Spring 2021

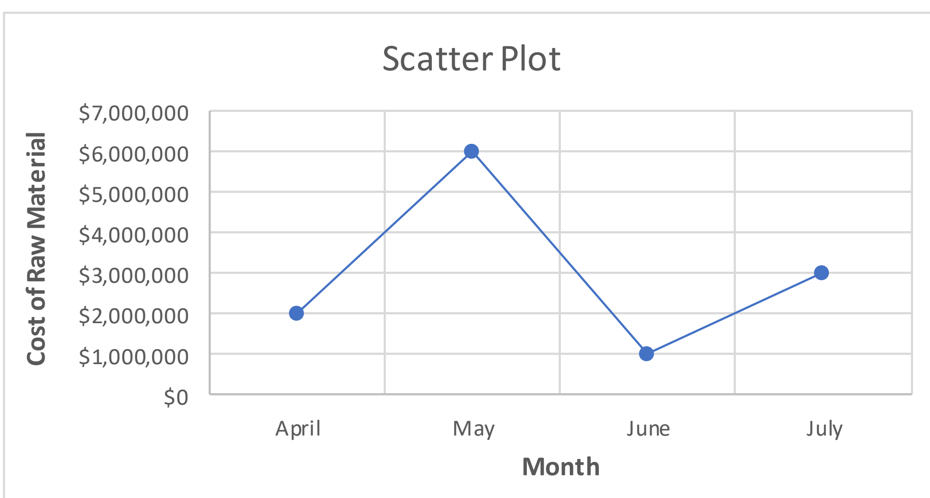
Section 6 (DSSA)

Forecasting – Part 1

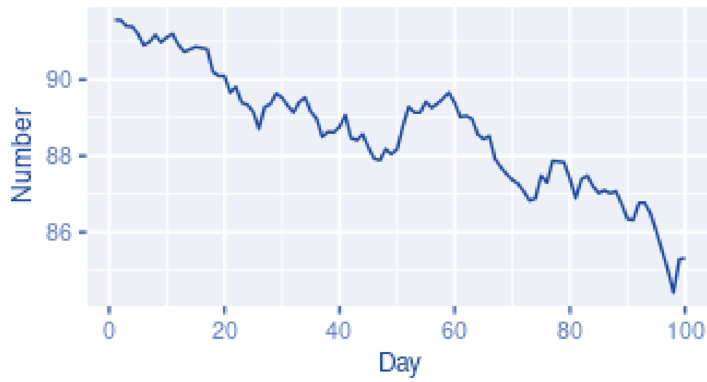
First: Forecasting Overview

- Goal/Objective of Forecasting
  - The idea is that we want to *predict* what will happen in the *future*
  - If we *reduce the uncertainty* about the future, then we can *make better decisions* in the present
  - Example:
    1. Past:
       - Cost of Raw Material in the last 3 months:
         - April: $ 2M
         - May: $ 6M
         - June: $ 1M
    2. Forecast:
       - Cost of Raw Material next month:
         - July: $ 3M
    3. Decision:
       - Currently we can buy the Raw Material for *$ 1M*, or we can wait and buy it next month at *$ 3M*
       - Buy the Raw Material *now* and store it to use it next month!

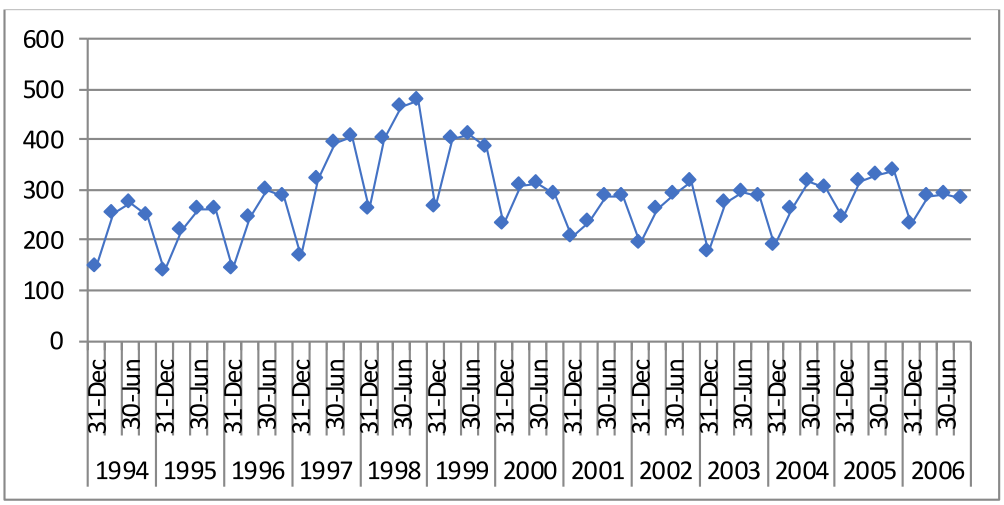
- A plot of the data *"A Scatter Diagram"* may highlight its relationships
  - A *Scatter Diagram* of time-series data is a graph that shows the forecasted variable on the vertical axis and the time variable on the horizontal axis.
  - Trend, cyclical, or seasonal patterns can be recognized.



1. Trend
    - A trend in time series is a persistent, long-term growth or decline.
    - Example: Attendance in FCAI Lectures
        - Downward trend:
            - Attendance decreases with time (low as we approach final exams and graduation project discussion)

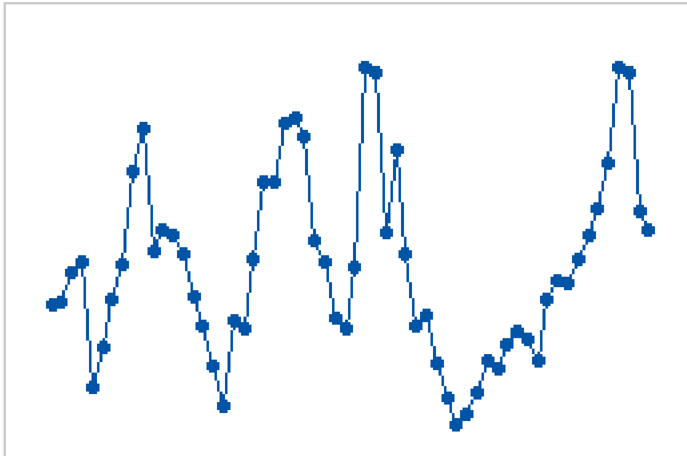


2. Seasonality
    - A seasonal time series is a series with a pattern of change that repeats itself at regular intervals.
    - Example: Ice-cream sales
        - Every year:
            - High mid-year (Summer)
            - Low at the beginning and the end (Winter)



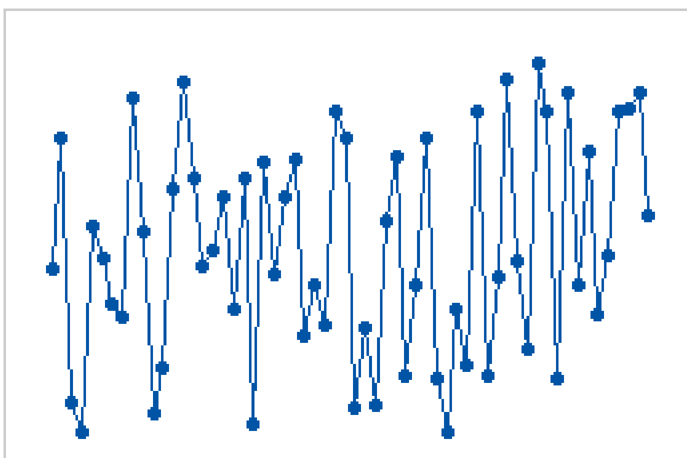
3. Cyclic

- A cyclical effect is as wavelike fluctuation around the trend.
- Patterns in annual data that occur every several years.
- Data exhibit rises and falls that are not of fixed period.
- Example: “Borsa”



4. Randomness

- There are random fluctuations which do not appear to be very predictable, and no strong patterns.
- These variations may be due to floods, famines, earthquakes, strikes, etc.



<u>Example 1</u>

Which of the following is an example of time series problem?

1. Estimating number of hotel rooms booking in next 6 months.
2. Estimating the total sales in next 3 years of an insurance company.
3. Estimating the number of calls for the next one week.

A) Only 3
B) 1 and 3
C) 1,2 and 3

Solution: (C) All the above options have a time component associated.

<u>Second: Forecasting Techniques</u>

- Qualitative Methods
  - Based on expert opinions, industry estimates, or input from customer surveys.
  - Useful when subjective factors are thought to be important or when accurate quantitative data is difficult to obtain.

- Causal Methods
  - Based on finding relationships between the forecast variable and the independent variable(s).
  - Example: Regression Models.

- Time-Series Methods
  - Based on using historical data - "Predicting the Future based on the Past"

1. <u>Naïve Method:</u>

- New Value = Old Value
  - Example:
    - Cost of Raw Material in the last 3 months:

April	$ 2M
May	$ 6M
June	$ 1M

- Cost of Raw Material next month = {July: $ 1M}

2. Moving Average Method:

- New Value = $\frac{\sum previous\ n\ periods}{n}$
  - Example:
    - Cost of Raw Material in the last 3 months:

April	$ 2M
May	$ 6M
June	$ 1M

    - Cost of Raw Material next month
      = {July: $\frac{2+6+1}{3} = \frac{9}{3} = \$\ 3M$}

3. Weighted Moving Average Method:

- New Value = $\frac{\sum(weight\ of\ period)*(value\ in\ period)}{\sum weights}$
  - Example:
    - Cost of Raw Material in the last 3 months:

April	$ 2M
May	$ 6M
June	$ 1M

    - Let the weights be: 3 for the most recent observation, 2 for the next observation, and 1 for the most distant observation
    - Cost of Raw Material next month
      = {July: $\frac{(1)2+(2)6+(3)1}{6} = \frac{17}{6} = \$\ 5.6M$}

4. Exponential Smoothing Method:

- We want to incorporate a fraction of the error
- New Value = $old\ forecast + \alpha\,(old\ actual - old\ forecast)$
  α: smoothing constant, between 0 and 1
  forecast error
- The new estimate is the old estimate plus some fraction of the error in the last period.
- Therefore as α increases, the model gives more weight to recent data
  - Example:
    - Cost of Raw Material in the last 3 months:

April	$ 2M
May	$ 6M
June	$ 1M

- Cost of Raw Material forecast
  – {July: \$ 5.6M}
- If actual Cost of Raw Material in July was \$ 7M
- Let α = 0.5
- Using Exponential Smoothing:

  New Value $= old\ forecast + \ \alpha\ (old\ actual - old\ forecast)$

  August $= July\ forecast + \ \alpha\ (July\ actual - July\ forecast)$

  August $= 5.6\ + 0.5\ (7 - \ 5.6) = \$\ 6.3M$

5. Exponential Smoothing with Trend Adjustment Method:

- Previous methods provide stable statistics, but they do not pick up trends; they forecast new data values within the past data values
- We want to incorporate adjustment such that the model responds to trend
- Steps:
  - 1st: Exponential Smoothing:
    - New Value = $old\ forecast + \ \alpha\ (old\ actual - old\ forecast)$
  - 2nd: Trend Adjustment:
    - New Trend = $old\ trend + \beta(\underbrace{new\ forecast - old\ forecast}_{\text{Excess trend}})$

      β: smoothing constant, between 0 and 1

      Excess trend
  - 3rd: Forecast including Trend:
    - New Forecast = Exponential Smoothing + Trend Adjustment

- Therefore as β increases, the model responds more to trend
- Sometimes called 2nd order smoothing or double smoothing, because we have two smoothing constants; α and β

  - Example:
    - Cost of Raw Material in the last 3 months:

April	\$ 2M
May	\$ 6M
June	\$ 1M

    - Cost of Raw Material forecast
      – {July: \$ 5.6M}
    - If actual Cost of Raw Material in July was \$ 7M and Trend = 1.2
    - Let α = 0.5, β = 0.6
    - Using Exponential Smoothing with Trend Adjustment:

      Forecast: $5.6\ + 0.5(7 - \ 5.6) = \$\ 6.3M$

      Trend: $1.2\ + 0.6(6.3 - \ 5.6) = \$\ 1.62M$

      Forecast w/ Trend: $6.3 + 1.62 = \$\ 7.92M$

Summary of Rules:

- Moving Average: $F_{t+1} = \frac{Y_t + Y_{t-1} + \ldots + Y_{t-n+1}}{n}$
- Weighted Moving Average: $F_{t+1} = \frac{w_1 Y_t + w_2 Y_{t-1} + \ldots + w_n Y_{t-n+1}}{w_1 + w_2 + \ldots + w_n}$
- Exponential Smoothing: $F_{t+1} = F_t + \alpha\,(Y_t - F_t)$
- Exponential Smoothing with Trend Adjustment:

1. $F_{t+1} = FIT_t + \alpha\,(Y_t - FIT_t)$
2. $T_{t+1} = T_t + \beta\,(F_{t+1} - FIT_t)$
3. $FIT_{t+1} = F_{t+1} + T_{t+1}$

Where

F_{t+1} : the new forecast (for time period t+1)
Y_t : the actual value in time period t
n : the number of periods to average
w_i : the weight for i^{th} observation
F_t : the previous forecast (for time period t)
α : smoothing constant ($0 \le \alpha \le 1$)
FIT_t : the forecast including trend for time period t
T_t : the smoothed trend for time period t
β : smoothing constant ($0 \le \beta \le 1$)

Example 2

You are given the data below, for the monthly sales (in millions) of the Lenovo ThinkPad Laptop.

1. Use the Exponential Smoothing method to forecast sales for months 2 to 13, assuming $F_1 = 1306$ and letting $\alpha = 0.8$

2. You should have realized that the sales for Lenovo ThinkPad Laptops have been increasing over the past years. We just learned that the Double Exponential Smoothing method can account for such a trend in the data. Use Double (Trend-Adjusted) Exponential Smoothing method to forecast sales for months 2 to 13, assuming $F_1 = 1306$, $T_1 = 15$ and letting $\alpha = 0.8$, $\beta = 0.2$

3. Compute the MAD and MAPE for months 2 to 12 for the double exponential smoothing forecast and compare them with those from simple exponential smoothing. Based on your calculations, which approach is better in this particular case, single or double exponential smoothing? Explain.

Month	Sales (in millions)
1	1306
2	1305
3	1311
4	1313
5	1324
6	1329
7	1346
8	1347
9	1378
10	1394
11	1441
12	1469

Solution:

1. Exponential Smoothing method to forecast sales for months 2 to 13, assuming F_1 = 1306 and letting α = 0.8

Month	Sales Actual	Forecast $F_{t+1} = F_t + \alpha\,(Y_t - F_t)$ $new\ forecast = old\ forecast + 0.8\,(old\ actual - old\ forecast)$
1	1306	F_1 = 1306 "Given"
2	1305	F_2 = 1306 + 0.8 (1306 – 1306) = 1306
3	1311	F_3 = 1306 + 0.8 (1305 – 1306) = 1305.2
4	1313	F_4 = 1305.2 + 0.8 (1311 – 1305.2) = 1309.8
5	1324	F_5 = 1309.84 + 0.8 (1313 – 1309.84) = 1312.4
6	1329	F_6 = 1312.4 + 0.8 (1324 – 1312.4) = 1321.7
7	1346	F_7 = 1321.7 + 0.8 (1329 – 1321.7) = 1327.5
8	1347	F_8 = 1327.5 + 0.8 (1346 – 1327.5) = 1342.3
9	1378	F_9 = 1342.3 + 0.8 (1347 – 1342.3) = 1346.1
10	1394	F_{10} = 1346.1 + 0.8 (1378 – 1346.1) = 1371.6
11	1441	F_{11} = 1371.6 + 0.8 (1394 – 1371.6) = 1389.5
12	1469	F_{12} = 1389.5 + 0.8 (1441 – 1389.5) = 1430.7
13		F_{13} = 1430.7 + 0.8 (1469 – 1430.7) = 1461.3

2. Exponential Smoothing with Trend Adjustment method to forecast sales for months 2 to 13, assuming $F_1 = 1306$, $T_1 = 15$ and letting $\alpha = 0.8$, $\beta = 0.2$

Month	**Sales Actual**	**Smoothed Forecast** $F_{t+1} = FIT_t + \alpha\,(Y_t - FIT_t)$	**Trend** $T_{t+1} = T_t + \beta\,(F_{t+1} - FIT_t)$	**Forecast including Trend** $FIT_{t+1} = F_{t+1} + T_{t+1}$
1	1306	F_1=1306 "Given"	T_1 = 15 "Given"	FIT_1 =1306 +15 = 1321
2	1305	F_2=1321 + 0.8 (1306 – 1321) = 1309	T_2 = 15 + 0.2 (1309 – 1321) = 12.6	FIT_2 =1309 +12.6 =1321.6
3	1311	F_3 = 1321.6 + 0.8 (1305 – 1321.6) = 1308.3	T_3 = 12.6 + 0.2 (1308.3 –1321.6) = 9.9	FIT_3 =1308.3 +9.9 = 1318.2
4	1313	1312.5	8.8	1321.2
5	1324	1314.6	7.5	1322.1
6	1329	1323.6	7.8	1331.4
7	1346	1329.5	7.4	1336.9
8	1347	1344.2	8.8	1353.0
9	1378	1348.2	7.9	1356.1
10	1394	1373.6	11.4	1385.0
11	1441	1392.2	12.8	1405.0
12	1469	1433.8	18.6	1452.4
13		1465.7	21.2	1486.9

Third: Forecasting Accuracy

- Mean Absolute Deviation

$$MAD = \frac{\sum |Forecsting\ Error|}{n}$$

  - Forecasting error = actual value – forecast value
  - When comparing different values for α in an Exponential Smoothing Forecasting Model, we choose the α value associated with the least MAD

- Mean Squared Error

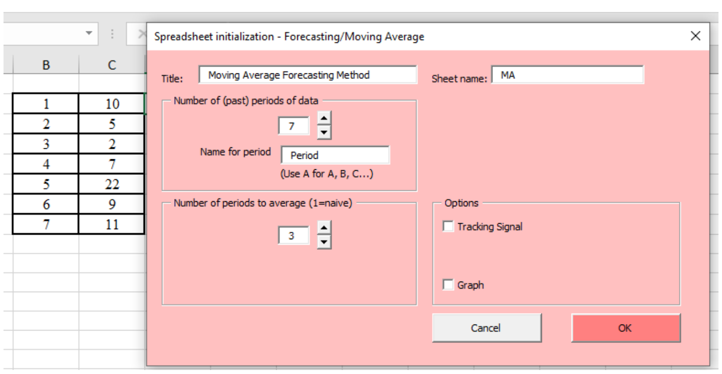
$$MSE = \frac{\sum (Forecasting\ Error)^2}{n}$$

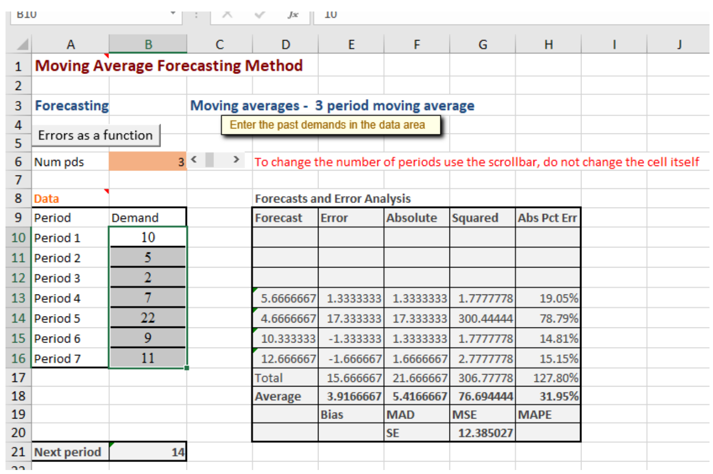
- Mean Absolute Percentage Error

$$MAPE = \frac{\sum \left| \frac{Forecasting\ Error}{Actual\ Value} \right|}{n} * 100\%$$

Utilize excel QM to build Forecasting Models

1. Open the Excel QM.
2. Click on the “By Chapter Tab” and choose (Chapter 5: Forecasting), then choose the requested method.
   - For Moving Average:
     - Enter the number of periods and the number of periods to average (n), then click ok
     - In the new sheet, enter the problem data in the column “Demand
     - F_{t+1} will appear (called Next Period), along with all the error calculations
     - You can change n using the scroll bar





- For Weighted Moving Average:
  - Enter the number of periods and the number of periods to average (n), then click ok
  - In the new sheet, enter the problem data in the column “Demand
  - Enter the period weights in their respective shaded cells
  - F_{t+1} will appear (called Next Period), along with all the error calculations

	A	B	C	D	E	F	G	H	I
1	Weighted Moving Average Forecasting Method								
2									
3	Forecasting		Weighted moving averages - 3 period moving average						
4									
5									
6									
7	Data				Forecasts and Error Analysis				
8	Period	Demand	Weights		Forecast	Error	Absolute	Squared	Abs Pct Err
9	Period 1	10	1	3 periods ago					
10	Period 2	5	2	2 periods ago					
11	Period 3	2	3	1 periods ago					
12	Period 4	7			4.3333333	2.6666667	2.6666667	7.1111111	38.10%
13	Period 5	22			5	17	17	289	77.27%
14	Period 6	9			13.666667	-4.666667	4.6666667	21.777778	51.85%
15	Period 7	11			13	-2	2	4	18.18%
16					Total	13	26.333333	321.88889	185.40%
17					Average	3.25	6.5833333	80.472222	46.35%
18						Bias	MAD	MSE	MAPE
19							SE	12.686388	
20	Next period	12.16666667							
21									

Enter the data in the shaded area. Enter weights in INCREASING order from top to bottom.

- For Exponential Smoothing:
  - Enter the number of periods and click ok
  - In the new sheet, enter the value of alpha in the shaded cell and enter the problem data in the column “Demand”
  - F_{t+1} will appear, along with all the error calculations

- For Trend Adjusted Exponential Smoothing:
  - Enter the number of periods and click ok
  - In the new sheet, enter the values of alpha and beta in their respective shaded cells and enter the problem data in the column “Demand”
  - Enter the given value of T_1 at period 1 (initial Trend)
  - F_{t+1} will appear, along with all the error calculations

Continue Example 2

3. Compute the MAD and MAPE for both methods. Based on your calculations, which approach is better in this particular case, single or double exponential smoothing? Explain.

First Method: Exponential Smoothing Forecasting Method

Month	Sales Actual	Exponential Smoothing Forecast $F_{t+1} = F_t + \alpha\,(Y_t - F_t)$	$\lvert Error \rvert$	$\lvert \frac{Error}{Actual\ Value} \rvert * 100\%$
1	1306	F_1 = 1306 "Given"	|1306-1306| = 0	0%
2	1305	F_2 = 1306 + 0.8 (1306 – 1306) = 1306	|1305-1306| = 1	0.08%
3	1311	F_3 = 1306 + 0.8 (1305 – 1306) = 1305.2	|1311-1305.2| = 5.8	0.44%
4	1313	F_4 = 1305.2 + 0.8 (1311 – 1305.2) = 1309.8	|1313-1309.8| = 3.2	0.24%
5	1324	F_5 = 1309.84 + 0.8 (1313 – 1309.84) = 1312.4	|1324-1312.4| =11.6	0.88%
6	1329	F_6 = 1312.4 + 0.8 (1324 – 1312.4) = 1321.7	7.3	00.55%
7	1346	F_7 = 1321.7 + 0.8 (1329 – 1321.7) = 1327.5	18.5	01.37%
8	1347	F_8 = 1327.5 + 0.8 (1346 – 1327.5) = 1342.3	4.7	00.35%
9	1378	F_9 = 1342.3 + 0.8 (1347 – 1342.3) = 1346.1	31.9	02.32%
10	1394	F_{10} = 1346.1 + 0.8 (1378 – 1346.1) = 1371.6	22.4	01.61%
11	1441	F_{11} = 1371.6 + 0.8 (1394 – 1371.6) = 1389.5	51.5	03.57%
12	1469	F_{12} = 1389.5 + 0.8 (1441 – 1389.5) = 1430.7	38.3	02.61%
13		F_{13} = 1430.7 + 0.8 (1469 – 1430.7) = 1461.3		
		Summation	196.2	14.01%

$$MAD = \frac{\sum \lvert Forecsting\ Error \rvert}{n} = \frac{196.2}{12} = 16.3$$

$$MAPE = \frac{\sum \lvert \frac{Forecasting\ Error}{Actual\ Value} \rvert}{n} * 100\% = \frac{14.01}{12} = 1.17\%$$

Second Method: Exponential Smoothing with Trend Adjustment Forecasting Method

Month	**Sales Actual**	**Forecast including Trend** $FIT_{t+1} = F_{t+1} + T_{t+1}$	$\lvert Error \rvert$	$\lvert \frac{Error}{Actual\ Value} \rvert * 100\%$
1	1306	FIT_1 =1306 +15 = 1321	|1306-1321| = 15	1.1%
2	1305	FIT_2 =1309 +12.6 =1321.6	|1305-1321.6| = 16.6	1.3%
3	1311	FIT_3 =1308.3 +9.9 = 1318.2	|1311-1318.2| = 7.2	0.55%
4	1313	1321.2	8.2	0.63%
5	1324	1322.1	1.9	0.14%
6	1329	1331.4	2.4	0.18%
7	1346	1336.9	9.1	0.68%
8	1347	1353.0	6.0	0.45%
9	1378	1356.1	21.9	1.59%
10	1394	1385.0	9.0	0.65%
11	1441	1405.0	36.0	2.50%
12	1469	1452.4	16.6	1.13%
13		1486.9		
		Summation	150	10.91%

$$MAD = \frac{\sum \lvert Forecsting\ Error \rvert}{n} = \frac{150}{12} = 12.5$$

$$MAPE = \frac{\sum \lvert \frac{Forecasting\ Error}{Actual\ Value} \rvert}{n} * 100\% = \frac{10.91}{12} = 0.91\%$$

Conclusion: Exponential Smoothing with Trend Adjustment is better in this case, since the MAD of the Trend Adjusted Forecasting Model is less than the MAD of the Exponential Smoothing with no Trend Adjustment. This confirms our intuition that the data had a positive trend.