

Linear Regression Numerical

Exercise: Ice-Cream Sales vs. Temperature

Dataset :

Temperature(°C)	Ice Cream Sales(\$)
14	215
16	325
20	400
24	515
28	590

Now, we will apply Linear Regression on the dataset.

Step-By-Step Calculation

Temperature(x)	Ice cream Sales(y)	$x - \bar{x}$	$y - \bar{y}$	$(x - \bar{x})(y - \bar{y})$	$(x - \bar{x})^2$	y^{\wedge}	$e = (y - y^{\wedge})$	e^2	$ e $
14	216	-6.4	-194	1241.6	46.96	251.752	-36.752	1350.71	36.752
16	325	-4.4	-84	369.6	19.96	300.892	24.108	581.20	24.108
20	400	-0.4	-9	3.6	0.16	399.172	0.828	0.69	0.828
24	515	3.6	106	381.6	12.96	497.452	17.548	307.93	17.548
28	590	7.6	181	1375.6	57.76	595.732	-5.732	32.86	5.732
				$\Sigma=674.4$	$\Sigma=27.44$			$\Sigma= 84.968$	$\Sigma= 227.39$

1. Calculate the Mean of X (Temperature) and Mean of Y (Ice Cream Sales):

Mean of X (\bar{x}) = $(14 + 16 + 20 + 24 + 28) / 5 = 20.4$

Mean of Y (\bar{y}) = $(215 + 325 + 400 + 515 + 590) / 5 = 409$

2. Slope=

- $m = \Sigma((x - \bar{x}) * (y - \bar{y})) / \Sigma((x - \bar{x})^2) = 674.4/27.44 = 24.57$

3. Y-intercept(b)

Use the formula:

$$b = \bar{y} - m\bar{x} = 409 - 24.57*20.4 = - 92.228$$

4. y^{\wedge} :predicted value for each x

The formula is: $y^{\wedge} = mx + b$

This is the required final linear regression equation.

The required line on the graph by plotting the predicted and x values is called the linear regression line.

5. Error= $e = y - y^{\wedge}$

6. Square Error formula = $e^{**2} = (y - y^{\wedge})^{**2}$

7. Absolute Error formula = $|e| = |y - y^{\wedge}|$

8. Mean Square Error = $\Sigma (y - y^{\wedge}) / n = 84.968/5 = 16.9936$

9. Mean Absolute Error = $\Sigma |y - y^{\wedge}| / n = 2273.39/5 = 45.478$

Define the following terms:

Linear Regression:

A statistical method is used to model the relationship between a dependent variable and one or more independent variables by fitting a linear equation to observed data.

Slope:

In the context of linear regression, the slope is the rate at which the dependent variable changes to changes in the independent variable. It indicates how much the dependent variable is expected to increase (or decrease) when the independent variable increases by one unit

Error(Positive and negative Error):

Error is the difference between the actual observed values and those predicted by the regression model. A **positive error** occurs when the expected value is less than the actual value, and a **negative error** occurs when the predicted value is more than the actual value

Square Error and Mean Square Error:

The square error is the square of the difference between the observed values and the predicted values. Mean square error (MSE) is the average of these square errors across all observations, providing a measure of the quality of the estimator—it reflects the average magnitude of the errors squared.

Absolute Error and Mean Absolute Error:

Absolute error is the absolute value of the difference between the observed value and the predicted value, disregarding the error's direction. Mean absolute error (MAE) is the average of these absolute values across all observations and gives an idea of how big the errors are on average.