

Report regarding the ongoing research

1 Main

Definition 1 (Extended social argumentation frameworks). *An extended social argumentation framework is a 4-tuple $F = \langle \mathcal{A}, \mathcal{R}, V_{\mathcal{A}}, V_{\mathcal{R}} \rangle$, where*

- \mathcal{A} is the set of arguments,
- $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is a binary attack relation between arguments,
- $V_{\mathcal{A}} : \mathcal{A} \rightarrow \mathbb{N} \times \mathbb{N}$ stores the crowd's pro and con votes for each argument.
- $V_{\mathcal{R}} : \mathcal{R} \rightarrow \mathbb{N} \times \mathbb{N}$ stores the crowd's pro and con votes for each attack.

Definition 2. [Vote Aggregation Function] *Given a totally ordered set L with top and bottom elements \top, \perp , a vote aggregation function τ over L is any function such that $\tau : \mathbb{N}^2 \times \mathbb{N} \rightarrow L$.*

Definition 3 (Semantic Framework). *A semantic framework is a 6-tuple $\langle L, \lambda_1, \lambda_2, \Upsilon, \neg, \tau, \rangle$ where:*

- L is a totally ordered set with top and bottom elements \top, \perp , containing all possible valuations of an argument.
- $\lambda_{\mathcal{A}}, \lambda_{\mathcal{R}} : L \times L \rightarrow L$, are two binary algebraic operations used to restrict strengths to given values.
- $\Upsilon : L \times L \rightarrow L$, is a binary algebraic operation on argument valuations used to combine or aggregate valuations and strengths.
- $\neg : L \rightarrow L$ is a unary algebraic operation for computing a restricting value corresponding to a given valuation or strength.
- τ is a vote aggregation function, which aggregates positive and negative votes and the maximum number of total votes into a social support value.

Notation 1. *Let $F = \langle \mathcal{A}, \mathcal{R}, V_{\mathcal{A}}, V_{\mathcal{R}} \rangle$ be an ESAF, $\mathcal{S} = \langle L, \lambda_1, \lambda_2, \Upsilon, \neg, \tau, \rangle$ a semantic framework. Then, let*

- $\mathcal{R}^-(a) \triangleq \{a_i \in \mathcal{A} : (a_i, a) \in \mathcal{R}\}$ be the set of direct attackers of an argument $a \in \mathcal{A}$,

- $- V_{\mathcal{A}}^+(a) \triangleq x$ denote the number of positive votes for argument a ,
 $- V_{\mathcal{A}}^-(a) \triangleq y$ denote the number of negative votes for argument a ,
whenever $V_{\mathcal{A}}(a) = (x, y)$,
- $v^r : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$ be a function s.t. $v^r(v^+, v^-) \triangleq \frac{v^+}{v^-}$ where $v^+, v^- \in \mathbb{N}$, that denotes the ratio of positive votes to negative votes for some argument or attack relation,
- $v^t : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$ be a function s.t. $v^t(v^+, v^-) \triangleq v^+ + v^-$ where $v^+, v^- \in \mathbb{N}$, that denotes the total number of votes for some argument or attack relation,
- $v_{\max} \triangleq v^t(V_{\mathcal{A}}(b))$ where $b \in \mathcal{A}$ and $\forall a \in \mathcal{A}, v^t(V_{\mathcal{A}}(b)) \geq v^t(V_{\mathcal{A}}(a))$ denote the maximum total number of votes for an argument,
- $\tau(a) \triangleq \tau(V_{\mathcal{A}}(a), v_{\max})$ denote the social support for an argument a via utilizing a vote aggregation function τ ,
- $- V_{\mathcal{R}}^+((a, b)) \triangleq x$ denote the number of positive votes for an attack relation between arguments a and b ,
 $- V_{\mathcal{R}}^-((a, b)) \triangleq y$ denote the number of negative votes for an attack relation between arguments a and b ,
whenever $V_{\mathcal{R}}((a, b)) = (x, y)$,
- $v_{r\max} \triangleq v^t(V_{\mathcal{R}}(y))$ where $y \in \mathcal{R}$ and $\forall x \in \mathcal{R}, v^t(V_{\mathcal{R}}(y)) \geq v^t(V_{\mathcal{R}}(x))$ denote the maximum total number of votes for an attack relation,
- $\tau((a, b)) \triangleq \tau(V_{\mathcal{R}}((a, b)), v_{r\max})$ denote the social support for an attack relation between arguments a and b via utilizing a vote aggregation function τ ,
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$$\bigvee_{x \in R} x \triangleq (((x_1 \vee x_2) \vee \dots) \vee x_n)$$

$R = \{x_1, x_2, \dots, x_n\}$ denote the aggregation of a multiset of elements of L .

Definition 4 (Model). Let $F = \langle \mathcal{A}, \mathcal{R}, V_{\mathcal{A}}, V_{\mathcal{R}} \rangle$ be a social argumentation framework, $\mathcal{S} = \langle L, \lambda_1, \lambda_2, \vee, \neg, \tau, \rangle$ be a semantic framework. A \mathcal{S} -model of F is a total mapping $M : \mathcal{A} \rightarrow L$ such that for all $a \in \mathcal{A}$,

$$M(a) = \tau(a) \wedge_{\mathcal{A}} \neg \bigvee_{a_i \in \mathcal{R}^-(a)} (\tau((a_i, a)) \wedge_{\mathcal{R}} M(a_i))$$

Notation 2. Whenever a single spesific system $F = \langle \mathcal{A}, \mathcal{R}, V_{\mathcal{A}}, V_{\mathcal{R}} \rangle$ with some semantic framework $\mathcal{S} = \langle L, \lambda_1, \lambda_2, \vee, \neg, \tau, \rangle$ is considered, we implicitly consider v_{\max} and $v_{r\max}$ are constants and drop them from the respective function's argument list. Thus we update the two function definitions from Notation 1 as follows: $\tau(a) \triangleq \tau(V_{\mathcal{A}}(a))$ and $\tau((a, b)) \triangleq \tau(V_{\mathcal{R}}((a, b)))$, where $a \in \mathcal{A}$.

Definition 5. *[Enhanced Vote Aggregation]*

Enhanced vote aggregation function $\tau_e : \mathbb{N} \times \mathbb{N} \rightarrow [0, 1]$ is the vote aggregation function such that

$$\tau_e(v^+, v^-) = \begin{cases} 0 & v_{max} = 0 \\ \frac{v^+}{v^+ + v^- + \frac{1}{v_{max}}} & \text{otherwise} \end{cases}$$

Property 1. *[Absolute argument freeness]*

Let τ be a vote aggregation function given L . We say that τ is 'absolute argument free' if

$$\forall n_1, n_2 \in \mathbb{N}, \tau(n_1, n_2) \neq \top.$$

The gist of the property can be captured in a verbal context with the following sentence: *No argument enjoys perfect Social Support.*

Property 2. *[Precedence of the vote ratio]*

Let τ be a vote aggregation function given L . We say that τ is 'vote ratio precedent' if

$$\begin{aligned} & \forall n_1, n_2, n_3, n_4 \in \mathbb{N}, \\ & (v^r(n_1, n_2) \geq v^r(n_3, n_4)) \implies (\tau(n_1, n_2) \geq \tau(n_3, n_4)). \\ & \text{just for convenience:} \\ & (v^r(a_1) \geq v^r(a_2)) \implies (\tau(a_1) \geq \tau(a_2)). \end{aligned}$$

The gist of the property can be captured in a verbal context with the following sentence: *If the value of the ratio of positive votes to the negative votes is higher for an argument a than an argument b ; a 's social support value exceeds the social support of b .*

Property 3. *[Precedence of the total number of votes]*

Let τ be a vote aggregation function given L . We say that τ is 'total votes precedent' if

$$\begin{aligned} & \forall n_1, n_2, n_3, n_4 \in \mathbb{N}, \\ & ((v^r(n_1, n_2) = v^r(n_3, n_4)) \wedge (v^t(n_1, n_2) \geq (v^t(n_3, n_4)))) \implies \\ & (\tau(n_1, n_2) \geq \tau(n_3, n_4)). \\ & \text{just for convenience:} \\ & ((v^r(a_1) = v^r(a_2)) \wedge (v^t(a_1) \geq (v^t(a_2)))) \implies (\tau(a_1) \geq \tau(a_2)). \end{aligned}$$

The gist of the property can be captured in a verbal context with the following sentence: *When the ratios are equal, the function should return a higher social support value for the one with the higher number of total votes.*