Definition 1 (Extended social argumentation frameworks)**.** *An* extended social argumentation framework *is a 4-tuple* $F = \langle \mathcal{A}, \mathcal{R}, V_{\mathcal{A}}, V_{\mathcal{R}} \rangle$*, where*

- $\mathcal{A}$ *is the set of arguments,*
- $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ *is a binary attack relation between arguments,*
- $V_{\mathcal{A}} : \mathcal{A} \to \mathbb{N} \times \mathbb{N}$ *stores the crowd's pro and con votes for each argument.*
- $V_{\mathcal{R}} : \mathcal{R} \to \mathbb{N} \times \mathbb{N}$ *stores the crowd's pro and con votes for each attack.*

Definition 2. *[Vote Aggregation Function] Given a totally ordered set* L *with top and bottom elements* $\top$*,* $\bot$*, a vote aggregation function* τ *over* L *is any function such that* $\tau : \mathbb{N} \times \mathbb{N} \times 2^{\mathbb{N} \times \mathbb{N}} \to L$*.*

Definition 3 (Semantic Framework)**.** *A semantic framework is a 6-tuple* $\langle L, \curlywedge_1, \curlywedge_2, \curlyvee, \neg, \tau, \rangle$ *where:*

- L *is a totally ordered set with top and bottom elements* $\top$*,* $\bot$*, containing all possible valuations of an argument.*
- $\curlywedge_{\mathcal{A}}, \curlywedge_{\mathcal{R}} : L \times L \to L$*, are two binary algebraic operations used to restrict strengths to given values.*
- $\curlyvee : L \times L \to L$*, is a binary algebraic operation on argument valuations used to combine or aggregate valuations and strengths.*
- $\neg : L \to L$ *is a unary algebraic operation for computing a restricting value corresponding to a given valuation or strength.*
- τ *is a vote aggregation function, which aggregates positive and negative votes and the maximum number of total votes into a social support value.*

Notation 1. *Let* $F = \langle \mathcal{A}, \mathcal{R}, V_{\mathcal{A}}, V_{\mathcal{R}} \rangle$ *be an ESAF,* $\mathcal{S} = \langle L, \curlywedge_1, \curlywedge_2, \curlyvee, \neg, \tau, \rangle$ *a semantic framework. Then, let*

- $\mathcal{R}^{-}(a) \triangleq \{a_i \in \mathcal{A} : (a_i, a) \in \mathcal{R}\}$ *be the set of direct attackers of an argument* $a \in \mathcal{A}$*,*
- $v^r : \mathbb{N} \times \mathbb{N} \to \mathbb{R}$ *be a function s.t.* $v^r(v^+, v^-) \triangleq \frac{v^+}{v^-}$ *where* $v^+, v^- \in \mathbb{N}$*,*
- $v^t : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ *be a function s.t.* $v^t(v^+, v^-) \triangleq v^+ + v^-$ *where* $v^+, v^- \in \mathbb{N}$*,*
- - $V_{\mathcal{A}}^{+}(a) \triangleq x$ *denote the number of positive votes for argument* a*,*
  - $V_{\mathcal{A}}^{-}(a) \triangleq y$ *denote the number of negative votes for argument* a*,*

  whenever $V_{\mathcal{A}}(a) = (x, y)$*,*
  denote the maximum total number of votes for an argument,
- $p_{\mathcal{A}} : 2^{\mathcal{A}} \to 2^{\mathbb{N} \times \mathbb{N}}$ *be a function s.t.* $p_{\mathcal{A}}(\mathcal{B}) = \{V_{\mathcal{A}}(a) \mid a \in \mathcal{B}\}$ *where* $\mathcal{B} \subseteq \mathcal{A}$

- $\tau(a, \mathcal{B}) \triangleq \tau(V_{\mathcal{A}}(a), p_{\mathbb{A}}(\mathcal{B}))$ *denote the social support for an argument a via utilizing a vote aggregation function* τ *(where* $\mathcal{B} \subseteq \mathcal{A}$*),*

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  - $V^{+}_{\mathcal{R}}((a,b)) \triangleq x$ *denote the number of positive votes for an attack relation between arguments a and b,*
  - $V^{-}_{\mathcal{R}}((a,b)) \triangleq y$ *denote the number of negative votes for an attack relation between arguments a and b,*

  whenever $V_{\mathcal{R}}((a,b)) = (x,y)$*,*
  denote the maximum total number of votes for an attack relation,

- $p_{\mathcal{R}} : 2^{\mathcal{R}} \rightarrow 2^{\mathbb{N}\times\mathbb{N}}$ *be a function s.t.* $p_{\mathcal{R}}(\mathcal{T}) = \{V_{\mathcal{R}}((a,b)) \mid (a,b) \in \mathcal{T}\}$ *where* $\mathcal{T} \subseteq \mathcal{R}$*,*

- $\tau((a,b), \mathcal{T}) \triangleq \tau(V_{\mathcal{R}}((a,b)), p_{\mathcal{R}}(\mathcal{T}))$ *denote the social support for an attack relation between arguments a and b via utilizing a vote aggregation function* τ *(where* $\mathcal{T} \subseteq \mathcal{R}$*),*

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$$\bigcurlyvee_{x\in R} x \triangleq (((x_1 \curlyvee x_2) \curlyvee ...) \curlyvee x_n)$$

  $R = \{x_1, x_2, ..., x_n\}$ *denote the aggregation of a multiset of elements of L.*

- *bimax* $: \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$ *be a function s.t.*

$$bimax(x,y) = \begin{cases} x & x \geq y \\ y & otherwise \end{cases}$$

- $max : 2^{\mathbb{N}} \rightarrow \mathbb{N}$ *be a function s.t.*
  $max(X) \triangleq bimax(...bimax(bimax(bimax(x_1, x_2), x_3)...), x_n)$ *where* $x_i \in X$ *for* $i = [1..n]$*.*

Definition 4 (Model)**.** $\mathbb{L}$ *Let* $F = \langle \mathcal{A}, \mathcal{R}, V_{\mathcal{A}}, V_{\mathcal{R}} \rangle$ *be a social argumentation framework,* $\mathcal{S} = \langle L, \curlywedge_1, \curlywedge_2, \curlyvee, \neg, \tau, \rangle$ *be a semantic framework. A* $\mathcal{S}$*-model of F is a total mapping* $M : \mathcal{A} \rightarrow L$ *such that for all* $a \in \mathcal{A}$*,*

$$M(a) = \tau(a) \curlywedge_{\mathcal{A}} \neg \bigcurlyvee_{a_i \in \mathcal{R}^{-}(a)} (\tau((a_i, a)) \curlywedge_{\mathcal{R}} M(a_i))$$

Definition 5. *[Enhanced Vote Aggregation]*
Enhanced vote aggregation function $\tau_e : \mathbb{N} \times \mathbb{N} \times 2^{\mathbb{N}\times\mathbb{N}} \rightarrow [0,1]$ *is the vote aggregation function such that*

$$\tau_e(v^+, v^-, \mathcal{X}) = \begin{cases} 0 & v_{max} = 0 \\ \frac{v^+}{v^+ + v^- + \frac{1}{max(\mathcal{X})}} & otherwise \end{cases}$$