Definition 1 (Extended social argumentation frameworks)**.** *An* extended social argumentation framework *is a 4-tuple $F = \langle \mathcal{A}, \mathcal{R}, \mathcal{O}, V \rangle$, where*

- *$\mathcal{A}$ is the set of arguments,*
- *$\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is a binary attack relation between arguments,*
- *$\mathcal{O} = \mathcal{A} \cup \mathcal{R}$ is the set of objects, composed by the union of the sets of arguments and attack relations,*
- *$V : \mathcal{O} \to \mathbb{N} \times \mathbb{N}$ stores the crowd's pro and con votes for each object.*

Definition 2. *[Vote Aggregation Function] Given a totally ordered set L with top and bottom elements $\top$, $\bot$, a vote aggregation function τ over L is any function such that $\tau : \mathcal{O} \times 2^{\mathcal{O}} \to L$.*

Definition 3 (Semantic Framework)**.** *A semantic framework is a 6-tuple $\langle L, \curlywedge_{\mathcal{A}}, \curlywedge_{\mathcal{R}}, \curlyvee, \neg, \tau \rangle$ where:*

- *L is a totally ordered set with top and bottom elements $\top$, $\bot$, containing all possible valuations of an argument.*
- *$\curlywedge_{\mathcal{A}}, \curlywedge_{\mathcal{R}} : L \times L \to L$, are two binary algebraic operations used to restrict strengths to given values.*
- *$\curlyvee : L \times L \to L$, is a binary algebraic operation on argument valuations used to combine or aggregate valuations and strengths.*
- *$\neg : L \to L$ is a unary algebraic operation for computing a restricting value corresponding to a given valuation or strength.*
- *τ is a vote aggregation function which, given the votes, determines the social support of an object within a set of objects.*

Notation 1. *Let $F = \langle \mathcal{A}, \mathcal{R}, \mathcal{O}, V \rangle$ be an ESAF, $\mathcal{S} = \langle L, \curlywedge_{\mathcal{A}}, \curlywedge_{\mathcal{R}}, \curlyvee, \neg, \tau \rangle$ a semantic framework. Then, let*

- *$\mathcal{R}^{-}(a) \triangleq \{a_i \in \mathcal{A} : (a_i, a) \in \mathcal{R}\}$ be the set of direct attackers of an argument $a \in \mathcal{A}$,*
- *$v^r : \mathbb{N} \times \mathbb{N} \to \mathbb{R}$ be a function s.t. $v^r(x, y) \triangleq \frac{x}{y}$ where $x, y \in \mathbb{N}$,*
- *$v^t : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ be a function s.t. $v^t(x, y) \triangleq x + y$ where $x, y \in \mathbb{N}$,*
-   - *$V_{\mathcal{A}}^{+}(a) \triangleq x$ denote the number of positive votes for argument a,*
    - *$V_{\mathcal{A}}^{-}(a) \triangleq y$ denote the number of negative votes for argument a,*

    whenever $V_{\mathcal{A}}(a) = (x, y)$,
- *$V_{\mathcal{A}} : 2^{\mathcal{A}} \to 2^{\mathbb{N} \times \mathbb{N}}$ be a function s.t. $V_{\mathcal{A}}(\mathcal{A}') = \{V_{\mathcal{A}}(a) \mid a \in \mathcal{A}'\}$,*

- $\tau(a, \mathcal{A}') \triangleq \tau(V_{\mathcal{A}}(a), V_{\mathcal{A}}(\mathcal{A}'))$ *denote the social support for an argument* a*, within a set of arguments* A'*, via utilizing a vote aggregation function* τ*,*

- - $V^{+}_{\mathcal{R}}((a, b)) \triangleq x$ *denote the number of positive votes for an attack relation between arguments* a *and* b*,*
  - $V^{-}_{\mathcal{R}}((a, b)) \triangleq y$ *denote the number of negative votes for an attack relation between arguments* a *and* b*,*

  whenever $V_{\mathcal{R}}((a, b)) = (x, y)$*,*

- $V_{\mathcal{R}} : 2^{\mathcal{R}} \to 2^{\mathbb{N}\times\mathbb{N}}$ *be a function s.t.* $V_{\mathcal{R}}(\mathcal{R}') = \{V_{\mathcal{R}}((a, b)) \mid (a, b) \in \mathcal{R}'\}$*,*

- $\tau((a, b), \mathcal{R}') \triangleq \tau(V_{\mathcal{R}}((a, b)), V_{\mathcal{R}}(\mathcal{R}'))$ *denote the social support for an attack relation between arguments* a *and* b*, within a set of attack relations* R'*, via utilizing a vote aggregation function* τ*,*

- $$\bigcurlyvee_{x \in X} x \triangleq (((x_1 \curlyvee x_2) \curlyvee ...) \curlyvee x_n)$$

  $X = \{x_1, x_2, ..., x_n\}$ *denote the aggregation of a multiset of elements of* L*.*

- $max : 2^{\mathbb{N}} \to \mathbb{N}$ *be a function s.t. it returns the maximum value amongst the natural numbers from the non-empty multiset given as the input.*

- $\omega : 2^{\mathbb{N}\times\mathbb{N}} \to 2^{\mathbb{N}}$ *be a function s.t.* $\omega(X) = \{a_1 + a_2 \mid (a_1, a_2) \in X\}$*,*

Definition 4 (Model)**.** *Let* $F = \langle \mathcal{A}, \mathcal{R}, V_{\mathcal{A}}, V_{\mathcal{R}} \rangle$ *be a social argumentation framework,* $\mathcal{S} = \langle L, \curlywedge_{\mathcal{A}}, \curlywedge_{\mathcal{R}}, \curlyvee, \neg, \tau \rangle$ *be a semantic framework. An* $\mathcal{S}$*-model of* F *is a total mapping* $M : \mathcal{A} \to L$ *such that for all* $a \in \mathcal{A}$*,*

$$M(a) = \tau(a, \mathcal{A}) \curlywedge_{\mathcal{A}} \neg \bigcurlyvee_{a_i \in \mathcal{R}^{-}(a)} \left(\tau\left((a_i, a), \mathcal{R}\right) \curlywedge_{\mathcal{R}} M\left(a_i\right)\right)$$

Definition 5. *[Enhanced Vote Aggregation]*
Enhanced vote aggregation function $\tau_e : \mathbb{N} \times \mathbb{N} \times 2^{\mathbb{N}\times\mathbb{N}} \to [0, 1]$ *is the vote aggregation function such that*

$$\tau_e(v^+, v^-, X) = \begin{cases} 0 & \forall x \in X,\ x = (0, 0) \\ \frac{v^+}{v^+ + v^- + \frac{1}{max(\omega(X))}} & otherwise \end{cases}$$

Property 1. *[Absolute argument freeness]*
Let τ *be a a vote aggregation function given* L *and* $X = \{(x_1, x_2) \mid x_1, x_2 \in X',$ $X' \subseteq \mathbb{N}\}$*. We say that* τ *is 'absolute argument free' if*

$$\forall n_1, n_2 \in \mathbb{N},\ \tau(n_1, n_2, X) \neq \top.$$

Property 2. *[Precedence of the vote ratio]*
Let τ *be a a vote aggregation function given* L *and* $X = \{(x_1, x_2) \mid x_1, x_2 \in X',$ $X' \subseteq \mathbb{N}\}$*. We say that* τ *is 'vote ratio precedent' if*

$$\forall n_1, n_2, n_3, n_4 \in \mathbb{N},$$
$$(v^r(n_1, n_2) \geq v^r(n_3, n_4)) \implies (\tau(n_1, n_2, X) \geq \tau(n_3, n_4, X)).$$

Property 3. *[Precedence of the total number of votes]*
Let τ be a a vote aggregation function given L and $X = \{(x_1, x_2) \mid x_1, x_2 \in X^{'}$, $X^{'} \subseteq \mathbb{N}\}$. We say that τ is 'total votes precedent' if

$$\forall n_1, n_2, n_3, n_4 \in \mathbb{N},$$
$$((v^r(n_1, n_2) = v^r(n_3, n_4)) \wedge (v^t(n_1, n_2) > (v^t(n_3, n_4))) \implies$$
$$(\tau(n_1, n_2, X) > \tau(n_3, n_4, X)).$$

Proposition 1. *Enhanced Vote Aggregation function respects Property 1.*

Proof. Let $F = \langle \mathcal{A}, \mathcal{R}, V_{\mathcal{A}}, V_{\mathcal{R}} \rangle$ be an extended social argumentation framework with a semantic framework $\mathcal{S} = \langle L, \curlywedge_{\mathcal{A}}, \curlywedge_{\mathcal{R}}, \curlyvee, \neg, \tau_e \rangle$ and $X = \{(x_1, x_2) \mid x_1, x_2 \in X^{'}, X^{'} \subseteq \mathbb{N}\}$ and two arbitrary naturals $n_1, n_2 \in \mathbb{N}$.

We consider the two main cases for the enhanced vote aggregation function with respect to the multiset X.

Firstly, if X is an empty set or X does not have any elements other than the pair $(0, 0)$, by definition $\tau_e(n_1, n_2, X) = 0 < 1$.

We know that for an arbitrary multiset of naturals Y, $max(X) \in \mathbb{N}^+$, and consequently $\frac{1}{max(X)} > 0$. Since the denominator of the term in the social support function is simply the sum of the numerator and some $r \in R^+$, the numerator is strictly less than the denominator. Hence it follows that:

$$\tau_e(n_1, n_2, X) = \frac{n_1}{n_1 + n_2 + \frac{1}{v_{max}}} < 1.$$

□

Conjecture 1. *Enhanced Vote Aggregation function respects Property 2.*

Proposition 2. *Vote Aggregation function[1] does not respect Property 2.*

Proof. It's sufficient to display a counter example.

Let $\epsilon = 0.1, v_1^+ = 1, v_1^- = 1, v_2^+ = 499, v_2^- = 501$.

Since $v^r(v_1^+, v_1^-) \geq v^r(v_2^+, v_2^-)$ via the property the following should hold $\tau(v_1^+, v_1^-) \geq \tau(v_2^+, v_2^-)$.

However $\tau(v_1^+, v_1^-) = \tau(1, 1) \cong 0.4761$ and $\tau(v_2^+, v_2^-) = \tau(499, 501) \cong 0.4989$. Hence, $\tau(v_1^+, v_1^-) < \tau(v_2^+, v_2^-)$.

We conclude a contradiction. □

Conjecture 2. *Enhanced Vote Aggregation function respects Property 3.*

References

[1] S.Egilmez, J. Martins, and J. Leite. Extending social abstract argumentation with votes on attacks. In *Procs. of TAFA 2013*. TBA, 2013.

Pointers for discussion:

- Rationale for the potential property regarding $M(x) > 0.5$.
- Problem concerning comparing the new&old vote aggregation functions wrt. the properties.
  e.g. Proposition 2, old one missing the third argument.
- Type matching in vote aggregation functions when the domain is a bag consisting the union of $\mathcal{A}$ and $\mathcal{R}$.