JOE SEIDEL

MATH CAMP 2016 EXERCISES

Exercise 4

Compute
$$\int_0^1 (e^x + x)dx.$$
First find $F(x)$ the anti derivative of $(e^x + x)$.
$$F(t) = e^t - \frac{t^2}{2}.$$
Apply the fundemental theorem of calculus
$$\int_0^1 (e^x + x)dx = F(1) - F(0) = e - \frac{1}{2}.$$

Exercise 5

Revisit chapter 2, exercise 3(ii). Let
$$f(x) = \begin{cases} -\cos\frac{1}{x} + 2\sin\frac{1}{x}, & 0 < x \leq 1 \\ 0, & x = 0 \end{cases}$$
Compute $\int_0^1 f(x)dx$.
First, find the $F(x)$. Write
$$\int f(x)dx = \int 2x\sin\frac{1}{x} + \int -\cos\frac{1}{x}dx.$$
Integrate $\int(-\cos\frac{1}{x})dx$ by parts where

$\int fdg = fg - \int gdf$

$$\begin{aligned} f &= x^2 \\ dg &= -\frac{\cos\frac{1}{x}}{x^2}dx \\ g &= \sin\frac{1}{x} \\ df &= 2xdx. \end{aligned}$$

Now
$$\int(-\cos\frac{1}{x}) = x^2\sin\frac{1}{x} - \int 2x\sin\frac{1}{x}dx$$
therefore
$$F(x) = \begin{cases} x^2\sin\frac{1}{x} & 0 < x \leq 1 \\ 0 & x = 0 \end{cases}$$
Applying the fundemental theorem of calculus.
$$\int_0^1 f(x)dx = F(1) - F(0) = \sin 1$$