Stat 201: Statistics I
Chapter 10

Metropolitan State University

date

Chapter 10
Correlation and Regression

Comparing samples from two populations

Example

Consider the following data:

X	83	85	66	89	96	78
Y	90	90	66	86	99	85

How should this data be analyzed? It depends on the context of the data or what the data represents.

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  - One possible analysis would be a two-sample t-test comparing the mean test scores from each class.
- Suppose the data are scores from the midterm and the final for one set of students.
  - One possible analysis would be a matched pairs t-test comparing the mean difference between the midterm and the final for each student.

Comparing samples from two populations, cont.

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- Suppose X is a students score on the statistics final and Y is that students yearly salary, in thousands of dollars, a year later.

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  - It doesn't really make sense to compare means. The two samples represent entirely different kinds of data. There is no meaningful way to compare means.
  - It is useful to examine the association between the data. Is a higher test score associated with a higher salary? Or are the samples independent, values of one having no effect on values of the other?

Section 10.1
Correlation

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- A positive correlation indicates that as one value in a pair increases the other will tend to increase.
- A negative correlation indicates that as one value in a pair increases the other will tend to decrease.
- No correlation indicates that the two values of a pair have no relationship with each other. They are independent.

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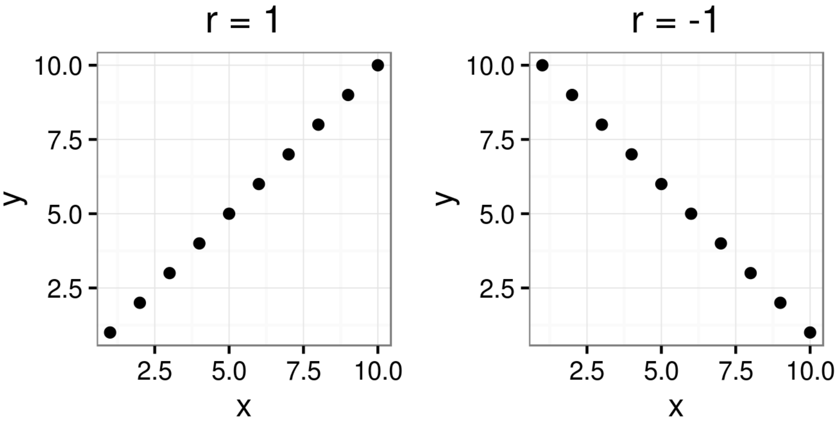
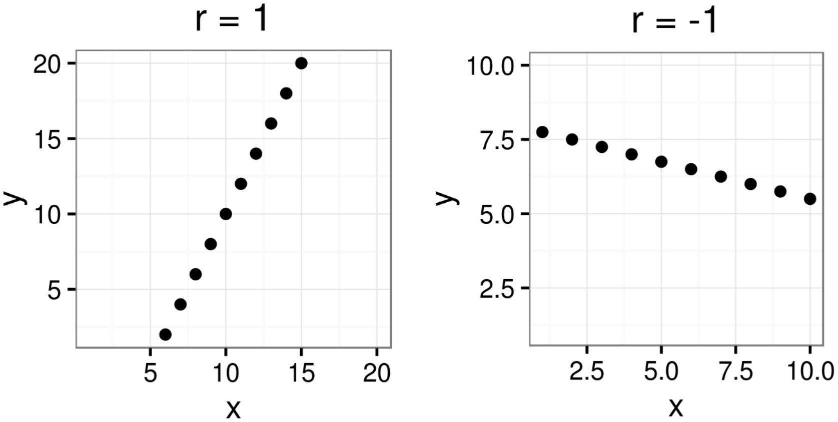
- Most samples will have some other value for r.
- The order of the values, i.e. (x, y) vs. (y, x), have no effect of the value of r.
- The units of the values also have no effect on r. A correlation on height will be the same whether it is measured in inches, feet or meters, for example.

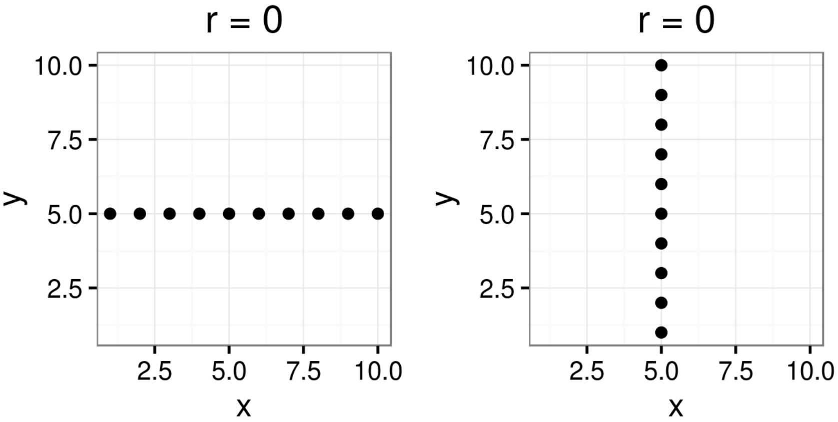
Perfect correlation



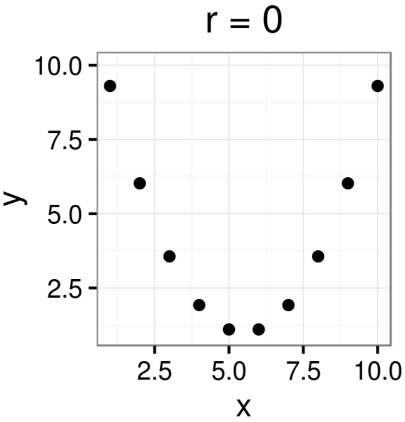
Perfect correlation, cont.



Zero correlation



Non-linear correlation



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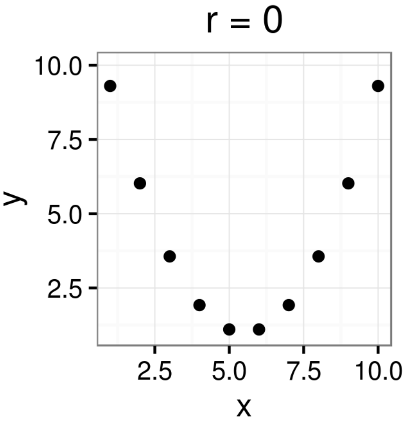
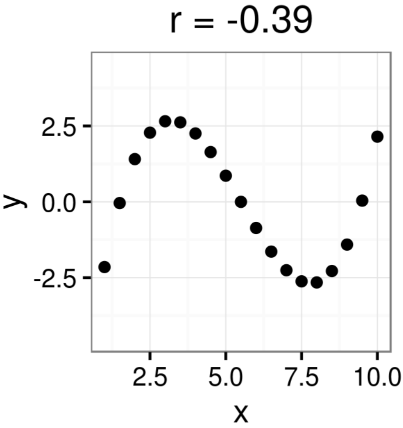
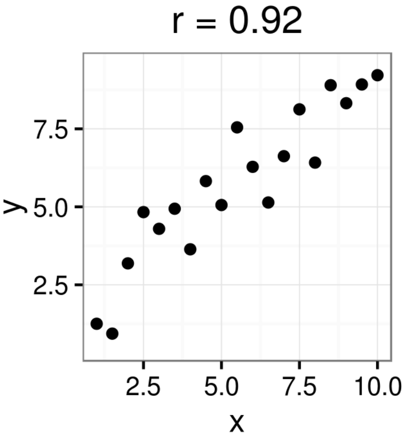




"Real world" correlation



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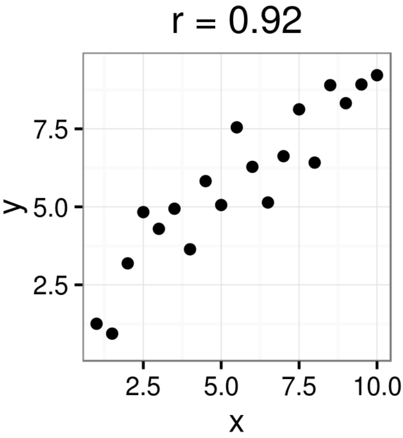
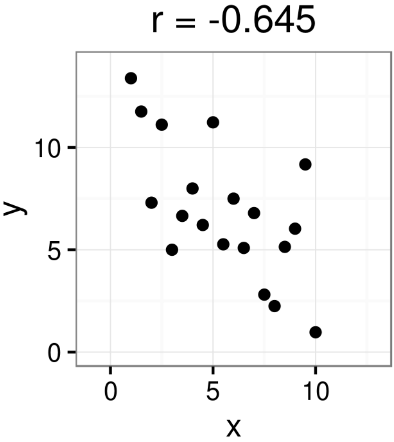
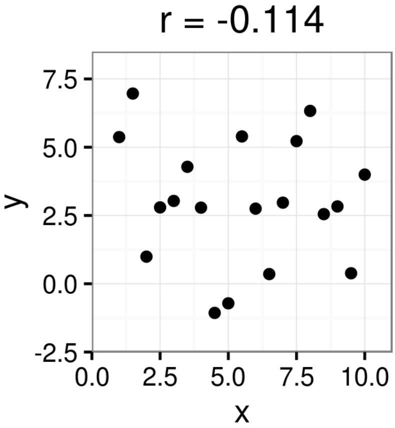




"Real world" correlation, cont.



Correlation and outliers

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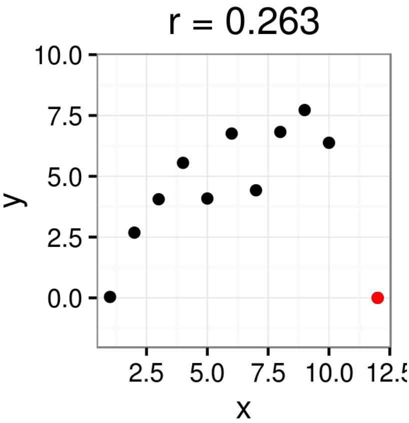

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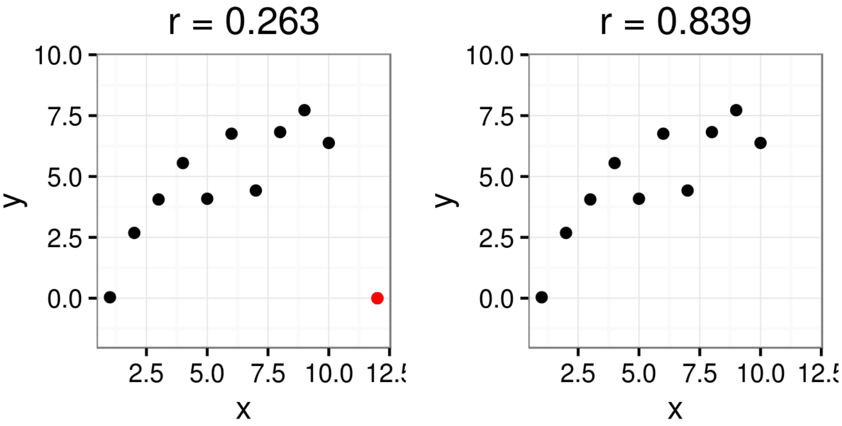

Calculating correlation coefficient

The book uses the following formula for calculating r:

$$r = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^2\right) - \left(\sum x\right)^2}\sqrt{n\left(\sum y^2\right) - \left(\sum y\right)^2}}$$

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- Since technology should be used to calculate the correlation calculation anyway, it would be useful to look at r defined with a more informative formula.

Calculating correlation coefficient, cont.

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- If when x values are far away from $\bar{x}$ then y values are also far away from $\bar{y}$ (whether positive or negative), then r is proportional to the sum of larger numbers and will be closer to 1 or -1.

Calculating correlation coefficient, example

Example

X	2	4	6	8	$\bar{x} = 5$
Y	1	6	7	12	$\bar{y} = 6.5$

U	2	4	6	8	$\bar{u} = 5$
W	7	12	1	6	$\bar{w} = 6.5$

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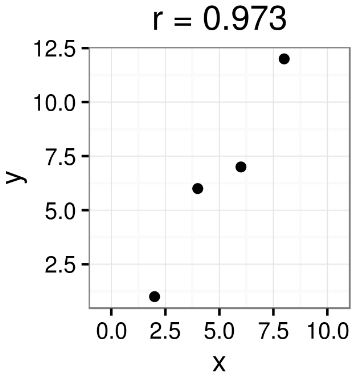

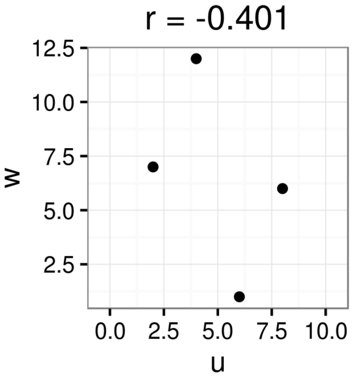

Calculating correlation coefficient, example

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x	y	$(x-\bar{x})$	$(y-\bar{y})$	$(x-\bar{x})(y-\bar{y})$
2	1	-3	-5.5	16.5
4	6	-1	-0.5	0.5
6	7	1	0.5	0.5
8	12	3	5.5	16.5
				$r \propto 34$

Calculating correlation coefficient, example

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u	w	$(u-\bar{u})$	$(w-\bar{w})$	$(u-\bar{u})(w-\bar{w})$
2	7	-3	0.5	-1.5
4	12	-1	5.5	-5.5
6	1	1	-5.5	-5.5
8	6	3	-0.5	-1.5
				$r \propto -14$

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- Test statistic: $t = \dfrac{r}{\sqrt{\frac{1-r^2}{n-2}}}$ $\qquad df = n - 2$
- Make a decision, two ways:
  - P-value: If $p < \alpha$, reject null hypothesis
  - Critical value (from table A-6): If $|t| > t_{crit}$, reject null hypothesis

Correlation coefficient and p-value in StatCrunch

- Stat → Summary Stats → Correlation
- Select the columns which contain the data
- Check "Two-sided P-value"
- Click "Compute!"
- The correlation coefficient r is given, the p-value is in parentheses

Note: The test statistic for the hypothesis test can be found in StatCrunch by doing a regression, discussed in the next section.

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Thus, $r = 0.5$ indicates a moderate positive correlation and $r = -0.8$ indicates a strong negative correlation.

Correlation, example

Example

In 1886, Sir Francis Galton, a British sociologist, published the paper "Regression towards Mediocrity in Hereditary Statures", in which he examined the heights of parents and their adult children. The core of modern uses of correlation and regression come from this paper (he also invented standard deviation). His data for fathers and sons is in the file "Galton-father-son.csv" on D2L.

Is there correlation between heights of fathers and their adult sons? Test at $\alpha = 0.05$ level of significance.

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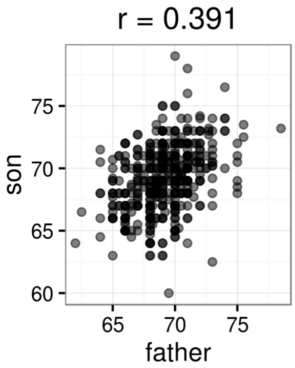
- $H_0 : \rho = 0$
  $H_a : \rho \neq 0$
- $r = 0.39131736$
  $p < 0.0001$

Correlation, example

Example

- $p < 0.0001 < \alpha = 0.5$. Reject null hypothesis
- There is evidence that there is a correlation between the heights of fathers and their adult sons. However, $r < 0.4$ indicates weak correlation.



Coefficient of determination

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Example

In the previous example, heights of fathers and sons had a correlation coefficient of $r = 0.391$. Thus, $R^2 = (0.391)^2 = 0.153$.

About 15% of the variation of the heights of adult men can be explained by the association with their fathers heights.

Cautions

If a sample of paired data has a correlation coefficient that is zero or very low, that does not necessarily mean that there is not a association between the variables, only that there is not a *linear* association.

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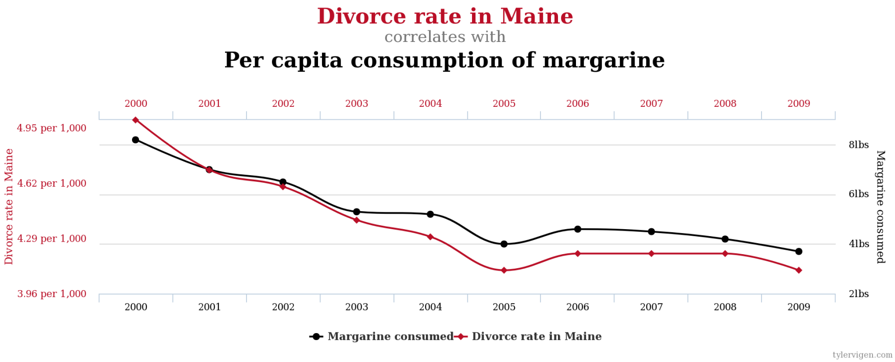
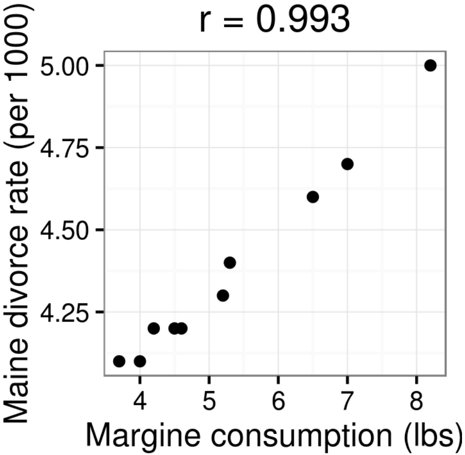
Correlation does not imply causation.

Spurious correlations

From the *Spurious Correlations* website:
`http://www.tylervigen.com/spurious-correlations`



Spurious correlations, cont.



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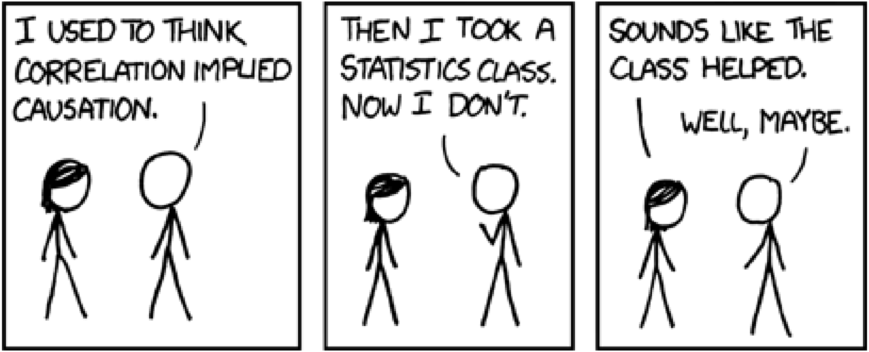

Group work

- For all the questions, complete parts (a) and (b).

Section 10.2
Regression

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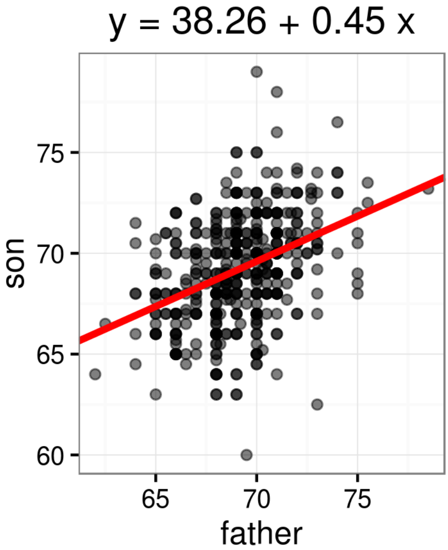
The line found is known as the **regression line** or the line of best fit.

Regression, example



Algebra review, lines

The equation for a line generally has the following form:

$$y = b + mx$$

- b is the y-intercept, or where the line crosses the y-axis ($x = 0$).
- m is the slope of the line. It is the amount the y value increases as the x value increase by one.

Regression population models

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- X is known as the predictor variable, or the explanatory variable, or the independent variable.
- Y is the response variable, or the dependent variable.
- β_0 and β_1 are the intercept and slope of a line describing the association of X and Y.

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- ϵ (epsilon) is a random error term. It is usually assumed that $\epsilon \sim N(0, \sigma^2)$.

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- b_0, the y-intercept of the regression line, is usually not of much interest.

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However, if there is a valid regression equation, it can be used to make predictions of the response variable for given values of the predictor variable. Replace x in the equation with the given predictor value and calculate the predicted response $\hat{y}$.

Predictions, example

Example

The regression line equation from the Galton data, for fathers height as predictor x and sons height as response y, is

$$\hat{y} = 38.26 + 0.45x$$

What is the predicted adult height of a son whose father is 68 inches tall?

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What is the predicted adult height of a son whose father is 68 inches tall?

- $\hat{y} = 38.26 + 0.45 \times 68 = 68.86$ inches

Extrapolating

Predictions for predictor values outside the range of x values used to find the regression line are highly suspect. This is known as **extrapolating** and should be avoided.

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$$\hat{y} = -208.69 + 5.46x$$

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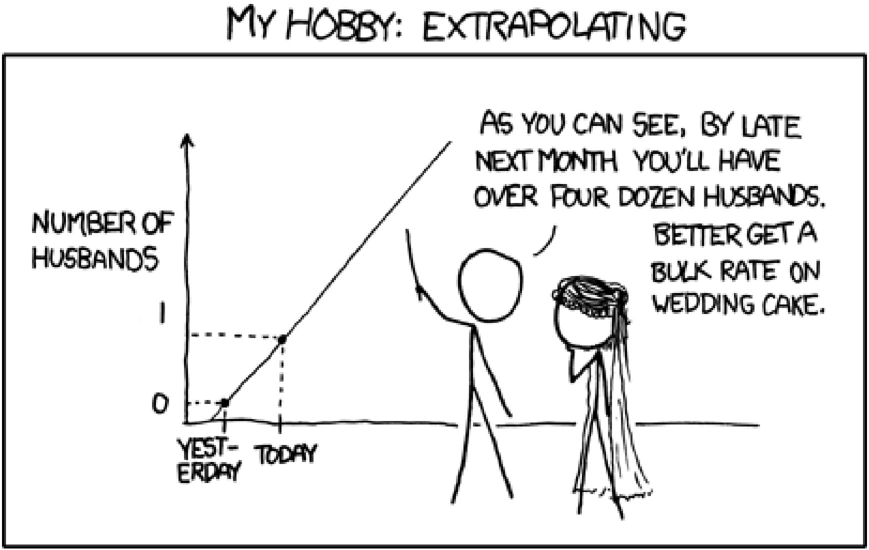
- $\hat{y} = -208.69 + 5.46 \times 60 = 118.91$ games won

Extrapolating, example



Regression in StatCrunch

- Stat → Regression → Simple Linear
- Select columns for X and Y variables
- Select "Hypothesis tests" (default values are fine)
- If desired, enter "X value(s)" for "Prediction of Y"
- If desired, select graphs to generate (the default "Fitted line plot" is usually best)
- Click "Compute!"

Interpreting regression results in StatCrunch

- "Simple linear regression results" section contains:
  - The regression equation in the form of "YVAR = intercept + slope XVAR"
  - The correlation coefficient r as "R (correlation coefficient)"
  - The coefficient of determination R^2 as "R-sq"
- The "Parameter estimates" table contains:
  - The estimates for intercept (b_0) and slope (b_1)
  - The t statistic and p-value for the slope are the same as for a correlation hypothesis test with this data
- Ignore "Analysis of variance table for regression model"
- If predicted values were asked for, they will be in the "Predicted values" table
- Click on the right arrow at the bottom of the results window to get to any graphs that were selected

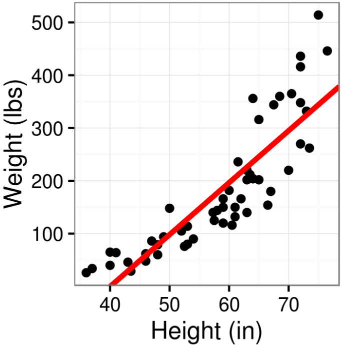
Regression, example

Example

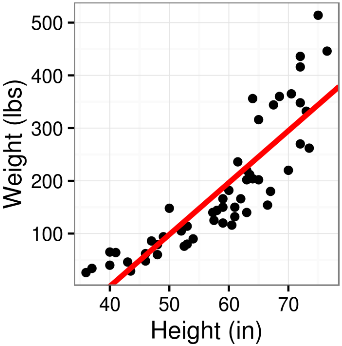
The Department of Natural Resources wishes to track the weight of bears in the wild. While it is very difficult to weigh a bear, it is fairly easy to estimate the length of bear using photos. The data set "bears.csv" on D2L contains measurements made from anesthetized wild bears.

- Find a relationship, if any, between the length and weight of bears using the data.
- What is the best predicted weight of a bear thought to be 71 inches long?
- Would it be appropriate to predict the weight of a bear 39 inches long? 89 inches?

Regression, example



Regression, example



Example

From the StatCrunch results:

- $r = 0.864,\ R^2 = 0.747.$
- About 75% of the variation in bear weight is explained by the association with bear height.

Regression, example

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- Regression line equation: $\hat{y} = -393.84 + 9.84x$

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- A bear that is 71 inches long should weigh: $\hat{y} = -393.84 + 9.84 \times 71 = 304.7282$ lbs.
- It would be appropriate to predict the weight of a bear that is 39 inches long, because 39 is within the range of lengths used to find the regression line (36, 76.5).
- 89 inches is not in that range, so it would not be appropriate to try to predict the weight of a 89 inch long bear.

Group work

- For all the questions, complete parts (c) and (d).