Weight of Statistical Evidence

Detection and Correction of Publication Bias

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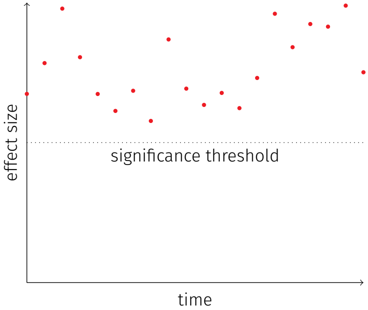
Écublens, July 9th

Master's Programme in Computational Science and Engineering

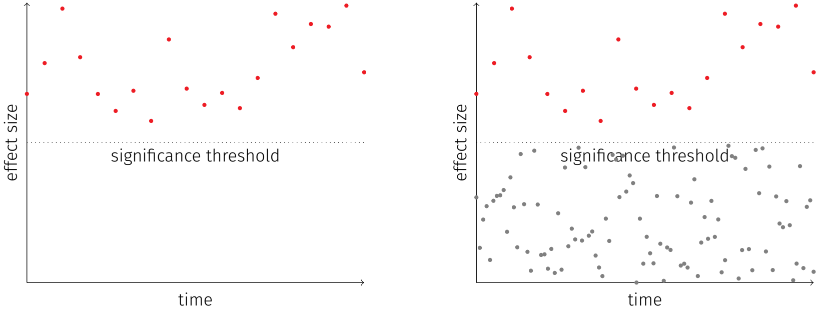
The Woozle effect

Pooh and Piglet tracking down the elusive Woozle (Image: Ernest H. Shepard)

The Woozle effect—why care?



The Woozle effect—why care?



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Analysing and Testing Evidence

Hypothesis Testing

$$H_0 : \mu \leq \mu_0$$
$$H_1 : \mu > \mu_0$$

Making evidence comparable

To test hypotheses, we need a test statistic V_n.

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$$E_2: \; V_n \sim \mathcal{N}(\tau, 1);$$

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$$\begin{aligned}
E_1 &: \ V_n = h_n(S_n) \text{ is monotonically increasing in } S_n; \\
E_2 &: \ V_n \sim \mathcal{N}(\tau, 1); \\
E_3 &: \ \mathrm{Var}[V_n] = 1; \\
E_4 &: \ \mathrm{E}_\mu[V_n] = \tau(\mu) \text{ is monotonically increasing in } \mu \text{ from } \\
&\quad\ \tau(0) = 0.
\end{aligned}$$

Example case: difference in means

Let $X \sim \mathcal{N}(\mu, \sigma^2)$ with μ and σ^2 unknown.

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with

$$\hat{\mu} = \bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i; \quad \hat{\sigma}^2 = s_n^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X}_n)^2$$

Variance stabilising transformation

For n large enough, T_n fulfills all properties E_1 to E_4 by virtue of the central limit theorem.

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Solution: Transform T_n!

Variance stabilising transformation

$V_n = h_n(T_n) = \sqrt{2n}\,\sinh^{-1}(T_n/\sqrt{2n}) \dot{\sim} \mathcal{N}(0,1)$ (Kulinskaya, Morgenthaler, and Staudte, 2008).

Variance stabilising transformation

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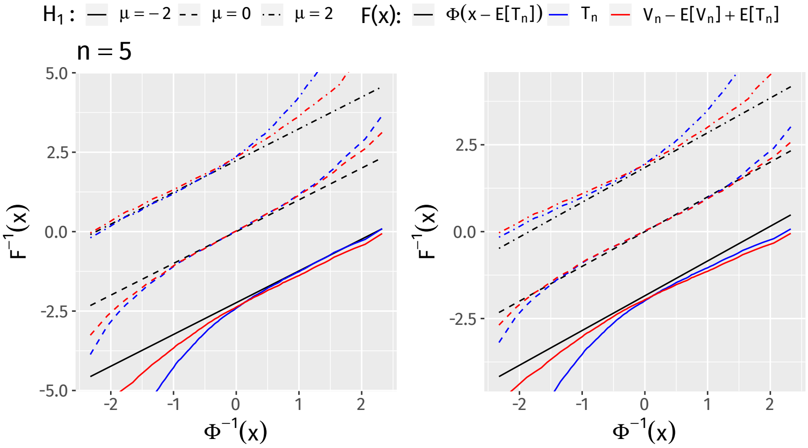
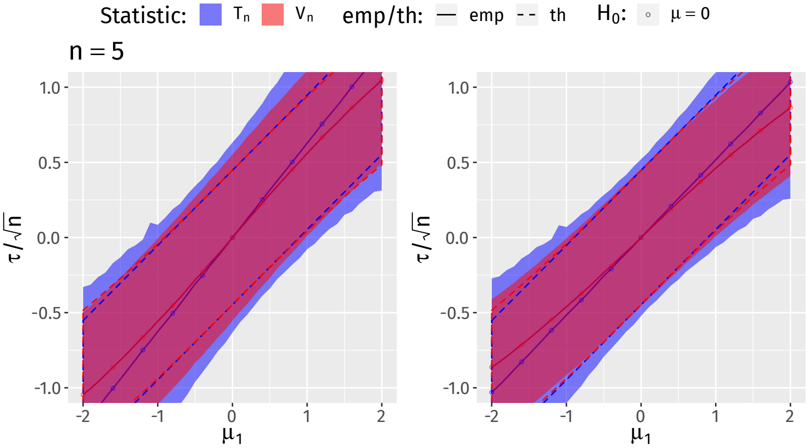
Further improvement by finite sample correction:

$$V_n^* = \frac{n-1.7}{n-1}\sqrt{2n}\,\sinh^{-1}(T_n/\sqrt{2n})$$

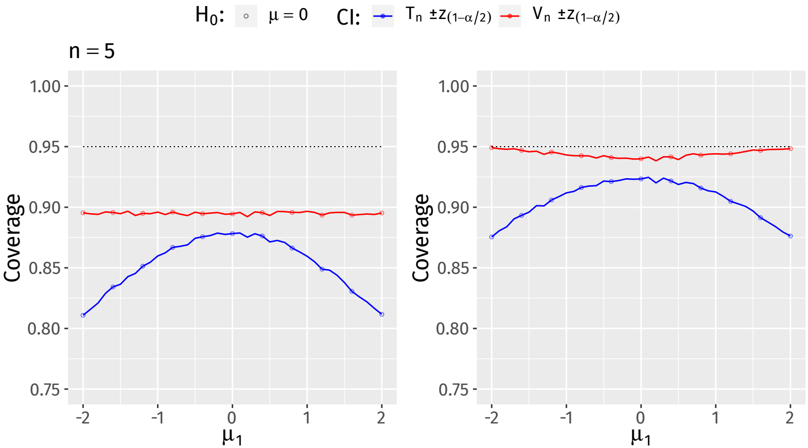
h_n improves normal fit



h_n stabilises variance



h_n improves empirical coverage probability of confidence intervals



Aggregate study effects in the absence of publication bias

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Global effect size estimate across k studies without publication bias:

$$\bar{X}_N = \frac{\sum_{j=1}^{k} \bar{X}_{n_j} w_j}{\sum_{j=1}^{k} w_j} \sim \mathcal{N}(\mu, \sigma^2/N); \quad N = \sum_{j=1}^{k} n_j; \quad w_j = n_j/\sigma^2$$

Detecting Publication Bias

Methods to detect publication bias

- The funnel plot

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- File drawer calculation

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Funnelling statistical evidence

Funnel plots: draw effect size against precision, look for gaps.

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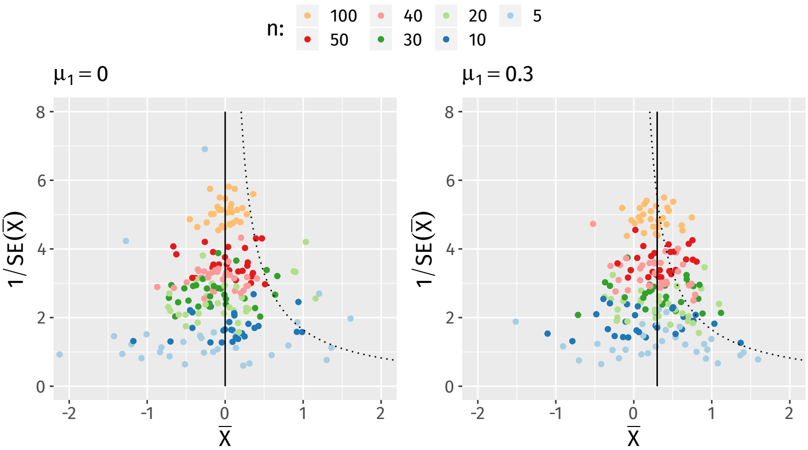
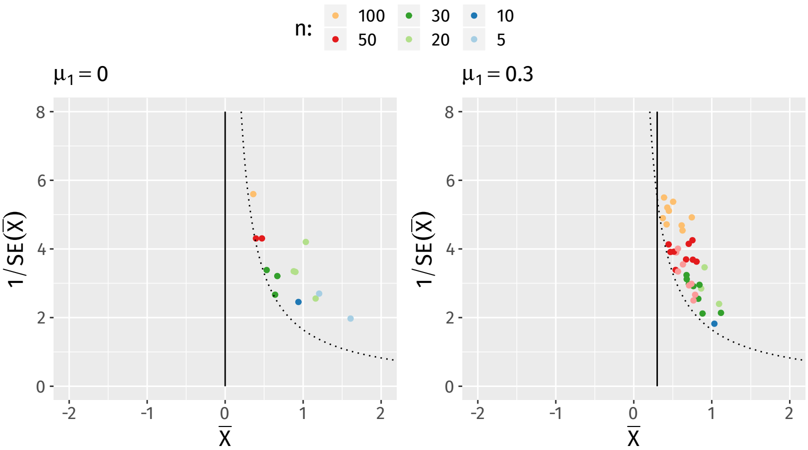
Idea proposed by Light and Pillemer (1984, p. 64–69).

Funnelling statistical evidence



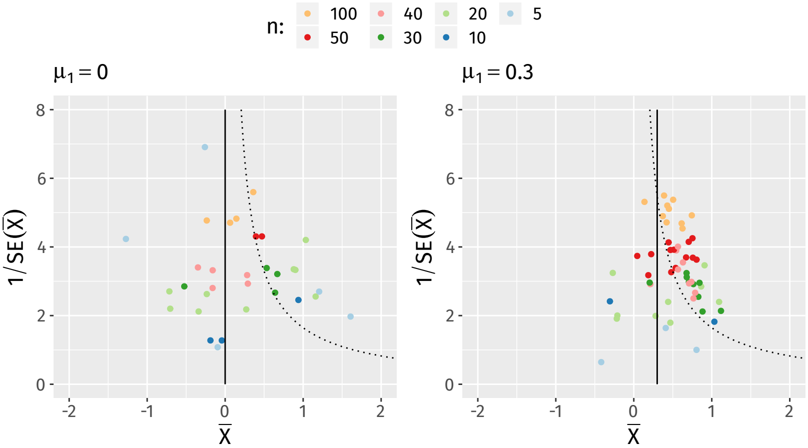
No publication bias

Funnelling statistical evidence



Significant studies only

Funnelling statistical evidence



Significant studies and 10% of non-significant studies

The file drawer problem

Many studies land in the file drawer (Image: Geckoboard)

The file drawer problem

Worst case assumption by Rosenthal (1979): Published results are Type I error only.

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Calculate number of omitted studies needed to make findings non-significant:

$$z_{(1-\alpha)} = \frac{k\bar{z}_k + o\bar{z}_o}{\sqrt{k+o}}$$

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$$
\begin{aligned}
z_{(1-\alpha)} &= \frac{k\bar{z}_k + o\bar{z}_o}{\sqrt{k+o}} \\
\iff \quad o &= \frac{(k\bar{z}_k + o\bar{z}_o)^2}{z^2_{(1-\alpha)}} - k.
\end{aligned}
$$

The file drawer problem

Assume that $\bar{z}_o = 0$:

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More accurate is $\bar{z}_o = \mathrm{E}[Z \mid Z < z_{(1-\alpha)}]$:

$$o^* = \frac{-2k\bar{z}_k\bar{z}_o + z^2_{(1-\alpha)} - z_{(1-\alpha)}\sqrt{4k\bar{z}_o^2 - 4k\bar{z}_k\bar{z}_o + z^2_{(1-\alpha)}}}{2\bar{z}_o^2}.$$

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Check whether $o^* < 5k + 10$.

The file drawer problem

	k	μ_1	o	o^*	bias detected?
full sample (no bias)	200	0	-141	-181	o : no o^*: no
	200	0.3	8123	884	o : no o^*: no

Critical value: $5k + 10$

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full sample (no bias)	200	0	-141	-181	o : no o^*: no
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significant studies	13	0	320	109	o : no o^*: no
	38	0.3	2705	457	o : no o^*: no

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significant studies	13	0	320	109	o : no o^*: no
	38	0.3	2705	457	o : no o^*: no
significant studies and 10% of non-significant studies	32	0	99	43	o : yes o^*: yes
	55	0.3	3086	494	o : no o^*: no

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How many significant studies ought to be expected?

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Assuming σ^2 to be known, the (overestimated) power of the jth study is:

$$1 - \beta_j = \Phi(\sqrt{n_j}\frac{\bar{X}_N}{\sigma} - z_{(1-\alpha)})$$

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Expected number of significant studies in the absence of publication bias (Ioannidis and Trikalinos, 2007):

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Expected number of significant studies in the absence of publication bias (Ioannidis and Trikalinos, 2007):

$$E = \sum_{j=1}^{k}(1 - \beta_j).$$

Compare to observed number of studies O via χ^2-test:

$$A = [(O - E)^2/E + (O - E)^2/(k - E)] \sim \chi^2_1.$$

How many significant studies ought to be expected?

	k	μ_1	$\bar{x}_N$	A	bias detected?
full sample (no bias, A)	200	0	-0.02	1.70	no
	200	0.3	0.27	0.08	no

Critical value: 3.84 (95%-quantile of χ^2_1-distribution)

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	200	0.3	0.27	0.08	no
significant studies (B)	13	0	0.69	11.67	yes
	38	0.3	0.61	19.94	yes

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	k	μ_1	$\bar{x}_N$	A	bias detected?
full sample (no bias, A)	200 200	0 0.3	-0.02 0.27	1.70 0.08	no no
significant studies (B)	13 38	0 0.3	0.69 0.61	11.67 19.94	yes yes
significant studies and 10% of non-significant studies (C)	32 55	0 0.3	0.15 0.51	28.64 8.21	yes yes

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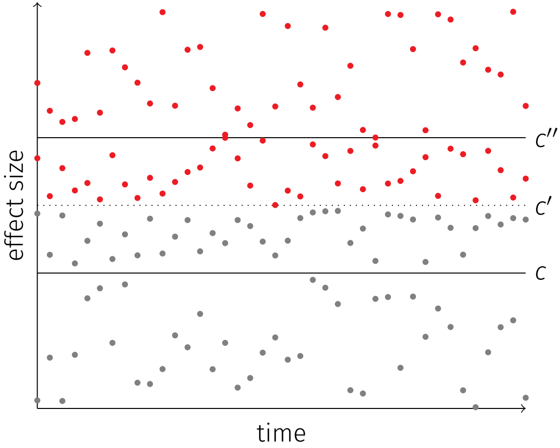
The calliper test

$$c' = z_{(1-\alpha)}; \quad c'' = c + e; \quad c = c' - e; \quad k' \in [c, c'']; \quad k'' \in [c', c''].$$



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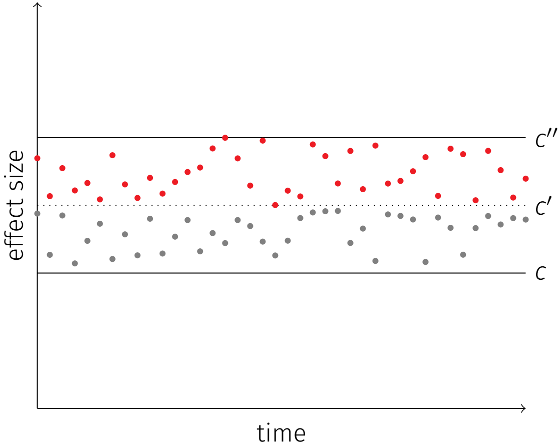

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$$p = \Pr[Z \in [c, c'] \mid Z \in [c, c'']] = \frac{\Phi(c') - \Phi(c)}{\Phi(c'') - \Phi(c)} \simeq \frac{(c' - c)\phi(c)}{(c'' - c)\phi(c)}$$

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$K'' \sim \mathrm{Bin}(k', p = 0.5)$

Proposed by Gerber and Malhotra (2006)

The calliper test

	k	μ_1	k'	k''	$q_{(1-\alpha/2)}$	bias detected?
full sample (no bias, A)	200	0	4	3	4	no
	200	0.3	15	6	11	no

Critical value: 97.5%-quantile of Bin(k', 0.5)-distribution

The calliper test

	k	μ_1	k'	k''	$q_{(1-\alpha/2)}$	bias detected?
full sample (no bias, A)	200	0	4	3	4	no
	200	0.3	15	6	11	no
significant studies (B)	13	0	3	3	3	no
	38	0.3	6	6	5	yes

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	k	μ_1	k'	k''	$q_{(1-\alpha/2)}$	bias detected?
full sample (no bias, A)	200	0	4	3	4	no
	200	0.3	15	6	11	no
significant studies (B)	13	0	3	3	3	no
	38	0.3	6	6	5	yes
significant studies and 10% of non-significant studies (C)	32	0	3	3	3	no
	55	0.3	7	6	6	no

Critical value: 97.5%-quantile of Bin(k', 0.5)-distribution

Correcting Publication Bias

Methods to correct publication bias

- Reweight by publication probability

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- Maximise the truncated likelihood function

Publication probabilities and truncated distributions

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Publication probabilities and truncated distributions

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- expected publication probability:

$$\text{E}[\text{ppr}(S_n, \pi)] = \pi \Pr(\text{ppr} = \pi) + \Pr(\text{ppr} = 1)$$

- truncated probability density function of S_n:

$$f^*_{S_n}(s_n) = \frac{\text{ppr}(s_n, \pi)}{\text{E}[\text{ppr}(S_n, \pi)]} f_{S_n}(s_n)$$

Reweight by publication probability

Let

$$\mu_{\text{sig}} = \mathrm{E}[\bar{X}_{n_j} \mid S_{n_j} > q_{(1-\alpha)}]; \quad \mu_{\text{ns}} = \mathrm{E}[\bar{X}_{n_j} \mid S_{n_j} \leq q_{(1-\alpha)}]$$

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$$\sigma^2_{\text{sig}} = \mathrm{Var}[\bar{X}_{n_j} \mid S_{n_j} > q_{(1-\alpha)}]; \quad \sigma^2_{\text{ns}} = \mathrm{Var}[\bar{X}_{n_j} \mid S_{n_j} \leq q_{(1-\alpha)}].$$

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$$\sigma^2_{\text{sig}} = \mathrm{Var}[\bar{X}_{n_j} \mid S_{n_j} > q_{(1-\alpha)}]; \quad \sigma^2_{\text{ns}} = \mathrm{Var}[\bar{X}_{n_j} \mid S_{n_j} \leq q_{(1-\alpha)}].$$

Then

$$\bar{X}_{n_j} \mid S_n \sim \begin{cases} \mathcal{N}\left(\mu_{\text{sig}}\,\mathrm{ppr}_j, \sigma^2_{\text{sig}} \frac{\mathrm{ppr}_j^2}{n_j}\right), & \text{if } S_n > q_{(1-\alpha)}; \\ \mathcal{N}\left(\mu_{\text{ns}}\,\mathrm{ppr}_j, \sigma^2_{\text{ns}} \frac{\mathrm{ppr}_j^2}{n_j}\right), & \text{if } S_n \leq q_{(1-\alpha)}. \end{cases}$$

Reweight by publication probability

The biased weight $w_j^* = w_j \, \mathrm{ppr}_j$ yields biased estimator

$$\bar{X}_{\mathrm{N}} = \frac{\sum_{j=1}^{k} \bar{X}_{n_j} w_j^*}{\sum_{j=1}^{k} w_j^*} \sim \mathcal{N}(\mu^*, \sigma^*)$$

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with

$$\mu^* = \frac{\sum_{j=1}^{s} w_j^* \mu_{\mathrm{sig}} + \sum_{j=s+1}^{k} w_j^* \mu_{\mathrm{ns}}}{\sum_{j=1}^{k} w_j^*}$$

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with

$$\mu^* = \frac{\sum_{j=1}^{s} w_j^* \mu_{\mathrm{sig}} + \sum_{j=s+1}^{k} w_j^* \mu_{\mathrm{ns}}}{\sum_{j=1}^{k} w_j^*}$$

$$\sigma^* = \frac{\sum_{j=1}^{s} (w_j^*)^2 \sigma_{\mathrm{sig}}/n_j + \sum_{j=s+1}^{k} (w_j^*)^2 \sigma_{\mathrm{ns}}/n_j}{(\sum_{j=1}^{k} w_j^*)^2}$$

Reweight by publication probability

Reweight biased weight w_j^* by $1/\,\mathrm{ppr}_j$ to get unbiased estimator:

$$\bar{X}_{\mathrm{N}}^* = \frac{\sum_{j=1}^{k} \bar{X}_{n_j} w_j^* /\,\mathrm{ppr}_j}{\sum_{j=1}^{k} w_j^* /\,\mathrm{ppr}_j} \sim \mathcal{N}\left(\mu, \sigma^2/N\right)$$

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Reweight biased weight w_j^* by $1/\operatorname{ppr}_j$ to get unbiased estimator:

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Inspired by Hansen and Hurwitz (1943).

Reweight by publication probability

	k	μ_1	$\overline{X}_N$	$\overline{X}_N^*$
full sample (no bias, A)	200	0	-0.02	-0.07
	200	0.3	0.27	0.15

Reweight by publication probability

	k	μ_1	$\bar{x}_N$	$\bar{x}_N^*$
full sample (no bias, A)	200	0	-0.02	-0.07
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significant studies (B)	13	0	0.69	0.69
	38	0.3	0.61	0.61

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significant studies (B)	13	0	0.69	0.69
	38	0.3	0.61	0.61
significant studies and 10% of non-significant studies (C)	32	0	0.15	-0.17
	55	0.3	0.51	0.27

Maximise truncated likelihood

Truncated likelihood of μ given by

$$\mathcal{L}^*(\mu \mid v_{n_1}, \ldots, v_{n_k}) = \frac{\mathcal{L}(\mu \mid v_{n_1}, \ldots, v_{n_k})}{\mathrm{E}[\mathrm{ppr}(V_{n_j}, \pi) \mid \mu]} \prod_{j=1}^{k} \mathrm{ppr}(V_{n_j}, \pi).$$

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Can be maximised by grid search.

Maximise truncated likelihood

	k	μ_1	$\bar{x}_N$	$\hat{\mu}$	$\hat{\pi}$
full sample (no bias, A)	200	0	-0.02	-0.02	0.66
	200	0.3	0.27	0.27	1

Maximise truncated likelihood

	k	μ_1	$\bar{x}_N$	$\hat{\mu}$	$\hat{\pi}$
full sample (no bias, A)	200	0	-0.02	-0.02	0.66
	200	0.3	0.27	0.27	1
significant studies (B)	13	0	0.69	0.31	0
	38	0.3	0.61	0.31	0

Maximise truncated likelihood

	k	μ_1	$\bar{x}_N$	$\hat{\mu}$	$\hat{\pi}$
full sample (no bias, A)	200	0	-0.02	-0.02	0.66
	200	0.3	0.27	0.27	1
significant studies (B)	13	0	0.69	0.31	0
	38	0.3	0.61	0.31	0
significant studies and 10% of non-significant studies (C)	32	0	0.15	-0.07	0.05
	55	0.3	0.51	0.33	0.16

Conclusion

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- Variety of methods available—none is a silver bullet.
- Reliable empirical data about origin of publication bias is key.
- Pre-registration can both reduce publication bias and improve correction accuracy.

Next Steps

- Systematically evaluate performance of methods

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- Extend assessment to additional methods

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- Systematically evaluate performance of methods
- Extend assessment to additional methods
- Create ensemble models

Coming Full Circle

Fighting publication bias is like hunting the Woozle—all too often it forces you to go in circles

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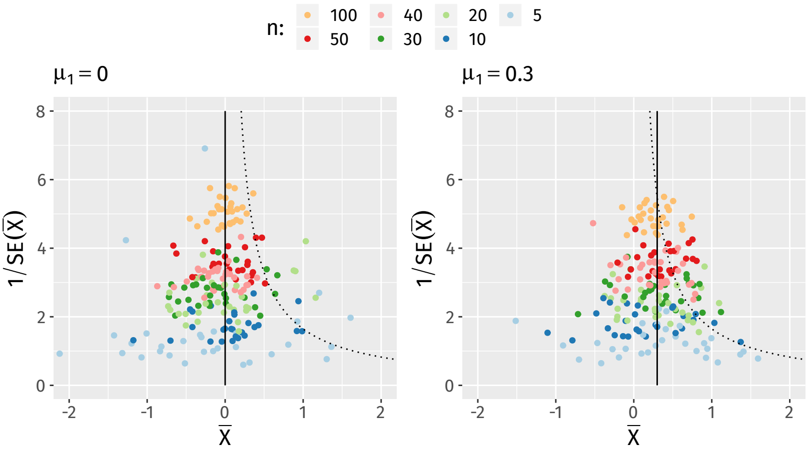
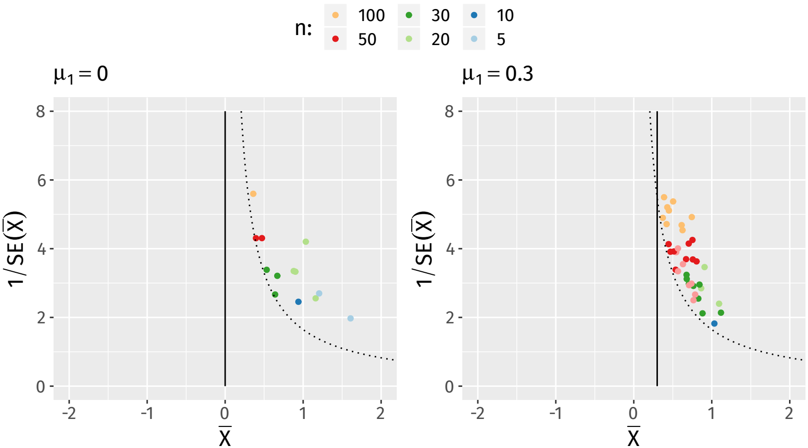
Quantify correlation between effect size and standard error



No correlation

Quantify correlation between effect size and standard error



Negative correlation

Rank correlation between effect size and standard error

Non-parametric test proposed by Begg and Mazumdar (1994):

Rank correlation between effect size and standard error

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$$Z_j = \frac{\bar{X}_{n_j} - \bar{X}_N}{\sqrt{v_j}} \sim \mathcal{N}(0,1);$$

Rank correlation between effect size and standard error

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$$Z_j = \frac{\bar{X}_{n_j} - \bar{X}_N}{\sqrt{v_j}} \sim \mathcal{N}(0,1); \quad v_j = 1/w_j - 1/\sum_{i=1}^{k} w_i.$$

Rank correlation between effect size and standard error

Non-parametric test proposed by Begg and Mazumdar (1994):

$$Z_j = \frac{\bar{X}_{n_j} - \bar{X}_N}{\sqrt{v_j}} \sim \mathcal{N}(0,1); \quad v_j = 1/w_j - 1/\sum_{i=1}^{k} w_i.$$

$$Z_k = \frac{\sum_{i=1}^{k-1}\sum_{j=i+1}^{k} \operatorname{sgn}(Z_i - Z_j)\operatorname{sgn}(v_i - v_j)}{\sqrt{(2k+5)k(k-1)/18}} \dot{\sim} \mathcal{N}(0,1).$$

Rank correlation between effect size and standard error

	k	μ_1	$\bar{x}_N$	$\lvert\tau/\sigma_\tau\rvert$	bias detected?
full sample (no bias, A)	200	0	-0.02	1.33	no
	200	0.3	0.27	0.51	no

Critical value: 1.96 (97.5%-quantile of $\mathcal{N}(0,1)$-distribution)

Rank correlation between effect size and standard error

	k	μ_1	$\bar{x}_N$	$\lvert\tau/\sigma_\tau\rvert$	bias detected?
full sample (no bias, A)	200	0	-0.02	1.33	no
	200	0.3	0.27	0.51	no
significant studies (B)	13	0	0.69	2.81	yes
	38	0.3	0.61	5.70	yes

Critical value: 1.96 (97.5%-quantile of $\mathcal{N}(0, 1)$-distribution)

Rank correlation between effect size and standard error

	k	μ_1	$\bar{x}_N$	$\lvert\tau/\sigma_\tau\rvert$	bias detected?
full sample (no bias, A)	200	0	-0.02	1.33	no
	200	0.3	0.27	0.51	no
significant studies (B)	13	0	0.69	2.81	yes
	38	0.3	0.61	5.70	yes
significant studies and 10% of non-significant studies (C)	32	0	0.15	0.36	no
	55	0.3	0.51	1.60	no

Critical value: 1.96 (97.5%-quantile of $\mathcal{N}(0,1)$-distribution)

Regress effect size against standard error

Regression-based test proposed by Egger et al. (1997)

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$$Z_j = \frac{\bar{X}_{n_j} - \bar{X}_N}{\sqrt{v_j}} \sim \mathcal{N}(0,1); \quad v_j = 1/w_j - 1/\sum_{i=1}^{k} w_i; \quad Z_j \sim \beta_0 + \beta_1/\sqrt{v_j}.$$

$$T_k = \frac{\hat{\beta}_0 - \beta_0}{s_{\hat{\beta}_0}} = \frac{\hat{\beta}_0}{s_{\hat{\beta}_0}} \sim t(\nu = k - 2)$$

Rank correlation between effect size and standard error

	k	μ_1	$\bar{x}_N$	$\lvert t \rvert$	$q_{(1-\alpha/2)}$	bias detected?
full sample (no bias, A)	200	0	-0.02	1.30	1.97	no
	200	0.3	0.27	0.95	1.97	no

Critical value: 97.5%-quantile of $t(\nu)$-distribution

Rank correlation between effect size and standard error

	k	μ_1	$\bar{x}_N$	$\|t\|$	$q_{(1-\alpha/2)}$	bias detected?
full sample (no bias, A)	200	0	-0.02	1.30	1.97	no
	200	0.3	0.27	0.95	1.97	no
significant studies (B)	13	0	0.69	3.77	2.20	yes
	38	0.3	0.61	8.54	2.03	yes

Critical value: 97.5%-quantile of $t(\nu)$-distribution

Rank correlation between effect size and standard error

	k	μ_1	$\bar{x}_N$	$\lvert t \rvert$	$q_{(1-\alpha/2)}$	bias detected?
full sample (no bias, A)	200	0	-0.02	1.30	1.97	no
	200	0.3	0.27	0.95	1.97	no
significant studies (B)	13	0	0.69	3.77	2.20	yes
	38	0.3	0.61	8.54	2.03	yes
significant studies and 10% of non-significant studies (C)	32	0	0.15	1.02	2.04	no
	55	0.3	0.51	4.05	2.01	yes

Critical value: 97.5%-quantile of $t(\nu)$-distribution

Maximise truncated likelihood

Grid search:

1. Define a set of candidate values $\{\mu_1, \ldots, \mu_m\}$ and $\{\pi_1, \ldots, \pi_n\}$ for μ and π, respectively.

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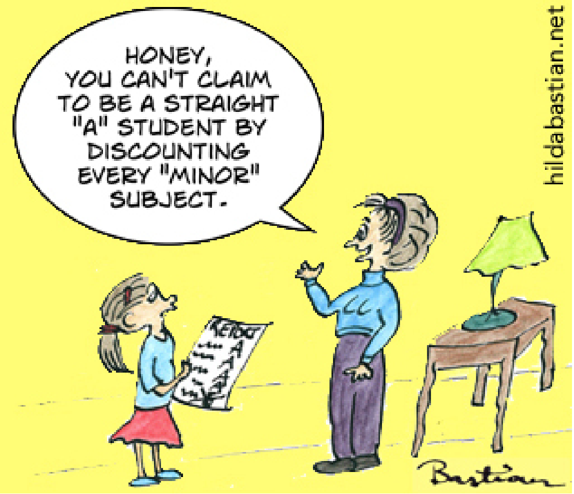
Grid search:

1. Define a set of candidate values $\{\mu_1, \ldots, \mu_m\}$ and $\{\pi_1, \ldots, \pi_n\}$ for μ and π, respectively.
2. For each combination of μ and π, calculate the likelihood.
3. Choose the candidate value for μ and π that yields the highest likelihood.

Publication bias—the bane of scientific publishing



Publication bias in a nutshell (Image: Hilda Bastian)

Trim-and-fill: Closing gaps in funnel plots