

Definition 1 (Extended social argumentation frameworks). An extended social argumentation framework is a 4-tuple $F = \langle \mathcal{A}, \mathcal{R}, V_{\mathcal{A}}, V_{\mathcal{R}} \rangle$, where

- \mathcal{A} is the set of arguments,
- $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is a binary attack relation between arguments,
- $V_{\mathcal{A}} : \mathcal{A} \rightarrow \mathbb{N} \times \mathbb{N}$ stores the crowd's pro and con votes for each argument.
- $V_{\mathcal{R}} : \mathcal{R} \rightarrow \mathbb{N} \times \mathbb{N}$ stores the crowd's pro and con votes for each attack.

Definition 2. [Vote Aggregation Function] Given a totally ordered set L with top and bottom elements \top, \perp , a vote aggregation function τ over L is any function such that $\tau : \mathbb{N} \times \mathbb{N} \times 2^{\mathbb{N} \times \mathbb{N}} \rightarrow L$.

Definition 3 (Semantic Framework). A semantic framework is a 6-tuple $\langle L, \lambda_1, \lambda_2, \Upsilon, \neg, \tau, \rangle$ where:

- L is a totally ordered set with top and bottom elements \top, \perp , containing all possible valuations of an argument.
- $\lambda_{\mathcal{A}}, \lambda_{\mathcal{R}} : L \times L \rightarrow L$, are two binary algebraic operations used to restrict strengths to given values.
- $\Upsilon : L \times L \rightarrow L$, is a binary algebraic operation on argument valuations used to combine or aggregate valuations and strengths.
- $\neg : L \rightarrow L$ is a unary algebraic operation for computing a restricting value corresponding to a given valuation or strength.
- τ is a vote aggregation function, which aggregates positive and negative votes and the maximum number of total votes into a social support value.

Notation 1. Let $F = \langle \mathcal{A}, \mathcal{R}, V_{\mathcal{A}}, V_{\mathcal{R}} \rangle$ be an ESAF, $\mathcal{S} = \langle L, \lambda_1, \lambda_2, \Upsilon, \neg, \tau, \rangle$ a semantic framework. Then, let

- $\mathcal{R}^-(a) \triangleq \{a_i \in \mathcal{A} : (a_i, a) \in \mathcal{R}\}$ be the set of direct attackers of an argument $a \in \mathcal{A}$,
- $v^r : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{R}$ be a function s.t. $v^r(v^+, v^-) \triangleq \frac{v^+}{v^-}$ where $v^+, v^- \in \mathbb{N}$,
- $v^t : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$ be a function s.t. $v^t(v^+, v^-) \triangleq v^+ + v^-$ where $v^+, v^- \in \mathbb{N}$,
- $- V_{\mathcal{A}}^+(a) \triangleq x$ denote the number of positive votes for argument a ,
 $- V_{\mathcal{A}}^-(a) \triangleq y$ denote the number of negative votes for argument a ,
whenever $V_{\mathcal{A}}(a) = (x, y)$,
denote the maximum total number of votes for an argument,
- $p_{\mathcal{A}} : 2^{\mathcal{A}} \rightarrow 2^{\mathbb{N} \times \mathbb{N}}$ be a function s.t. $p_{\mathcal{A}}(\mathcal{B}) = \{V_{\mathcal{A}}(a) \mid a \in \mathcal{B}\}$ where $\mathcal{B} \subseteq \mathcal{A}$

- $\tau(a, \mathcal{B}) \triangleq \tau(V_{\mathcal{A}}(a), p_{\mathcal{A}}(\mathcal{B}))$ denote the social support for an argument a via utilizing a vote aggregation function τ (where $\mathcal{B} \subseteq \mathcal{A}$),
 - $- V_{\mathcal{R}}^+((a, b)) \triangleq x$ denote the number of positive votes for an attack relation between arguments a and b ,
 - $- V_{\mathcal{R}}^-((a, b)) \triangleq y$ denote the number of negative votes for an attack relation between arguments a and b ,
- whenever $V_{\mathcal{R}}((a, b)) = (x, y)$,
denote the maximum total number of votes for an attack relation,
- $p_{\mathcal{R}} : 2^{\mathcal{R}} \rightarrow 2^{\mathbb{N} \times \mathbb{N}}$ be a function s.t. $p_{\mathcal{R}}(\mathcal{T}) = \{V_{\mathcal{R}}((a, b)) \mid ((a, b)) \in \mathcal{T}\}$ where $\mathcal{T} \subseteq \mathcal{R}$,
 - $\tau((a, b), \mathcal{T}) \triangleq \tau(V_{\mathcal{R}}((a, b)), p_{\mathcal{R}}(\mathcal{T}))$ denote the social support for an attack relation between arguments a and b via utilizing a vote aggregation function τ (where $\mathcal{T} \subseteq \mathcal{R}$),

- $$\bigvee_{x \in R} x \triangleq (((x_1 \vee x_2) \vee \dots) \vee x_n)$$
- $R = \{x_1, x_2, \dots, x_n\}$ denote the aggregation of a multiset of elements of L .
- $bimax : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$ be a function s.t.

$$bimax(x, y) = \begin{cases} x & x \geq y \\ y & \text{otherwise} \end{cases}$$

- $max : 2^{\mathbb{N}} \rightarrow \mathbb{N}$ be a function s.t.
 $max(X) \triangleq bimax(\dots bimax(bimax(bimax(x_1, x_2), x_3) \dots), x_n)$ where $x_i \in X$ for $i = [1..n]$.

Definition 4 (Model). \mathbb{L} Let $F = \langle \mathcal{A}, \mathcal{R}, V_{\mathcal{A}}, V_{\mathcal{R}} \rangle$ be a social argumentation framework, $\mathcal{S} = \langle L, \lambda_1, \lambda_2, \vee, \neg, \tau, \rangle$ be a semantic framework. A \mathcal{S} -model of F is a total mapping $M : \mathcal{A} \rightarrow L$ such that for all $a \in \mathcal{A}$,

$$M(a) = \tau(a) \lambda_{\mathcal{A}} \neg \bigvee_{a_i \in \mathcal{R}^-(a)} (\tau((a_i, a)) \lambda_{\mathcal{R}} M(a_i))$$

Definition 5. [Enhanced Vote Aggregation]

Enhanced vote aggregation function $\tau_e : \mathbb{N} \times \mathbb{N} \times 2^{\mathbb{N} \times \mathbb{N}} \rightarrow [0, 1]$ is the vote aggregation function such that

$$\tau_e(v^+, v^-, \mathcal{X}) = \begin{cases} 0 & v_{max} = 0 \\ \frac{v^+}{v^+ + v^- + \frac{1}{max(\mathcal{X})}} & \text{otherwise} \end{cases}$$