Stat 201: Statistics I
Week 6

Week 6
Binomial and Normal Distributions

Section 6.1
Binomial Probability Distributions

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    - Passing the midterm is a "success", failing it is a "failure"

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- The number of heads in three coin flips
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- The number of of car crashes that result in fatalities
  - A fatality is a "success", no fatalities is a "failure"

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The last two requirements are often summarized as “independent and identically distributed” and abbreviated as “iid”.

Requirements for binomial distributions, example

Example

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- No. The probability of success changes (hopefully) with each test.

Requirements for binomial distributions, example

Example

Recall the Youth Risk Behavior Survey (YRBS) which found the probability of a teenaged driver had texted or emailed while driving was 0.404. Suppose this is the probability for the population of teenaged drivers. Suppose 30 teenaged drivers are selected at random. Does the number of those drivers that had texted or emailed while driving follow a binomial distribution?

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- Yes.
    - There is a fixed number of trials (30).
    - Each trial has only two possible outcomes (had or had not texted).
    - Each trial is independent.
    - Each trial has the same probability of "success."

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The probability a teenaged driver had texted or emailed while driving is 0.404. If the random variable Y is the number of teenaged drivers who had texted or emailed while driving out of a sample of 30,

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- $p^x q^{n-x}$ is the probability of getting x successes and $n - x$ failures in one particular order.

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- $P(X = 5) = 0.165$

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- Standard deviation:

$$\sigma = \sqrt{\sigma^2} = \sqrt{npq}$$

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- $\sigma = \sqrt{\sigma^2} = \sqrt{7.22} = 2.69$

Unusual values

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- The lower bound for unusual values is $\mu - 2\sigma = 6.74$
- The upper bound for unusual values is $\mu + 2\sigma = 17.5$
- In a random sample of 30 teenaged drivers, it would be unusual to get 6 or fewer, or 18 or more, drivers who had texted or emailed while driving.

Group work

- Work on question 1, all parts.

Section 6.2
Normal Distributions

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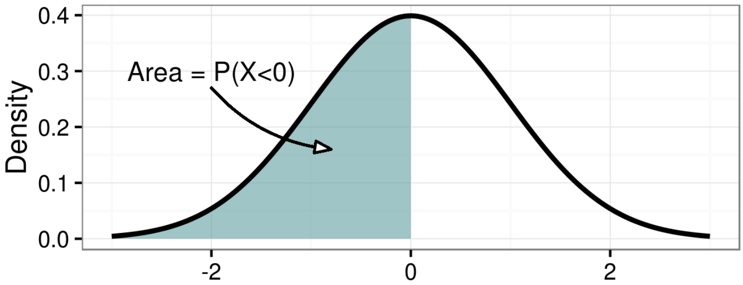
- Only probabilities of ranges of values have meaning.

Density curves

A continuous probability distribution is visualized by a **density curve**, a graph of the probability density function.

- The total area under the graph is always 1.
- Probabilities are defined as the area under the curve for the range of values of the random variable.



Normal distributions

Recall, normal distributions were defined as having a particular shape.

- Start with low values, rise to a maximum value, and end with low values.
- Distribution is symmetric (mirror image) around maximum.
- "Bell curve"

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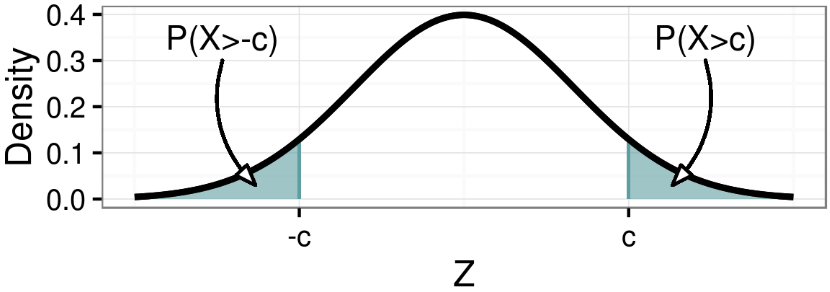
- $P(a < x < b \mid \mu, \sigma) = \dfrac{1}{\sqrt{2\pi}\sigma} \displaystyle\int_a^b e^{-\frac{(x-\mu)^2}{2\sigma}}\, dx$

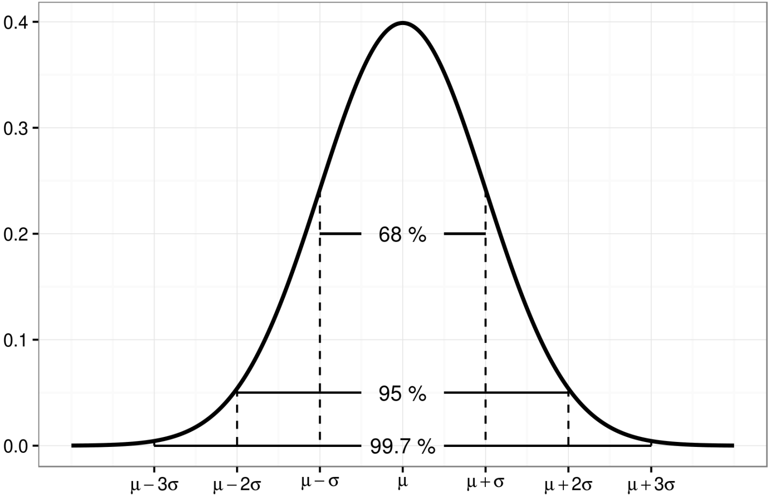
Symmetry of normal distribution

Normal distributions are perfectly symmetrical, mathematically speaking. That means, the probability a value is greater than some number is equal to the probability of being below the negative of that number.

- $P(X > c) = P(X < -c)$



Distribution of normal distributions



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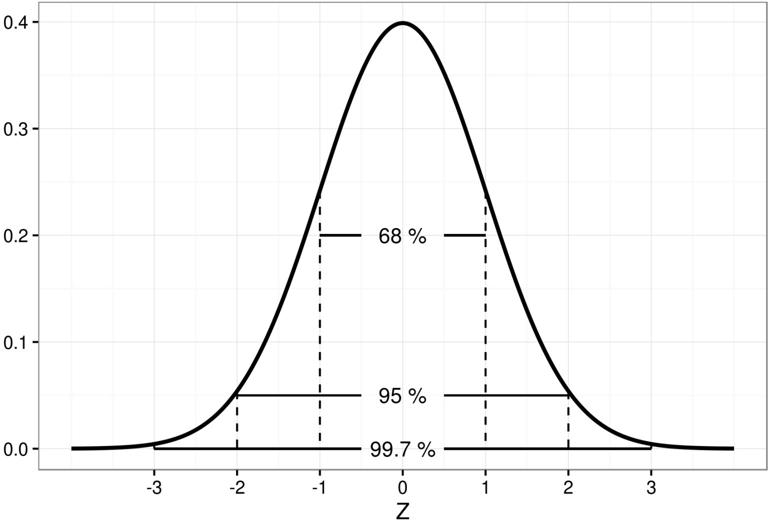
- Values of the standard normal are known as z-scores.
- A z-score of 1 ($z = 1$) is one standard deviation above the mean, $z = -2$ is two standard deviations below the mean, etc.

Z distribution



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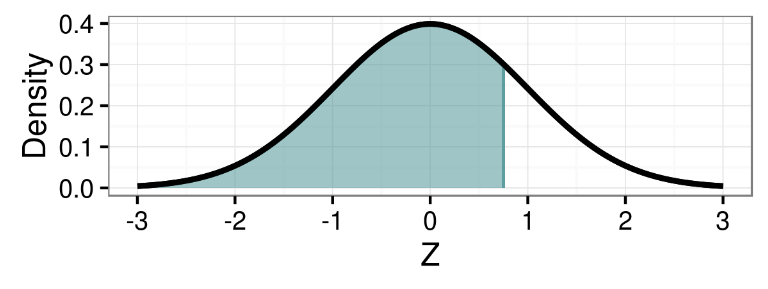
However, using technology is usually quicker and more accurate.

Probabilities, example

Example

Using the standard normal distribution, find the probability a value is less than 0.75 standard deviations above the mean, $P(Z < 0.75)$



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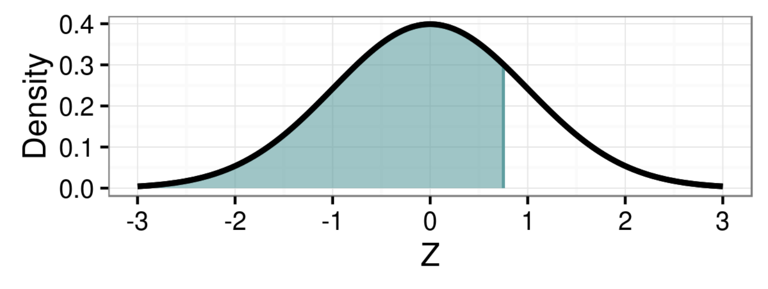
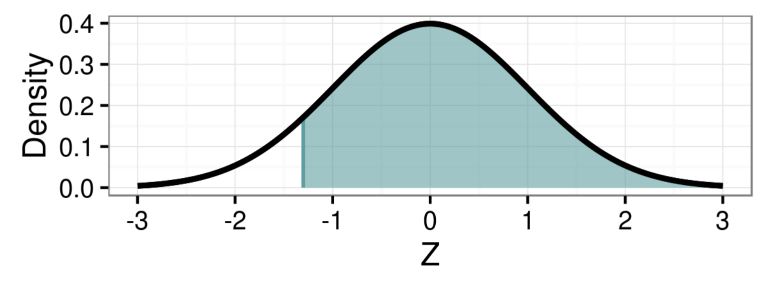


- $P(Z < .75) = \mathbf{0.773}$

Probabilities, example

Example

Using the standard normal distribution, find the probability a value is greater than 1.3 standard deviations below the mean, $P(Z > -1.3)$



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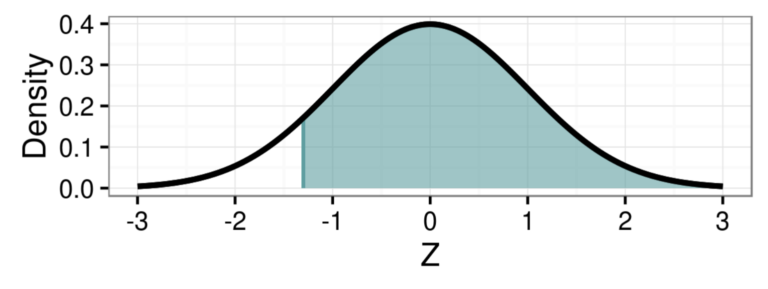
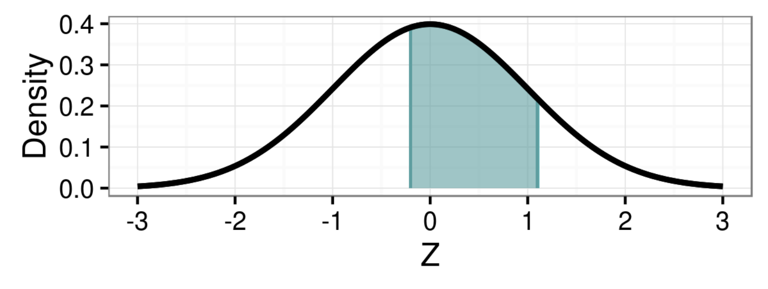


- $P(Z > -1.3) = \mathbf{0.903}$

Probabilities, example

Example

Using the standard normal distribution, find the probability a value is between -0.2 and 1.1 standard deviations, $P(-0.2 < Z < 1.1)$



Probabilities, example

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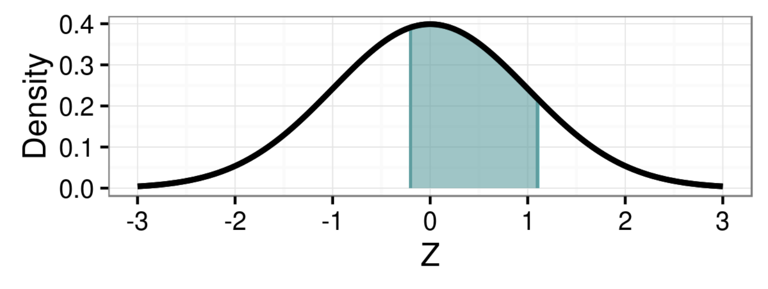


- $P(-0.2 < Z < 1.1) = \mathbf{0.444}$

Finding percentiles

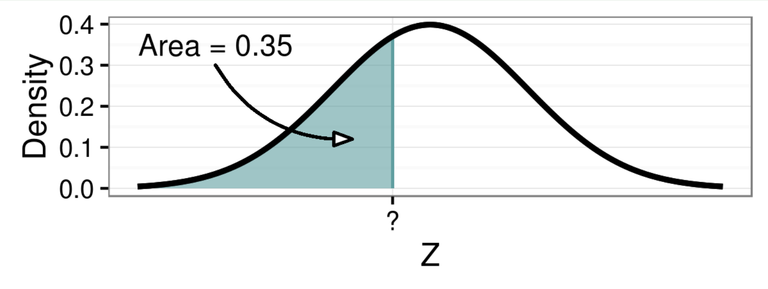
Often it is desirable to find a z-score that is greater than a specified probability, in other words, a percentile. This can be accomplished with the table by locating the desired probability and finding the corresponding z-score.

Again, technology provides an easier and more accurate method.

Finding percentiles, example

Example

What is the z-score greater than 35% of values? What is P_{35}? For what z-score is there a 0.35 probability of being less than $P(Z < z) = 0.35$?



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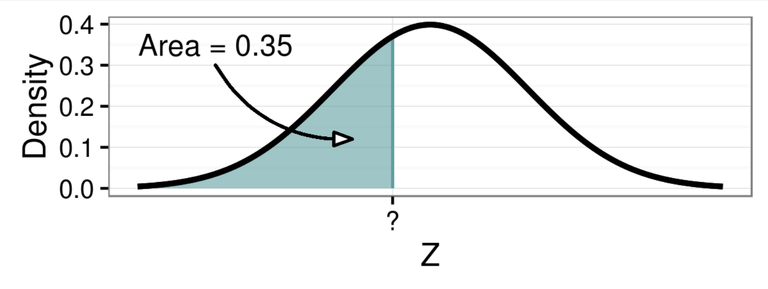
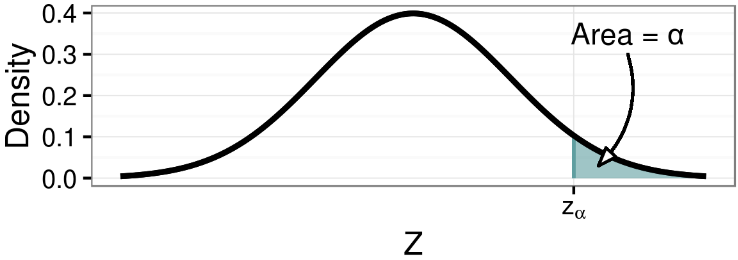


- $P(Z < \mathbf{-0.385}) = 0.35$

Critical values

In a standard normal distribution, the z-score separating usual outcomes from unusual outcomes is known as a **critical value**.

- The probability denoting unusual events is designated with α (alpha).
- Then z_α is the critical value such that $P(Z > z_\alpha) = \alpha$



Critical values, example

Example

Let $\alpha = 0.05$.

Find the critical value for α. That is, find z_α or $z_{0.05}$.

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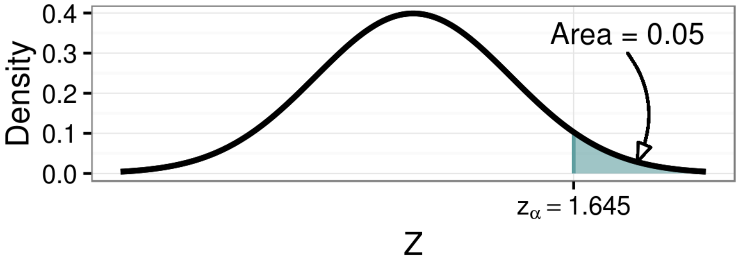
- $z_\alpha = 1.645$
- $P(Z > z_\alpha) = \alpha$ or $P(Z < -z_\alpha) = \alpha$



Critical values, example

Example

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Find the critical value for $\alpha/2$. That is, find $z_{\alpha/2}$ or $z_{0.025}$.

Critical values, example

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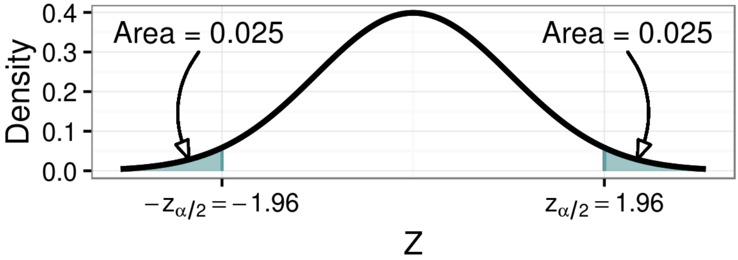
Find the critical value for $\alpha/2$. That is, find $z_{\alpha/2}$ or $z_{0.025}$.

- $z_{\alpha/2} = 1.96$
- $P(Z < -z_{\alpha/2}) + P(Z > z_{\alpha/2}) = \alpha$



Non-standard normal distributions

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- However, it is still useful to use z-scores for comparing values from different distributions.
- Recall, to convert a value x from a non-standard normal distribution to a z-score, and vice versa,

$$z = \frac{x - \mu}{\sigma} \qquad \text{and} \qquad x = \mu + z\sigma$$

Non-standard normal distributions, example

Example

An amusement park has made safety their highest priority. They design all their rides to have zero chance of causing serious head trauma, as long as the rider is under 78 inches tall. What proportion of men are in danger at this park?

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- $P(X_m > 78) = ?$

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- $X_m \sim N(69.2, 5.79)$
- $P(X_m > 78) = \mathbf{?}$
- $P(X_m > 78) = \mathbf{0.064}$

Non-standard normal distributions, example

Example

The amusement park is designing a new ride. It wants make sure that 85% of adult women can ride it safely. What is the maximum height the ride should be designed for?

Non-standard normal distributions, example

Example

The amusement park is designing a new ride. It wants make sure that 85% of adult women can ride it safely. What is the maximum height the ride should be designed for?

- $X_f \sim N(63.7, 5.96)$

Non-standard normal distributions, example

Example

The amusement park is designing a new ride. It wants make sure that 85% of adult women can ride it safely. What is the maximum height the ride should be designed for?

- $X_f \sim N(63.7, 5.96)$
- $P(X_f < \textbf{?}) = 0.85$

Non-standard normal distributions, example

Example

The amusement park is designing a new ride. It wants make sure that 85% of adult women can ride it safely. What is the maximum height the ride should be designed for?

- $X_f \sim N(63.7, 5.96)$
- $P(X_f < \mathbf{?}) = 0.85$
- $P(X_f < \mathbf{69.88}) = 0.85$

Non-standard normal distributions, example

Example

The amusement park is growing weary of accommodating the very tall and the very short. It has decided to exclude the most extreme heights among adult men. But it doesn't want to lose to much business, so it will only exclude 5% of the adult male population. What are the critical values for the tallest and shortest men, for a total of 5%?

Non-standard normal distributions, example

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- $X_m \sim N(69.2, 5.79), \qquad \alpha = 0.05$

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- $X_m \sim N(69.2, 5.79), \qquad \alpha = 0.05$
- $z_{\alpha/2} = z_{0.025} = 1.96$

Non-standard normal distributions, example

Example

The amusement park is growing weary of accommodating the very tall and the very short. It has decided to exclude the most extreme heights among adult men. But it doesn't want to lose to much business, so it will only exclude 5% of the adult male population. What are the critical values for the tallest and shortest men, for a total of 5%?

- $X_m \sim N(69.2, 5.79), \qquad \alpha = 0.05$
- $z_{\alpha/2} = z_{0.025} = 1.96$
- $P(X_m < \mathbf{57.85}) = 0.025, \qquad P(X_m > \mathbf{80.55}) = 0.025$

Group work

- Complete question 2.