Weight of Statistical Evidence

Detection and Correction of Publication Bias

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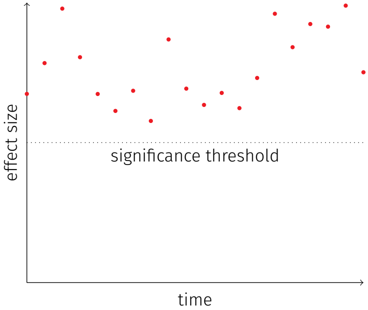
Master's Programme in Computational Science and Engineering

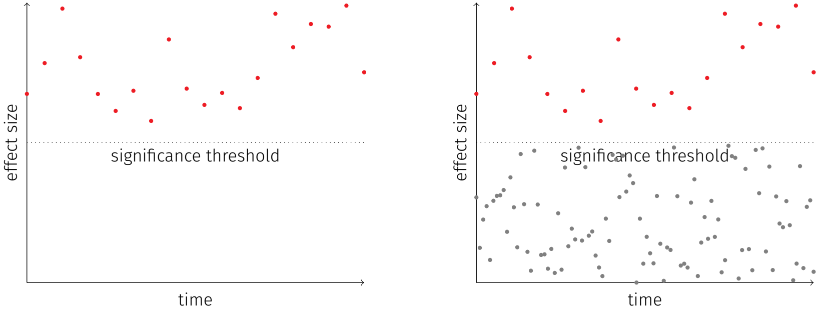
The Woozle effect

Pooh and Piglet tracking down the elusive Woozle (Image: Ernest H. Shepard)

The Woozle effect—why care?



The Woozle effect—why care?



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Analysing and Testing Evidence

Hypothesis Testing

$$H_0 : \mu \leq \mu_0$$
$$H_1 : \mu > \mu_0$$

Making evidence comparable

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E_1 &: \ V_n \text{ is monotonically increasing;} \\
E_2 &: \ V_n \sim \mathcal{N}(\tau, 1); \\
E_3 &: \ \mathrm{Var}[V_n] = 1; \\
E_4 &: \ \mathrm{E}_\mu[V_n] = \tau(\mu) \text{ is monotonically increasing in } \mu \text{ from} \\
&\quad\ \tau(0) = 0.
\end{aligned}$$

Example case: difference in means

Let $X \sim \mathcal{N}(\mu, \sigma^2)$ with μ and σ^2 unknown.

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with

$$\hat{\mu} = \bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i; \quad \hat{\sigma}^2 = s_n^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X}_N)^2$$

Variance stabilising transformation

If n is large enough, T_n fulfills all properties E_1 to E_4 by virtue of the central limit theorem.

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For small n, however, E_2 and E_3 are violated.

Solution: Transform T_n!

Variance stabilising transformation

$V_n = h_n(T_n) = \sqrt{2n}\,\sinh^{-1}(T_n/\sqrt{2n})$ yields improved approximation to standard normal distribution Kulinskaya, Morgenthaler, and Staudte (2008).

Variance stabilising transformation

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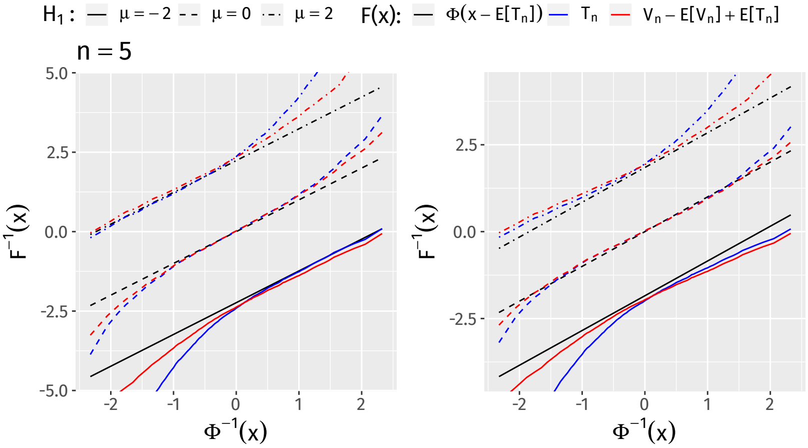
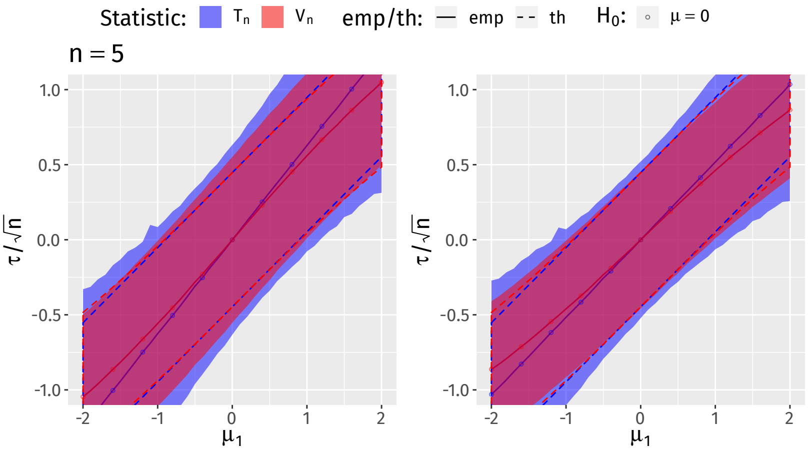
Further improvement by finite sample correction:

$$V_n^* = \frac{n-1.7}{n-1}\sqrt{2n}\sinh^{-1}(T_n/\sqrt{2n})$$

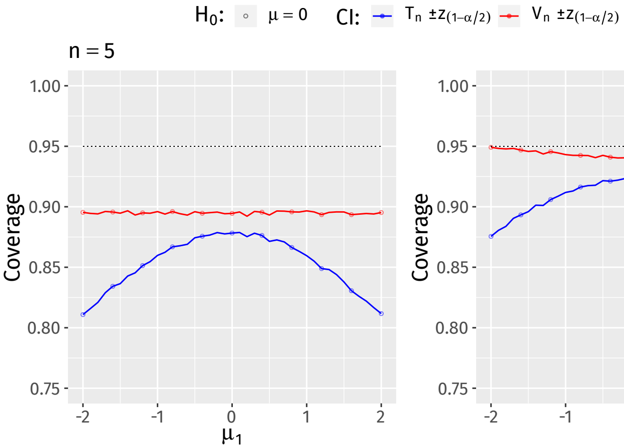
h_n improves normal fit



h_n stabilises variance



h_n improves empirical coverage probability of confidence intervals



Detecting Publication Bias

Aggregate study effects in the absence of publication bias

Without publication bias, the global effect size across k studies can be estimated by:

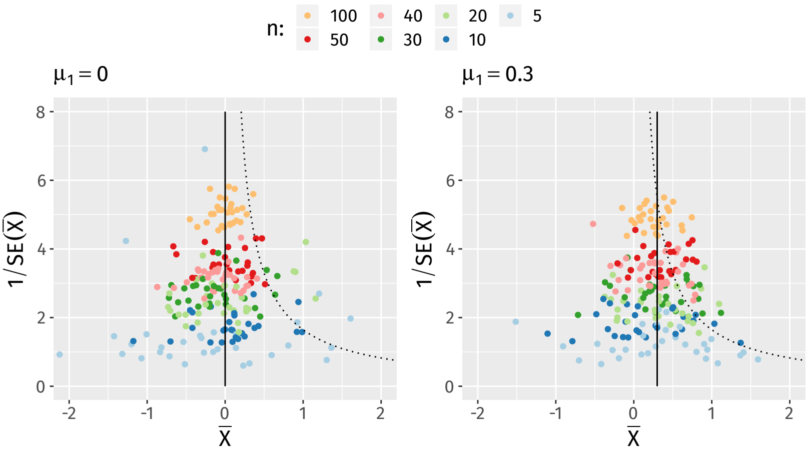
$$\bar{X}_N = \frac{\sum_{j=1}^{k} \bar{X}_{n_j} w_j}{\sum_{j=1}^{k} w_j} \sim \mathcal{N}(\mu, \sigma^2/N); \quad N = \sum_{j=1}^{k} n_j; \quad w_j = n_j/\sigma^2$$

Funnelling statistical evidence

Funnel plots: draw effect size against precision, look for gaps.

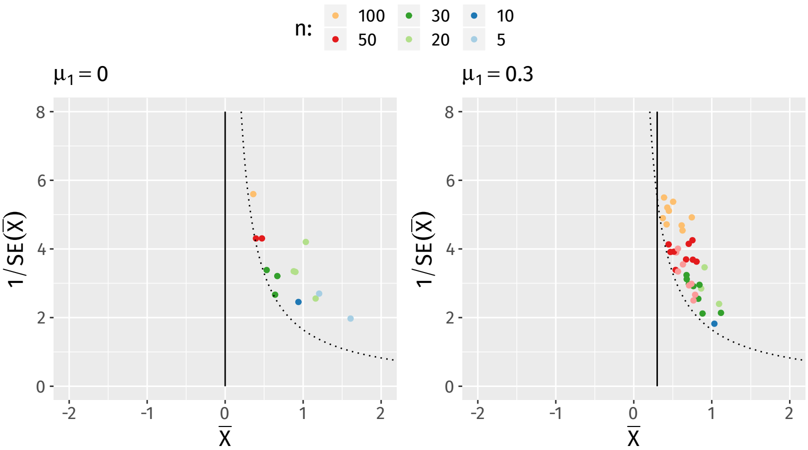
Idea proposed by Light and Pillemer (1984, p. 64–69).

Funnelling statistical evidence



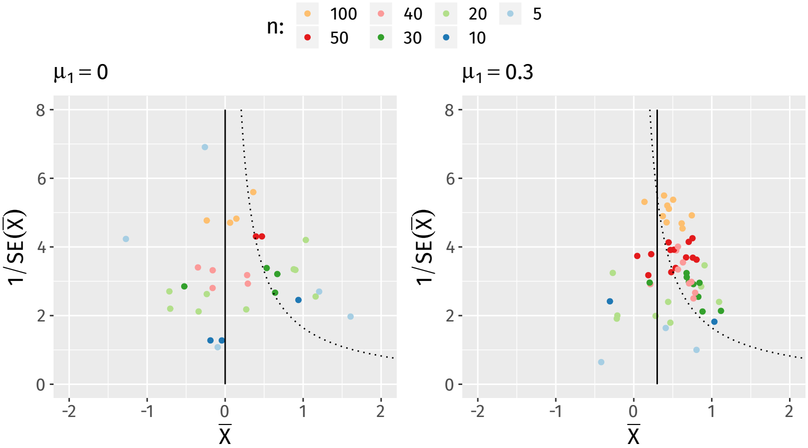
No publication bias

Funnelling statistical evidence



Significant studies only

Funnelling statistical evidence



Significant studies and 10% of non-significant studies

The file drawer problem

Many studies land in the file drawer (Image: Geckoboard)

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Calculate number of omitted studies needed to make findings non-significant:

$$z_{(1-\alpha)} = \frac{k\bar{z}_k + o\bar{z}_o}{\sqrt{k+o}}$$

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Calculate number of omitted studies needed to make findings non-significant:

$$
\begin{aligned}
z_{(1-\alpha)} &= \frac{k\bar{z}_k + o\bar{z}_o}{\sqrt{k+o}} \\
\iff \quad o &= \frac{(k\bar{z}_k + o\bar{z}_o)^2}{z^2_{(1-\alpha)}} - k.
\end{aligned}
$$

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Assume that $\bar{z}_o = 0$:

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Assume that $\bar{z}_o = 0$:

$$o = \frac{(k\bar{z}_k)^2}{z^2_{(1-\alpha)}} - k.$$

More accurate is $\bar{z}_o = \mathrm{E}[Z \mid Z < z_{(1-\alpha)}]$:

$$o^* = \frac{-2k\bar{z}_k\bar{z}_o + z^2_{(1-\alpha)} - z_{(1-\alpha)}\sqrt{4k\bar{z}_o^2 - 4k\bar{z}_k\bar{z}_o + z^2_{(1-\alpha)}}}{2\bar{z}_o^2}.$$

Check whether $o^* < 5k + 10$.

The file drawer problem

	k	μ_1	o	o^*	bias detected?
full sample (no bias)	200	0	-141	-181	o : no o^*: no
	200	0.3	8123	884	o : no o^*: no

Critical value: $5k + 10$

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full sample (no bias)	200	0	-141	-181	o : no o^*: no
	200	0.3	8123	884	o : no o^*: no
significant studies	13	0	320	109	o : no o^*: no
	38	0.3	2705	457	o : no o^*: no

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significant studies	13	0	320	109	o : no o^*: no
	38	0.3	2705	457	o : no o^*: no
significant studies and 10% of non-significant studies	32	0	99	43	o : yes o^*: yes
	55	0.3	3086	494	o : no o^*: no

Critical value: $5k + 10$

How many significant studies ought to be expected?

Assuming σ^2 to be known, the (overestimated) power of the jth study is:

$$1 - \beta_j = \Phi(\sqrt{n_j}\frac{\bar{X}_N}{\sigma} - z_{(1-\alpha)})$$

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Expected number of significant studies in the absence of publication bias (Ioannidis and Trikalinos, 2007):

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Expected number of significant studies in the absence of publication bias (Ioannidis and Trikalinos, 2007):

$$E = \sum_{j=1}^{k}(1 - \beta_j).$$

Compare to observed number of studies O via χ^2-test:

$$A = [(O - E)^2/E + (O - E)^2/(k - E)] \sim \chi^2_1.$$

How many significant studies ought to be expected?

	k	μ_1	$\bar{x}_N$	A	bias detected?
full sample (no bias, A)	200	0	-0.02	1.70	no
	200	0.3	0.27	0.08	no

Critical value: 3.84 (95%-quantile of χ^2_1-distribution)

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	k	μ_1	$\bar{x}_N$	A	bias detected?
full sample (no bias, A)	200	0	-0.02	1.70	no
	200	0.3	0.27	0.08	no
significant studies (B)	13	0	0.69	11.67	yes
	38	0.3	0.61	19.94	yes

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significant studies (B)	13	0	0.69	11.67	yes
	38	0.3	0.61	19.94	yes
significant studies and 10% of non-significant studies (C)	32	0	0.15	28.64	yes
	55	0.3	0.51	8.21	yes

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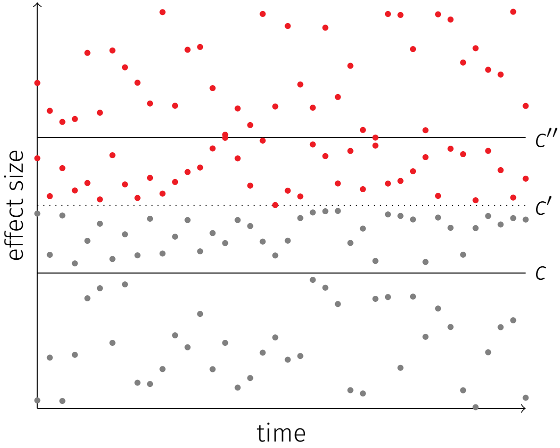
The calliper test

$$c' = z_{(1-\alpha)}; \quad c'' = c + e; \quad c = c' - e; \quad k' \in [c, c'']; \quad k'' \in [c', c''].$$



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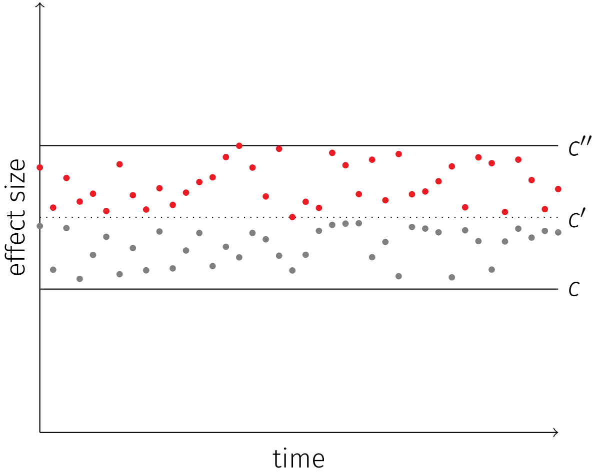

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$$p = \Pr[Z \in [c, c'] \mid Z \in [c, c'']] = \frac{\Phi(c') - \Phi(c)}{\Phi(c'') - \Phi(c)} \simeq \frac{(c' - c)\phi(c)}{(c'' - c)\phi(c)}$$

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$K'' \sim \mathrm{Bin}(k', p = 0.5)$

The calliper test

	k	μ_1	k'	k''	$q_{(1-\alpha/2)}$	bias detected?
full sample (no bias, A)	200	0	4	3	4	no
	200	0.3	15	6	11	no

Critical value: 97.5%-quantile of Bin(k', 0.5)-distribution

The calliper test

	k	μ_1	k'	k''	$q_{(1-\alpha/2)}$	bias detected?
full sample (no bias, A)	200	0	4	3	4	no
	200	0.3	15	6	11	no
significant studies (B)	13	0	3	3	3	no
	38	0.3	6	6	5	yes

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	k	μ_1	k'	k''	$q_{(1-\alpha/2)}$	bias detected?
full sample (no bias, A)	200	0	4	3	4	no
	200	0.3	15	6	11	no
significant studies (B)	13	0	3	3	3	no
	38	0.3	6	6	5	yes
significant studies and 10% of non-significant studies (C)	32	0	3	3	3	no
	55	0.3	7	6	6	no

Critical value: 97.5%-quantile of Bin(k', 0.5)-distribution

Correcting Publication Bias

Publication probabilities and truncated distributions

- publication probability:

$$\mathrm{ppr}(S_n, \pi) = \pi + (1 - \pi)\delta(S_n)$$

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- expected publication probability:

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Publication probabilities and truncated distributions

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- expected publication probability:

$$\text{E}[\text{ppr}(S_n, \pi)] = \pi \Pr(\text{ppr} = \pi) + \Pr(\text{ppr} = 1)$$

- truncated probability density function of S_n:

$$f^*_{S_n}(s_n) = \frac{\text{ppr}(s_n, \pi)}{\text{E}[\text{ppr}(S_n, \pi)]} f_{S_n}(s_n)$$

Reweight by publication probability

Let

$$\mu_{\text{sig}} = \mathrm{E}[\bar{X}_{n_j} \mid S_{n_j} > q_{(1-\alpha)}]; \quad \mu_{\text{ns}} = \mathrm{E}[\bar{X}_{n_j} \mid S_{n_j} \leq q_{(1-\alpha)}]$$

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$$\sigma^2_{\text{sig}} = \mathrm{Var}[\bar{X}_{n_j} \mid S_{n_j} > q_{(1-\alpha)}]; \quad \sigma^2_{\text{ns}} = \mathrm{Var}[\bar{X}_{n_j} \mid S_{n_j} \leq q_{(1-\alpha)}].$$

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$$\sigma^2_{\text{sig}} = \mathrm{Var}[\bar{X}_{n_j} \mid S_{n_j} > q_{(1-\alpha)}]; \quad \sigma^2_{\text{ns}} = \mathrm{Var}[\bar{X}_{n_j} \mid S_{n_j} \leq q_{(1-\alpha)}].$$

Then

$$\bar{X}_{n_j} \mid S_n \sim \begin{cases} \mathcal{N}\left(\mu_{\text{sig}}\,\mathrm{ppr}_j, \sigma^2_{\text{sig}} \frac{\mathrm{ppr}_j^2}{n_j}\right), & \text{if } S_n > q_{(1-\alpha)}; \\ \mathcal{N}\left(\mu_{\text{ns}}\,\mathrm{ppr}_j, \sigma^2_{\text{ns}} \frac{\mathrm{ppr}_j^2}{n_j}\right), & \text{if } S_n \leq q_{(1-\alpha)}. \end{cases}$$

Reweight by publication probability

The biased weight $w_j^* = w_j \, \mathrm{ppr}_j$ yields biased estimator

$$\bar{X}_{\mathrm{N}} = \frac{\sum_{j=1}^{k} \bar{X}_{n_j} w_j^*}{\sum_{j=1}^{k} w_j^*} \sim \mathcal{N}(\mu^*, \sigma^*)$$

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with

$$\mu^* = \frac{\sum_{j=1}^{s} w_j^* \mu_\mathrm{sig} + \sum_{j=s+1}^{k} w_j^* \mu_\mathrm{ns}}{\sum_{j=1}^{k} w_j^*}$$

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with

$$\mu^* = \frac{\sum_{j=1}^{s} w_j^* \mu_{\mathrm{sig}} + \sum_{j=s+1}^{k} w_j^* \mu_{\mathrm{ns}}}{\sum_{j=1}^{k} w_j^*}$$

$$\sigma^* = \frac{\sum_{j=1}^{s} (w_j^*)^2 \sigma_{\mathrm{sig}}/n_j + \sum_{j=s+1}^{k} (w_j^*)^2 \sigma_{\mathrm{ns}}/n_j}{(\sum_{j=1}^{k} w_j^*)^2}$$

Reweight by publication probability

Reweight each biased weight w_j^* by inverse of the publication probability to get unbiased estimator:

$$\bar{X}_{\mathrm{N}}^* = \frac{\sum_{j=1}^{k} \bar{X}_{n_j} w_j^* \,/\, \mathrm{ppr}_j}{\sum_{j=1}^{k} w_j^* \,/\, \mathrm{ppr}_j} \sim \mathcal{N}\left(\mu, \sigma^2/N\right)$$

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Inspired by Hansen and Hurwitz (1943).

Reweight by publication probability

	k	μ_1	$\bar{x}_N$	$\bar{x}_N^*$
full sample (no bias, A)	200	0	-0.02	-0.07
	200	0.3	0.27	0.15

Reweight by publication probability

	k	μ_1	$\bar{x}_N$	$\bar{x}_N^*$
full sample (no bias, A)	200	0	-0.02	-0.07
	200	0.3	0.27	0.15
significant studies (B)	13	0	0.69	0.69
	38	0.3	0.61	0.61

Reweight by publication probability

	k	μ_1	$\bar{x}_N$	$\bar{x}_N^*$
full sample (no bias, A)	200	0	−0.02	−0.07
	200	0.3	0.27	0.15
significant studies (B)	13	0	0.69	0.69
	38	0.3	0.61	0.61
significant studies and 10% of	32	0	0.15	−0.17
non-significant studies (C)	55	0.3	0.51	0.27

Maximise truncated likelihood

Truncated likelihood of μ given by

$$\mathcal{L}^*(\mu \mid v_{n_1}, \ldots, v_{n_k}) = \frac{\mathcal{L}(\mu \mid v_{n_1}, \ldots, v_{n_k})}{\mathrm{E}[\mathrm{ppr}(V_{n_j}, \pi) \mid \mu]} \prod_{j=1}^{k} \mathrm{ppr}(V_{n_j}, \pi).$$

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Can be maximised by grid search.

Maximise truncated likelihood

	k	μ_1	$\bar{x}_N$	$\hat{\mu}$	$\hat{\pi}$
full sample (no bias, A)	200	0	-0.02	-0.02	0.66
	200	0.3	0.27	0.27	1

Maximise truncated likelihood

	k	μ_1	$\bar{x}_N$	$\hat{\mu}$	$\hat{\pi}$
full sample (no bias, A)	200	0	-0.02	-0.02	0.66
	200	0.3	0.27	0.27	1
significant studies (B)	13	0	0.69	0.31	0
	38	0.3	0.61	0.31	0

Maximise truncated likelihood

	k	μ_1	$\bar{x}_N$	$\hat{\mu}$	$\hat{\pi}$
full sample (no bias, A)	200	0	-0.02	-0.02	0.66
	200	0.3	0.27	0.27	1
significant studies (B)	13	0	0.69	0.31	0
	38	0.3	0.61	0.31	0
significant studies and 10% of non-significant studies (C)	32	0	0.15	-0.07	0.05
	55	0.3	0.51	0.33	0.16

Conclusion

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Reliable empirical data about origin of publication bias is key.

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Reliable empirical data about origin of publication bias is key.

Pre-registration can both reduce publication bias and improve correction accuracy.

Next Steps

- Systematically evaluate performance of methods
- Extend assessment to additional methods
- Create ensemble models

Coming Full Circle

Fighting publication bias is like hunting the Woozle—all too often it forces you to go in circles

References i

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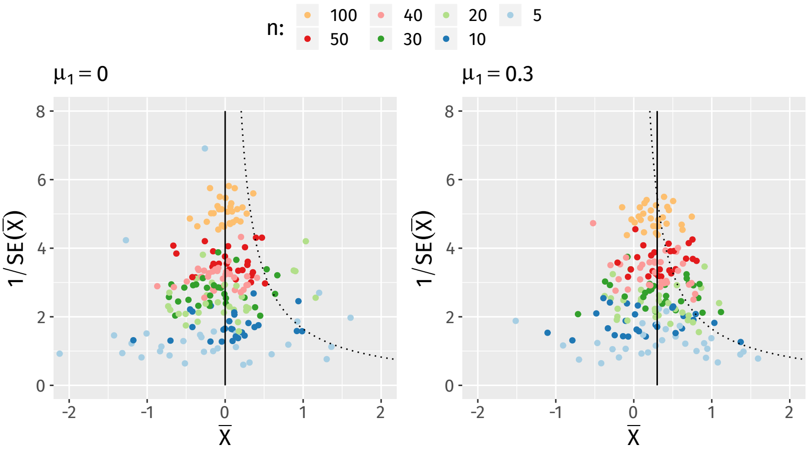
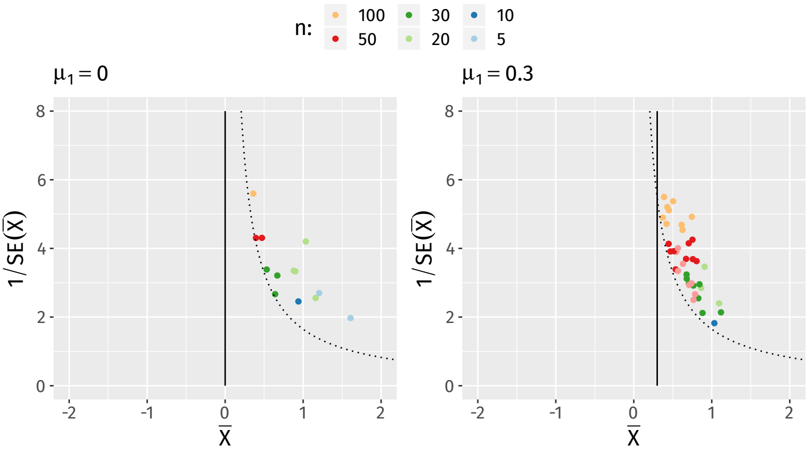
Quantify correlation between effect size and standard error



No correlation

Quantify correlation between effect size and standard error



Negative correlation

Rank correlation between effect size and standard error

Non-parametric test proposed by Begg and Mazumdar (1994):

Rank correlation between effect size and standard error

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$$Z_j = \frac{\bar{X}_{n_j} - \bar{X}_N}{\sqrt{v_j}} \sim \mathcal{N}(0, 1);$$

Rank correlation between effect size and standard error

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$$Z_j = \frac{\bar{X}_{n_j} - \bar{X}_N}{\sqrt{v_j}} \sim \mathcal{N}(0,1); \quad v_j = 1/w_j - 1/\sum_{i=1}^{k} w_i.$$

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$$Z_k = \frac{\sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \text{sgn}(Z_i - Z_j)\text{sgn}(v_i - v_j)}{\sqrt{(2k+5)k(k-1)/18}} \;\dot{\sim}\; \mathcal{N}(0,1).$$

Rank correlation between effect size and standard error

	k	μ_1	$\bar{x}_N$	$\|\tau/\sigma_\tau\|$	bias detected?
full sample (no bias, A)	200	0	-0.02	1.33	no
	200	0.3	0.27	0.51	no

Critical value: 1.96 (97.5%-quantile of $\mathcal{N}(0,1)$-distribution)

Rank correlation between effect size and standard error

	k	μ_1	$\bar{x}_N$	$\lvert\tau/\sigma_\tau\rvert$	bias detected?
full sample (no bias, A)	200	0	-0.02	1.33	no
	200	0.3	0.27	0.51	no
significant studies (B)	13	0	0.69	2.81	yes
	38	0.3	0.61	5.70	yes

Critical value: 1.96 (97.5%-quantile of $\mathcal{N}(0,1)$-distribution)

Rank correlation between effect size and standard error

	k	μ_1	$\bar{x}_N$	$\lvert\tau/\sigma_\tau\rvert$	bias detected?
full sample (no bias, A)	200	0	-0.02	1.33	no
	200	0.3	0.27	0.51	no
significant studies (B)	13	0	0.69	2.81	yes
	38	0.3	0.61	5.70	yes
significant studies and 10% of non-significant studies (C)	32	0	0.15	0.36	no
	55	0.3	0.51	1.60	no

Critical value: 1.96 (97.5%-quantile of $\mathcal{N}(0, 1)$-distribution)

Regress effect size against standard error

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$$T_k = \frac{\hat{\beta}_0 - \beta_0}{s_{\hat{\beta}_0}} = \frac{\hat{\beta}_0}{s_{\hat{\beta}_0}} \sim t(\nu = k - 2)$$

Rank correlation between effect size and standard error

	k	μ_1	$\bar{x}_N$	$\lvert t \rvert$	$q_{(1-\alpha/2)}$	bias detected?
full sample (no bias, A)	200	0	-0.02	1.30	1.97	no
	200	0.3	0.27	0.95	1.97	no

Critical value: 97.5%-quantile of $t(\nu)$-distribution

Rank correlation between effect size and standard error

	k	μ_1	$\bar{x}_N$	$\lvert t \rvert$	$q_{(1-\alpha/2)}$	bias detected?
full sample (no bias, A)	200	0	-0.02	1.30	1.97	no
	200	0.3	0.27	0.95	1.97	no
significant studies (B)	13	0	0.69	3.77	2.20	yes
	38	0.3	0.61	8.54	2.03	yes

Critical value: 97.5%-quantile of $t(\nu)$-distribution

Rank correlation between effect size and standard error

	k	μ_1	$\bar{x}_N$	$\lvert t\rvert$	$q_{(1-\alpha/2)}$	bias detected?
full sample (no bias, A)	200	0	-0.02	1.30	1.97	no
	200	0.3	0.27	0.95	1.97	no
significant studies (B)	13	0	0.69	3.77	2.20	yes
	38	0.3	0.61	8.54	2.03	yes
significant studies and 10% of non-significant studies (C)	32	0	0.15	1.02	2.04	no
	55	0.3	0.51	4.05	2.01	yes

Critical value: 97.5%-quantile of $t(\nu)$-distribution

Maximise truncated likelihood

Grid search:

1. Define a set of candidate values $\{\mu_1, \ldots, \mu_m\}$ and $\{\pi_1, \ldots, \pi_n\}$ for μ and π, respectively.

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2. For each combination of μ and π, calculate the likelihood.
3. Choose the candidate value for μ and π that yields the highest likelihood.

Publication bias—the bane of scientific publishing

Publication bias in a nutshell (Image: Hilda Bastian)

Trim-and-fill: Closing gaps in funnel plots