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Physical sizes of Mg II haloes beyond $z = 5$: a statistical study

1 DERIVATIONS

Brief outline of derivation following Churchill+99:

$$\frac{dN}{dX} = \frac{c\sigma n}{H_0}$$

The 'column density' $n\sigma$ is given by

$$n\sigma = \pi \int_{L_{\min}}^{\infty} f_R(L)\Phi(L)R^2(L)dL$$

$\Phi(L)$ and $R(L)$ are givan by a Schechter and a PL, respectively:

$$\Phi(L)dL = \Phi^*(L/L^*)^{\alpha}\exp(-L/L^*)dL$$

$$R(L) = R^*(L/L^*)^{\beta}$$

Grouping together:

$$n\sigma = \pi \int_{L_{\min}}^{\infty} \Phi^* {R^*}^2 (L/L^*)^{\alpha+2\beta}\exp(-L/L^*)dL$$

$$= \pi\Phi^* {R^*}^2 \Gamma(\alpha + 2\beta + 1, L_{\min}/L^*)$$

And finally

$$\frac{dN}{dX} = f_R(L)\frac{c\pi}{H_0}\Phi^* {R^*}^2 \Gamma(\alpha + 2\beta + 1, L_{\min}/L^*)$$

and now only the Γ function depends on $L_{\min}$.

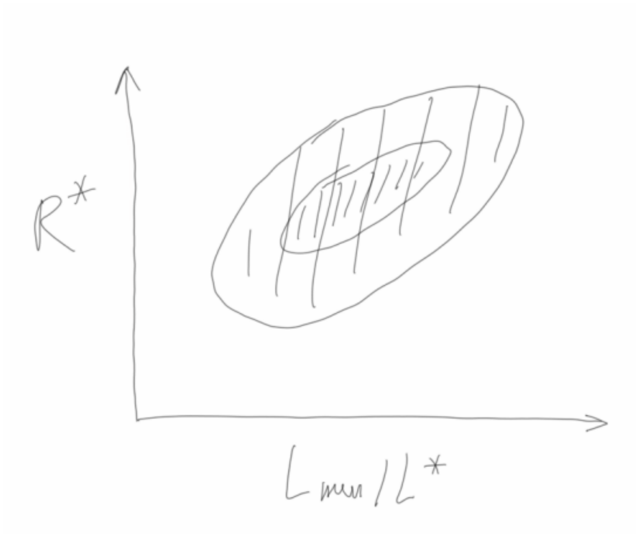

Figure 1. What we would like to get

2 CONSTRAINTS ON PARAMETERS

We would hope to achieve a posterior like Figure 1 where the shaded area is the 'allowed region' of parameter space based on uncertainties in $\beta, dN/dX$, and the luminosity function.

But the previous equation can be inverted to give

$${R^*}^2 = \frac{dN}{dX}\frac{H_0}{c\pi} f^{-1} {\phi^*}^{-1} \Gamma^{-1}(\alpha + 2\beta + 1, L_{min}/L^*)$$

which means that for each value of L_{min}, it is always possible to find a value of β which reproduces the observations. Doing things this way, we can input the measurements of dN/dX (with uncertainties) and deduce R^* as a function of β and L_{min}. The results at $z = 5$ are shown in Fig 2.

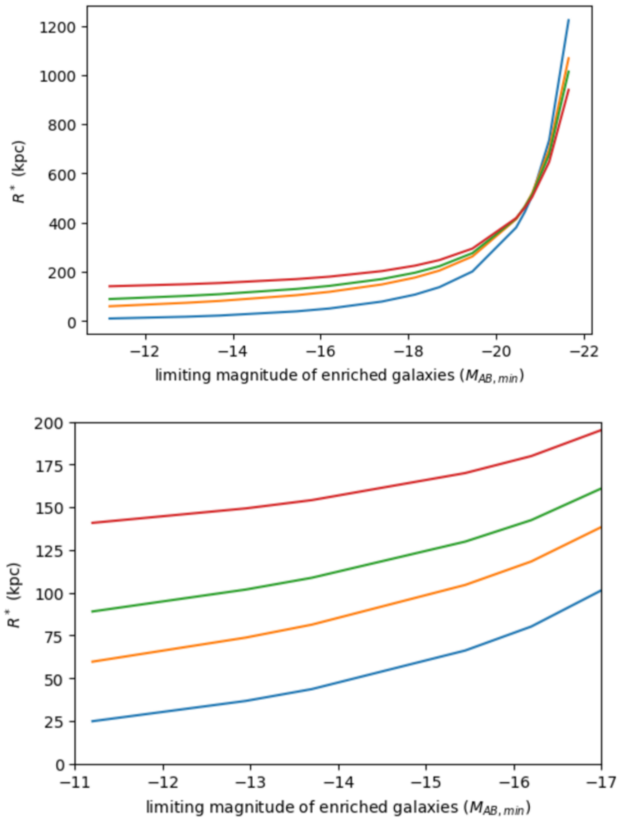

Figure 2. What the 'posterior' actually looks like: colors blue to red show $\beta = 0.1, 0.23, 0.3, 0.4$. Bottom is a zoom on the top panel.

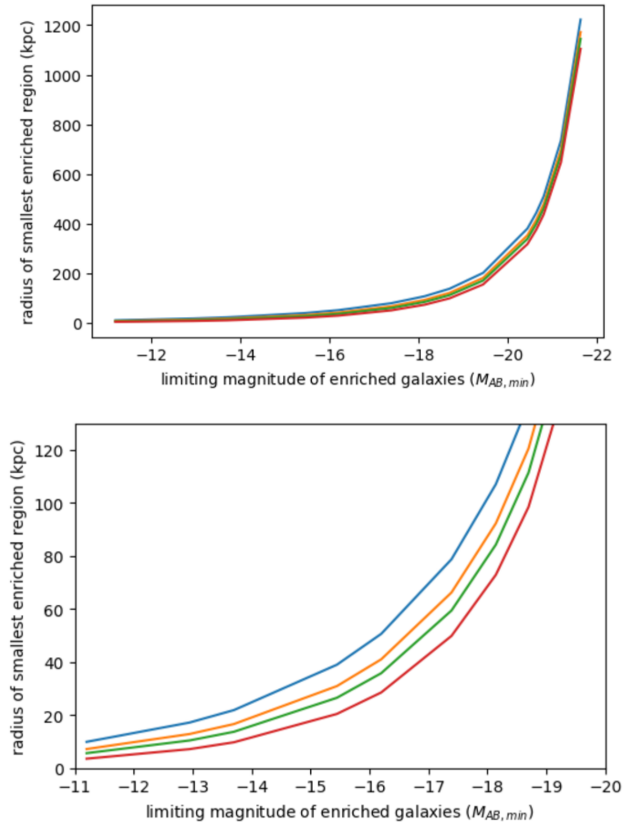

Figure 3. Sizes of the smallest enriched haloes: colors blue to red show $\beta = 0.1, 0.23, 0.3, 0.4$. Bottom is a zoom on the top panel.

The problem is that the relation $R = R^*(L/L^*)^\beta$ is extremely uncertain at high redshift. There are no obvious priors as to what this should be, although some scalings of R^* with $(1+z)^3$ have been suggested which we can look into (but this only holds at very low redshift according to the existing litt.). Therefore, a ML analysis would just recover any priors we choose to input into the calculation and isn't particularly justified or necessary.

There are still a wide range of physical conclusions to be drawn without priors. For instance, the size of the *smallest enriched regions*, $R_{min} = R^*(L_{min}/L^*)^\beta$, is (surprisingly) stable to varying β as shown in Fig 3. This is a useful plot for theorists who run simulations: given a choice of halo mass enrichment threshold, it tells the sizes which the simulations should resolve.

Finally, and in my opinion the most useful aspect of this, is the bottom line of $R(L)$. The relation $R = R^*(L/L^*)^\beta$ actually only depends on two parameters: L_{min} and β, with R^* being obtainable by using our measurements of dN/dX. This results in Figure 4, a good summary of all the constraints currently available at $z = 5$ with no priors. The figure really shows how good the constraints already are on the sizes of enriched haloes around faint objects: even 1 detection of a halo larger than these limits would raise very serious problems. It also presents an honest perspective on how uncertain the predictions are for the brightest objects. This figure will be extremely useful to observers looking for associated metal absorbers at high redshift.

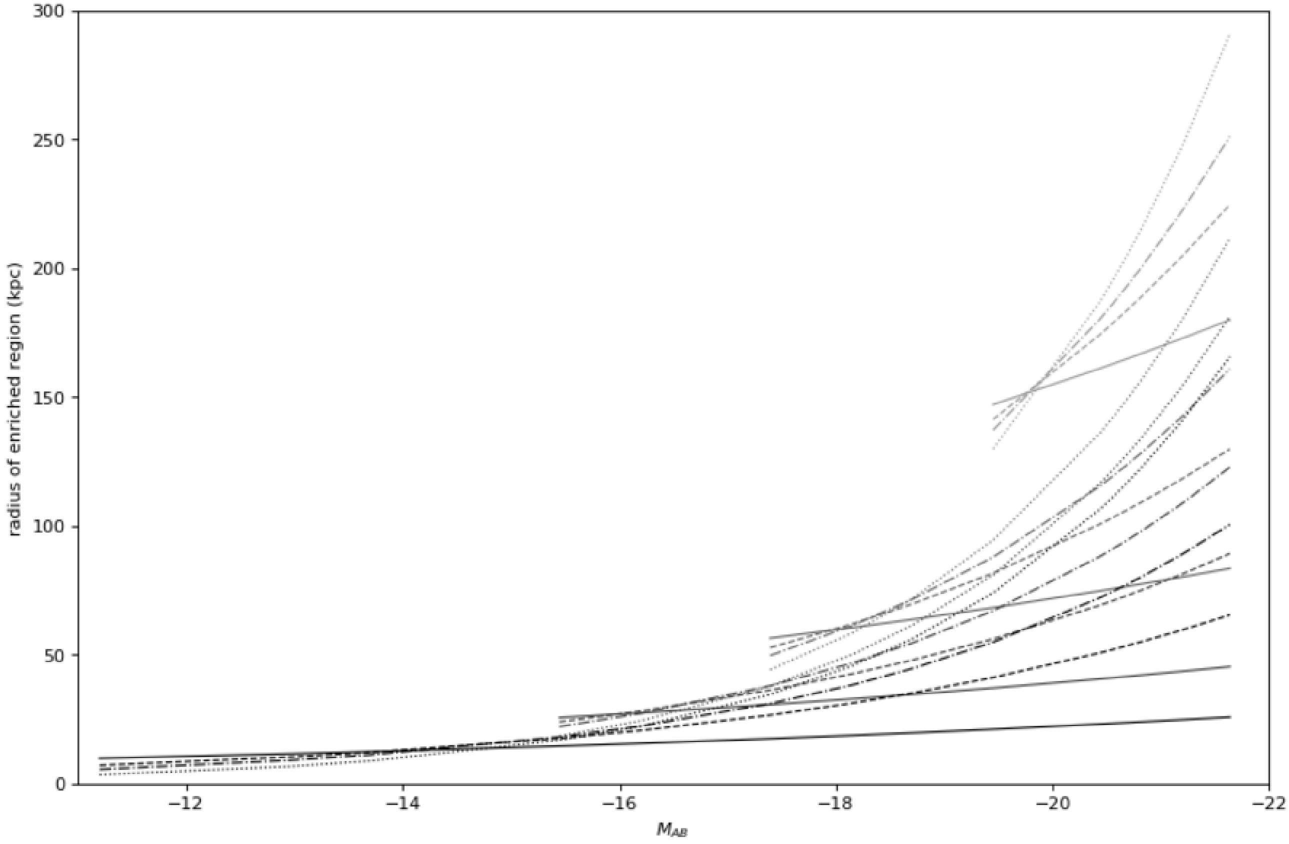

Figure 4. Relation between an object's AB magnitude and the expected size of the surrounding Mg II enriched region. Shades of grey correspond to varying $L_{min}/L^* = 0.0001, 0.001, 0.01, 0.1$ corresponding to limiting magnitudes of $M_{min} = -11.2, -13.7, -16.2, -18.7$, from dark to light. Line shape shows the effect of varying $\beta = 0.1, 0.23, 0.3, 0.4$, from continuous to dashed to dotted. Not all lines extend to low luminosities, since those systems are not enriched if M_{min} is high.

3 TODO

(i) Add errors
(ii) Do other redshift ranges
(iii) Turn discrete models in Fig4 into a continuum of models?