (O_x, O_y) be the coordinates of the principle point with respect to the top left corner of image plane. We can the write:

$$u = m_x x_i = m_x f \frac{x_c}{z_c} + O_x \qquad v = m_y y_i = m_y f \frac{y_c}{z_c} + O_y$$

Chapitre 2. Camera Calibration

2.2 Linear Camera Model

Let $(f_x = f \times m_x, f_y = f \times m_y)$ be the focal lengths in pixels in x and y directions. We can then write the non linear equations for perspective projection:

$$u = f_x \frac{x_c}{z_c} + O_x \qquad\qquad v = f_y \frac{y_c}{z_c} + O_y$$

The intrinsic parameters of the camera are: (f_x, O_x, f_y, O_y)

$$u = \boxed{f_x} \frac{x_c}{z_c} + \boxed{O_x} \qquad v = \boxed{f_y} \frac{y_c}{z_c} + \boxed{O_y}$$

It is convenient to express these equations linearly.

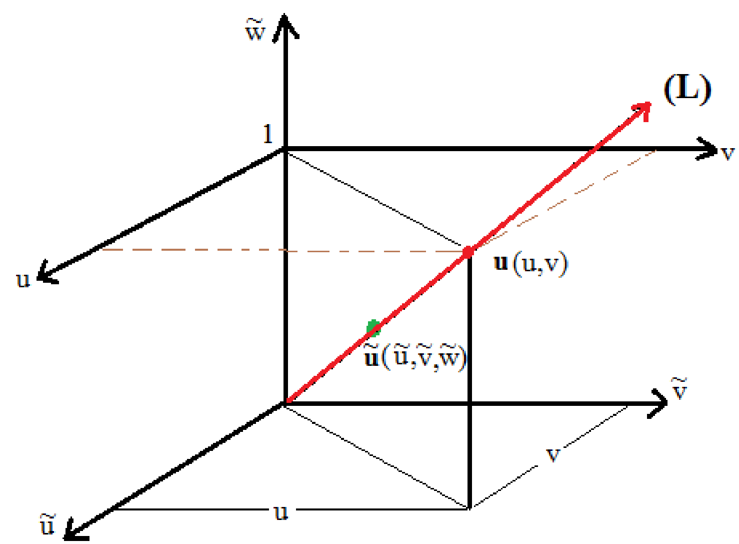
Chapitre 2. Camera Calibration

2.2 Linear Camera Model

Homogeneous coordinates

The Homogeneous coordinates of a 2D point (u, v) is a 3D point $(\tilde{u}, \tilde{v}, \tilde{w})$ such that $(\mathrm{u} = \widetilde{u}/\tilde{w}, v = \tilde{v}/\tilde{w})$

$$\boldsymbol{u} \equiv \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{w}u \\ \tilde{w}v \\ \tilde{w} \end{bmatrix} \equiv \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} \equiv \tilde{\boldsymbol{u}}$$



Each point belonging to (L) has the Homogeneous coordinates of the 2D point $\mathbf{u}(u, v)$.

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2.2 Linear Camera Model

Homogeneous coordinates

The Homogeneous coordinates of a 3D point (x, y, z) is a 4D point $(\tilde{x}, \tilde{y}, \tilde{z}, \tilde{w})$ such that $(x = \frac{\tilde{x}}{\tilde{w}}, y = \frac{\tilde{y}}{\tilde{w}}, z = \frac{\tilde{z}}{\tilde{w}})$

$$\boldsymbol{u} \equiv \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \\ \mathbf{z} \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{w}\,\mathbf{x} \\ \tilde{w}\,\mathbf{y} \\ \tilde{w}\,\mathbf{z} \\ \tilde{w} \end{bmatrix} \equiv \begin{bmatrix} \tilde{\mathbf{x}} \\ \tilde{\mathbf{y}} \\ \tilde{\mathbf{z}} \\ \tilde{w} \end{bmatrix} \equiv \tilde{\boldsymbol{u}}$$

Chapitre 2. Camera Calibration

2.2 Linear Camera Model

Rewriting the equations of perspective projection using the homogenous coordinates of (u, v):

$$z_c u = f_x x_c + z_c O_x \qquad z_c v = f_y y_c + z_c O_y$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} \equiv \begin{bmatrix} z_c u \\ z_c v \\ z_c \end{bmatrix} = \begin{bmatrix} f_x x_c + z_c o_x \\ f_y y_c + z_c o_y \\ z_c \end{bmatrix} = \begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

2.2 Linear Camera Model

Rewriting the equations of perspective projection using the homogenous coordinates of (u, v):

$$z_c u = f_x x_c + z_c O_x \qquad\qquad z_c v = f_y y_c + z_c O_y$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

Calibration matrix

Intrinsic matrix

$$M_{int} = [K \mid 0], \quad K = \begin{bmatrix} f_x & 0 & o_x \\ 0 & f_y & o_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\tilde{\mathbf{u}} = M_{int}\, \tilde{\mathbf{x}}_{\mathbf{c}}$$

Chapitre 2. Camera Calibration

2.2 Linear Camera Model

Homogeneous coordinates

The external parameters:

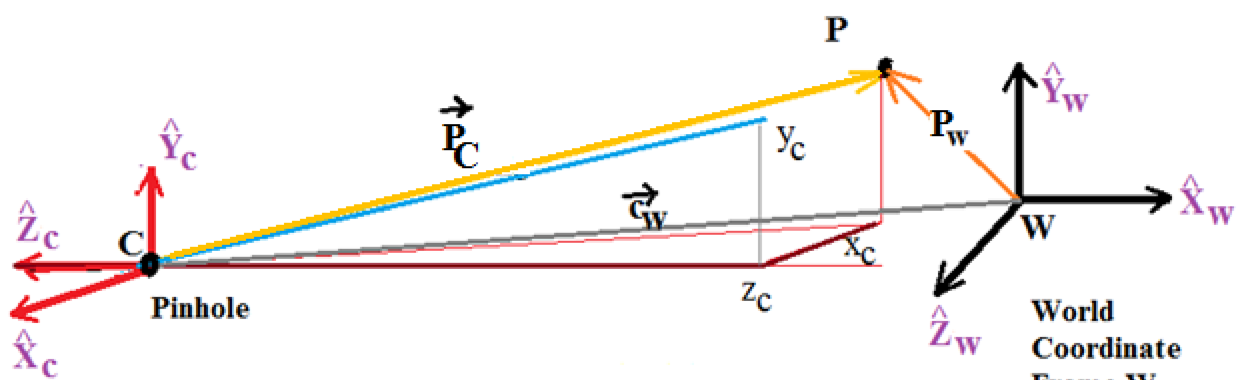
- The vector $\overrightarrow{C_w}$ (indicates the position of C with respect to the world coordinate frame)
- The orientation (Rotation) of the camera with respect to the world coordinate frame)

$r_{11}\ \ r_{12}\ \ r_{13}$ is the direction of $\tilde{x}_c$ in the world coordinate frame

$r_{21}\ \ r_{22}\ \ r_{23}$ is the direction of $\tilde{y}_c$ in the world coordinate frame

$r_{31}\ \ r_{32}\ \ r_{33}$ is the direction of $\tilde{z}_c$ in the world coordinate frame

$$R=\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$



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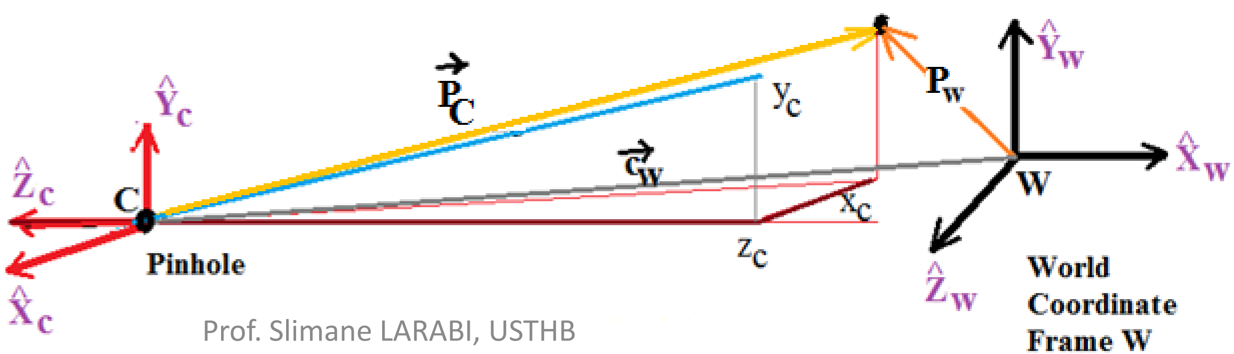
2.2 Linear Camera Model

$$R=\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

Rotation ω about the *x* axis, rotation φ about the new *y* axis, and rotation κ, about the new *z* axis.
Angle ω is the pitch (vertical angle) of the optical axis, angle φ is the yaw (horizontal angle) of the optical axis, and angle κ is the roll or twist about the optical axis.

$$\begin{aligned}
r_{11} &= \cos\phi\cos\kappa \\
r_{12} &= \sin\omega\sin\phi\cos\kappa + \cos\omega\sin\kappa \\
r_{13} &= -\cos\omega\sin\phi\cos\kappa + \sin\omega\sin\kappa \\
r_{21} &= -\cos\phi\sin\kappa \\
r_{22} &= -\sin\omega\sin\phi\sin\kappa + \cos\omega\cos\kappa \\
r_{23} &= \cos\omega\sin\phi\sin\kappa + \sin\omega\cos\kappa \\
r_{31} &= \sin\phi \\
r_{32} &= -\sin\omega\cos\phi \\
r_{33} &= \cos\omega\cos\phi.
\end{aligned}$$

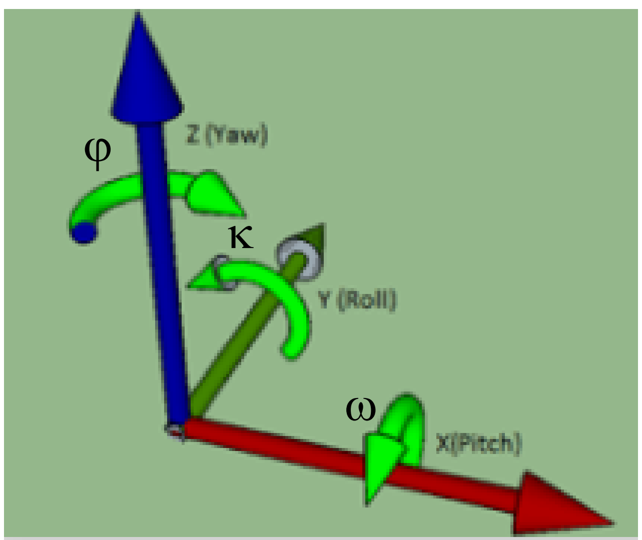


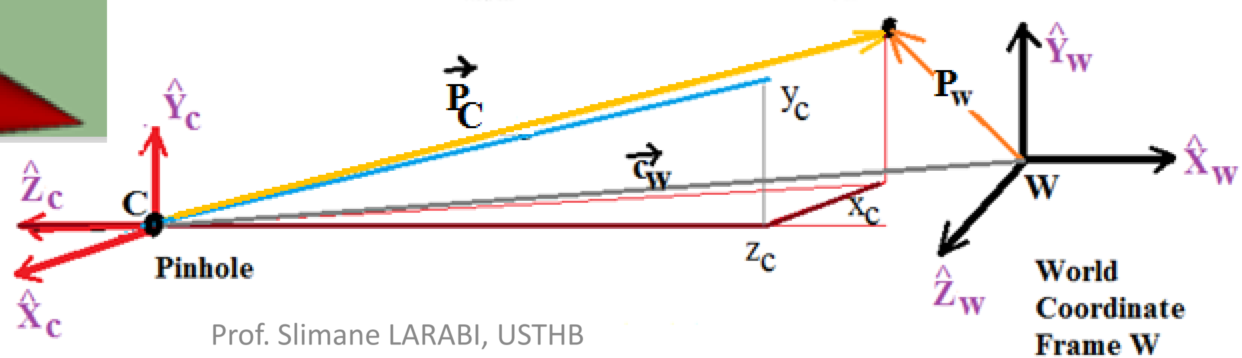
2.2 Linear Camera Model

$$R=\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

Angle ω is the pitch (vertical angle) of the optical axis, angle φ is the yaw (horizontal angle) of the optical axis, and angle κ is the roll or twist about the optical axis.

$$\begin{aligned}
r_{11} &= \cos\phi\cos\kappa \\
r_{12} &= \sin\omega\sin\phi\cos\kappa + \cos\omega\sin\kappa \\
r_{13} &= -\cos\omega\sin\phi\cos\kappa + \sin\omega\sin\kappa \\
r_{21} &= -\cos\phi\sin\kappa \\
r_{22} &= -\sin\omega\sin\phi\sin\kappa + \cos\omega\cos\kappa \\
r_{23} &= \cos\omega\sin\phi\sin\kappa + \sin\omega\cos\kappa \\
r_{31} &= \sin\phi \\
r_{32} &= -\sin\omega\cos\phi \\
r_{33} &= \cos\omega\cos\phi.
\end{aligned}$$





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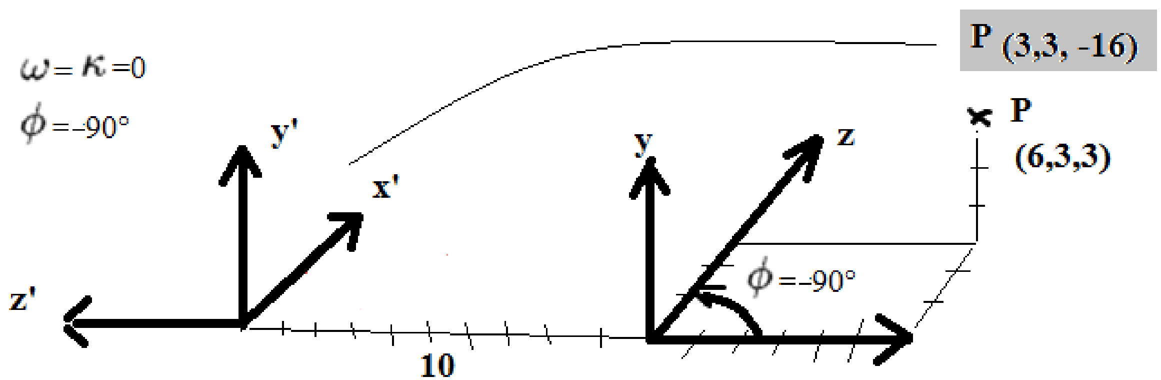
2.2 Linear Camera Model



$$
\begin{aligned}
r_{11} &= \cos\phi\cos\kappa = 0 \\
r_{12} &= \sin\omega\sin\phi\cos\kappa + \cos\omega\sin\kappa = 0 \\
r_{13} &= -\cos\omega\sin\phi\cos\kappa + \sin\omega\sin\kappa = 1 \\
r_{21} &= -\cos\phi\sin\kappa = 0 \\
r_{22} &= -\sin\omega\sin\phi\sin\kappa + \cos\omega\cos\kappa = 1 \\
r_{23} &= \cos\omega\sin\phi\sin\kappa + \sin\omega\cos\kappa = 0 \\
r_{31} &= \sin\phi = -1 \\
r_{32} &= -\sin\omega\cos\phi = 0 \\
r_{33} &= \cos\omega\cos\phi = 0
\end{aligned}
$$

$$
\mathbf{R.P+t} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 6 \\ 3 \\ 3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -10 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ -16 \end{bmatrix}
$$

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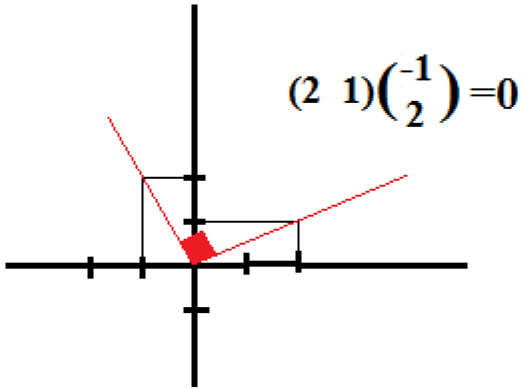
2.2 Linear Camera Model

Definitions

Two vectors are orthonormals if the dot product is equal to zero

Example (the x- , y-, z- axes of R^3 Euclidean space.

$\mathrm{d}ot(u, v) = u\ v \cos\theta = u^T v = 0$



A square matrix R is orthonormal if $R^{-1} = R^T$

The rotation matrix R is orthonormal.

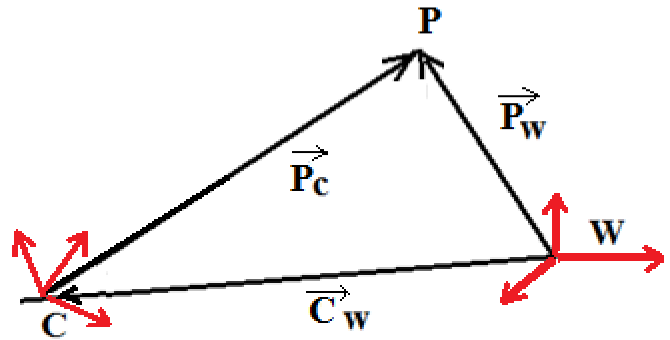
$$R^{-1}R = R^T R = I$$

Chapitre 2. Camera Calibration

2.2 Linear Camera Model

Let the vector $\overrightarrow{P_W} = \overrightarrow{WP}$ where W is the center of the world coordinate frame,
Let the vector $\overrightarrow{C_W} = \overrightarrow{WC}$ where C is the center of the world coordinate frame
Let the vector $\overrightarrow{P_C} = \overrightarrow{CP}$ where C is the center of the world coordinate frame

$P_w = C_w + P_c$, then: $P_c = P_w - C_w$



The camera centric-location of the point P in the world coordinate frame (wcf) is given by:

$$(P_c)in\ wcf = RP_c = R(P_w - C_w) = RP_w - RC_w = RP_w + t, where\ t = -RC_w$$

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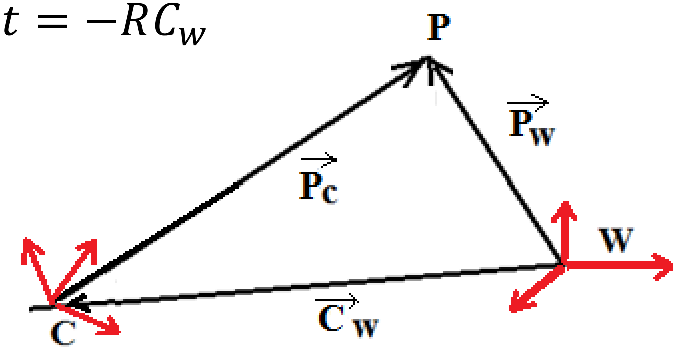
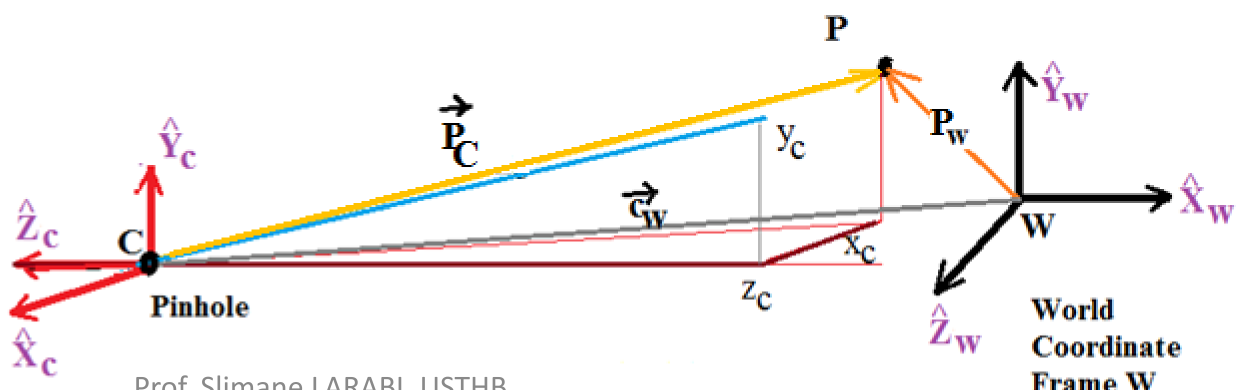
2.2 Linear Camera Model

$$P_c = RP_c = R(P_w - C_w) = RP_w - RC_w = RP_w + t, where\ t = -RC_w$$



$$P_w(x_w, y_w, z_w)$$
$$P_c(x_c, y_c, z_c)$$

$$(P_c) in\ wcf = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} (\text{in wcf}) = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$



Chapitre 2. Camera Calibration

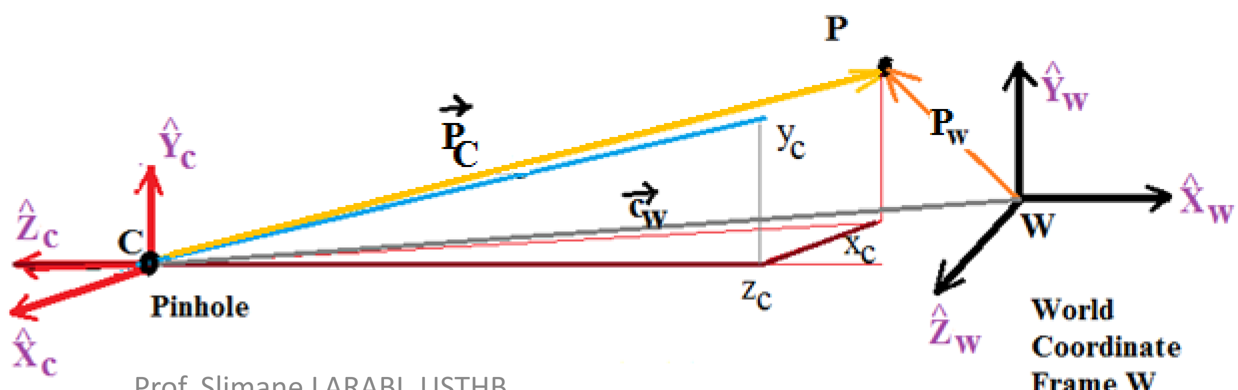
2.2 Linear Camera Model

$$\tilde{P}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

The extrinsic matrix:

$$M_{ext} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\tilde{P}_c = M_{ext}\tilde{P}_w$$



Chapitre 2. Camera Calibration

2.2 Linear Camera Model
Projection matrix P

Camera to pixel

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{w} \end{bmatrix} = \begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

$$\tilde{\mathbf{u}} = M_{int} \tilde{\mathbf{P}}_{\mathbf{c}}$$

World to camera

$$\tilde{P}_c = \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

$$\tilde{P}_c = M_{ext} \tilde{P}_w$$

$$\tilde{u} = M_{int} M_{ext} \tilde{P}_w = P \tilde{P}_w$$

P is the projection matrix

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix}$$

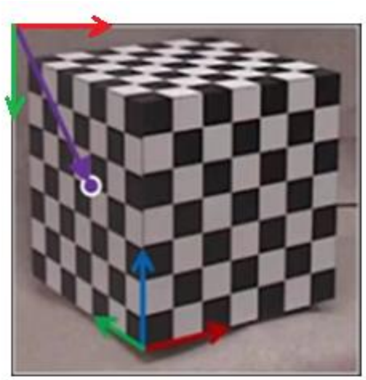
Chapitre 2. Camera Calibration

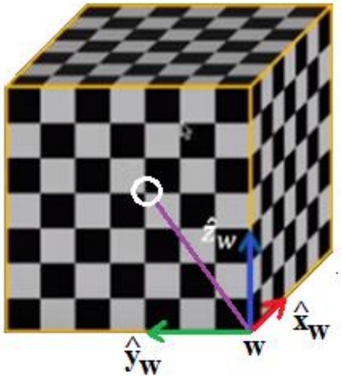
2.3 Calibrate a Camera

The procedure

1- Capture an image of an object with known geometry

2- Identify the correspondence between 3D scene point and image points.





$$x_w = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$$

$$u = \begin{bmatrix} 56 \\ 115 \end{bmatrix}$$

Chapitre 2. Camera Calibration

2.3 Calibrate a Camera

The procedure

3- For each pair of corresponding points (scene-image) we have two equations:

$$\tilde{u} = P\tilde{P}_w \qquad \begin{bmatrix} u^i \\ v^i \\ 1 \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

$$u^i = \frac{P_{11}x_w + P_{12}y_w + P_{13}z_w + P_{14}}{P_{31}x_w + P_{32}y_w + P_{33}z_w + P_{34}} \qquad v^i = \frac{P_{21}x_w + P_{22}y_w + P_{23}z_w + P_{24}}{P_{31}x_w + P_{32}y_w + P_{33}z_w + P_{34}}$$

2.3 Calibrate a Camera

The procedure:

Resolve AP=0

$$\begin{bmatrix} x^1_w & y^1_w & z^1_w & 1 & 0 & 0 & 0 & 0 & -u_1 x^1_w & -u_1 y^1_w & -u_1 z^1_w & -u_1 \\ 0 & 0 & 0 & 0 & x^1_w & y^1_w & z^1_w & 1 & -v_1 x^1_w & -v_1 y^1_w & -v_1 z^1_w & -v_1 \\ \ldots\ldots & & & & & & & & & & & \\ \ldots\ldots & & & & & & & & & & & \\ x^n_w & y^n_w & z^n_w & 1 & 0 & 0 & 0 & 0 & -u_n x^n_w & -u_n y^n_w & -u_n z^n_w & -u_n \\ 0 & 0 & 0 & 0 & x^n_w & y^n_w & z^n_w & 1 & -v_n x^n_w & -v_n y^n_w & -v_n z^n_w & -v_n \end{bmatrix} \begin{bmatrix} P_{11} \\ P_{12} \\ P_{13} \\ P_{14} \\ P_{21} \\ P_{22} \\ P_{23} \\ P_{24} \\ P_{31} \\ P_{32} \\ P_{33} \\ P_{34} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ . \\ . \\ . \\ . \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

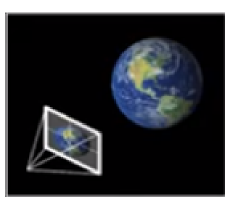
$$A \; P = \; 0$$

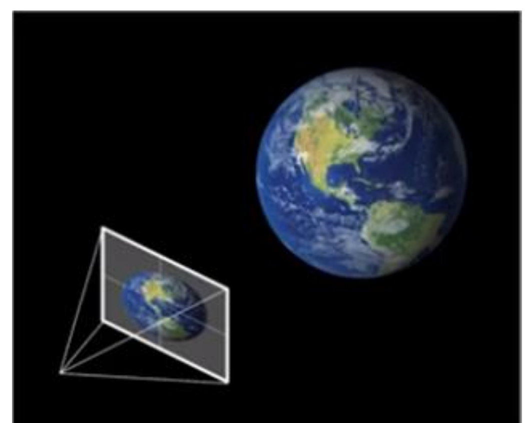
Chapitre 2. Camera Calibration

2.3 Calibrate a Camera

The projection matrix P is defined only up to a scale because P and kP produce the same homogenous coordinate.
Scaling P implies simultaneously the scaling of the world and camera which not change the image.

$$\begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix} = P\tilde{P}_w = \text{k}\, P\tilde{P}_w$$





2.3 Calibrate a Camera

In order to determine the projection matrix P, we have two choices:

- Set the scale so that $P_{34} = 1$
- Set scale so that $\|P\|^2 = 1$

We need to resolve:

$$\min_P \|AP\|^2 \; so\ that \|P\|^2 = 1$$

$$\min_P (P^T A^T A P) so\ that\ P^T P = 1$$

We define the loss function:

$$L(P, \lambda) = P^T A^T A P - \lambda(P^T P\text{-}1)$$

If we derive with respect to p we obtain:

$$2A^T A P - 2\lambda P\text{=}0$$

$$A^T A P - \lambda P\text{=}0 \quad \text{Eigen value problem}$$

We search the eigenvector p with smallest value λ of the matrix $A^T A$ which minimize the loss function L.

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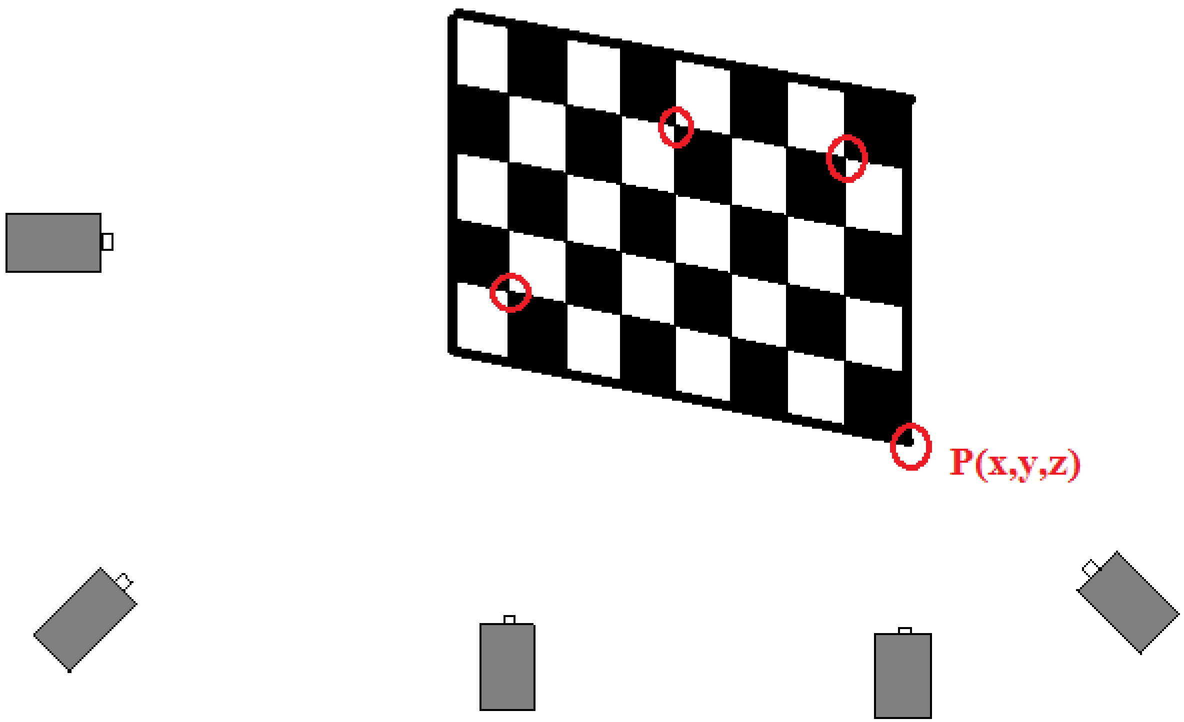
2.3 Calibrate a Camera



Z. Zhang. A flexible new technique for camera calibration. IEEE Trans. Pattern Analysis and Machine Intelligence, 22(11):1330–1334, 2000.

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2.3 Calibrate a Camera

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2.3 Calibrate a Camera

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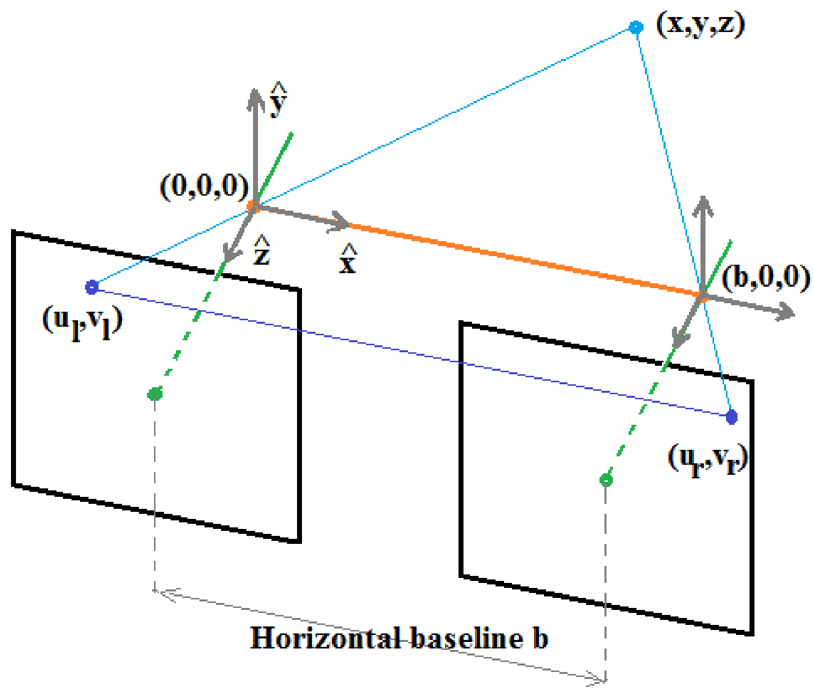
2.4 Simple Stereo

We assume that the projection planes of the two cameras (left and right) are planar.



Example of simple stereo device: Fujifilm FinePix REAL 3D W3

Chapitre 2. Camera Calibration

2.4 Simple Stereo

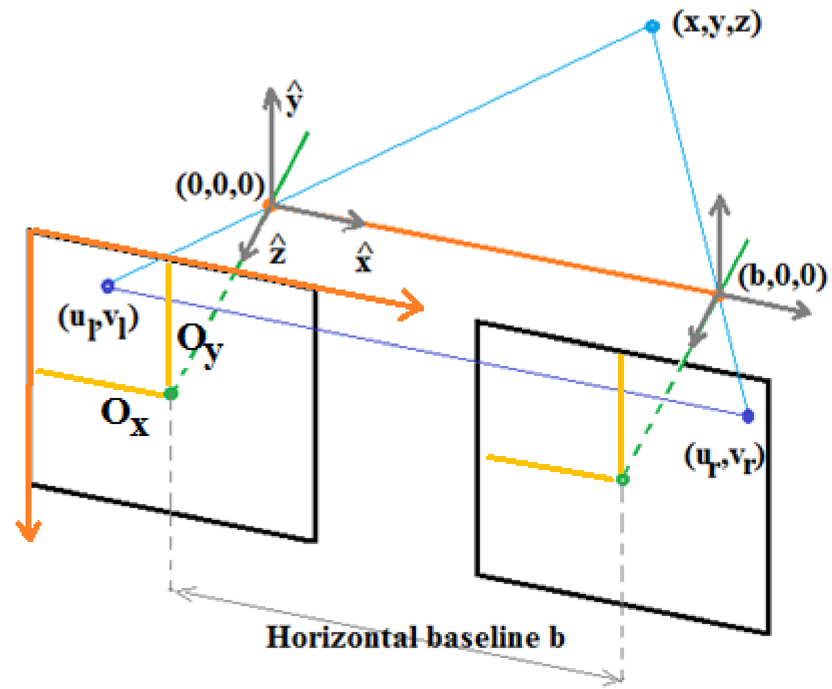
We assume that the projection planes of the two cameras (left and right) are planar.
In this case, we can compute the (x,y,z) of the scene point if the cameras are calibrated as follow:

For the left camera:

$$u_l = f_x \frac{x}{z} + O_x \qquad v_l = f_y \frac{y}{z} + O_y$$

For the right camera:

$$u_r = f_x \frac{x-b}{z} + O_x \qquad v_r = f_y \frac{y}{z} + O_y$$



The parameters f_x , f_y, O_x, O_y b are $known.$

Chapitre 2. Camera Calibration

2.4 Simple Stereo

We compute x, y, z as follow:

$$zu_l = xf_x + zO_x \qquad zv_l = yf_y + zO_y$$

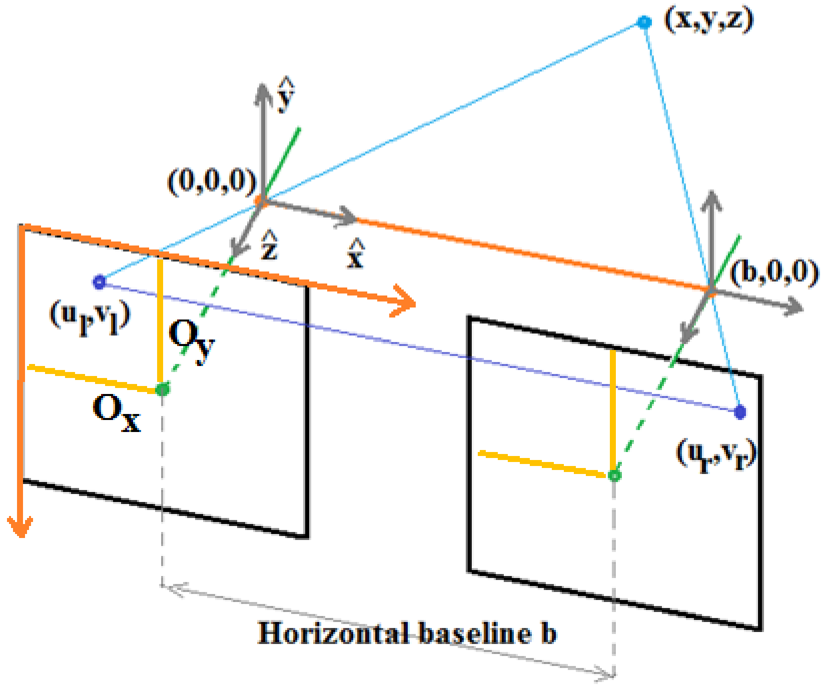
$$zu_r = (x-b)f_x + zO_x \qquad zv_r = yf_y + zO_y$$

$$z(u_l - u_r) = bf_x$$

$$z = \frac{bf_x}{(u_l - u_r)}$$

$$\frac{bf_x}{(u_l - u_r)}u_l = xf_x\frac{(u_l - u_r)}{(u_l - u_r)} + \frac{bf_x}{(u_l - u_r)}O_x$$

$$x = \frac{b(u_l - O_x)}{(u_l - u_r)} \qquad y = \frac{bf_x(u_l - O_x)}{f_y(u_l - u_r)}$$



2.4 Simple Stereo

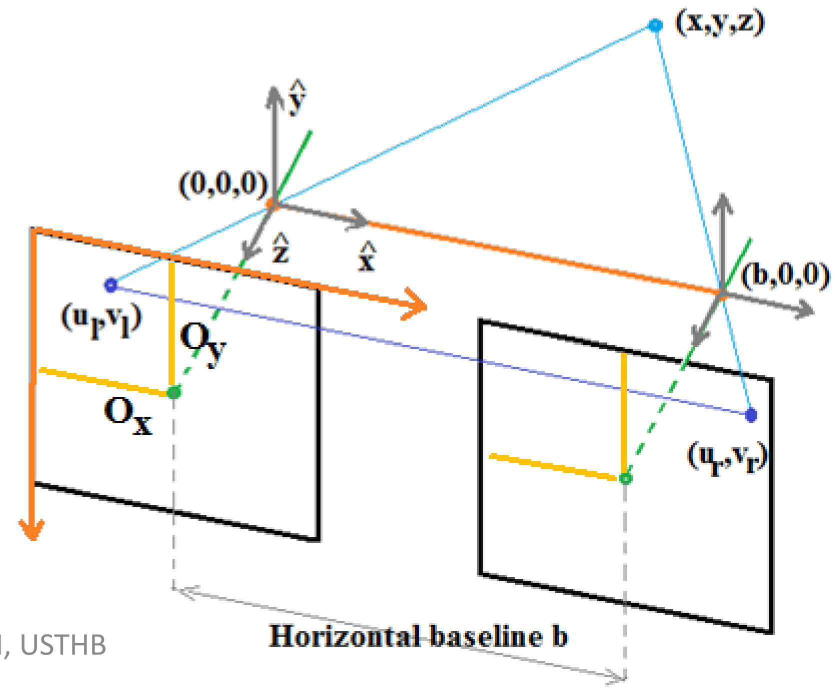
We call the disparity: $(u_l - u_r)$

The depth z is inversely proportional to the disparity.

The disparity is proportional to the baseline b

$$x = \frac{b(u_l - O_x)}{(u_l - u_r)} \qquad y = \frac{bf_x(u_l - O_x)}{f_y(u_l - u_r)}$$

$$z = \frac{bf_x}{(u_l - u_r)}$$



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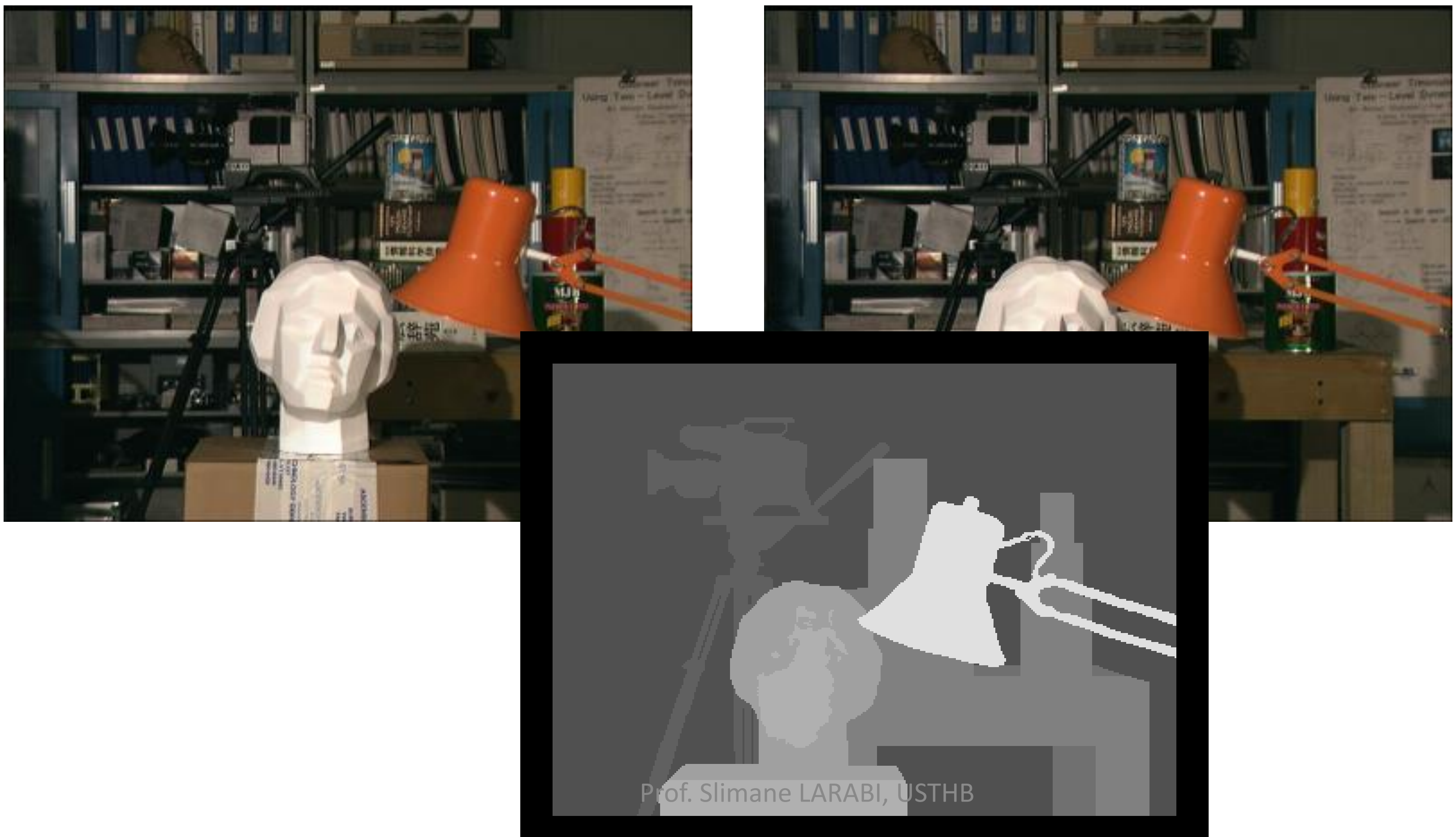
2.4 Simple Stereo

How to find disparity between the left and right stereo pairs:



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2.4 Simple Stereo

How to find disparity between the left and right stereo pairs:

From the equations of v_l and v_r, the images of scene point lie on the same horizontal line.

$$v_l = f_y \frac{y}{z} + O_y \qquad v_r = f_y \frac{y}{z} + O_y$$

We determine disparity using template matching:

$$z = \frac{b f_x}{(u_l - u_r)} \qquad disparity\ d = (u_l - u_r)$$

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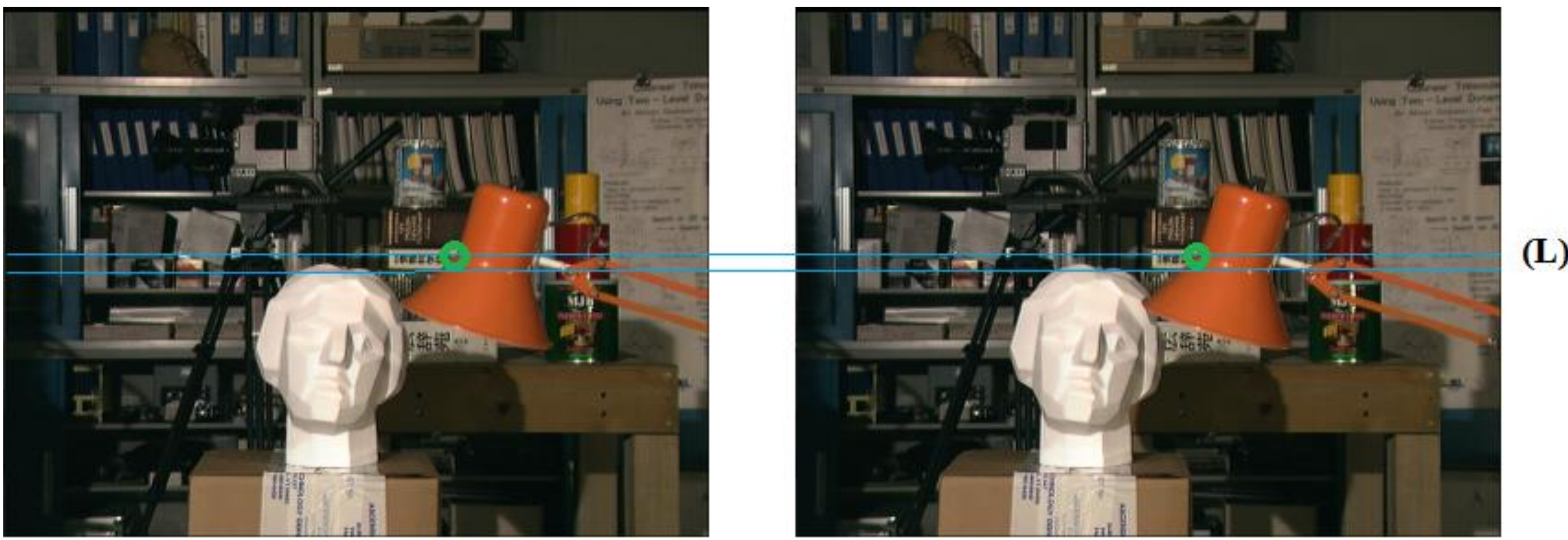
2.4 Simple Stereo

How to find disparity between the left and right stereo pairs:

From the equations of v_l and v_r, the images of scene point lie on the same horizontal line.



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2.4 Simple Stereo

Similarity metrics for template matching:

Find (k,l) with minimum Sum of Absolute Differences

$$\boldsymbol{SAD(k,l) = \sum_{(i,j)\in T} |E_l(i,j) - E_r(i+k, j+l)|}$$

Find (k,l) with minimum Sum of Squared Differences

$$SAD(k,l) = \sum_{(i,j)\in T} |E_l(i,j) - E_r(i+k, j+l)|^2$$

Find (k,l) with maximum Normalized Cross-Correlation

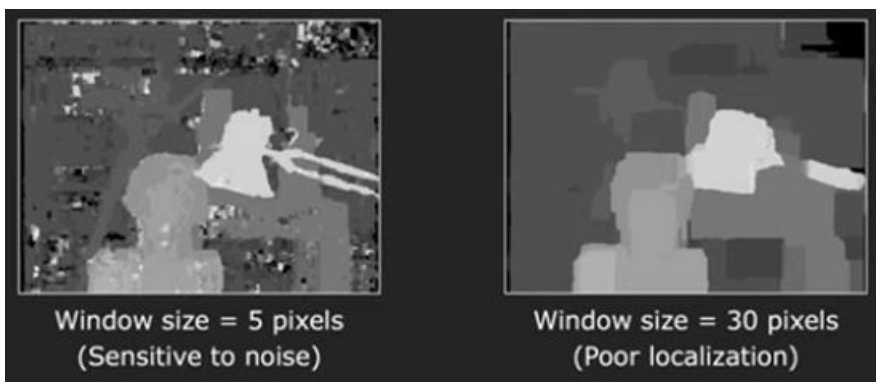
$$NCC(k,l) = \frac{\sum_{(i,j)\in T} E_l(i,j) E_r(i+k,j+l)^2}{\sqrt{\sum_{(i,j)\in T} E_l(i,j)^2 \quad \sum_{(i,j)\in T} E_r(i+k,j+l)^2}}$$

Chapitre 2. Camera Calibration

2.4 Simple Stereo

Similarity metrics for template matching

How large should be the window?
Adaptative window method solution:
For each point match using windows of multiple sizes and use the disparity that is a result of the best similarity measure.



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2.4 Simple Stereo

Similarity metrics for template matching