SLP
Kinematic Analysis of Harmonic Drives

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Contents

Introduction

Introduction

Gears play a crucial role in mechanical engineering as they are fundamental components used for transmitting power and motion between rotating shafts. There is a huge variety of gears; spur, helical, bevel, worm, and planetary gears, each offering unique advantages in terms of efficiency, torque transmission, and design complexity.

Figure: *Planetary gear*

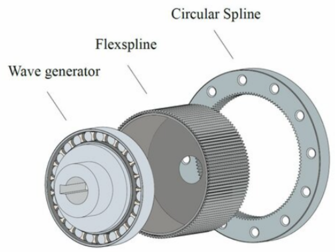
Figure: *Spur gear*

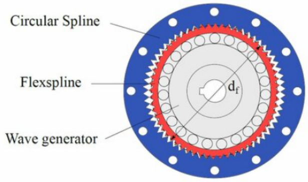
Figure: *Worm Gear*

Figure: *Bevel gear*

Harmonic Drive

- **Wave Generator (WG):** An elliptical cam that deforms the flexspline as it rotates. It is the input element of the harmonic drive.
- **Flexspline (FS):** A thin, flexible cylindrical cup with external teeth, it deforms under loading and acts as output
- **Circular Spline (CS):** A rigid circular ring with internal teeth. It has two more teeth than the flexspline and remains fixed
- **Working Principle:** As the wave generator rotates, it continuously deforms the flexspline, causing its teeth to engage progressively with those of the CS. Harmonic Drive Animation – Wikipedia





Harmonic Drive

Five coordinate systems

- **S(O,X,Y)**: Global coordinate system
- S_W: Attached to the wave generator
- S_C: Attached to the circular spline
- S_F: Attached to the closed end of the cup
- S_f: Attached to the referred tooth on the FS, O_f is the origin

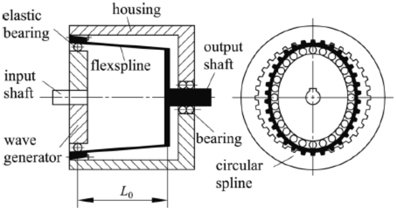


Figure: *Structural diagram of a HD*

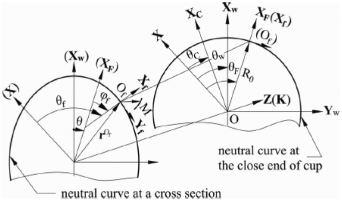


Figure: *Coordinate systems of HDs*

Harmonic Drive

- θ_W: Angular displacement of the wave generator
- θ_f: Angular displacement of the flexspline tooth
- θ_F: Angular displacement of the flexspline's output end
- θ_C: Angular displacement of the circular spline
- O_f: Point on the neutral layer of the referred cross section of the FS (open end)

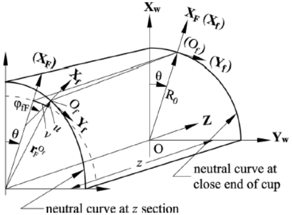


Figure: *Relative motion of S_f to S_F system*

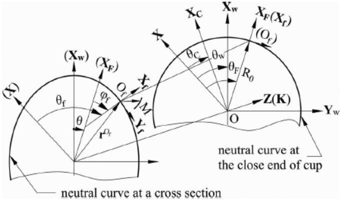


Figure: *Coordinate systems of HDs*

Harmonic Drive

- The images below show the 2-D sketches of the HD for reference

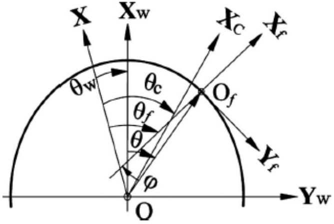


Figure: Relative position between the HD components

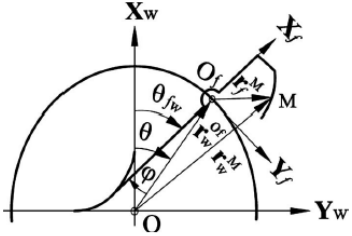


Figure: A point on the FS tooth curve

Why Harmonic Drives?

- **High Precision and Zero Backlash:** Harmonic drives offer extremely accurate motion control, making them ideal for robotics, aerospace, and medical devices.
- **Compact and Lightweight:** They provide high reduction ratios in a small footprint, crucial where space and weight constraints exist.
- **High Torque Capacity:** Despite their size, they can transmit large torques, which is valuable in applications like robotic arms and satellite actuators.
- **Smooth and Silent Operation:** The compliant flexspline ensures smooth transmission with minimal noise and vibration.
- **Widely Used in Cutting-Edge Technology:** Found in NASA space missions, surgical robots, humanoid joints, and precision automation, studying them opens doors to high-impact engineering fields.

Our Objective

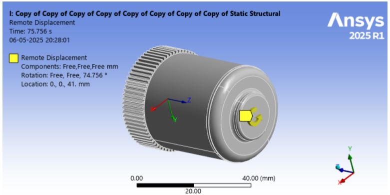
Goal: To study and verify the kinematic mathematical model formulated by Prof. Huimin Dong in his paper "Kinematic Fundamentals of Planar Harmonic Drives"
Steps:

- To observe how motion of a point on flexspline (base of a tooth, point O_f) changes if we give a constant angular velocity at the cup output end through Ansys simulations
- To test the above given point using analytical approach in MATLAB for verification
- This gives a function between the θ_f and θ_F
- Now we use the mathematical model given in the paper to compute θ_f, and using the function found above, we can find θ_F
- If the angular speed at the output (cup end) comes out to be constant, the model proposed in the paper is correct!

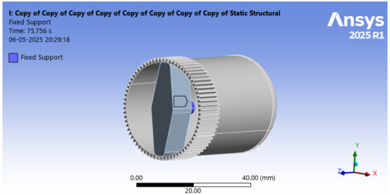
Methodology

FEA Setup

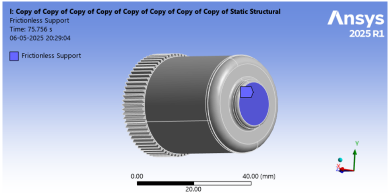
Boundary Conditions



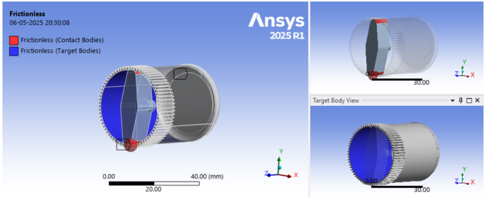
Remote Displacement 90^o



Fixed Support



Frictionless support



Frictionless Contact

FEA Setup

A convergence study was attempted, but due to device limitation, had to restrict to element size of 0.6 mm

Mesh Statistics

- **Elements:** 51,555
- **Nodes:** 94,226

A sphere of influence was added to get a finer mesh near the contact points

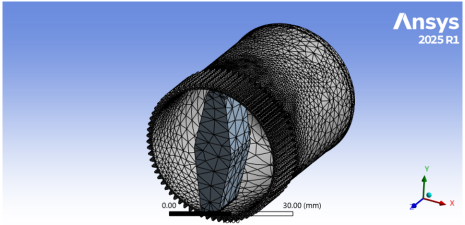


Figure: Meshed Geometry

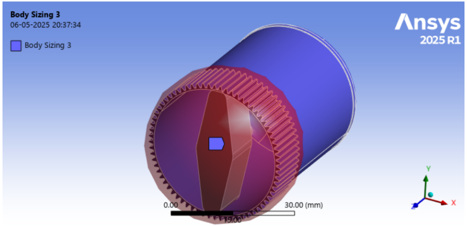


Figure: Sphere of influence

FEA Setup

Named selections

Two nodes were selected as named selections, and their x,y and z displacements were noted

- **Node 1**: This is at front, placed on the tooth base, initially aligning with the y-axis $(\theta = 0)$
- **Node 2:** This node is the back, at the cup end

A sphere of influence was added to get a finer mesh near the contact points

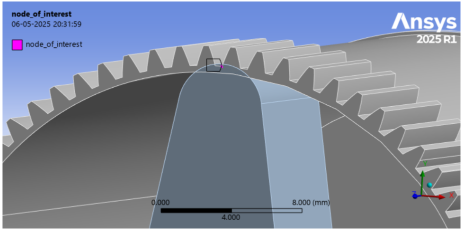


Figure: Node 1

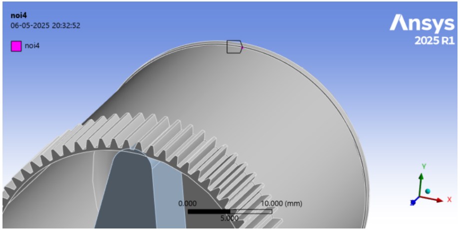


Figure: Node 2

Analytical Formulation

The following equation is used to find the arc length of a polar curve R(θ):

$$\sigma = \int_0^{\theta} \sqrt{R^2 + \dot{R}^2}\, d\theta$$

Since the arc length remains the same before and after deformation, as mentioned in *Kondo, 1990*, we can write:

$$\sigma = R_0 \theta_F$$

where, R_0 is the radius at the cup end and θ_F is the angle at the cup end.
Using these two equations, we can find θ. Now a comparison plot can be made between θ and θ_F, which can be used to validate the FEA results.

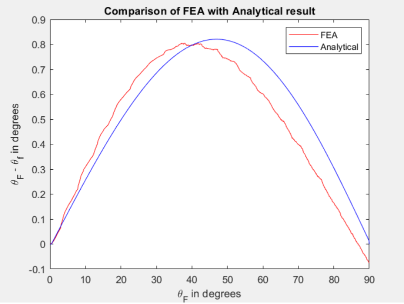


Figure: θ_F - θ_f comparison from FEA and Analytical formulation

Kinematic Model

- Since HD is compliant mechanism, hard to predict kinematics
- The paper proposes a combination of disk cams with a translating knife-edge follower and an oscillating flat face follower to make a kinematic model of HD

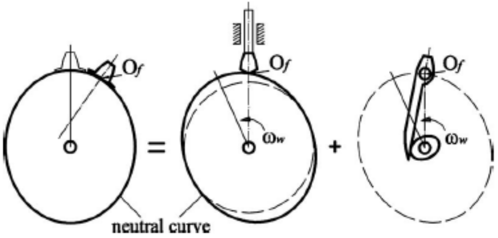


Figure: The analogy to the combination of disk cams with a translating knife-edge follower and an oscillating flat face follower

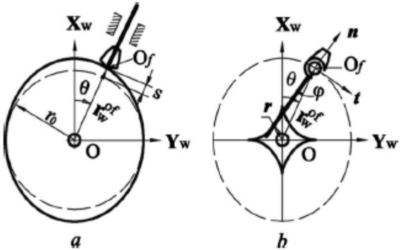


Figure: Modeling a FS tooth and the WG: (a) the translating knife-edge follower model and (b) the oscillating flat-face follower model

Kinematic Model

Major Assumptions

- The FS tooth is considered rigid
- CS and WG are completley rigid
- There is no load at the output

The images below show the 2-D sketches of the HD for reference

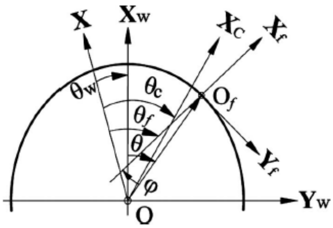


Figure: Relative position between the HD components

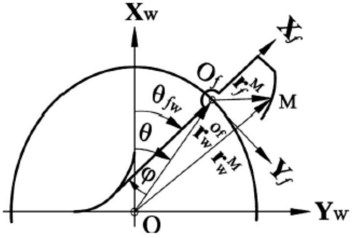


Figure: A point on the FS tooth curve

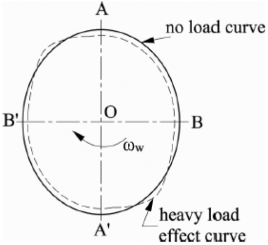


Figure: The FS neutral curve on the WG transverse section

Kinematic Model

Comparison with Planetary Gear Train

- The subscripts C, W, and F refer to the Circular Spline (CS), Wave Generator (WG), and Flexspline (FS), respectively.
- Similar to planetary gear systems, if angular velocities of any two components are known, the third can be determined using the kinematic relation.

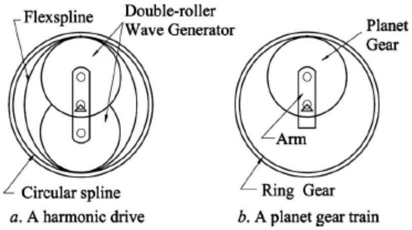


Figure: Kinematic diagrams of HD and PGT

$$\frac{\overline{\omega}_F - \omega_W}{\omega_C - \omega_W} = \frac{N_C}{N_F}$$

where:

- ω_C: Angular speed of the circular spline (analogous to the ring gear)
- ω_W: Angular speed of the wave generator (analogous to the carrier)
- $\overline{\omega}_F$: Average angular speed of the flexspline (analogous to the planet gear)

Kinematic Model

$$R(\theta) = r_0 + w_0(1 + \cos 2\theta)$$

$$\varphi = \cos^{-1}\left(\frac{R}{\sqrt{R^2 + \dot{R}^2}}\right)$$

$$\theta_C = \theta_W + \theta_{CW}$$

$$\theta_{CW} = \frac{2N_F}{D_{F0}N_C}\int_0^{\theta}\sqrt{R^2 + \dot{R}^2}\,d\theta$$

$$\theta_f = \theta_w + \theta_{fw} = \theta_W + \theta + \varphi$$

Variable Definitions:

- $R(\theta)$: Radial position of the flexspline
- r_0: Radius of the prime circle of cam FS
- w_0: Amplitude of deformation
- φ: Oscillating angle of tooth (follower)
- $N_F(N_C)$: Number of teeth on the FS (CS)
- D_{F0}: Initial pitch diameter of flexspline
- θ: Angular position on flexspline
- θ_W: Angular position of the WG
- θ_{CW}: Angular pos. of CS relative to WG
- θ_{fw}: Rel. angular pos. of tooth normal from WG
- θ_f: Final angular position of the joint
- θ_C: Final angular position of the CS

Methodology

- r_0 and w_0 are known; R is a function of θ, so compute $\dot{R}$
- Find φ using the analytical expression
- The input is given to the wave generator and the circular spline is fixed, so $\theta_{CW} = \theta_C - \theta_W = -\theta_W$
- Using equation 3, compute θ
- Compute θ_f from equation 4
- Now that θ_f is known, from the previous analytical formulation, θ_F can be found
- Compute ω_f and ω_F, and plot them

Results

HD Animation

- The kinematic model was used to generate a sample animation. The tooth profiles for both CS and FS was known
- A harmonic drive has $N_F = 240$, $N_C = 242$, $D_{F0} = 121.46\,\text{mm}$, and $r_0 = 60.31\,\text{mm}$, with the neutral curve prescribed using $w_0 = 0.42\,\text{mm}$

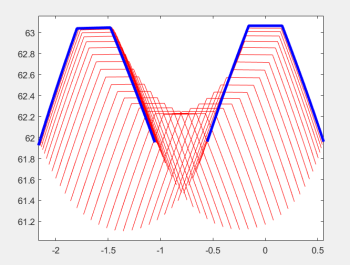


Figure: Our animation

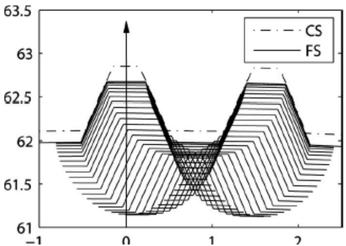
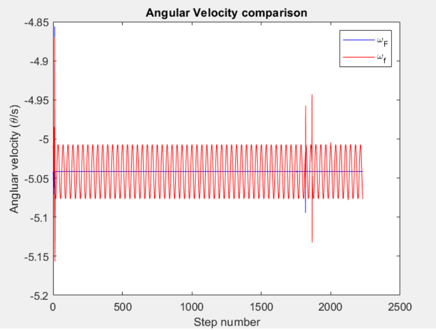


Figure: Animation from paper

HD Animation

- As mentioned earlier, our goal is to see the variation of ω_F and ω_f
- It can be clearly observed that ω_f shows a periodic variation, while ω_F remains constant
- This proves the validity of the kinematic model proposed in the paper



Conclusions

Final Thoughts

- An analytical formulation was done to find relation between θ_f and θ_F
- This analytical formulation was verified with Ansys simulation
- The kinematic model from the paper was then used to compute θ_f
- θ_F was computed from θ_f using the analytical formulation, and consequently, ω_F was found
- ω_F turns out to be constant, which means we get a constant output, thus proving the validity of the model

Acknowledgement

I would like to express my sincere gratitude to my project advisor, Prof. Prasanna Gandhi, for their constant guidance, encouragement, and insightful feedback throughout the course of this work.

Thank you!

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References

- Huimin Dong, *Kinematic Fundamentals of Planar Harmonic Drives*, 2011.
- Huimin Dong, *Kinematic Effect of the Compliant Cup in Harmonic Drives*, 2011.
- K. Kondo & J. Takada 1990, *Study on Tooth Profiles of the Harmonic Drive, 131-137*.