JOE SEIDEL

REAL ANALYSIS SIXTH WEEK

Exercise 3.5.9

Suppose that (X, d) and (X', d') are metric spaces and that $f : X \to X'$ is continuous. For each of the following statements, determine whether or not is true. If the assertion is true, prove it. If it is not true, give a counter example.

1. If A is an open subset of X, then $f(A)$ is an open subset of X'; Not neccessarily true. Consider the constant function $f : \mathbb{R} \to \mathbb{R}$, $f(x) = c$. Let A be an open subset of $\mathbb{R}$, then $f(A)$ is a closed subset of $\mathbb{R}$.

2. If A is a closed subset of X, then $f(A)$ is a closed subset of X'; Not neccessarily true. Consider the function $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \frac{x}{x+1}$. If $A = [0, \infty)$ then $f(A) = [0, 1)$ which is not closed.

3. If B is a closed subset of X', then $f^{-1}(B)$ is a closed subset of X; True. First note that $f^{-1}(S^c) = (f^{-1}(S))^c$. Since $B \subset X'$ is closed, $B^c \subset X'$ is open. From Theorem 3.5.5. a function $f : X \to X'$ is continuous iff for any open set $V \in X'$, the set $f^{-1}(V)$ is open in X. Thefore, if B^c is open then $f^{-1}(B^c)$ is open so $f^{-1}(B^c) = (f^{-1}(B))^c$ then $((f^{-1}(B))^c)^c = (f^{-1}(B))$ is closed.

4. A is a bounded subset of X, then $f(A)$ is a bounded subset of X';

Exercise 3.5.13

Exercise 3.5.15

Exercise 3.5.23(sans isometry part)

Exercise 3.5.30

Exercise 3.5.33