Definition 1 (Extended social argumentation frameworks). *An extended social argumentation framework is a 4-tuple $F = \langle \mathcal{A}, \mathcal{R}, V_\mathcal{A}, V_\mathcal{R} \rangle$, where*

- *$\mathcal{A}$ is the set of arguments,*

- *$\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is a binary attack relation between arguments,*

- *$V_\mathcal{A} : \mathcal{A} \to \mathbb{N} \times \mathbb{N}$ stores the crowd's pro and con votes for each argument.*

- *$V_\mathcal{R} : \mathcal{R} \to \mathbb{N} \times \mathbb{N}$ stores the crowd's pro and con votes for each attack.*

Definition 2. *[Vote Aggregation Function] Given a totally ordered set L with top and bottom elements $\top$, $\bot$, a vote aggregation function τ over L is any function such that $\tau : \mathbb{N} \times \mathbb{N} \times 2^{\mathbb{N} \times \mathbb{N}} \to L$.*

Definition 3 (Semantic Framework). *A semantic framework is a 6-tuple $\langle L, \curlywedge_\mathcal{A}, \curlywedge_\mathcal{R}, \curlyvee, \neg, \tau \rangle$ where:*

- *L is a totally ordered set with top and bottom elements $\top$, $\bot$, containing all possible valuations of an argument.*

- *$\curlywedge_\mathcal{A}, \curlywedge_\mathcal{R} : L \times L \to L$, are two binary algebraic operations used to restrict strengths to given values.*

- *$\curlyvee : L \times L \to L$, is a binary algebraic operation on argument valuations used to combine or aggregate valuations and strengths.*

- *$\neg : L \to L$ is a unary algebraic operation for computing a restricting value corresponding to a given valuation or strength.*

- *τ is a vote aggregation function which given some context, aggregates positive and negative votes into a social support value.*

Notation 1. *Let $F = \langle \mathcal{A}, \mathcal{R}, V_\mathcal{A}, V_\mathcal{R} \rangle$ be an ESAF, $\mathcal{S} = \langle L, \curlywedge_\mathcal{A}, \curlywedge_\mathcal{R}, \curlyvee, \neg, \tau \rangle$ a semantic framework. Then, let*

- *$\mathcal{R}^-(a) \triangleq \{a_i \in \mathcal{A} : (a_i, a) \in \mathcal{R}\}$ be the set of direct attackers of an argument $a \in \mathcal{A}$,*

- *$v^r : \mathbb{N} \times \mathbb{N} \to \mathbb{R}$ be a function s.t. $v^r(x,y) \triangleq \frac{x}{y}$ where $x, y \in \mathbb{N}$,*

- *$v^t : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ be a function s.t. $v^t(x,y) \triangleq x + y$ where $x, y \in \mathbb{N}$,*

-     – *$V_\mathcal{A}^+(a) \triangleq x$ denote the number of positive votes for argument a,*
      – *$V_\mathcal{A}^-(a) \triangleq y$ denote the number of negative votes for argument a,*
      whenever $V_\mathcal{A}(a) = (x,y)$,

- *$V_\mathcal{A} : 2^\mathcal{A} \to 2^{\mathbb{N} \times \mathbb{N}}$ be a function s.t. $V_\mathcal{A}(\mathcal{A}') = \{V_\mathcal{A}(a) \mid a \in \mathcal{A}'\}$,*

- $\tau(a, \mathcal{A}') \triangleq \tau(V_\mathcal{A}(a), V_\mathcal{A}(\mathcal{A}'))$ denote the social support for an argument a, within a set of arguments $\mathcal{A}'$, via utilizing a vote aggregation function τ,

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  - $V_\mathcal{R}^+((a,b)) \triangleq x$ denote the number of positive votes for an attack relation between arguments a and b,
    - $V_\mathcal{R}^-((a,b)) \triangleq y$ denote the number of negative votes for an attack relation between arguments a and b,

  whenever $V_\mathcal{R}((a,b)) = (x,y)$,

- $V_\mathcal{R} : 2^\mathcal{R} \to 2^{\mathbb{N} \times \mathbb{N}}$ be a function s.t. $V_\mathcal{R}(\mathcal{R}') = \{V_\mathcal{R}((a,b)) \mid (a,b) \in \mathcal{R}'\}$,

- $\tau((a,b), \mathcal{R}') \triangleq \tau(V_\mathcal{R}((a,b)), V_\mathcal{R}(\mathcal{R}'))$ denote the social support for an attack relation between arguments a and b, within a set of attack relations $\mathcal{R}'$, via utilizing a vote aggregation function τ,

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$$\bigvee_{x \in X} x \triangleq (((x_1 \curlyvee x_2) \curlyvee ...) \curlyvee x_n)$$

$X = \{x_1, x_2, ..., x_n\}$ denote the aggregation of a multiset of elements of L.

- $max : 2^\mathbb{N} \to \mathbb{N}$ be a function s.t. it returns the maximum value amongst the natural numbers from the non-empty set given as the input.

- $\omega : 2^{\mathbb{N} \times \mathbb{N}} \to 2^\mathbb{N}$ be a function s.t. $\omega(\mathcal{X}) = \{a_1 + a_2 \mid (a_1, a_2) \in \mathcal{X}\}$,

Definition 4 (Model). Let $F = \langle \mathcal{A}, \mathcal{R}, V_\mathcal{A}, V_\mathcal{R} \rangle$ be a social argumentation framework, $\mathcal{S} = \langle L, \curlywedge_\mathcal{A}, \curlywedge_\mathcal{R}, \curlyvee, \neg, \tau \rangle$ be a semantic framework. An $\mathcal{S}$-model of F is a total mapping $M : \mathcal{A} \to L$ such that for all $a \in \mathcal{A}$,

$$M(a) = \tau(a, \mathcal{A}) \curlywedge_\mathcal{A} \neg \bigvee_{a_i \in \mathcal{R}^-(a)} (\tau((a_i, a), \mathcal{R}) \curlywedge_\mathcal{R} M(a_i))$$

Definition 5. *[Enhanced Vote Aggregation]*
Enhanced vote aggregation function $\tau_e : \mathbb{N} \times \mathbb{N} \times 2^{\mathbb{N} \times \mathbb{N}} \to [0, 1]$ is the vote aggregation function such that

$$\tau_e(v^+, v^-, \mathcal{X}) = \begin{cases} 0 & v_{max} = 0 \\ \frac{v^+}{v^+ + v^- + \frac{1}{max(\omega(\mathcal{X}))}} & otherwise \end{cases}$$