ECON526: Quantitative Economics with Data Science Applications

Directed Graphical Models and Causality

Phil Solimine

philip.solimine@ubc.ca

University of British Columbia

Table of contents

Overview

Summary

- Previously in the course, we talked at a high level about some of the barriers to causal inference
- We used the potential outcomes framework to discuss the idea of a treatment effect
- Then we discussed the idea of a randomized experiment as a way to identify a treatment effect
  - → However, we mentioned that there are many situations where we cannot run a randomized experiment
- Today, we will discuss the idea of using a **graphical model** as a way to analyze whether you can truly identify a treatment effect

Directed Graphical Models

Conditional Independence

- Recall that two random variables X and Y are **conditionally independent** given a third random variable Z if and only if the following holds:
  - $\rightarrow$ $P(X|Z) = P(X|Z \cap Y)$
  - $\rightarrow$ Equivalently, $P(X \cap Y|Z) = P(X|Z)P(Y|Z)$
  - $\rightarrow$ We will denote this as $X \perp Y|Z$
- In the context of potential outcomes, we require that $(Y_0, Y_1) \perp T|X$
  - $\rightarrow$ This means that the potential outcomes are independent of the treatment assignment, given the covariates

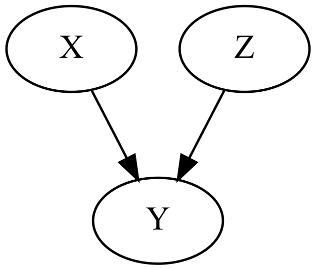
Directed Graphical Models

- Complete independence is rare in complex systems. However, we can often find conditional independence relationships, that help inform our choice of statistical model.
- We can visualize conditional independence relationships using a **Bayesian** or **directed graphical model**
- A Bayesian graphical model is a directed graph where:
  - → The nodes represent random variables
  - → The edges represent conditional independence relationships

Directed Graphical Models

- A directed graphical model might look something like this:

```python
import graphviz as gr

g = gr.Digraph()
g.node('X')
g.node('Y')
g.node('Z')
g.edge('X', 'Y')
g.edge('Z', 'Y')
g
```

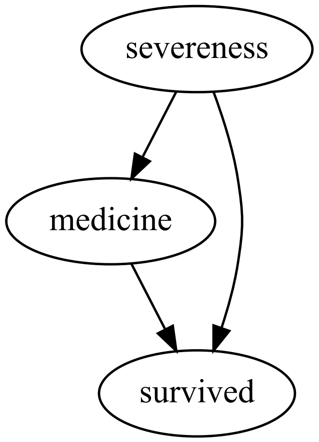


- Here, we have three random variables: X, Y, and Z
  - $\longrightarrow$ X and Z are independent
  - $\longrightarrow$ Y depends on both X and Z

Directed Graphical Models

- Here is a more realistic example:

```python
g = gr.Digraph()
g.edge("medicine", "survived")
g.edge("severeness", "survived")
g.edge("severeness", "medicine")
g
```



- We use arrows to indicate the direction of the conditional independence relationship
    - $\longrightarrow$ For example, $medicine \to survived$ means that $survived$ depends on $medicine$, but not the other way around

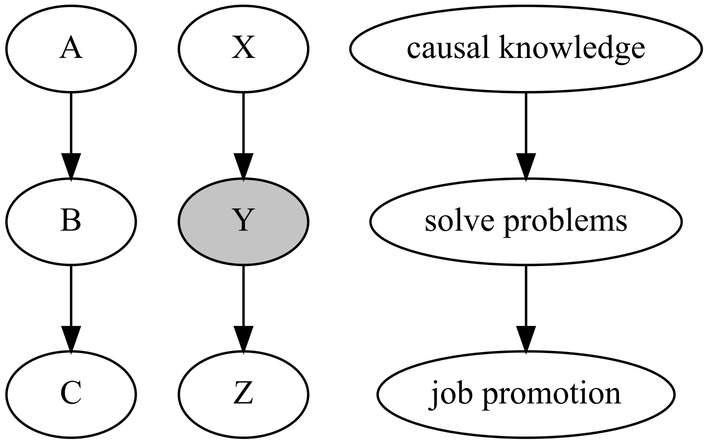
Directed Graphical Models

- Directed graphical models are useful because they allow us to visualize conditional independence relationships, which are often difficult to keep track of
- However, they are also useful because they allow us to determine whether or not we can identify a treatment effect.
- There are three very common sub-structures that appear in graphical models, that inform how dependence will flow through the model.

Directed Graphical Models

Let's look at a really simple example, which we could call a directed path:

```
g = gr.Digraph()
g.edge("A", "B")
g.edge("B", "C")

g.edge("X", "Y")
g.edge("Y", "Z")
g.node("Y", "Y", fillcolor="#5f5f5f5f", style="fille


g.edge("causal knowledge", "solve problems")
g.edge("solve problems", "job promotion")

g
```



- In this stylized example, we are postulating that knowing causal inference is the only way to solve business problems
  - This is obviously not true, but it is a useful example for our purposes

Directed Graphical Models

- This model highlights the following statistical process:
  - → Knowing causal inference gives you the ability to solve business problems
  - → Solving business problems makes you more likely to get a job promotion
- Notice that this does not imply that causal knowledge is independent of job promotion
  - → That is, if we know the value of job promotion, we can still learn something about causal knowledge
  - → If we observe that a promotion happens in this model, this tells us that it is more likely that the person knows causal inference

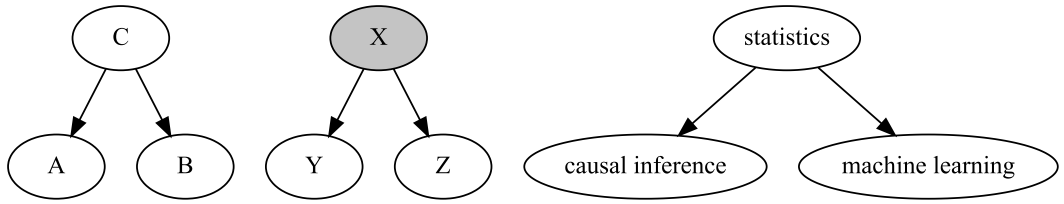
Directed Graphical Models

- Now let's condition on the intermediate variable Y
  - $\longrightarrow$ This is the variable that represents the ability to solve business problems
  - $\longrightarrow$ In the graph, we have colored this variable grey to indicate that we are conditioning on it
- Conditioning on Y means that we are assuming that we know whether or not the person solved a business problem
- In this case, conditioning on Y breaks the dependence relationship between X and Z
  - $\longrightarrow$ That is, X and Z are now independent, given Y
  - $\longrightarrow$ Mathematically, $A \perp C$, but $X \not\perp Z|Y$

Directed Graphical Models

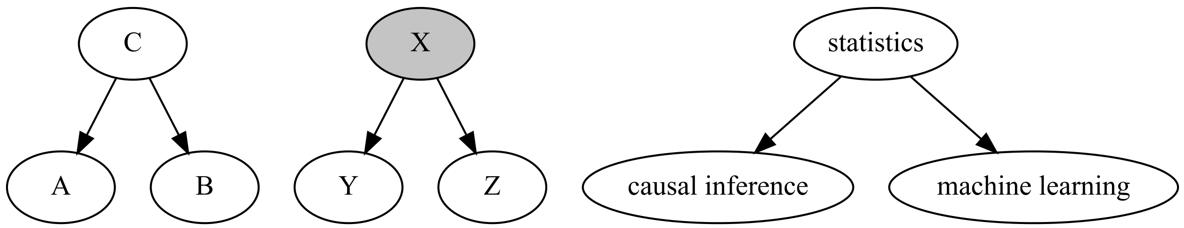
Now let’s look at another common structure:

```
g = gr.Digraph()
g.edge("C", "A")
g.edge("C", "B")

g.edge("X", "Y")
g.edge("X", "Z")
g.node("X", "X", fillcolor="#5f5f5f5f", style="fille

g.edge("statistics", "causal inference")
g.edge("statistics", "machine learning")

g
```



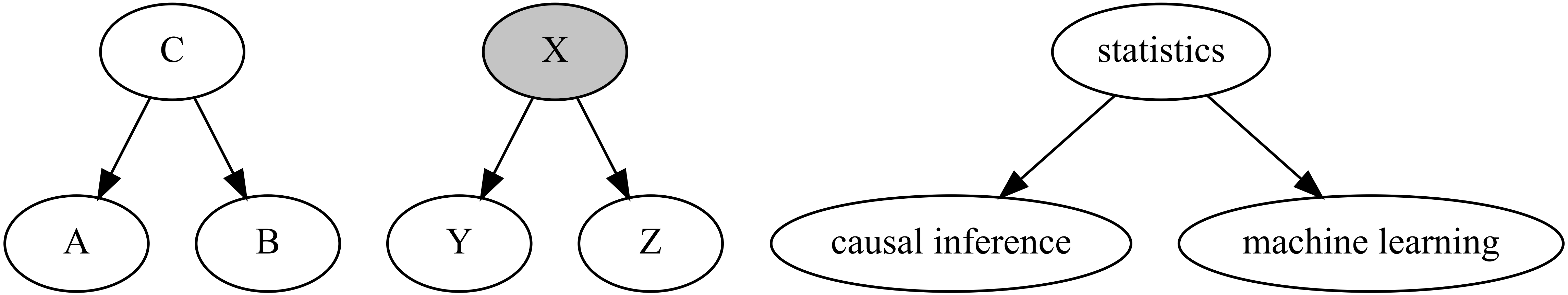
- In this model, we are postulating that statistics is a prerequisite for both causal inference and machine learning.

Directed Graphical Models



- This model highlights the following statistical process:
  - $\longrightarrow$ Knowing statistics gives you the ability to do causal inference
  - $\longrightarrow$ Knowing statistics gives you the ability to do machine learning
- When we don't condition for the root node, C, there is still a dependence relationship between A and B
  - $\longrightarrow$ That is, if we know that an individual has causal knowledge, this tells us that they are more likely to know statistics, and thus to also know machine learning

Directed Graphical Models



- By conditioning on X, we break the dependence relationship between Y and Z

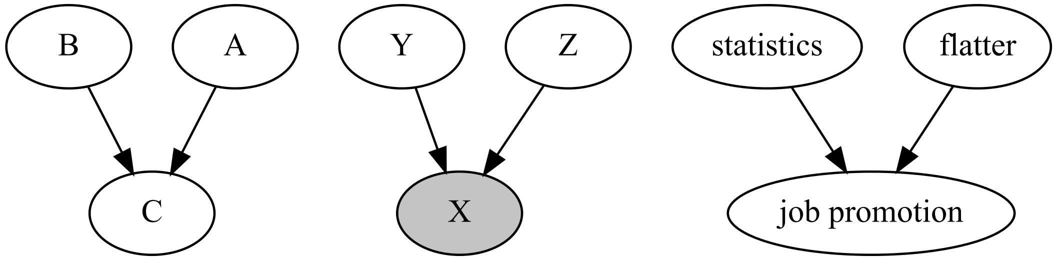
  $\rightarrow$ That is, $A \not\perp B$, but $Y \perp Z | X$

- We would call this a **fork** structure

Directed Graphical Models

Finally, let's look at a third common structure, called a **collider**:

```
g = gr.Digraph()
g.edge("B", "C")
g.edge("A", "C")

g.edge("Y", "X")
g.edge("Z", "X")
g.node("X", "X", fillcolor="#5f5f5f5f", style="fille

g.edge("statistics", "job promotion")
g.edge("flatter", "job promotion")

g
```

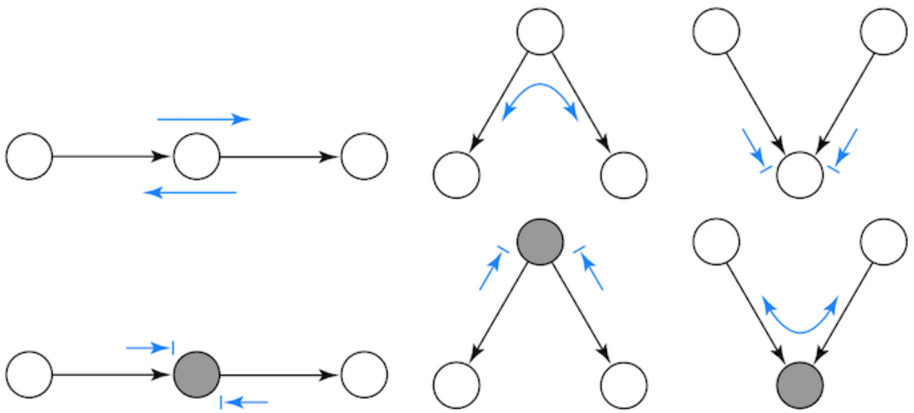


- In this model, we are postulating that statistics and flattery are both determinants of getting a job promotion

Dependence Flows

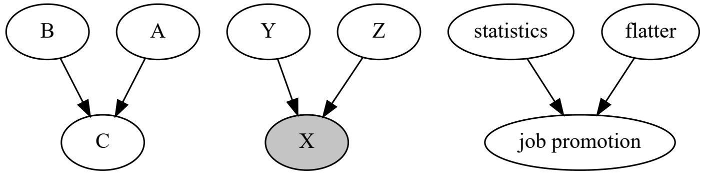
The Rules of Bayes-Ball

- We can think about the flow of dependence as a game of “Bayes-ball”
- The rules of Bayes-ball are reasonably simple. A dependence path is blocked if and only if:
  1. It contains a *non-collider* that is conditioned on
  2. It contains a *collider* that is not conditioned on, and neither are any of its descendants



Directed Graphical Models

Turning back to our collider example:



- Notice that when we do not condition on C, A and B are independent.
- However, somewhat unintuitively, when we condition on X, Y and Z become dependent.

  $\longrightarrow$ This is because conditioning on X opens the flow of dependence from Y to Z.

  $\longrightarrow$ In any case that is not a collider, conditioning on a node blocks the flow of dependence.

Viualizing Bias

Bias and Causality

- In a causal inference framework, we can use graphical models to determine whether or not we can identify a treatment effect, and which covariates we need to condition on.
- Typically, drawing out a graphical model is not necessary, but it can be a useful exercise to help you think through the problem.
  - → The links you draw represent the assumptions you are making about the data generating process

Bias and Causality

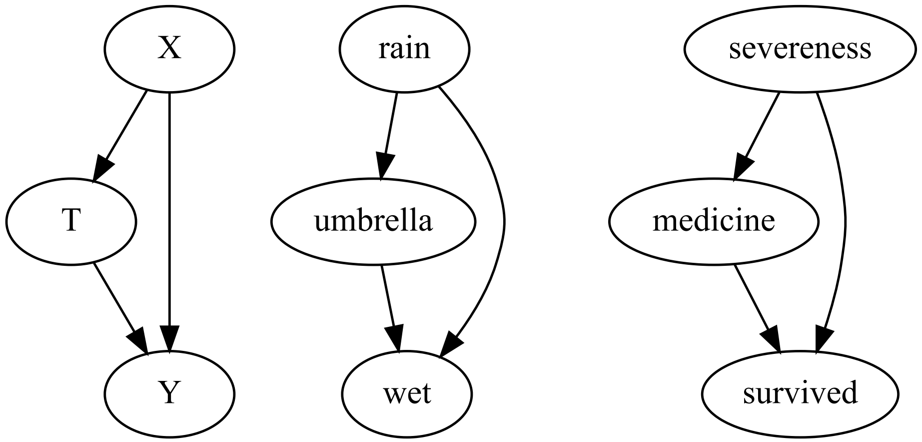
- There are two major types of bias that we need to worry about in causal inference:
  - → **Confounding**: When there is an unobserved variable that is a common cause of both the treatment and the outcome
  - → **Selection**: When there is an unobserved variable that is a common cause of both the treatment and the selection into the sample
- Both of these types of bias can be represented using a graphical model

Confounding

Confounding

- Let's look at an example of confounding:

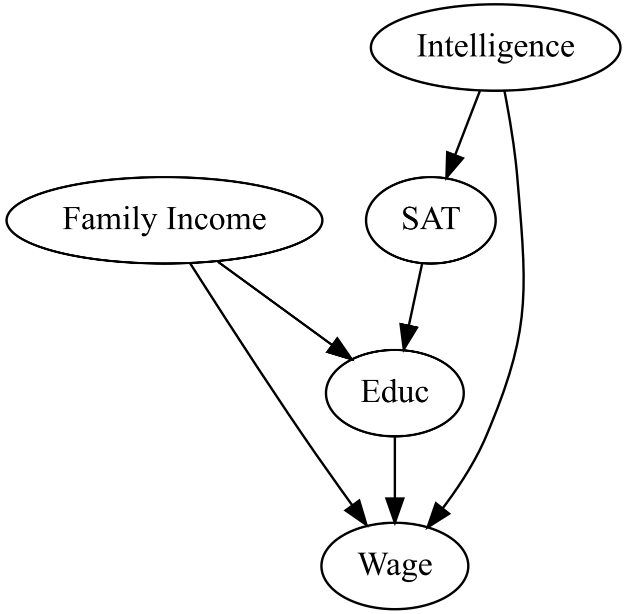
```python
g = gr.Digraph()
g.edge("X", "T")
g.edge("X", "Y")
g.edge("T", "Y")

g.edge("rain", "umbrella")
g.edge("rain", "wet")
g.edge("umbrella", "wet")

g.edge("severeness", "medicine")
g.edge("severeness", "survived")
g.edge("medicine", "survived")
g
```



- To control for confounding, we need to condition on all of the common causes of the treatment and the outcome.

Confounding

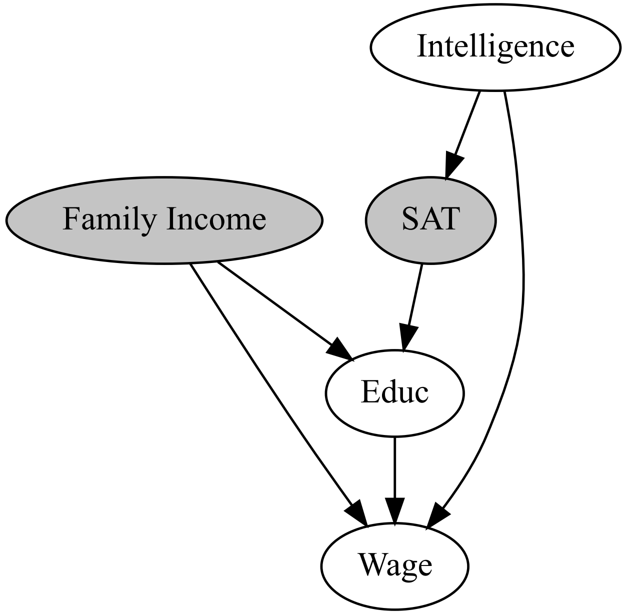
```
g = gr.Digraph()

g.node("Family Income")
g.edge("Family Income", "Educ")
g.edge("Educ", "Wage")

g.node("SAT")
g.edge("SAT", "Educ")

g.node("Family Income")
g.edge("Family Income", "Wage")

g.edge("Intelligence", "SAT")
g.edge("Intelligence", "Wage")
g
```



- Often, there are confounding variables that we cannot observe
  - → For example, we cannot observe intelligence, but it is a common cause of both education (the treatment) and wages

Confounding

```
g = gr.Digraph()

g.node("Family Income", fillcolor="#5f5f5f5f", style
g.edge("Family Income", "Educ")
g.edge("Educ", "Wage")

g.node("SAT", fillcolor="#5f5f5f5f", style="filled")
g.edge("SAT", "Educ")

g.node("Family Income", fillcolor="#5f5f5f5f", style
g.edge("Family Income", "Wage")

g.edge("Intelligence", "SAT")
g.edge("Intelligence", "Wage")
g
```



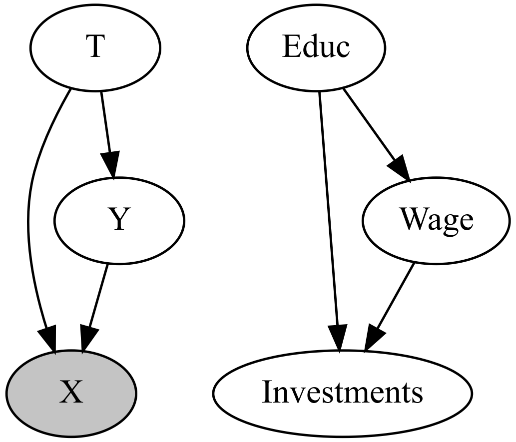
- Often, there are confounding variables that we cannot observe
  - → For example, we cannot observe intelligence, but it is a common cause of both education (the treatment) and wages

Selection

Selection

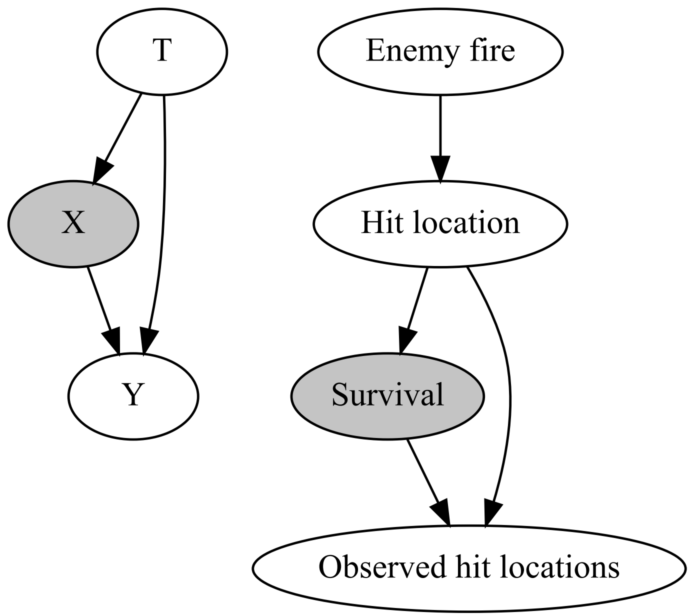
- Selection bias is the other fundamental type of bias that we need to worry about in causal inference
- Selection bias occurs when there is an unobserved variable that is a common cause of both the treatment and the selection into the sample

```
g = gr.Digraph()
g.node("X", fillcolor="#5f5f5f5f", style="filled")
g.edge("T", "X")
g.edge("T", "Y")
g.edge("Y", "X")

g.edge("Educ", "Investments")
g.edge("Educ", "Wage")
g.edge("Wage", "Investments")

g
```



Selection

- Selection bias can also occur when controlling for a **mediator** between the treatment and the outcome



Credits

This lecture draws heavily from Causal Inference for the Brave and True: Chapter 1 - Introduction to Causality by Matheus Facure.

There is also material from A Short Course on Graphical Models Chapter 2: Structured Representations by Mark Paskin