

Stat 201: Statistics I

Chapter 3



Chapter 3

Statistics for Describing, Exploring, and Combining Data

Section 3.1

Measures of Center

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Perhaps the most important of these, **measures of center** are a way of representing the value of the middle of the data.

There are four measures of center discussed in this section:

- mean
- median
- mode
- midrange

Mean

The **mean** (the arithmetic mean) is the measure of center calculated by adding the values of the data set and dividing by the size of the data set. Also known as the average.

- Only makes sense with quantitative data
- Sensitive to outliers (extreme or unusual values) and skewed data distributions.

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To calculate

Let X be sample of size n of quantitative data with values x_1, \dots, x_n . Then,

$$\bar{x} = \frac{\sum x_i}{n}$$

\bar{x} is the mean of the sample.

- \sum means add all the x_i 's, where i is between 1 and n

Mean, example

Example

Suppose we find a sample of 8 students and ask for their ages.

- The sample is $X = \{22, 32, 46, 50, 33, 38, 20, 24\}$

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- The sample is $X = \{22, 32, 46, 50, 33, 38, 20, 24\}$
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- The sum of the data is

$$\sum x_i = 22 + 32 + 46 + 50 + 33 + 38 + 20 + 24 = 265$$

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- The sample is $X = \{22, 32, 46, 50, 33, 38, 20, 24\}$
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- The sum of the data is

$$\sum x_i = 22 + 32 + 46 + 50 + 33 + 38 + 20 + 24 = 265$$

- The mean is

$$\bar{x} = \frac{\sum x_i}{n} = \frac{265}{8} = 33.125 \text{ years}$$

Median

The **median** is the value that is greater than or equal to at least 50% of the data and less than or equal to at least 50% of the data.

- Can be used with quantitative and ordinal data
- Not sensitive to extreme values (**resistant** measure of center)

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To calculate

Arrange the data in order, from lowest to highest.

- If n is odd, the middle value is the median.
- If n is even, the median is the mean of the two middle values.

Median, example

Example

Returning to the ages of 8 students.

- The sample is $X = \{22, 32, 46, 50, 33, 38, 20, 24\}$

Median, example

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20 22 24 32 33 38 46 50

Median, example

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- Since n is even, find the mean of the two middle values.

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$$(32+33)/2=32.5$$

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- Since n is even, find the the mean of the two middle values.

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$$(32+33)/2=32.5$$

- The median is $\tilde{x} = 32.5$ years.

Mean vs. Median

Suppose in our age data set, we replaced the 50 with a 85.

- Mean goes from 33.125 to 37.5
- Median remains unchanged at 32.5

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- Median remains unchanged at 32.5

This is why median is called a **resistant** statistic.

- Median is used when we don't want a few extreme values to distort a more reasonable middle, such as house prices or incomes.

Mean vs. Median, cont.

Suppose instead of calculating a grade point average (mean), we calculated a grade point median. Consider a student who got A's in 3 classes and D's in 2.

- The median grade point is 4, an A.
- The GPA for such a student would be 2.8.

Mean vs. Median, cont.

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- The median grade point is 4, an A.
- The GPA for such a student would be 2.8.

The median does not consider all values of a data set. The mean does.

- Mean is used when all values are important or when we expect to have roughly symmetric data.

The **mode** is the data value with highest frequency.

- Can be used with any kind of data.
- A data set might have more than one mode, or there might not be any mode.

Mode, example

Examples

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Mode, example

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- From a TACO survey, favorite kind of taco had these responses:
 $\{\text{Beef, Beef, Fish, Shrimp, Beef, Pork, Chicken, Beef, Chicken, Beef}\}$
The mode is “Beef” with a frequency of five.

Mode, example

Examples

- The age data, $\{22, 32, 46, 50, 33, 38, 20, 24\}$, has no mode.
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The mode is “Beef” with a frequency of five.

- Suppose a sample from a class got the following grades on a quiz:

$$\{A, C, B, A, A, B, C, B\}$$

The modes are A and B, with frequencies of three each.

Midrange

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Example

The age data is $X = \{22, 32, 46, 50, 33, 38, 20, 24\}$.

- The minimum age is 20 and the maximum age is 50.
- The midrange is

$$\frac{\min(X) + \max(X)}{2} = \frac{20 + 50}{2} = 35$$

Group work

- For each question, complete part (a).

Section 3.2

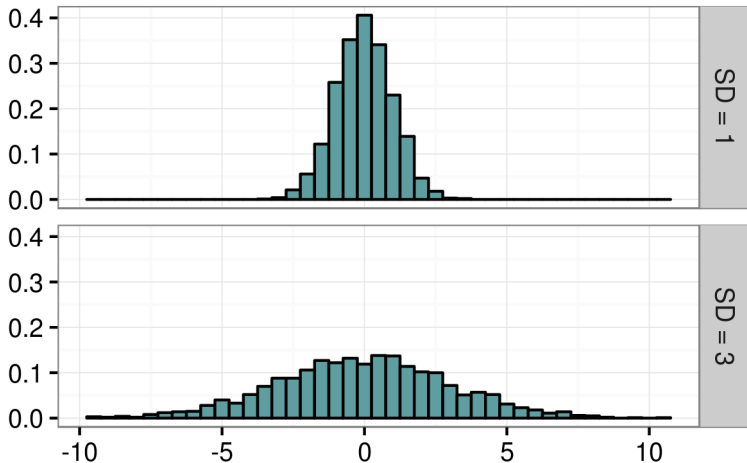
Measures of Variation

Other measures

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Measures of variation

Another important class of descriptive statistics are **measures of variation** which describe how much the data is spread out.

There are three measures of variation discussed in this section:

- Range
- Variance
- Standard deviation

Range

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Example

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- The minimum age is 20 and the maximum age is 50.

Range

The **range** is the difference between the maximum and minimum values.

- Like the midrange, very sensitive to extreme values.

Example

The age data is $X = \{22, 32, 46, 50, 33, 38, 20, 24\}$.

- The minimum age is 20 and the maximum age is 50.
- The range is

$$\max(X) - \min(X) = 50 - 20 = 30 \text{ years}$$

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- Always non-negative. A zero standard deviation means all the data are the same value.
- Sensitive to extreme values.
- The units of standard deviation are the same as the data. Variance units are the data units squared.

Variance and standard deviation, calculation

To calculate

Let X be sample of size n of quantitative data with values x_1, \dots, x_n and sample mean \bar{x} . Then,

$$\text{Var}(X) = s^2 = \frac{\sum (x_i - \bar{x})^2}{n - 1} \quad \text{and} \quad \text{SD}(X) = s = \sqrt{s^2}$$

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- Note: Never calculate this by hand. Use technology.

Variance and standard deviation, example

Example

The age data is $X = \{22, 32, 46, 50, 33, 38, 20, 24\}$. The sample size is $n = 8$ and the sample mean is $\bar{x} = 33.125$

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- The variance is

$$\begin{aligned}s^2 &= \frac{\sum (x_i - \bar{x})^2}{n - 1} \\&= \frac{(22 - 33.125)^2 + \cdots + (24 - 33.125)^2}{7} \\&= 122.125 \text{ years}^2\end{aligned}$$

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- The standard deviation is

$$s = \sqrt{s^2} = \sqrt{122.125} = 11.05 \text{ years}$$

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Notation

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Property	Parameter	Statistic
Mean	μ (mu)	\bar{x}
Variance	σ^2 (sigma-squared)	s^2
Standard deviation	σ (sigma)	s

Group work

- For each question, complete part (b).

Section 3.3

Measures of Relative Standing and Boxplots

Measures of relative standing

Measures of relative standing describe the location of a given data value with a data distribution or a data set.

Two measures of relative standing are discussed in this section:

- Z-scores
- Percentiles

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- Another way to put it is a z-score is the number of standard deviations that a particular value is above or below the mean.
- Z-scores are standardized and unit-less, so they can be used to compare values from different populations.
- A positive z-score means the value is greater than the mean and a negative z-score means that it is below the mean.
- Z-scores can be calculated for samples or populations, if the population mean and standard deviation are known.

Z-scores, calculations

To calculate

- For a sample X with sample mean \bar{x} and standard deviation s , the z-score for a value x is

$$z = \frac{x - \bar{x}}{s}$$

Z-scores, calculations

To calculate

- For a sample X with sample mean \bar{x} and standard deviation s , the z-score for a value x is

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- For a population with population mean μ and standard deviation σ , the z-score for value x is

$$z = \frac{x - \mu}{\sigma}$$

Z-scores, example

Example

The age data has mean of $\bar{x} = 33.125$ and SD of $s = 11.05$.

Z-scores, example

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The age data has mean of $\bar{x} = 33.125$ and SD of $s = 11.05$.

- Suppose a new student joins the class. His age is 61. He has an age z -score of

$$z = \frac{x - \bar{x}}{s} = \frac{61 - 33.125}{11.05} = \frac{27.875}{11.05} = 2.52$$

Z-scores, example

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His age two and a half standard deviations above the class mean.

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- Another student joins the class. Her age is 27. She has an age z -score of

$$z = \frac{x - \bar{x}}{s} = \frac{27 - 33.125}{11.05} = \frac{-6.125}{11.05} = -0.554$$

Z-scores, example

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$$z = \frac{x - \bar{x}}{s} = \frac{27 - 33.125}{11.05} = \frac{-6.125}{11.05} = -0.554$$

Her age is about a half standard deviation below the class mean.

Values from z-scores

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- After some algebra,

$$x = \bar{x} + zs \quad \text{or} \quad x = \mu + z\sigma$$

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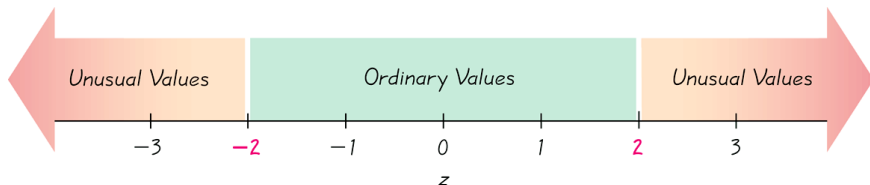
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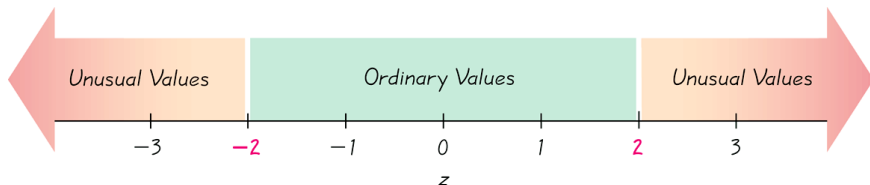
- $x = \bar{x} + zs = 33.125 + 2 \times 11.05 = 55.225$ years

Unusual values



A value is called **unusual** if it has a z-score z such that $z < -2$ or $z > 2$. A value is **ordinary** if z is between -2 and 2 .

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Example

Consider the new students to the class:

- The 61 year old ($z = 2.52$) has an unusual age for the class.
- The 27 year old ($z = -0.554$) has an ordinary age for the class.

Percentiles

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To calculate

- To find the percentile of a value x in a data set,

$$\%ile = \frac{\text{number of values} < x}{n} \times 100\%$$

If percentile is not a whole number, round up.

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To calculate

- To find the percentile of a value x in a data set,

$$\%ile = \frac{\text{number of values} < x}{n} \times 100\%$$

If percentile is not a whole number, round up.

- To find the value of P_p (the p th percentile), calculate the rank,

$$r = \frac{p}{100} \times n$$

If r is a whole number, P_p is the mean of the r th and $(r + 1)$ th values. If not, round up. Then, P_p is the r th value in an ordered list.

Percentile, example

Example

The age data, in order is,

20 22 24 32 33 38 46 50

Percentile, example

Example

The age data, in order is,

20 22 24 32 33 38 46 50

- The percentile of the value 38 is

$$\frac{\text{number of values} < x}{n} \times 100\% = \frac{5}{8} \times 100\% = 62.5\% \Rightarrow P_{63}$$

Percentile, example

Example

The age data, in order is,

20 22 24 32 33 38 46 50

- The percentile of the value 38 is

$$\frac{\text{number of values} < x}{n} \times 100\% = \frac{5}{8} \times 100\% = 62.5\% \Rightarrow P_{63}$$

- To find the 30th percentile, P_{30} , calculate rank

$$r = \frac{p}{100} \times n = \frac{30}{100} \times 8 = 2.4$$

Round up r to 3. P_{30} is the 3rd value, 24.

Quartiles

The **quartiles** are values that divide the data set into 4 parts, or quarters.

$$Q_1 = P_{25} \quad Q_2 = P_{50} \quad Q_3 = P_{75}$$

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- Note: The median is equivalent to Q_2 and P_{50} .

5 number summary

The **5 number summary** summarize the distribution of a data set.

The 5 numbers are:

- Minimum
- Q_1
- Median (or Q_2)
- Q_3
- Maximum

5 number summary, example

Example

The age data, in order is,

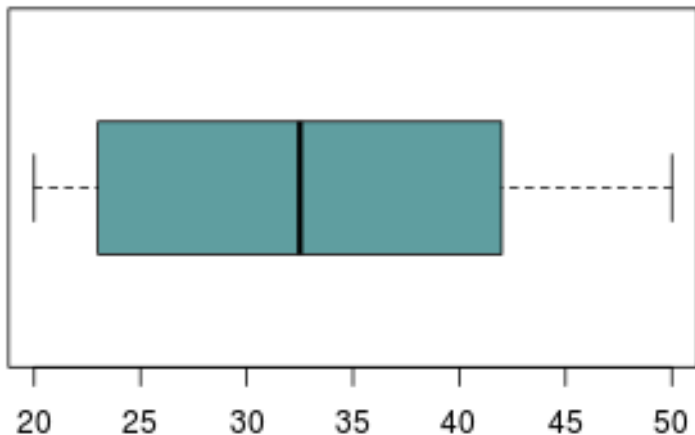
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The 5 number summary is

20	23	32.5	42	50
				
min	Q_1	med	Q_3	max

Boxplots

A **boxplot** is a graph depicting the 5 number summary.



Group work

- For each question, complete part (c).