PROJECT 1: MARTINGALE

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1 EXPERIMENT 1

First, we implement the roulette with American wheel. According to google, the probability of winning any color with the American wheel is 0.474. The idea in our implementation is that if we lose this time, we will win most likely the next time. Therefore, if we lose, we double the bet. Also, if we win this time, most likely we will lose next time. Therefore, we bet only 1 USD if we win. The idea behind this is that winning twice consecutively has a probability of 0.474 × 0.474 = 0.22. Further, losing twice consecutively has a probability of 0.526 × 0. 526 = 0.28. Therefore, we wouldn't have the same outcome consecutively most of the time.

Figure 1 shows the winnings for every episode. As requested, x-axis is limited in between [0,300]. Similarly, y-axis is bounded between [-260,100]. We can see that the winnings value goes to very negative values occasionally, because we are doubling our bets every time we lose. The dips occur because of losing several times consecutively. However, the next time we win, the bet is very high, and the winnings go back to positive. By 160 spins, we achieve 80 USD and after that we do forward fill without playing.

Figure 2 shows the mean value for every spin. Mean is taken along the columns i.e. episodes. Figure 2 also plots the mean +/- std. We can see that mean and std values can vary significantly if there are consecutive losses in a particular episode. However, eventually std declines to 0 and the winning converge to 80 USD for 1000 episodes.

Figure 3 plots the median value +/- std. We can see that median value is much smoother because the outliers' effect is much smaller.

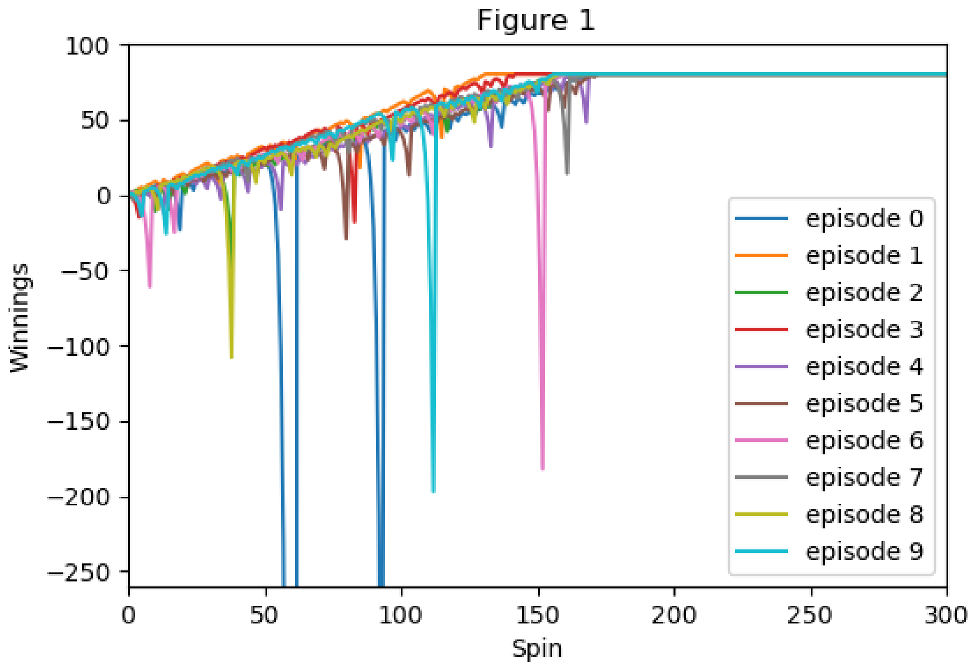

Figure 1 — Winning values for 10 episodes numbered from 0 to 9.

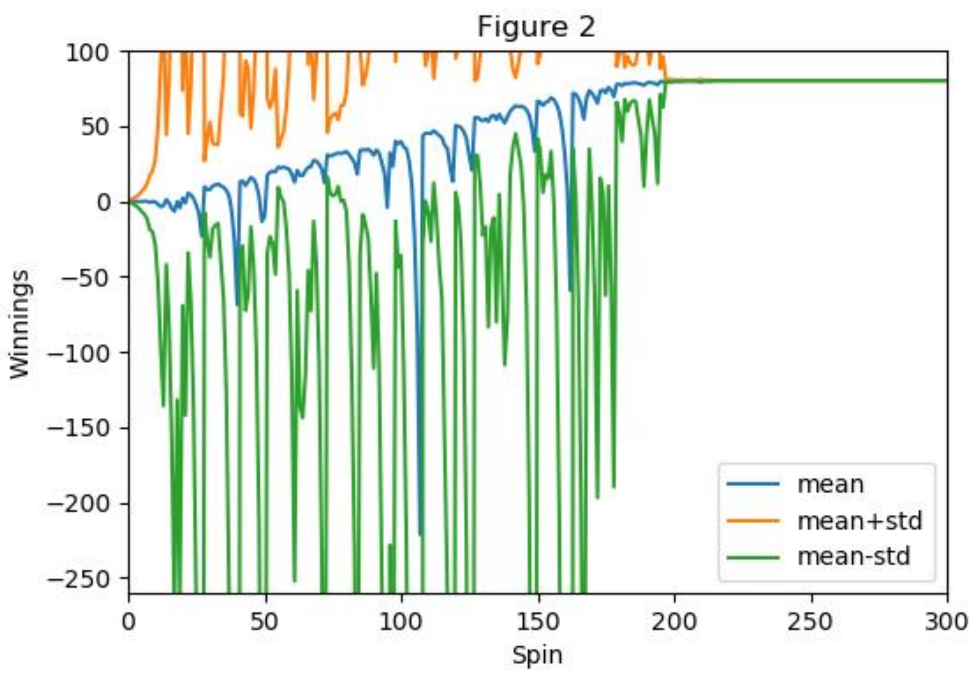

Figure 2 — Mean, mean+std and mean-std winning values for 300 spins for 1000 episodes.

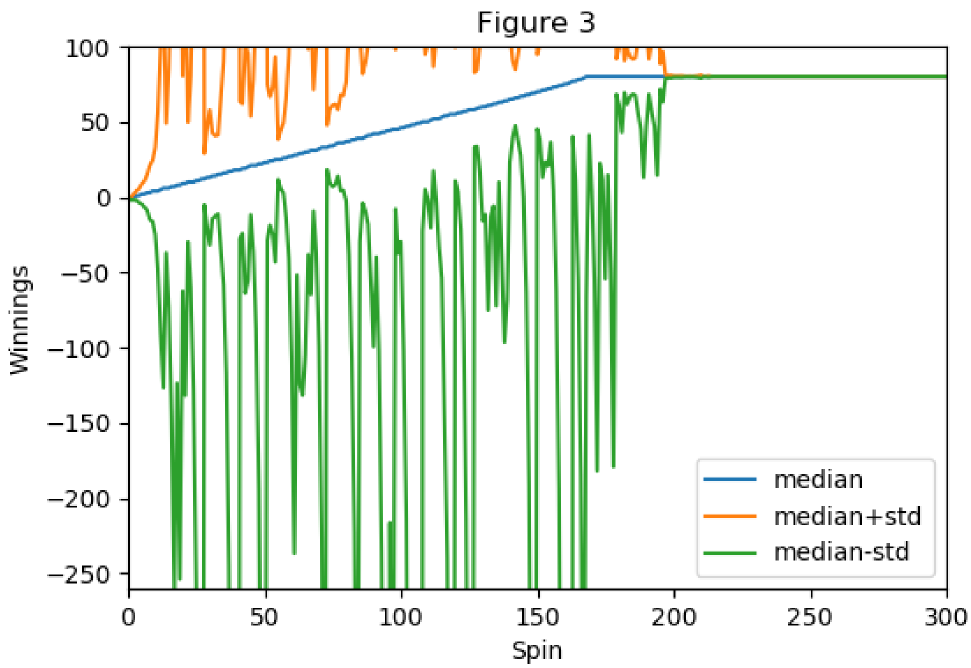

Figure 3 — Median, median+std and median-std winning values for 300 spins for 1000 episodes.

2 EXPERIMENT 2

This time, we quit as soon as we run out of 256 USD. For that reason, we do not see an eventual convergence to 80 USD. After several dozens of spins, the winnings value either goes to 80 USD or to -256 USD. Figure 4 shows that the mean eventually goes to -36.256 USD. Standard deviation doesn't converge to 0 because we have either 80 USD or -256 USD. Actually, 346 episodes converge to -256 USD and 654 episodes converge to 80 USD.

Figure 5 shows the median value which is 80 USD because it is the most common value. However, standard deviation is very high because the individual values are very far away from the mean.

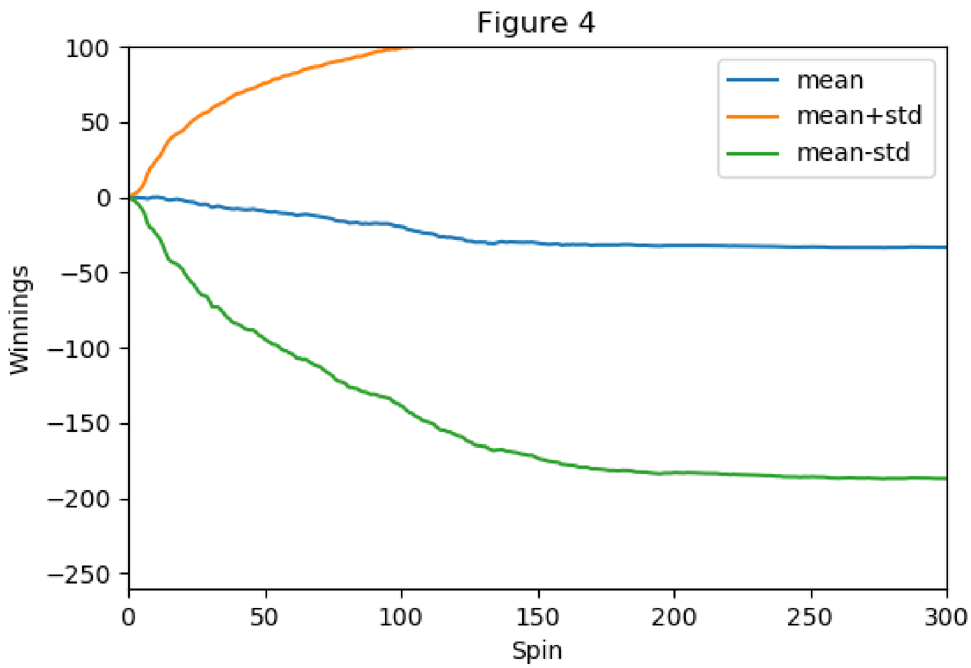

Figure 4 — Mean, mean+std and mean-std winning values for 300 spins for 1000 episodes.

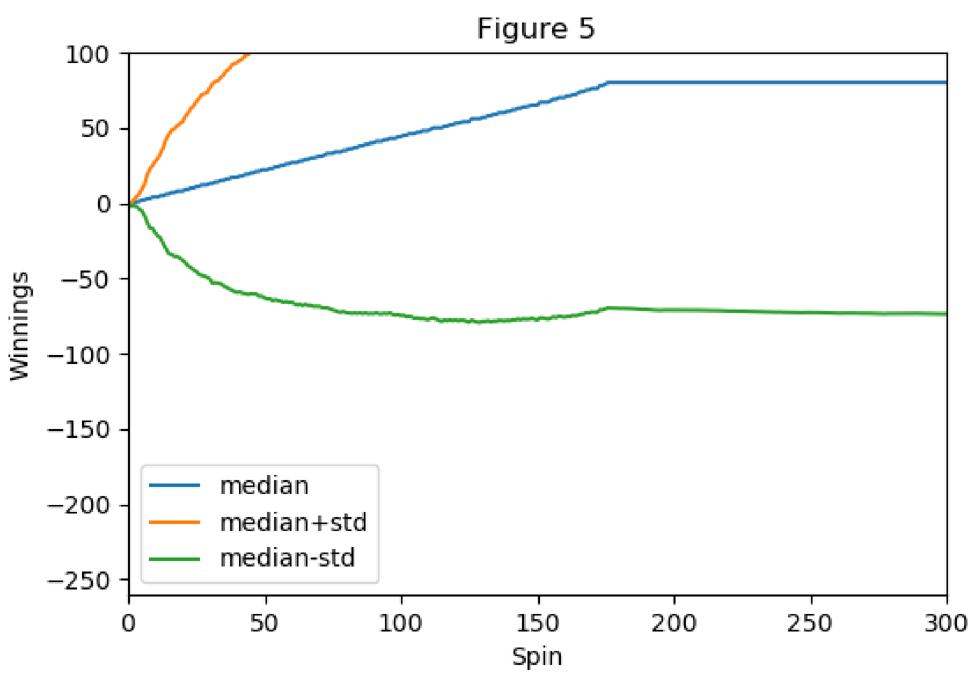

Figure 5 — Median, median+std and median-std winning values for 300 spins for 1000 episodes.

3 QUESTIONS

I have answered these questions mostly based on the empirical results as suggested in the office hour.

- **Question 1**: In Experiment 1, based on the experiment results calculate and provide the estimated probability of winning $80 within 1000 sequential bets. Thoroughly explain your reasoning for the answer using the experiment output. Your explanation should NOT be based on estimates from visually inspecting your plots.

The estimated probability of winning $80 is 100%. The reason is when we fail, we double the bet until we win. The moment we win, we cover all the losses and make some money. Therefore, all the episodes reach $80 by 160 spins. If we lose the first spin and win the next spin, then we make $1 every 2 spins. Therefore, it makes sense to make $80 in 160 spins. I can see in the numpy array that all the episodes go to $80.

- **Question 2**: In Experiment 1, what is the estimated expected value of winnings after 1000 sequential bets? Thoroughly explain your reasoning for the answer.

The estimated expected value after 1000 spins is $80 because the estimated probability of winning $80 is 1 as explained above and as can be seen in the empirical results.

- **Question 3**: In Experiment 1, do the upper standard deviation line (mean + stdev) and lower standard deviation line (mean – stdev) reach a maximum (or minimum) value and then stabilize? Do the standard deviation lines converge as the number of sequential bets increases? Thoroughly explain why it does or does not.

Standard deviation fluctuates a lot because sometimes the loss is very big after a few consecutive losses. However, standard deviation eventually reduces to 0 because all the episodes converge to $80.

- **Question 4**: In Experiment 2, based on the experiment results calculate and provide the estimated probability of winning $80 within 1000 sequential bets. Thoroughly explain your reasoning for the answer using the experiment output. Your explanation should NOT be based on estimates from visually inspecting your plots.

Out of 1000 episodes, 654 of them reach \$80. Therefore, the probability of winning \$80 within 1000 sequential bets is 65.4%

- **Question 5**: In Experiment 2, what is the estimated expected value of winnings after 1000 sequential bets? Thoroughly explain your reasoning for the answer.

654 episodes go to \$80 and 346 episodes goes to \$(-256). Therefore, the expected value is 0.654×80-0.346×256=-36.256. The estimated expected value is \$(-36.256).

- **Question 6**: In Experiment 2, do the upper standard deviation line (mean + stdev) and lower standard deviation line (mean – stdev) reach a maximum (or minimum) value and then stabilize? Do the standard deviation lines converge as the number of sequential bets increases? Thoroughly explain why it does or does not.

STD starts at 0, then goes up very fast and finally converges to 159.83. Final std is very high because we either go to a very large negative value or to a positive value.

- **Question 7**: What are some of the benefits of using expected values when conducting experiments instead of simply using the result of one specific random episode?

Individually, experiments can go in completely different directions and therefore the results will not be reliable. However, carrying out multiple experiments will represent the stochastic nature of the problem. Therefore, the results will be more reliable.