SLP
Parametric Study of Harmonic Drives using FEA

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Introduction

Gears play a crucial role in mechanical engineering as they are fundamental components used for transmitting power and motion between rotating shafts. There is a huge variety of gears; spur, helical, bevel, worm, and planetary gears, each offering unique advantages in terms of efficiency, torque transmission, and design complexity.

Figure: *Planetary gear*

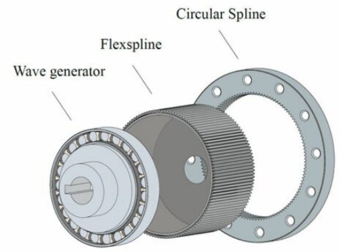
Figure: *Spur gear*

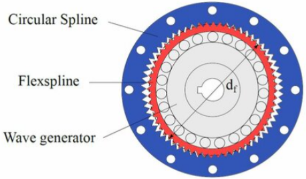
Figure: *Worm Gear*

Figure: *Bevel gear*

Harmonic Drive

- **Wave Generator (WG):** An elliptical cam that deforms the flexspline as it rotates. It is the input element of the harmonic drive.
- **Flexspline (FS):** A thin, flexible cylindrical cup with external teeth, it deforms under loading and acts as output
- **Circular Spline (CS):** A rigid circular ring with internal teeth. It has two more teeth than the flexspline and remains fixed
- **Working Principle:** As the wave generator rotates, it continuously deforms the flexspline, causing its teeth to engage progressively with those of the CS.
  Harmonic Drive Animation – Wikipedia





Objective

The aim of this study is to perform structural analysis on the flexspline when it is forced to deform while being pushed onto the wave generator. A parametric analysis is carried out by varying parameters such as:

- Thickness of the flexspline
- Inner diameter of the flexspline
- Major and minor axes of the wave generator

The effect of these parameters on the stress developed in the flexspline is then studied. The simulation consists only of the wave generator and flexspline. Bearings and the circular spline are not included to keep the model simple and computationally efficient.

Model

Before beginning the parametric analysis, an initial model was created using the following parameters for mesh convergence study:

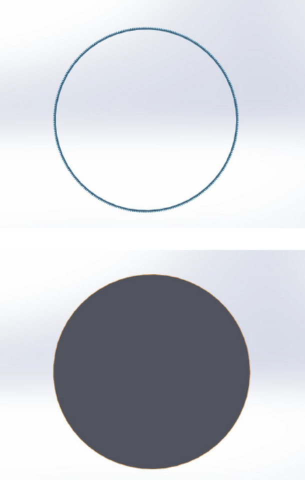
- **Geometry Details**

  Flexspline

  - Number of teeth = 317
  - Module = 0.25
  - Inner diameter = 78.575 mm

  Wave Generator

  - Major diameter = 79.1724 mm
  - Minor diameter = 78.0114 mm



figureInitial model used for mesh convergence study

Model

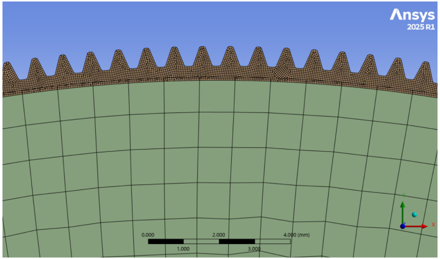
Material

Structural Steel is used, assumed to be linear elastic, isotropic and homogeneous with: Young's Modulus = 200 GPa and poisson's Ratio = 0.3

Meshing

PLANE182 elements were used (4-node linear elements).

- Nodes: 119162
- Elements: 115226



figureMeshing
2

Boundary Conditions

1. Fixed support applied to the face of the wave generator and the outer wall of the circular spline.
2. Displacement BC applied to the flexspline face in cylindrical coordinates: tangential displacement = 0 (except Step-4), radial displacement free.

Analysis Steps

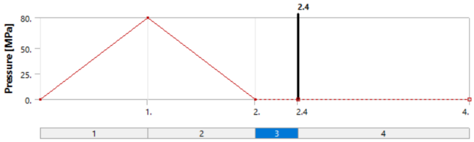
- Total 4 steps were used:

  Step 1:
  - Uniform pressure applied on inner periphery of circular flexspline to allow insertion of elliptical wave generator.
  - Pressure ramped to 70 MPa and then prepared for release.
  - Displacement BC active: radial movement allowed, tangential movement restricted.

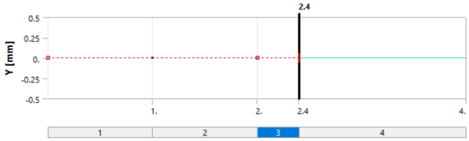
  Step 2:
  - Pressure is deactivated.
  - Flexspline deforms into elliptical shape due to constraint release.

  Step 3: Pressure remains zero and displacement BC still active.

  Step 4: Displacement BC removed, allowing free deformation.



Pressure variation



Displacement BC

Mesh Convergence Study

Mesh size of 1 mm was used on the wave generator.
On the flexspline, mesh size was successively refined as: 1 mm, 0.5 mm, 0.25 mm, 0.1 mm, 0.05 mm.

Table: Mesh convergence results considering large deflection

Flexspline mesh (mm)	Wave generator mesh (mm)	Max eqv stress (MPa)
1	1	69.89
0.5	1	71.123
0.25	1	102.8
0.1	1	102.14
0.05	1	123.06

Proper convergence beyond 0.05 mm could not be achieved due to computational limitations.
Hence, mesh size of 0.05 mm was chosen for all subsequent simulations.

Variation of Stress with Young's Modulus

Young's Modulus was varied by -10%, +10% and +20% to study its effect on stress. It was observed that stress varies linearly with Young's Modulus, as shown in the plot.

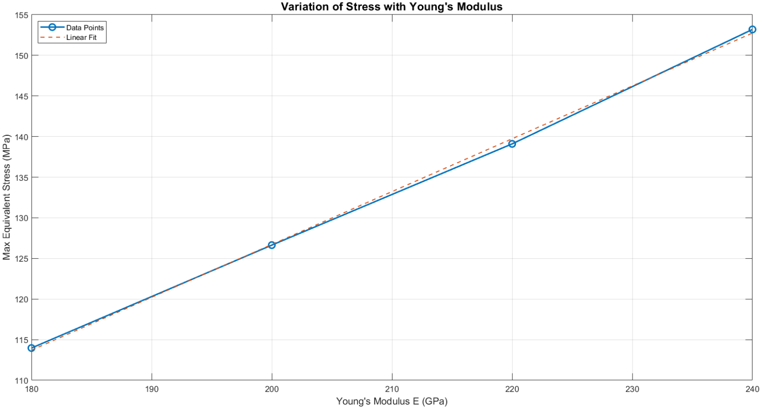


Figure: Young's Modulus

Parametric Analysis

To reduce the dimensionality of the problem, Buckingham π theorem was applied, resulting in the following non-dimensional parameters:

$$\pi_1 = \frac{\sigma_{eqv}}{E}$$
$$\pi_2 = \frac{t}{R}$$
$$\pi_3 = \frac{e}{R}$$
$$\pi_4 = \frac{P_{ellipse}}{\pi D}$$

Thus,

$$\pi_1 = f(\pi_2, \pi_3, \pi_4)$$

where:

- t = flexspline thickness
- D = inner Radius of flexspline
- e = eccentricity $(a - b)$
- $P_{ellipse} \approx \pi(a + b)$
- E = Young's Modulus of material

Parametric Analysis

To understand how the flexspline stress scales with geometry, the four non-dimensional groups are defined as:

$$\pi_1 = \frac{\sigma_{eqv}}{E}, \qquad \pi_2 = \frac{t}{D}, \qquad \pi_3 = \frac{e}{D}, \qquad \pi_4 = \frac{P_{ellipse}}{\pi D}.$$

Step 1: Fixing π_4

- We first isolate the effect of thickness and eccentricity by treating π_4 as a constant.
- Three fixed values are considered:

$$\pi_4 = 1.005, \qquad 1.015, \qquad 1.025.$$

- For each fixed value of π_4, we obtain a relation of the form

$$\pi_1 = f(\pi_2, \pi_3).$$

Step 2: Full three-parameter model

- After studying the above three cases individually, we combine all 27 simulations to obtain a general scaling of the form

Parametric Analysis

To determine the relationship between the non-dimensional stress π_1 and the geometric ratios (π_2, π_3, π_4), we assume a general power-law dependence. This form is widely used because many structural and mechanical relationships exhibit multiplicative scaling.

Power-law assumption

$$\pi_1 = C\,\pi_2^{\alpha}\,\pi_3^{\beta}\,\pi_4^{\gamma}$$

Linearizing the model

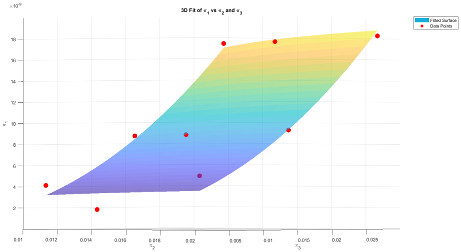
Taking the natural logarithm on both sides gives:

$$\ln(\pi_1) = \ln C + \alpha \ln(\pi_2) + \beta \ln(\pi_3) + \gamma \ln(\pi_4)$$

This converts the nonlinear multiplicative model into a *linear* relation in the transformed variables. Thus, the constants C, α, β, and γ can be obtained using ordinary least-squares regression on the log-transformed data.

Surface Fit for $\pi_4 = 1.005$

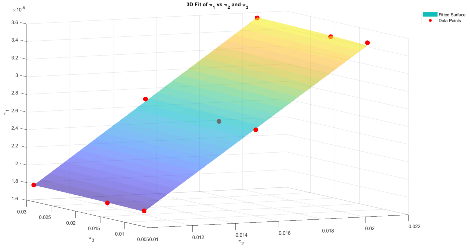
π_1	π_2	π_3
0.0089	0.0155	0.0075
0.0177	0.0208	0.0075
0.0176	0.0103	0.0075
0.0090	0.0155	0.0150
0.0178	0.0208	0.0151
0.0175	0.0103	0.0149
0.0093	0.0155	0.0300
0.0183	0.0208	0.0301
0.0177	0.0103	0.0298



$$\pi_1 = 288.8816\, \pi_2^{2.4306}\, \pi_3^{0.0603}$$

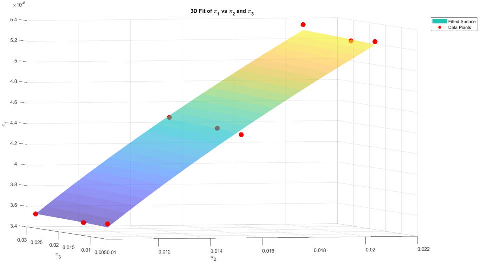
Surface Fit for $\pi_4 = 1.015$

π_1	π_2	π_3
0.0176	0.0103	0.0075
0.0261	0.0155	0.0075
0.0354	0.0207	0.0075
0.0175	0.0103	0.0150
0.0261	0.0155	0.0150
0.0351	0.0207	0.0150
0.0177	0.0103	0.0300
0.0268	0.0155	0.0300
0.0354	0.0207	0.0300



$$\pi_1 = 1.787023\, \pi_2^{1.0036}\, \pi_3^{0.0079}$$

Surface Fit for $\pi_4 = 1.025$



π_1	π_2	π_3
0.0354	0.0103	0.0075
0.0439	0.0155	0.0075
0.0528	0.0207	0.0075
0.0351	0.0103	0.0150
0.0442	0.0155	0.0150
0.0525	0.0207	0.0150
0.0353	0.0103	0.0300
0.0445	0.0155	0.0300
0.0533	0.0207	0.0300

$$\pi_1 = 0.5153390\, \pi_2^{0.5828}\, \pi_3^{0.0050}$$

Curve Fitting

Derived Empirical Relation (Buckingham Π Model):

$$\frac{\sigma}{E} = C \left(\frac{t}{R}\right)^{\alpha} \left(\frac{e}{R}\right)^{\beta} \left(\frac{P}{\pi D}\right)^{\gamma}$$

Substituting fitted constants:

$$\boxed{\frac{\sigma}{E} = 1.851 \left(\frac{t}{R}\right)^{1.3390} \left(\frac{e}{R}\right)^{0.0275} \left(\frac{P}{\pi D}\right)^{83.8221}}$$

High exponent β indicates strong sensitivity of perimeter ratio to the stress value.

Model Validation

Validation on Unseen Test Cases

Case	Actual π_1	Predicted π_1	Error (%)
1	4.139×10^{-3}	5.049×10^{-3}	23.40
2	6.300×10^{-4}	6.168×10^{-3}	879.10

Interpretation

- The model performs reasonably well when π_4 is **well above 1.05**.
- For values of π_4 close to 1, the prediction error becomes large because:
  - the training data near $\pi_4 \approx 1$ was obtained from **non-converged simulations**,
  - the fitted exponent for π_4 is very high, causing strong sensitivity.

Large prediction error reflects both model sensitivity and the limited reliability of low-π_4 data.

Inferences

The relation

$$\pi_1 = C\,\pi_2^{\alpha}\,\pi_3^{\beta}$$

was fitted separately for three fixed values of π_4 (1.005, 1.015, 1.025). This yielded different values of C, α, and β for each case **Key Observations**

- **The fitted exponents α and β change with π_4.** This shows that the stress response cannot be fully captured by only π_2 and π_3; π_4 plays a significant role.
- **As π_4 increases**, the sensitivity of stress to
  - flexspline thickness (π_2) and
  - eccentricity ratio (π_3)

  gradually decreases. This appears as smaller values of α and β.

Inferences

- Physically, increasing

$$\pi_4 = \frac{P_{\text{ellipse}}}{\pi D}$$

  means the **wave generator perimeter becomes larger** compared to the flexspline diameter. Thus, the flexspline deformation is governed more by the overall imposed wave shape rather than small changes in thickness or eccentricity.
- Hence, π_4 behaves like a **global deformation or stretching parameter**. As π_4 increases, the influence of local geometric variations such as

$$\pi_2 = \frac{t}{D} \quad \text{and} \quad \pi_3 = \frac{e}{D}$$

  reduces.

Conclusion:

π_4 is a dominant non-dimensional parameter and it modulates the relative sensitivity of stress to thickness and eccentricity in the flexspline.

Thank you!

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