Electricity and Magnetism

ELTN 2112

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Electricity

- Field of Electricity consists of two areas of study.
- Electrostatics and Current Electricity
- Electrostatics deals with stationary (fixed) charges
- Current Electricity deals with moving charges

We will first study Basic Concepts and Laws in Electrostatics and their applications

Electrostatics

- Charge Distributions
- Coulomb's Law
- Electric Field

Electrostatics

Electric charge -properties of electric charge

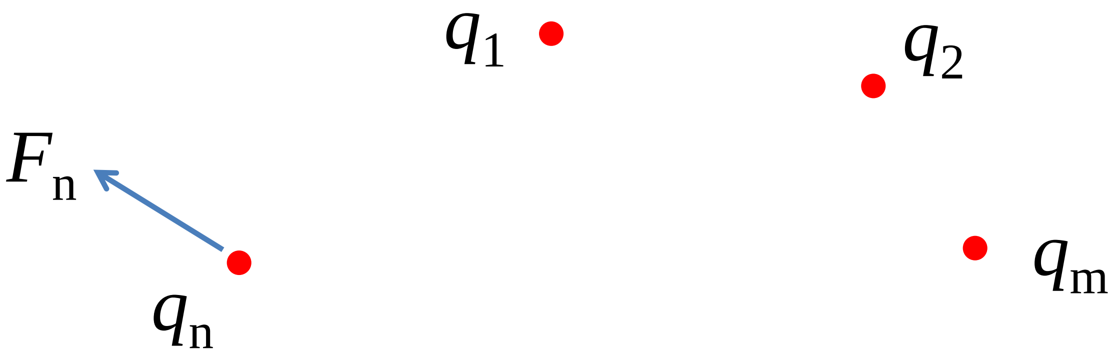
- Charge is a fundamental property of matter
- Has two types: positive and negative
- Smallest freely existing amount = 1.602×10^{-19} C

  ($+1.602 \times 10^{-19}$ C = e)
- Charge is quantized (charge exists in multiples of e);

  that is charge $Q=Ne$ *(N is a positive or negative integer)*
- Charge is conserved in chemical and nuclear reactions

Charge Distributions

(a) Point charge distribution:



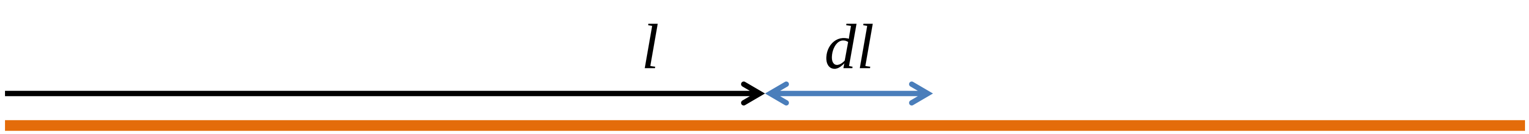
(b) Continuous charge distributions:

1. Linear Charge Distributions

(Eg: a charged wire)

If a length dl of a wire has a charge dq, then the linear charge density of the wire ,

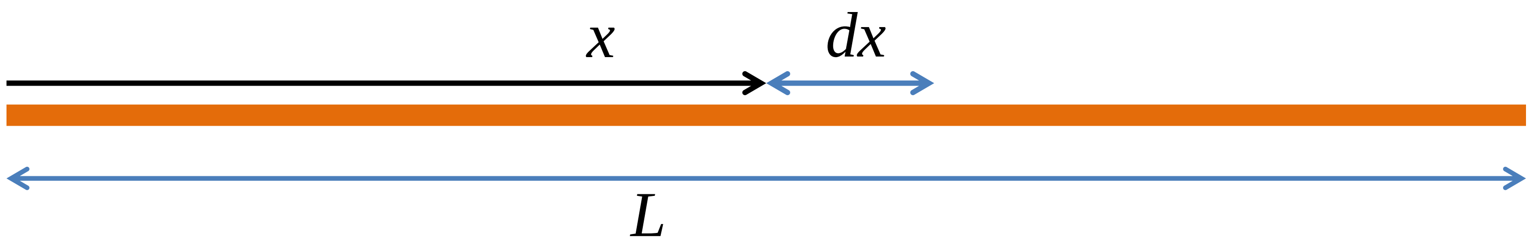
$$\lambda = \frac{dq}{dl}$$



Units of λ : Cm^{-1}

λ could be constant or it could be a function of l; $\lambda = \lambda(l)$

Ex: A wire has a linear charge density $\lambda(x) = 2kx$. k is a constant. The wire has a total length L. Calculate the total charge on this wire.



Charge of length dx of the wire , $dq = \lambda\, dx$

Total charge of the wire, $Q = \int dq = \int_0^L \lambda(x)dx$

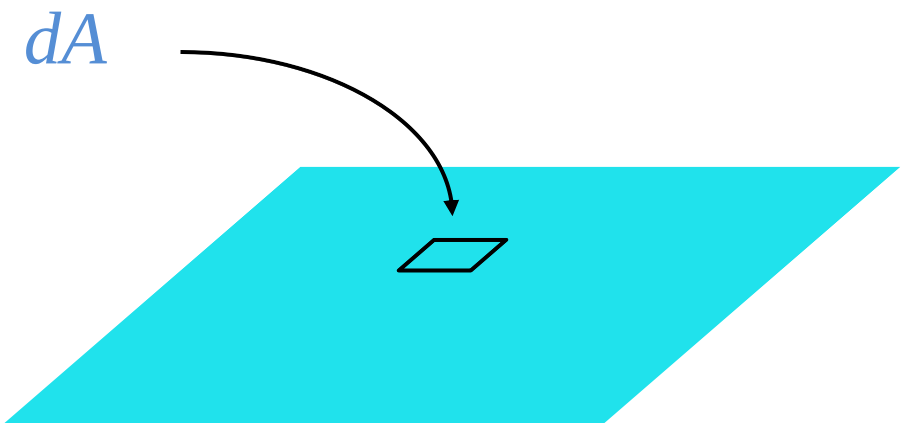
$$= \int_0^L 2kxdx = kL^2$$

2. Surface Charge Distributions

(Eg: a charged metal sheet)

If a small area dA of a surface has a charge dq,

the surface charge density, $\sigma = \dfrac{dq}{dA}$



Units of surface charge density, σ : Cm^{-2}

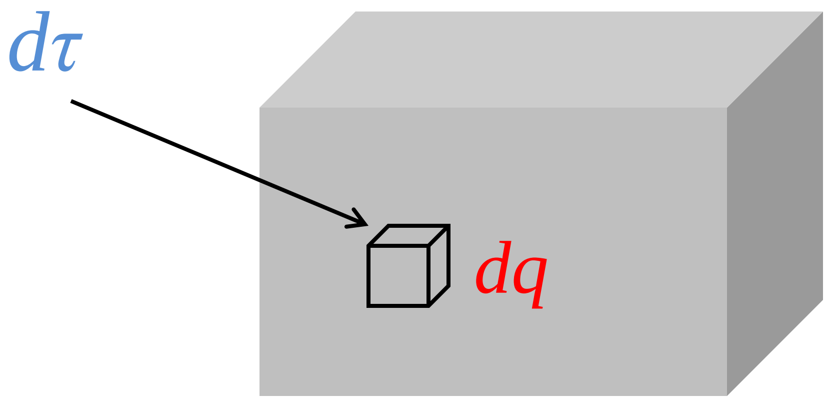
3. Volume Charge Distributions

(Eg: a charged sphere, charged cloud)

Volume charge density, $\boldsymbol{\rho}$ = charge per unit volume

If a small volume $\boldsymbol{d\tau}$ has a charge dq,

$$\rho = \frac{dq}{d\tau}$$

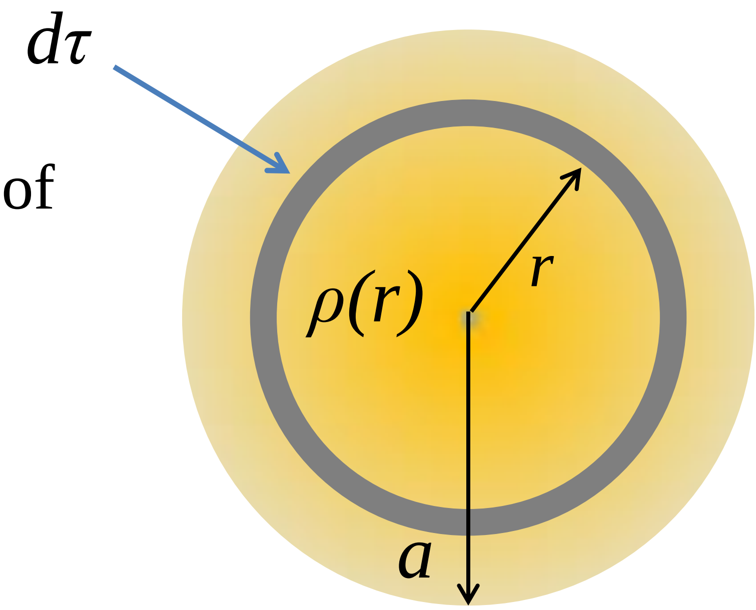


Units of volume charge density $\boldsymbol{\rho}$: Cm^{-3}

$\rho=\rho(r)$ or $\rho=\rho(x,y,z)$

Ex: A spherical charge distribution has a volume charge density $\rho(r) = k/r$; $(0 < r \leq a)$, r is the distance from the center of the charge distribution, k and a are constants. Calculate the total charge of this charge distribution.

$$\rho(r) = {}^{k}\!/_{r}$$



Consider a very thin spherical shell of radius r and thickness dr. Let the volume of this shell be $d\tau$, and the charge be dq.

$$d\tau = 4\pi r^2 dr$$

Total charge of the system

$$Q = \int dq = \int \rho(r) d\tau = \int_0^a \left(\frac{k}{r}\right)(4\pi r^2 dr)$$

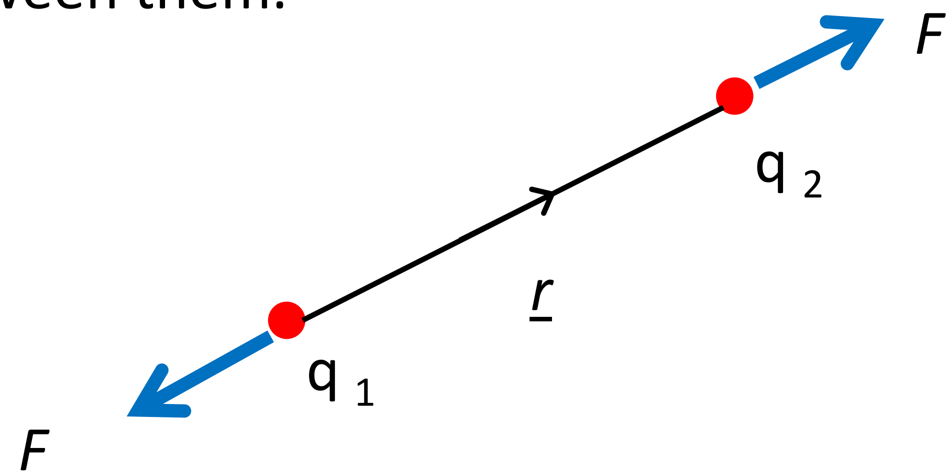
Coulomb's Law

(Fundamental Law in Electrostatics -1785)

The electrostatic force between two point charges acts along the line joining the two charges, is proportional to the magnitude of each charge and is inversely proportional to the square of the distance between them.

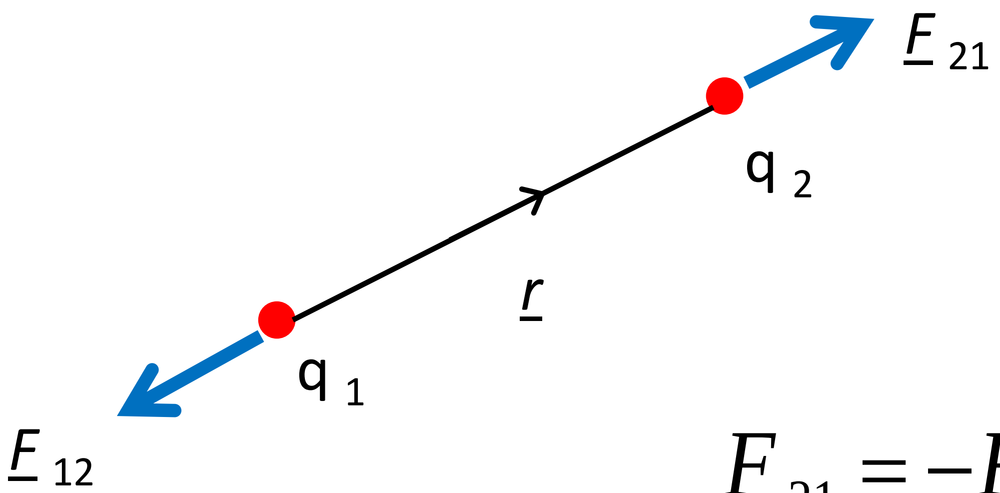
$$F = k\frac{q_1 q_2}{r^2}$$

$$F = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}$$



Electrostatic force on point charge q_2 due to point charge q_1 can be given by the vector equation,

$$\underline{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{\underline{r}}$$



$$\underline{F}_{21} = -\underline{F}_{12}$$

$$F_{21} = F_{12}$$

$\hat{\underline{r}}$ is the unit vector in the direction of $\underline{r}$

Permittivity ε

ε = permittivity of the medium

ε_o = permittivity of free space (vacuum)

$$\varepsilon_o = 8.854 \times 10^{-12} C^2 N^{-1} m^{-2} (Fm^{-1})$$

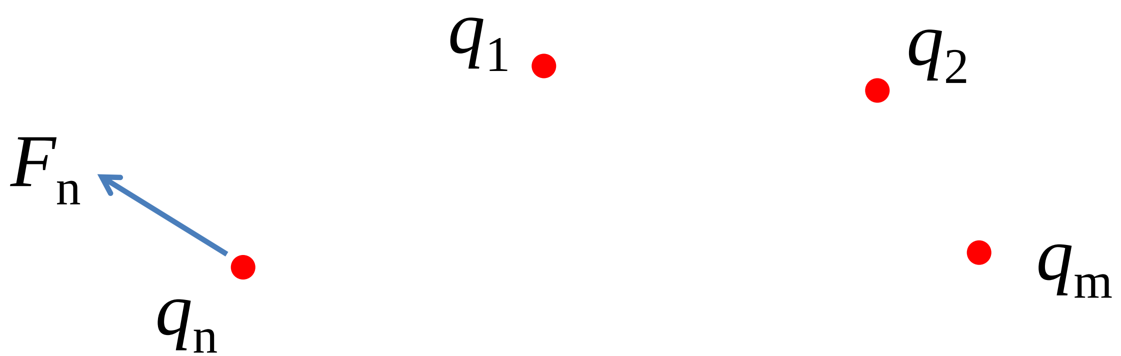
$$\frac{1}{4\pi\varepsilon_o} = 9 \times 10^9 Nm^2 C^{-2}$$

$\varepsilon = \varepsilon_r \varepsilon_o$

ε_r = relative permittivity of the medium

Principle of superposition of electrostatic forces.

Electrostatic force on a point charge q_{n} due to a group of point charges $q_1, q_2, q_3, \ldots\ldots, q_{\mathrm{m}}$



$$\underline{F}_n = \underline{F}_{n1} + \underline{F}_{n2} + \underline{F}_{n3} + \ldots\ldots\ldots = \sum_{i=1}^{m} \underline{F}_{ni}$$

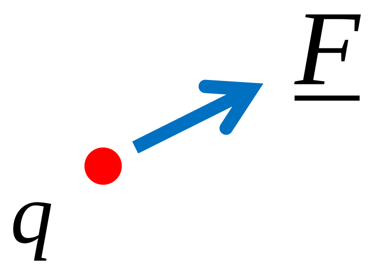
Electric Field

Electric field is a region in space where a stationary charge experiences a force due to another charge distribution.

Electric Field is a Vector Field
(see MIT OpenCourseWare for more details on Vector fields, Electric Fields and comparison with gravitational Field)

Electric Field Intensity $\underline{E}$

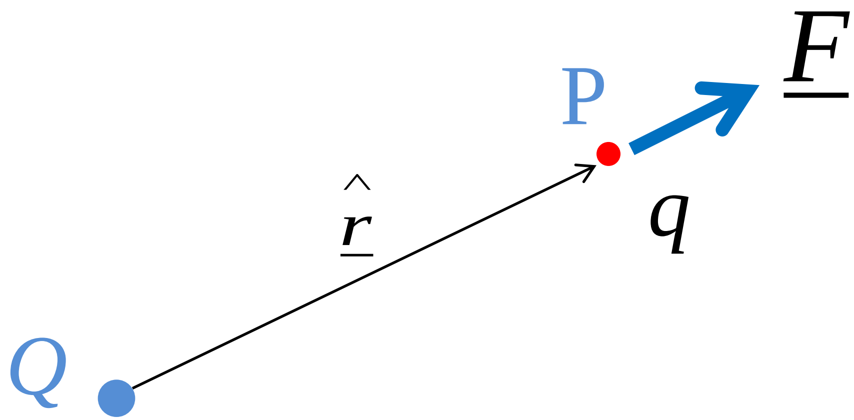
The Electric Field **Intensity** (or Electric Field Strength) at a point is defined as the electric force per unit charge at that point.



If the total electrostatic force on a charge q is $\underline{F}$,

then the Electric Field at that point, $\underline{E} = \dfrac{\underline{F}}{q}$

Electric field at a point P due to a point charge Q



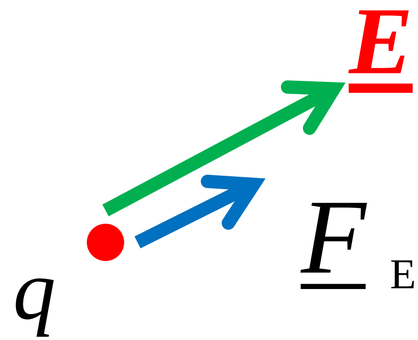
Force on a point charge q at P ,
$$\underline{F} = \frac{1}{4\pi\varepsilon}\frac{Qq}{r^2}\hat{\underline{r}}$$

$\therefore$ Electric field at P,
$$\underline{E} = \frac{\underline{F}}{q}$$
$$= \frac{1}{4\pi\varepsilon}\frac{Q}{r^2}\hat{\underline{r}}$$

Electric force on a point charge

According to the definition of the E. Field, **electric force on a point charge *q*** placed at a point where electric field intensity is E,

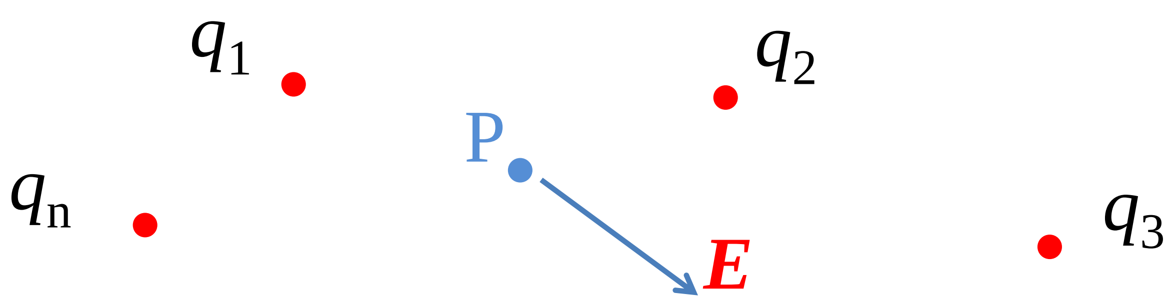
$$\underline{F}_E = \underline{E}q$$



Units of Electric Field : $NC^{-1} = Vm^{-1}$

Electric Field due to a Group of Point Charges

Electrostatic Field at a point (P) due to a group of point charges $q_1, q_2, q_3, \ldots\ldots\ldots, q_n$



$$\underline{E} = \underline{E}_1 + \underline{E}_2 + \underline{E}_3 + \cdots\cdots\cdots + \underline{E}_n = \sum_{i=1}^{n} \underline{E}_i$$

where E_i is the Electric field at P due to charge q_i.

Point charges q_1, q_2, q_3 and q_4 are placed at the corners of a square of side a. $q_1 = q_3$ and $q_2 = q_4$. Calculate the electric field intensity at a point on the central axis of the square at a distance d from the center.

Electric Field at **P**

$$\underline{E} = \underline{E}_1 + \underline{E}_2 + \underline{E}_3 + \underline{E}_4$$

Also, $\underline{E} = \underline{E}_x + \underline{E}_y$

$$\underline{E}_x = \underline{E}_{1x} + \underline{E}_{2x} + \underline{E}_{3x} + \underline{E}_{4x} = 0$$

(*from symmetry*)

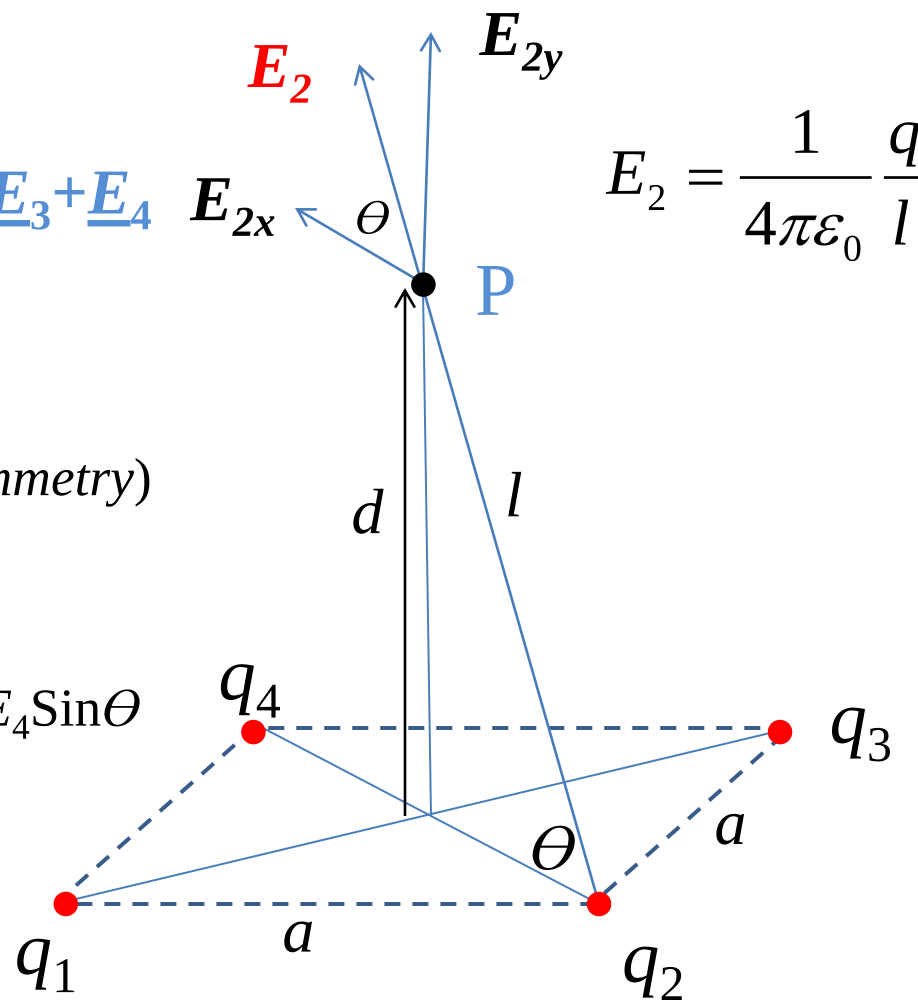
$$\underline{E}_y = \underline{E}_{1y} + \underline{E}_{2y} + \underline{E}_{3y} + \underline{E}_{4y}$$

$$E_y = E_{1y} + E_{2y} + E_{3y} + E_{4y}$$

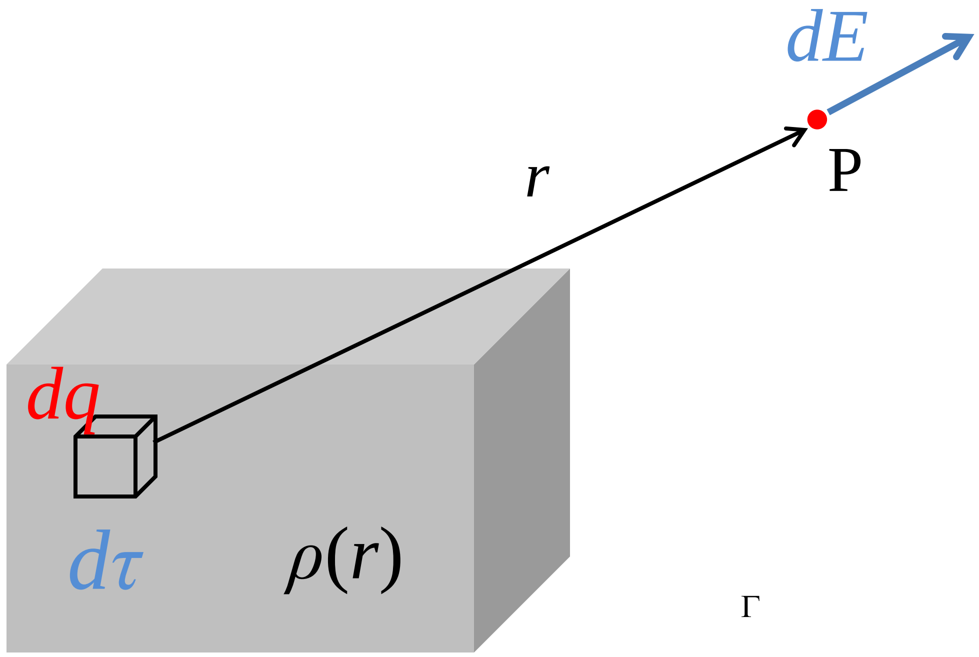
$$= E_1 \mathrm{Sin}\Theta + E_2 \mathrm{Sin}\Theta + E_3 \mathrm{Sin}\Theta + E_4 \mathrm{Sin}\Theta$$

$$\therefore E = E_y = \frac{(q_1 + q_2 + q_3 + q_4)}{4\pi\varepsilon_0} \left(\frac{d}{l^3} \right)$$

$$E_2 = \frac{1}{4\pi\varepsilon_0} \frac{q_2}{l^2}$$



Electric Field at a point due to a Continuous charge distribution



Charge distribution has volume charge density $\rho(r)$. Identify a suitable volume element (very small volume) of the charge distribution $d\tau$.

Let the charge of this volume element be dq and the Electric field at P due to this volume element be dE.

Electric Field at a point due to a Continuous charge distribution . . . .

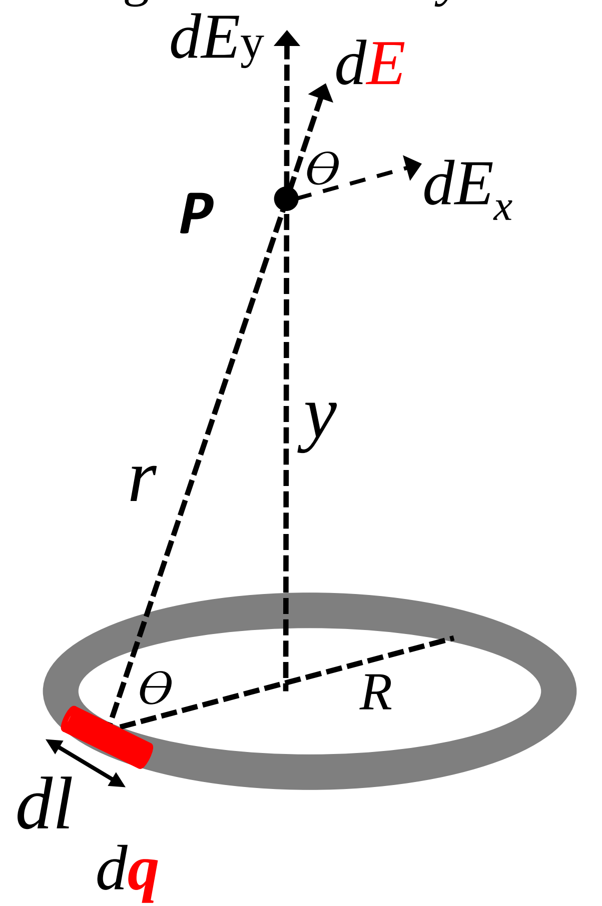
Then $$d\underline{E} = \frac{1}{4\pi\varepsilon}\frac{dq}{r^2}\hat{\underline{r}}$$

$$dq = \rho d\tau \qquad \therefore d\underline{E} = \frac{1}{4\pi\varepsilon}\frac{\rho d\tau}{r^2}\hat{\underline{r}}$$

Total Electric field at P, $$\underline{E} = \int d\underline{E} = \int \frac{1}{4\pi\varepsilon}\frac{\rho d\tau}{r^2}\hat{\underline{r}}$$

$$= \frac{1}{4\pi\varepsilon}\int_{\Gamma} \frac{\rho d\tau}{r^2}\hat{\underline{r}}$$

Ex.1. A thin charged ring of radius *R* has a linear charge density λ Cm^{-1}. Calculate the Electric field intensity on the axis of the ring at a distance *y* from the center.



Consider a length *dl* of the ring.
Charge of the length *dl*, $dq = \lambda dl$.

Electric field at P due to length *dl* $= d\underline{E}$

Electric field at P due to whole ring, $\underline{E} = \underline{E}_x + \underline{E}_y$

From Symmetry, $E_x = 0$

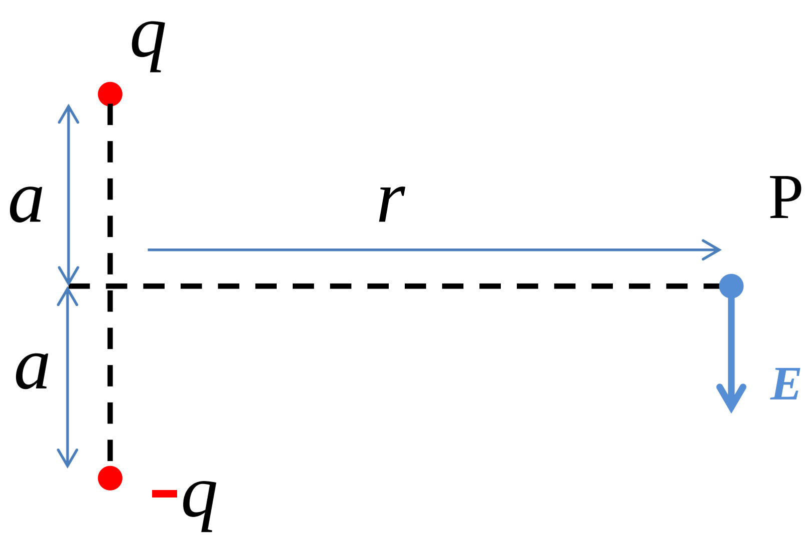
$$E_y = \int dE_y = \int dE \, Sin\theta$$

$$= \int_0^{2\pi R} \frac{\lambda dl}{4\pi\varepsilon_0 r^2} Sin\theta$$

$$\therefore E = E_y = \frac{\lambda Sin\theta}{4\pi\varepsilon_0 r^2} \int_0^{2\pi R} dl = \frac{\lambda R y}{2\varepsilon_0 \left(R^2 + y^2\right)^{3/2}}$$

Electric Dipole

Two equal and opposite charges placed close to each other forms an Electric Dipole



Electric field at point P, a distance *r* along the perpendicular bisector of the line joining the two charges when $r \gg a$

$$E \cong \frac{1}{4\pi\varepsilon_0}\frac{2aq}{r^3}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$$

$(2aq)$ is called the electric dipole moment $\boldsymbol{p}$.