Spring 2021

Section 2 (DSSA)

Decision Trees

- A decision tree can be used to visually and explicitly represent decisions and decision making.
- Any problem that is represented in a decision table can also be represented graphically using decision tree.
- Decision trees are most beneficial when a sequence of decisions must be made.
- All decision trees contain *decision points* or nodes and *states of nature points* or nodes.
  - A decision node from which one of several alternatives may be chosen.
  - A state of nature node out of which one state of nature will occur.

5 Steps to decision tree analysis:

- Define the problem.
- Structure or draw the decision tree.
- Estimate payoffs for each possible combination of alternatives and states of nature.
- Solve the problem by computing expected monetary values (EMV) for each state of nature node.

Structure of decision trees:

- They start from left to right
- Represent decision and outcomes in a sequential order.
- Decision nodes are represented by *squares*.
- States of nature nodes are represented by *circles*.
- Lines or branches connect the decision and the states of nature nodes.

Example 1:

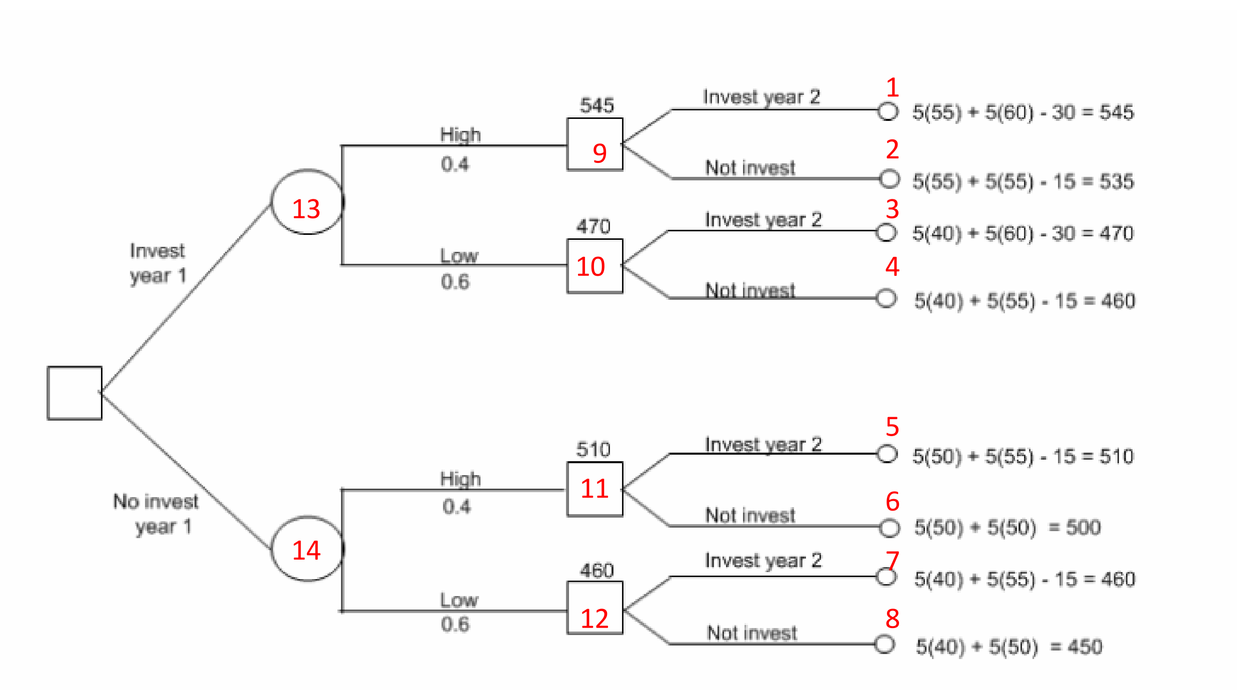
A company faces a decision as to whether to invest (or not) in a particular facility. In each of the next two years they can spend £15K in expanding the capacity of the facility by 5000 units. The current capacity is 50000 units of a particular product X. So, for example, if they invest in years 1 and 2 the capacity will increase to 55000 in year 1 and to 60000 in year 2.

The demand for product X is forecast to be high (70000 units) or low (40000 units) in each of the next two years. The probability of the demand being high is 0.4 in year 1. The probability of the demand being low in year 2 is zero. The company sells the product for a profit per unit of £5 (after accounting for fixed and variable production costs). If the company cannot satisfy market demand then customers buy the product from overseas suppliers for a price much higher than the company charges.

Assume that any investment in expanding capacity in a particular year takes effect instantaneously at the start of that year.

Analyze this situation using a decision tree. What would you suggest the company should do?

Answer:



We have numbered the nodes in the decision tree for easier calculations.

1) Start by calculating the payoffs of the decision tree from right to left (backwards).

- Payoff of node 1 = 5(55) + 5(60) – 30 = 545
- Payoff of node 2 = 5(55) + 5(55) – 15 = 535
- Payoff of node 3 = 5(40) + 5(60) – 30 = 470
- Payoff of node 4 = 5(40) + 5(55) – 15 = 460
- Payoff of node 5 = 5(50) + 5(55) – 15 = 510
- Payoff of node 6 = 5(50) + 5(50) = 500
- Payoff of node 7 = 5(40) + 5(55) – 15 = 460
- Payoff of node 8 = 5(40) + 5(50) = 450

2) For any state of nature node, calculate its EMV

3) For any state of decision node, choose the best value

- Decision of node 9 = max [545, 535] = 545
- Decision of node 10 = max [470, 460] = 470
- Decision of node 11 = max [510, 500] = 510
- Decision of node 12 = max [460, 450] = 460

- EMV of node 13 = (0.4) (545) + (0.6) (470) = 500
- EMV of node 14 = (0.4) (510) + (0.6) (460) = 480

- Decision of node 15 = max [500, 480] = 500

Therefore, the optimal decision is to invest in year 1

Expected Value of sample information:

EVSI = [expected value with sample information assuming no cost to gather it] – expected value of best decision without sample information]

= (EV with sample information + cost) – (EV without sample information)

= (500 + 15) – (480) = 35

This means, that we can pay up to £35 K and still the information is worthwhile.

Sensitivity Analysis:

- How sensitive are the decision to changing the probabilities of the states of nature nodes?
- If we change a probability, would we make the same decision?
- How much could it change before we would make a different decision?

Back to our previous example:

We are indifferent if:

EMV (node 13) = EMV (node 14)

Let:

- Probability of the demand being high = P
- Probability of the demand being low = 1-P

Therefore,

$$(P)\ (545) + (1\text{-}P)\ (470) = (P1)\ (510) + (1\text{-}P)\ (460)$$

$$545P + 470 - 470P = 510P + 460 - 460P$$

$$75P + 470 = 50P + 460$$

P = -0.4 (which will never exist)

So, no matter what value P will have, the optimal decision will always be to invest in year 1.

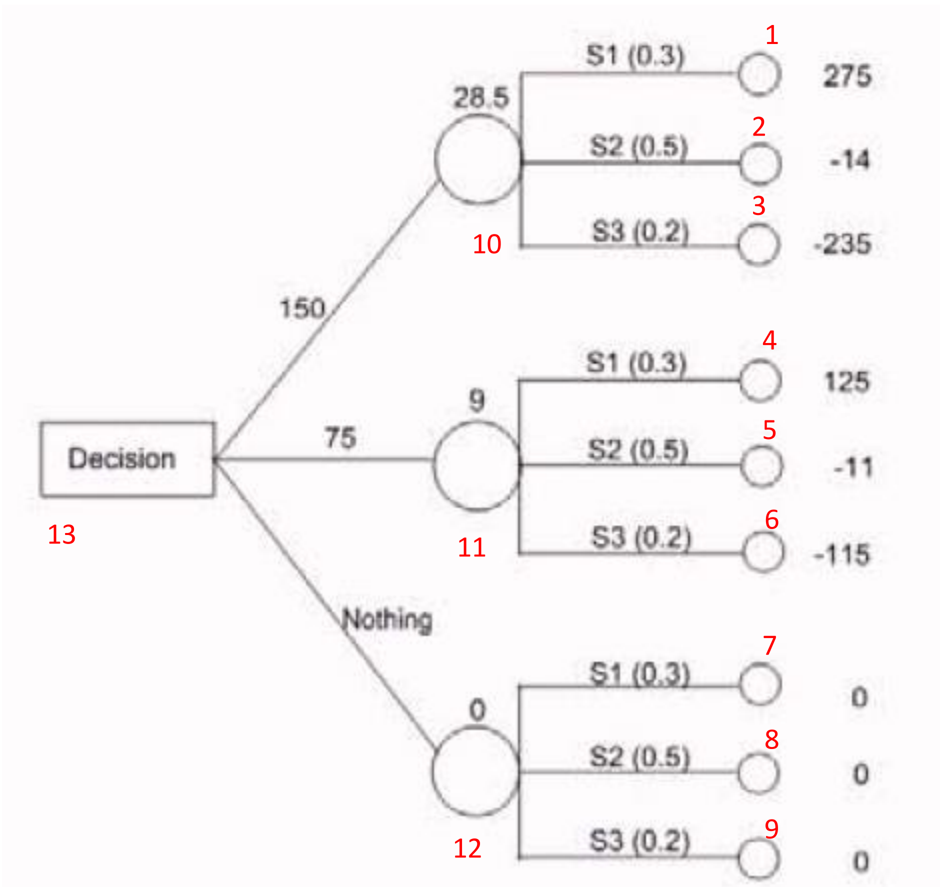
Example 2(Practice Problem):

A company faces three choices with regard to a potential project they are considering with a partner company. They can choose to invest £150 million and this will entitle them to 85% of any profits made. Alternatively they could invest £75 million, but this would only entitle them to 40% of any profits made. In either case if a negative profit (loss) is made the company has to fund the appropriate percentage (85% and 40%) of the loss. Of course the company could choose not to invest at all.

There are three scenarios as to the performance of the project. It may be very successful (probability 0.3) giving a total profit of £500 million to be shared between the companies. Alternatively it may be only moderately successful (probability 0.5) giving a total profit of £160 million to be shared between the companies. The final alternative is that the project will be unsuccessful, giving a total loss of £100 million to be shared between the companies. In all three cases the total profit to be shared does not include the initial investment made by the company.

Analyze this situation using a decision tree. What would you suggest the company should do?

Answer:



We have numbered the nodes in the decision tree for easier calculations.

4) Start by calculating the payoffs of the decision tree from right to left (backwards).

- Payoff of node 1 = -150 + (0.85) (500) = 275
- Payoff of node 2 = -150 + (0.85) (160) = -14
- Payoff of node 3 = -150 + (0.85) (-100) = -235
- Payoff of node 4 = -75 + (0.4) (500) = 125
- Payoff of node 5 = -75 + (0.4) (160) = -11
- Payoff of node 6 = -75 + (0.4) (-100) = -115
- Payoff of node 7 = 0
- Payoff of node 8 = 0
- Payoff of node 9 = 0

5) For any state of nature node, calculate its EMV

- EMV of node 10 = (0.3) (275) + (0.5) (-14) + (0.2) (-235) = 28.5
- EMV of node 11 = (0.3) (125) + (0.5) (-11) + (0.2) (-115) = 9
- EMV of node 12 = 0

6) For any state of decision node, choose the best value

- Since, we are talking about profit
- Thus, Max [28.5, 9, 0] = 28.5

Therefore, the optimal decision is to invest $150 m

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