Reinforcement Learning

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A new kind of learning

- Supervised learning: matching features to labels

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- Unsupervised learning: clustering data

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- Unsupervised learning: clustering data
- **Reinforcement learning**: learning from rewards

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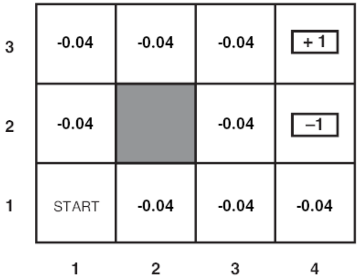
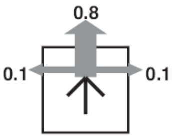
- Markov property: we can view problems as **Markov decision processes** (MDPs)

A framework: Markov decision processes

- Actions map states to states with a probability distribution
- Transition reward: $R(s, a, s')$
- Transition probability: $P(s' \mid s, a)$



(a)



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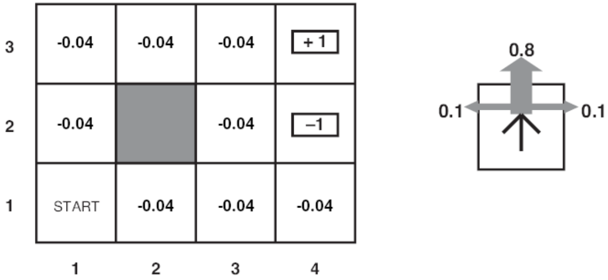

(a) (b)

Bellman's equation

Given a discount factor $\gamma \in [0, 1]$, the utility is given by:

$$U(s) = \max_{a \in A(s)} \sum_{s'} P(s' \mid s, a) \left[R(s, a, s') + \gamma U(s') \right]$$

Model-based reinforcement learning

We maintain a transition model (an **MDP**):

- Reward function: $R(s, a, s')$
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- Utility function: U(s), maps states to **utility**

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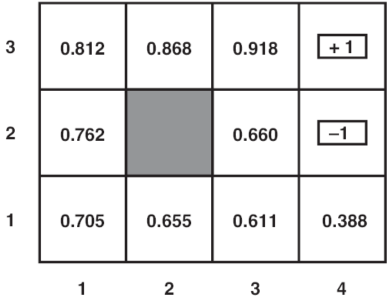
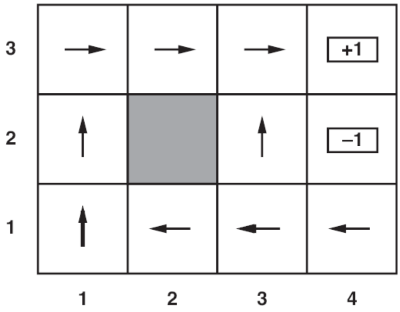
Once the model is learned, we can **maximize utility**



Model-free reinforcement learning

- Action-utility function: $Q(s, a)$ maps **actions** to **utility**
- Policy search: maps **states** to **actions**, essentialy a reflex agent



Passive reinforcement learning

We can't modify the policy, but we can figure out the utilities $U(s)$

- We use **policy iteration**:

$$U^{\pi}(s) = E\left[\sum_{t=0}^{+\infty} \gamma^{t} R(s_t, \pi(s_t), s_{t+1})\right]$$

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Three approaches for approximation:

- Direct estimation
- Adaptive dynamic programming (ADP)
- Temporal difference learning (TD)

Direct estimation

The goal is to approximate state utility under the given policy

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We calculate utilities for $(1,2)$ given the trial:

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- This is inefficient! We can **exploit** the Markov property

Adaptive dynamic programming

We can approximate the transition reward and probability functions (P and R) and apply **simplified Bellman's equation**:

$$U(s) = \sum_{s'} P(s' \mid s, \pi(s)) \left[R(s, \pi(s), s') + \gamma U(s') \right]$$

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- Calculating P and R is easy when the environment is **fully observable**

Temporal difference learning

Still taking trials, but we **update** the utility to match frequently observed transitions

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Then we can update with learning rate α:

$$U^{\pi}(s) \leftarrow U^{\pi}(s) + \alpha \left[R(s, \pi(s), s') + \gamma U^{\pi}(s') - U^{\pi}(s) \right]$$

- From TD's point of view, ADP is TD with *simulated experiences*

Active reinforcement learning

Utility functions give the best (expected) policy
We can decide to improve utility (**explore**) or keep maximizing utility (**exploit**)

- This is a **bandit problem** (set of Markov reward processes)

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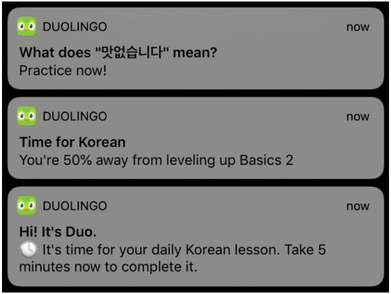
Example

Duolingo's recurring notifications use a bandit problem solver:



Template	Eligibility Criteria
You're on fire Continue your 109 day Spanish streak on Duolingo.	*Must have a 3+ day streak.*[1]
Streak wager reminder You're on day 2 of your 7-day streak wager! Now get to day 3!	*Must have a streak wager.*[2]
Ready for your trip? Take 5 minutes to practice Italian now	*User's profile must indicate travel motivation.*

Greedy in the limit of infinite exploration (GLIE)

A greedy algorithm could ignore higher utility actions. We need a GLIE algorithm:

- Each eaction in a each state is tried an unbounded amount of times
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We can try different approaches:

- Random sampling
- Minimum trials

Random sampling

At step t, do the following:

- Choose a random action with probability $\frac{1}{t}$
- Otherwise, follow the greedy policy

This **does** eventually converge, but can be slow: a better approach would use some heuristic on state utility

Minimum trials

Have an **optimistic estimate** of utility ($U^+(s)$):

$$U^+(s) \leftarrow \max_a f\left(\sum_{s'} P(s' \mid s, a)\left[R(s, a, s') + \gamma U^+(s')\right], N(s, a)\right)$$

- $N(s, a)$ is the number of times action a was tried from state s
- f is defined as:

$$f(u, n) = \begin{cases} R^+, & n < N_e \\ u, & n \geq N_e \end{cases}$$

  Here, R^+ is an optimistic utility for actions tried less than N_e times

Basically, we are estabilishing an optimistic **prior** that assigns higher utilities to actions tried less than N_e times

Safe exploration

In the real world, we have to look out for:

- Large negative rewards
- Absorbing states
- Permanent damage (e.g. long-term utility losses)

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Neither the greedy nor the exploratory approach are optimal! Two ways to solve the problem:

- Bayesian reinforcement learning
- Robust control theory approaches

Bayesian reinforcement learning

We take a probabilistic approach, starting with a **prior** probability $P(h)$ on h hypotheses about what the model **is**: the policy is then given by the posterior $P(h \mid e)$:

$$\pi^* = \underset{\pi}{\operatorname{argmax}} \sum_h P(h \mid e) U_h^{\pi}$$

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- We turned the problem into an exploration **partially observable Markov decision process** (POMDP)
- Note that this approach assumes that the prior contains good estimates of reality

The robust control theory approach

We define a set of models $\mathcal{H}$, and always take the policy that maximizes utility in the **worst case** over $\mathcal{H}$:

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- This is closely tied to bayesian reinforcement learning: the set $\mathcal{H}$ is the set of models that exceed some likelihood threshold on $P(h \mid e)$
- Again, we assume the models to be good estimates of reality

Temporal difference Q-learning

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We substitute $Q(s, a)$ into Bellman's equation:

$$Q(s, a) = \sum_{s'} P(s'|s, a) \left[R(s, a, s') + \gamma \max_{a'} Q(s', a') \right]$$

Now we can derive an update:

$$Q(s, a) = Q(s, a) + \alpha \left[R(s, a, s') + \gamma \max_{a'} Q(s', a') - Q(s, a) \right]$$

SARSA learning

SARSA is a variation of TD Q-learning: instead of assuming the best possible action under known action-utility functions to be taken (**off-policy**), we wait for the model to actually execute actions (**on-policy**)

- We act after s, a, r, s', a' quintuplets

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TD and SARSA are subtly different:

- TD learning asks: "What does this action give if i stop using my policy, and take the best action (according to my estimates) from there onwards?"
- SARSA asks: "What did I get when i followed my policy and took this action?"