

## Time Complexity

1. for ( $i=0, i < n, i++$ )  
    { stmt —  $n$   
    }  
     $\rightarrow O(n)$

2. for ( $i=n, i > 0, i--$ )  
    { stmt } —  $n \rightarrow O(n)$

3. for ( $i=1; i < n; i = i+2$ )  
    { stmt —  $n/2$   
    }  
     $f(n) = n/2 \rightarrow O(n)$

4. for ( $i=0, i < n, i++$ ) —  $n+1$   
    { for ( $j=0, j < n, j++$ ) —  $n(n+1)$   
        { stmt }  
    }  
     $\rightarrow O(n^2)$

5. for ( $i=0; i < n, i++$ )  
    { for ( $j=0, j < i, j++$ )  
        { stmt }  
    }  
     $\rightarrow O(n^2)$

6.  $P=0$   
    for ( $i=1, i \leq n, i++$ )  
        {  $P = P+i$   
        }  
     $P > n$   
     $\therefore P = \frac{K(K+1)}{2} > n$   
     $K^2 > n$   
     $K > \sqrt{n} \rightarrow O(\sqrt{n})$

7. for ( $i=1, i < n, i = i * 2$ )

{ stmt

}

$\rightarrow O(\log_2 n)$

Assume  $i > n$

$\therefore i = 2^K$

$\therefore K = \log_2 n$

$$1 \times 2 = 2$$

$$2 \times 2 = 4$$

$$2^2 \times 2 = 2^3$$

$$\vdots$$

$$2^K$$

8. for ( $i=n, i \geq 1, i = i/2$ )

{ stmt

}

$\rightarrow O(\log_2 n)$

Assume  $i < 1$

then  $i/n$

$$\therefore \frac{n}{2^K} \leq 1$$

$$\Rightarrow K = \log_2 n$$

9. for ( $i=0, i * i < n, i++$ )

{ stmt

}

$\rightarrow O(\sqrt{n})$

$$i * i < n$$

$$i * i \geq n$$

$$i^2 = n$$

$$i = \sqrt{n}$$

10. for ( $i=0, i < n, i++$ )

{ stmt —  $n$

for ( $j=0, j < n, j++$ )

{ stmt

{ —  $\frac{n}{2n}$

$\rightarrow O(n)$

11.  $P=0$

for ( $i=1, i < n, i = i * 2$ )

{  $P++$  }  $P = \log n$

for ( $j=1, j < P, j = j * 2$ )

{ stmt —  $\log P$

$O(\log(\log n))$

12. for ( $i=0, i < n, i++$ ) —  $n$   
 { for ( $j=1, j < n, j=j*2$ )  
 { stmt —  $n \times \log n$   
 }  $O(n \log n)$

1 Summary

for ( $i=0, i < n, i++$ ) —  $O(n)$   
 for ( $i=0, i < n, i=i+2$ ) —  $n/2 - O(n)$   
 for ( $i=n, i > 1, i--$ ) —  $O(n)$   
 for ( $i=1, i < n, i=i*2$ ) —  $O(\log_2 n)$   
 for ( $i=1; i < n, i=i*3$ ) —  $O(\log_3 n)$   
 for ( $i=n, i > 1, i=i/2$ ) —  $O(\log_2 n)$

13.  $i=0$   
 while ( $i < n$ ) —  $n+1$   
 { stmt —  $n$   
 $i++$  —  $n$   
 }  $T(n) = 3n+2$   
 $\rightarrow \Theta(n)$

for ( $i=0, i < n, i++$ )  
 { stmt  
 }  
 $\rightarrow O(n)$

14.  $a=1$   
 while ( $a < b$ )  
 { stmt;  
 $a=a*2$  }  
 $\rightarrow O(\log n)$

$\frac{a}{1 \times 2 = 2}$   
 $2 \times 2 = 2^2$   
 $2^2 \times 2 = 2^3$   
 $\vdots$   
 $2^k$   
 $a > b$   
 $\therefore a = 2^k$   
 $2^k \geq b$   
 $\Rightarrow k = \log_2 b$

for ( $a=1, a < b, a=a*2$ )  
 { stmt;  
 ~~$i=i/2$~~  }  
 $O(\log n)$

15.  $i = n$ ;  
 while ( $i > 1$ ):  
 { stmt  
 $i = i/2$  }  
 $\rightarrow O(\log n)$

16.  $i = 1$   
 $K = 1$   
 while ( $K < n$ )  
 { stmt  
 $K = K + i$   
 $i++$  }

$\rightarrow O(\sqrt{n})$

i	K
1	1
2	$1+1=2$
3	$2+2=4$
4	$2+2+3=$
$\vdots$	
n	$1+2+3+\dots+n = \frac{n(n+1)}{2}$

for ( $K=1, i=1, K < n, i++$ )  
 { stmt  
 $K = K + i$  }

$$K \geq n$$

$$\frac{n(n+1)}{2} \geq n$$

$$n \geq \sqrt{n}$$

$$n = 16$$

$$14$$

$$12$$

$$10$$

$$\vdots$$

$$2$$

$$n = 2$$

$$2$$

$$2$$

$$2$$

$$2$$

17. while ( $m \neq n$ )

{ if ( $m > n$ ):

$m = m - n$

else:  $n = n - m$ }

min  $O(1)$ ,  $O(n)$

18. Algorithm test ( $n$ )

{ if ( $n < 5$ )

printf ("%.1",  $n$ ) } — 1

else: { for ( $i=0, i < n, i++$ )

{ printf ("%.d",  $i$ ) —  $n$

{

best  $O(1)$

worst  $O(n)$