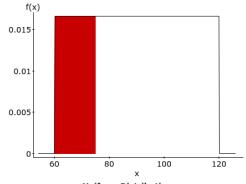
## Group Work - Chapter 6

1

(a) The time it takes to finish a statistics midterm in a certain class is uniformly distributed between 1 and 2 hours (60 to 120 minutes). What is the probability that a randomly selected student will finish the exam in less than 75 minutes? What is the probability they will complete it between 100 and 110 minutes?

f(x)



0.015 0.005 0.005 0.005

**Uniform Distribution** 

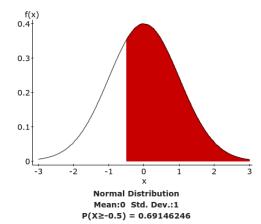
Lower limit:60 Upper limit:120 P(100≤X≤110) = 0.16666667

Uniform Distribution
Lower limit:60 Upper limit:120  $P(X \le 75) = 0.25$ 

 $P(X < 75) = \frac{75 - 60}{120 - 60} = \frac{15}{60} = \frac{1}{4} = 0.25$ 

 $P(100 < X < 110) = \frac{110 - 100}{120 - 60} = \frac{10}{60} = \frac{1}{6} = 0.167$ 

(b) The scores of the midterm are normally distributed and reported as z-scores. What is the probability of a random student getting a grade greater than z = -0.5? What is the grade (z-score) that separates the bottom 90% of the class from the top 10%?

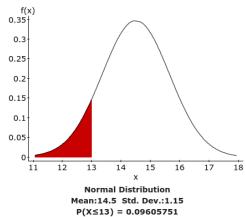


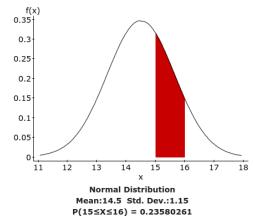
0.3 0.2 0.1 0.1 0.2 0.1 0.2 0.1 0.2 0.1 0.2 0.1 0.3 Normal Distribution Mean:0 Std. Dev.:1 P(X≤1.2815516) = 0.9

P(Z > -0.5) = 0.691

 $P(Z < z) = 0.9, \qquad z = 1.28$ 

- 2 A new breakfast cereal "Super Fruity Taco Bombs" is packaged by a machine which puts the cereal in the bag. The amount loaded by the machine is normally distributed with a mean of 14.5 ounces with a standard deviation of 1.15 ounces. A bag is rejected if it weighs less than 13 ounces.
- (a) What is the proportion of bags of cereal that are rejected? What is the probability that a bag weighs between 15 and 16 ounces?





$$P(X < 13) = 0.096$$

$$P(15 < X < 16) = 0.235$$

(b) The company doesn't want to check each bag individually, so it begins weighing cases of 16 bags of cereal, rejecting a case if the mean weight of the bags is less than 13 ounces. Can we apply the CLT to this procedure? What proportion of cases get rejected? Is this procedure fair to the consumer?

We can use the Central Limit Theorem because the population of bag weights is normally distributed.

$$\mu_{ar{x}}=\mu=14.5,$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.15}{\sqrt{16}} = \frac{1.15}{4} = 0.2875$$

$$f(x)$$

$$1.4$$

$$1.2$$

$$1$$

$$0.8$$

$$0.6$$

$$0.4$$

$$0.2$$

$$0$$
Normal Distribution
$$\text{Mean:} 14.5 \text{ Std. Dev.:} 0.2875$$

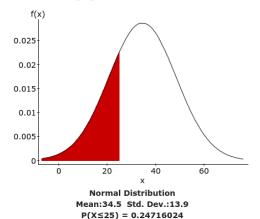
$$P(x \le 13) = 9e-8$$

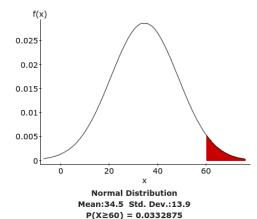
$$P(\bar{x} < 13) = 0.9 \times 10^{-8}$$

This method would not be fair because it is possible to get a case which is accepted with one or more boxes that are under weight.

2

- **3** A certain midwestern city has a population with ages that are normally distributed with a mean of 34.5 years and a standard deviation of 13.9 years.
- (a) What is the probability of a randomly selected resident being younger that 25 years old? What proportion of the population is over 60?





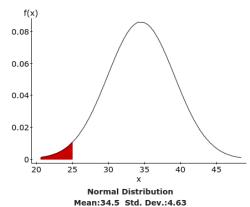
$$P(X < 25) = 0.247$$

$$P(X > 60) = 0.033$$

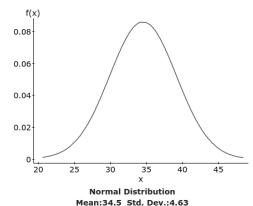
(b) A marketing research firm wants to study characteristics of residents of the city. So they randomly select 9 residents to complete a survey. Is the CLT theorem applicable here? What is the probability the sample is "young" (mean age 25 or under)? What is the probability to mean age is over 60?

We can use the Central Limit Theorem because the population of ages is normally distributed.

$$\mu_{ar{x}} = \mu = 34.5, \qquad \sigma_{ar{x}} = rac{\sigma}{\sqrt{n}} = rac{13.9}{\sqrt{9}} = rac{13.9}{3} = 4.63$$



 $P(X \le 25) = 0.02009281$ 



 $P(X \ge 60) = 2e-8$ 

 $P(\bar{x} < 25) = 0.020$ 

$$P(\bar{x} > 60) = 2 \times 10^{-8}$$