Definition 1 (Extended social argumentation frameworks)**.** *An* extended social argumentation framework *is a 4-tuple $F = \langle \mathcal{A}, \mathcal{R}, \mathcal{O}, V \rangle$, where*

- *$\mathcal{A}$ is the set of arguments,*
- *$\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is a binary attack relation between arguments,*
- *$\mathcal{O} = \mathcal{A} \cup \mathcal{R}$ is the set of objects, composed by the union of the sets of arguments and attack relations,*
- *$V : \mathcal{O} \to \mathbb{N} \times \mathbb{N}$ stores the crowd's pro and con votes for each object.*

Definition 2. *[Vote Aggregation Function] Given a totally ordered set L with top and bottom elements $\top$, $\bot$, a voting function V and a set of objects $\mathcal{O}$, a vote aggregation function τ is any function such that $\tau : \mathcal{O} \times 2^{\mathcal{O}} \to L$.*

Definition 3 (Semantic Framework)**.** *A semantic framework is a 6-tuple $\langle L, \curlywedge_{\mathcal{A}}, \curlywedge_{\mathcal{R}}, \curlyvee, \neg, \tau \rangle$ where:*

- *L is a totally ordered set with top and bottom elements $\top$, $\bot$, containing all possible valuations of an argument.*
- *$\curlywedge_{\mathcal{A}}, \curlywedge_{\mathcal{R}} : L \times L \to L$, are two binary algebraic operations used to restrict strengths to given values.*
- *$\curlyvee : L \times L \to L$, is a binary algebraic operation on argument valuations used to combine or aggregate valuations and strengths.*
- *$\neg : L \to L$ is a unary algebraic operation for computing a restricting value corresponding to a given valuation or strength.*
- *τ is a vote aggregation function which, given the votes, determines the social support of an object within a set of objects.*

Notation 1. *Let $F = \langle \mathcal{A}, \mathcal{R}, \mathcal{O}, V \rangle$ be an ESAF, $\mathcal{S} = \langle L, \curlywedge_{\mathcal{A}}, \curlywedge_{\mathcal{R}}, \curlyvee, \neg, \tau \rangle$ a semantic framework. Then, let*

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  - *$V^{+}(o) \triangleq x$ denote the number of positive votes for object o,*
  - *$V^{-}(o) \triangleq y$ denote the number of negative votes for object o,*
  - *$v^{r} : \mathcal{O} \to \mathbb{R}$ be a function s.t. $v^{r}(o) \triangleq \frac{x}{x+y}$,*
  - *$v^{t} : \mathcal{O} \to \mathbb{N}$ be a function s.t. $v^{t}(o) \triangleq x + y$,*

  whenever $V(o) = (x, y)$,

- *$V : 2^{\mathcal{O}} \to 2^{\mathbb{N} \times \mathbb{N}}$ be a function s.t. $V(\mathcal{O}') = \{V(o) \mid o \in \mathcal{O}'\}$,*
- *$V^{t} : 2^{\mathcal{O}} \to 2^{\mathbb{N} \times \mathbb{N}}$ be a function s.t. $V^{t}(\mathcal{O}') = \{V^{t}(o) \mid o \in \mathcal{O}'\}$,*
- *$max : 2^{\mathbb{N}} \to \mathbb{N}$ be a function s.t. it returns the maximum value amongst the natural numbers from the non-empty multiset given as the input.*

- $\mathcal{R}^{-}(a) \triangleq \{a_i \in \mathcal{A} : (a_i, a) \in \mathcal{R}\}$ *be the set of direct attackers of an argument* $a \in \mathcal{A}$*,*

- $$\bigvee_{x \in X} x \triangleq (((x_1 \curlyvee x_2) \curlyvee ...) \curlyvee x_n)$$

  $X = \{x_1, x_2, ..., x_n\}$ *denote the aggregation of a multiset of elements of* L*.*

Definition 4 (Model)**.** *Let* $F = \langle \mathcal{A}, \mathcal{R}, \mathcal{O}, V \rangle$ *be a social argumentation framework,* $\mathcal{S} = \langle L, \curlywedge_{\mathcal{A}}, \curlywedge_{\mathcal{R}}, \curlyvee, \neg, \tau \rangle$ *be a semantic framework. An* $\mathcal{S}$*-model of* F *is a total mapping* $M : \mathcal{A} \to L$ *such that for all* $a \in \mathcal{A}$*,*

$$M(a) = \tau(a, \mathcal{A}) \curlywedge_{\mathcal{A}} \neg \bigvee_{a_i \in \mathcal{R}^{-}(a)} (\tau((a_i, a), \mathcal{R}) \curlywedge_{\mathcal{R}} M(a_i))$$

Definition 5. *[Enhanced Vote Aggregation]*
Given a voting function V *and a set of objects* $\mathcal{O}$*, enhanced vote aggregation function* $\tau_e : \mathcal{O} \times 2^{\mathcal{O}} \to [0, 1]$ *is the vote aggregation function such that*

$$\tau_e(o, \mathcal{O}) = \begin{cases} 0 & V(o) = (0,0) \\ \frac{v^+(o)}{v^t(o) + \frac{1}{max(v^t(o \cup \mathcal{O}))}} & \textit{otherwise} \end{cases}$$

Property 1. *[Absolute argument freeness]*
Let τ *be a a vote aggregation function given a set of values* L*, a set of objects* $\mathcal{O}$ *and a value function* V*. We say that* τ *is 'absolute argument free' if*

$$\forall o \in \mathcal{O},\ \tau(o, \mathcal{O}) \neq \top.$$

Property 2. *[Precedence of the vote ratio]*
Let τ *be a a vote aggregation function given a set of values* L*, a set of objects* $\mathcal{O}$ *and a value function* V*. We say that* τ *is 'vote ratio precedent' if*

$$\forall o_1, o_2 \in \mathcal{O},$$
$$(v^r(o_1) \geq v^r(o_2)) \implies (\tau(o_1, \mathcal{O}) \geq \tau(o_2, \mathcal{O})).$$

Property 3. *[Precedence of the total number of votes]*
Let τ *be a a vote aggregation function given a set of values* L*, a set of objects* $\mathcal{O}$ *and a value function* V*. We say that* τ *is 'total votes precedent' if*

$$\forall o_1, o_2 \in \mathcal{O},$$
$$((v^r(o_1) = v^r(o_2)) \wedge (v^t(o_1) > (v^t(o_2))) \implies (\tau(o_1, \mathcal{O}) > \tau(o_2, \mathcal{O})).$$

Proposition 1. *Enhanced Vote Aggregation function respects Property 1.*

Proof. Let L be a totally ordered set with top and bottom elements $\top$, $\bot$, V a voting function, $\mathcal{O}$ a set of objects and an arbitrary $o \in \mathcal{O}$.

We consider the two main cases for the enhanced vote aggregation function with respect to o.

Firstly, if $V(o) = (0, 0)$, by definition $\tau_e(o, \mathcal{O}) = 0 < 1$.

Now assume $V(o) \neq (0,0)$. We know that for an arbitrary multiset of naturals X where $\exists(n_1, n_2) \in X$ s.t. $(n_1, n_2) \neq (0,0)$, $max(X) \in \mathbb{N}^+$. And consequently $\frac{1}{max(X)} > 0$. Furthermore we have $v^t(o) = v^+(o) + v^-(o)$.

From the two inferences we get: $(v^t(o) + \frac{1}{max(v^t(o \cup \mathcal{O}))}) > v^+$.

Hence it follows that:

$$\tau_e(o, \mathcal{O}) = \frac{v^+(o)}{v^t(o) + \frac{1}{max(v^t(o \cup \mathcal{O}))}} < 1.$$

□

Conjecture 1. *Enhanced Vote Aggregation function respects Property 2.*

Proposition 2. *Vote Aggregation function[1] does not respect Property 2.*

Proof. It's sufficient to display a counter example.

Let L be a totally ordered set with top and bottom elements $\top$, $\bot$, V a voting function, $\mathcal{O}$ a set of objects and $o_1, o_2 \in \mathcal{O}$.

EXTEND PART

Furthermore assume $\epsilon = 0.1, V(o_1) = (1,1)$ and $V(o_2) = (499, 501)$.

Since $v^r(o_1) \geq v^r(o_2)$ via the property the following should hold $\tau(o_1, \mathcal{O}) \geq \tau(o_2, \mathcal{O})$.

However $\tau(v_1^+, v_1^-) = \tau(1,1) \cong 0.4761$ and $\tau(v_2^+, v_2^-) = \tau(499, 501) \cong 0.4989$. Hence, $\tau(v_1^+, v_1^-) < \tau(v_2^+, v_2^-)$.

We conclude a contradiction. □

Conjecture 2. *Enhanced Vote Aggregation function respects Property 3.*

References

[1] S.Egilmez, J. Martins, and J. Leite. Extending social abstract argumentation with votes on attacks. In *Procs. of TAFA 2013*. TBA, 2013.