Report on ongoing research

1 Main

Definition 1 (Extended social argumentation frameworks)**.** *An* extended social argumentation framework *is a 4-tuple $F = \langle \mathcal{A}, \mathcal{R}, V_{\mathcal{A}}, V_{\mathcal{R}} \rangle$, where*

- *$\mathcal{A}$ is the set of arguments,*
- *$\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ is a binary attack relation between arguments,*
- *$V_{\mathcal{A}} : \mathcal{A} \to \mathbb{N} \times \mathbb{N} \times \mathbb{N}$ stores the crowd's pro and con votes for each argument together with the maximum number of votes for an argument in the system.*
- *$V_{\mathcal{R}} : \mathcal{R} \to \mathbb{N} \times \mathbb{N}$ stores the crowd's pro and con votes for each attack.*

Definition 2 (Semantic Framework)**.** *A semantic framework is a 6-tuple $\langle L, \curlywedge_1, \curlywedge_2, \curlyvee, \neg, \tau, \omega \rangle$ where:*

- *L is a totally ordered set with top and bottom elements $\top$, $\bot$, containing all possible valuations of an argument.*
- *$\curlywedge_{\mathcal{A}}, \curlywedge_{\mathcal{R}} : L \times L \to L$, are two binary algebraic operations used to restrict strengths to given values.*
- *$\curlyvee : L \times L \to L$, is a binary algebraic operation on argument valuations used to combine or aggregate valuations and strengths.*
- *$\neg : L \to L$ is a unary algebraic operation for computing a restricting value corresponding to a given valuation or strength.*
- *$\tau : \mathbb{N} \times \mathbb{N} \times \mathbb{N}^+ \to L$ is a function that aggregates positive and negative votes and the maximum number of total votes into a* social support *value.*
- *$\omega : (\mathbb{N} \times \mathbb{N})^n \to \mathbb{N}^+$ is a function given a list of tuples of positive and negative votes that computes the maximum total number of votes amongst the tuples, where $n \in \mathbb{N}^+$.*

Definition 3. *[Vote Aggregation] Given a totally ordered set L with top and bottom elements $\top$, $\bot$, a vote aggregation function τ over L is any function such that $\tau : \mathbb{N} \times \mathbb{N} \times \mathbb{N}^+ \to L$.*

Notation 1. *Let* $F = \langle \mathcal{A}, \mathcal{R}, V_{\mathcal{A}}, V_{\mathcal{R}} \rangle$ *be an ESAF,* $\mathcal{S} = \langle L, \curlywedge_1, \curlywedge_2, \curlyvee, \neg, \tau, \omega \rangle$ *a semantic framework and*

- $\mathcal{R}^{\text{-}}(a) \triangleq \{a_i \in \mathcal{A} : (a_i, a) \in \mathcal{R}\}$ *be the set of direct attackers of an argument* $a \in \mathcal{A}$*,*
- $V_{\mathcal{A}}^{+}(a) \triangleq x$ *denote the number of positive votes for argument* a*,*
- $V_{\mathcal{A}}^{-}(a) \triangleq y$ *denote the number of negative votes for argument* a*,*
- $v_{max} \triangleq \omega(\mathcal{A})$ *denote the parameter for the maximum total number of votes for an argument (attack relations are handled similarly).*
- $\tau(a, v_{max}) \triangleq \tau(V_{\mathcal{A}}(a), v_{max}) = \tau(x, y, v_{max})$ *denote the social support for an argument* a *via utilizing a vote aggregation function* τ *(attack relations are handled similarly),*
- $v^{r}(a) = \frac{V_{\mathcal{A}}^{+}(a)}{V_{\mathcal{A}}^{-}(a)}$ *be a function that computes the ratio of positive votes to negative votes for argument* a*,*
- $v^{t}(a) = V_{\mathcal{A}}^{+}(a) + V_{\mathcal{A}}^{-}(a)$ *be a function that computes the total number of votes for argument* a*,*
- $$\bigcurlyvee_{x \in R} x \triangleq (((x_1 \curlyvee x_2) \curlyvee ...) \curlyvee x_n)$$

  $R = \{x_1, x_2, ..., x_n\}$ *denote the aggregation of a multiset of elements of* L*.*

Definition 4 (Model)**.** *Let* $F = \langle \mathcal{A}, \mathcal{R}, V_{\mathcal{A}}, V_{\mathcal{R}} \rangle$ *be a social argumentation framework,* $\mathcal{S} = \langle L, \curlywedge_1, \curlywedge_2, \curlyvee, \neg, \tau, \omega \rangle$ *be a semantic framework. A* $\mathcal{S}$*-model of* F *is a total mapping* $M : \mathcal{A} \to L$ *such that for all* $a \in \mathcal{A}$*,*

$$M(a) = \tau(a, v_{max_1}) \curlywedge_{\mathcal{A}} \neg \bigcurlyvee_{a_i \in \mathcal{R}^{\text{-}}(a)} \left(\tau\left((a_i, a), v_{max_2}\right) \curlywedge_{\mathcal{R}} M\left(a_i\right)\right)$$

Definition 5. *[Enhanced Vote Aggregation]*
Enhanced vote aggregation function $\tau_e : \mathbb{N} \times \mathbb{N} \times \mathbb{N}^{+} \to [0, 1]$ *is a vote aggregation function such that*

$$\tau_e\left(v^{+}, v^{-}, v_{max}\right) = \frac{v^{+}}{v^{+} + v^{-} + \frac{1}{v_{max}}}$$

Property 1. *[Absolute argument freeness]*
Let τ *be a a vote aggregation function. We say that* τ *is 'absolute argument free' if*

$$\forall n_1, n_2, n_3 \in \mathbb{N},\ \tau\left(n_1, n_2, n_3\right) \neq \top.$$

The gist of the property can be captured in a verbal context with the following sentence: *No argument enjoys perfect Social Support.*

Proof. Trivial.
Suppose an arbitrary $a \in \mathcal{A}$ of an extended social argumentation framework $\mathcal{F}$ with some well-behaved semantics $\mathcal{S}$:
If $v^+ = 0$ then $\tau(a) = \tau(v^+, v^-, v_{max}) = 0 \leq 1$.
Else if $v^+ \neq 0$, then $v_{max} \neq 0$ and since $v_{max} \in \mathcal{Z}^+$, then $\frac{1}{v_{max}} > 0$. Since denominator equals to the addition of the numerator and some $r \in R^+$, denominator is bigger than the numerator and thus
$\frac{v^+}{v^+ + v^- + \frac{1}{v_{max}}} = \tau(v^+, v^-, v_{max}) \leq 1$.

$\square$

Property 2. *[Precedence of the vote ratio]*
Let τ be a a vote aggregation function. We say that τ is 'vote ratio precedent' if

$$\forall n_1, n_2, n_3, n_4, n_5 \in \mathbb{N},$$
$$(v^r(n_1, n_2) \geq v^r(n_3, n_4)) \implies (\tau(n_1, n_2, n_5) \geq \tau(n_3, n_4, n_5)).$$
just for convenience:
$$(v^r(a_1) \geq v^r(a_2)) \implies (\tau(a_1, v_{max}) \geq \tau(a_2, v_{max})).$$

The gist of the property can be captured in a verbal context with the following sentence: *If the value of the ratio of positive votes to the total amount of votes is higher for an argument a than an argument b; a's social support value exceeds the social support of b.*

Property 3. *[Precedence of the total number of votes]*
Let τ be a a vote aggregation function. We say that τ is 'vote ratio precedent' if

$$\forall n_1, n_2, n_3, n_4, n_5 \in \mathbb{N},$$
$$((v^r(n_1, n_2) = v^r(n_3, n_4)) \wedge (v^t(n_1, n_2) \geq (v^t(n_3, n_4))) \implies$$
$$(\tau((n_1, n_2, n_5) \geq \tau(n_3, n_4, n_5)).$$
just for convenience:
$$((v^r(a_1) = v^r(a_2)) \wedge (v^t(a_1) \geq (v^t(a_2))) \implies (\tau(a_1, v_{max}) \geq \tau(a_2, v_{max})).$$

The gist of the property can be captured in a verbal context with the following sentence: *When the ratios are equal, the function should return a higher social support value for the one with the higher number of total votes.*