Definition 1 (Extended social argumentation frameworks)**.** *An* extended social argumentation framework *is a 4-tuple* $F = \langle \mathcal{A}, \mathcal{R}, V_{\mathcal{A}}, V_{\mathcal{R}} \rangle$*, where*

- $\mathcal{A}$ *is the set of arguments,*
- $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ *is a binary attack relation between arguments,*
- $V_{\mathcal{A}} : \mathcal{A} \to \mathbb{N} \times \mathbb{N}$ *stores the crowd's pro and con votes for each argument.*
- $V_{\mathcal{R}} : \mathcal{R} \to \mathbb{N} \times \mathbb{N}$ *stores the crowd's pro and con votes for each attack.*

Definition 2. *[Vote Aggregation Function] Given a totally ordered set L with top and bottom elements $\top$, $\bot$, a vote aggregation function τ over L is any function such that $\tau : \mathbb{N} \times \mathbb{N} \times 2^{\mathbb{N} \times \mathbb{N}} \to L$.*

Definition 3 (Semantic Framework)**.** *A semantic framework is a 6-tuple* $\langle L, \curlywedge_1, \curlywedge_2, \curlyvee, \neg, \tau, \rangle$ *where:*

- *L is a totally ordered set with top and bottom elements $\top$, $\bot$, containing all possible valuations of an argument.*
- *$\curlywedge_{\mathcal{A}}, \curlywedge_{\mathcal{R}} : L \times L \to L$, are two binary algebraic operations used to restrict strengths to given values.*
- *$\curlyvee : L \times L \to L$, is a binary algebraic operation on argument valuations used to combine or aggregate valuations and strengths.*
- *$\neg : L \to L$ is a unary algebraic operation for computing a restricting value corresponding to a given valuation or strength.*
- *τ is a vote aggregation function which given some context, aggregates positive and negative votes into a social support value.*

Notation 1. *Let $F = \langle \mathcal{A}, \mathcal{R}, V_{\mathcal{A}}, V_{\mathcal{R}} \rangle$ be an ESAF, $\mathcal{S} = \langle L, \curlywedge_1, \curlywedge_2, \curlyvee, \neg, \tau, \rangle$ a semantic framework. Then, let*

- *$\mathcal{R}^{-}(a) \triangleq \{a_i \in \mathcal{A} : (a_i, a) \in \mathcal{R}\}$ be the set of direct attackers of an argument $a \in \mathcal{A}$,*
- *$v^r : \mathbb{N} \times \mathbb{N} \to \mathbb{R}$ be a function s.t. $v^r(x, y) \triangleq \frac{x}{y}$ where $x, y \in \mathbb{N}$,*
- *$v^t : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ be a function s.t. $v^t(x, y) \triangleq x + y$ where $x, y \in \mathbb{N}$,*
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  - *$V_{\mathcal{A}}^{+}(a) \triangleq x$ denote the number of positive votes for argument a,*
  - *$V_{\mathcal{A}}^{-}(a) \triangleq y$ denote the number of negative votes for argument a,*

  whenever $V_{\mathcal{A}}(a) = (x, y)$,
- *$V_{\mathcal{A}} : 2^{\mathcal{A}} \to 2^{\mathbb{N} \times \mathbb{N}}$ be a function s.t. $V_{\mathcal{A}}(\mathcal{A}') = \{V_{\mathcal{A}}(a) \mid a \in \mathcal{A}'\}$,*

- $\tau(a, A^{'}) \triangleq \tau(V_{\mathcal{A}}(a), V_{\mathcal{A}}(A^{'}))$ *denote the social support for an argument* a*, within a set of arguments* $A^{'}$*, via utilizing a vote aggregation function* τ*,*

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  - $V_{\mathcal{R}}^{+}((a,b)) \triangleq x$ *denote the number of positive votes for an attack relation between arguments* a *and* b*,*
  - $V_{\mathcal{R}}^{-}((a,b)) \triangleq y$ *denote the number of negative votes for an attack relation between arguments* a *and* b*,*

  whenever $V_{\mathcal{R}}((a,b)) = (x,y)$*,*

- $V_{\mathcal{R}} : 2^{\mathcal{R}} \rightarrow 2^{\mathbb{N}\times\mathbb{N}}$ *be a function s.t.* $V_{\mathcal{R}}(R^{'}) = \{V_{\mathcal{R}}((a,b)) \mid (a,b) \in R^{'}\}$*,*

- $\tau((a,b), R^{'}) \triangleq \tau(V_{\mathcal{R}}((a,b)), V_{\mathcal{R}}(R^{'}))$ *denote the social support for an attack relation between arguments* a *and* b*, within a set of attack relations* $R^{'}$*, via utilizing a vote aggregation function* τ*,*

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  $$\bigcurlyvee_{x\in X} x \triangleq (((x_1 \curlyvee x_2) \curlyvee ...) \curlyvee x_n)$$

  $X = \{x_1, x_2, ..., x_n\}$ *denote the aggregation of a multiset of elements of* L*.*

- $max : 2^{\mathbb{N}} \rightarrow \mathbb{N}$ *be a function s.t. it returns the maximum value amongst the non-empty set of natural numbers given as the input.*

- $\omega : 2^{\mathbb{N}\times\mathbb{N}} \rightarrow 2^{\mathbb{N}}$ *be a function s.t.* $\omega(\mathcal{X}) = \{v^t(a,b) \mid (a,b) \in \mathcal{X}\}$*,*

Definition 4 (Model)**.** *Let* $F = \langle \mathcal{A}, \mathcal{R}, V_{\mathcal{A}}, V_{\mathcal{R}} \rangle$ *be a social argumentation framework,* $\mathcal{S} = \langle L, \curlywedge_1, \curlywedge_2, \curlyvee, \neg, \tau, \rangle$ *be a semantic framework. An* $\mathcal{S}$*-model of* F *is a total mapping* $M : \mathcal{A} \rightarrow L$ *such that for all* $a \in \mathcal{A}$*,*

$$M(a) = \tau(a, \mathcal{A}) \curlywedge_{\mathcal{A}} \neg \bigcurlyvee_{a_i \in \mathcal{R}^{-}(a)} (\tau((a_i, a), \mathcal{R}) \curlywedge_{\mathcal{R}} M(a_i))$$

Definition 5. *[Enhanced Vote Aggregation]*
Enhanced vote aggregation function $\tau_e : \mathbb{N} \times \mathbb{N} \times 2^{\mathbb{N}\times\mathbb{N}} \rightarrow [0,1]$ *is the vote aggregation function such that*

$$\tau_e(v^+, v^-, \mathcal{X}) = \begin{cases} 0 & v_{max} = 0 \\ \frac{v^+}{v^+ + v^- + \frac{1}{max(\omega(\mathcal{X}))}} & otherwise \end{cases}$$