City Location Choice and Productivity

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1 Joint distribution of productivity across cities

$$P[Z_c \leq z] = e^{-G^c T_c z^{-\theta}} \tag{1}$$

$$P[Z_1 \leq z, \ldots, Z_c \leq z] = exp[-\sum_{c=1}^{N} (G^c T_c Z_c^{-\theta})^{\frac{1}{1-\sigma}}]^{1-\sigma} \tag{2}$$

G^c is the tail dependence correlation funciton. σ determines the substitutability between cities. T_c is the scale parameter that can be subtituted for amenities later on. θ is the shapre parameter.

2 Tail dependence correlation function

$$G^c(x_1, \ldots, x_c) = \sum_{k=1}^{K} [\sum_{s=1}^{N} (w_{sk} x_{sc})^{\frac{1}{1-\rho_k}}]^{1-\rho_k} \tag{3}$$

w_{sk} is the weight of technology k for sector s which is common between cities. ρ_k is the substitutability of technologies. x_s^c is the expenditure in sector s for city c, this can be analogous to endowments for each city.

3 Individual distributions

$$P[Z_{csk} \leq z] = exp[-((w_{sk} x_{sc})^{\frac{1}{1-\rho_k}} T_c Z_c^{-\theta})^{\frac{1}{1-\sigma}}] \tag{4}$$

Specific Fréchet distribution for city c, sector s and technology k.

$$\phi_c = \begin{pmatrix} z_{11} & \cdots & z_{1k} \\ \vdots & \ddots & \vdots \\ z_{s1} & \cdots & z_{sk} \end{pmatrix} \tag{5}$$

ϕ_c is the matrix of productivity draws from their respective Frechet distributions for each sector and technology in city c.

4 Individual specific technology endowments

$$\omega_p = \begin{pmatrix} v_1 \\ \vdots \\ v_k \end{pmatrix} \tag{6}$$

ω_p is the vector of technology endowments for each person p. For now, each endowment is assumed to be drawn from a normal distribution.

5 Expected City Wage Realization

$$E[w_c|\omega_p, \phi_c] = A_c[\sum_s (T_s \sum_k v_{sk} z_{sk})^{\frac{\eta}{\eta-1}}]^{\frac{\eta-1}{\eta}} \tag{7}$$

Realised wage for person p in city c given their technology draws and the productivity draws for the city.

T_s will be a sector specific scale parameter that is analogous to a city's amenities which is determined by a person's preference towards a specific sector. A_c is a city specific amenity shifter.

6 Worker problem

$$\max_w [E[w_1|\omega_p, \phi_1], \ldots, E[w_N|\omega_p, \phi_N]] \tag{8}$$

The worker will choose the city that maximizes their expected wage.