Normalization of Social Models in Social Argumentation Frameworks

1 Preliminaries

Definition 1 (Social argumentation frameworks)**.** *A* social argumentation framework *is a 3-tuple $F = \langle \mathcal{A}, \mathcal{R}, V \rangle$, where*

- *$\mathcal{A}$ is the set of arguments,*
- *$\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is a binary attack relation between arguments,*
- *$V : \mathcal{A} \cup \mathcal{R} \to \mathbb{N} \times \mathbb{N}$ is a voting function keeping the crowd's pro and con votes for each argument and attack relation.*

Definition 2 (Vote Aggregation Function)**.** *Given a totally ordered set L with top and bottom elements $\top$, $\bot$, and a framework $F = \langle \mathcal{A}, \mathcal{R}, V \rangle$, τ is any function such that $\tau : (\mathcal{A} \cup \mathcal{R}) \times 2^{(\mathcal{A} \cup \mathcal{R})} \to L$.*

Definition 3 (Semantic Framework)**.** *A semantic framework is a 6-tuple $\mathcal{S} = \langle L, \curlywedge_1, \curlywedge_2, \curlyvee, \neg, \tau \rangle$ where:*

- *L is a totally ordered set with top and bottom elements $\top$, $\bot$, containing all possible valuations of an argument.*
- *$\curlywedge_{\mathcal{A}}, \curlywedge_{\mathcal{R}} : L \times L \to L$, are two binary algebraic operations used to restrict strengths to given values.*
- *$\curlyvee : L \times L \to L$, is a binary algebraic operation on argument valuations used to combine or aggregate valuations and strengths.*
- *$\neg : L \to L$ is a unary algebraic operation for computing a restricting value corresponding to a given valuation or strength.*
- *τ is a vote aggregation function which, given the votes, determines the social support of an object within a set of objects.*

Notation 1. *Let $F = \langle \mathcal{A}, \mathcal{R}, V \rangle$ be a SAF, $\mathcal{S} = \langle L, \curlywedge_{\mathcal{A}}, \curlywedge_{\mathcal{R}}, \curlyvee, \neg, \tau \rangle$ a semantic framework. Then, let*

- *$\mathcal{O} = \mathcal{A} \cup \mathcal{R}$ be the set of objects, composed by the union of the sets of arguments and attack relations,*

- - $V^+(o) \triangleq x$ *denote the number of positive votes for object* o,
  - $V^-(o) \triangleq y$ *denote the number of negative votes for object* o,
  - $V^t : \mathcal{O} \to \mathbb{N}$ *be a function s.t.* $V^t(o) \triangleq x + y$,

  whenever $V(o) = (x, y)$,

- $V : 2^{\mathcal{O}} \to 2^{\mathbb{N}\times\mathbb{N}}$ *be a function s.t.* $V(\mathcal{O}') = \{V(o) \mid o \in \mathcal{O}'\}$,
- $V^t : 2^{\mathcal{O}} \to 2^{\mathbb{N}\times\mathbb{N}}$ *be a function s.t.* $V^t(\mathcal{O}') = \{V^t(o) \mid o \in \mathcal{O}'\}$,
- $max : 2^{\mathbb{N}} \to \mathbb{N}$ *be a function s.t. it returns the maximum value amongst the natural numbers from the non-empty multiset given as the input.*
- $min : 2^{\mathbb{N}} \to \mathbb{N}$ *be a function s.t. it returns the minimum value amongst the natural numbers from the non-empty multiset given as the input.*
- $\tau : 2^{\mathcal{O}} \to 2^L$ *be a function s.t.* $\tau(A) = \{\tau(a) | a \in A\}$.
- $\mathcal{R}^{\text{-}}(a) \triangleq \{a_i \in \mathcal{A} : (a_i, a) \in \mathcal{R}\}$ *be the set of direct attackers of an argument* $a \in \mathcal{A}$,
-
$$\bigcurlyvee_{x \in X} x \triangleq (((x_1 \curlyvee x_2) \curlyvee \ldots) \curlyvee x_n)$$
  $X = \{x_1, x_2, \ldots, x_n\}$ *denote the aggregation of a multiset of elements of* L.

Definition 4 (Model)**.** *Let* $F = \langle \mathcal{A}, \mathcal{R}, V \rangle$ *be a social argumentation framework,* $\mathcal{S} = \langle L, \curlywedge_{\mathcal{A}}, \curlywedge_{\mathcal{R}}, \curlyvee, \neg, \tau \rangle$ *be a semantic framework. An* $\mathcal{S}$*-model of* F *is a total mapping* $M : \mathcal{A} \to L$ *such that for all* $a \in \mathcal{A}$,

$$M(a) = \tau(a, \mathcal{A}) \curlywedge_{\mathcal{A}} \neg \bigcurlyvee_{a_i \in \mathcal{R}^{\text{-}}(a)} \left(\tau\left((a_i, a), \mathcal{R}\right) \curlywedge_{\mathcal{R}} M\left(a_i\right)\right)$$

Definition 5 (Enhanced Vote Aggregation)**.** *Given a SAF* $F = \langle \mathcal{A}, \mathcal{R}, V \rangle$ *and a semantic framework* $\mathcal{S} = \langle L, \curlywedge_{\mathcal{A}}, \curlywedge_{\mathcal{R}}, \curlyvee, \neg, \tau \rangle$*, enhanced vote aggregation function* $\tau_e : \mathcal{O} \times 2^{\mathcal{O}} \to [0, 1]$ *is the vote aggregation function such that*

$$\tau_e(o, \mathcal{O}) = \begin{cases} 0 & V(o) = (0, 0) \\ \frac{V^+(o)}{V^t(o) + \frac{1}{max(V^t(o \cup \mathcal{O}))}} & otherwise \end{cases}$$

Definition 6 (Enhanced Product Semantics)**.** *An enhanced product semantic framework is any* $\mathcal{S}^{\cdot}_e = \langle [0, 1], \tau_e, \curlywedge^{\cdot}, \curlywedge^{\cdot}, \curlyvee^{\cdot}, \neg \rangle$ *where 1)* $x_1 \curlywedge^{\cdot} x_2 = x_1 \cdot x_2$*, 2)* $x_1 \curlyvee^{\cdot} x_2 = x_1 + x_2 - x_1 \cdot x_2$*, 3)* $\neg x_1 = 1 - x_1$

Conjecture 1. *Let* $F = \langle \mathcal{A}, \mathcal{R}, V \rangle$ *be a social argumentation framework,* $\mathcal{S}^{\cdot}_e = \langle [0, 1], \tau_e, \curlywedge^{\cdot}, \curlywedge^{\cdot}, \curlyvee^{\cdot}, \neg \rangle$ *be a semantic framework. Then,* F *has one and only one* $\mathcal{S}^{\cdot}_e - model$.

Notation 2. *Let* $F = \langle \mathcal{A}, \mathcal{R}, V \rangle$ *be a social argumentation framework,* $\mathcal{S} = \langle L, \curlywedge_{\mathcal{A}}, \curlywedge_{\mathcal{R}}, \curlyvee, \neg, \tau_e \rangle$ *be a semantic framework. Then, under the assumption that Conjecture 1 holds, let*

- $\mathcal{D} \triangleq \{M(a) | a \in \mathcal{A}\}$ *be the multiset of model evaluations of all arguments.*

2 Normalization of Model Evaluations

2.1 Problem domain

We have some dataset $\mathcal{D}$ containing the multiset of model evaluation values for all the arguments of a social abstract argumentation framework with product semantics. The values $\forall d \in \mathcal{D}$ lay between the interval $[0, 1] \in \mathbb{R}$.

Under the aforementioned semantics, the values in our candidate datasets tend to form big clusters, especially close to zero. The main goal of this study is adjusting the original distributions by some mapping $\sigma : \mathcal{D} \to [0, 1]$, in order to spread high-density areas (increasing the values in most cases), so that arguments with distinct values can be distinguished better.

The method of remapping may introduce some distortions or biases into the data. However in this case distortions are deliberately introduced to the system, in order to better expose the information content.

2.1.1 Characterization of Normalized Sets

In this section we try to find out the parameters that define whether a dataset is *normalized* or not. However the findings are at a preliminary level.

At this point, two values that I believe to be relevant are:

- The ratio between the range of the model evaluations and the range of the social support values

$$\frac{max(\mathcal{D}) - min(\mathcal{D})}{max(\mathcal{T}) - min(\mathcal{T})}$$

- The ratio between the standard deviation of the model evaluations and the standard deviation of the social support values

$$\frac{\sigma(\mathcal{D})}{\sigma(\mathcal{T})}$$

In the following subsection you may find the narrative of the thought process behind determining up with these two elements. But before that, below with Definition 7 we take our first shot at defining what a normalized set is, by incorporating the aforementioned the parameters.

Definition 7 (Normalized dataset)**.** *Given a SAF* $F = \langle \mathcal{A}, \mathcal{R}, V \rangle$ *and a semantic framework* $\mathcal{S} = \langle L, \curlywedge_{\mathcal{A}}, \curlywedge_{\mathcal{R}}, \curlyvee, \neg, \tau \rangle$*, the generic standard deviation mapping* $\sigma : 2^{\mathbb{N}} \to \mathbb{N}$ *and constants* $r_1, r_2, r_3, r_4 \in \mathbb{R}^+$ *where* $r_1 \geq r_2$ *and* $r_3 \geq r_4$*, the multiset of model evaluations* $\mathcal{D}$ *is said to be normalized dataset if the following conditions hold:*

- $r_1 \geq \frac{max(\mathcal{D})-min(\mathcal{D})}{max(\mathcal{T})-min(\mathcal{T})} \geq r_2$,
- $r_3 \geq \frac{\sigma(\mathcal{D})}{\sigma(\mathcal{T})} \geq r_4$.

2.1.2 Discussion on the Characterization of Normalized Sets

Our initial suspicion was that whenever all model evaluations are very close to zero, this problem originates because the argumentation graph is strongly connected and this collection of datapoints $\mathcal{D}$ needs normalization. However consider the following setting:

Example 1. *Let $F = \langle \mathcal{A}, \mathcal{R}, V \rangle$ be a SAF, $\mathcal{S} = \langle L, \curlywedge_{\mathcal{A}}, \curlywedge_{\mathcal{R}}, \curlyvee, \neg, \tau \rangle$ a semantic framework, $\mathcal{A} = \{a_1, a_2, a_3\}$ and $\mathcal{R} = \emptyset$.*

In addition assume that all the social support values are extremely low. Since the attack set is empty, the model evaluation values for the arguments will be equal their social support values. So here the structure of the graph is not to blame. The model evaluations are low because of the popular opinion and thus probably the dataset should be left this way, it should not be normalized.

In order to exaggerate the example and display the situation clearer, we have taken the attack set to be empty. One may argue that to identify such a scenario it's enough to check how dense the argumentation graph is. However even if $\mathcal{R}$ all possible edges at full strength, the resulting model evaluations would be pretty close to the previous values since the social support is low for all the arguments. *Thus the graph structure is not a reliable indicator in identifying whether a dataset that requires normalization.*

At this point I thought comparing the ranges of the social support values and the model evaluations might give us an idea. From Notation 1&2 please remember that $\mathcal{T}$ is the multiset of all social supports and $\mathcal{D}$ is the multiset of model evaluations of all arguments in the framework. In this particular example it would follow that $\frac{max(\mathcal{D})-min(\mathcal{D})}{max(\mathcal{T})-min(\mathcal{T})} \simeq 1$. This would hint that the model evaluations have not shifted too far from the social support values.

However it appears that considering the ranges by itself is not sufficient as well. To see this, consider the following example:

Example 2. *Let $F = \langle \mathcal{A}, \mathcal{R}, V \rangle$ be a SAF, $\mathcal{S} = \langle L, \curlywedge_{\mathcal{A}}, \curlywedge_{\mathcal{R}}, \curlyvee, \neg, \tau \rangle$ a semantic framework, $\mathcal{A} = \{a_1, a_2, a_3\}$ and $\mathcal{R} = \{(a_1, a_2), (a_2, a_3), (a_3, a_1)\}$.*

Assume all the arguments are at full strength. Then under product semantics the model evaluations would follow as $M(a_1) = M(a_2) = M(a_3) \simeq 0.33$.

Now let's add another argument to the system (also with perfect social support) that only attacks to the first argument:

Example 3. *Let $F = \langle \mathcal{A}, \mathcal{R}, V \rangle$ be a SAF, $\mathcal{S} = \langle L, \curlywedge_{\mathcal{A}}, \curlywedge_{\mathcal{R}}, \curlyvee, \neg, \tau \rangle$ a semantic framework, $\mathcal{A} = \{a_1, a_2, a_3, a_4\}$ and $\mathcal{R} = \{(a_1, a_2), (a_2, a_3), (a_3, a_1), (a_4, a_1)\}$.*

Now with addition of a single argument and attack relation we have $M(a_4) = M(acc_2) = 1$ and $M(a_1) = M(a_3) = 0$. The range of the social supports is the same, since all of them are at the same value. However the range of the model evaluations increased from a minimum of 0 to a maximum of 1. So on top of the comparison of ranges, we need some additional concept that would capture the essence of the distribution i.e. how datapoints are spread in the interval.

Since standard deviation measures how far a set of numbers is spread out, the relative measure of the two multisets' standard deviations, $\frac{\sigma(\mathcal{D})}{\sigma(\mathcal{T})}$ may overcome the shortcoming of only taking ranges into consideration.

With these two ratio values, we obtain information on both the relative size of the sub-interval the datapoints are located at and also on the pattern they are spread.

As you might see in Definition 7, we've tried to restrain the envisioned values of the two rations with some constants. Currently we've not settled on these values, and maybe in future we may choose to replace these constants with some other terms.

One thing that comes to mind is some sort of a supervized learning method. Perhaps we may assume a certain set of datasets as our training set. Then we may parameterize the constants with regards to our training set. That way we may fix the interval for the ratios of ranges and standard deviations.

Let us see one example on this notion to make it more clear. For simplicity, we will only consider the standard deviation of the model evaluations when we are talking about the characterization of normalized sets (i.e. instead of comparing the ratios of range and standard deviations of social support values and model evaluations as we originally intend, we just take into account the standard deviations of the model evaluations in this example).

Example 4. *Assume we have the following three datasets that are known to be normalized:*

- $D_1 = \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$
- $D_2 = \{0.05, 0.1, 0.3, 0.45, 0.75, 0.8, 0.9, 0.95\}$
- $D_3 = \{0.25, 0.35, 0.4, 0.5, 0.6, 0.7, 0.75\}$

And we would like to figure out whether the following two data sets are normalized or not with respect to the training set.

- $D_4 = \{0.05, 0.1, 0.1, 0.15, 0.2, 0.2, 0.85, 0.9, 0.95\}$
- $D_5 = \{0.25, 0.3, 0.4, 0.6, 0.75, 0.9, 0.9\}$

Then the standard deviations for the training set follow as: $\sigma_1 \cong 0.287, \sigma_2 \cong 0.337, \sigma_3 \cong 0.172$.

Thus we may conclude that for a dataset D_i*, the corresponding standard deviation value should fall into the interval* $0.172 \leq \sigma_i \leq 0.337$ *for* D_i *to be regarded as normalized.*

The standard deviations for the test set follow as: $\sigma_4 \cong 0.364, \sigma_5 \cong 0.254$.

Consequently we may determine that D_5 is a normalized dataset, on the other hand D_4 is not.

Undoubtedly we've to study this subject in more detail to find out the optimal range for these values. We might also uncover more parameters with respect to the notion of normalized sets.

2.2 Characterization of Normalizing Mappings

The approach we adopt in this section is first defining a list of desirable properties that a normalizing mapping $\sigma_X : X \to [0, 1]$ may possess, where X is a fixed multiset. Then in the next section we will follow by defining some concrete classes of normalizing mappings which contain a subset the properties defined in the current section.

Before we move on, here we may better take a small pause to discuss the structure of the normalizing mapping. The mapping of each datapoint to a value in the unit interval is carried out with respect to the whole dataset. So the information that the dataset if fixed should be included in the mapping symbol. Indeed, that's what the subscript in the function definition above stands for.

For example, assume two sets $A = \{1, 3, 5, 7\}$ and $B = \{5, 50, 500, 5000\}$. Undoubtedly it would follow as $\sigma_A(5) \neq \sigma_B(5)$.

For the upcoming property definitions, let $F = \langle \mathcal{A}, \mathcal{R}, V \rangle$ be a SAF, $\mathcal{S} = \langle L, \curlywedge_{\mathcal{A}}, \curlywedge_{\mathcal{R}}, \curlyvee, \neg, \tau \rangle$ a semantic framework, $\sigma_{\mathcal{D}} : \mathcal{D} \to [0, 1]$ a normalizing function, $\triangle : L \times L \to \mathbb{R}$ a metric, $M(a) = d \in \mathcal{D}$ be the model evaluation of an argument $a \in \mathcal{A}$.

Property 1 (Bottom argument)**.** *The argument of a context with the bottom value does not attain any strength in the normalized mutliset.*

$$d = 0 \Rightarrow \sigma_{\mathcal{D}}(d) = 0$$

Property 2 (Guarantee for the existence of arguments)**.** *An argument with some social strength is never diminished to the value of zero through normalization.*

$$d \neq 0 \Rightarrow \sigma_{\mathcal{D}}(d) \neq 0$$

Property 3 (Decisiveness of Popular Opinion)**.** *Normalized value of a model evaluation is limited by the original argument's social support.*

$$\sigma_{\mathcal{D}}(d) \leq \tau(a, \mathcal{A})$$

Property 4 (Conservation of relative ordering)**.** *The relative order between the pairs of normalized model evaluations is preserved.*

$$d_1 > d_2 \Longrightarrow \sigma_{\mathcal{D}}(d_1) > \sigma_{\mathcal{D}}(d_2)$$

Property 5 (Conservation of distance ordering)**.** *The relative order between the distance of pairs of normalized model evaluations is preserved.*

$$\triangle(d_1, d_2) > \triangle(d_3, d_4) \Longrightarrow \triangle(\sigma_{\mathcal{D}}(d_1), \sigma_{\mathcal{D}}(d_2)) > \triangle(\sigma_{\mathcal{D}}(d_3), \sigma_{\mathcal{D}}(d_4))$$

Even all together, these properties still give way to very flexible mapping definitions. For instance the identity mapping $id : x \mapsto x$ would interestingly satisfy all of the aforementioned properties.

Thus we have to define some property based on a concept that could capture the essence of specifically dense areas of the set of model evaluations. In the light of this, we benefit from the study of cluster analysis.

Very crudely, our objective via utilizing cluster analysis is to identify groups of objects that are very similar with regard to their values. Before going into more detail, firstly we need to formally define what a *cluster* is. In the literature, there are many definitions with respect to clusters. In our context we will define them closely to the *partitions* from the classical set theory via updating the definition accordingly with respect to multisets.

Definition 8 (Cluster)**.** *A family of multisets $\mathcal{C}$ is a clustering of a multiset X and every element C of $\mathcal{C}$ is a cluster if and only if all of the following conditions hold:*

- *Set of clusters does not contain the empty set.*

$$\emptyset \notin \mathcal{C}$$

- *(Collectively exhaustive) The sum of the multisets in $\mathcal{C}$ is equal to X.*

$$\biguplus_{C \in \mathcal{C}} C = X$$

- *(Mutually exclusive) Distinct multisets do not contain shared elements.*

$$(C_i, C_j \in \mathcal{C}) \wedge (C_i \neq C_j) \Longrightarrow (C_i \wedge C_j = \emptyset)$$

As mentioned clustering is the task of grouping a set of objects in such a way that objects in the same cluster are more similar (in some sense or another) to each other than to those in other cluster. So we should also define the concept of *similarity* in a formal manner. Different procedures adopt different metrics, $\triangle : S \times S \to \mathbb{R}$ where S is some set of valuations, when grouping the most similar objects into clusters. We have a single clustering variable, the model evaluation of arguments. Thus using the generic distance function as the similarity measure is a possibility.

We continue by stating two more preliminary definitions that we will utilize in our last property.

Definition 9 (Separated points)**.** *Let $\mathcal{C}$ be a clustering of some multiset X and $C_i, C_j \in \mathcal{C}$. Two distinct datapoints p, q are said to be separated if $p \in C_i$ and $q \in C_j$ when $i \neq j$.*

Notation 3. *Let $Separated(p, q)$ be the shortcut of notation where p and q are two separated points from some clustering $\mathcal{C}$.*

Definition 10 (Spacing)**.** *Given a metric* $\triangle$*, the spacing of a clustering* $\mathcal{C}$ *is the minimum distance between any two separated points:*

$\triangle(X,Y) = min_{x \in X, y \in Y} \triangle(x,y)$ *where* X *and* Y *are any two distinct clusters, and d(x,y) denotes the distance between the two elements x and y.*

One last discussion before we continue to the last property is k, the number of clusters. Some clustering procedures require k to be defined by the user manually, and the rest compute it as the starting step of the procedure. The problem with the family of clustering methods that compute the number of clusters by themselves is that they don't scale well. They have a high time-complexity, in all cases exceeding the cubic time. Thus we may either use a method like Monte Carlo to generate an artificial k and clustering centers then continue with the efficient methods, or use the inefficient but *self-computing* methods initially and then switch to the efficient ones.

Property 6 (Max spacing over k-clustering)**.** *Given constant* $k \in \mathbb{N}$*, a normalizing function* $\sigma_{\mathcal{D}} : \mathcal{D} \to [0,1]$ *maximizes the spacing over a clustering* $\mathcal{C}$ *with* k*-clusters.*

2.3 The Algorithm of a Concrete Normalizing Mapping

input : $\mathcal{D}$ (instances set), k (# of clusters))
output: $\mathcal{D}$ (normalized instances set)

$cluster\_counter = 0$;
Clusters $\leftarrow \emptyset$;
foreach $d_i \in \mathcal{D}$ **do**
    $newCluster \leftarrow \{d_i\}$;
    $instance\_counter(newCluster) = 1$;
    $Clusters \leftarrow Clusters \cup newCluster$;
    $cluster\_counter++$;
end
foreach $C_i, C_j \in Clusters$ **do**
    Compute $\triangle(C_i, C_j)$
end
while $cluster\_counter \neq k$ **do**
    $cluster\_counter--$;
    (C_m, C_n) = two clusters closest together in $Clusters$
end

Algorithm 1: A Clustering Based Normalizing Algorithm

Theorem 1. *The algorithm satisfies properties...*

Proof. □