AM 106: Algebra for Models and Data

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Algebra is the study of operations (such as addition and multiplication) on objects (such as numbers, polynomials, and matrices). This course is an introduction to abstract algebra and its applications. Algebraic structure appears in problems in data analysis, modeling, and optimization. We will cover the abstract algebra theory required to extract insights in such contexts. The course is in four parts.

1. Introduction to abstract algebra [CLOS97] [8 lectures]:
   (i) polynomials and the fundamental theorem of algebra
   (ii) rings and ideals
   (iii) properties of (monomial) ideals: dimension, degree, etc.
   (iv) Newton polytopes and Bézout's theorem
   (v) the Nullstellensatz
   (vi) factorizing matrices
2. Exact and numerical algorithms for solving equations [CLO05] [6 lectures]:
   (i) saturation, elimimation, and implicitization
   (ii) Gröbner bases
   (iii) homotopy continutation: theory and practice
3. Optimization and parameter estimation in statistical, physical, and generative models through the lens of algebra [MS21, Sul18] [8 lectures]:
   (i) Lagrange multipliers
   (ii) parameter estimation
   (iii) testing goodness of fit with equations; Markov bases
   (iv) the method of moments
   (v) applications to statistical models (such as Hardy–Weinberg from population genetics), physical models (such as Ising models from statistical physics) and generative models (such as in representation learning)

4. Factorizing data: algebraic decompositions and their applications to data analysis [LRR04] [4 lectures]:
   (i) ideals of minors
   (ii) dimensionality reduction via matrix decomposition

This is an undergraduate course in applied algebra. The course combines mathematical theory, computational and numerical experiments, and exploration of real world applications.

Prerequisites. Recommended preparation for the course is familiarity with linear algebra (at the level of Math 21b) and proofs (at the level of Math 22a, Math 101, or CS 20). Programming experience is helpful but not required.

Assessment. Class attendance (10%), quizzes (50%), final exam (40%).

Learning outcomes. Students will learn the theory of polynomials, rings, ideals and their uses. They will learn how to solve systems of equations by hand and on a computer. They will be able to identify and use the algebraic structure that appears in areas of data analysis, modeling, and optimization.

References

[CLO05] David A Cox, John Little, and Donal O'Shea. *Using algebraic geometry*, volume 185. Springer Science & Business Media, 2005.

[CLOS97] David Cox, John Little, Donal O'Shea, and Moss Sweedler. *Ideals, varieties, and algorithms*, volume 3. Springer, 1997.

[LRR04] Brigitte Le Roux and Henry Rouanet. *Geometric data analysis: from correspondence analysis to structured data analysis*. Springer Science & Business Media, 2004.

[MS21] Mateusz Michałek and Bernd Sturmfels. *Invitation to nonlinear algebra*, volume 211. American Mathematical Soc., 2021.

[Sul18] Seth Sullivant. *Algebraic statistics*, volume 194. American Mathematical Soc., 2018.