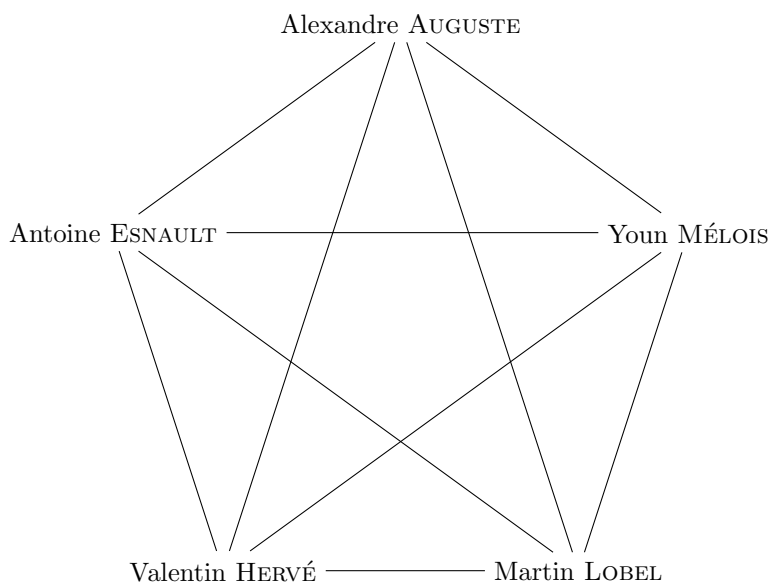


---

# Final Project

## The Maximum Edge Weight Clique Problem

---



**Abstract** The Maximum Edge Weight Clique (MEWC) problem is an optimization problem in graph theory that asks for the clique (a subset of vertices, all adjacent to one another) with the maximum total weight in an edge-weighted undirected graph. In the MEWC problem, each edge has a weight, and the weight of a clique is the sum of the weights of its edges. The goal is to find a clique with the maximum possible weight.

*Class group :*  
CIR3 - Team 1

*Teacher :*  
Leandro MONTERO

January 7, 2023

## Contents

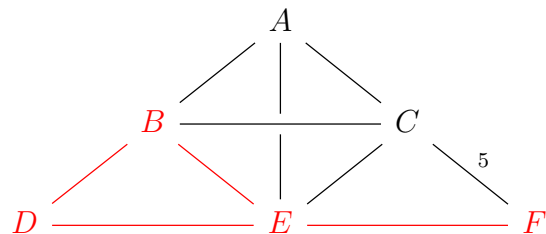
<b>1</b>	<b>Introduction</b>	<b>3</b>
1.1	Presentation of the subject . . . . .	3
1.2	Configuration . . . . .	3
1.3	Example of real-life situations . . . . .	4
<b>2</b>	<b>Exact Algorithm</b>	<b>6</b>
2.1	Presentation . . . . .	6
2.2	How it works . . . . .	6
2.3	Pseudo code . . . . .	6
2.4	Complexity . . . . .	6
2.5	Instance . . . . .	6
2.6	Experiments . . . . .	6
2.7	Analysis . . . . .	6
<b>3</b>	<b>Constructive Algorithm</b>	<b>7</b>
3.1	Presentation . . . . .	7
3.2	How it works . . . . .	7
3.3	Pseudo code . . . . .	7
3.4	Complexity . . . . .	7
3.5	Instance . . . . .	7
3.6	Experiments . . . . .	7
3.7	Analysis . . . . .	7
<b>4</b>	<b>Local Search Algorithm</b>	<b>8</b>
4.1	Presentation . . . . .	8
4.2	How it works . . . . .	8
4.3	Pseudo code . . . . .	8
4.4	Complexity . . . . .	8
4.5	Instance . . . . .	8
4.6	Experiments . . . . .	8
4.7	Analysis . . . . .	8
<b>5</b>	<b>Grasp Algorithm</b>	<b>9</b>
5.1	Presentation . . . . .	9
5.2	How it works . . . . .	9
5.3	Pseudo code . . . . .	9
5.4	Complexity . . . . .	9
5.5	Instance . . . . .	9
5.6	Experiments . . . . .	9
5.7	Analysis . . . . .	9
<b>6</b>	<b>Conclusion</b>	<b>10</b>
	<b>References</b>	<b>11</b>

## 1 Introduction

### 1.1 Presentation of the subject

Graph theory is a branch of mathematics that deals with the study of graphs, which are mathematical structures used to model pairwise relationships between objects. Graphs consist of vertices (also called nodes) that are connected by edges. The edges can be directed (one-way) or undirected (two-way) and can have weight. An induced subgraph of a graph is a subgraph that includes all the vertices and edges of the original graph that are present in the subset of vertices chosen for the subgraph. A clique is a complete maximal induced subgraph of a graph. It means, that it will take its most improved form possible and that it cannot be extended to include any more vertices or edges.

For example, in the following graph,  $A$  is a vertex, the link between  $A$  and  $B$  is an edge. The edge  $(CF)$  have a weight of 5. The graph colored in red is an example of induced graph (there are a lot of them).  $ABC$  is an induced graph and a clique, the maximal clique here is  $ABEC$ .



The Maximum Clique Problem (MCP) consists in finding a clique of maximum size. This is a classic graph theory problem that has many-real life applications in various fields such as social networks, chemistry, bioinformatics. It has been studied since the early 1990s [1].

In addition, this problem is *NP*-Hard. It means, this is a problem where the veracity of any solution can be verified quickly (that is, in polynomial time) and a brute-force search algorithm can find a solution by trying all possible solutions. Moreover, NP-complete problems are a type of problem in computer science that are known to be very difficult to solve. These problems are special because if we can find a solution to any one of them, we can use that solution to quickly solve all other problems of the same type. This is because the solutions to NP-complete problems can be easily checked or verified. So, NP-complete problems are kind of like the "benchmark" for how difficult it is to solve a problem.

As a result, it has been studied as a combinatorial optimization problem, being very important in operations research and theoretical computer science.

The Maximum Edge Weight Clique Problem (MEWC) consists in finding a clique that maximises the sum of the weights of its edges. Clearly MEWC is also a *NP*-Hard problem since the Maximum Clique Problem is the special case in which all weights are equal.

### 1.2 Configuration

- Ajouter les configurations que l'on a utilise pour les tests du style la config

On this project, we decide to use **the C++ language** to developp and implement our

algorithms. Nevertheless, a debate took place, especially on the choice of the language. We hesitated between Python and C++. The first one was for us easier to handle and was a tool in which we had more confidence in our ability to use it efficiently. The latter was nevertheless preferred because of its speed of execution, which was an important criterion for the study. We did have some problems with memory allocation, which made us regret this choice at times.

To share the code between us, we used **Github**<sup>1</sup>. It is a web-based platform for version control, collaboration, and sharing of code, as well as a community of developers who contribute to open source projects and share their knowledge. It was a tool that was difficult for some to get used to quickly, especially on the configuration of the project at home, but which brought us a significant gain in efficiency once we had understood how to use it. To share information and communicate between us, we used **Discord**.

### 1.3 Example of real-life situations

As we said, the MEWC has many real-life applications in various fields such as social networks, chemistry, bioinformatics. We could model this problem with :

- We can model this problem on social **social networks**, indeed we can represent each **users** as a **vertex** in a graph and add an **edge** between two vertices if the corresponding individuals **have some kinds of relationships together**. The weight of the edge could represent **the degree of relationships** between the two users.
- Furthermore, we can model the MEWC problem on **chemistry**, indeed we can represent each

We will take a practical example which could include and involve ISEN students in our future. We will reuse the presented case to illustrate the different algorithms that we will implement later in the report.

**An example of real-life situations that can be modelled as MEWC is the team formation process during a project, or in the search for a particular social group.**

We can imagine that the Student Office of ISEN Nantes is looking to reinforce the video games club of its school. Indeed, the latter has no succession for the following year and is thus led to die if no member presents himself. The future members of the office will have to be in contact with each other during a whole year and it is thus important to find people with common interests so that no tension is formed during their studies. The office has access to the Steam profiles of the students within ISEN (Steam is a video game digital distribution service that gives information about the games played by each one) as well as a record made by the gaming club of the different games played by their members. The fact of playing games in common could bring some people closer, and this makes it a good criterion to create a group that could take over the club because it would share common interests. Otherwise, it would allow to see which games and which group could be present at different events they could organize. Here, forming a team from a group of individuals can be considered as a maximum edge weight clique problem because the goal

<sup>1</sup><https://github.com/sehnryr/Final-Graph-Project-ISEN-CIR3>

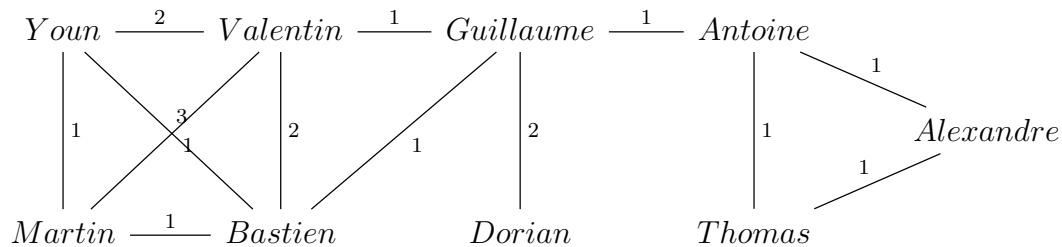
is to select a subset of individuals such that the members share a common interest that could.

To model this problem as a maximum edge weight clique problem, we can represent each **individual** as a **vertex** in a graph and add an **edge** between two vertices if the corresponding individuals **share at least one game on Steam**. The weight of the edge could represent **the number of game** that they have in common.

For example, on a small scale we could imagine :

Students	Game played
Youn	Minecraft, Civilisations, Lost ARK, Among US
Martin	Minecraft
Valentin	Genshin, Minecraft, Civilisations
Maxence	Genshin, Minecraft, Lost ARK, Among US
Aubin	CSGO, Genshin, Overwatch, Stardew Valley
Dorian	CSGO, Paladins, Overwatch
Antoine	League of Legends, Stardew Valley
Thomas	League of Legends, The Last of US
Alexandre	League of Legends, Dofus

Which would give us this graph :



The goal of the maximum edge weight clique problem in this context would be to find a complete subset of individuals such that the sum of the weights of the edges between the individuals is maximized. In this example, the maximum weight clique would be the clique consisting of nodes Youn, Valentin, Martin, Maxence with a total weight of  $10(2 + 1 + 1 + 1 + 3 + 2)$ .

## 2.2 How it works

## 2.7 Analysis

### **3 Constructive Algorithm**

#### **3.1 Presentation**

#### **3.2 How it works**

#### **3.3 Pseudo code**

#### **3.4 Complexity**

#### **3.5 Instance**

#### **3.6 Experiments**

#### **3.7 Analysis**

## 4 Local Search Algorithm

### 4.1 Presentation

### 4.2 How it works

### 4.3 Pseudo code

### 4.4 Complexity

### 4.5 Instance

### 4.6 Experiments

### 4.7 Analysis



## 5 Grasp Algorithm

### 5.1 Presentation

### 5.2 How it works

### 5.3 Pseudo code

### 5.4 Complexity

### 5.5 Instance

### 5.6 Experiments

### 5.7 Analysis

## 6 Conclusion

## References

- [1] Cham Springer. The maximum edge weight clique problem: Formulations and solution approaches. [https://doi.org/10.1007/978-3-319-68640-0\\_10](https://doi.org/10.1007/978-3-319-68640-0_10), 2017.