

Stat 201: Statistics I

Week 6



June 26, 2017

Chapter 7

Estimates and Samples Sizes

Confidence intervals

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Recall, sample statistics can be used as estimators for population parameters. However, estimators are rarely exactly equal to the parameter.

It is more informative to calculate a **confidence interval**, a range of values that is likely to contain the value of the parameter.

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The **point estimate** is the value the estimator, the sample statistic used to estimate the population parameter. For example, \bar{x} .

The **margin of error** is the amount the lower and upper bounds of the interval differ from the point estimate.

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The confidence level is the probability that a confidence interval constructed from a random sample actually contains the true population parameter.

- Expressed as a percent, in terms of α as $(1 - \alpha)\%$
- It would be incorrect to say: “There is a 95% chance the true parameter is in the interval.”
- Rather: “We are 95% confident that the interval contains the true parameter.”
- Or: “When constructing intervals from random samples with this method, 95% of the time the interval will contain the true parameter.”

Margin of error

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$$ME = z_{\alpha/2} \left(\frac{s}{\sqrt{n}} \right)$$

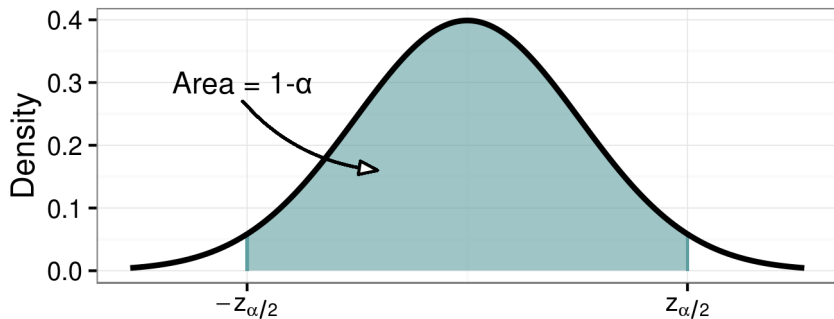
Where...

- $z_{\alpha/2}$ is the two sided critical z value at α level of significance
- s is the sample standard deviation
- n is the sample size

Critical values

Recall, for a significance level α , the critical values $z_{\alpha/2}$ and $-z_{\alpha/2}$ separate the bulk of the distribution from the lowest and highest values comprising a total proportion of α of the distribution.

Thus, between the critical values is an area or probability of $(1 - \alpha)$.



Standard normal critical values

Sig. Level (α)	Conf. Level ($1 - \alpha$)	Critical Value ($z_{\alpha/2}$)
0.10	90%	1.645
0.05	95%	1.96
0.01	99%	2.576

Confidence interval definition

A confidence interval at confidence level $(1 - \alpha)\%$, given a sample of size n with point estimate x and standard deviation s , is

$$CI(1 - \alpha)\% = x \pm ME = x \pm z_{\alpha/2} \left(\frac{s}{\sqrt{n}} \right)$$

or

$$\left(x - z_{\alpha/2} \left(\frac{s}{\sqrt{n}} \right), x + z_{\alpha/2} \left(\frac{s}{\sqrt{n}} \right) \right)$$

Inference using confidence intervals

If a confidence interval does not contain a value of interest, then it can be said there is evidence that the sample was drawn from a population whose parameter is different than the value of interest.

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Example

Recall, in the United States, adult men have a mean height of 69.2 in with a standard deviation of 5.79 in.

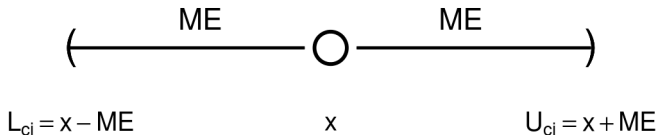
If a confidence interval for mean height calculated from a sample of male Metro State students is $(62.3, 67.9)$, then there is evidence that male Metro State students are shorter than the general US population.

Find point estimate and margin of error

Given a confidence interval (L_{ci}, U_{ci}) , the point estimate and margin of error can be calculated.

$$\text{Point estimate: } x = \frac{L_{ci} + U_{ci}}{2}$$

$$\text{Margin of error: } ME = \frac{U_{ci} - L_{ci}}{2}$$



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Using the previous definition of margin of error,

$$ME = z_{\alpha/2} \left(\frac{s}{\sqrt{n}} \right)$$

after some algebra,

$$n = \left(\frac{s \times z_{\alpha/2}}{ME} \right)^2$$

Section 7.2

Estimating a Population Proportion

Population proportions

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- The variance of \hat{p} is $s^2 = \hat{p}(1 - \hat{p}) = \hat{p}\hat{q}$
- The standard deviation of \hat{p} is $s = \sqrt{\hat{p}\hat{q}}$

Confidence intervals of proportions

A confidence interval of a population proportion with confidence level $(1 - \alpha)\%$ from a sample of size n and sample proportion \hat{p} is

$$CI = \hat{p} \pm ME = \hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

Confidence intervals of proportions, example

Example

Suppose 100 Metro State students were asked if they had eaten a taco in the past week. 36 students responded they had in fact eaten a taco. What is a 95% confidence interval for the proportion of all Metro State students who have eaten a taco in the past week.

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- $CI = \hat{p} \pm ME = 0.36 \pm 0.094 = (0.266, 0.454)$

We are 95% confident that the true proportion of Metro State students who have eaten a taco in the last week is between 0.266 and 0.454.

Confidence intervals of proportions in StatCrunch

See “STATCRUNCH Tutorial for confidence intervals”

- Stat → Proportion Stats → One Sample → With Summary
- Enter “# of successes” (36) and “# of observations” (100)
- Select “Confidence interval for p”
- Enter confidence level if different than 0.95.
- Click “Compute!”
- The confidence interval is found in “L. Limit” and “U. Limit”

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If the known parameter value (the proportion of the larger population, or the proportion before the intervention) **is not** contained in the confidence interval, it can be said there is evidence of a change.

If the known value **is** within the interval, then there is not evidence of a difference.

Inference of population proportion, example

Example

It is thought that 30% of people will have eaten at least one taco in a given week. The Tortilla And Cheese Organization (TACO) would like to increase that. After an intensive taco promotion campaign, they survey a random sample of 55 Metro State students. 38% of them report eating a taco in the previous week. Was the campaign successful? (Test at a 95% confidence level.)

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- Number of successes: $(0.38) \times 55 = 20.9 \approx 21$
- Confidence interval (from StatCrunch): $(0.253, 0.510)$
- The interval contains 30% (0.3). Thus, there is no evidence the taco promotion campaign was successful.

Point estimate and margin of error, example

Example

The survey to test the effectiveness of the taco promotion campaign found a confidence interval of $(0.253, 0.510)$.

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What was the point estimate and margin of error for this confidence interval?

- Point estimate: $\hat{p} = \frac{L_{ci} + U_{ci}}{2} = \frac{0.253 + 0.510}{2} = 0.3815$
- Margin of error: $ME = \frac{U_{ci} - L_{ci}}{2} = \frac{0.510 - 0.253}{2} = 0.1285$

Find needed sample size

The minimum sample size needed to obtain a specific margin of error is calculated by,

$$n = \left(\frac{z_{\alpha/2} \times s}{ME} \right)^2 \quad \text{where} \quad s = \sqrt{\hat{p}\hat{q}}$$

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Then,

$$n = \left(\frac{z_{\alpha/2} \times \sqrt{\hat{p}\hat{q}}}{ME} \right)^2$$

Find needed sample size, example

Example

TACO, disappointed by the large confidence interval of their first survey, decide to do another. This time they wish to get an estimate with a 4% margin of error with 95% confidence level. That is, they want a confidence interval of $\hat{p} \pm 0.04$.

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- The pre-intervention population proportion of 30% can be used as \hat{p} for this calculation. Then, $\hat{p} = 0.3$ and $\hat{q} = 0.7$.

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What is wrong with this plan?

- Retesting, drawing another sample, changes the probabilities of the outcome. The inference is no longer valid.

Find needed sample size with StatCrunch

See “STATCRUNCH Tutorial for sample size”

- Stat → Proportion Stats → One Sample → Width/Sample Size
- Enter “Confidence level” if different than 0.95.
- Enter estimated \hat{p} as “Target Proportion” (0.3)
- Enter twice desired margin of error as “Width” (0.08)
- Click “Compute!”
- The needed sample size is found in “Sample size”

Section 7.3

Estimating a Population Mean

Population means

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Under those conditions, the point estimate for the population mean is the sample mean and a confidence interval can be constructed.

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Recall, a sampling distribution of sample means is normal if the population is normally distributed or, by the Central Limit Theorem, approximately normal if the sample size is 30 or greater.

Under those conditions, the point estimate for the population mean is the sample mean and a confidence interval can be constructed.

However, there is one more factor to consider...

Population standard deviation

If the population standard deviation σ is known, confidence intervals are calculated with critical values from the standard normal distribution and the population standard deviation. That is,

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If the population standard deviation σ is not known, the sample standard deviation is used and critical values are pulled from *Student's t distribution*.

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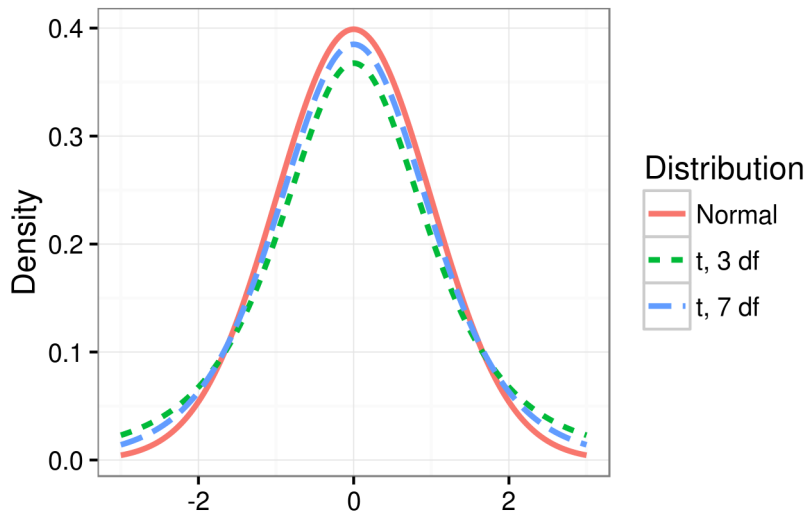
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- The degrees of freedom for a sampling distribution is defined as sample size minus one ($df = n - 1$).

Student's t distribution

Student's t distribution is similar to a normal distribution, except with an adjusted shape to account for an estimated standard deviation.

- The t distribution has an added parameter known as the degrees of freedom (df).
- The degrees of freedom for a sampling distribution is defined as sample size minus one ($df = n - 1$).
- As degrees of freedom increases, the t distribution approaches a normal distribution.

Student's t distribution



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Critical values from t distributions can be found in tables (Table A-3) or in the StatCrunch “T” calculator.

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Then, confidence interval can be calculated by,

$$CI = \bar{x} \pm t_{\alpha/2, df} \left(\frac{s}{\sqrt{n}} \right)$$

where $t_{\alpha/2, df}$ is the critical t value at $\alpha/2$ and $df = n - 1$ and s is the sample standard deviation.

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Otherwise, if population is not normally distributed and sample size is less than 30, valid confidence intervals can not be constructed using these methods.

Confidence intervals for means, example

Example

TACO would like to know, among people who eat tacos, how many tacos per week they eat. They survey 36 taco eaters and get a sample mean of 5.4 tacos per week with a standard deviation of 2.7. What is a 90% confidence interval for the population mean number of tacos eaten?

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- It is unknown if number of tacos eaten is normally distributed (probably not), but our sample size is above 30, so we can treat the sampling distribution as normal.
- Since population standard deviation is unknown, we will use t distribution critical values with degrees of freedom of $n - 1 = 35$, $t_{\alpha/2, df} = t_{0.05, 35} = 1.69$.

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- $CI = \bar{x} \pm t_{\alpha/2} \left(\frac{s}{\sqrt{n}} \right) = 5.4 \pm (1.69) \left(\frac{2.7}{\sqrt{36}} \right) = 5.4 \pm 0.76$

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(4.64, 6.16)

Confidence intervals for means, example

Example

Recall, in the United States adult men have a mean height of 69.2 inches with a standard deviation of 5.79 inches.

The heights from a sample of 40 male Metro State students are measured. The mean height from the sample is 66.3 inches. What is a 95% confidence interval for the mean height of Metro State students?

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- We can assume that the heights of Metro State students have the same standard deviation as the general US population, $\sigma = 5.79$.

Confidence intervals for means, example

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(64.506, 68.094)

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With a confidence interval of $(64.51, 68.01)$, can we conclude that the heights of Metro State students are different than the general US population?

- Since the US mean height of 69.2 inches is not in our interval, we can conclude (with 95% certainty) that the heights of Metro State students differ from the general population.

Confidence intervals of means in StatCrunch

See “STATCRUNCH Tutorial for confidence intervals”

- Stat → Z Stats **or** T Stats → One Sample → With Summary
- Enter “Sample mean” (66.3), “Standard deviation” (if known) **or** “Sample std. dev.” (5.79) and “Sample Size” (40) (not degrees of freedom)
- Select “Confidence interval for μ ”
- Enter confidence level if different than 0.95.
- Click “Compute!”
- The confidence interval is found in “L. Limit” and “U. Limit”

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- Because t distribution values depend of sample size, they are difficult to work with while calculating sample size. Thus, z critical values are generally used.

Find needed sample size with StatCrunch

Example

Recall, in the United States, adult women have a mean height of 63.7 in with a standard deviation of 5.96 in. If we wanted to find the mean height of female students at Metro State within plus or minus 1.5 inches at a 99% confidence level, how many students would need to be included in the sample?

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- A 99% level of confidence means that $\alpha = 0.01$.
- $$n = \left(\frac{z_{\alpha/2} \times \sigma}{ME} \right)^2 = \left(\frac{2.576 \times 5.96}{1.5} \right)^2 = 104.76 \rightarrow 105$$

Find needed sample size with StatCrunch

See “STATCRUNCH Tutorial for sample size”

- Stat → Z Stats → One Sample → Width/Sample Size
- Enter “Confidence level” if different than 0.95.
- Enter estimated or given standard deviation as “Std. dev.” (5.96)
- Enter twice desired margin of error as “Width” (3.0)
- Click “Compute!”
- The needed sample size is found in “Sample size”