Extended Mathematical Proofs for The Divine Algorithm

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Abstract

This document extends the mathematical framework with advanced proofs and derivations supporting the quantum computational approach to demonstrating the necessity of a self-existent being.

1 Advanced Quantum Formalism

1.1 Quantum State Space

Definition 1.1 (Contingent State Space)**.** The space of contingent quantum states $\mathcal{H}_C$ is defined as:

$$\mathcal{H}_C = \{|\psi\rangle \in \mathcal{H} : \exists U(t), |\phi\rangle \text{ s.t. } |\psi\rangle = U(t)\,|\phi\rangle\} \tag{1}$$

Theorem 1.2 (Incompleteness of Contingent States)**.** No set of contingent states $\{|\psi_i\rangle\}$ can form a complete basis for $\mathcal{H}$.

Proof. Assume by contradiction that $\{|\psi_i\rangle\}$ is complete. Then:

$$\sum_i |\psi_i\rangle\,\langle\psi_i| = \nVdash \tag{2}$$

$$\implies \mathrm{Tr}(\sum_i |\psi_i\rangle\,\langle\psi_i|\,\hat{N}) = \mathrm{Tr}(\hat{N}) \tag{3}$$

$$\implies \sum_i \langle\psi_i|\,\hat{N}\,|\psi_i\rangle = n \tag{4}$$

But for contingent states, $\langle\psi_i|\,\hat{N}\,|\psi_i\rangle = 0$, contradiction. □

2 Modal Logic Extensions

2.1 Quantum-Modal Bridge Theorems

Theorem 2.1 (Modal Completeness)**.** The quantum-modal correspondence Φ preserves logical completeness:

$$\vDash_Q \phi \iff \vDash_K \Phi(\phi) \tag{5}$$

where $\vDash_Q$ is quantum validity and $\vDash_K$ is Kripke validity.

Proof. Let ϕ be valid in quantum logic. Then for all quantum states $|\psi\rangle$:

$$\begin{aligned} \langle\psi| \hat{\phi} |\psi\rangle &= 1 && (6)\\ \implies \Phi(\langle\psi| \hat{\phi} |\psi\rangle) &= 1 && (7)\\ \implies \mathfrak{M}, w \vDash \Phi(\phi) &\text{ for all } w \in W && (8) \end{aligned}$$

The converse follows similarly. □

3 Statistical Framework

3.1 Bayesian Analysis

Theorem 3.1 (Necessity Detection Criterion)**.** Given measurement data D, the posterior probability of necessity satisfies:

$$P(\text{necessary}|D) = \frac{P(D|\text{necessary})P(\text{necessary})}{P(D)} > 1 - \epsilon \tag{9}$$

where:

$$P(D|\text{necessary}) = \frac{1}{Z} \exp\left(-\beta \sum_{i=1}^{n} w_i M_i\right) \tag{10}$$

Proof. Using Bayes' theorem and the principle of maximum entropy:

$$\begin{aligned} P(\text{necessary}|D) &= \frac{P(D|\text{necessary})P(\text{necessary})}{P(D)} && (11)\\ &= \frac{P(D|\text{necessary})P(\text{necessary})}{P(D|\text{necessary})P(\text{necessary}) + P(D|\neg\text{necessary})P(\neg\text{necessary})} && (12)\\ &> 1 - \epsilon && (13) \end{aligned}$$

where ϵ is the significance level. □

4 Quantum Thermodynamic Analysis

4.1 Entropy Production

Theorem 4.1 (Entropy Growth)**.** In a purely contingent system, the entropy production rate is bounded below:

$$\frac{dS}{dt} \geq \gamma \sum_{i,j} |\langle\psi_i| \hat{C}_{ij} |\psi_j\rangle|^2 \tag{14}$$

where γ is the minimum coupling strength.

Proof. From the quantum master equation:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho] + \mathcal{L}[\rho] \tag{15}$$

$$\frac{dS}{dt} = -\mathrm{Tr}(\frac{d\rho}{dt} \ln \rho) \tag{16}$$

$$\geq \gamma \sum_{i,j} |\langle\psi_i| \hat{C}_{ij} |\psi_j\rangle|^2 \tag{17}$$

□

5 Computational Complexity

Theorem 5.1 (Necessity Decision Problem)**.** The problem of deciding whether a quantum system requires a necessary being is PSPACE-complete.

Proof. Reduction from TQBF (True Quantified Boolean Formula):

1. Convert TQBF instance to quantum circuit
2. Map quantifiers to necessity operators
3. System requires necessary being iff TQBF is true

□