Stat 201: Statistics I
Chapter 9

date

Chapter 9
Inferences from Two Samples

Two sample hypothesis testing

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Hypothesis testing can be used to compare two unknown populations using samples drawn from each population. This is unsurprisingly known as **two sample hypothesis testing**. These tests compare two unknown population parameters.

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For two sample tests, the null hypothesis states that the two unknown parameters are the same.

For example, if comparing means, $H_0 : \mu_1 = \mu_2$, or the mean of population 1 is the same as the mean of population 2.

Alternative hypotheses

The alternative hypothesis is then the claim that something interesting has occurred. For one sample tests, the alternative hypothesis states that the unknown population parameter is somehow different from the known parameter. The unknown parameter might be less than, greater than or simple not equal to the known value.

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For example, if comparing means, $H_a : \mu_1 < \mu_2$ or $\mu_1 > \mu_2$ or $\mu_1 \neq \mu_2$.

Conducting two sample hypothesis test

Once the null and alternative hypotheses are determined, two sample hypothesis tests are carried out almost identically to one sample hypothesis tests.

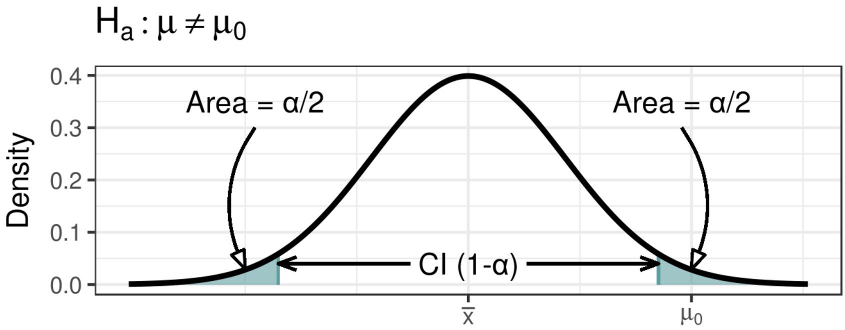
Assuming the null hypothesis is true, a test statistic representing the place of the samples within the appropriate sampling distribution is calculated. A p-value representing the probability of obtaining the test statistic or one more extreme is calculated. If the p-value is below a pre-determined threshold (the significance level), then the null hypothesis is rejected and it can be said that there is evidence for the alternative hypothesis.

Steps for hypothesis test

1. Identify null and alternative hypotheses from research question
2. Determine appropriate sampling distribution
3. Calculate test statistic
4. Calculate p-value
5. Compare p-value to significance level α and report decision
6. State conclusion in terms of original research question

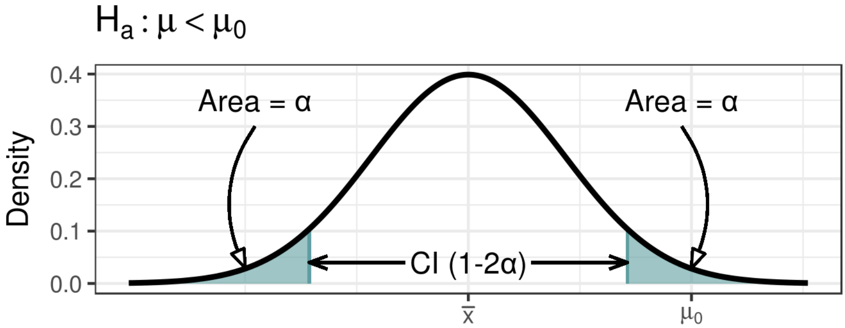
Confidence intervals as hypothesis test

Recall, a confidence interval can be used as an equivalent to a hypothesis test with the area outside the interval as rejection region. For two-sided tests, given a $(1-\alpha)\%$ confidence interval, reject the null if the null parameter is outside of the interval.



Confidence intervals for one-sided tests

However, for one-sided tests the rejection region is only on one side of the distribution. Thus, to get a one-sided rejection region with area α a $(1-2\alpha)\%$ confidence interval is needed.



Section 9.1
Two Proportions

Two sample hypothesis tests for proportions

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Notationally, p_1 is the population proportion from the first population and p_2 is the population proportion from the second population. Similarly, $\hat{p}_1 = x_1/n_1$, the sample proportion of the sample from the first population is the number of successes in sample 1 divided by the sample size of sample 1, etc.

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It is usually not important which of the two populations is "first", but it is very important that which ever designation is used, it is used consistently through the hypothesis test process.

Requirements

- Both samples must meet the requirements of a binomial distribution (simple random samples, consistent probability of success for each trial, etc.)

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- For each sample, the number of successes and the number of failures must both be 5 or greater.

Null hypotheses

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Thus, under the null hypothesis, the two population proportions are the same and the common proportion can be estimated by

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

Alternative hypotheses

The alternative hypothesis for a two sample proportion test is that proportion 1 differs from proportion 2. Like one sample tests, two sample alternative hypotheses can be one-sided or two-sided. And like the null hypothesis for two sample tests, each alternative hypothesis can be written in two ways.

$$\begin{array}{rl|l} \text{Less than:} & H_a : p_1 < p_2 & H_a : p_1 - p_2 < 0 \\ \text{Greater than:} & H_a : p_1 > p_2 & H_a : p_1 - p_2 > 0 \\ \text{Not equal:} & H_a : p_1 \neq p_2 & H_a : p_1 - p_2 \neq 0 \end{array}$$

Test statistic

Like with one sample proportion tests, two sample proportion tests use the standard normal sampling distribution. As always, z-scores are calculated by

$$z = \frac{x - \mu}{\sigma}$$

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- x is the difference of the sample proportions, or $\hat{p}_1 - \hat{p}_2$
- the expected value is the difference of the population proportions, or $p_1 - p_2$, which under the null is 0
- σ (standard deviation) is... complicated...

$$\sigma = \sqrt{(\bar{p}\bar{q})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

Test statistic, cont.

The test statistic for two sample proportion tests is

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{(\bar{p}\bar{q})\left(1/n_1 + 1/n_2\right)}}$$

As before, z-scores and p-values can be found using technology.

Hypothesis tests for two proportions in StatCrunch

- Stat $\rightarrow$ Proportion Stats $\rightarrow$ Two Samples $\rightarrow$ With Summary
- Enter "# of successes" and "# of observations" for sample 1 and sample 2
- Select "Hypothesis test for $p_1 - p_2$"
- The null hypothesis should always be $H_0 : p_1 - p_2 = 0$
- Enter the appropriate value for the alternative hypothesis.
- Click "Compute!"
- The test statistic and p-value are found in "Z-Stat" and "P-value"

Confidence interval for difference of proportions

- A confidence interval for the difference of population proportions $p_1 - p_2$, is defined as

$$CI_{1-\alpha} = (\hat{p}_1 - \hat{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

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- A confidence interval that does not contain zero is evidence that the population proportions are not equal.
- Because hypothesis tests and confidence intervals use different estimates for standard deviation, it is possible to get different results.
- Remember, for one-sided tests, construct a $(1 - 2\alpha)$% confidence interval.

Confidence intervals for difference of proportions in StatCrunch

- Stat $\rightarrow$ Proportion Stats $\rightarrow$ Two Samples $\rightarrow$ With Summary
- Enter "# of successes" and "# of observations" for sample 1 and sample 2
- Select "Confidence interval for $p_1 - p_2$"
- Enter the appropriate confidence level.
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- The confidence interval bounds are found in "L. Limit" and "U. Limit"

Hypothesis test for two proportions, example

Example

A school district in an effort to reduce the number of teen drivers who text or email while driving creates an education program that half the high school students in the district attend. Two months after the program, the district surveys a random sample of teen drivers. The survey finds that 62 out of 212 teens surveyed who had attended the program (population 1) had texted or emailed while driving during the previous month, while among those that did not attend the program (population 2), 59 out of 173 did so.

Test whether the program was successful at a 0.05 level of significance.

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Test whether the program was successful at a 0.05 level of significance.

1. Identify null and alternative hypotheses from research question
   $H_0 : p_1 = p_2$ or $H_0 : p_1 - p_2 = 0$
   $H_a : p_1 < p_2$ or $H_a : p_1 - p_2 < 0$

Hypothesis test for two proportions, example

Example

2. Check the requirements for using normal distribution

Hypothesis test for two proportions, example

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2. Check the requirements for using normal distribution
   - Simple random sample
   - Requirements of a binomial satisfied (fixed sample size, two outcomes, independent, constant probability of success)
   - Successes and failures from both samples exceed 5

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3. Calculate z test statistic

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   $z = -1.0215331$

Hypothesis test for two proportions, example

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4. Calculate p-value

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2. Check the requirements for using normal distribution
   - Simple random sample
   - Requirements of a binomial satisfied (fixed sample size, two outcomes, independent, constant probability of success)
   - Successes and failures from both samples exceed 5
3. Calculate z test statistic
   $z = -1.0215331$
4. Calculate p-value
   $p = 0.1535$

Hypothesis test for a proportion, example

Example

5. Compare p-value to significance level α and report decision

Hypothesis test for a proportion, example

Example

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   $p = 0.1535 > \alpha = 0.05$. Fail to reject null hypothesis.

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   $p = 0.1535 > \alpha = 0.05$. Fail to reject null hypothesis.
6. State conclusion in terms of original research question

Hypothesis test for a proportion, example

Example

5. Compare p-value to significance level α and report decision
   $p = 0.1535 > \alpha = 0.05$. Fail to reject null hypothesis.
6. State conclusion in terms of original research question
   There is not evidence that teen drivers who attended the program text and email while driving at a lower rate than those who did not attend.

Confidence interval for two means, example

Example

A $(1-2\alpha)\%$, or 90%, confidence interval for $p_1 - p_2$ is

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A $(1-2\alpha)$%, or 90%, confidence interval for $p_1 - p_2$ is

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Confidence interval for two means, example

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A $(1 - 2\alpha)$%, or 90%, confidence interval for $p_1 - p_2$ is

$$CI_{0.90} = (-0.127,\, 0.0299)$$

Since zero is in the interval, there is not evidence that $p_1 - p_2 > 0$ or that teen drivers who attended the program text and email while driving at a lower rate than those who did not attend.

Group work

- Complete question 1

Section 9.2
Two Means: Independent Samples

Two sample hypothesis tests for means

A two sample hypothesis test can be conducted to compare the means of two populations.

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Similar to proportion tests, parameters and statistics are designated with subscripts to identify which populations or samples they are from, μ_1 and μ_2, etc.

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Independent samples are samples that come from distinct populations where the values from one sample do not affect the values from the other samples.

Conversely, **dependent samples** are samples that often involve the same subjects with measurements taken at different times.

Independent vs dependent samples, example

Example

Independent samples:

- Scores on statistic final in one class vs another class
- Heights of men vs heights of women
- In a clinical trial, outcomes for patients given experiment treatment vs patients given placebo (control)

Independent vs dependent samples, example

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- Scores on statistic final in one class vs another class
- Heights of men vs heights of women
- In a clinical trial, outcomes for patients given experiment treatment vs patients given placebo (control)

Dependent samples:

- Scores on midterm exam vs final exam
- Heights of husbands vs heights of wives
- In a case control study, risk factors for subjects with disease vs matched subjects without disease

Null and alternative hypotheses

Similar to two sample proportion tests, the null hypothesis for two sample mean tests is always $H_0 : \mu_1 = \mu_2$ or $H_0 : \mu_1 - \mu_2 = 0$.

Alternative hypotheses can be any of

$$H_a : \mu_1 < \mu_2,\ \mu_1 > \mu_2,\ \mu_1 \neq \mu_2$$

or the equivalent forms of

$$H_a : \mu_1 - \mu_2 < 0,\ \mu_1 - \mu_2 > 0,\ \mu_1 - \mu_2 \neq 0$$

Requirements

Both samples must be independent and simple random samples drawn from normal populations or have a sample size of at least 30.

Test statistic

As with one sample mean tests, two sample mean tests use a t distribution. Similar to two sample proportion tests, a t statistic is calculated by

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

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Recall, the degrees of freedom of a t distribution is defined by sample size minus one $(n-1)$. For a two sample test, a conservative approach is to use the smaller of the degrees of freedom from the two samples.

$$df = \min\left[(n_1 - 1), (n_2 - 1)\right]$$

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$$df = \min\left[(n_1 - 1), (n_2 - 1)\right]$$

When using technology, a more complicated, but accurate value will be used for the degrees of freedom.

Hypothesis tests for two means in StatCrunch

- Stat $\rightarrow$ T Stats $\rightarrow$ Two Samples $\rightarrow$ With Summary
- Enter "Sample mean", "Sample std. dev." and "Sample size" for both samples
- Leave "Pool variances" unchecked
- Select "Hypothesis test for $\mu_1 - \mu_2$"
- The null hypothesis should always be $H_0 : \mu_1 - \mu_2 = 0$
- Enter the appropriate value for the alternative hypothesis.
- Click "Compute!"
- The test statistic and p-value are found in "T-Stat" and "P-value"

Confidence intervals for difference of means in StatCrunch

- Stat → T Stats → Two Samples → With Summary
- Enter "Sample mean", "Sample std. dev." and "Sample size" for both samples
- Leave "Pool variances" unchecked
- Select "Confidence interval for $\mu_1 - \mu_2$"
- Enter the appropriate confidence level.
- Click "Compute!"
- The confidence interval bounds are found in "L. Limit" and "U. Limit"

Hypothesis test for two means, example

Example

A study is conducted of the heights of Metro State students. The heights of a sample of 32 male students taking statistics (population 1) have a mean of $\bar{x} = 70.1$ with a standard deviation of $s = 3.5$. The heights of a sample of 38 male students taking accounting (population 2) have a mean of $\bar{x} = 67.9$ with a standard deviation of $s = 3.1$.

Conduct a test at $\alpha = 0.05$ level of significance of the claim that statistics students are taller than accounting students.

Hypothesis test for two means, example

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Conduct a test at $\alpha = 0.05$ level of significance of the claim that statistics students are taller than accounting students.

1. Identify null and alternative hypotheses from research question

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Conduct a test at $\alpha = 0.05$ level of significance of the claim that statistics students are taller than accounting students.

1. Identify null and alternative hypotheses from research question
   $H_0 : \mu_1 = \mu_2$ of $H_0 : \mu_1 - \mu_2 = 0$
   $H_a : \mu_1 > \mu_2$ or $H_a : \mu_1 - \mu_2 > 0$

Hypothesis test for two means, example

Example

2. Use a t distribution

Hypothesis test for two means, example

Example

2. Use a t distribution
3. Calculate t test statistic

Hypothesis test for two means, example

Example

2. Use a t distribution
3. Calculate t test statistic
   $t = 2.7592694$

Hypothesis test for two means, example

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   $t = 2.7592694$
4. Calculate p-value

Hypothesis test for two means, example

Example

2. Use a t distribution
3. Calculate t test statistic
   $t = 2.7592694$
4. Calculate p-value
   $p = 0.0038$

Hypothesis test for two means, example

Example

2. Use a t distribution
3. Calculate t test statistic
   $t = 2.7592694$
4. Calculate p-value
   $p = 0.0038$
5. Compare p-value to significance level α and report decision

Hypothesis test for two means, example

Example

2. Use a t distribution
3. Calculate t test statistic
   $t = 2.7592694$
4. Calculate p-value
   $p = 0.0038$
5. Compare p-value to significance level α and report decision
   $p = 0.0038 < \alpha = 0.05$. Reject the null hypothesis.

Hypothesis test for two means, example

Example

2. Use a t distribution
3. Calculate t test statistic
   $t = 2.7592694$
4. Calculate p-value
   $p = 0.0038$
5. Compare p-value to significance level α and report decision
   $p = 0.0038 < \alpha = 0.05$. Reject the null hypothesis.
6. State conclusion in terms of original research question

Hypothesis test for two means, example

Example

2. Use a t distribution
3. Calculate t test statistic
   $t = 2.7592694$
4. Calculate p-value
   $p = 0.0038$
5. Compare p-value to significance level α and report decision
   $p = 0.0038 < \alpha = 0.05$. Reject the null hypothesis.
6. State conclusion in terms of original research question
   There is evidence that male statistics students are taller than male accounting students.

Confidence interval for two means, example

Example

A $(1-2\alpha)\%$, or 90%, confidence interval for $\mu_1 - \mu_2$ is

Confidence interval for two means, example

Example

A $(1-2\alpha)$%, or 90%, confidence interval for $\mu_1 - \mu_2$ is

$$CI_{0.90} = (0.869,\ 3.531)$$

Confidence interval for two means, example

Example

A $(1-2\alpha)$%, or 90%, confidence interval for $\mu_1 - \mu_2$ is

$$CI_{0.90} = (0.869,\ 3.531)$$

Since zero is not in the interval and both bounds are positive, there is evidence that $\mu_1 - \mu_2 > 0$ or male statistics students are taller than male accounting students.

Group work

- Complete question 2

Section 9.3
Two Dependent Samples (Matched Pairs)

Hypothesis tests for dependent samples

Conceptually, when working with two dependent samples, also known as matched pairs, a new value $d = x_1 - x_2$ is calculated for each pair. Then, the null hypothesis $H_0 : \mu_1 = \mu_2$ becomes $H_0 : \mu_D = 0$ and the test is essentially a one sample hypothesis test for the mean of d.

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Matched pairs tests, when appropriate, are preferable to independent samples tests because the variability between subjects is almost eliminated. Thus, the total variability is reduced resulting in a more accurate test.

Conducting a dependent samples test

Since a dependent samples test is essentially a one sample test of the sample of difference values d, null and alternative hypotheses should be stated in terms of μ_D the population mean of the differences.

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For example, $H_0 : \mu_D = 0$ and $H_a : \mu_D < 0$.

Conducting a dependent samples test

Since a dependent samples test is essentially a one sample test of the sample of difference values d, null and alternative hypotheses should be stated in terms of μ_D the population mean of the differences.

For example, $H_0 : \mu_D = 0$ and $H_a : \mu_D < 0$.

The test statistic is calculated as a one sample statistic from the sample of differences

$$t = \frac{\bar{d}}{\frac{s_d^2}{\sqrt{n}}}$$

Hypothesis tests for matched pairs in StatCrunch

- Stat $\rightarrow$ T Stats $\rightarrow$ Paired
- Select columns of data for both samples
- Select "Hypothesis test for $\mu_D = \mu_1 - \mu_2$"
- The null hypothesis should always be $H_0 : \mu_D = 0$
- Enter the appropriate value for the alternative hypothesis.
- Click "Compute!"
- The test statistic and p-value are found in "T-Stat" and "P-value"

Confidence intervals for matched pairs in StatCrunch

- Stat $\rightarrow$ T Stats $\rightarrow$ Paired
- Select columns of data for both samples
- Select "Confidence interval for $\mu_D = \mu_1 - \mu_2$" item Enter the appropriate confidence level.
- Click "Compute!"
- The confidence interval bounds are found in "L. Limit" and "U. Limit"

Hypothesis test for matched pairs, example

Example

A study is conducted to see if statistics students change their scores from the midterm exam to the final exam. 42 students who took both exams are randomly selected. Their scores are found in the file "scores.csv".

Test at significance level of 0.01 whether scores changed between the midterm and the final.

Hypothesis test for matched pairs, example

Example

A study is conducted to see if statistics students change their scores from the midterm exam to the final exam. 42 students who took both exams are randomly selected. Their scores are found in the file "scores.csv".

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1. Identify null and alternative hypotheses from research question

Hypothesis test for matched pairs, example

Example

A study is conducted to see if statistics students change their scores from the midterm exam to the final exam. 42 students who took both exams are randomly selected. Their scores are found in the file "scores.csv".

Test at significance level of 0.01 whether scores changed between the midterm and the final.

1. Identify null and alternative hypotheses from research question
   $H_0 : \mu_D = 0$
   $H_a : \mu_D \neq 0$

Hypothesis test for matched pairs, example

Example

2. Use a t distribution

Hypothesis test for matched pairs, example

Example

2. Use a t distribution
3. Calculate t test statistic

Hypothesis test for matched pairs, example

Example

2. Use a t distribution
3. Calculate t test statistic
   $t = -3.8176772$

Hypothesis test for matched pairs, example

Example

2. Use a t distribution
3. Calculate t test statistic
   $t = -3.8176772$
4. Calculate p-value

Hypothesis test for matched pairs, example

Example

2. Use a t distribution
3. Calculate t test statistic
   $t = -3.8176772$
4. Calculate p-value
   $p = 0.0004$

Hypothesis test for matched pairs, example

Example

2. Use a t distribution
3. Calculate t test statistic
   $t = -3.8176772$
4. Calculate p-value
   $p = 0.0004$
5. Compare p-value to significance level α and report decision

Hypothesis test for matched pairs, example

Example

2. Use a t distribution
3. Calculate t test statistic
   $t = -3.8176772$
4. Calculate p-value
   $p = 0.0004$
5. Compare p-value to significance level α and report decision
   $p = 0.0004 < \alpha = 0.01$. Reject the null hypothesis.

Hypothesis test for matched pairs, example

Example

2. Use a t distribution
3. Calculate t test statistic
   $t = -3.8176772$
4. Calculate p-value
   $p = 0.0004$
5. Compare p-value to significance level α and report decision
   $p = 0.0004 < \alpha = 0.01$. Reject the null hypothesis.
6. State conclusion in terms of original research question

Hypothesis test for matched pairs, example

Example

2. Use a t distribution
3. Calculate t test statistic
   $t = -3.8176772$
4. Calculate p-value
   $p = 0.0004$
5. Compare p-value to significance level α and report decision
   $p = 0.0004 < \alpha = 0.01$. Reject the null hypothesis.
6. State conclusion in terms of original research question
   There is evidence that statistics exam scores change from the midterm to the final.

Confidence interval for matched pairs, example

Example

A $(1-\alpha)\%$, or 99%, confidence interval for μ_D is

Confidence interval for matched pairs, example

Example

A $(1-\alpha)$%, or 99%, confidence interval for μ_D is

$$CI_{0.99} = (-6.871,\ -1.177)$$

Confidence interval for matched pairs, example

Example

A $(1-\alpha)\%$, or 99%, confidence interval for μ_D is

$$CI_{0.99} = (-6.871,\ -1.177)$$

Since zero is not in the interval, there is evidence that $\mu_d \neq 0$ or statistics exam scores change from the midterm to the final.

Group work

- Complete question 3

Teaching the null hypothesis