Stat 201: Statistics I
Chapter 5

September 25, 2017

Chapter 5
Discrete Probability Distributions

Section 5.1
Probability Distributions

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Example

- X = the number of heads from three coin flips
- Y = the sum of two dice
- Z = the midterm score of a randomly selected student
  - A students grade (A, A-, B+, etc.) can not be used as a random variable because it is not numeric,
  - ... Unless, the grade is coded as a number (i.e. A = 4.0, A- = 3.7, etc.)

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- Number of defective insulin test strips in a box of 50
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Continuous random variables:

- Height or weight of a test subject
- Survival time of a cancer patient
- Price of a company's stock at a particular moment

Probability distributions

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- The probabilities must add up to 1
- Often displayed in tables (if practical)

Probability distributions, example

Example

A restaurant wants to track its taco sales. It records how many tacos customers order with each visit. The results are in the table,

Number of tacos	Probability
0	0.35
1	0.2
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Probability distributions, example

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The restaurant introduces a new "Super Taco" (beef, chicken and shrimp). It wonders if larger groups are more likely to order the new item. The results are in the table,

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What is the probability of a customer ordering less than two tacos?

$$P(X < 2) = P(X = 0 \text{ or } 1) = P(0) + P(1) = 0.35 + 0.2 = 0.55$$

Weighted means

A **weighted mean** is the mean of values that are not considered equally, or do not have equal importance.

- Each value has an associated weight, which is its relative importance.
- To calculate, let x_i be a value and w_i its weight.

$$\mu_w = \frac{\sum w_i \times x_i}{\sum w_i}$$

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Example

Fiona buys 4 lbs. of hamburger at $4.89 / lb. and 2 lbs. of steak at $11.99 / lb. What is the average price per pound she is paying?

$$\$/lb. = \frac{4 \times 4.89 + 2 \times 11.99}{6} = \frac{43.54}{6} = 7.26$$

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The mean of a probability distribution is also known as the **expected value** of the random variable.

- Denoted with an "E", as in

$$\mathbb{E}(X) = \mu$$

Mean, exmaple

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$$\begin{aligned}\mathbb{E}(X) = \mu = & \sum x_i \cdot P(x_i) \\ = & \ 0 \cdot 0.35 + 1 \cdot 0.2 + 2 \cdot 0.3 + 3 \cdot 0.1 + 4 \cdot 0.05 \\ = & \ 0 + 0.2 + 0.6 + 0.3 + 0.2 \\ = & \ 1.3\end{aligned}$$

Standard deviation of probability distributions

Similarly, variance of a probability distribution is the weighted mean of difference from the mean squared and standard deviation is the square root of variance.

Thus,

$$\sigma^2 = \sum (x_i - \bar{x})^2 \cdot P(x_i)$$

$$\sigma = \sqrt{\sigma^2}$$

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$$\begin{aligned} \sigma^2 = & \sum (x_i - \bar{x})^2 \cdot P(x_i) \\ = & (0 - 1.3)^2 \cdot 0.35 + \cdots + (4 - 1.3)^2 \cdot 0.05 \\ = & 0.5915 + \cdots + 0.3645 \\ = & 1.41 \end{aligned}$$

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\begin{aligned}
\sigma^2 &= \sum (x_i - \bar{x})^2 \cdot P(x_i) \\
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&= 0.5915 + \cdots + 0.3645 \\
&= 1.41 \\
\sigma = \sqrt{\sigma^2} &= 1.19
\end{aligned}
$$

Mean and var. of random variables in StatCrunch

- Stat $\rightarrow$ Calculators $\rightarrow$ Custom
- Under "Values in:" select the column which contains the random variable values
- Under "Weights in:" select the column which contains the random variable probabilities
- Click "Compute!"
- Mean and standard deviation will be displayed
- Variance can be calculated by squaring the standard deviation

Unusual events rule of thumb

Recall, the range rule of thumb for unusual values are those values more than 2 standard deviations away from the mean. In other words, x is unusual if

$$x < \mu - 2\sigma \qquad \text{or} \qquad x > \mu + 2\sigma$$

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- The lower bound for unusual values is $\mu - 2\sigma = -1.08$. Since you can't order negative tacos, there is not an unusually low number of tacos to order.
- The upper bound for unusual values if $\mu + 2\sigma = 3.86$. Thus, 4 (or more) tacos is an unusually high number of tacos to order.

More precise definition for unusual events

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That is, if

$$P(X \leq x \text{ or } X \geq x) < 0.05$$

then x is an extreme value.

Unusual events, example

Example

Tom and Barbara collect eggs from their chickens everyday. The number of eggs they collect follows the following distribution,

Eggs (x)	0	1	2	3	4	5	6	7	8	9
$P(x)$	0.01	0.03	0.1	0.2	0.3	0.2	0.1	0.04	0.02	0

Is collecting only one egg unusual? Is collecting 7 eggs unusual?

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- $P(X \leq 1) = P(0) + P(1) = 0.01 + 0.03 = 0.04 < 0.05$
- $P(X \geq 7) = P(7) + P(8) + P(9) = 0.06 \not< 0.05$

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- Getting 523 heads is not unusual ($P(X \geq 523) = 0.077$). There is no reason to think the coin is not fair.
- Getting 46 heads is unusual ($P(X \leq 46) = 6.23 \times 10^{-222}$). We would be justified in questioning the assumption that the coin is fair.

Group work

- Work on question 1, all parts.

Section 5.2
Binomial Probability Distributions

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  - Passing the midterm is a "success", failing it is a "failure"
- The number of heads in three coin flips
  - Getting a head is a "success", getting a tail is a "failure"
- The number of of car crashes that result in fatalities
  - A fatality is a "success", no fatalities is a "failure"

Requirements for binomial distributions

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The last two requirements are often summarized as "independent and identically distributed" and abbreviated as "iid".

Requirements for binomial distributions, example

Example

Recall the Metro State Statistics Club, with 6 male members and 4 females members. If the three officer positions are selected randomly, does the number of women selected follow a binomial distribution?

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- No. The probability of success changes (hopefully) with each test.

Requirements for binomial distributions, example

Example

Recall the Youth Risk Behavior Survey (YRBS) which found the probability of a teenaged driver had texted or emailed while driving was 0.404. Suppose this is the probability for the population of teenaged drivers. Suppose 30 teenaged drivers are selected at random. Does the number of those drivers that had texted or emailed while driving follow a binomial distribution?

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- Yes.
    - There is a fixed number of trials (30).
    - Each trial has only two possible outcomes (had or had not texted).
    - Each trial is independent.
    - Each trial has the same probability of "success."

Notation for binomial distributions

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Example

The probability a teenaged driver had texted or emailed while driving is 0.404. If the random variable Y is the number of teenaged drivers who had texted or emailed while driving out of a sample of 30,

$$Y \sim \mathrm{Binom}\,(30, 0.404)$$

Probabilities for binomial distributions

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- $\binom{n}{x}$ is read as "n choose x". It is the number of ways to get x successes in n trials. For example, we previously determined that there were three ways to get two heads in three flips, { HHT, HTH, THH }. Thus, $\binom{3}{2}$ is 3.

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- $p^x q^{n-x}$ is the probability of getting x successes and $n - x$ failures in one particular order.

Binomial probabilities in StatCrunch

- Stat $\rightarrow$ Calculators $\rightarrow$ Binomial
- Enter sample size ("n") and probability ("p")
- Select appropriate comparison symbol and numeric value
- Click "Compute" if necessary
- Probability will be displayed

Practice: cancer screening

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- $P(X = 5) = 0.165$

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- Standard deviation:

$$\sigma = \sqrt{\sigma^2} = \sqrt{npq}$$

Parameters, example

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- The lower bound for unusual values is $\mu - 2\sigma = 6.74$
- The upper bound for unusual values is $\mu + 2\sigma = 17.5$
- In a random sample of 30 teenaged drivers, it would be unusual to get 6 or fewer, or 18 or more, drivers who had texted or emailed while driving.

Group work

- Work on question 2, all parts.

Section 5.3
Poisson Probability Distributions

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Example

- The number of fish caught in the next hour
- The number of customers to come into a store between 1 and 6 pm
- The number of insects found in a square foot of grassland

Poisson notation

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- The random variable X has a Poisson distribution with an mean rate of events of λ (lambda).
- The mean rate (λ) is scalable. That is, if λ is the mean number of events per hour, $\mathrm{Pois}\left(\lambda/2\right)$ models the number of events in a half hour interval.

Poisson probability and parameters

The probability of x events in an interval,

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

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- Variance:

$$\sigma^2 = \lambda$$

- Standard deviation:

$$\sigma = \sqrt{\lambda}$$

Poisson probabilities in StatCrunch

- Stat → Calculators → Poisson
- Enter the rate value λ ("Mean")
- Select appropriate comparison symbol and numeric value
- Click "Compute" if necessary
- Probability will be displayed

Poisson, example

Example

Paul fishes for an hour every morning. Over the past month (30 days) he has caught 62 fish. Paul is having friends over for dinner tonight, so he needs to catch 4 fish.

What is the probability he catches 4 fish in an hour? Is that an unusually high number?

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- Probability of catching four or more fish: $P(X \geq 4) = 0.156$
- Rule of thumb boundary: $\mu = 2\sigma = 2.07 + 2\sqrt{2.07} = 4.95$

Group work

- Work on question 3, all parts.