A Modified BGM-PHD Filter With Unknown Detection Probability

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***Abstract*—The standard Probability Hypothesis Density (PHD) filter requires manually presetting the detection probability before tracking. In addition, incorrect settings would lead to deterioration in filter performance. Beta GM-PHD filter introduces the unknown detection probability modeled by Beta distribution into the Bayes recursion to enhance the robustness under the lack of detection model parameters. However, the limitation that biases the target number and detection probability estimations significantly affects the application of the Beta GM-PHD filter. In this paper, the mechanism of the estimation biases is analyzed and a simple but effective modification is presented. Simulation results demonstrate that the modified filter provides more accurate detection probability and target state estimates in unknown and space-varying detection probability scenarios.**

***Index Terms*—multi-target tracking, PHD filter, unknown detection probability, parameter estimation**

I. Introduction

Multi-Target Tracking (MTT) method could simultaneously estimate the time-varying number of targets and states from noisy observations and is of significant importance in many areas [1] [2] [3]. Recently, the MTT method based on Random Finite Sets (RFS) [4] has led to substantial research interest. The filters such as PHD [5], Cardinality PHD (CPHD) [6], and Labeled Multi-Bernoulli (LMB) [7] [8] [9] are sequentially proposed. Among them, the PHD filter implemented by Gaussian Mixture (GM) [10] achieves the lowest computational complexity. However, several prior parameters and birth models are required for these standard filters.

Detection probability is one of the parameters required to be known as a priori. In practical applications, the detection probability cannot be accurately known since it depends on the target characteristics, the surveillance environment, and the sensor capabilities [11]. Meanwhile, manual presetting based on experience could lead to severely deteriorated filtering performance due to the detection model mismatch. To address this problem, [12] first augments the single target state with unknown detection probability, then model the augmented state by the Beta-Gaussian Mixture (BGM). Via introducing the augmented state model into the recursion, the robustness of the CPHD filter under unknown detection probability has been enhanced. However, applying the above thoughts to the PHD filter results in an overestimation of the target number. In [13], the dependence of PHD filters on the priori detection probability is treated from a different perspective. Considering that the signal-to-noise ratio (SNR) is the most common feature influencing detection, [13] exploits the inverse gamma component to propagate the SNR through time. Then, the SNR is utilized to incorporate the detection probability during filtering. This method achieves better tracking accuracy than the BGM-CPHD filter but requires additional SNR information. [14] models the detection probability as a time-varying Markov process, and exploits an adaptive tracker based on the Bayesian framework to achieve target tracking with unknown detection probability. Nevertheless, the tracker can only be applied to single-target tracking.

In this paper, the standard BGM-PHD filter is modified to perform better on the target state estimate. Specifically, we first analyze the BGM-PHD recursion under an ideal condition and find that the component merging process will inevitably lead to the detection probability underestimation. In light of this problem, a simple and effective modification of the BGC merging process is performed. Finally, the effectiveness of the modified BGM-PHD method is verified through simulation experiments under two unknown detection probability scenarios.

The rest of the paper is as organized as follows. Section II provides the necessary background of the BGM-PHD filter. In section III, the mechanism of the BGM-PHD filter overestimated target number is analyzed, and the specific amendment is presented. Section IV evaluates the performance of the modified BFM-PHD filter under multi-target tracking scenarios. Finally, the conclusion is given in section V.

II. Background

This section provides background knowledge related to standard PHD recursion, PHD recursion with unknown detection probability, and its implementation based on BGM.

A. The PHD recursion

Denote the single-target state as x, the PHD filter propagates the intensity function $v(x)$ in the recursions. At time k, the prediction of the prior intensity $v_{k-1}(x)$ is given as

$$v_{k|k-1}(x) = \int p_{S,k}(\xi)\phi_{k|k-1}(x|\xi)v_{k-1}(\xi)d\xi + v_{\gamma,k}(x), \quad (1)$$

where $p_{S,k}(\cdot)$ is the survival probability. $\phi_{k|k-1}(\cdot|\cdot)$ is the target state transition density, and $v_{\gamma,k}(\cdot)$ denotes the prior PHD of the targets birth at time k.

Then, the predicted intensity $v_{k|k-1}$ is updated by measurement set Z_k collected at time k according to the following equation

$$\begin{aligned} v_k(x) = [1 - p_{D,k}(x)]v_{k|k-1}(x) + \\ \sum_{z \in Z_k} \frac{p_{D,k}(x)g_k(z|x)v_{k|k-1}(x)}{\kappa_k(z) + \int p_{D,k}(\xi)g_k(z|\xi)v_{k|k-1}(\xi)d\xi}, \end{aligned} \quad (2)$$

where $g_k(\cdot|\cdot)$ is the likelihood function. $P_{D,k}$ is the probability of detection and $\kappa_k(\cdot)$ is the clutter density. In the rest of the paper, the first and the second terms in $v_k(x)$ are denoted as missed detection and detection term, respectively.

B. Detection Probability Modeling

In the condition of unknown detection probability, the probability of detection can be considered a parameter to be estimated. That is, the target state x is augmented with the unknown detection probability a. The integral operation $\int \cdot dx$ is replaced by $\iint \cdot da dx$. Define the transition density as

$$\phi_{k+1|k}(x, a|\xi, a') = \phi_{k+1|k}(x|\xi)\phi_{k+1|k}(a|a'). \quad (3)$$

As a result, the prediction equation (1) is re-given by

$$\begin{aligned} v_{k|k-1}(x, a) = v_{\gamma,k}(x, a) + \\ \iint p_{S,k}(\xi)\phi_{k|k-1}(x|\xi)\phi_{k|k-1}(a|a')v_{k-1}(\xi, a')d\xi da'. \end{aligned} \quad (4)$$

The updating equation (2) is rewritten as

$$\begin{aligned} v_k(x, a) = (1 - a)v_{k|k-1}(x, a) + \\ \sum_{z \in Z_k} \frac{a \cdot g_k(z|x)v_{k|k-1}(x, a)}{\kappa_k(z) + \iint a' g_k(z|\xi)v_{k|k-1}(\xi, a')d\xi da'}. \end{aligned} \quad (5)$$

C. The Standard BGM-PHD Filter

Considering the computational efficiency and the closed-form of intensity distribution during the recursion, the intensity function is approximated by the BGM as follows

$$v_k(x, a) = \sum_{i=1}^{J_k} w_k^{(i)} \beta(a; u_k^{(i)}, v_k^{(i)})\mathcal{N}(x; m_k^{(i)}, P_k^{(i)}). \quad (6)$$

where $\beta(a; u, v)$ is the beta distribution given as

$$\beta(a; u, v) = \frac{a^{u-1}(1-a)^{v-1}}{\int_0^1 a^{u-1}(1-a)^{v-1}da}. \quad (7)$$

The mean and variance of $\beta(a, u, v)$ is given as

$$\begin{aligned} \mu_\beta &= u/u + v, \\ \sigma_\beta^2 &= uv/(u+v)^2(u+v+1). \end{aligned} \quad (8)$$

It implies that given μ_β and σ_β, parameters u and v can obtained by $u = \theta\mu_\beta$ and $v = \theta(1 - \mu_\beta)$, where

$$\theta = \mu_\beta(1 - \mu_\beta)/\sigma_\beta^2 - 1. \quad (9)$$

Particularly, the beta distribution has the following properties

$$a \cdot \beta(a; u, v) = \mu_\beta \cdot \beta(a; u + 1, v), \quad (10)$$

$$(1 - a) \cdot \beta(a; u, v) = (1 - \mu_\beta) \cdot \beta(a; u, v + 1). \quad (11)$$

The above properties ensure the closed-form of the beta distribution in the updating stage.

III. The Modified BGM-PHD Filter

In this section, to identify the critical determinants affecting the performance of the BGM-PHD filter, an ideal single-target tracking scenario is considered for simplifying the recursive process. Then, the main reason causing the underestimation of the detection probability is analyzed. Finally, a heuristic improvement made to the standard BGM-PHD filter is given, together with the specific implementation details.

A. Theoretical analysis

Consider an ideal single-target tracking scenario that ignores the clutter, target birth, process noise, and measurement noise. Suppose that the survival probability in the BGM-PHD filter constantly holds 1. The intensity function (6) is given as

$$v_k(x, a) = w_k\beta(a; u_k, v_k)\mathcal{N}(x; m_k, P_k). \quad (12)$$

Then the prediction formulation (4) for the BGM-PHD filter is further simplified as

$$\begin{aligned} v_{k|k-1}(x, a) = w_{k|k-1}\beta(a; u_{k|k-1}, v_{k|k-1}) \\ \cdot \mathcal{N}(x; m_{k|k-1}, P_{k|k-1}) \end{aligned} \quad (13)$$

where the Gaussian weight $w_{k|k-1}$ and the mean of beta distribution remain unchanged, and the variance of beta distribution is enlarged by $|k_\beta|$, i.e.,

$$\begin{aligned} \mu_{\beta,k|k-1} &= \mu_{\beta,k-1}, \\ \sigma_{\beta,k|k-1} &= |k_\beta|\sigma_{\beta,k-1}^2. \end{aligned} \quad (14)$$

The parameters $u_{k|k-1}$ and $v_{k|k-1}$ are determined by (9).

The updating formulation (5) can be simplified as

$$\begin{aligned} v_k(x, a) = (1 - \mu_{\beta,k|k-1})w_{k|k-1} \\ \cdot \beta(a; u_{k|k-1}, v_{k|k-1} + 1)\mathcal{N}(x; m_k, P_k) \\ + \beta(a; u_{k|k-1} + 1, v_{k|k-1})\mathcal{N}(x; m_k, P_k). \end{aligned} \quad (15)$$

Further, the final intensity $v_k(x, a)$ after merging is given as

$$v_k(x, a) = w_k\beta(a; u_k, v_k)\mathcal{N}(x; m_k, P_k), \quad (16)$$

where $w_k = (1 - \mu_{\beta,k|k-1})w_{k|k-1} + 1$. The parameters u_k and v_k are determined by

$$\begin{aligned} \mu_{\beta,k} &= ((1 - \mu_{\beta,k|k-1})w_{k|k-1}\mu_{\beta,1} + \mu_{\beta,2})/w_k, \\ \sigma_{\beta,k}^2 &= ((1 - \mu_{\beta,k|k-1})w_{k|k-1}\sigma_{\beta,1}^2 + \sigma_{\beta,2}^2)/w_k, \end{aligned} \quad (17)$$

where

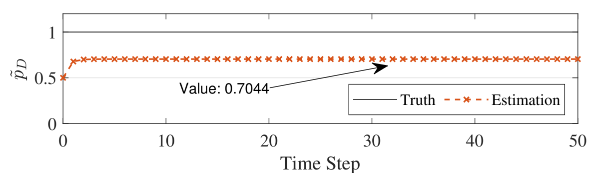
$$\mu_{\beta,1} = u_{k|k-1}/(u_{k|k-1} + v_{k|k-1} + 1),$$
$$\mu_{\beta,2} = (u_{k|k-1} + 1)/(u_{k|k-1} + v_{k|k-1} + 1), \tag{18}$$

and

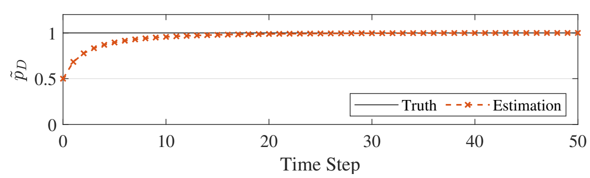
$$\sigma^2_{\beta,1} = \frac{u_{k|k-1}(v_{k|k-1} + 1)}{(u_{k|k-1} + v_{k|k-1} + 1)^2(u_{k|k-1} + v_{k|k-1} + 2)},$$
$$\sigma^2_{\beta,2} = \frac{(u_{k|k-1} + 1)v_{k|k-1}}{(u_{k|k-1} + v_{k|k-1} + 1)^2(u_{k|k-1} + v_{k|k-1} + 2)}, \tag{19}$$

From equations (14) and (17), the recursion of beta distribution $\beta(a; u, v)$ and Gaussian weight w are obtained.

Initialize the parameter as $u_0 = v_0 = 1$, $|k_\beta| = 1.1$, and $w_0 = 0.03$, performing 50 times recursion. The detection probability estimation $\tilde{p}_D$ at each time step is recorded in Fig. 1 (a). Since the $\beta(a; u, v)$ has the properties shown in (10) and (11), μ_β serves as the detection probability in the updating process. Hence, we choose μ_β as the detection probability estimation $\tilde{p}_D$.



(a) $\tilde{p}_D$ with β distribution merging.



(b) $\tilde{p}_D$ without β distribution merging.

Fig. 1. Detection probability estimate.

As can be seen from Fig. 1 (a), even in the perfectly ideal single-target tracking scenario, the detection probability estimates of the BGM-PHD filter already appear to be significantly underestimated. The reason is that the weighted average is performed on μ_β between missed detection and detection terms when merging the BGM components. As shown in (17), the mean $\mu_{\beta,1}$ of missed detection term will pull down the detection probability estimates. In the case shown in Fig. 1(b), the beta distribution in the detection term is directly retained instead of merging with the beta distribution in the missed detection term, so that the underestimate of $\tilde{p}_D$ is avoided.

B. Implementation details

Inspired by the analysis provided in III-A, it is necessary to reconsider the merging procedure for the Beta Gaussian components in the BGM-PHD filter.

Suppose that the updated intensity $v(x, a)$ is approximated as (6). As we can see, a BGM component consists of a Gaussian component $\mathcal{N}(x; m, P)$ and a beta distribution $\beta(a; u, v)$. Thereinto, the former implies the target kinematic state, and the latter contains the information on detection probability.

We suggest the following scheme for the fusion of BGM components. First, the BGM components to be merged are identified by the Mahalanobis distance between the Gaussian components $\mathcal{N}(x; m, P)$. Then the Gaussian components and the beta distribution of the BGM components are merged separately. The merging of Gaussian components adopts the same way as the standard GM-PHD filter [10]. In contrast, the beta distribution $\beta(a; u, v)$ merging is performed after determining the source of the BGM component. Particularly, if the BGM component from the detection term exists, only the beta distributions from the detection term are merged. Otherwise, all the beta distributions are merged directly.

The reason for this treatment is listed as follows.

- Since the ultimate purpose of target tracking is to obtain estimates of the kinematic states, the BGM components invoved in the merging procedure only require a similarity on the kinematic states.
- For a BGM component, the mean μ_β of the beta distribution reflects the frequency of the single-target state updated by the measurement. However, the missed detection term does not record the measurement update at the current time step and therefore the BGM components from missed detection term have a lower mean μ_β. Merging the beta distribution from the detection and missed detection term will pull down the mean μ_β, resulting in the detection probability underestimates.

The implementation of modified BGM merging procedure is elaborated by the pseudo-code in Algorithm 1.

Algorithm 1 The modified BGM merging procedure.

Require: $\{w_k^{(i)}, m_k^{(i)}, P_k^{(i)}, u_k^{(i)}, v_k^{(i)}\}_{i=1}^{J_k}$, a merging threshold U_{th}, the number of BGM components belonging to missed detection term after pruning J_p.
Ensure: $\{\tilde{w}_k^{(i)}, \tilde{m}_k^{(i)}, \tilde{P}_k^{(i)}, \tilde{u}_k^{(i)}, \tilde{v}_k^{(i)}\}_{i=1}^{l}$.
Set $l = 0$ and $I = \{1, \cdots, J_k\}$.
repeat
  $l = l + 1$.
  $j = \arg\max_{i \in I} w_k^{(i)}$.
  $L = \{i \in I | (m_k^{(i)} - m_k^{(j)})^{\mathrm{T}}(P_k^{(i)})^{(-1)}(m_k^{(i)} - m_k^{(j)}) \leq U_{th}\}$.
  Calculate $\{\tilde{w}_k^{(l)}, \tilde{m}_k^{(l)}, \tilde{P}_k^{(l)}\}$ according to [10].
  $I = I \setminus L$.
  if $J_p < \max(L)$ **then**
    $L = L(L > J_p)$.
  end if
  Calculate $\mu_{\beta,k}^{(i)}$ and $\sigma_{\beta,k}^{2(i)}$ according to (8), where $i \in L$.
  $\tilde{\mu}_{\beta,k}^{(l)} = \sum_{i \in L}(w_k^{(i)}\mu_{\beta,k}^{(i)}) / \sum_{i \in L} w_k^{(i)}$.
  $\tilde{\sigma^2_{\beta,k}}^{(l)} = \sum_{i \in L}(w_k^{(i)}\sigma_{\beta,k}^{2(i)}) / \sum_{i \in L} w_k^{(i)}$.
  Obtain $\{\tilde{u}_k^{(l)}, \tilde{v}_k^{(l)}\}$ according to (9).
until $I = \emptyset$

Since the updated BGM components belonging to detection terms are strictly arranged behind the components of the missed detection term, Jp can be used to determine whether the current BGM component belongs to a missed detection term or a detection term.

The implementation of other stages in the modified BGM-PHD filter such as prediction, updating, pruning, and capping remain unchanged. Therefore, the computational complexity of the proposed method is the same as that of the standard BGM-PHD filter.

IV. SIMULATION

In this section, a planar multi-target tracking scenario shown in Fig. 2 is adopted to assess the performance of the Modified-BGM-PHD (M-BGM-PHD) filter. Each target $x_k = [p_{x,k}, \dot{p}_{x,k}, p_{y,k}, \dot{p}_{y,k}]^{\mathrm{T}}$ follows a Constant Velocity (CV) motion given by

$$x_k = F^{CV} x_{k-1} + G^{CV} w_{k-1}. \tag{20}$$

Here

$$F^{CV} = \begin{bmatrix} 1 & T_s \\ 0 & 1 \end{bmatrix} \otimes I_2, \quad G^{CV} = \begin{bmatrix} T_s^2/2 \\ T \end{bmatrix} \otimes I_2, \tag{21}$$

$\otimes$ denotes the Kronecker product, I_2 is a 2 dimension identity matrix, $T_s = 1$ is the sampling period. $w_{k-1} \sim N(0, \sigma_w^2 I_2)$ with $\sigma_w = 5\mathrm{m/s}^2$ is a zero-mean Gaussian process noise.

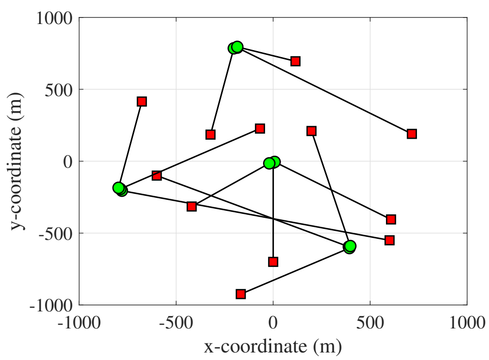


Fig. 2. Ground truth trajectories of targets. Start and end points of the trajectories are denoted by circles and squares.

The observation is given by $z = [x, y]^T + v$, where Gaussian noise $v \sim N(0, \sigma^2 I_2)$ with $\sigma = 10$m. Further, clutter follows the Poisson model with an average of 10 clutters per scan in the surveillance area.

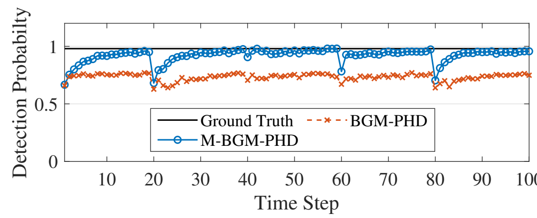
The comparison among the M-BGM-PHD, BGM-PHD, and GM-PHD (with known p_D) filters is implemented in two different case studies. In the first case, the detection probability is fixed all over the surveillance area. On the contrary, the detection probability is space-varying in the second case.

For both cases, the average of the Optimal Sub-Pattern Assignment (OSPA) [15] metric (with $c = 100$m, $p = 1$) under 100 times Monte Carlo runs is used to evaluate the state estimation accuracy of the filter.

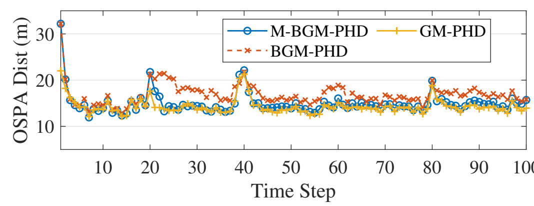
A. Fixed Detection Probability

In this case, the detection probability is set as $p_D = 0.98$ over the surveillance area. Fig. 3 (a) presents the average detection probability estimate comparison over 100 Mont Carlo runs. As we can see, the modified BGM-PHD attains closer estimation compared to the standard filter.

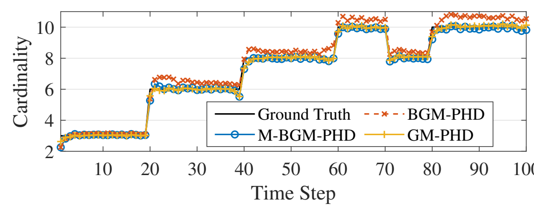
Fig. 3 (b) and (c) depict the OSPA distance and cardinality estimation across the M-BGM-PHD, BGM-PHD, and GM-PHD (with known p_D) filters. The result shows that M-BGM-PHD has a more accurate state estimation performance. Moreover, the M-BGM-PHD fixes the problem of target number overestimation for BGM-PHD.



(a) Detection probability estimations.



(b) OSPA distance.



(c) Cardinality estimations

Fig. 3. Comparison result for case 1.

B. Space-varying Detection Probability

In this scenario, different positions in the surveillance area have different detection probabilities. Specifically, the detection probability is negatively correlated with the distance from the current position to the sensor, as shown in Fig. 4.

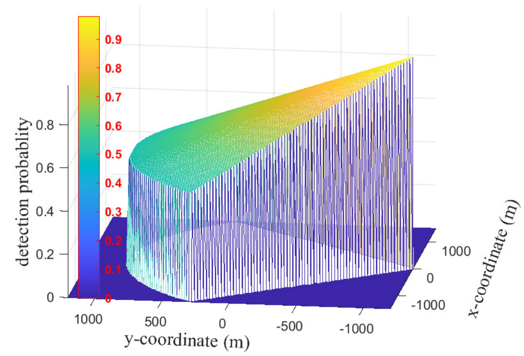


Fig. 4. The space-varying detection probability distribution.

Utilizing the M-BGM-PHD and BGM-PHD filter for multi-target tracking under the scenario with unknown and space-varying detection probability, the obtained multi-target state and detection probability estimation are recorded in Fig. 5. The multi-target position estimations at each time are represented by the position of the solid circle, meanwhile, the detection probability estimations are indicated by the color of the circle. In conjunction with Fig. 4, it can be seen that the M-BGM-PHD has fewer false alarms and a more accurate detection probability estimate than the BGM-PHD filter.

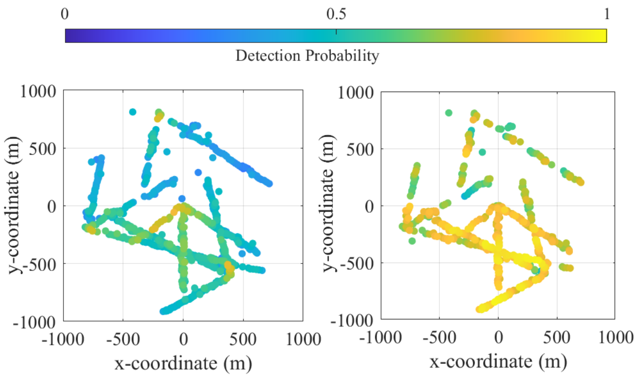
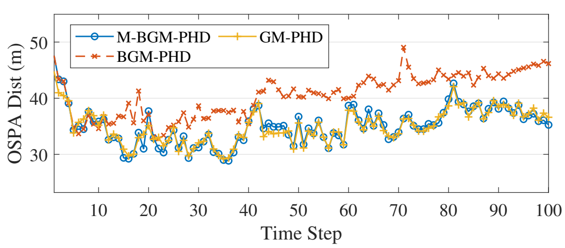
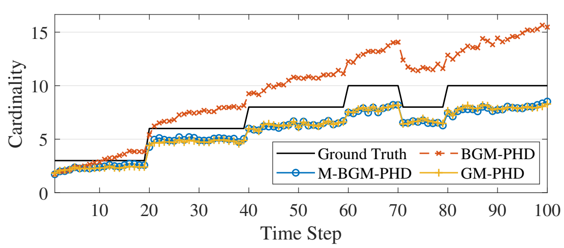


Fig. 5. Multi-target augmented state estimates using BGM-PHD filter (left-hand) and M-BGM-PHD (right-hand).

Fig.6 plots the performance comparisons among three GM-PHD based filters in terms of OSPA distance and cardinality estimations. It is suggested that the tracking accuracy of the M-BGM-PHD filter is higher than that of the BGM-PHD filter and is basically close to that of the GM-PHD filter with known p_D. Moreover, compared with the case 1, the performance of target number estimates of the BGM-PHD filter decreases badly in the space-varying detection probability scenario. At the same time, the target number estimate obtained by M-BGM-PHD is close to the GM-PHD filter with known p_D.



(a) OSPA distance.



(b) Cardinality estimations.

Fig. 6. Comparison result for case 2.

V. Conclusion

In this paper, we modified the standard BGM-PHD filter to address the problem that the BGM-PHD filter underestimates the detection probability and overestimates the target number. First, by analyzing the recursion under the ideal target tracking scenario, it can be found that the detection probability estimate of the standard BGM-PHD filter inevitably converges to a value that is lower than the true detection probability. The explanation for this phenomenon lies in the merging process of the beta-Gaussian components. To this end, a heuristic modification for the merging process of the BGM-PHD filter is presented. Simulation results show that the proposed M-BGM-PHD filter achieves a significant improvement in both detection probability and target state estimation.

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