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Firm dynamics and financial development

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ABSTRACT

Using comprehensive firm-level datasets, this paper studies the impact of cross-country variation in financial market development on firms' financing choices and growth. In less financially developed economies, small firms grow faster and have lower leverage than large firms. As financial development improves, the growth difference between small and large firms shrinks, while the leverage difference rises. The paper then develops a quantitative model where financial frictions drive firm growth and debt financing through the availability of credit and default risk. The model explains the observed cross-country variations in firm size, leverage and growth in response to changes in financial frictions.

1. Introduction

Financial restrictions can hinder firms' ability to use inputs efficiently and affect firm growth. Recent theoretical models of firm dynamics predict that limited credit makes inefficiently small and young firms grow faster than large firms.[1] However, evidence for the magnitude of these effects in actual firm-level data is scarce.[2] The central goal of this paper is to use cross-country variation in financial market development to evaluate empirically and quantitatively the impact of financial frictions on firms' financing choices and growth rates with firm-level datasets.

Consider two countries with varying financial market development: the United Kingdom and Bulgaria. Fig. 1 plots the growth–size and leverage–size relations for firms in the two countries.[3] In both countries, small firms grow faster than large firms, but the difference in growth rates is larger in Bulgaria with worse financial market development. The difference in leverage ratios across firms and countries is striking. Small firms in Bulgaria have lower leverage ratios than large firms, whereas in the United Kingdom the relation is reversed.

This paper documents that these patterns of financial development with firm size, growth and leverage are robust across many countries. We use comprehensive firm-level data from 27 European countries and focus on the *relative* behavior of firms of different sizes across countries with *varying* financial development, as indicated by the ratio of private credit to GDP, the banks' overhead costs relative to assets, and the coverage of credit information for consumers and firms.

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[1] Cooley and Quadrini (2001), Albuquerque and Hopenhayn (2004), Quadrini (2004), Clementi and Hopenhayn (2006), and DeMarzo and Fishman (2007), among others.

[2] Two exception are Huynh and Petrunia (2010) and Midrigan and Xu (2010), who document for Canadian, Colombian, and Korean firms that financial factors, such as leverage, impact growth rates for new firms.

[3] Growth is measured by annual sales growth, leverage is measured by the ratio of total debt to total assets, and size is measured by five asset quantiles. For more details on these definitions see Section 2.

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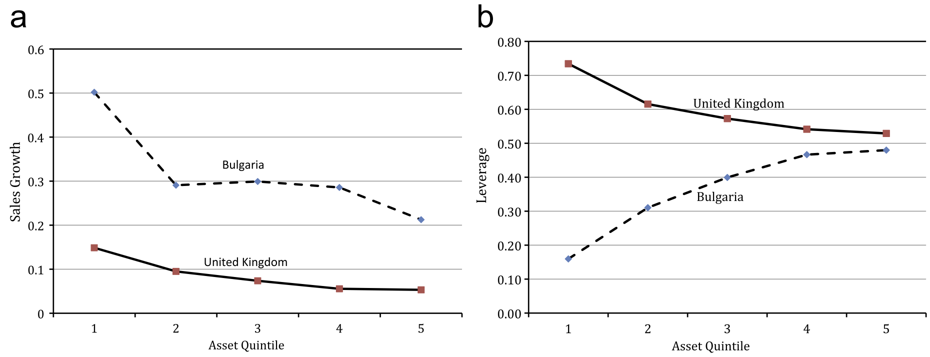

Fig. 1. Firm size, leverage and sales growth. (a) Size and growth. (b) Size and leverage.

Consistent with theories of financial frictions, small and new entrant firms grow disproportionately faster than large and mature firms especially in less financially developed countries. Small firms also tend to have lower leverage ratios than large firms on average. However as financial markets improve, the leverage ratio of small relative to large firms increases, although by less for new entrant firms. The relations among size, growth, leverage and financial development are not only statistically significant but also economically important. For example, consider a 120 percentage points difference in the ratio of credit to GDP as found between the United Kingdom and Bulgaria. The difference in growth rates between two firms with assets equal to 1% and 0.01% of the economy's assets is 54 percentage points across these two economies. Importantly, all these findings are robust to controlling for country, industry, or age-specific characteristics.

The paper then develops a model to highlight the mechanisms that link firm growth to financial conditions, and perform a counterfactual exercise as well as a quantitative assessment of the theory. Credit restrictions arise in our model because firms can default and lenders incur a fixed cost when issuing debt. A large fixed credit cost induces high default risk, and in turn limits credit, which proxies a low degree of financial market development. Debt is restricted disproportionately for small firms in less financially developed economies and these restrictions make their scale inefficient. These small firms grow faster because they can expand their scale. Modeling financial frictions with a fixed credit cost allows the model to account for the empirical findings that small firms in less financially developed economies have disproportionately less debt financing and higher growth rates.

The framework is a dynamic stochastic model where firms use a decreasing returns to scale technology to transform capital into output and face uncertain productivity. They finance investment and dividends with debt and profits and have the option to default on their debt. Firms face debt schedules that encode their default risk net of any recovery value as well as the economy wide credit cost. These schedules impact firms' debt financing and capital choices. Increasing debt is useful for financing investment and dividends, but larger loans are costly because of higher default risk. Hence, firms prefer to shrink their size and become inefficiently small to avoid excessively large loans, especially after a history of low shocks. However, small loans are costly due to the fixed credit cost. Small firms that are particularly financially constrained prefer to shrink even more to avoid credit markets completely.

The firm-specific debt schedules together with the dynamics of debt determine firms' size, growth, and leverage. Small firms are more likely to be inefficient in scale because they face more restricted schedules or are closer to their borrowing limits. Small firms grow faster in response to good shocks because they use the additional output to increase their scale to a more efficient level. In terms of leverage, small firms have on average low leverage due to their tight constraints and the fact that they avoid borrowing at all. In economies with better financial development, loans become more accessible and small firms can respond to low productivity shocks by building up debt and hence leverage. Moreover, with better financial development, small firms have more efficient scales, which implies that growth rates are more equal among all firms.

The paper quantitatively evaluates the model implications in rationalizing the cross-sectional financing and growth patterns jointly. The calibration uses the firm-level data of Bulgaria and chooses parameters capturing the financial frictions to match the averages and standard deviations of growth and leverage. The calibrated credit cost relative to loan equals 1.3%. The calibrated model can account well for the observed variation in leverage across firms but it overestimates the variation in growth rates. Nevertheless in the model as in the data, small firms have higher growth rates and lower leverage ratios.

With the calibrated model, the paper analyzes the consequences of improving the development of financial markets in Bulgaria by reducing the credit cost to zero. Consistent with the data, following this experiment the size–leverage relation and the size–growth relation becomes flatter. In particular, the difference in growth rates between small and large firms declines from 57% to 8%, and the difference in leverage ratios increases from -32% to -5%.

Varying financial markets also has a differential effect on the growth and leverage of entrant versus incumbent firms. The model predicts that in less financially developed economies, the relation of size and growth is more negative and the relation of size and leverage is more positive for entrants than for incumbents. For economies with better financial markets, the model predicts that for entrants the relations of growth–size and leverage–size are flatter. These predictions are consistent with the empirical evidence: as financial development improves, the difference in sales growth and leverage of small and large entrant firms relative to incumbents shrink.

Our empirical findings are novel because we are the first to examine the cross-sectional firm financing and growth patterns simultaneously across countries with a broad coverage of firms. In regard to growth, the cross-section firm-level analyses have considered only one country, as in Rossi-Hansberg and Wright (2007) for the United States.[4] In regard to firms' financing patterns, cross-country comparisons have been studied only for large public firms; Rajan and Zingales (1995) examine G7 countries, and Booth et al. (2001) study 10 developing countries. Public firms, however, constitute a small percentage of firms in all countries, which limits the scope of these previous findings.[5]

The theoretical model is related to the literature that studies the implications of financial frictions on firm growth. Our theory is closest to Cooley and Quadrini (2001) who develop a model where financing restrictions arise from limited commitment in debt contracts. They show that these frictions can potentially deliver large differences in growth rates between small and large firms. Our paper uses firm-level data to quantify the extent to which financial considerations impact growth rates and focuses on how differences in financial market development can explain firm financing and growth patterns across countries. Our work is also closely related to Albuquerque and Hopenhayn (2004), who analyze the effects of enforcement problems under a full set of state-contingent assets. In our model, incomplete markets allow firms with a history of bad shocks to decrease their value and for precautionary savings to play a role.

The paper is also related to the literature in corporate finance on the capital structure of firms.[6] Gomes and Schmid (2010) develop a model to analyze the relation between firms' leverage and stock returns. In their model, mature firms have higher leverage because they face lower default risk. In our model, large mature firms also have lower default risk because financial restrictions are less severe for them. Our paper shows that empirically and theoretically their findings become more pronounced in less financially developed countries as the differential default risk between small and large firms widens.

The rest of the paper is organized as follows. Section 2 presents empirical findings on firm growth and leverage across countries with varying financial development. Section 3 introduces the model. Section 4 presents the quantitative analysis and counterfactual experiments. Section 5 concludes.

2. Cross-country empirical facts

This section first describes the database that provides information on firm-level balance sheets in European countries. It then presents the main empirical findings. Small firms grow faster and use less debt financing than large firms. When financial development improves, the growth of small relative to large firms decreases, especially for young firms; while the leverage of small relative to large firms increases, but by less for young firms. Finally the section establishes robustness of the results by controlling for alternative explanations and exploring various years.

2.1. Data description

The data source is Amadeus, which is a comprehensive European database. Amadeus contains financial data of over 7 million firms in 38 European countries covering all sectors in the economy. The analysis uses firms' balance sheet data in 2004 and 2005.[7] Firm size is measured by the book value of the firm's total asset in 2004.[8] To measure debt financing, the analysis uses the firm's leverage ratio in 2004. Leverage is defined as the ratio of total debt and total asset. Total debt includes short-term and long-term debt as well as short-term loans from suppliers. Firm growth is measured by the growth rate of sales, de-meaned with the aggregate sales growth rate, from 2004 to 2005.

To clean the dataset, firms in the financial and government sectors are excluded following Rajan and Zingales (1995). The sample of firms is restricted to those reporting positive assets, non-negative liabilities in 2004 and non-negative sales in both 2004 and 2005. Firms with growth rates or leverage ratios in the top one percentile in each country are removed. Finally, the countries that have less than 1000 observations after cleaning are dropped.[9] These criteria leave us with about 2.6 million firms in 27 countries: Belgium, Bosnia and Herzegovina, Bulgaria, Croatia, Czech Republic, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Latvia, Lithuania, the Netherlands, Poland, Portugal, Romania, Russian Federation, Serbia, Slovakia, Spain, Sweden, Ukraine, and the United Kingdom.[10] The datasets for these 27 countries are quite representative of the universe as reported by Eurostat (2007) as shown in the online appendix.

[4] The cross-country analysis of growth has been restricted to industry-level data, as in Rajan and Zingales (1998).

[5] For example, in the United Kingdom less than 1% of firms in our dataset are public firms.

[6] Hennessy and Whited (2005) and Miao (2005) develop dynamic models of debt financing. They show that tax considerations and default risk are important determinants of leverage. Harris and Raviv (1991) provide for a comprehensive review of this literature.

[7] Data for 2004 and 2005 are used as a benchmark because these two years offer the most extensive coverage and are less affected by backlog reporting.

[8] Book value is used instead of market value because less than 1% of the firms in the sample are public firms.

[9] See the online supplementary materials at Elsevier's website: http://www.journals.elsevier.com/journal-of-monetary.-economics/.

[10] The countries that are excluded in this analysis are Austria, Belarus, Cyprus, Denmark, Liechtenstein, Luxembourg, Macedonia, Moldova, Monaco, Norway, and Switzerland. The threshold of 1000 is not critical: only the Switzerland will be included in our sample if 500 is used as the threshold instead.

The fraction of small firms does vary some across countries, but this variation is uncorrelated with the measures of financial development.

The development of financial markets is measured using three statistics. The first one is the average private credit to GDP ratio over 2000–2004 taken from the *World Development Indicators*. The second one is the banks' overhead costs as a share of the total bank assets in 2004 taken from Beck and Demirgüç-Kunt (2009). The third measure is the coverage of credit bureaus (the percentage of adults included in the public and private credit bureaus) in 2004 taken from the *Doing Business* publications of the World Bank.[11] Higher private credit to GDP ratios, lower overhead costs, or larger credit bureau coverage indicate better financial development.

Table 1 reports descriptive statistics for the firm-level datasets and the three measures of financial markets development for each country. Countries are ordered by their level of private credit to GDP. The table shows that the variability of financial development is large across these 27 countries. For example, the private credit to GDP ratio is 143% in the Netherlands and only 18% in Russia; the overhead costs as a share of banks' assets is 2.4% in Ireland and 8.21% in Russia; the credit bureau coverage is 98% in Sweden and 0% in Croatia. As expected, these financial development indices are correlated in our sample. The correlations of the private credit to GDP ratio with the credit bureau coverage and the overhead costs are 0.81 and -0.29, respectively.

The mean and median level of assets for firms in each country are reported for 2004 in terms of thousand current euros in the table. Firm asset levels vary across countries, and they tend to be larger in countries with more developed financial markets. Moreover, the distribution of firms in all countries is highly skewed, as the mean asset levels are much larger than the median asset levels.[12] The table also reports the average leverage ratio and the average net growth rate (CPI-adjusted) across all firms in each country. Both mean leverage and mean growth vary substantially across countries. The mean leverage ratio is 0.6 in the United Kingdom, but only 0.01 in Hungary; the mean net growth rate is 3% in the Netherlands, but 47% in Estonia. Finally, the table reports the number of firms in the clean datasets of each country, which is the sample used in the main regression results that follow.

Overall these aggregate statistics are systematically related to financial market development. First, firms in countries with more developed financial markets tend to have larger leverage ratios. The correlations of mean leverage with private credit to GDP, overhead costs and credit bureau coverage are 0.19, -0.33 and 0.20, respectively. Second, firm growth is on average smaller in countries with better financial development. The correlations of mean growth with private credit to GDP, overhead costs and credit bureau coverage are -0.42, 0.45 and -0.35, respectively. Third, firms in countries with more developed financial markets are larger. The correlations of mean asset with private credit to GDP, overhead costs and credit bureau coverage are 0.6, -0.12 and 0.53, respectively.

2.2. Firm size, growth, and leverage

Our hypothesis is that in countries with more developed financial markets, small firms have higher leverage ratios and lower growth rates relative to large firms. Therefore, we pool all the countries together and estimate two regressions of the following form:

$$Leverage_{k,c}(\text{or } Growth_{k,c}) = \beta_0 + \beta_1 Size_{k,c} + \gamma_1 Size_{k,c} \times FD_c + Dummy + v_{k,c}, \tag{1}$$

where c denotes the country, and k the firm. The dependent variable is firm i's leverage for leverage regressions and firm i's sales growth for growth regressions. $Size_{k,c}$ is the log of the share of firm k's assets in the total assets of country c. Given the highly skewed firm size distribution, the log of the firm asset share is used as firm size. FD_c corresponds to the three measures of financial development in country c. The term *Dummy* corresponds to fixed effects at the country × industry × age level.

The regression specification controls for country-specific effects, 2-digit industry-specific effects, and seven age-group-specific effects. Country effects control for any country characteristic, for instance, business cycles, institutional quality, the legal system, the political system, and many others. Industry effects are at the 2-digit level constructed with NACE codes. They control for any inherent features of industries, including capital intensity, competition structure, liquidity needs, and tradability. The seven age groups are constructed, using the information on "Date of Incorporation", at 5-year intervals up to 30 years and a final group for firms with age greater than 30 years. Age effects control for any inherent life cycle features of firms, such as market share and technological development.

As discussed in Rajan and Zingales (1998), the use of fixed effects enables us to control for a much wider array of omitted variables. These dummy variables capture the peculiar features of each age group within each sector of each country, such as particular technological characteristics or specific tax treatments varying at the country × industry × age level. Only additional explanatory variables that vary within each of the industry-country-age groups need to be included. These are firm size and the primary variable of interest, the interaction between size and financial market development. According to our hypothesis, the coefficient estimate for the interaction between size and financial development should be negative in the leverage regression and positive in the growth regression.

[11] The statistics for Iceland and the UK correspond to data in 2005 because this statistic is not available for them in 2004.

[12] Cabral and Mata (2003) find the similar pattern of firm size distribution in the universe of Portuguese manufacturing firms. Quintin (2008) emphasizes that enforcement constraints might account for the difference in firm size distribution across countries.

Table 1
Summary of firm-level datasets and financial development.

	Firm-level datasets					Financial development %		
	Mean asset	Median asset	Mean leverage	Mean growth	No. firms	Overhead costs	Credit coverage	Credit to GDP
Netherlands	263 724	24 124	0.24	0.03	5077	2.63	65	143
United Kingdom	71 260	849	0.60	0.09	67 748	3.95	76	142
Portugal	17 939	787	0.51	0.03	19 784	2.05	64	138
Iceland	2017	85	0.59	0.29	4096	2.40	100	120
Germany	198 267	3205	0.44	0.05	20 225	5.12	86	116
Ireland	128 762	3417	0.39	0.06	1807	1.10	100	115
Spain	5694	365	0.22	0.11	437 405	3.93	39	109
Sweden	12 296	323	0.36	0.04	93116	5.87	98	89
France	7621	220	0.32	0.04	637 764	3.37	2	87
Italy	7740	659	0.14	0.06	414 447	4.20	57	81
Belgium	22 789	393	0.46	−0.01	41 995	2.31	53	75
Greece	9484	1535	0.50	0.08	20 191	2.93	11	60
Finland	16 201	284	0.40	0.06	26 154	1.13	15	60
Estonia	560	37	0.33	0.47	34 187	3.90	10	46
Croatia	656	115	0.46	0.02	6922	3.95	0	42
Slovakia	9649	1556	0.39	0.07	4511	3.02	18	38
Hungary	375	30	0.01	0.45	207 207	5.59	3	38
Czech Republic	3436	221	0.27	0.30	40 429	2.40	25	37
Latvia	3712	588	0.59	0.10	3142	2.77	1	34
Bosnia	2791	379	0.47	0.05	2660	5.35	16	33
Poland	23 451	3624	0.38	0.02	8044	4.01	38	27
Serbia	1300	70	0.52	0.11	29 385	–	0	27
Bulgaria	1463	91	0.36	0.32	17 894	4.13	1	22
Lithuania	8556	1738	0.49	0.17	2237	3.23	12	19
Russia	2484	55	0.43	0.48	237 639	8.21	0	19
Ukraine	6618	705	0.28	0.05	15 594	5.47	0	18
Romania	326	14	0.00	0.68	269 044	6.12	0	11

Note: Firm asset is in thousand 2004 euros. Leverage is defined as the ratio of total debt and total asset. Firm growth is measured by the CPI-adjusted growth rate of sales, de-meaned by the aggregate sales growth rate, from 2004 to 2005. Overhead Cost denotes the banks' overhead costs as a share of the total bank assets in 2004. Credit Coverage denotes the percentage of adults included in the public and private credit bureaus in 2004. Credit to GDP denotes the average private credit to GDP ratio over 2000–2004. – denotes that the data is not available.

Table 2
Firm leverage, growth and financial development.

	Leverage			Sales growth		
	(1)	(2)	(3)	(1)	(2)	(3)
Size	0.021***	0.014***	0.018***	−0.134***	0.024***	−0.082***
	(0.0002)	(0.0003)	(0.0001)	(0.0016)	(0.0011)	(0.0010)
FD × *Size*	−0.006***	0.050***	−0.005***	0.097***	−1.880***	0.051***
	(0.0002)	(0.0048)	(0.0002)	(0.0013)	(0.0310)	(0.0008)
Adjusted R^2	0.28	0.27	0.28	0.06	0.06	0.06
Observations	2 621 201	2 606 324	2 621 201	2 621 201	2 606 324	2 621 201

Notes: *Size* is measured by the logged asset share of a firm. *FD* denotes financial development, measured by private credit to GDP (1), overhead costs (2) or credit bureau coverage (3). All regressions have a fixed effect at the country × industry × age level. The standard errors reported in parentheses are robust to heteroskedasticity. *** denotes significant at 1%.

Table 2 reports the regression results using the three measures of financial development. The first three columns report the leverage regressions, and the last three columns report the growth regressions. Results with private credit to GDP are presented in columns (1), results with overhead costs are in columns (2), and results with credit bureau coverage are in columns (3). The standard errors of the regression coefficients are reported in parentheses and are robust to heteroskedasticity throughout the paper. Let us start with the regression that analyzes the size–leverage relation. For a country with the median level of financial development small firms have lower leverage ratios than large firms. To see this, consider for example regression (1) and the median level of credit to GDP across countries of 47%. The size–leverage slope for this country equals $0.021-0.006 \times 0.47 = 0.018$. The coefficient on the interaction variable of size and financial development has the expected sign and statistically significant at the 1% level under all three measures of financial market

Table 3
Entrant firm leverage, growth and financial development.

	Leverage regressions			Sales growth regressions		
	(1)	(2)	(3)	(1)	(2)	(3)
Size	0.022***	0.014***	0.018***	−0.113***	0.019***	−0.071***
	(0.0002)	(0.0003)	(0.0001)	(0.0016)	(0.0012)	(0.0010)
FD × *Size*	−0.007***	0.054***	−0.005***	0.083***	−1.532***	0.049***
	(0.0002)	(0.0048)	(0.0002)	(0.0012)	(0.0309)	(0.0008)
Entry × *Size*	−0.002***	0.000**	−0.001***	−0.086***	0.064***	−0.039***
	(0.0001)	(0.0001)	(0.0000)	(0.0011)	(0.0014)	(0.0004)
Entry × *FD* × *Size*	0.001***	−0.022***	0.001***	0.072***	−2.219***	0.034***
	(0.0001)	(0.0030)	(0.0002)	(0.0012)	(0.0367)	(0.0008)
Adjusted R^2	0.28	0.27	0.28	0.07	0.07	0.07
Observations	2 621 201	2 606 324	2 621 201	2 621 201	2 606 324	2 621 201

Notes: *Size* is measured by total asset of a firm. *FD* denotes financial development, measured by private credit to GDP (1), overhead costs (2) or credit bureau coverage (3). *Entry* is a dummy variable, which equals one for new entrants defined as firms with age less than or equal to two years. All regressions have a fixed effect at the country × industry × age level. The standard errors reported in parentheses are robust to heteroskedasticity. *** denotes significant at 1% and ** denotes significant at 5%.

development. This means that when private credit to GDP or credit bureau coverage increases, or when overhead costs decrease, the leverage ratios of small firms relative to large firms increase.

The interaction term is a second-order cross-partial derivative. To interpret its magnitude, let us look at the regression with private credit to GDP and compare a small firm with an asset share equal to 0.01% to a large firm with an asset share equal to 1% in Bulgaria and the United Kingdom. As private credit to GDP is higher in the UK by about 120 percentage points, the leverage difference between these comparable small and large firms is $0.006 \times \ln 100 \times 120\% = 3.3\%$ higher in the UK than in Bulgaria. These numbers are economically significant given that the mean leverage ratio for Bulgaria equals 0.36.

Let us next look at the regressions that analyze the size–growth relation. For a country with the median level of financial development, small firms grow faster than large firms. For example, in regression (2) the median overhead costs across countries of 3.9% means that the slope of size and growth for this country equals $0.024-1.88 \times 0.039 = -0.049$. The coefficient on the interaction variable of size and financial development has again the expected sign and statistically significant at the 1% level under all three measures of financial market development. Thus, the growth difference between small and large firms decreases with both private credit to GDP and credit bureau coverage and increases with overhead costs. The interpretation of the coefficient on the interaction of private credit to GDP and size is as follows. The difference in growth rates of a small firm with an asset share equal to 0.01% relative to a large firm with an asset share equal to 1% is $0.097 \times \ln 100 \times 120\% = 54\%$ less in the United Kingdom than in Bulgaria.

2.3. Firm size, growth, and leverage: new entrants

We now examine whether the relations of size, growth and leverage are different for newly established firms relative to older firms across countries. Cooley and Quadrini (2001) document that younger firms tend to grow faster than older firms even conditional on size. This section tests whether a country's financial development influences differently the dynamics and financing patterns of young firms. With a pooled sample of all the countries, the following regressions are estimated :

$$
\begin{aligned}
Leverage_{k,c} \text{ (or } Growth_{k,c}) = \beta_0 + \beta_1 Size_{k,c} + \gamma_1 Size_{k,c} \times FD_c + \beta_2 Entry_{k,c} \times Size_{k,c} + \gamma_2 Entry_{k,c} \times Size_{k,c} \\
\times FD_c + Dummy + v_{k,c}, \qquad (2)
\end{aligned}
$$

where $Entry_{k,c}$ is a dummy variable that equals one for new entrants, defined as firms with age less than or equal to two years. In a country with financial development FD_c, the difference in the slopes of the size–leverage (size–growth) relation for entrants relative to incumbent firms is governed by $\beta_2+\gamma_2 FD_c$. Across countries, the response of this difference to financial development is governed by γ_2; a positive γ_2 in regressions (1) and (3) and a negative γ_2 in regression (2) imply that the difference in the slopes rises as financial development improves.

Table 3 reports the estimated coefficients. The coefficients on the interaction between size and financial development are similar to those in the main regressions. Thus, the cross-country size–leverage and size–growth relations of incumbent firms are similar to those of all firms. The first three columns in the table show the leverage regressions. For a country with the median level of financial development the slope of leverage and size is less positive for entrant than for incumbent firms. This means that small entrant firms have relatively higher leverage ratios than small incumbent firms. Moreover, as financial development improves, although all small firms increase their leverage relative to large firms, the entrants increase by less. The sales growth regressions shown in the last three columns of Table 3 indicate that for a country with the median level of financial development the slope of growth and size is more negative for entrants than for incumbents. As financial development improves, the growth differential across small and large firms shrinks and by more for entrant firms. Hence, these results suggest that financial development affects more the relative leverage of incumbent firms and the relative growth of entrant firms.

2.4. Robustness tests

This subsection provides robustness on the main results. First it examines whether our findings are robust to controlling for alternative explanations for the negative relation between firm size and growth. To do so, three additional interaction terms of size with volatility of GDP growth rates, GDP per capita, and two-digit industry categories are added to the main regressions. Second, it considers alternative time periods, namely the years 1999–2003. Extensive analysis in both dimensions confirms the robustness of our results.

2.4.1. Robustness to other explanations

One important theoretical explanation of the growth–size relation is the *selection* theory: small firms are more likely to exit under adverse shocks and thus tend to have higher growth rates conditional on survival.[13] If selection differs across countries, one concern is whether our results are robust when such variation controlled for. The degree of selection in each country is proxied by the standard deviation of GDP growth rates. Specifically, an interaction term between firm size and volatility of GDP growth is added to the main regressions.

In a recent work, Rossi-Hansberg and Wright (2007) propose another theory for the relation between firm size and growth based on mean reversion in the accumulation of factors. In their model the growth difference between small and large firms is larger in sectors that use physical capital more intensively. To control industry, an additional interaction term between firm size and two-digit industry categories is introduced to the main regressions.

Finally, the interaction term between firm size and the log of the country's GDP per capita is added. This variable allows for the relation of size with sales growth to vary with the country's GDP per capita. Table 4 reports the results where financial development is measured by private credit to GDP. Results with overhead costs and credit bureau coverage are similar and reported in the Appendix.

In the growth regressions, the coefficient of the interaction between size and GDP volatility is significantly negative as expected by the selection theory. However, the selection effect is not robust to the inclusion of the interaction between size and GDP per capita. As the income rises, the negative size–growth relation becomes less pronounced. The size and growth relation varies across industries: the coefficients of the interaction between size and industry dummies are significantly positive for some industries and significantly negative for others. Nevertheless, even after selection, industry variation and GDP per capita are controlled for, the coefficients of the interaction between size and financial market development remain significantly positive as in the main regressions. In addition, the coefficients of the interaction terms with entry are consistent with those in Table 3. Similarly, our main leverage regression results are robust to the inclusion of these additional interaction terms.

2.4.2. Robustness to other years

The benchmark regression results use data from 2004 and 2005 because Amadeus offers the most extensive and stable coverage of firm-level balance sheet information for many countries in these two years. Though the data coverage for some earlier years is limited, this section explores these data and shows that our findings are robust across time. Basically, the same regression analysis of leverage and sales growth is done using data for 1999, 2000, 2001, 2002 and 2003. The coefficients on size and the interaction between size and financial market development are similar across specifications (1) and (2). Hence, for brevity only the regression coefficients of specification (2) are reported in Table 5 with the private credit to GDP ratio as the measure of financial development.

The upper panel of the table reports the leverage regressions across all years. The coefficients on the interaction between size and financial development have similar sign and significance to the benchmark results. The coefficients on the interaction terms with the entry dummy are less robust. This might be due to a smaller and more random sample of entrants in the early periods. The growth regressions for all the sample years, reported in the lower panel, feature similar coefficients on all the interaction variables to those in the benchmark year. Similar results are found when financial development is defined by overhead costs and credit bureau coverage. (See the online appendix.)

In summary, small firms use disproportionately less debt financing and grow disproportionately faster than large firms in countries with worse credit bureau coverage, larger overhead costs, and lower ratios of private credit to GDP. As financial development rises, the differences in the size–leverage and size–growth slopes of entrant and incumbents both rise. These empirical findings are robust and provide a comprehensive picture of the relations of financial market development with financing and growth across firms and across countries.

3. Model economy

To study theoretically firms' financing choices and dynamics, this section presents a dynamic model of heterogeneous firms that face default risk. The model builds on Cooley and Quadrini (2001) and Albuquerque and Hopenhayn (2004)

[13] See for example Hopenhayn (1992) and Luttmer (2007).

Table 4
Robustness with additional interactions: Credit over GDP.

	Leverage regressions		Sales growth regressions	
FD × *Size*	−0.009***	−0.011***	0.073***	0.020***
	(0.0003)	(0.0005)	(0.0014)	(0.0013)
Entry × *Size*	−0.001***	−0.001***	−0.086***	−0.085***
	(0.0001)	(0.0001)	(0.0011)	(0.0011)
Entry × *FD* × *Size*	0.001***	0.001***	0.072***	0.071***
	(0.0001)	(0.0001)	(0.0012)	(0.0012)
GDPVOL × *Size*	−0.090***	−0.065***	−0.372***	0.271***
	(0.0041)	(0.0051)	(0.0215)	(0.0294)
GDP per capita × *Size*		0.001***		0.036***
		(0.0002)		(0.0008)
Industry × *Size*	Yes	Yes	Yes	Yes
Adjusted R^2	0.28	0.28	0.06	0.07
Number of observations	2 621 201	2 621 201	2 621 201	2 621 201

Notes: *Size* is measured by total asset of a firm. *FD* denotes financial development, measured by private credit to GDP. *Entry* is a dummy variable, which equals one for new entrants defined as firms with age less than or equal to two years. *GDPVOL* is the standard deviation of GDP growth rates. All regressions have a fixed effect at the country × industry × age level. The standard errors reported in parentheses are robust to heteroskedasticity. *** denotes significant at 1%.

Table 5
Robustness with additional years.

	1999	2000	2001	2002	2003
	Leverage regressions				
Size	0.032***	0.033***	0.034***	0.033***	0.020***
	(0.0002)	(0.0002)	(0.0003)	(0.0002)	(0.0002)
FD × *Size*	−0.016***	−0.017***	−0.019***	−0.018***	−0.007***
	(0.0003)	(0.0003)	(0.0004)	(0.0003)	(0.0002)
Entry × *Size*	−0.001***	−0.002***	−0.002***	0.001***	−0.002***
	(0.0002)	(0.0002)	(0.0002)	(0.0001)	(0.0001)
Entry × *FD* × *Size*	0.001***	0.001***	0.001***	−0.001***	0.002***
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)
Adjusted R^2	0.20	0.17	0.17	0.21	0.22
	Sales growth regressions				
Size	−0.151***	−0.153***	−0.156***	−0.145***	−0.143***
	(0.0030)	(0.0026)	(0.0026)	(0.0027)	(0.0020)
FD × *Size*	0.128***	0.123***	0.124***	0.112***	0.104***
	(0.0025)	(0.0021)	(0.0022)	(0.0022)	(0.0015)
Entry × *Size*	−0.074***	−0.078***	−0.085***	−0.088***	−0.100***
	(0.0021)	(0.0021)	(0.0021)	(0.0018)	(0.0017)
Entry × *FD* × *Size*	0.060***	0.065***	0.071***	0.072***	0.084***
	(0.0022)	(0.0021)	(0.0022)	(0.0019)	(0.0017)
Adjusted R^2	0.08	0.08	0.08	0.07	0.08
Observations	1 282 817	1 409 644	1 595 747	1 857 510	2 124 784

Notes: *Size* is measured by total asset of a firm. *FD* is the private credit to GDP ratio. *Entry* is a dummy variable, which equals one for new entrants defined as firms with age less than or equal to two years. All regressions have a fixed effect at the country × industry × age level. The standard errors reported in parentheses are robust to heteroskedasticity. *** denotes significant at 1%.

while incorporating differential degrees of financial market development across economies. In the model, entrepreneurs decide on the level of capital and debt financing for their firms. Credit restrictions arise because debt is unenforceable and firms can default. Lenders offer firm-specific debt schedules that compensate for default risk and for a fixed credit cost they incur when issuing debt. The degree financial market development is proxied with the size of the credit cost; large costs induce high default risk and thus limit the economy-wide credit.

3.1. Firms

Entrepreneurs are infinitely lived and have access to a mass one of risky project opportunities, which are referred to as firms. Each entrepreneur owns at most one firm and decides on entry, exit, production, and financing plans to maximize the present value of dividends. An operating firm starts the period with capital K and debt B_R. It produces output with a stochastic decreasing returns technology with capital: $y = zK^{\alpha}$, where $0 < \alpha < 1$ and productivity z follows a Markov process given by $f(z',z)$. Capital depreciates at rate δ. The firm finances investment $K' - (1-\delta)K$, capital adjustment costs $\phi(K'-K)^2/K$, and dividends D with internal funds which consist of the firm's output net of debt repayment $zK^{\alpha} - B_R$ and with external funds by acquiring a new loan B'. The firm can also obtain resources from equity holders when $D < 0$, however, doing so carries a proportional cost γ.

Debt contracts are not enforceable as entrepreneurs can default on their debt. Debt contracts (B', B'_R) are firm specific and incorporate the firm specific default risk. Let $\Omega(K',z)$ denote the set of debt schedules available to a firm with next-period capital K' and productivity z. Each contract $(B', B'_R) \in \Omega(K',z)$ maps a current loan B' to a repayment amount B'_R.

The recursive formulation for operating a firm follows. Upon observing the shock realization, the value of the firm is $V(K,B_R,z)$ and the entrepreneur decides whether to default or not. In case of default, the entrepreneur gets zero while the lender gets a fraction of the capital stock of the firm. Thus,

$$V(K,B_R,z) = \max_{d \in \{0,1\}} (1-d)V^c(K,B_R,z), \tag{3}$$

where V^c is the value of firm conditional on repaying the debt and $d(K,B_R,z)$ is a binary variable that represents the default decision; it equals 1 if default is chosen and 0 if repayment is chosen.

An operating firm decides on production and financing. The entrepreneur chooses capital K', dividends D, and a loan contract (B', B'_R) from the available schedule $\Omega(K',z)$ to maximize the repayment value:

$$V^c(K,B_R,z) = \max_{D,K',(B',B'_R) \in \Omega} (1+\gamma I_{\{D<0\}})D + \beta E_z[V(K',B'_R,z')] \tag{4}$$

subject to

$$D = zK^{\alpha} - B_R + B' - K' + (1-\delta)K - \phi(K'-K)^2/K, \tag{5}$$

where $\beta < 1$ is the discount rate of the entrepreneur, and the expectation is taken over the conditional probability distribution of z'.[14]

Enforcement frictions limit firms' ability to equate the marginal product of capital to the risk free rate. Thus, investment depends on the set of loan contracts available and is distorted downward. For example, if a firm starts with large debt, it might want to borrow a big loan B' to keep the investment level at the unconstrained optimal while avoiding the equity issuance costs. Nonetheless, given the bounded set of loans due to possible defaults, such a big loan might not be offered to the entrepreneur. Hence, the entrepreneur might choose to reduce investment, making the project inefficiently small.

When an idle entrepreneur receives a project opportunity, he will start a new firm if the value of the entry V^e is nonnegative. Entrants start with K_0 and decide on the optimal capital and debt holding before they know their future productivity which is drawn from an endogenously determined probability distribution $g(z')$. The value for a potential entrant is given by

$$V^e = \max_{D,K',(B',B'_R) \in \Omega^e(K')} (1+\gamma_e I_{\{D<0\}})D + \beta E[V(K',B'_R,z')] \tag{6}$$

subject to

$$D = B' - K' - \phi(K'-K_0)^2/K_0. \tag{7}$$

The initial equity issuance costs γ_e differ between incumbents and entrants and debt schedules for new entrants Ω^e depend only on their capital choice K'.

3.2. Debt schedules

Firms borrow from a debt schedule that depends on their default decisions.[15] Creditors have to pay a fixed credit cost ξ for every loan they offer. For each loan contract (B', B'_R), the creditors transfer B' to the firms today, and receive B'_R if the firms repay and a recovery value of the loan $R(K')$ if the firms default next period. The recovery of the loan is given by $R(K') = \max\{(1-\psi)(1-\delta)K' - \phi K', 0\}$, as in Hennessy and Whited (2007) and Gomes and Schmid (2010). Creditors extract a $1-\psi$ fraction of the depreciated capital minus the capital adjustment costs. Debt schedules $\Omega(K',z)$ include all contracts (B', B'_R) that allow creditors to break even in expected value:

$$B' + \xi = \frac{B'_R\left(1 - \int d(K',B'_R,z')f(z',z)\,dz'\right) + \int d(K',B'_R,z')R(K')f(z',z)\,dz'}{1+r}. \tag{8}$$

[14] In the layout of the model, the probability distribution $f(z',z)$ is assumed to be actual probability distribution which by construction abstracts from risk premia. However, the probability distribution can also be interpreted as a risk neutral measure in which case the valuation of firms contains a risk premium component. Such interpretation is common in models of investment under uncertainty as in Pindyck (1991) and also in defaultable bonds pricing models as in Duffie and Singleton (1999).

[15] Similarly as in Bai and Zhang (2012), the endogenous debt schedules developed in Chatterjee et al. (2007) and Arellano (2008) in their study of unsecured consumer credit and sovereign default are generalized by adding an interaction of capital and default risk.

The left-hand side of Eq. (8) is the resources creditors spend today. The right-hand side is the expected repayment discounted by the risk-free rate $1+r$. The debt schedules for entrants also satisfy (8) except that the expectation is taken over the unconditional probability distribution.

The availability and the terms of debt contracts are determined by both default risk and credit costs. Absent of credit costs, default risk generates debt restrictions which depend on the firms' choice of capital and their productivity. Credit costs further limit the economy wide availability of credit by making financing more costly for all firms. Nevertheless, the impact of credit cost ξ is different across firms with varying capital and productivity. In general, a high ξ increases default risk disproportionately for firms with low capital and productivity.

One can interpret the expense of ξ as any financial intermediation costs lenders pay when making loans. These expenses can include overhead costs or costs to obtain information about firms' default likelihood. The parameter ξ controls financial market development of our model economy and can be naturally linked to the coverage of credit registries, lenders' overhead costs as well as to the aggregate level of credit across countries.[16] As documented in the empirical section, these variables vary substantially across countries and are directly linked to the way firms finance their assets and grow.

Entrepreneurs in our model can also save. When the entrepreneur saves creditors do not need to pay ξ. Savings contracts satisfy the following condition: $B' = B'_R/(1+r)$ for $B' \leq 0$.

3.3. Equilibrium

The *stationary recursive equilibrium* consists of the policy and value functions of firms, the loan contracts offered by creditors, the distribution of firms, and the productivity distribution of entrants such that (i) given the schedule of loan contracts offered, the policy and value functions of firms satisfy their optimization problem; (ii) loan contracts reflect the firm's default probabilities such that with every contract creditors break even in expected value; (iii) the distribution of firms $\Upsilon(K,B_R,z)$ and the productivity distribution of entrants $g(z)$ satisfy

$$\Upsilon(K',B'_R,z') = \int d(K,B_R,z)Q_e(K',B'_R,z')g(z')\,\Upsilon(K,B_R,z)\,d(K \times B_R \times z)$$
$$+ \int (1-d(K,B_R,z))Q((K,B_R,z),(K',B'_R,z'))]f(z',z)\,\Upsilon(K,B_R,z)\,d(K \times B_R \times z)$$

and

$$g(z) = \int d(K,B_R,z)\,\Upsilon(K,B_R,z)\,d(K \times B_R),$$

where $Q(\cdot)$ denotes a transition function that maps current states into future states given by

$$Q((K,B_R,z),(K',B'_R,z')) = \begin{cases} 1 & \text{if } B'_R(K,B_R,z) = B'_R, K'(K,B_R,z) = K', \\ 0 & \text{elsewhere,} \end{cases}$$

and $B'_R(K,B_R,z)$ and $K'(K,B_R,z)$ are the policy rules for firms. Similarly, $Q_e(K',B'_R,z')$ is 1 if entrants' optimal choice is (K',B'_R) and 0 otherwise.

The evolution of the distribution of firms depends on the decisions of firms to borrow and invest. Whenever existing firms exit, their z projects become available to potential entrant entrepreneurs such that the mass of projects is always equal to one.

3.4. Borrowing limits and financial development

Limited enforceability of debt contracts generates endogenous borrowing limits which play a key role in determining optimal debt and vary across firms and with financial market development. In particular, weak financial development limits borrowing relative to assets. And this limitation is more severe for small firms than for large firms.

This section provides an analytical characterization of these findings by considering the case when firms are heterogeneous with respect to z yet this productivity is constant over the firm's lifetime. In addition, for simplicity the analysis assumes full depreciation of capital, no capital adjustment costs, no equity issuance, no debt recovery after default, and that entrants start with zero debt and zero capital. The following assumptions guarantee that firms have an incentive to borrow and that the borrowing limit is at least as large as the efficient capital for all firms.[17]

Assumption 1. $\beta(1+r) < 1$, and $\xi \leq (\alpha z/(1+r))^{1/(1-\alpha)}(1-\alpha)/\alpha$ for all z.

[16] This specification of credit issuance costs is similar to the one used in Livshits et al. (2008). They document that improvements in credit scoring in the United States are important for understanding the rise in bankruptcies and volume of debt.

[17] In our general model with stochastic productivity, firms may not maintain the efficient capital stock even when debt schedules allow for it.

In this environment, firms always invest capital to the level at which the marginal product of capital equals the interest rate. Let us call this capital level the first best capital $K_{fb}(z)$, which solves $z\alpha K_{fb}(z)^{\alpha-1}=1+r$. Let $\overline{B}(z)$ denote the debt limit of a firm with productivity z and $\overline{B}_R(z)=(1+r)(\overline{B}(z)+\xi)$ denote the associated debt repayment. The firm borrows to the debt limit given $\beta(1+r)<1$. The value of an existing firm with productivity z and debt repayment $\overline{B}_R(z)$ is

$$V^c(K_{fb}(z),\overline{B}_R(z),z)=[zK_{fb}(z)^{\alpha}-K_{fb}(z)-r\overline{B}(z)-(1+r)\xi]/(1-\beta),$$

while the value of a new entrant firm with productivity z is

$$V^c(0,0,z)=[\overline{B}(z)-K_{fb}(z)]+\beta V^c(K_{fb}(z),\overline{B}_R(z),z).$$

The borrowing limit for a firm with productivity z is the level of debt that makes the contract value equal to the default value of zero, and is given by $V^c(K_{fb}(z),\overline{B}_R(z),z)=0$ which implies that the debt limit is

$$\overline{B}(z)=\frac{(1+r-\alpha)}{r\alpha}K_{fb}(z)-\frac{\alpha(1+r)}{r\alpha}\xi.$$

More productive and larger firms (bigger z) have looser borrowing limits than small firms, independent of the degree of financial market development. Also, stronger financial market development (lower ξ) increases the loan availability for all firms, independent of productivity.

The leverage ratio, defined as debt relative to assets, of a firm with productivity z is

$$\frac{\overline{B}(z)}{K_{fb}(z)}=\frac{(1+r-\alpha)}{r\alpha}-\frac{\alpha(1+r)}{r\alpha}\frac{\xi}{K_{fb}(z)}. \tag{9}$$

The relation between debt limits to assets and size is affected by the fixed credit cost. Credit costs make small firms more constrained in borrowing relative to large firms because credit costs increase default risk disproportionately for them. Moreover, the disadvantage of small firms relative to large firms becomes more pronounced as ξ increases. Clearly, when the fixed credit cost ξ is zero, the leverage ratios are independent of size. Modeling financial development with fixed credit costs is essential to match our empirical finding that small firms have lower leverage than large firms in less financially developed countries. The following proposition summarizes this finding.

Proposition 1. *In the case without uncertainty*, $\delta=1$, $\varphi=0$, $\gamma=0$, $\psi=1$ *and under Assumption*1, *the relation between debt limits to assets and firm size is decreasing in the degree of financial development*: $d^2(\overline{B}(z)/K(z))/(dK(z)d\xi)>0$.

Proof. Direct differentiation of Eq. (9) delivers the result. □

Deriving analytical expressions for debt limits in the case of stochastic productivity is difficult due to lack of analytical solutions for the firm's decision rules of debt and investment. However, all the results regarding borrowing limits, sizes, and financial development carry through in the numerical analysis of the model for the general case with uncertainty.

4. Model quantitative implications

This section assesses quantitatively how financial development and default risk shape the patterns of firms' financing and growth across countries.[18] The model is calibrated to Bulgaria. Financial frictions can account for the relation of firm size with growth and leverage found in this country. Improving financial markets in the model reduces the difference in both growth rates and leverage ratios of small versus large firms, consistent with the empirical evidence. The distinct behavior of entrant and incumbent firms in the model as financial development varies also mirrors the findings in the data.

4.1. Calibration

The model is calibrated to match Bulgarian data in 2004 and 2005. The following parameters are chosen independently of the model equilibrium. The interest rate r is set at 4% per annum to match the real interest rate in Bulgaria.[19] The capital depreciation rate δ is set at 10% per year and the annual discount factor β is set at 0.96, which are standard values for annual RBC models. Following Cooley and Quadrini (2001), the equity issuance cost γ is 30%. Following Gomes and Schmid (2010), the decreasing returns parameter α is set to 0.65 and ψ is set to 25%, which is consistent with estimates on deadweight costs on capital after default.

The firms' idiosyncratic productivity consists of a permanent shock μ_z and a stochastic shock ε such that the productivity for firm i equals $z_t^i=\mu_z^i\cdot\varepsilon_t^i$. At birth, the firm draws its permanent productivity μ_z from a Pareto distribution $\Pr(x)=cx^{-c-1}$. In each period, there is a probability θ that the firm's permanent productivity turns zero forever, in which case the firm exits. θ equals 7.2% to match the average firm exit rate in euro countries reported in Eurostat (2007).[20]

[18] The online appendix describes the computation algorithm.

[19] According to *International Financial Statistics*, the real interest rate, constructed as the difference between the annual nominal lending rate and the annual inflation rate, is 4% in Bulgaria in 2004.

[20] Although the model abstracts from any life-cycle patterns of exit, it contains the right number of small relative to large firms because the observed firm size distribution is one of the targets in the calibration.

Table 6
Benchmark parameters and target moments.

Calibrated parameters			
Discount factor	β	0.96	
Interest rate	r	0.04	
Capital depreciation rate	δ	0.10	
Technology	α	0.65	
Equity issuance cost	γ	0.30	
Capital loss after default	ψ	0.25	
Death rate	θ	0.072	
Shock persistence	ρ	0.86	
Estimated parameters			
Permanent productivity	c	0.550	
Stochastic shock variance	σ	0.525	
Capital adjustment cost	ϕ	0.001	
Credit cost	ξ	0.010	
Entrant starting capital	K_0	0.002	
Entrant equity issuance cost	γ_e	0.130	
Target moments		Data	Model
Coef of variation of log assets		1.96	1.84
Mean sales growth		0.32	0.34
Std of sales growth		1.52	1.04
Mean leverage		0.36	0.48
Std of leverage		0.35	0.31
Mean entrant growth		0.95	1.13
Mean entrant leverage		0.51	0.49

The stochastic shock ϵ for continuing firms is assumed to be lognormal with persistence ρ and standard deviation σ. The persistence parameter of 0.86 follows firm level estimates from Foster et al. (2008). The permanent productivity is discretized into five levels, which are given by the 5th, 25th, 50th, 75th, and 95th percentile of the Pareto distribution. The stochastic productivity is discretized into two points ε_l and ε_h, following Tauchen and Hussey (1991).

Six parameters $\{c, \sigma, \phi, K_0, \gamma_e, \xi\}$ are jointly calibrated to best match seven moments in the data: the coefficient of variation of log assets, the means of leverage and sales growth across firms, the standard deviations of leverage and growth, and the mean leverage and growth of entrants. In the model, all these parameters affect all the target moments in a non-linear fashion. Nevertheless, each parameter impacts some more particular moments. Specifically, c determines the standard deviation of asset levels. σ and ϕ affect the mean and standard deviation of sales growth. ξ affects the mean and standard deviation of leverage ratios. K_0 and γ_e affect the mean leverage and growth of entrants. A lower ξ increases mean leverage and decreases the standard deviation of leverage. The calibration requires a positive fixed credit costs for the economy to replicate the observed values in Bulgaria.

Table 6 summarizes the parameter values and the target moments in the data and in the model. Overall, the calibration is successful in matching the target moments in the data. The model generates a tight fit in terms of the average sales growth rate, the coefficient of variation of log assets, and the mean growth and leverage of entrant firms. The standard deviation of sales growth however is matched less well. The calibrated ξ parameter equals 0.01, which corresponds to 1.3% of the loan value for the average firm.

4.2. Debt schedules and dynamics

Before presenting the quantitative results, it is informative to understand how default risk affects firms' debt schedules and how these restrictions on credit impact firms' choices of debt, capital, and dividends.

Let us start by looking at the equilibrium debt schedules $\Omega(z,K')$ that arise due to default risk shown in Fig. 2. The left panel of the figure plots the effective interest rate $(B'_R/B'-1)$ for every contract (B',B'_R) as a function of the debt repayment B'_R (relative to the average capital) for a firm with median permanent productivity μ_z^3, low stochastic shock ε_l, and capital choice K' equal to the average capital across μ_z^3-firms. The figure shows that small and large loans are the most expensive. Small loans have large effective interest rates due to the fixed cost; large loans are expensive because of higher default risk. For loans with B'_R less than 1.3 the firm only defaults when it gets the θ shock. For larger values of B'_R between 1.3 and 1.9 the firm defaults under the low shock the following period and the creditor gets the recovery value. For even larger values of B'_R, the firm defaults with probability and B' is bounded by expected recovery value of the firm.

Default risk not only increases with loan size but also with small choices for capital. The left panel of Fig. 2 plots the effective interest rates of an otherwise identical firm but with a capital choice equal to 80% of the average

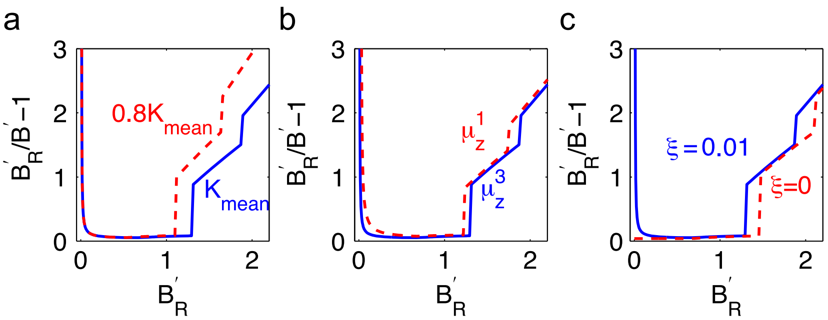

Fig. 2. Debt schedules. *Note*: The left panel plots the interest rate schedules for two firms with median permanent productivity μ_z^3, low stochastic shock ε_l, and capital choice K' equal to 100% and 80% the average capital K_{mean} across μ_z^3-firms. The middle panel plots the interest rate schedules for two firms with mean productivity μ_z^1 and μ_z^3 when the stochastic shock is ϵ_l and capital K' equals the average capital across firms with the same permanent productivity. The right panel plots the interest rate schedules with and without a positive fixed credit cost for a firm with median permanent productivity, low stochastic shock, and mean capital. All values on the axis are relative to the corresponding average capital.

capital.[21] This firm faces a tighter debt schedule and higher interest rates because low capital makes default more attractive and lowers recovery rates.

Debt schedules are also more lenient for firms with higher productivity. The middle panel of Fig. 2 plots the effective interest rates for two firms with mean productivity μ_z^1 and μ_z^3 when the stochastic shock is ϵ_l and capital K' equals the average capital across firms with the same permanent productivity. Firms with higher productivity can borrow *relatively* larger loans at both the risk free rate and risky rates. The *absolute* differences in risk-free and risky debt capacities across these two firms are very large given that the ratio of the two average capital stocks is about 10. Similar patterns hold for stochastic shocks; debt schedules under ϵ_h are more lenient than those under ϵ_l.

The right panel of Fig. 2 compares the effective interest rates across economies with different credit costs. Fixed credit costs make borrowing costly especially for the very small loans and larger loans. In the benchmark economy, a firm can borrow up to 1.3 of its capital stock without defaulting, whereas in an economy with a zero credit cost it can borrow without defaulting up to 1.5.

The limitations on debt contracts affect the way firms respond to shocks. When experiencing sequences of bad shocks, firms tend to reduce their scale to avoid equity issuance costs and increase their debt financing, climbing up their debt schedules. When experiencing a good shock, these inefficient firms expand their scale and reduce their debt, sliding down their debt schedules. These dynamics imply that firms with the same permanent productivity display different sizes that depend on the history of shocks. Across these firms, inefficiently small firms tend to have higher growth rates and higher leverage ratios.

To illustrate these dynamics, consider the decision rules for a firm with median permanent productivity μ_z^3, stochastic shock ϵ_l and debt at 43% of the average capital of μ_z^3. Fig. 3 plots the optimal capital choice, dividends, and debt relative to the capital choice as a function of capital. For convenient interpretation of the values, all these statistics are normalized by the average capital across the μ_z^3-firms. With capital larger than 75% of the average capital, the firm chooses a large future capital stock, distributes dividends, and holds a low level of debt. The low debt level is due to a precautionary motive, similarly as in standard precautionary savings models (Aiyagari, 1994; Huggett, 1993). With uncertainty, the firm may not find it optimal to exhaust its borrowing opportunities because large debt increases default risk and the likelihood of costly equity issuance next period. With intermediate levels of capital between 20% and 75% of the average capital, the firm stops paying dividends, increases debt and decreases investment. Although choosing larger capital allows firms to face more lenient debt schedules, it might also require firms to choose larger loans or to incur equity issuance costs. With low levels of capital less than 20% of the average capital, the firm starts to issue equity, shrinks its capital, and increases its leverage.[22]

4.3. Quantitative results

Let us now examine the model's quantitative results. After computing the model, the model is simulated to obtain a model economy with 15,000 firms over 500 periods. The model delivers in the long run a cross-sectional distribution of firms, which is used to compute the model's statistics. In the model, firm size equals the assets of the firm: capital K, plus savings B_R if $B_R < 0$. For each period, the cross section of firms is divided into two groups (small and large) according to their starting asset levels: below the median and above the median. Average sales growth rates, leverage ratios, and asset levels are computed for each group and for the entire distribution of firms. Specifically, the leverage of a firm equals the

[21] Our model shares this additional benefit of capital of relaxing borrowing constraints with Bai and Zhang (2010) and many models of collateral constraints such as Kyotaki and Moore (1997) and Cagetti and De Nardi (2006).

[22] The decision rules as a function of the level of (negative) debt have similar qualitative features. Firms with less debt choose larger capital and dividends and smaller new debt.

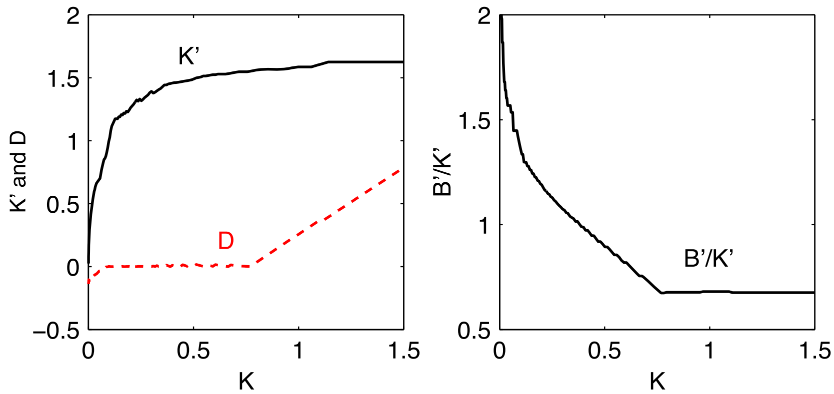

Fig. 3. Policy rules. *Note*: This figure plots the optimal capital choice K', dividends D, and the ratio of the loan choice relative to the capital choice B'/K' as a function of the beginning capital K for a firm with median permanent productivity μ_z^3, stochastic shock ϵ_l and debt at 43% of the average capital across the μ_z^3-firms. All values on the axis are relative to the average capital across the μ_z^3-firms.

Table 7
Quantitative model results.

	Bulgaria data		Bulgaria benchmark		Zero credit cost	
	Growth	Leverage	Growth	Leverage	Growth	Leverage
All firms						
Mean	0.32	0.36	0.34	0.48	0.30	0.68
Small firms	0.37	0.26	0.62	0.32	0.34	0.65
Large firms	0.26	0.46	0.05	0.64	0.26	0.71
Difference	0.11	−0.20	0.57	−0.32	0.08	−0.06
Entrants						
Mean	0.95	0.51	1.13	0.49	1.12	0.54
Small firms	1.50	0.37	1.40	0.35	0.30	0.43
Large firms	0.73	0.57	0.32	0.89	2.34	0.72
Difference	0.77	−0.20	1.08	−0.54	−2.04	−0.29
Regression coefficient	−0.06	0.05	−0.12	0.10	−0.08	0.02

Notes: The panel under Bulgaria data reports data statistics for Bulgaria. The panel under Bulgaria benchmark reports model statistics under the benchmark calibration. The panel under Zero credit cost reports the model statistics for the counterfactual experiment in which the fixed credit cost is set at zero. Small firms are firms with asset below the median, and large firms are those with asset above the median. Entrant firms are firms with age less than or equal to two. Regression coefficient denotes the regression coefficient β_1 in the following regression: sales growth$_i$ (or leverage$_i$) $= \beta_0 + \beta_1 Size_i$, where i denotes the firm index, and *Size* denotes the log asset share. The sales growth rates in the Bulgaria data is de-meaned using the aggregate real sales growth rate of 11% in 2004–2005.

ratio of outstanding debt to capital installed B_R/K if $B_R \geq 0$. If a firm starts with assets $B_R < 0$, it has no liabilities due, and thus its leverage ratio is equal to zero. The sales growth rate is given by the ratio of next-period sales $z'(K')^\alpha$ and current sales zK^α.

4.3.1. Benchmark results

The model results are reported in the middle of Table 7, together with the data statistics on the left. Consistent with the data, the model generates a downward sloping relation of growth and size and an upward sloping relation of leverage and size. The sales growth rate is 0.62 for small firms and 0.05 for large firms in the model, and it is 0.37 for small firms and 0.26 for large firms in the data. Similarly, the leverage ratio is 0.32 for small firms and 0.64 for large firms in the model, and it is 0.26 and 0.46 in the data. Our model matches well the patterns of leverage in the data. However, it over-predicts the differences of sales growth between small and large firms in these statistics.

To understand these results, let us consider the two determinants of firm size: permanent productivity and stochastic productivity. Small firms tend to have low levels of permanent productivity (*unproductive*). As shown in the right panel of Fig. 2, the unproductive small firms have disproportionately tighter interest rate schedules, which induces them to have low leverage ratios. The unproductive small firms have inefficient scales and thus grow fast when hit with good stochastic shocks, as these shocks alleviate their needs of credit and they can expand their size to a more efficient level. Firm size also depends on its history of stochastic shocks. Firms tend to be particularly small after a sequence of bad shocks (*unlucky*) because they are closer to their borrowing limits. Thus, similarly as the unproductive small firms, these unlucky small firms grow fast when hit with good shocks. However, different from the unproductive small firms, unlucky small firms tend to have high leverage ratios as a result of the bad shocks.

The benchmark results are dominated by the variation in debt schedules across firms with different size. The unfavorable debt schedules for small firms induce them to have low leverage ratios and high growth rates. The reason why the model overestimates the differences in growth and leverage between small and large firms is that the calibration requires a relatively large fixed credit cost to match average leverage.

The lower panel of Table 7 reports the behavior of entrants relative to incumbents. Firms with age less than or equal to two are classified as entrants in the model, following the classification in the empirical analysis. The model matches well the higher average growth and leverage for entrants which are targets in the calibration. The model also matches the more negative relation between size and growth for entrants relative to incumbents. The model also predicts a more positive relation of size–leverage whereas in the data for Bulgaria the relation of size–leverage is similar across incumbents and entrants.

To ensure that the model implications across firm size are robust to the number of groups, let us consider the following regressions both in the model and in the data:

$$Leverage_k(\text{or } Growth_k) = \beta_0 + \beta_1 Size_k + \nu_k,$$

where k denotes the firm index, and *Size* denotes the log asset share. The slope coefficients β_1 are reported in the last row of Table 7. In the model, the slope coefficient is -0.12 for the growth regression and 0.10 for the leverage regression. In the data, the slope regression coefficient is -0.06 in the growth regression and 0.05 in the leverage regression. Thus, confirming our results across two bins, the model generates an upward sloping size–leverage relation and a downward sloping size–growth relation as in the data, though the model over-predicts the magnitudes of these relations.

The risk of default shapes firms' choices of debt and capital, but in equilibrium the parameterized model does not generate endogenous default. With slow depreciation of capital and the option to issue equity, firms avoid default completely because the value of firms are very large. Default only occurs when firms experience a perfectly persistent zero productivity shock θ, akin to a death shock.

4.3.2. *Improving financial development*

We use our calibrated model to analyze the consequences of improving financial markets in Bulgaria by reducing the credit cost to zero. The model statistics for this experiment are reported in the right panel of Table 7. When financial markets improve, both the size–leverage relation and the size–growth relation become flatter. In particular, the difference in growth rates of the small and large firms declines from 57% to 8%, and the difference in leverage ratios increases from -32% to -6%. Lowering credit costs increases the mean leverage from 48% to 68%, and decreases the mean growth from 34% to 30%. The regression coefficient on size rises from -0.12 to -0.08 in the growth regression, and decreases from 0.1 to 0.02 in the leverage regression. All these implications are fully consistent with our empirical findings.[23]

To understand these results, recall that in the benchmark, the debt schedules are more restrictive for firm with lower permanent productivity, which pushes down the leverage ratio and pushes up the growth rate of small firms relative to large firms. When the fixed credit cost is reduced to zero, the variation in the debt schedules across permanent productivity disappears. Consequently, both the size–growth and size–leverage patterns become flatter than in the benchmark.[24]

The above model regression coefficients are used to construct a counterpart of the empirical coefficients on the interaction between financial development and size in Table 3. The change in the fixed credit cost to loan ratio across the model economies is interpreted as a change in overhead. Overhead costs relative to average loan are 1.3% lower in the model counterfactual. Hence, our model predicts that an increase in overhead costs relative to loans of 1% changes the slope of growth on size by $(-0.12+0.08)/1.3 = -0.03$. In the empirical overhead regression, an increase of 1% in overhead changes the slope of growth by -0.02. For the leverage regression, the model predicts that an increase of 1% in overhead changes the slope by 0.06, whereas in the data the estimate indicates a 0.001 change in slope. Thus, the model predicts the interaction effect between financial development and size on firm growth in line with the data, but it overestimates the interaction effect on leverage.

Not only do credit costs make debt schedules more restrictive for firms with lower permanent productivity but also ration small loans. Small loans are unattractive due to large effective interest rates and hence these loans are not observed in equilibrium. Credit rationing is an important channel for the quantitative results because they are more relevant for small firms who borrow small loans. In the benchmark 47% of small firms have zero leverage, while only 9% of large firms have zero leverage. When the credit cost is reduced to zero, all firms have positive leverage. Our data provides empirical support that a larger fraction of small firms have zero leverage in less financially developed countries. The following regression reports these findings:

$$\text{ZeroLeverage}_{k,c} = \text{constant} - 0.054^{***} \times \text{Size}_{k,c} + 0.014^{***} \times \text{Size}_{k,c} \times (\text{Credit/GDP})_c + \nu_k,$$

[23] The relation between size and leverage in the model with a zero fixed cost depends on parameter values. For example in the case of i.i.d. transitory shocks or higher capital adjustment costs the relationship is negative.

[24] The online appendix highlights that modeling financial development with a fixed credit cost is essential in accounting for the cross country patterns of size and leverage. If improved financial development is modeled with lower proportional costs, i.e. lower interest rates for loans, the model predicts a change in the size–leverage patterns inconsistent with the data.

where Zero Leverage is a dummy variable that equals 1 only when leverage is zero. The negative and significant sign on the coefficient on size implies that small firms are more likely to have zero leverage. The positive and significant sign on the interaction coefficient between size and credit to GDP imply that in more financially developed countries it is less likely that small firms have zero leverage. Both predictions are consistent with the model.

Finally, let us examine the model implications on the behavior of entrants relative to incumbents as financial market development improves. The difference in the growth rates of small and large entrant firms decreases from 108% to -204%. The magnitude of this decline is much larger than the corresponding decline for incumbent firms. The difference in leverage of small and large entrant firms rises from -54% to -29%. The magnitude of this change is slightly smaller than that of the change for incumbent firms. These implications are consistent with the data.

5. Conclusion

Using a broad and comprehensive firm-level database from 27 European countries, this paper documents that small firms grow faster and use less debt financing than large firms. More importantly, as financial development improves, the growth rate of small firms decreases, but the leverage ratio of small firms increases, relative to large firms, especially for new entrant firms. Our empirical analysis provided a new picture of the relation of financial market development with debt financing and growth across firms and countries. The paper then developed a quantitative dynamic model of heterogeneous firms where financial development affects firm financing and growth through the availability of credit. Financial market development is important in accounting for the difference in growth rates and debt financing across firms and across countries.

A contribution of the paper is to use micro firm-level data in a quantitative model to study the growth and financing patterns in the cross section of firms of multiple countries. A natural next step is to analyze the time dimension by introducing aggregate fluctuations in the model to study the cyclical features of firm dynamics. Moscarini and Postel-Vinay (2009) document that for the United States, the variance in the firm size distribution is pro-cyclical, and the early phases of booms are mainly driven by the expansion of small firms. Our framework can prove useful in analyzing the impact of financial frictions on the cyclical cross-sectional firm dynamics. More generally, we view our quantitative methodology that combines firm-level data with theory as a useful tool for analyzing the interaction of micro-decisions with macro-implications.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found in the online version at http://dx.doi.org.10.1016/j.jmoneco.2012.06.006.

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