1. Blood testing Suppose a blood test for a disease is given to a population of N people,  
   where N is large. At most, N individual blood tests must be done. The following  
   strategy reduces the number of tests. Suppose 100 people are selected from the  
   population and their blood samples are pooled. One test determines whether any of the  
   100 people test positive. If that test is positive, those 100 people are tested individually,  
   making 101 tests necessary. However, if the pooled sample tests negative, then 100  
   people have been tested with one test. This procedure is then repeated. Probability  
   theory shows that if the group size is x (for example, x = 100, as described here), then  
   the average number of blood tests required to test N people is N\*(1- 𝑞^𝑥 + 1/𝑥 ), where q is  
   the probability that any one person tests negative. What group size x minimizes the  
   average number of tests in the case that N = 100 and q = 0.95? Assume x is a real  
   number between 1 and 150 in Excel or Python program.

Ans:

A screenshot of a computer program

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2. Modified Newton’s method The function ƒ has a root of multiplicity2 at r if 𝑓(𝑟) = 𝑓′

(𝑟) = 0 and 𝑓"(𝑟) ≠ 0. In this case, a slight modification of Newton’s method, known

as the modified (or accelerated) Newton’s method, is given by the formula

𝑥𝑛+1 = 𝑥𝑛 ― 2𝑓(𝑥𝑛)/𝑓′(𝑥𝑛) , for 𝑛 = 0, 1, 2, ......

This modified form generally increases the rate of convergence. Please complete the

following questions in Excel or Python program.

a. Verify that 0 is a root of multiplicity 2 of the function

𝑓(𝑥) = 𝑒^2sin (𝑥) ― 2𝑥 ― 1

b. Apply Newton’s method and the modified Newton’s method using 𝑥0 = 0.1 to find

the value of 𝑥9 in each case. Compare the accuracy of these values of 𝑥9 .

c. Consider the function 𝑓(𝑥) = 8𝑥^2/3𝑥^2 +1. Use the modified Newton’s method to find the value of 𝑥9 using 𝑥0 = 0.15. Compare this value to the value of 𝑥9 found

in Newton’s method.

Ans:

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