Stat 201: Statistics I
Week 3

Metropolitan State University

Week 3
More Probability, Sampling methods and Types of Studies

Section 3.1
Conditional Probability

Formal definition of conditional probability

Recall the multiplication rule,

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From this, the formal definition of conditional probability is

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

Intuitive approach to conditional probability

An intuitive approach to $P(B \mid A)$ is to assume A has occurred, then count the instances of B. A is, in a sense, the new sample space.

$$P(B \mid A) = \frac{\text{number of } B \text{ and } A}{\text{number of } A}$$

Practice: Cancer screening

	Positive	Negative	Total
Cancer	74 (0.074)	13 (0.013)	87 (0.087)
No cancer	26 (0.026)	887 (0.887)	913 (0.913)

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Many diagnostic tests work by measuring the level of a certain chemical and returning a positive result if it is above a designated threshold. Adjusting this threshold to increase sensitivity will decrease specificity, and vice versa. There is always a trade-off.

Sensitivity and specificity, examples

Example

Screening tests for prostate cancer measure levels of Prostate Specific Antigen (PSA). The sensitivity and specificity of the test depends on the cutoff point used.

	< 4.0 ng/mL	< 3.0 ng/mL
Sensitivity (%)	21	32
Specificity (%)	91	85

Sensitivity and specificity, examples

Example

Accuracy of tests often depend also on the population being screened. The sensitivity of mammograms is different for different age groups.

	40-49 years	50-59 years
Sensitivity (%)	77	88

Screening tests for rare events

Example

Suppose there is a screening test for a rare disease which has a prevalence of 0.3%. The screening test has 99% sensitivity and 99% specificity. 100,000 people are screened.

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Total	1294	98706	100,000

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- The complement, $P(\text{disease} \mid \text{positive}) = 0.23$, is known as the **precision** or the **positive predictive value (PPV)** of the test.

Screening tests for rare events, cont.

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  - Drug screening for jobs
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  - etc.
- It is important to remember that no test is perfect and there are often trade-offs (sensitivity / specificity).

Group work

- Complete all parts of question 1.
- Probabilities can be expressed as fractions.

Section 3.2
Sampling Methods and Types of Studies

Samples

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Example

- Suppose an organization is interesting in the taco consumption by Metro State students. It would be difficult, if not impossible, to ask every student about their taco eating habits. A sample is needed.

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Example

- Given an alphabetical list of students, use a random number generator to select a sample.

Types of samples: Systematic sampling

Systematic sampling is a method where every kth member of of a population is selected.

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Example

- Given an alphabetical list of students, select every fifth student until you have a sample of the desired size.

Types of samples: Convenience sampling

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- The easiest of all methods, but by far the lowest quality data for producing results.
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Example

- Wander the halls before class, asking students who happen to walk by.
- Put a poll on the Metro State website.
- Everyone who is diagnosed with a rare disease at a particular clinic.

Types of samples: Stratified sampling

Stratified sampling is a method where the population is divided into groups and samples are selected from each group.

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Example

- If we have particular interest in the taco consuming difference between graduate students and undergrads, select a sample from each group.

Types of samples: Cluster sampling

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Example

- Choose 5 random classes, and survey all the students in those classes.

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Example

- Choose random classes by cluster sampling, and then take a simple random sample of students from each chosen class.

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In an **experiment** a change (treatment) is made to some or all of sample and then data is collected in order to detect changes.

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A **prospective** study follows subjects into the future to measure and collect data.

- Also know as: longitudinal study, cohort study

Experimental design: Controlling

An experiment is **controlled** when at least one group of subjects are not given any experimental treatments. The control group might receive no treatments, a placebo treatment (see blinding) or a standard-of-care treatment. Controlling an experiment allows a direct measurement of any possible treatment effects.

Example

The World Health Organization says the average case fatality rate for Ebola virus disease (EVD) is 50%, with fatality rates of individual breakouts ranging from 25% to 90%.

PREVAIL II, a controlled trial of a new drug cocktail for EVD, found a fatality rate in the control group was 37% and 22% in the treatment group.

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The **placebo effect** is a phenomenon where people who believe they are being treated demonstrate improvement.

Experiment design: Replication

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- Experimental studies should have adequate sample sizes to ensure that observed effects are "true" effects and not due to individual characteristics or chance.
- Experimental studies should be, but rarely are, repeated by different researchers to verify results.

Experimental design: Randomization

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Confounding variables (or just confounders) are unmeasured and possible unknown factors that affect the experimental outcome.

Group work

- Answer all parts of question 2.