Spring 2021

Section 7 (DSSA)

Forecasting – Part 2

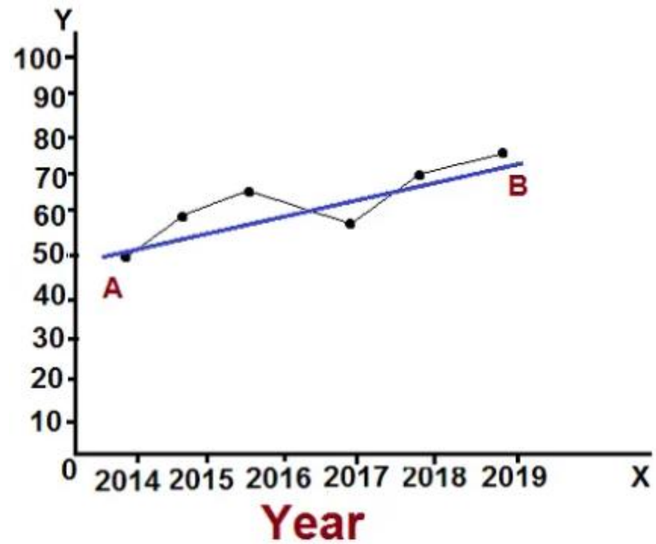
Trend Projection:

- It fits a trend line to a series of historical data points.
- The line is projected into the future for medium to long range forecasts.
- The simplest is a linear model developed using regression analysis.

$$Y^{\wedge} = b_0 + b_1 x$$

Where:

- $Y^{\wedge}$ = Predicted value
- b_0 = intercept
- b_1 = slope of the line
- X = time period



Seasonal Variations:

- Recurring variations over time may indicate the need for seasonal adjustments in the trend line
- Analyzing data in monthly or quarterly terms usually makes it easy to spot seasonal patterns.
- A seasonal index indicates how a particular season (e.g., month or quarter) compares with an average season.
- When no trend is present, the seasonal index can be found by dividing the average value for a particular season by the average of all the data

1. Get the average of the same period over all the years
2. Get the total (summation) of all the averages.
3. Divide the total of averages by number of periods of time.
4. Calculate the seasonal index of each period by: $\frac{Step\ 1}{Step\ 3}$

Example 1

Quarter	***Year***			***Average 3-year sales***	***Quarterly sales***	***Average seasonal index***
	2003	***2004***	***2005***			
Quarter 1	72	75	76	(72+75+76)/3 = 74.33	291.32/4 = 72.83	1.02
Quarter 2	64	66	68	66	72.83	0.906
Quarter 3	63	64	67	64.66	72.83	0.887
Quarter 4	75	89	95	86.33	72.83	1.185
Total				291.32		

If the average demand for year 2006 is forecasted to be 350, then what is the forecasted value of each quarter, taken into consideration the calculated seasonal index?

Quarter	**Forecasted Seasonal value**
Quarter 1	$\frac{350}{4} * 1.02 = 89.25$
Quarter 2	79.275
Quarter 3	77.6
Quarter 4	103.687

Seasonal variations with Trend:

- When both trend and seasonal components present, the forecasting task is more complex.
- Seasonal indices should be computed using a ***centered moving average (CMA)*** approach.
- **Steps:**

1) Compute the CMA for each observation (where possible).
2) Compute the seasonal ratio = observation / CMA for that observation.
3) Average seasonal ratios to get seasonal indices.
4) If seasonal indices do not add to the number of seasons, multiply each index by (Number of seasons/sum of indices)

Example 2

Time Period	Quarter	Sales (in millions)
1	1	1306
2	2	1305
3	3	1311
4	4	1313
5	1	1324
6	2	1329
7	3	1346
8	4	1347
9	1	1378
10	2	1394
11	3	1441
12	4	1469

Solution:

- The centered moving average is the average of two moving averages. Therefore, we have to calculate the moving average first.

- We are dealing here with 4-MA, because the year is divided into 4 quarters (timestamps) or based on what will be required in the problem.

- Average of the first 4 periods = [1306 + 1305 + 1311 + 1313] / 4 = 1308.75
- Average of the second 4 periods = [1305 + 1311 + 1313 + 1324] / 4 = 1313.25

- Since, we got now two moving averages, we can calculate the first centered moving average which is = [1308.75 + 1313.25] / 2 = 1311

- Now, we need to know the location of this centered moving average or what we call centered location = the average of the centers of the moving averages.

- The center of the first 4 periods equals [(1 + 4) / 2] = 2.5
- The center of the second 4 periods equals [(2 + 5) / 2] = 3.5
- Therefore, the first centered location of the CMA is [(2.5 + 3.5) / 2] = 3
- The second centered moving average will be the average of (the second 4 periods and the third 4 periods)
- Average of the second 4 periods = [1305 + 1311 + 1313 + 1324] / 4 = 1313.25
- Average of the third 4 periods = [1311 + 1313 + 1324 + 1329] / 4 = 1319.25
- Therefore, the 2^{nd} centered moving average = [1313.25 + 1319.25] / 2=1316.25
- You don't need here to calculate the centered location, because it will be automatically period 4.

And so on, until you cover all the data

Time Period	**Quarter**	**Sales (in millions)**	**CMA**	**Seasonal Ratio**
1	1	1306		
2	2	1305		
3	3	1311	1311	1
4	4	1313	1316.25	0.997531
5	1	1324	1323.625	1.000283
6	2	1329	1332.25	0.997561
7	3	1346	1343.25	1.002047
8	4	1347	1358.125	0.991809
9	1	1378	1378.125	0.999909
10	2	1394	1405.25	0.991994
11	3	1441		
12	4	1469		

Therefore, the seasonal indices of the 4 quarters are:

- SA of Quarter 1 = (1.000283+ 0.999909) / 2 = 1.0000963
- SA of Quarter 2 = (0.997561+ 0.991994) / 2 = 0.994777
- SA of Quarter 3 = (1 + 1.002047) / 2 = 1.001024
- SA of Quarter 4 = (0.997531+ 0.991809) / 2 = 0.99467
- Summation approximately equals 4.

The Decomposition method for Forecasting:

- Decomposition is the process of isolating linear trend and seasonal factors to develop more accurate forecasts.

- There are 5 main steps of decomposition:
  - Compute seasonal indices using CMA approach.
  - De-seasonalize the data by dividing each number by its seasonal index.
  - Find the equation of a trend line using the de-seasonalized data.
  - Forecast for the future periods using the trend line.
  - Multiply the trend line forecast by the appropriate seasonal index.

Example 3

Year	Period	Quarter	Data
1997	1	1	7130
	2	2	6940
	3	3	7354
	4	4	7556
1998	5	1	7673
	6	2	7332
	7	3	7662
	8	4	7809
1999	9	1	7872
	10	2	7551
	11	3	7989
	12	4	8143
2000	13	1	8167
	14	2	7902
	15	3	8268
	16	4	8436

Step 1: calculate the seasonal indices using centered moving average:

YEAR	PERIOD	QUARTER	Data	CENTERED MOVE AVG.
1997	1	1	7130	
	2	2	6940	
	3	3	7354	7312.875
	4	4	7556	7429.75
1998	5	1	7673	7517.25
	6	2	7332	7587.375
	7	3	7662	7643.875
	8	4	7809	7696.125
1999	9	1	7872	7764.375
	10	2	7551	7847
	11	3	7989	7925.625
	12	4	8143	8006.375
2000	13	1	8167	8085.125
	14	2	7902	8156.625
	15	3	8268	
	16	4	8436	

Year	Period	Quarter	Data	CMA	Seasonal Ratio
1997	1	1	7130		
	2	2	6940		
	3	3	7354	7312.875	1.005
	4	4	7556	7429.75	1.0169
1998	5	1	7673	7517.25	1.0207
	6	2	7332	7587.375	0.966
	7	3	7662	7643.875	1.00237
	8	4	7809	7696.125	1.0146
1999	9	1	7872	7764.375	1.01386
	10	2	7551	7847	0.96
	11	3	7989	7925.625	1.0079
	12	4	8143	8006.375	1.017
2000	13	1	8167	8085.125	1.01
	14	2	7902	8156.625	0.9687
	15	3	8268		
	16	4	8436		

Therefore, the seasonal indices of the 4 quarters are:

- SA of Quarter 1 = (1.0207 + 1.01386 + 1.01) / 3 = 1.01485
- SA of Quarter 2 = (0.966+ 0.96 + 0.9687) / 3 = 0.9649
- SA of Quarter 3 = (1.005 + 1.00237 + 1.0079) / 3 = 1.00509
- SA of Quarter 4 = (1.0169 + 1.0146 + 1.017) / 3 = 1.016
- Summation approximately equals 4.

Step 2: De-seasonalize the data:

Year	Period	Quarter	Data	De-Seasonalized Data
1997	1	1	7130	7130 / 1.01485 = 7025.668
	2	2	6940	7192.45
	3	3	7354	7316.757
	4	4	7556	7437
1998	5	1	7673	7560.7
	6	2	7332	7598.714
	7	3	7662	7623.1979
	8	4	7809	7686
1999	9	1	7872	7756.8
	10	2	7551	7825.68
	11	3	7989	7948.54
	12	4	8143	8014.76
2000	13	1	8167	8047.49
	14	2	7902	8189.45
	15	3	8268	8226.129
	16	4	8436	8303.15

Step 3: Find equation of the trend line of the de-seasonalized data:

- Using Excel QM add-in normally or manually, to find the regression trend line equation.
- The final equation is:
- $Y^{\wedge} = 7068.28 + 78.38\,X$, where X is the time.
- Therefore, the forecast for period 17 will be = (7068.28) + (78.38) (17) = 8400.78
- **Take care:** this forecasted value is deseasonalized.
- So, in order to add the seasonality part, multiply by the appropriate seasonal index.
- Since, it is quarter 1 in year 2001 → $Y^{\wedge} = (8400.78)(1.01485) = 8525.532$

Utilize excel QM to build Forecasting Models

1. Open the Excel QM.
2. Click on the “By Chapter Tab” and choose (Chapter 5: Forecasting), then choose the requested method (Decomposition).
    - Enter the number of periods and the number of seasons, then click ok