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Assignment of master's thesis

Title:	Estimation of detection probability in multitarget filters using object advanced image processing techniques
Student:	Bc. Michal Seibert
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Study program:	Informatics
Branch / specialization:	Knowledge Engineering
Department:	Department of Applied Mathematics
Validity:	until the end of summer semester 2024/2025

Instructions

Abstract: The subject of the thesis is the dynamic adjustment of the detection probability in random finite set (RFS)-based filters. These filters can track objects in noisy environments from imprecise measurements. However, they rely on the knowledge of the object (mis)detection probability. If this quantity is set inappropriately, the filters are oversensitive and prone to track loss. However, there is no convenient methodology for a consistent estimation of the detection probability. The present thesis aims to focus on its inference using algorithms for object detection and image segmentation. For object detection, the YOLO model should be used, for segmentation, the Segment anything from Meta AI is a possible way towards a solution. These models can recognize various objects in the image. It is conjectured that the combination of these algorithms can yield a filter with good robustness to target misdetections.

The goals are as follows:
- study the principles of the multitarget tracking algorithms
- study the principles of image segmentation and object detection
- propose a technique for estimation of object detection probability
- perform assessment of the proposed algorithm and discuss the obtained results

Literature:
[1] A. F. Garcia-Fernandez, A. S. Rahmathullah, and L. Svensson, "A Metric on the Space of Finite Sets of Trajectories for Evaluation of Multi-Target Tracking Algorithms," IEEE Trans.

Electronically approved by Ing. Magda Friedjungová, Ph.D. on 31 January 2024 in Prague.

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Master’s thesis

ESTIMATION OF DETECTION PROBABILITY IN MULTITARGET FILTERS USING OBJECT ADVANCED IMAGE PROCESSING TECHNIQUES

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March 20, 2024

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Declaration

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In Praze on March 20, 2024

Abstract

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Abstrakt

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Seznam zkratek

DFA	Deterministic Finite Automaton
FA	Finite Automaton
LPS	Labelled Prüfer Sequence
NFA	Nondeterministic Finite Automaton
NPS	Numbered Prüfer Sequence
XML	Extensible Markup Language
XPath	XML Path Language
XSLT	eXtensible Stylesheet Language Transformations
W3C	World Wide Web Consortium

Introduction

Multi-target tracking (MTT), a fundamental aspect of surveillance and monitoring systems, has undergone significant advancements in recent years, transforming it into a critical field with diverse applications across various domains. The scope of MTT extends beyond mere tracking, encompassing tasks such as object detection, identification, and trajectory prediction. The primary goal is to maintain a comprehensive situational awareness, providing invaluable information for decision-making processes in various applications, ranging from defense and surveillance to autonomous systems and robotics. This section provides an overview of the evolution, applications, significance, and current research focus of multi-target tracking.

0.1 Evolution and development

The roots of MTT can be traced back to the early 20th century when radar technology emerged during World War II [1]. Initially developed for single-target detection, radar systems laid the groundwork for subsequent advancements in multi-target tracking. As scenarios evolved and became more complex, the need for advanced tracking capabilities grew, prompting the development of more sophisticated algorithms.

The 1970s and 1980s witnessed the emergence of basic tracking algorithms, marking the initial forays into the field. Subsequent decades saw the integration of probabilistic techniques, such as the Kalman filter [2], which significantly enhanced tracking accuracy. . The 2000s marked the transition to data-driven approaches, with particle filters gaining popularity due to their ability to handle non-linear and non-Gaussian tracking scenarios [3]. However, due to the high computational complexity of particle filters, much attention is paid to data association filters such as JPDA [4] or RFS based filters [5].

0.2 Applications of multi-target tracking

The versatility of MTT is reflected in its diverse applications across various domains. In defense, MTT plays a pivotal role in monitoring and tracking multiple targets simultaneously, aiding in threat assessment, target prioritization, and distinguishing friend from foe.

The advent of autonomous systems, particularly in vehicles, has heightened the importance of MTT in predicting and tracking the movements of pedestrians, vehicles, and other obstacles. This application enhances the safety and efficiency of autonomous vehicles by providing real-time awareness of the surrounding environment [6].

Surveillance systems rely on multi-target tracking for monitoring activities in crowded environments and identifying suspicious behavior

Moreover, in fields such as robotics, defense, healthcare monitoring, and wildlife conservation, multi-target tracking systems contribute significantly to enhancing situational awareness, enabling real-time decision-making, improving resource allocation efficiency, and supporting various mission-critical tasks.

0.3 Multi target algorithms

The field of multi-target tracking is marked by a rich and diverse landscape of algorithms, each tailored to address specific challenges inherent in tracking multiple objects. These algorithms can be broadly categorized into association-based methods and Random Finite Set (RFS) based methods, each offering unique advantages and trade-offs.

Association-based methods form a foundational category in multi-target tracking, emphasizing the linking of measurements to existing tracks or the creation of new tracks. The well-established Kalman filter and its variants, such as the Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF), fall under this category. More advanced methods considering clutter such as Probabilistic Association (PDA) filter, Joint Probabilistic Association (JPDA) filter or Multiple Hypothesis Tracker (MHT) filter are another examples of filters from this category.

In contrast to association-based methods, Random Finite Set (RFS) based methods provide a probabilistic framework to model multiple target states simultaneously. These methods operate on sets of possible target states, allowing for a more comprehensive representation of uncertainty and variability in tracking scenarios. The Probability Hypothesis Density (PHD) filter, Cardinalized Probability Hypothesis Density (CPHD) or Poisson multi-Bernoulli mixture (PMBM) filter are prominent examples of an RFS-based approach.

0.4 Research interests

Contemporary research in multi-target tracking is characterized by a strong emphasis on addressing key challenges to further enhance the performance and robustness of tracking systems. Current research focus deald with problems such as:

- **Data Association in complex environment:** Handling scenarios with high target density, occlusions, and clutter remains a complex issue. Robust methods for accurate and efficient data association in crowded and dynamic environments are still actively researched.
- **Handling non-linear and non-gaussian dynamics:** Real-world scenarios often exhibit non-linear and non-Gaussian characteristics. Improving tracking algorithms to effectively handle these complexities, possibly through the integration of advanced probabilistic models is an ongoing area of research.
- **Real-time processing and computational efficiency:** Many MTT algorithms, especially those with high computational demands, face challenges in meeting real-time processing requirements. Efficient algorithms that strike a balance between computational complexity and tracking accuracy are continuously sought.
- **Sensor fusion and heterogeneous data integration:** Integrating information from various sensors, each with its own characteristics and limitations, is an ongoing challenge. Developing robust methods for sensor fusion to improve tracking accuracy and reliability is an active research area.
- **Handling variability in target behavior:** Targets in real-world scenarios may exhibit diverse and unpredictable behaviors. Adapting tracking algorithms to handle varying target speeds, accelerations, and maneuvers remains an unsolved problem.

- **Online learning and adaptive algorithms:** Designing algorithms that can adapt and learn online as they encounter new scenarios or dynamic changes in the environment is a topic of interest. Adaptive tracking systems that can continuously improve their performance without extensive retraining are sought after.

The probabilistic formulation of the RFS-based filters and the inherent Bayesian processing of available information allow to accommodate the uncertainty arising from the presence of false detections, missed detections, and data association ambiguities. Nevertheless, a fundamental challenge persists: The performance of the filters is highly sensitive to the accurate setting of the target detection probability. This quantity, representing the likelihood of correctly identifying and associating observations with actual targets, is a critical parameter. It has a substantial impact on the Bayesian updating of the prior information. However, in real-world scenarios, the sensor performance is susceptible to various environmental conditions. Adverse weather, occlusions, or just the nature of the current scenario can lead to variations in detection probabilities. A mismatch between assumed and actual detection probabilities can result in a suboptimal tracking performance, leading to missed detections, false alarms, or inaccurate target state estimates [7].

The RFS-based formulation of the PHD or (P)MBM filters naturally takes the uncertainty about the detection probability into account [7]. The update formulae involve it as a function of the target state. In the figurative sense, this allows to model it as a function of the spatial and temporal properties of the environment. Still, two difficulties arise. First, the (Gaussian) filters are analytically tractable only if the detection probabilities are scalar numbers. Second, the nature of the detection probabilities differs from scenario to scenario.

In general, several methods have been proposed to deal with unknown detection probabilities. A Gaussian-beta modeling of a slowly-varying detection profile in the Cardinalized PHD filter is reported in [8]; its alternative for MBM filters follows in [9], and for PMBM filter in [10]. In [11], the authors propose to overcome some deficiencies in the CPHD filter [8] by different clutter/detection probability models. Another variant was recently proposed in [12]. A track-state augmentation with an amplitude offset-based prediction of the detection probability appeared in [13], however, this method suffers difficulties in multistatic fields. An automatic identification system-based sensor performance assessment for clutter-free environments is developed in [14]. A recent paper [15] deals with the unknown detection profile in the trajectory PHD/CPHD filters. There, the algorithm learns from the history of the unknown target detection probability.

This thesis focuses on tracking targets in video data. This allows to avoid the generic solutions and focus on the peculiarities associated with this specific data type. In particular, advantage of YOLO (You Only Look Once) is taken, offering high-performance real-time object detection with high accuracy and efficiency [16]. Its ability to simultaneously predict multiple bounding boxes and class probabilities within an image is used, providing a streamlined and efficient approach to object detection. However, YOLO-based multi-target tracking systems can still be compromised under conditions such as adverse weather, low lighting, or scenarios with occlusions. Sensors may encounter difficulties in accurately detecting and localizing targets, leading to gaps or errors in the tracking process.

0.5 Structure

The structure of this thesis is as follows:

Chapter 1

Chapter 2

Chapter 3

Chapter 4

Chapter 1
Background

Multi-target tracking relies heavily on mathematical concepts from probability theory, statistics, and linear algebra to model and infer the state of multiple objects over time. Key mathematical concepts involved in MTT include Bayesian inference, which provides a framework for updating estimates based on observed evidence, and the use of probabilistic models such as the Gaussian distribution and its extensions, such as Gaussian mixture models. In addition, MTT commonly employs state space models to represent the dynamics of object motion, with popular models including the constant velocity and constant acceleration models. These models are integrated into Bayesian filters, such as the Kalman filter, which recursively estimates the state of multiple targets given noisy measurements.

1.1 Bayesian inference

Bayesian inference stands as a foundational pillar within the realm of probabilistic reasoning, offering a fundamental methodology for systematically updating beliefs in response to observed evidence. At its heart stands Bayes' rule, a fundamental theorem in probability theory that formalizes the process of revising prior beliefs in light of new data. The essence of Bayesian inference transcends mere statistical calculations; it embodies a philosophical stance towards uncertainty, emphasizing the incorporation of prior knowledge and the iterative refinement of beliefs through the assimilation of empirical observations. Within the context of multi-target tracking (MTT), Bayesian inference assumes a paramount role, providing a principled framework for joining information from disparate sources, such as sensor measurements, historical data, and domain expertise. By embracing Bayesian principles, MTT algorithms gain the capacity to model and quantify uncertainty inherent in tracking scenarios, thereby fostering robustness and adaptability in the face of dynamic and complex environments. Moreover, Bayesian inference empowers MTT systems to exploit contextual cues and domain-specific knowledge, enhancing their ability to discern meaningful patterns among noise and uncertainty. Due to the Bayesian inference, MTT researchers and practitioners are equipped with a powerful tool for navigating the intricacies of multi-target tracking, facilitating informed decision-making and advancing the frontier of robust systems. Thus, Bayesian inference emerges not merely as a mathematical construct but as a guiding philosophy underpinning the quest for understanding and reasoning in the face of uncertainty.

1.2 Bayes' rule

Bayes' rule, a cornerstone of Bayesian inference, embodies a fundamental principle in probability theory that underpins the systematic revision of beliefs in the face of new evidence. Mathematically expressed as a simple formula, Bayes' rule encapsulates the process of updating prior probabilities based on observed data, thereby yielding posterior probabilities that reflect the incorporation of new information. At its basis, Bayes' rule provides a formal mechanism for quantifying the impact of new evidence on the likelihood of various hypotheses or states of nature. The rule states that the posterior probability of a hypothesis given observed data is proportional to the product of the likelihood of the data given the hypothesis and the prior probability of the hypothesis, divided by the marginal likelihood of the data. In essence, Bayes' rule facilitates a principled approach to inference, allowing practitioners to integrate prior knowledge with empirical observations to arrive at more informed and reliable conclusions. In the context of multi-target tracking (MTT), Bayes' rule serves as the bedrock upon which tracking algorithms are built, enabling the continuous refinement of estimates about the state of multiple targets based on sensor measurements and historical data. By adhering to the principles of Bayes' rule, MTT systems can effectively navigate the inherent uncertainty and complexity of tracking scenarios, thereby enhancing their robustness and accuracy.

▶ **Definition 1.1** (Conditional distribution)**.** *Let X and Y be jointly continuous random variables, f_Y continuous at y and $f_Y(y) > 0$. Then the conditional distribution function of X, given condition $Y = y$ is defined by*

$$F_{X|Y}(x|y) := lim_{\epsilon \to 0}\mathbf{P}\{X \leq x | Y \in (y, y+\epsilon)\} = \frac{\partial F(x,y)/\partial y}{f_Y(y)}. \tag{1.1}$$

Differentiating this, the conditional density function of X, given the condition $Y = y$ is

$$f_{X|Y}(x|y) := \frac{\partial^2 F(x,y)/[\partial x \partial y]}{f_Y(y)} = \frac{f(x,y)}{f_Y(y)}. \tag{1.2}$$

Naturally, fixing y, $F_{X|Y}(\cdot|y)$ and $f_{X|Y}(\cdot|y)$ are proper distribution and density functions, respectively. It is also clear that X and Y are independent if and only if $F_{X|Y}(x|y) = F_X(x)$ and $f_{X|Y}(x|y) = f_X(x)$ for all x and y for which these quantities are defined.

Bayes' theorem is defined as follows:

▶ **Definition 1.2** (Bayes' theorem)**.** *Let x and x be random variables with densities $f(y|x)$ and $f(x)$. Then Bayes' theorem is defined as:*

$$f(x|y) = \frac{f(y|x)f(x)}{f(y)}, \quad f(y) > 0, \tag{1.3}$$

where

- $f(x|y)$ is the conditional aposterior density of x,
- $f(x)$ is aprior density,
- $f(y|x)$ is likelihood and $f(x)$ is marginal density of X, also called *evidence*, and is given by

$$f(y) = \int f(x,y)\partial x. \tag{1.4}$$

Combining equations 1.1, 1.2 and 1.4 to formula 1.3 we get complete formula for Bayes' theorem:

$$f(x|y) = \frac{f(y|x)f(x)}{f(y)} = \frac{f(y|x)f(x)}{\int f(y|x)f(x)\partial x}. \tag{1.5}$$

The denominator is only the normalizing constant independent of x, often only proportionality is used:

$$f(x|y) \propto f(y|x)f(x). \tag{1.6}$$

1.3 Multivariate Gaussian distribution

In the realm of multi-target tracking, various probability distributions are employed to model the uncertainty associated with target states and measurements. However, among these distributions, the multivariate Gaussian distribution holds a preeminent position due to its versatility, mathematical tractability, and empirical relevance. While other distributions may capture specific aspects of target behavior or measurement noise, the Gaussian distribution emerges as the cornerstone of MTT due to its ability to characterize complex probability distributions in multi-dimensional spaces. As a result common assumption among MTT filters is, that the targets follows linear Gaussian dynamic and measuremets models, as in [1], [17] or [18]. The Gaussian distribution finds ubiquitous application across various components of tracking algorithms, including:

- State Representation: Gaussian distributions are used to model the probability distributions of target states, allowing for efficient representation and propagation of uncertainty over time.
- Measurement Model: Gaussian distributions are employed to model the likelihood of sensor measurements given the true target state, facilitating the incorporation of sensor data into the tracking process.
- Filtering Algorithms: Gaussian-based filters leverage the Gaussian assumption to derive recursive estimation algorithms for tracking multiple targets with optimal efficiency and accuracy.
- Data Association: Gaussian mixture models (GMMs), which represent mixtures of Gaussian distributions, are utilized for probabilistic data association in MTT, enabling robust handling of measurement uncertainty and target ambiguity.

▶ Note 1.3. The multivariate Gaussian distribution is a generalization of the univariate Gaussian distribution, but instead of a scalar mean and variace, there are mean vector and covariance matrix, that describes correlations between variables. The distribution of a $k-$dimensional random vector $\mathbf{X} = (X_1, \dots, X_k)^T$ and can be written in the following notation:

$$\mathbf{X} \sim \mathcal{N}(\mu, \mathbf{\Sigma}),$$

with $k-$dimensional mean vector $\mu = \mathsf{E}[\mathbf{X}] = (\mathsf{E}[X_1], \mathsf{E}[X_1], \dots, \mathsf{E}[X_k])^T$ and $k \times k$ positive semi-definite covariance matrix $\mathbf{\Sigma}_{ij} = \mathsf{E}[(X_i - \mu_i)(X_j - \mu_j)] = Cov[X_i, X_j]$

The probability density function (PDF) is given by formula

$$\mathcal{N}(\mathbf{x}; \mu, \mathbf{\Sigma}) = \frac{1}{\sqrt{(2\pi)^k |\mathbf{\Sigma}|}} \exp(-\frac{1}{2}(\mathbf{x} - \mu)^T \mathbf{\Sigma}^{-1}(\mathbf{x} - \mu)), \tag{1.7}$$

where k is the length of the vector $\mathbf{x}$ and $|\cdot|$ denotes the determinant of a matrix. It should be noted that the exponent is known as Mahalanobis distance and is given by:

▶ **Definition 1.4** (Mahalanobis distance). *For vectors x and y and a positive semi-definite matrix S, the Mahalanobis distance between two objects is defined as*

$$d(\mathbf{x}, \mathbf{y}) = \sqrt{(\mathbf{x} - \mathbf{y})^T S^{-1} (\mathbf{x} - \mathbf{y})} \tag{1.8}$$

The Mahalanobis distance (MD) is the distance between two points in multivariate space and unlike Eucledian distance, it measures distances even between correlated points for multiple variables.

Target tracking filters regularly use conditional probabilities and joint distributions, especially Gaussian.

▶ **Theorem 1.5** (Conditional joint Gaussian distribution). *Let $\mathbf{x}$ and $\mathbf{y}$ are Gaussian random variables with distributions $\mathcal{N}(\mathbf{x}; \mu_{\mathbf{x}}, \mathbf{\Sigma_{xx}})$ and $\mathcal{N}(\mathbf{y}; \mu_{\mathbf{y}}, \mathbf{\Sigma_{y}})$, respectively. Let their joint probability is given by:*

$$p(\mathbf{x}, \mathbf{y}) = \mathcal{N}\left(\begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix}; \begin{bmatrix} \mu_{\mathbf{x}} \\ \mu_{\mathbf{y}} \end{bmatrix}, \begin{bmatrix} \mathbf{\Sigma_{xx}} & \mathbf{\Sigma_{xy}} \\ \mathbf{\Sigma_{xy}} & \mathbf{\Sigma_{yy}} \end{bmatrix} \right). \tag{1.9}$$

Then the conditional distribution of $\mathbf{x}$ given by $\mathbf{y}$ is defined as:

$$p(\mathbf{x}|\mathbf{y}) = \mathcal{N}(\mathbf{x}; \mu_{\mathbf{x}|\mathbf{y}}, \mathbf{\Sigma_{x|y}}), \tag{1.10}$$

where

$$\mu_{\mathbf{x}|\mathbf{y}} = \mu_{\mathbf{x}} + \mathbf{\Sigma_{xy}} \mathbf{\Sigma_{yy}}^{-1} (\mathbf{y} - \mu_{\mathbf{y}}), \tag{1.11}$$

$$\mathbf{\Sigma_{x|y}} = \mathbf{\Sigma_{xx}} - \mathbf{\Sigma_{xy}} \mathbf{\Sigma_{yy}^{-1}} \mathbf{\Sigma_{xy}^{T}}. \tag{1.12}$$

1.4 Gaussian mixture

Gaussian Mixture Models offer a flexible and powerful framework for representing complex probability distributions by combining multiple Gaussian components. In a Gaussian mixture model, a vector of parameters (e.g., observations of a signal) is modeled using a mixture distribution comprising several Gaussian components. Mathematically, this is represented as:

$$p(\Theta) = \sum_{i=1}^{k} w_i \mathcal{N}(\mu_{\mathbf{i}}, \mathbf{\Sigma_i}), \tag{1.13}$$

where the i^{th} vector component is characterized by normal distributions with weights w_i, means $\mu_{\mathbf{i}}$ and covariance matrices $\mathbf{\Sigma_i}$.

In the context of multi-target tracking, Gaussian mixture models find utility in capturing the complex nature of target states and measurements. MTT often involves dealing with linear Gaussian models, where the posterior density is represented as a mixture of one or more Gaussian components. This representation enables MTT algorithms to probabilistically model uncertainties associated with target dynamics, sensor measurements, and data association.

The probability density function (pdf) of a Gaussian mixture distribution is a simple sum of each Gaussian component, expressed as:

$$p(x) = \sum_{i=1}^{k} w_i \mathcal{N}(\mathbf{x}; \mu_{\mathbf{i}}, \mathbf{\Sigma_i}), \tag{1.14}$$

where k is the number of Gaussian component, $w_i > 0$ is the weight of i^{th} component and $\sum_{i=1}^{k} w_i = 1$.

In MTT, Gaussian mixture models are particularly useful for tasks such as data association, where they probabilistically assign measurements to existing tracks or create new tracks based on the likelihood of observations given the target states. By modeling complex distributions of target states and measurements, Gaussian mixture models enable MTT algorithms to handle uncertainties and ambiguities inherent in real-world tracking scenarios, including occlusions, clutter, and target interactions.

1.5 State space model

State-space models serve as a fundamental framework for describing the evolution of dynamic systems over time. In the context of multi-target tracking, state-space models provide a formalism for representing motion dynamics of targets and the measurement process, enabling efficient and accurate inference of target states from sensor data.

At the core of a state-space model lies a set of latent variables, known as the state vector, which encapsulates the unobservable quantities of interest, such as the position, velocity, and acceleration of targets in MTT. The dynamics governing the evolution of the state vector are typically described by a transition model, which specifies how the state evolves over time according to a probabilistic process. This transition model can take various forms depending on the nature of the system dynamics, ranging from simple linear or nonlinear models to more complex stochastic processes.

In addition to the transition model, state-space models incorporate an observation model that describes the relationship between the observed measurements and the underlying state variables. This observation model accounts for the uncertainties and noise inherent in the measurement process, allowing for the probabilistic mapping of observed data to the latent state space.

S state-space model can be represented as a pair of stochastic equations:

1. **State transition equation:**

$$\mathbf{x_k} = f(\mathbf{x_{k-1}}, \mathbf{u_k}, \mathbf{w_k}), \tag{1.15}$$

   where $\mathbf{x_k}$ represents the state vector at time k, f denotes the transition function describing the evolution of the state, $\mathbf{u_k}$ represents optional control inputs, and $\mathbf{w_k}$ denotes process noise.

2. **Observation equation:**

$$\mathbf{y_k} = h(\mathbf{x_k}, \mathbf{v_k}), \tag{1.16}$$

   where $\mathbf{y_k}$ represents the observed measurements at time k, h denotes the observation function mapping the state to measurements, and v_k represents measurement noise.

In MTT, state-space models provide a natural framework for representing the motion dynamics of multiple targets and the sensor measurements associated with each target. Each target is typically associated with its own state vector, allowing for simultaneous tracking of multiple objects within the same probabilistic framework.

State-space models enable MTT algorithms to perform a range of tasks, including target prediction, data association, and state estimation, by propagating the state forward in time using the transition model and updating the state based on observed measurements using the observation model. By incorporating uncertainty explicitly into the tracking process, state-space models facilitate robust inference of target states in complex and dynamic tracking scenarios.

While state-space models offer a powerful framework for multi-target tracking, they also present several challenges and considerations:

- **Model complexity:** Designing an appropriate state-space model requires careful consideration of the underlying dynamics and measurement process, which can be challenging in complex tracking scenarios with non-linearities and uncertainties.
- **Parameter estimation:** Estimating the parameters of a state-space model from data, such as the transition and observation matrices, can be computationally demanding and prone to issues such as overfitting or underfitting.
- **Computational Complexity:** Performing inference in state-space models often involves recursive algorithms such as the Kalman filter or particle filter, which can be computationally intensive, especially in high-dimensional or real-time tracking applications.

Despite these challenges, state-space models remain a cornerstone of multi-target tracking, offering a principled and flexible framework for representing and reasoning about dynamic systems in the presence of uncertainty. There are many state space models used in MTT. Before we define two of the most common ones in next sections, it should be noted, that this definition is too general, because f_k and h_k could by any functions.

To get a closed-form solution in the Bayesian inference framework, we need to choose conjugate distributions for the likelihood and the prior. The Kalman filter (see Section 1.8 for more) works for the Gaussian-linear case, where the functions f_t and h_t are linear and noise variables are distributed as Gaussian with zero mean, i. e. $w_k, v_k \sim \mathcal{N}(0, \Sigma)$. The Gaussian linear state space model has the following formulation:

$$p(x_k|x_{k-1}) = Fx_{k-1} + Bu_k + w_k \qquad w_k \sim \mathcal{N}(0, Q), \tag{1.17}$$

$$p(z_k|x_k) = Hx_k + v_k \qquad w_k \sim \mathcal{N}(0, R), \tag{1.18}$$

$$p(x_0) \sim \mathcal{N}(\hat{x}_o, P_0), \tag{1.19}$$

where

- F is the transition matrix of appropriate dimension,
- B is the input matrix of appropriate dimension,
- H is the measurement matrix of appropriate dimension,
- Q is the symmetric positive semi-definite matrix that desribes the statistical properties of motion noise w_k,
- R is the symmetric positive semi-definite matrix that desribes the statistical properties of measurement noise v_k,
- $\hat{x}_0$ and P_0 are mean and covariance matrix of the prior state.

The control variable u k , in general, represents some input signal from the environment, and B specifies how the input signal affects the dynamic system. This variable is ususally not considered in MTT scenarios. In multi-target tracking, most often with Gaussian linear models is worked, thus following formulation is instead used:

$$p(x_k|x_{k-1}) = \mathcal{N}(x_k; Fx_{k-1}, Q), \tag{1.20}$$

$$p(z_k|x_k) = \mathcal{N}(z_k; Hx_k, R), \tag{1.21}$$

$$p(x_0) \sim \mathcal{N}(x_0; \hat{x}_o, P_0). \tag{1.22}$$

1.5.1 Constant velocity model

The constant velocity model (CVM) is a fundamental component of multi-target tracking systems, providing a simplified yet effective representation of target motion dynamics over time. This model assumes that the target's velocity remains constant between consecutive time steps, making it particularly suitable for tracking objects with relatively smooth and predictable motion patterns. In this section, we explore the conceptual basis, mathematical formulation, and practical implications of the constant velocity model in the context of MTT.

At its core, the constant velocity model embodies the notion of inertia, where a target maintains a constant velocity unless acted upon by external forces. This conceptual simplicity allows for a straightforward representation of target motion, making the constant velocity model a popular choice for MTT applications where targets exhibit relatively uniform and predictable movement behaviors.

Mathematically, the constant velocity model describes the evolution of a target's state vector over time in terms of its position and velocity. At each time step k, the state vector $\mathbf{x_k}$ comprises the position $[x_{1,k}, x_{2,k}]$ and velocity v of the target:

$$\mathbf{x_k} = [x_{1,k}, x_{2,k}, v_{1,k}, v_{2,k}]^T. \tag{1.23}$$

The dynamics of the constant velocity model can be represented using a state transition matrix F and a process noise vector w_k, where the state transition matrix captures the relationship between the target's state at consecutive time steps:

$$\mathbf{x_{k+1}} = \mathbf{F}\mathbf{x_k} + \mathbf{w_k} \tag{1.24}$$

All together, vector $\mathbf{x_k}$, matrices $\mathbf{F}$ and $\mathbf{Q}$, might, for example, look like this:

$$\mathbf{x_k} = \begin{bmatrix} x_{1,k} \\ x_{2,k} \\ v_{1,k} \\ v_{2,k} \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} 1 & 0 & dt & 0 \\ 0 & 1 & 0 & dt \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{Q} = q^2 \begin{bmatrix} \frac{dt^3}{3} & 0 & \frac{dt^2}{2} & 0 \\ 0 & \frac{dt^3}{3} & 0 & \frac{dt^2}{2} \\ \frac{dt^2}{2} & 0 & dt & 0 \\ 0 & \frac{dt^2}{2} & 0 & dt \end{bmatrix}, \tag{1.25}$$

where dt stands fot delta time, i. e., elapsed time between the last estimatioin and the current one, q is the motion noise parameter, which represents the uncertainty in the state transition. The measurement model transforms a state vector from the state space into the measurement space. Since filters derived from Kalman filter assumes only linear models, the measurement model for CVM assumes the same space as in the state vectors. As a result, the measurement matrix $\mathbf{H}$ and the noise matrix $\mathbf{R}$ can be formulated as:

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad \mathbf{R} = r^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{1.26}$$

where r is the measurement noise, which determines the variance of the measurement noise.

To obtain optimal results given by filters, it is essential to set parameters r and q appropriately. There are three main ways to dot it. The first one requires the knowledge of motion noise given by tracked object and the knowledge of sensor's error when detecting targets, i. e. the noise measurement. This method is often used in simulations and experiments. The second one is simple trial and error method with reasonable choices. The last one is using automated methods, like in [19].

In the context of MTT, the constant velocity model serves as a fundamental building block for tracking algorithms, providing a simplified yet effective representation of target motion. By assuming constant velocity, MTT algorithms can predict the future positions of targets based on their current state and velocity, facilitating trajectory estimation and target prediction.

One common application of the constant velocity model in MTT is in the design of prediction algorithms, where future positions of targets are estimated based on their current state and velocity information. These predictions are essential for anticipating target movements and facilitating data association, enabling MTT algorithms to maintain track continuity and adapt to changes in target behavior over time.

Moreover, the constant velocity model can be seamlessly integrated into recursive Bayesian filters, such as the Kalman filter, for state estimation in MTT. By incorporating the constant velocity model into the state-space representation of the tracking problem, Kalman filter-based algorithms can effectively fuse measurement information with dynamic predictions to estimate the most likely trajectories of targets over time.

1.5.2 Constant acceleration model

The Constant Acceleration Model (CAM) stands as a sophisticated extension of the state-space model in multi-target tracking, offering a more comprehensive representation of target motion dynamics. Unlike simpler models such as the Constant Velocity Model, which assumes a constant velocity for targets over time, the CAM acknowledges the potential for changes in target acceleration, allowing for more accurate and flexible trajectory estimation. This section delves into the conceptual basis, mathematical formulation, and practical implications of the Constant Acceleration Model in the context of MTT.

In MTT scenarios, targets often exhibit varying degrees of acceleration due to factors such as changes in speed, direction, or environmental influences. Ignoring these accelerative effects can lead to biased trajectory estimates and diminished tracking accuracy. The CAM addresses this limitation by incorporating an additional acceleration component into the state-space model, enabling more faithful representation of target motion dynamics. By accounting for changes in acceleration, the CAM provides MTT algorithms with greater flexibility and adaptability in tracking targets with non-uniform motion profiles.

Mathematically, the Constant Acceleration Model extends the state transition function of the state-space model to accommodate changes in acceleration over time. At each time step k, the evolution of the target's state vector x_k is governed by a set of dynamic equations that describe the position, velocity, and acceleration of the target:

$$\mathbf{x_k} = [x_{1,k}, x_{2,k}, v_{1,k}, v_{2,k}, a_{1,k}, a_{2,k}]^T, \tag{1.27}$$

where, as in CVM, $x_{1,k}$, $x_{2,k}$ is the position of an target in two-dimensional space, $v_{1,k}, v_{2,k}$ is the velocity and $a_{1,k}, a_{2,k}$ are the accelerations in both directions. Matrices F and Q can then look like this:

$$\mathbf{x_k} = \begin{bmatrix} x_{1,k} \\ x_{2,k} \\ v_{1,k} \\ v_{2,k} \\ a_{1,k} \\ a_{2,k} \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} 1 & 0 & dt & 0 & \frac{1}{2}dt^2 & 0 \\ 0 & 1 & 0 & dt & 0 & \frac{1}{2}dt^2 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{Q} = q^2 \begin{bmatrix} \frac{dt^4}{4} & 0 & \frac{dt^3}{3} & 0 & \frac{dt^2}{2} & 0 \\ 0 & \frac{dt^4}{4} & 0 & \frac{dt^3}{3} & 0 & \frac{dt^2}{2} \\ \frac{dt^3}{3} & 0 & \frac{dt^2}{2} & 0 & dt & 0 \\ 0 & \frac{dt^3}{3} & 0 & \frac{dt^2}{2} & 0 & dt \\ \frac{dt^3}{3} & 0 & \frac{dt^2}{2} & 0 & dt & 0 \\ 0 & \frac{dt^3}{3} & 0 & \frac{dt^2}{2} & 0 & dt \end{bmatrix}. \tag{1.28}$$

The measurement model remains unchanged from the standard state-space model, relating the observed measurements $\mathbf{y_t}$ to the target's true state $\mathbf{x_t}$ through a measurement function $h(\cdot)$ corrupted by measurement noise v_t:

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{R} = r^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{1.29}$$

The Constant Acceleration Model offers several practical advantages in MTT applications:

- Improved Trajectory Estimation: By accounting for changes in acceleration, the CAM enables more accurate and realistic trajectory estimation, particularly for targets exhibiting non-uniform motion patterns or sudden changes in velocity.
- Enhanced Predictive Capability: The inclusion of acceleration dynamics allows MTT algorithms to make more informed predictions about future target states, improving tracking performance and reducing prediction errors.
- Robustness to Dynamic Environments: In dynamic environments with varying levels of congestion, obstacles, or traffic patterns, the CAM provides MTT algorithms with greater robustness and adaptability, enabling effective tracking even in challenging scenarios.
- Applications in Autonomous Systems: In autonomous systems such as self-driving cars, drones, or robotics, the CAM plays a crucial role in motion planning, obstacle avoidance, and trajectory prediction, enhancing the safety and efficiency of autonomous navigation.

In summary, the Constant Acceleration Model represents a significant advancement in multi-target tracking, offering a more nuanced and comprehensive approach to modeling target motion dynamics. By incorporating acceleration effects into the tracking process, the CAM enables MTT algorithms to achieve higher accuracy, robustness, and predictive capability, making it an indispensable tool for a wide range of applications.

1.6 Hidden Markov Model

In the domain of multi-target tracking, the Hidden Markov Model (HMM) serves as a framework for capturing the temporal dynamics of target behavior in complex environments. Building upon the principles of Markov processes, the HMM extends the state-space model by introducing hidden states that govern the evolution of observable measurements over time. Markov process of the first order is a model, in which current state depends only on the previous state

$$p(x_k|x_1,\cdots,x_{k-2},x_{k-1},z_1,\cdots,z_{k-2},z_{k-1}) = p(x_k|x_{k-1}) \quad \text{(transition probability)}, \tag{1.30}$$
$$p(z_k|x_1,\cdots,x_{k-2},x_{k-1},x_k,z_1,\cdots,z_{k-2},z_{k-1}) = p(z_k|x_k) \quad \text{(observation likelihood)}, \tag{1.31}$$
$$p(x_0) \quad \text{(initial state)}. \tag{1.32}$$

In models of higher order, the transition probability would be:

$$p(x_k|x_{k-1},\cdots,x_{k-n}), \tag{1.33}$$

where n is the number of the order.

In cases where the process is not didectly observable, but can be observed through another observable variable y_k, we often talk about Hidden Markov process.

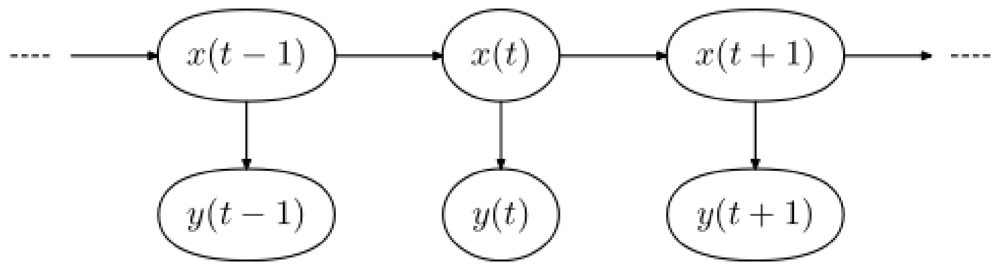

Figure 1.1 Demonstration of the course of the hidden Markov process.

At its core, the Hidden Markov Process represents a stochastic process characterized by a set of hidden states that transition probabilistically over time. While these hidden states are unobservable, they influence the generation of observable measurements, which are assumed to be conditionally independent given the hidden states. In the context of MTT, the hidden states may represent latent attributes of targets, such as their locations, velocities, or motion patterns, while the observable measurements correspond to sensor readings or detections obtained from surveillance systems.

1.7 Bayes' filter

The Bayes filter is a recursive framework that estimates an internal state of the system over time using measurements . The Bayes filter leverages both motion and measurement models, operating under the assumption that these models accurately describe the system's behavior. Operating within a recursive framework, the Bayes filter continually estimates the internal state of the system over time using available measurements. Each iteration of the filter entails two fundamental steps: prediction and update. During the prediction step, the filter anticipates the internal state x_k based on the preceding state x_{k-1} and the motion model $p(x_k|x_{k-1})$. This prediction step is commonly referred to as the Chapman-Kolmogorov equation.

▶ **Theorem 1.6** (The Chapman-Kologorov equation prediction step)**.** *Given the set of measurements $z_{0:k-1} = z_0, \cdots, z_{k-1}$, set of control variables $u_{0:k-1} = u_0, \cdots, u_{k-1}$ and current state x_{k-1} and the motion model $p(x_k|x_{k-1})$, the prediction step is as follows:*

$$p(x_k|z_{0:t-1}, u_{0:k}) = \int p(x_k|x_{k-1}, u_k)p(x_{k-1}|z_{0:k-1}, u_{0:k-1})dx_{k-1}, \tag{1.34}$$

where the integral is taken over the entire state space of x_{k-1}, and $p(x_{k-1}|z_{0:k-1}, u_{0:k-1})$ is the posterior density of the state at time $k-1$.

On the update step, the Bayes' filter corrects (updates) the prediction step with measurements z_k using the measurement model $p(z_k|x_k)$. This step uses common Bayes' rule.

▶ **Theorem 1.7** (The update step of Bayes' filter)**.** *Given the output of Bayes' filter's prediction step, the measurements z_k observed at time k and the measurement model $p(z_k|x_k)$, the update step of Bayes' filter is formulated as follows:*

$$p(x_k|z_{0:t}, u_{0:t}) = \frac{p(z_k|x_k)p(x_k|z_{0:k-1}, u_{0:k})}{p(z_k|z_{0:k-1})} \propto p(z_k|x_k)p(x_k|z_{0:k-1}, u_{0:k}) \tag{1.35}$$

These two steps work in an iterative method where in each iteration first prediction step is used, where we predict the state x_k and then update step is used to correct our quess with provided measurement.

1.8 Kalman filter

The Kalman filter, named after its developer Rudolf E. Kalman, has a rich history dating back to the early 1960s when Kalman first introduced the algorithm in a series of landmark papers. Initially developed for aerospace applications, the Kalman filter gained prominence for its ability to provide optimal state estimation in the presence of noisy measurements and uncertain dynamics.

Over the years, the Kalman filter has found widespread usage across diverse fields and industries, including aerospace, robotics, finance, and telecommunications. Its applications range from tracking spacecraft trajectories and guiding missiles to monitoring financial markets and controlling autonomous vehicles. The filter's versatility, efficiency, and effectiveness have made it a cornerstone of modern estimation and control systems.

The Kalman filter operates on the principles of recursive Bayesian estimation, utilizing a system dynamics model and noisy measurements to predict and update the state of a dynamic system over time. Its mathematical elegance and simplicity make it well-suited for real-time applications, enabling accurate and efficient state estimation even in complex and uncertain environments.

Despite its remarkable success, the Kalman filter continues to evolve, with extensions and variations such as the extended Kalman filter (EKF), unscented Kalman filter (UKF), and particle filter (PF) addressing more challenging scenarios involving nonlinear dynamics and non-Gaussian noise.

In summary, the Kalman filter stands as a seminal contribution to the field of estimation and control, with a rich history of development and widespread usage across diverse domains. Its continued relevance and versatility underscore its status as a foundational tool for state estimation and tracking in modern technological applications.

1.8.1 Kalman filter inference

In section 1.5 we talked about State space model, which is used by Kalman filter. Thus the equations look like this:

$$x_k = Fx_{k-1} + Bu_t + v_t \tag{1.36}$$
$$z_t = Hx_k + w_k, \tag{1.37}$$

where both noise variables are independent and with zero mean

$$v_k \sim \mathcal{N}(0, Q), \tag{1.38}$$
$$w_k \sim \mathcal{N}(0, R). \tag{1.39}$$

Then the state and measurement distributions follows:

$$x_k \sim \mathcal{N}(Fx_{k-1} + Bu_t, Q) \qquad \text{with density} \quad p(x_k|x_{k-1}, u_k), \tag{1.40}$$
$$z_k \sim \mathcal{N}(Hx_k, R) \qquad \text{with density} \quad p(z_k|x_k). \tag{1.41}$$

Because Kalman filter is bayesian, we need aprior distribution for x_k. Model z_k is Gaussian, conjugated apriori is then also Gaussian with mean x^+_{k-1} and covariance matrix $P_{k-1}+$,

$$p(x_k|z_{0:k-1}, u_{0:k-1}) = \mathcal{N}(x^+_{k-1}, P^+_{k-1}). \tag{1.42}$$

The prediction step is derived from formula (1.34). Note, that by multiplicating two gaussian distributions we again get gaussian distribution $\mathcal{N}(x^-_k, P^-_k)$ with hyperparemeters

$$x^-_k = Fx^+_{k-1} + Bu_k \tag{1.43}$$
$$P^-_k = FP^+_{k-1}F^T + Q. \tag{1.44}$$

By multiplicating two normal distributions and followed marginalization enumerates the state equation. The estimation of state x^-_k is just applying model variables into appropriate equations. The estimated covariance P^-_k expresses the degree of uncertainty of the estimation. By applying this step, the uncertainty grows.

The update step corrects the prediction step by applying new observed measurements z_k. For that, formula (1.35) is used. The model is transformed into exponential form a looks like this:

$$\begin{aligned} p(z_k|x_k) &\propto \exp\left\{-\frac{1}{2}(z_k - Hx_k)^T R^{-1}(z_k - Hx_k)\right\} \\ &= \exp\left\{Tr\left(\underbrace{-\frac{1}{2}\begin{bmatrix}-1\\ x_k\end{bmatrix}\begin{bmatrix}-1\\ x_k\end{bmatrix}^T}_{\eta}\underbrace{\begin{bmatrix}z_k^T\\ H^T\end{bmatrix}R^{-1}\begin{bmatrix}z_k^T\\ H^T\end{bmatrix}^T}_{T(z_k)}\right)\right\}. \end{aligned} \tag{1.45}$$

The conjugated shape has the form

$$
\begin{aligned}
p(x_k|z_{0:k-1},u_{0:t}) &\propto \exp\left\{-\frac{1}{2}(x_k - x_k^-)^T (P_k^-)^{-1}(x_k - x_k^-)\right\} \\
&= \exp\left\{ Tr\left(\underbrace{-\frac{1}{2}\begin{bmatrix}-1\\ x_k\end{bmatrix}\begin{bmatrix}-1\\ x_k\end{bmatrix}^T}_{\eta} \underbrace{\begin{bmatrix}(x_k^-)^T\\ I\end{bmatrix}(P_k^-)^{-1}\begin{bmatrix}(x_k^-)^T\\ I\end{bmatrix}^T}_{\xi_k} \right)\right\},
\end{aligned}
\tag{1.46}
$$

where I is identity matrix of appropriate shape.

The bayesian update is sum of the hyperparameter and sufficient statistic,

$$
\begin{aligned}
\xi_k &= \xi_{k-1} + T(z_k) \\
&= \begin{bmatrix}(x_k^-)^T (P_k^-)^{-1} x_k^- + z_k^T R^{-1} z_k, & (x_k^-)^T (P_k^-)^{-1} + z_k^T R^{-1} H \\ (P_k^-)^{-1}(x_k^-)^T + H^T R^{-1} z_k, & (P_k^-)^{-1} + H^T R^{-1} H.\end{bmatrix}
\end{aligned}
\tag{1.47}
$$

The aposteriori parameters are then derived as follows:

$$
\begin{aligned}
P_k^+ &= (\xi_{k;[2,2]})^{-1} \\
&= \left[(P_k^-)^{-1} + H^T R^{-1} H\right]^{-1} \\
&= (I - K_k H) P_k^- && (1.48)\\
x_k^+ &= (\xi_{k;[2,2]})^{-1} \xi_{k;[2,1]} \\
&= P_k^+ \left[(P_k^-)^{-1}(x_k^-)^T + H^T R^{-1} z_k\right] \\
&= x_k^- + P_k^+ H^T R^{-1}(z_k - H x_k^-), && (1.49)
\end{aligned}
$$

where

$$
K_k = P_k^- H^T (R + H P_k^- H^T)^{-1} \tag{1.50}
$$

is Kalman gain. This form is the optimal Kalman gain, as it minimizes the root mean squared error. In general, the greater the gain, the greater the emphasis of the new measurements. The filter is then more sensitive, but filters noise less easily.

In literature it is common to write each equation in this or similar form:

$$
\begin{aligned}
\hat{z}_k^- &= H\hat{x}_k^-, && \text{(measurement prediction)} && (1.51)\\
\nu_t &= z_t - \hat{z}_k^-, && \text{(innovation or prediction error } z_t) && (1.52)\\
S_t &= H P_k^- H^T + R, && \text{(covariance of innovation } \nu_t) && (1.53)\\
K_t &= P_k^- H^T S_t^{-1}, && \text{(Kalman gain)} && (1.54)\\
x_k^+ &= \hat{x}_k^- + K_t \nu_t, && \text{(aposteriori state estimation } x_t) && (1.55)\\
P_k^+ &= (I - K_t H) P_k^-. && \text{(aposteriori covariance)} && (1.56)
\end{aligned}
$$

1.8.1.1 Kalman filter algorithm

Algorithm 1 An algorithm with caption

Require: $n \geq 0$
Ensure: $y = x^n$
  $y \leftarrow 1$
  $X \leftarrow x$
  $N \leftarrow n$
  while $N \neq 0$ **do**
    if N is even **then**
      $X \leftarrow X \times X$
      $N \leftarrow \frac{N}{2}$ ▷ This is a comment
    else if N is odd **then**
      $y \leftarrow y \times X$
      $N \leftarrow N - 1$
    end if
  end while

Chapter 2

Target tracking

2.1 Clutter

2.2 Single target tracking

2.2.1 PDA filter

2.3 Multi target tracking

2.3.1 RFS statistics

2.3.1.1 PHD filter

2.3.1.1.1 Gaussian mixture PHD filter recursion

Chapter 3

Object detection and segmentation

3.1 YOLO

3.2 Segment Anything

Chapter 4

Dynamic time and state varying detection probability

Chapter 5

Experiments

Appendix A
Nějaká příloha

Sem přijde to, co nepatří do hlavní části.

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Obsah příloh

- readme.txt ..... stručný popis obsahu média
- exe ..... adresář se spustitelnou formou implementace
- src
  - impl ..... zdrojové kódy implementace
  - thesis ..... zdrojová forma práce ve formátu LaTeX
- text ..... text práce
  - thesis.pdf ..... text práce ve formátu PDF