

# Report on ongoing research

## 1 Main

**Definition 1** (Extended social argumentation frameworks). *An extended social argumentation framework is a 4-tuple  $F = \langle \mathcal{A}, \mathcal{R}, V_{\mathcal{A}}, V_{\mathcal{R}} \rangle$ , where*

- $\mathcal{A}$  is the set of arguments,
- $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$  is a binary attack relation between arguments,
- $V_{\mathcal{A}} : \mathcal{A} \rightarrow \mathbb{N} \times \mathbb{N}$  stores the crowd's pro and con votes for each argument.
- $V_{\mathcal{R}} : \mathcal{R} \rightarrow \mathbb{N} \times \mathbb{N}$  stores the crowd's pro and con votes for each attack.

**Definition 2.** [Vote Aggregation Function] *Given a totally ordered set  $L$  with top and bottom elements  $\top, \perp$ , a vote aggregation function  $\tau$  over  $L$  is any function such that  $\tau : \mathbb{N}^2 \times \mathbb{N} \rightarrow L$ .*

**Definition 3** (Semantic Framework). *A semantic framework is a 6-tuple  $\langle L, \lambda_1, \lambda_2, \Upsilon, \neg, \tau, \rangle$  where:*

- $L$  is a totally ordered set with top and bottom elements  $\top, \perp$ , containing all possible valuations of an argument.
- $\lambda_{\mathcal{A}}, \lambda_{\mathcal{R}} : L \times L \rightarrow L$ , are two binary algebraic operations used to restrict strengths to given values.
- $\Upsilon : L \times L \rightarrow L$ , is a binary algebraic operation on argument valuations used to combine or aggregate valuations and strengths.
- $\neg : L \rightarrow L$  is a unary algebraic operation for computing a restricting value corresponding to a given valuation or strength.
- $\tau$  is a vote aggregation function, which aggregates positive and negative votes and the maximum number of total votes into a social support value.

**Notation 1.** *Let  $F = \langle \mathcal{A}, \mathcal{R}, V_{\mathcal{A}}, V_{\mathcal{R}} \rangle$  be an ESAF,  $\mathcal{S} = \langle L, \lambda_1, \lambda_2, \Upsilon, \neg, \tau, \rangle$  a semantic framework. Then, let*

- $\mathcal{R}^-(a) \triangleq \{a_i \in \mathcal{A} : (a_i, a) \in \mathcal{R}\}$  be the set of direct attackers of an argument  $a \in \mathcal{A}$ ,

- $- V_{\mathcal{A}}^+(a) \triangleq x$  denote the number of positive votes for argument  $a$ ,  
 $- V_{\mathcal{A}}^-(a) \triangleq y$  denote the number of negative votes for argument  $a$ ,  
whenever  $V_{\mathcal{A}}(a) = (x, y)$ ,
- $v^r : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$  be a function s.t.  $v^r(v^+, v^-) \triangleq \frac{v^+}{v^-}$  where  $v^+, v^- \in \mathbb{N}$ , that denotes the ratio of positive votes to negative votes for some argument or attack relation,
- $v^t : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$  be a function s.t.  $v^t(v^+, v^-) \triangleq v^+ + v^-$  where  $v^+, v^- \in \mathbb{N}$ , that denotes the total number of votes for some argument or attack relation,
- $v_{\max} \triangleq v^t(V_{\mathcal{A}}(b))$  where  $b \in \mathcal{A}$  and  $\forall a \in \mathcal{A}, v^t(V_{\mathcal{A}}(b)) \geq v^t(V_{\mathcal{A}}(a))$  denote the maximum total number of votes for an argument,
- $\tau(a) \triangleq \tau(V_{\mathcal{A}}(a), v_{\max})$  denote the social support for an argument  $a$  via utilizing a vote aggregation function  $\tau$ ,
- $- V_{\mathcal{R}}^+((a, b)) \triangleq x$  denote the number of positive votes for an attack relation between arguments  $a$  and  $b$ ,  
 $- V_{\mathcal{R}}^-((a, b)) \triangleq y$  denote the number of negative votes for an attack relation between arguments  $a$  and  $b$ ,  
whenever  $V_{\mathcal{R}}((a, b)) = (x, y)$ ,
- $v_{\max} \triangleq v^t(V_{\mathcal{R}}(y))$  where  $y \in \mathcal{R}$  and  $\forall x \in \mathcal{R}, v^t(V_{\mathcal{R}}(y)) \geq v^t(V_{\mathcal{R}}(x))$  denote the maximum total number of votes for an attack relation,
- $\tau((a, b)) \triangleq \tau(V_{\mathcal{R}}((a, b)), v_{\max})$  denote the social support for an attack relation between arguments  $a$  and  $b$  via utilizing a vote aggregation function  $\tau$ ,
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$$\bigvee_{x \in R} x \triangleq (((x_1 \vee x_2) \vee \dots) \vee x_n)$$

$R = \{x_1, x_2, \dots, x_n\}$  denote the aggregation of a multiset of elements of  $L$ .

**Definition 4** (Model). Let  $F = \langle \mathcal{A}, \mathcal{R}, V_{\mathcal{A}}, V_{\mathcal{R}} \rangle$  be a social argumentation framework,  $\mathcal{S} = \langle L, \lambda_1, \lambda_2, \vee, \neg, \tau, \rangle$  be a semantic framework. A  $\mathcal{S}$ -model of  $F$  is a total mapping  $M : \mathcal{A} \rightarrow L$  such that for all  $a \in \mathcal{A}$ ,

$$M(a) = \tau(a) \lambda_{\mathcal{A}} \neg \bigvee_{a_i \in \mathcal{R}^-(a)} (\tau((a_i, a)) \lambda_{\mathcal{R}} M(a_i))$$

**Definition 5.** [Enhanced Vote Aggregation]

Enhanced vote aggregation function  $\tau_e : (\mathbb{N} \times \mathbb{N}) \times \mathbb{N} \rightarrow [0, 1]$  is the vote aggregation function such that

$$\tau_e((v^+, v^-), v_{\max}) = \begin{cases} 0 & v_{\max} = 0 \\ \frac{v^+}{v^+ + v^- + \frac{1}{v_{\max}}} & \text{otherwise} \end{cases}$$

**Property 1.** *[Absolute argument freeness]*

Let  $\tau$  be a vote aggregation function given  $L$ . We say that  $\tau$  is 'absolute argument free' if

$$\forall n_1, n_2, n_3 \in \mathbb{N}, \tau((n_1, n_2), n_3) \neq \top.$$

The gist of the property can be captured in a verbal context with the following sentence: *No argument enjoys perfect Social Support.*

**Property 2.** *[Precedence of the vote ratio]*

Let  $\tau$  be a vote aggregation function given  $L$ . We say that  $\tau$  is 'vote ratio precedent' if

$$\begin{aligned} &\forall n_1, n_2, n_3, n_4, n_5 \in \mathbb{N}, \\ &(v^r(n_1, n_2) \geq v^r(n_3, n_4)) \implies (\tau((n_1, n_2), n_5) \geq \tau((n_3, n_4), n_5)). \end{aligned}$$

The gist of the property can be captured in a verbal context with the following sentence: *If the value of the ratio of positive votes to the negative votes is higher for an argument  $a$  than an argument  $b$ ;  $a$ 's social support value exceeds the social support of  $b$ .*

**Property 3.** *[Precedence of the total number of votes]*

Let  $\tau$  be a vote aggregation function given  $L$ . We say that  $\tau$  is 'total votes precedent' if

$$\begin{aligned} &\forall n_1, n_2, n_3, n_4, n_5 \in \mathbb{N}, \\ &((v^r(n_1, n_2) = v^r(n_3, n_4)) \wedge (v^t(n_1, n_2) \geq v^t(n_3, n_4))) \implies \\ &(\tau((n_1, n_2), n_5) \geq \tau((n_3, n_4), n_5)). \end{aligned}$$

The gist of the property can be captured in a verbal context with the following sentence: *When the ratios are equal, the function should return a higher social support value for the one with the higher number of total votes.*