Homework - Week 6

Your name here

Questions marked with "(OS3: X.X)" are from the textbook with "X.X" as the exercise number. The answers to the odd questions (odd by book numbering that is) will be in the back of the book.

1. (OS3: 3.25) Data collected by the Substance Abuse and Mental Health Services Administration (SAMSHA) suggests that 69.7% of 18-20 year olds consumed alcoholic beverages in 2008.

    a. Suppose a random sample of ten 18-20 year olds is taken. Is the use of the binomial distribution appropriate for calculating the probability that exactly six consumed alcoholic beverages? Explain.

    b. Calculate the probability that exactly 6 out of 10 randomly sampled 18- 20 year olds consumed an alcoholic drink.

    c. What is the probability that exactly four out of ten 18-20 year olds have *not* consumed an alcoholic beverage?

    d. What is the probability that at most 2 out of 5 randomly sampled 18-20 year olds have consumed alcoholic beverages?

    e. What is the probability that at least 1 out of 5 randomly sampled 18-20 year olds have consumed alcoholic beverages?

2. (OS3: 3.26) The National Vaccine Information Center estimates that 90% of Americans have had chickenpox by the time they reach adulthood.

    a. Is the use of the binomial distribution appropriate for calculating the probability that exactly 97 out of 100 randomly sampled American adults had chickenpox during childhood.

    b. Calculate the probability that exactly 97 out of 100 randomly sampled American adults had chickenpox during childhood.

    c. What is the probability that exactly 3 out of a new sample of 100 American adults have *not* had chickenpox in their childhood?

    d. What is the probability that at least 1 out of 10 randomly sampled American adults have had chickenpox?

    e. What is the probability that at most 3 out of 10 randomly sampled American adults have *not* had chickenpox?

3. (OS3: 3.27) We learned in Exercise 1 that about 70% of 18-20 year olds consumed alcoholic beverages in 2008. We now consider a random sample of fifty 18-20 year olds.

    a. How many people would you expect to have consumed alcoholic beverages? And with what standard deviation?

    b. Would you be surprised if there were 45 or more people who have consumed alcoholic beverages?

    c. What is the probability that 45 or more people in this sample have consumed alcoholic beverages? How does this probability relate to your answer to part (b)?

4. (OS3: 3.32) A 2005 Gallup Poll found that 7% of teenagers (ages 13 to 17) suffer from arachnophobia and are extremely afraid of spiders. At a summer camp there are 10 teenagers sleeping in each tent. Assume that these 10 teenagers are independent of each other.

    a. Calculate the probability that at least one of them suffers from arachnophobia.

    b. Calculate the probability that exactly 2 of them suffer from arachnophobia.

    c. Calculate the probability that at most 1 of them suffers from arachnophobia.

d. If the camp counselor wants to make sure no more than 1 teenager in each tent is afraid of spiders, does it seem reasonable for him to randomly assign teenagers to tents?

5. (OS3: 3.34) Sickle cell anemia is a genetic blood disorder where red blood cells lose their flexibility and assume an abnormal, rigid, "sickle" shape, which results in a risk of various complications. If both parents are carriers of the disease, then a child has a 25% chance of having the disease, 50% chance of being a carrier, and 25% chance of neither having the disease nor being a carrier. If two parents who are carriers of the disease have 3 children, what is the probability that

   a. two will have the disease?

   b. none will have the disease?

   c. at least one will neither have the disease nor be a carrier?

   d. the first child with the disease will the be 3rd child?

6. (OS3: 3.3 a-g) Sophia who took the Graduate Record Examination (GRE) scored 160 on the Verbal Reasoning section and 157 on the Quantitative Reasoning section. The mean score for Verbal Reasoning section for all test takers was 151 with a standard deviation of 7, and the mean score for the Quantitative Reasoning was 153 with a standard deviation of 7.67. Suppose that both distributions are nearly normal.

   a. Write down the short-hand for these two normal distributions.

   b. What is Sophia's Z-score on the Verbal Reasoning section? On the Quantitative Reasoning section? Draw a standard normal distribution curve and mark these two Z-scores.

   c. What do these Z-scores tell you?

   d. Relative to others, which section did she do better on?

   e. Find her percentile scores for the two exams.

   f. What percent of the test takers did better than her on the Verbal Reasoning section? On the Quantitative Reasoning section?

   g. Explain why simply comparing raw scores from the two sections could lead to an incorrect conclusion as to which section a student did better on.

7. (OS3: 3.4 a-e) In triathlons, it is common for racers to be placed into age and gender groups. Friends Leo and Mary both completed the Hermosa Beach Triathlon, where Leo competed in the *Men, Ages 30 - 34* group while Mary competed in the *Women, Ages 25 - 29* group. Leo completed the race in 1:22:28 (4948 seconds), while Mary completed the race in 1:31:53 (5513 seconds). Obviously Leo finished faster, but they are curious about how they did within their respective groups. Can you help them? Here is some information on the performance of their groups:

   - The finishing times of the *Men, Ages 30 - 34* group has a mean of 4313 seconds with a standard deviation of 583 seconds.

   - The finishing times of the *Women, Ages 25 - 29* group has a mean of 5261 seconds with a standard deviation of 807 seconds.

   - The distributions of finishing times for both groups are approximately Normal.

   Remember: a better performance corresponds to a faster finish.

   a. Write down the short-hand for these two normal distributions.

   b. What are the Z-scores for Leo's and Mary's finishing times? What do these Z-scores tell you?

   c. Did Leo or Mary rank better in their respective groups? Explain your reasoning.

   d. What percent of the triathletes did Leo finish faster than in his group?

e. What percent of the triathletes did Mary finish faster than in her group?

8. (OS3: 3.5) In Exercise 6 we saw two distributions for GRE scores: $N(\mu = 151, \sigma = 7)$ for the verbal part of the exam and $N(\mu = 153, \sigma = 7.67)$ for the quantitative part. Use this information to compute each of the following:

   a. The score of a student who scored in the 80th percentile on the Quantitative Reasoning section.

   b. The score of a student who scored worse than 70% of the test takers in the Verbal Reasoning section.

9. (OS3: 3.10) Heights of 10 year olds, regardless of gender, closely follow a normal distribution with mean 55 inches and standard deviation 6 inches.

   a. What is the probability that a randomly chosen 10 year old is shorter than 48 inches?

   b. What is the probability that a randomly chosen 10 year old is between 60 and 65 inches?

   c. If the tallest 10% of the class is considered `very tall", what is  the height cutoff for`very tall"?

   d. The height requirement for *Batman the Ride* at Six Flags Magic Mountain is 54 inches. What percent of 10 year olds cannot go on this ride?