An Introduction to BRDF Models

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ABSTRACT

This document focuses on Bidirectional Reflectance Distribution Functions (BRDFs) in the context of computer based image synthesis. An introduction to basic radiometry and BRDF theory is given. Several well known BRDF models are reviewed and compared based on their physical plausibility and the types of surfaces that they are suited for.

Keywords

Computer Graphics, Rendering, Radiometry, BRDF Theory, Physically Based Shading

1. INTRODUCTION

One of the primary objectives in computer graphics is the synthesis of images based on digital models. Photorealistic rendering is a subfield in computer graphics that deals with the rendering of objects such that the produced image exhibits a degree of realism. Ideally, a photorealistic image would, for a human observer, be indistinguishable from a photograph of real objects.

Some degree of photorealism can be achieved by producing detailed objects and materials and to use these in a simulation that mimics the way light would naturally interact with the objects. The geometry of the object models and materials primarily determine how they interact with visible light. One option to generate images that exhibit a degree of photorealism is to use lighting models that are physically based. These models are often inspired by the physics of light, and specifically, how light interacts with objects.

The goal of this paper is to introduce readers to the basic concepts that are key in photorealistic computer based image synthesis, to review some well known reflectance models, and to provide an overview of the models that have been covered. In section 2, the field of radiometry is introduced. We define some basic radiometric quantities and concepts that will be used throughout the rest of the paper. Section 3 introduces the rendering equation. We dissect this equation and describe each of its components; one of them being the Bidirectional Reflectance Distribution Function (BRDF). We delve deeper into BRDF theory in section 4 and review several BRDF models in section 5.

A brief comparison between the discussed BRDF models is held in section 6. We summarise the models by describing whether or not they exhibit desirable BRDF properties (which have been defined in section 4). It should be noted that the comparison will make no effort to determine a single optimal model, as all of them have been developed for specific purposes and will excel in different situations. Instead, we describe for what material types the models are well suited and mention important aspects that should be taken into consideration when choosing BRDF models.

2. RADIOMETRY

Radiometry deals with the measurement of electromagnetic radiation which consists of a flow of photons. It provides a set of tools that can be used to describe light propagation and reflection. This section will provide a brief introduction into the field of radiometry and will serve as a foundation for the rest of the document.

Photons exhibit properties of both particles and waves, depending on circumstances. In computer-based rendering, photons can for the most part usually be treated as particles. This does mean that not all physical phenomena can be successfully modelled (e.g. polarisation and diffraction). One of the wave-like properties that cannot be fully ignored is that each photon has a specific wavelength (or equivalently, frequency), which influences the interaction with sensors such as the cones and rods of the human eye. The fields of photometry and colorimetry focus on the physical interactions between photons and the human sensors, and the perception of color. We will not delve into these fields as the physical complexities can for the most part be avoided by encoding color as RGB values.

2.1 Basic Radiometric Quantities

Radiant flux (sometimes also called *radiant power* and is denoted as Φ) is equal to the sum of energy passing through a surface over a period of time. Radiant flux is measured in watts, which is defined as joules per second. The radiant flux of a light source could be measured by summing the energy of all emitted photons in a second.

Irradiance (denoted as E) is the density of *incoming* radiant flux with respect to an area. The irradiance for a point inside a surface with area A is expressed as

$$E = \frac{d\Phi}{dA}. \tag{1}$$

The notion of measuring radiant flux over a surface area can also be extended to *outgoing* radiant flux. This quantity is called *radiant exitance* and is often denoted as M.

Intensity (denoted as I) describes the directional distribution of energy. Its definition includes the concept of a *solid angle*. A solid angle can perhaps most easily be visualised as the extension an an angle to a three-dimensional sphere. Solid angles are measured in steradians. An entire sphere subtends a solid angle of 4π and a hemisphere subtends a solid angle of 2π. Intensity can be defined as the density of radiant flux per solid angle $\boldsymbol{\omega}$

$$I = \frac{d\Phi}{d\boldsymbol{\omega}}. \tag{2}$$

Radiance (denoted as L) combines the ideas of the previously defined quantities and is defined as the radiant flux density with respect to both area and solid angle

$$L = \frac{d^2\Phi}{dA_{\text{proj}}d\boldsymbol{\omega}}, \tag{3}$$

where dA_{proj} represents the projection of dA onto a plane perpendicular the solid angle $\boldsymbol{\omega}$ ($dA_{\text{proj}} = dA|\cos\theta|$). Think of radiance as a measure of energy along a single ray. Radiance is the radiometric quantity that is most used in computer graphics.

3. RENDERING EQUATION

Almost all physically based lighting models are in some way an approximation of what is called the rendering equation. This equation was first presented by Kajiya [5] and is based only on physics; it can be seen as a standard by which all realistic computer graphics are measured. Unfortunately, this equation is difficult to compute as it features an integral over a hemisphere of directions and where L (radiance) appears on both sides of the equation. We present the equation below and will briefly describe each of the terms. Keep in mind that different forms have been used for this equation. This is only one of many, luckily the general idea and structure remains the same. We write the equation in the form

$$\begin{aligned} L_o(\boldsymbol{x}, \boldsymbol{\omega}_o, \lambda) &= L_e(\boldsymbol{x}, \boldsymbol{\omega}_o, \lambda) \\ &\quad + \int_\Omega f(\boldsymbol{x}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \lambda) L_i(\boldsymbol{x}, \boldsymbol{\omega}_i, \lambda) \cos\theta d\boldsymbol{\omega}_i, \end{aligned} \tag{4}$$

where $L_o(\boldsymbol{x}, \boldsymbol{\omega}_o, \lambda)$ is the radiance of wavelength λ leaving the surface point $\boldsymbol{x}$ along direction $\boldsymbol{\omega}_o$. The first term on the right hand side (i.e. $L_e(\boldsymbol{x}, \boldsymbol{\omega}_o, \lambda)$) is the radiance *emitted* from point $\boldsymbol{x}$ along direction $\boldsymbol{\omega}_o$ at wavelength λ. At a high level, we can view the integral as the radiance reflected from point $\boldsymbol{x}$ in the direction of $\boldsymbol{\omega}_o$.

As mentioned before, the rendering equation features an integral over the hemisphere of possible incoming directions ($d\boldsymbol{\omega}_i$). For each of these incoming directions a product of three factors is calculated. The first of these terms, $f(\boldsymbol{x}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o, \lambda)$, is also known as the bidirectional reflectance distribution function. We will go more into details on this in due time. For now, it suffices to know that it is a formalism for describing the reflectivity of a surface. $L_i(\boldsymbol{x}, \boldsymbol{\omega}_i, \lambda)$ denotes the radiance that is incident from the direction $\boldsymbol{\omega}_i$. The final term, which is the cosine of θ, is based on Lambert's Law; it states that – for perfectly diffuse surfaces – the outgoing radiance is proportional to the cosine of the angle θ between the incident radiance and the surface normal.

Now that we have covered all of the components of the rendering equation, we should note that it cannot be directly applied in software systems. The integral cannot be evaluated analytically, which means that it must be approximated by taking discrete samples over the hemisphere of possible incoming directions. Furthermore, the function is recursive; as we have mentioned before, the radiance function appears on both sides of the equation. This means that in order to evaluate the outgoing radiance at point $\boldsymbol{x}$, we require the incident radiance at $\boldsymbol{x}$ from all possible directions. Yet, the radiance incident on point $\boldsymbol{x}$ is equal to the outgoing radiance from all surfaces visible to $\boldsymbol{x}$. Analytically solving this equation is impossible. The integral can however be solved numerically; Monte Carlo integration is a method often used in ray tracing applications. Another option, more suited for real time rendering systems, is to model light sources as points and to only evaluate direct illumination (i.e. energy does not 'bounce' off of objects). Doing so would allow the integral to be substituted by a sum over all light sources in the scene. Doing so results in a performance gain at the expense of realism. Of course, there are many more options for evaluating the rendering equation. However, we will not go into more detail on this as the purpose of this section is only to provide context for the BRDFs that will be examined more in depth.

4. BRDF THEORY

The function that describes how a surface reflects light is called the bidirectional reflectance distribution function (BRDF). The mathematical definition of the BRDF is

$$f(\boldsymbol{x}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \frac{dL_o(\boldsymbol{x}, \boldsymbol{\omega}_o)}{dE(\boldsymbol{x}, \boldsymbol{\omega}_i)}, \tag{5}$$

which is to say that it is the ratio between the differential radiance (outgoing along $\boldsymbol{\omega}_o$) and the differential irradiance at surface point $\boldsymbol{x}$. In layman's terms we might say that the BRDF describes how much of the incoming light (irradiance) is reflected towards direction $\boldsymbol{\omega}_o$. Note that the value of the BRDF is dependent on wavelength (see definition of rendering equation), this means that it is usually represented as an RGB triplet. For a BRDF to be physically plausible, it has to exhibit two properties. These properties will be described in the following paragraphs.

The first property is that a physically plausible BRDF must be energy conserving. The law of conservation of energy dictates that energy does not simply appear or disappear, but that instead, it is transformed. In the context of computer based image synthesis this means that the radiant energy reflected at a surface cannot be more than the energy that is incident to the surface. Formally, this property can be

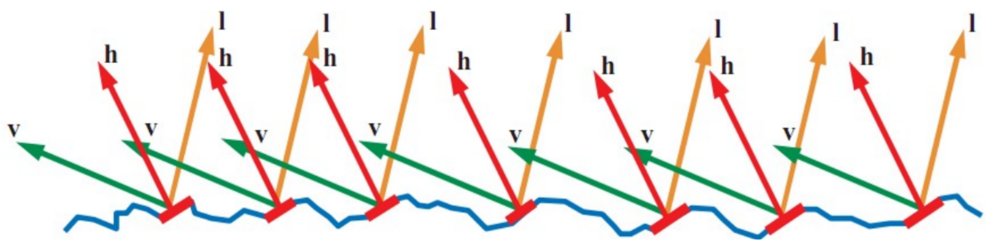

Figure 1: Surface consisting of microfacets. Source: [1]

described by

$$\forall \boldsymbol{\omega}_o, \int_\Omega f(\boldsymbol{x}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o) \cos\theta d\boldsymbol{\omega}_i \geq 1. \quad (6)$$

The second property refers to what is known as *Helmholtz reciprocity*, which is to say that the BRDF must be symmetric. In other words, the reversal of incoming and outgoing light must not affect the BRDF outcome. Formally, this property can be described as $f(\boldsymbol{x}, \boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = f(\boldsymbol{x}, \boldsymbol{\omega}_o, \boldsymbol{\omega}_i)$.

A BRDF is physically plausible when it is both energy conserving and reciprocal. Many BRDFs that are used in practise are not physically plausible however. Real-time rendering systems often violate Helmholtz reciprocity without noticeable artefacts [1]. Depending on circumstances, these properties may be required or not. For example, ray tracing systems require physically plausible BRDFs due to the fact that rays are traced in reversal and that a violation of energy conservation could result in unrealistic shading.

Another property that BRDFs have is whether or not they are *isotropic*. Isotropic BRDFs are used for regular surfaces where there is a rotational symmetry, that is, rotating the view vector around the surface normal does not influence BRDF outcome. The counterpart of isotropic BRDFs are *anisotropic* BRDFs. Anisotropic BRDFs do not exhibit the same rotational symmetry; rotating around the surface normal *does* influence the BRDF outcome. An example of a real-world surface that is often modelled with anisotropic BRDFs is brushed metal.

5. BRDF MODELS

In this section we review several BRDF models that have been proposed in literature. The review will be limited to physically-based BRDF models. Each model will be discussed and we will note how they relate to the properties that have been described in the previous section.

5.1 Cook-Torrance

The model proposed by Cook & Torrance in [3] describes a model that is based on geometrical optics and is applicable to a wide range of materials and lighting situations. The model splits up the BRDF into two components, specular and diffuse. Here the diffuse component models body reflectance and the specular component models surface reflectance. The Cook-Torrance BRDF is of the form

$$f = d f_{\text{diff}} + s f_{\text{spec}}, \quad (7)$$

where d and s are weighting parameters and $d + s = 1$. The diffuse term is independent of viewing direction and is therefore often a constant (bidirectional reflectance at normal incidence).

The specular term is based on microfacet theory, where it is assumed that a surface consists of many small mirror-like surfaces (see Figure 1). Correctly calculating how light interacts with these microfacets is not feasible, instead this is statistically approximated. Microfacets reflect light specularly, which means that the angle between the microfacet normal and the incident light vector is equal to the angle between the normal and the outgoing light vector. Due to the fact that the microfacets are modelled to only reflect light specularly, we can say that a microfacet will only contribute when its surface normal is equal to the vector that is halfway between the light vector $\boldsymbol{l}$ and the view vector $\boldsymbol{v}$. This halfway vector is denoted as $\boldsymbol{h}$ and is formally described as

$$\boldsymbol{h} = \frac{\boldsymbol{l} + \boldsymbol{v}}{|\boldsymbol{l} + \boldsymbol{v}|}. \quad (8)$$

An important phenomenon that occurs during the interaction of light with microgeometry is *shadowing* and *masking*. Shadowing is the occlusion of the light source by the microgeometry, masking is the visibility occlusion of a microfacet by other microgeometry (see Figure 2). In reality, there are also interreflections of light between microfacets, but this aspect is often ignored in microfacet BRDFs.

The general form of the Cook-Torrance specular term is

$$f_{\text{spec}}(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \frac{F}{\pi} \frac{GD}{(\boldsymbol{n} \cdot \boldsymbol{\omega}_i)(\boldsymbol{n} \cdot \boldsymbol{\omega}_o)}, \quad (9)$$

where F is the Fresnel term that describes how light is reflected from each microfacet, G is the geometrical attenua-

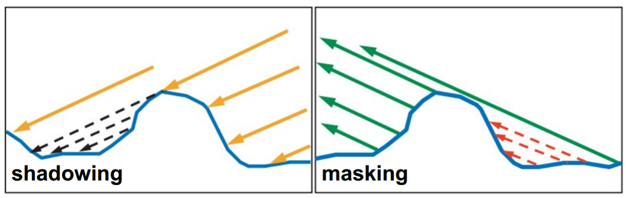

Figure 2: On the left, occlusion of light source by microgeometry (shadowing). On the right, visibility occlusion of microgeometry (masking). Source: [1]

tion factor that accounts for the shadowing and masking of microfacets, and D is the normal distribution function that represents the fraction of facets that are oriented in $\boldsymbol{h}$.

The Fresnel term that is proposed by Cook and Torrance is

$$F(\theta) = \frac{1}{2}\frac{(g-c)^2}{(g+c)^2}\left(1 + \frac{(c(g+c)-1)^2}{(c(g-c)+1)^2}\right), \tag{10}$$

where

$$\begin{aligned} c &= \cos\theta = \boldsymbol{v}\cdot\boldsymbol{h}, \\ g &= \sqrt{\eta^2 + c^2 - 1}, \end{aligned} \tag{11}$$

and η is the index of refraction of the medium. It is a function of incidence angle (θ) and wavelength (but we ignore the latter).

Shadowing and masking is modelled by the following equation:

$$G(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \min\left\{1, \frac{2(\boldsymbol{n}\cdot\boldsymbol{h})(\boldsymbol{n}\cdot\boldsymbol{\omega}_o)}{(\boldsymbol{\omega}_o\cdot\boldsymbol{h})}, \frac{2(\boldsymbol{n}\cdot\boldsymbol{h})(\boldsymbol{n}\cdot\boldsymbol{\omega}_i)}{(\boldsymbol{\omega}_o\cdot\boldsymbol{h})}\right\}. \tag{12}$$

The paper does not propose a new normal distribution function. Instead, they suggest to consider several and explicitly mention the Gaussian model and the Beckmann distribution which, for rough surfaces, is

$$D_{Beckmann} = \frac{1}{m^2\cos^4\alpha}e^{-[(\tan\alpha/m]^2}, \tag{13}$$

where α is the angle between the surface normal $\boldsymbol{n}$ and the halfway vector $\boldsymbol{h}$, and m is the root mean square slope which is a measure of roughness.

The foundations of the Cook-Torrance model (general form, G term) were already laid in the previous work of Torrance and Sparrow [12]. The main contribution of Cook and Torrance was the idea that only microfacets that are oriented in $\boldsymbol{h}$ contribute to the specular reflection. The proposed Fresnel term in [3] was also original work; it is an optimized formulation of the Fresnel equation, which increased its usefulness for computer graphics.

5.2 GGX

Walter et al. [13] propose a BSDF (Bidirectional Scattering Distribution Function), which they have called GGX. BSDFs can be considered a superset of the BRDF and the BTDF (Bidirectional Transmittance Distribution Function). BSDFs model both reflection and transmittance, and as such are the sum of a BRDF and a BTDF. We restrict ourselves to BRDFs in this study, which means that we will also only be looking at a subset of [13].

We will see that the GGX BRDF is heavily inspired by the microfacet model of Cook and Torrance. The general form of the GGX BRDF is

$$f(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \frac{FGD}{4(\boldsymbol{n}\cdot\boldsymbol{\omega}_i)(\boldsymbol{n}\cdot\boldsymbol{\omega}_o)}. \tag{14}$$

Note that this is exactly the same as the Cook-Torrance specular BRDF, except that the normalisation factor for the Fresnel term has been changed to 4 (from π). They mention that more recent literature agrees with their normalisation factor.

The paper does not propose a new equation for the Fresnel term. Instead, they recommend using the Fresnel term proposed by Cook and Torrance (shown in the previous section) for dielectrics with unpolarised light and mention that cheaper approximations are also available [3][10]. Walter et al. propose a new set of D and G functions that have been developed to better match the measurements they have done in relation to transmittance of energy. Aside from this, they also remark that the functions proposed by Cook and Torrance contain first derivative discontinuities and features not exhibited by real surfaces.

The GGX normal distribution function is defined as

$$D = \frac{\alpha_g^2\chi^+(\boldsymbol{h}\cdot\boldsymbol{n})}{\pi\cos^4\theta_h(\alpha_g^2 + \tan^2\theta_h)^2}, \tag{15}$$

where α_g is a width parameter for the specular highlight, θ_h is the angle between the half vector $\boldsymbol{h}$ and the surface normal $\boldsymbol{n}$, and χ^+ is a positive characteristic function that equals one if its parameter is greater than zero and zero if its parameter is lesser or equal to zero.

Walter et al. use a Smith shadowing-masking function for G that has been derived from D (as opposed to the Cook-Torrance G term, which you may recall is wholly independent of the normal distribution function). This G function approximates bidirectional shadowing and masking by splitting it into two separable monodirectional shadowing terms such that

$$G(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) \approx G_1(\boldsymbol{\omega}_i)G_1(\boldsymbol{\omega}_o). \tag{16}$$

Finally, the monodirectional shadowing term is

$$G_1(\boldsymbol{v}) = \chi^+\left(\frac{\boldsymbol{v}\cdot\boldsymbol{h}}{\boldsymbol{v}\cdot\boldsymbol{n}}\right)\frac{2}{1 + \sqrt{1 + \alpha_g^2\tan^2\theta_v}}. \tag{17}$$

Here, θ_v is the angle between the view vector $\boldsymbol{v}$ and the surface normal $\boldsymbol{n}$.

Visually, the GGX distribution has much stronger tails (or, a longer slope) than the Beckmann and Phong distributions. Walter et al. describe that the GGX distribution, at least for some surfaces, provides a closer match to their measurements than the Beckmann distribution.

5.3 Schlick

Schlick's paper [10] proposes a new inexpensive BRDF model as well as the optimisation technique that was used to develop the model. We will not discuss the technique used, but only the resulting model. Schlick briefly reviews several BRDF models (Cook-Torrance, HTSG and Ward) and notes several unsatisfactory points:

1. BRDFs are formulated as linear combinations with constant weights between diffuse and specular parts. The proportions of diffuse and specular components should not be constant, but rather a function of incident angle.

2. The geometrical attenuation terms in microfacet BRDFs do not model reflection towards any directions other than the outgoing light direction (in micro scale).

3. Theoretical BRDF models that are physically plausible involve complex mathematical expressions that were – at least, at the time – expensive to compute.

5.3.1 *Approximation of Existing Functions*

Based on these unsatisfactory points, the Rational Fraction Approximation technique has been used to approximate the F, D and G functions that are pervasive in microfacet theory.

The approximated Fresnel term is defined as

$$F(\theta) = f_0 + (1 - f_0)(1 - \cos\theta)^5, \tag{18}$$

where θ is the angle of incidence (angle between viewing direction $\boldsymbol{v}$ and half-angle direction $\boldsymbol{h}$) and f_0 is the measured spectral distribution of the Fresnel factor at normal incidence ($\cos\theta = 1$).

The geometrical attenuation term that has been approximated from the Smith formulation [11] is

$$G_1(\boldsymbol{v}) = \frac{\boldsymbol{v}}{\boldsymbol{v} - k\boldsymbol{v} + k}, \text{ where } k = \sqrt{\frac{2m^2}{\pi}}. \tag{19}$$

Beckmann's distribution is used as the normal distribution function, it is approximated by

$$\forall\alpha \in [1-m, 1], D(\alpha) = \frac{m^3 x}{t(mx^2 - x^2 + m^2)^2}, \tag{20}$$

where $x = t + m - 1$. In both equations 19 and 20, m is the root mean square slope which is a measure of surface roughness.

5.3.2 *Schlick's BRDF Model*

The approximations described in the previous section lead to an optimised Cook-Torrance model. This approach tackles the third unsatisfactory point that was noted, but leaves the others unresolved. However, the paper also proposes two new BRDF models. The first models homogeneous material surface reflection, and the second models heterogeneous surfaces using a two layered model. A single material layer is defined by the following set of properties:

- $C_\lambda \in [0, 1]$: Reflection factor at wavelength λ
- $r \in [0, 1]$: Roughness factor ($r = 0:$ perfectly specular, $r = 1$ perfectly diffuse)
- $p \in [0, 1]$: Isotropy factor ($p = 0:$ perfectly anisotropic, $p = 1$ perfectly isotropic)

The general form of Schlick's BRDF model is

$$f(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = R((\boldsymbol{n}\cdot\boldsymbol{h}), (\boldsymbol{\omega}_o\cdot\boldsymbol{h}), (\boldsymbol{\omega}_i\cdot\boldsymbol{n}), (\boldsymbol{\omega}_o\cdot\boldsymbol{n}), \phi_h), \tag{21}$$

where ϕ_h is the azimuthal angle of the half vector $\boldsymbol{h}$, and the definition of R depends on which of the two models is used.

The first model assumes that materials have homogeneous optical properties. Examples of materials that exhibit this property are metal, glass, paper and cotton. For this model, R is defined as

$$R(t, u, v, v', w) = S(u)D(t, v, v', w), \tag{22}$$

where $S(u)$ is the spectral term which depends on the angle of incidence (the definition of $S(u)$ is equivalent to the approximated Fresnel term $F(\theta)$), and where the directional factor $D(t, v, v', w)$ is defined as

$$D(t, v, v', w) = \frac{G_1(v)G_1(v')}{4\pi vv'} Z(t)A(w) + \frac{1 - G_1(v)G_1(v')}{4\pi vv'}. \tag{23}$$

Here, we can see that there is a dependence on the geometrical attenuation factor, and we note that anisotropy is modelled by a dependence on the zenith angle and the azimuthal angle. The definitions of the dependencies on the zenith and azimuthal angles are respectively

$$Z(t) = \frac{r}{(1 + rt^2 - t^2)^2}, \text{ and } A(w) = \sqrt{\frac{p}{p^2 - p^2w^2 + w^2}}. \tag{24}$$

The second model assumes materials exhibit heterogeneous optical properties and are usually composed of a somewhat translucent layer on top of an opaque layer. This model is essentially a sum of two single layered models with an added weighting term based of the spectral factor (which varies with angle of incidence). The double layered model is formally defined as

$$\begin{aligned} R(t, u, v, v', w) &= S(u)D(t, v, v', w) \\ &\quad + [1 - S(u)]S'(u)D'(t, v, v', w). \end{aligned} \tag{25}$$

To conclude, this model offers a definition for both homogeneous and heterogeneous material surfaces, offering a continuum between perfect diffuse and perfect specular as well as between perfect anisotropy and perfect isotropy. The model features a set of few parameters that have intuitive meaning. Aside from the proposed model, the paper also describes an approximation technique and has defined approximations for popular F, D and G terms (the approximated Fresnel term is still widely used today).

5.4 Oren-Nayar

The models that have been discussed so far model specular reflection (the exception being Schlick's model, where the opaque layer models diffuse subsurface scattering). These models are often accompanied by the Lambertian diffuse term, which is a constant term in BRDFs. Lambertian surfaces are ideally diffusing surfaces, whose radiance is independent of viewing direction; they appear equally bright from all directions.

Oren and Nayar have proposed a diffuse BRDF that models reflectance from rough diffuse surfaces [8]. The model is based on microfacet theory, specifically on the work by Torrance and Sparrow. The reasoning for this model is that while the Lambertian assumption is often reasonable when looking at single planar facets, reflectance is not Lambertian when a collection of facets is imaged onto a single pixel. Their measurements have shown that the deviation from Lambertian behaviour is significant – and clearly noticeable – for very rough surfaces and that it increases with angle of incidence. Figure 3 shows a comparison of the Lambertian model, the proposed Oren-Nayar model, and a real image of a clay vase.

Figure 3: Comparison of Lambertian and Oren-Nayar models to an image of a cylindrical clay vase. The light source is positioned in the viewer direction (i.e. angle of incidence is 0°). Source: [8]

The paper documents the development of the BRDF by detailing several of the model's iterations. We will however limit ourselves to only the most general form and an approximation of this form. The model is similar to the specular Cook-Torrance BRDF in the sense that surfaces are modelled as consisting of many small microfacets. The orientation of these microfacets is distrubuted according to a Gaussian function, with a standard deviation σ. This standard deviation is a measure of surface roughness and $\sigma \in [0, 1]$ (The visual effect of varying the surface roughness parameter is depicted in figure 4). The unapproximated Oren-Nayar BRDF is defined as

$$\begin{aligned} f(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \frac{\rho}{\pi} \Bigg[& C_1(\sigma) + \cos(\phi_{\boldsymbol{\omega}_o} - \phi_{\boldsymbol{\omega}_i}) \\ & \times C_2(\alpha, \beta, \phi_{\boldsymbol{\omega}_o} - \phi_{\boldsymbol{\omega}_i}, \sigma) \tan\beta \\ & + (1 - |\cos(\phi_{\boldsymbol{\omega}_o} - \phi_{\boldsymbol{\omega}_i})|) \\ & \times C_3(\alpha, \beta, \sigma) \tan\left(\frac{\alpha + \beta}{2}\right) \Bigg], \end{aligned} \tag{26}$$

where ρ is the spectral distribution at normal incidence, θ and ϕ represent zenith and azimuthal angles respectively, $\alpha = \max(\theta_{\boldsymbol{\omega}_o}, \theta_{\boldsymbol{\omega}_i})$ and $\beta = \min(\theta_{\boldsymbol{\omega}_o}, \theta_{\boldsymbol{\omega}_i})$. The coefficients C_1, C_2 and C_3 are further defined as

$$C_1 = 1 - 0.5\frac{\sigma^2}{\sigma^2 + 0.33}$$

$$C_2 = \begin{cases} 0.45\frac{\sigma^2}{\sigma^2+0.09} \sin\alpha, & \text{if } \cos(\phi_{\boldsymbol{\omega}_o} - \phi_{\boldsymbol{\omega}_i}) \geq 0 \\ 0.45\frac{\sigma^2}{\sigma^2+0.09} \left(\sin\alpha - \left(\frac{2\beta}{\pi}\right)^3\right), & \text{otherwise} \end{cases}$$

$$C_3 = 0.125\left(\frac{\sigma^2}{\sigma^2 + 0.09}\right)\left(\frac{4\alpha\beta}{\pi^2}\right)^2$$

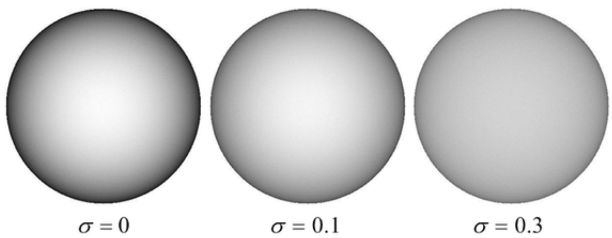

Figure 4: Influence of surface roughness (σ) on the Oren-Nayar reflectance model. Note that $\sigma = 0$ reduces the model to a Lambertian model. Source: [8]

This BRDF models the shadowing and masking phenomena exhibited by microfacets, but not interreflection between microfacets. While interreflection is included in the complete Oren-Nayar model, it is not included in their functional approximation. The approximation has been developed by analysing the relevant significance of individual terms. The simpler model is obtained by discarding the C_3 term and ignoring interreflections:

$$\begin{aligned} f(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) &= \frac{\rho}{\pi}(A + B \\ & \quad \max(0, \cos(\phi_{\boldsymbol{\omega}_o} - \phi_{\boldsymbol{\omega}_i})) \sin\alpha \tan\beta) \\ A &= 1.0 - 0.5\frac{\sigma^2}{\sigma^2 + 0.33} \\ B &= 0.45\frac{\sigma^2}{\sigma^2 + 0.09} \end{aligned} \tag{27}$$

Visual characteristics of simple objects rendered with the Oren-Nayar model are shown in figures 3 and 4. The model is a more general model than the ubiquitous Lambertian model for diffuse surfaces, as it models rough surfaces more realistically and still reduces to the Lambertian when $\sigma = 0$.

6. COMPARISON

In this section we will compare the BRDF models described previously. Here, the focus lies on determining in which circumstances the BRDFs are ideal. This means that we will be looking at desirable properties for BRDF models as well as the type of surfaces that they are well suited for.

In section 4 we identified several desirable properties of BRDF models. First, we look at the physical plausibility of the models that have been studied, that is to say that we will be looking at whether or not the models are energy conserving and reciprocal.

All of the models that have been covered in this paper are reciprocal. The Cook-Torrance model is not energy conserving under all angles [6], while on the other hand GGX, Schlick's model and Oren-Nayar are all energy conserving. Do note that – for the specular BRDFs – energy conservation applies to the specular BRDF terms. When adding a diffuse term and energy conservation is desired, there are a number of options that are available to balance the diffuse and specular terms [7]. Some of these option however result in the violation of Helmholtz reciprocity, which may be problematic for certain rendering techniques.

The Cook-Torrance model is primarily suited for the modelling of metallic and plastic surfaces, but it can be used to successfully model other types of surfaces as well. The main issue with this however is that the parameters are not as intuitive as those used in other models, which often makes experimentation necessary. The general form of the Cook-Torrance model is well suited for experimentation however, as the F, D and G terms can be substituted by other functions with relative ease.

It is difficult to fairly include the GGX model into this comparison as it is not only limited to reflection. The model was developed to simulate transmission through rough surfaces such as etched glass. By limiting this study purely to energy reflection we have reduced this model to an improved formulation of the Cook-Torrance model.

The Schlick model is designed to account for heterogeneous materials, where the material is composed of a translucent layer over an opaque layer. Additionally, the model also provides a continuum between isotropic and anisotropic behaviour. This allows the model to be used for a large variety of materials. Moreover, the parameters used by this model are well defined and intuitive, which is often a desirable trait in production environments.

The model proposed by Oren and Nayar is a more general diffuse model than the Lambertian model, and is suitable for rendering very rough surfaces. For very smooth surfaces the model is reduced to a Lambertian model. Whether or not the additional costs are worth it depends wholly on the type of surfaces that are being rendered. This model does not compete with the other specular BRDF models that have been discussed. In fact, it could very well be used in combination with other specular BRDFs, as it would only replace the diffuse BRDF term (which is often a constant term).

Choosing an optimal lighting model can be a difficult task that is dependent on a large set of parameters. As we have discussed, many of the models described in this paper are well suited for rendering of specific types of surfaces. Some development environments may have the luxury of using a large set of lighting models, using each of them to only render surfaces of specific material types. Other environments may require the use of a few general models that can be used to render a wide array of different surface types. Intuitiveness of lighting model parameters can certainly also be an important factor, especially if artists without much knowledge of lighting models are those that set the parameters. Real-time image synthesis also places additional requirements on the computational complexity of lighting models, which may further limit the number of suitable lighting models for the environment.

7. FUTURE WORK

This document has given a brief introduction into basic radiometry and BRDF theory. Only a few models have been discussed however. If more time was available, the roster of BRDF models would have been expanded as there is truly a wealth of literature available on this subject. Aside from this, the models would have been analysed from a computational complexity perspective and inserted in a comprehensive overview of BRDF models (e.g. physical properties, computational complexity, surface types).

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