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MATH CAMP 2016 EXERCISES

Exercise 3

Let $f(x, y) = x \ln(x^2 + y^2)$. Calculate its partial derivatives.
To find $\frac{\partial f}{\partial y}$, use the chain rule, holding x constant.

$$\frac{\partial f}{\partial y} = x\frac{2y}{x^2 + y^2}$$

To find $\frac{\partial f}{\partial x}$, use the product rule and the chain rule/

$$\frac{\partial f}{\partial x} = \ln(x^2 + y^2) + \frac{2x^2}{x^2 + y^2}$$

Exercise 4

Let $f(x, y, z) = (x^2 + y^2)z^2 + \sin x^2$. Calculate it's partial derivatives.
To find $\frac{\partial f}{\partial x}$ use the chain rule for each terms of the sum.

$$\frac{\partial f}{\partial x} = 2x(z^2 + \cos x^2)$$

To find $\frac{\partial f}{\partial y}$ use chain rule on $(x^2 + y^2)z^2$ and notice $\sin x^2$ is constant so it becomes 0.

$$\frac{\partial f}{\partial y} = 2yz^2$$

To find $\frac{\partial f}{\partial z}$ notice $\sin x^2$ and $(x^2 + y^2)$ are constants hence we have the form $Bz^2 + C$.

$$\frac{\partial f}{\partial z} = 2z(x^2 + y^2)$$

Exercise 5

Let $z = z(u, v) = v \ln u$, $u = x^2 + y^2$ and $v = \frac{y}{x}$. Calculate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.
Observe that $z = z(u, v) = \frac{y}{x} \ln(x^2 + y^2)$ hence

$$\frac{\partial z}{\partial x} = -\frac{y \ln(x^2 + y^2)}{x^2} + \frac{2y}{x^2 + y^2}$$

and

$$\frac{\partial z}{\partial y} = \frac{1}{x}(\ln(x^2 + y^2) + \frac{2y^2}{x^2 + y^2}).$$

Exercise 7

Let $z = e^{xy} \sin(x + y)$. Calculate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

$$\frac{\partial z}{\partial x} = e^{xy}(y \sin(x + y) + \cos(x + y)$$

and
$$\frac{\partial z}{\partial y} = e^{xy}(x\sin(x+y) + \cos(x+y).$$

Exercise 8

Let $f(x,y) = x^2 + 2xy + y^2$. Calculate $(\Delta f)(1,2)$.

First find
$$(\Delta f)(x,y) = (\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}) = (2(x+y), 2(x+y))$$
then
$$(\Delta f)(1,2) = (6,6).$$

Exercise 9

Let $F(x) = (x^2 + y^3, xy)$. Calculate the Jacobian J_F.
$$f_1 = x^2 + y^3 \text{ and } f_2 = xy$$
$$J_F = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix} = \begin{bmatrix} 2x & 3y^2 \\ y & x \end{bmatrix}$$

Exercise 11

Prove $(\ln x)' = \frac{1}{x}$ for $x > 0$, provided $(e^x)' = e^x$.

We are given the inverse function of $\ln x$ and it's derivative. Using the Inverse Function Theorem observe
$$(\ln x)' = \frac{1}{e^{\ln x}} = \frac{1}{x}.$$