Teaching video of Haoran Meng

- Target audience: graduate & senior undergraduate students.
- They have basic knowledge of seismology and statistics.
- Outline:
  - Finite source attributes of earthquakes
  - Concept of Second Seismic Moments
  - Examples
  - Method and its applications
  - Q & A

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July, 2021

Finite Source Attributes & Second Seismic Moments

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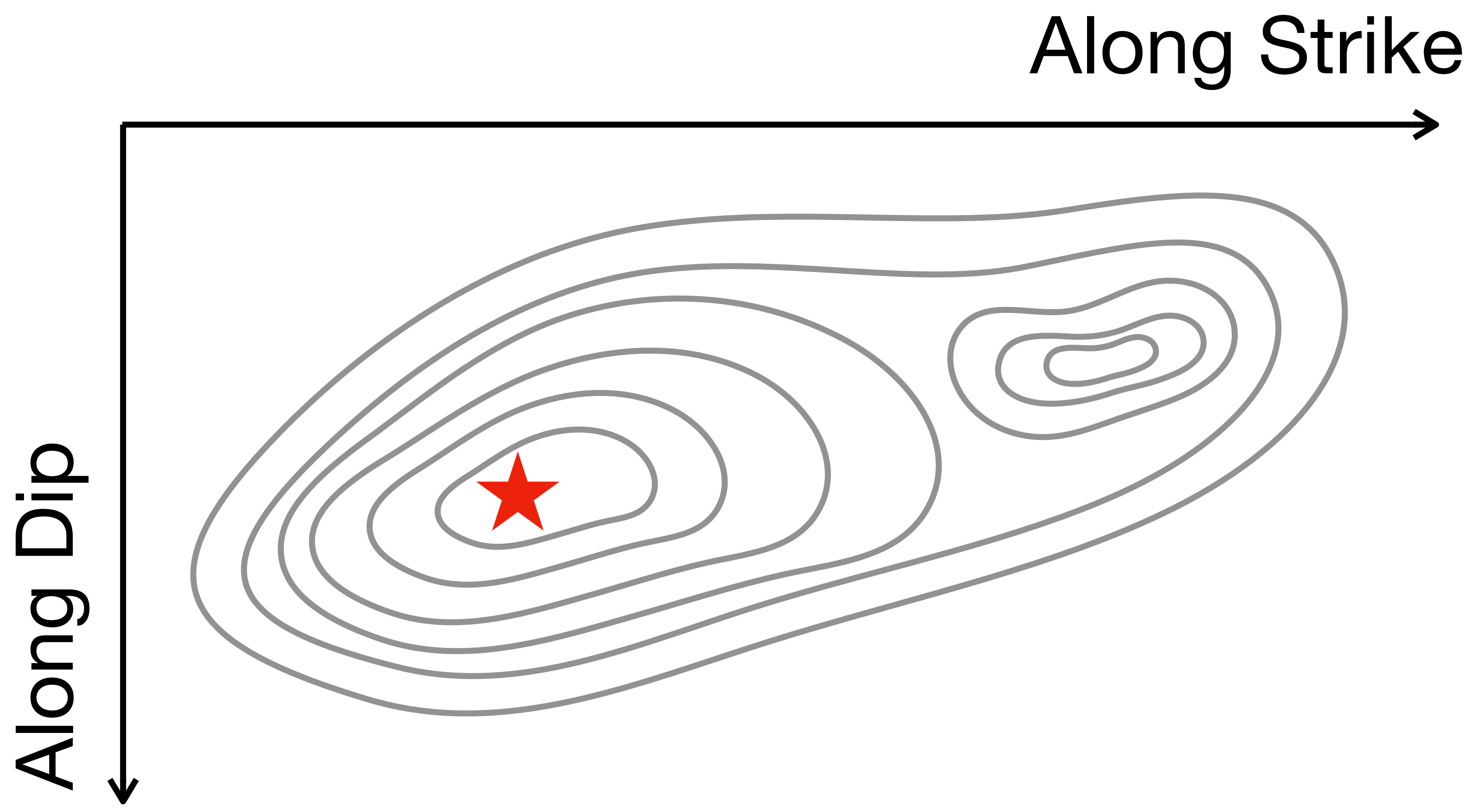
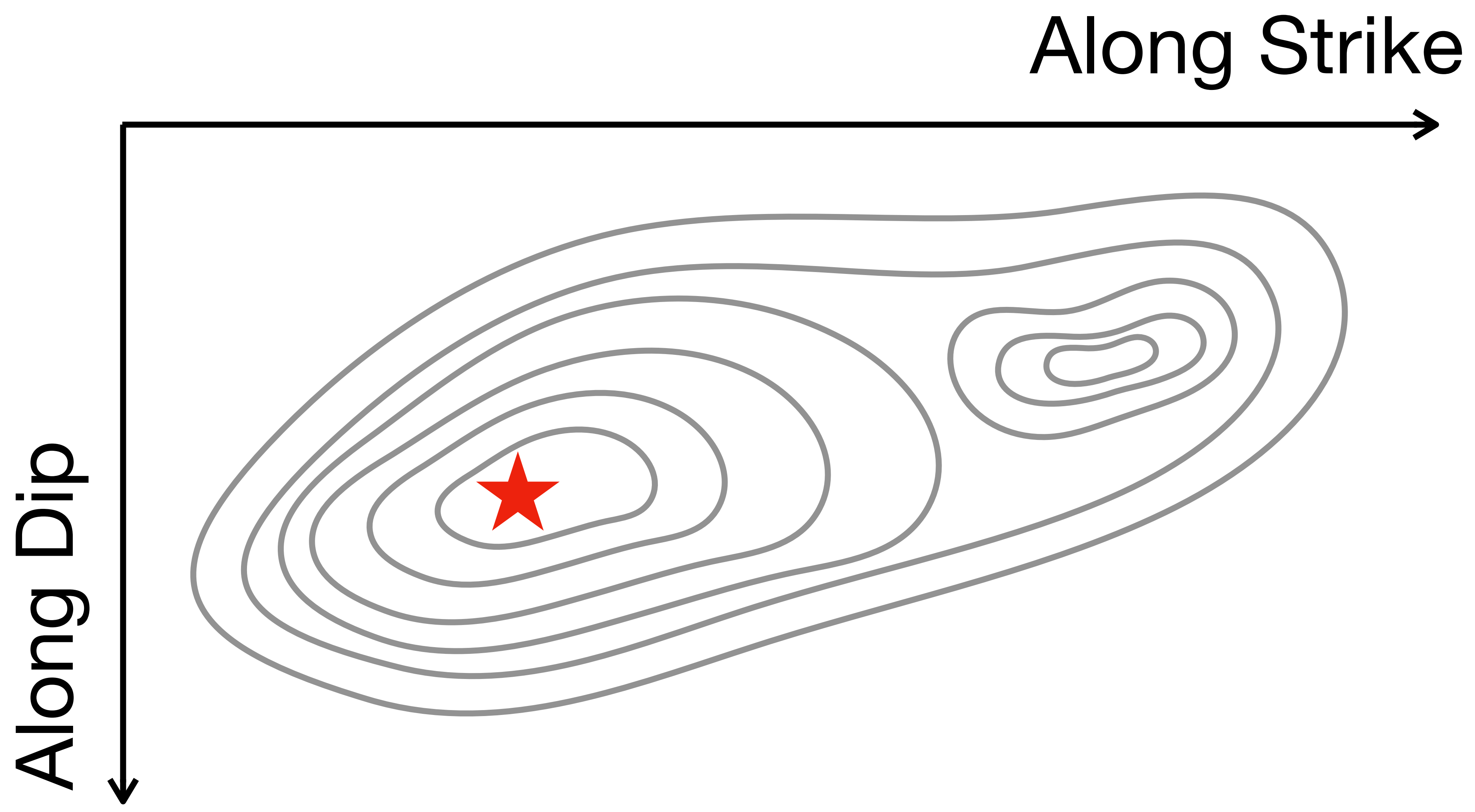
What are finite source attributes of an earthquake?

What are finite source attributes?



- Finite source attributes of an earthquake
  - Rupture extent (length, width)
  - Rupture duration
  - Rupture propagation velocity
  - Rupture directivity

What are finite source attributes?

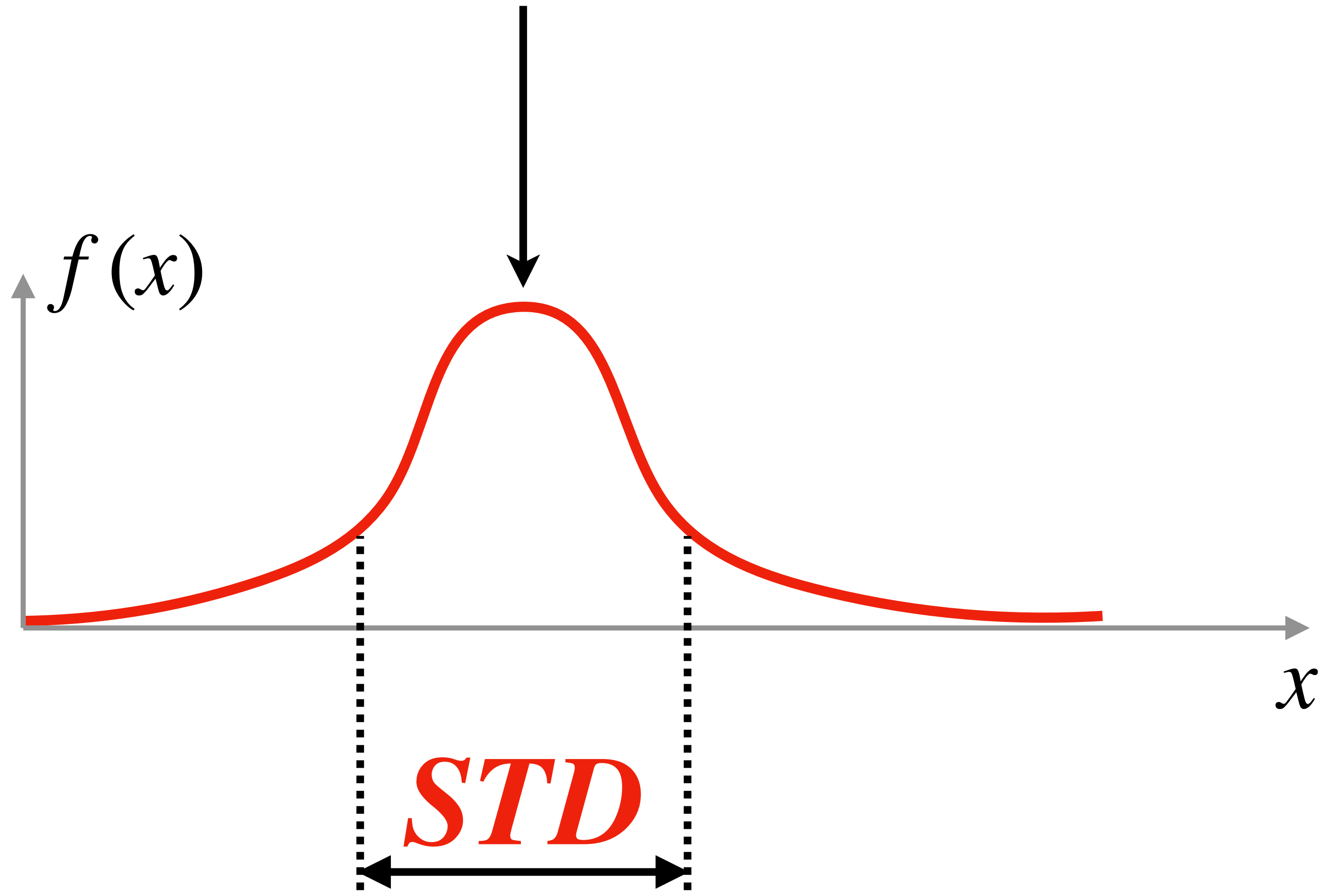


- Provide observational constraints
  - Fault zone properties
  - Rupture dynamics
- Hazard mitigation

What are second degree seismic moments?

Probability Density Function (PDF), mean, and variance

$\textit{mean} = \int f(x)\, x \, dx$

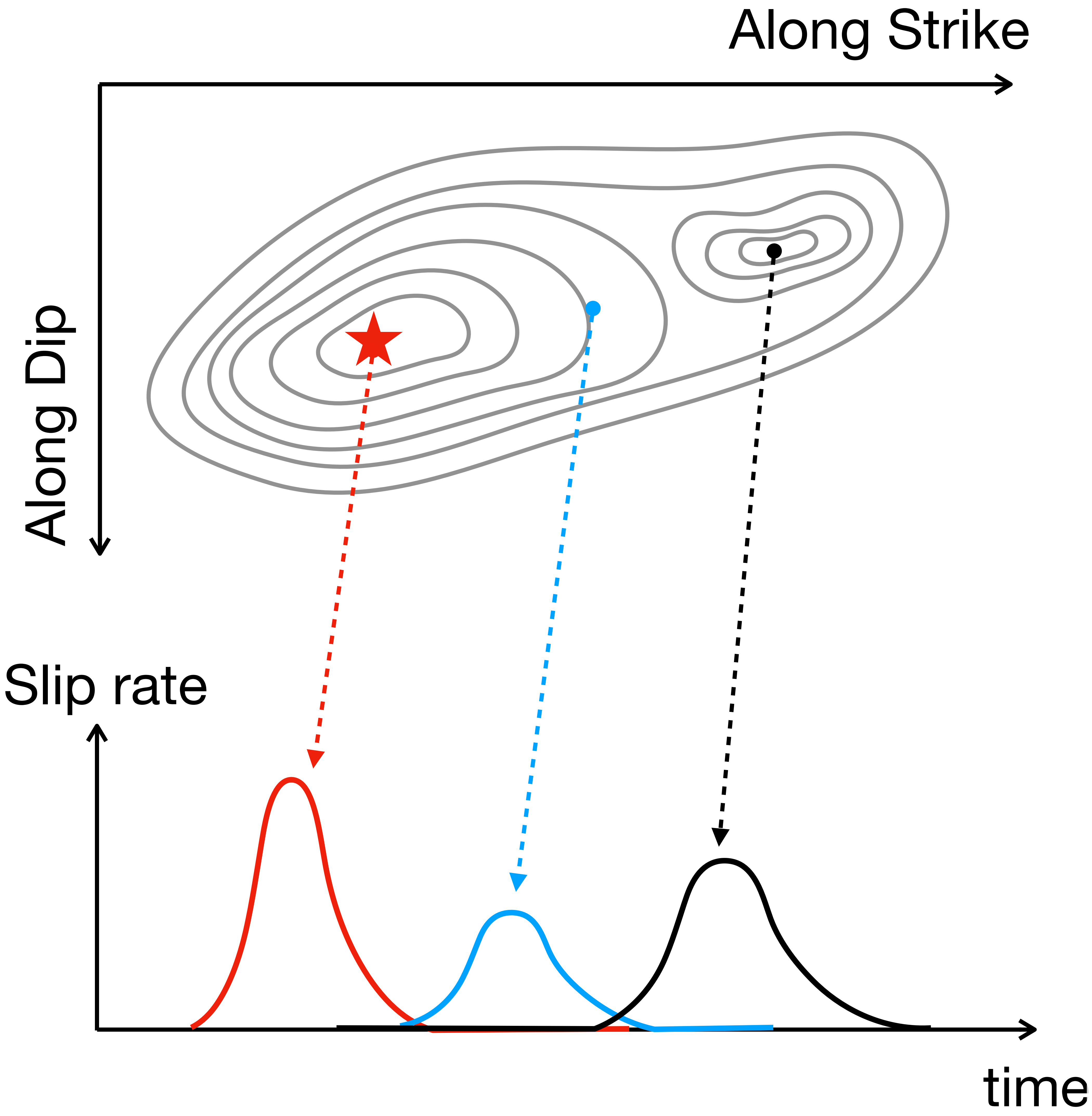


$\textit{variance} = \int f(x)\, (x - \textit{mean})^2 \, dx$

First degree moment: centroid

Second degree moment: width

Moment rate function



Moment rate density function

$$\boxed{\dot{f}(\underline{r}, t)}$$

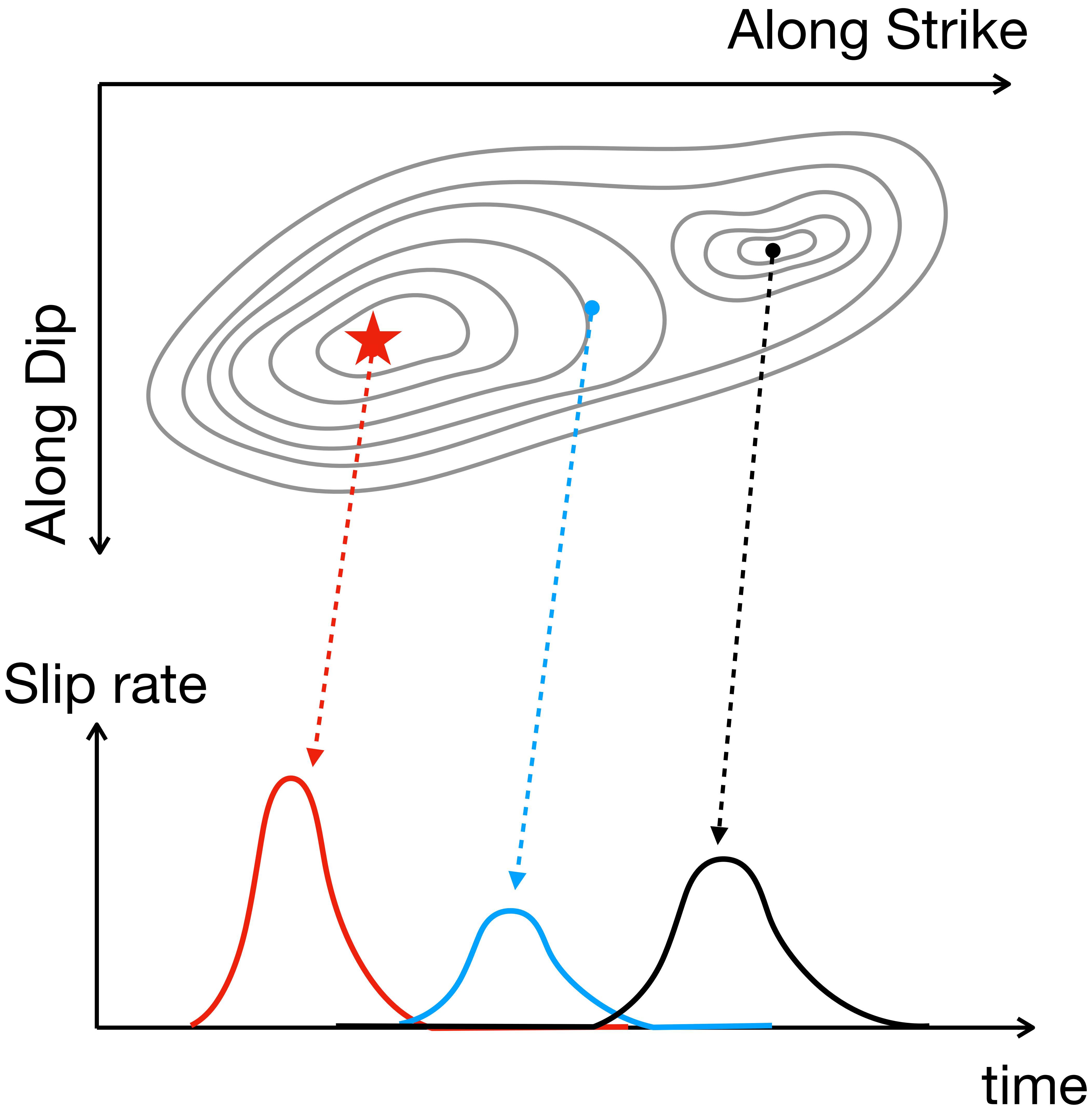
Integrate to unity

$$\boxed{\int\int \dot{f}(\underline{r}, t)\mathrm{d}V\mathrm{d}t = 1}$$

Source time function

$$\boxed{\int \dot{f}(\underline{r}, t)\mathrm{d}V = \mathrm{STF}(t)}$$

Moment rate function



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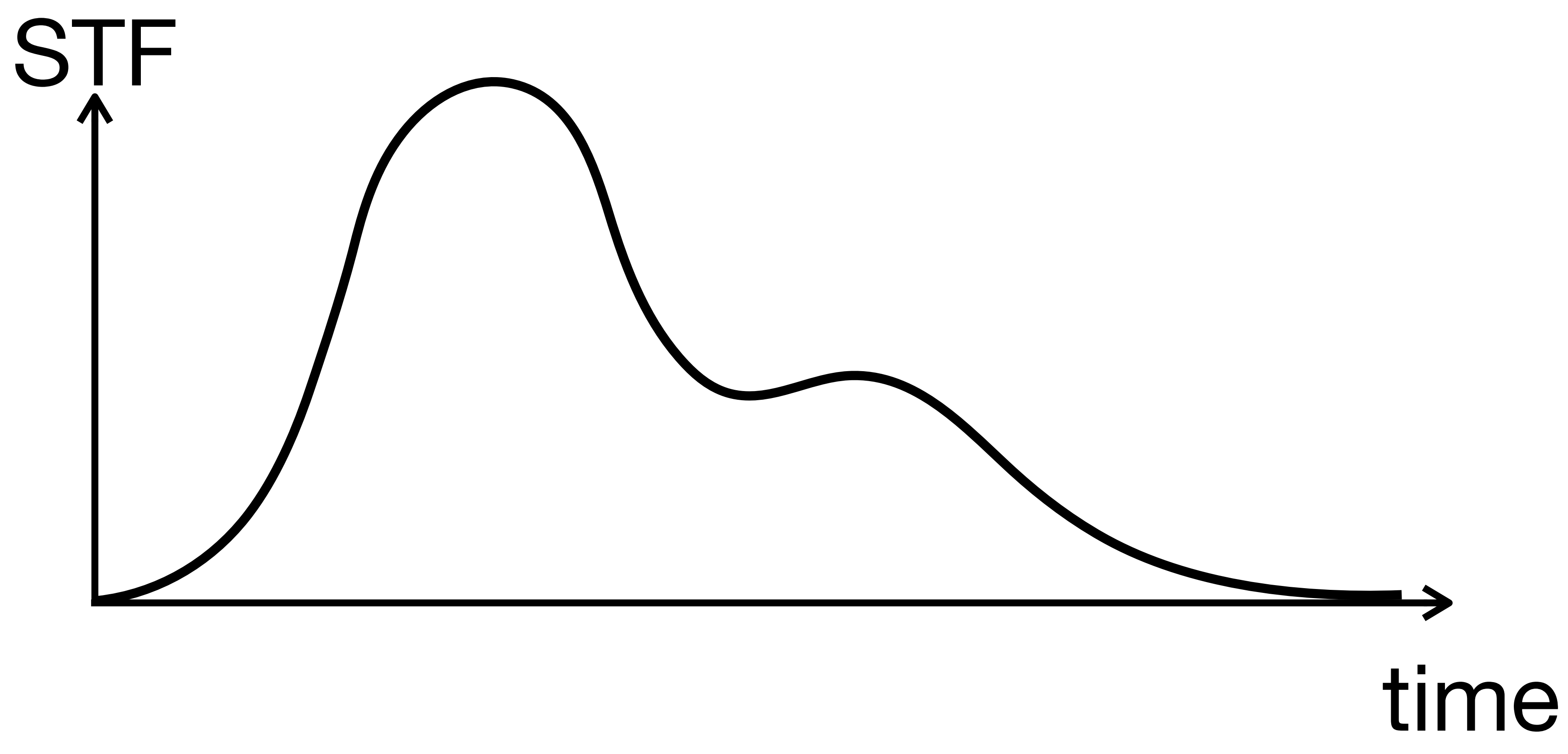
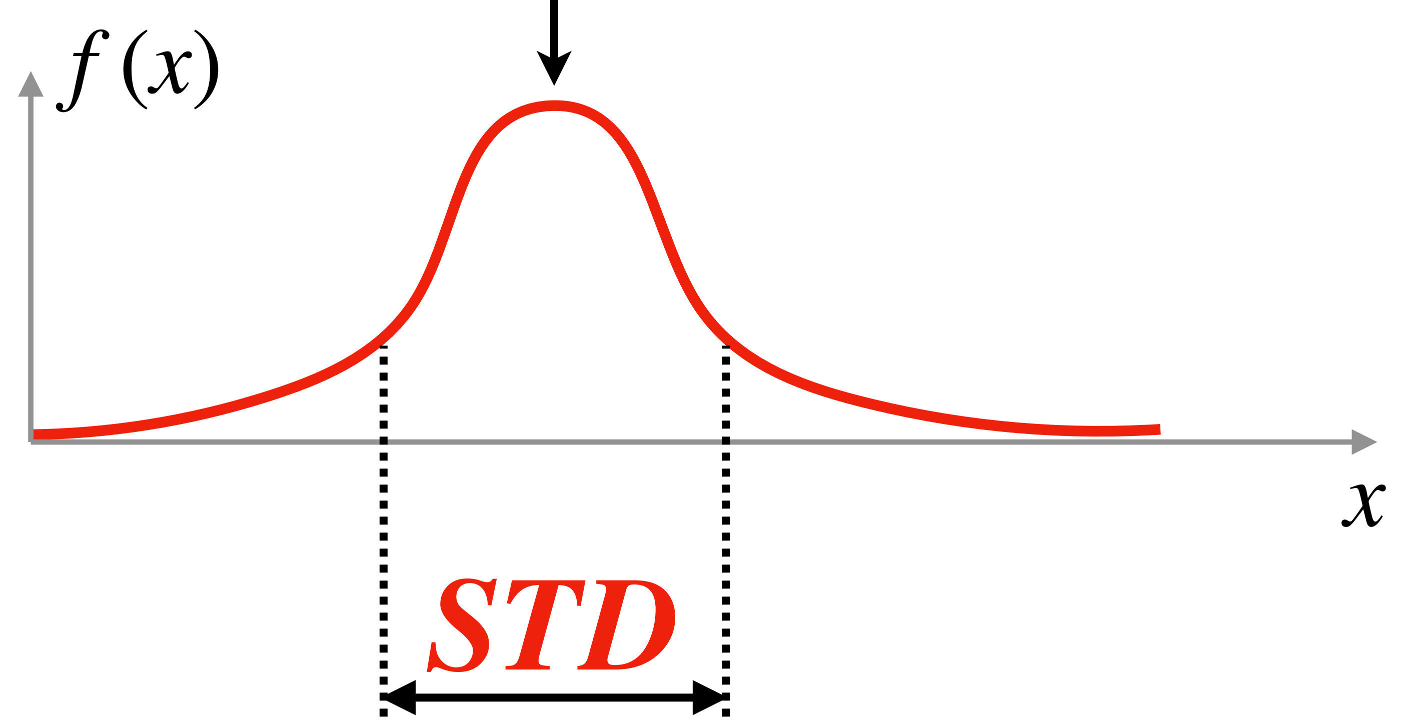


PDF

mean $= \int f(x)\, x \,\mathrm{d}x$



variance $= \int f(x)\,(x - \textit{mean})^2 \,\mathrm{d}x$

Space

Time

$$\dot{f}(\underline{r}, t)$$

Normalized moment rate density function

First degree moments

Centroid location $\underline{r}_0 = \int \dot{f}(\underline{r}, t)\,\underline{r}\,\mathrm{d}V\mathrm{d}t$

Centroid time $t_0 = \int \dot{f}(\underline{r}, t)\,t\,\mathrm{d}V\mathrm{d}t$

Second degree moments

Spatial moment

$$\hat{\underline{\underline{\mu}}}^{(2,0)} = \int\int \dot{f}(\underline{r}, t)(\underline{r} - \underline{r}_0)^{\mathrm{T}}(\underline{r} - \underline{r}_0)\mathrm{d}V\mathrm{d}t$$

Temporal moment

$$\hat{\mu}^{(0,2)} = \int\int \dot{f}(\underline{r}, t)(t - t_0)(t - t_0)\mathrm{d}V\mathrm{d}t$$

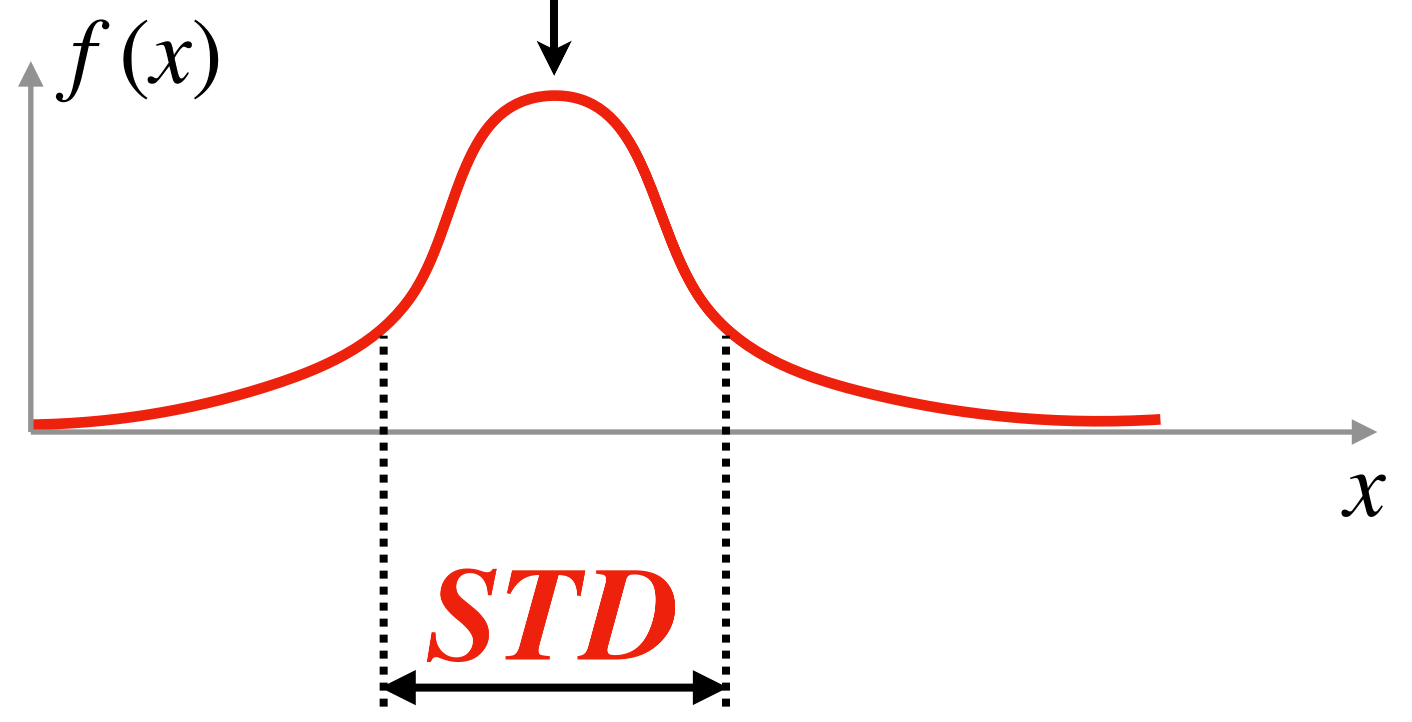
Spatiotemporal moment

$$\hat{\underline{\mu}}^{(1,1)} = \int\int \dot{f}(\underline{r}, t)(\underline{r} - \underline{r}_0)(t - t_0)\mathrm{d}V\mathrm{d}t$$

McGuire, 2014 & 2017

PDF

$$\textit{mean} = \int f(x)\, x \,\mathrm{d}x$$



$$\textit{variance} = \int f(x)\, (x - \textit{mean})^2 \,\mathrm{d}x$$

Space

Time

$$\dot{f}(\underline{r}, t)$$

Normalized moment rate density function

First degree moments

Centroid location $\underline{r}_0 = \int \dot{f}(\underline{r}, t)\underline{r}\,\mathrm{d}V\mathrm{d}t$

Centroid time $t_0 = \int \dot{f}(\underline{r}, t)t\,\mathrm{d}V\mathrm{d}t$

Second degree moments

Spatial moment

$$\underline{\underline{\widehat{\mu}}}^{(2,0)} = \int\int \dot{f}(\underline{r}, t)(\underline{r} - \underline{r}_0)^{\mathrm{T}}(\underline{r} - \underline{r}_0)\mathrm{d}V\mathrm{d}t$$

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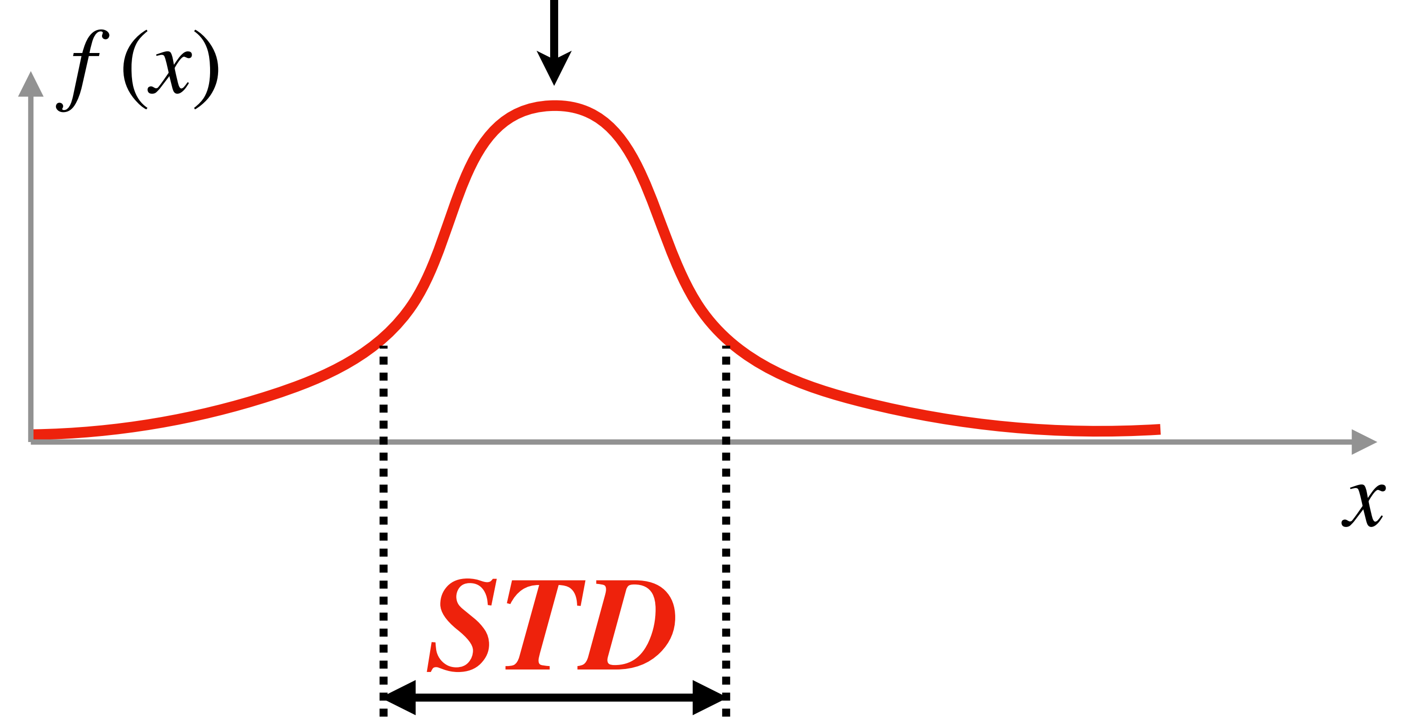
Spatiotemporal moment

$$\underline{\widehat{\mu}}^{(1,1)} = \int\int \dot{f}(\underline{r}, t)(\underline{r} - \underline{r}_0)(t - t_0)\mathrm{d}V\mathrm{d}t$$

McGuire, 2014 & 2017

PDF

$$\textit{mean} = \int f(x)\, x\, \mathrm{d}x$$



$$\textit{variance} = \int f(x)\, (x - \textit{mean})^2\, \mathrm{d}x$$

Space

Time

$$\dot{f}(\underline{r}, t)$$

Normalized moment rate density function

First degree moments

Centroid location $\underline{r}_0 = \iint \dot{f}(\underline{r}, t)\,\underline{r}\,\mathrm{d}V\mathrm{d}t$

Centroid time $t_0 = \iint \dot{f}(\underline{r}, t)\,t\,\mathrm{d}V\mathrm{d}t$

Second degree moments

Spatial moment

$$\underline{\underline{\widehat{\mu}}}^{(2,0)} = \iint \dot{f}(\underline{r}, t)\,(\underline{r} - \underline{r}_0)^{\mathrm{T}}\,(\underline{r} - \underline{r}_0)\,\mathrm{d}V\mathrm{d}t$$

Temporal moment

$$\widehat{\mu}^{(0,2)} = \iint \dot{f}(\underline{r}, t)\,(t - t_0)\,(t - t_0)\,\mathrm{d}V\mathrm{d}t$$

Spatiotemporal moment

$$\underline{\widehat{\mu}}^{(1,1)} = \iint \dot{f}(\underline{r}, t)\,(\underline{r} - \underline{r}_0)\,(t - t_0)\,\mathrm{d}V\mathrm{d}t$$

McGuire, 2014 & 2017

What can we infer from second moments?

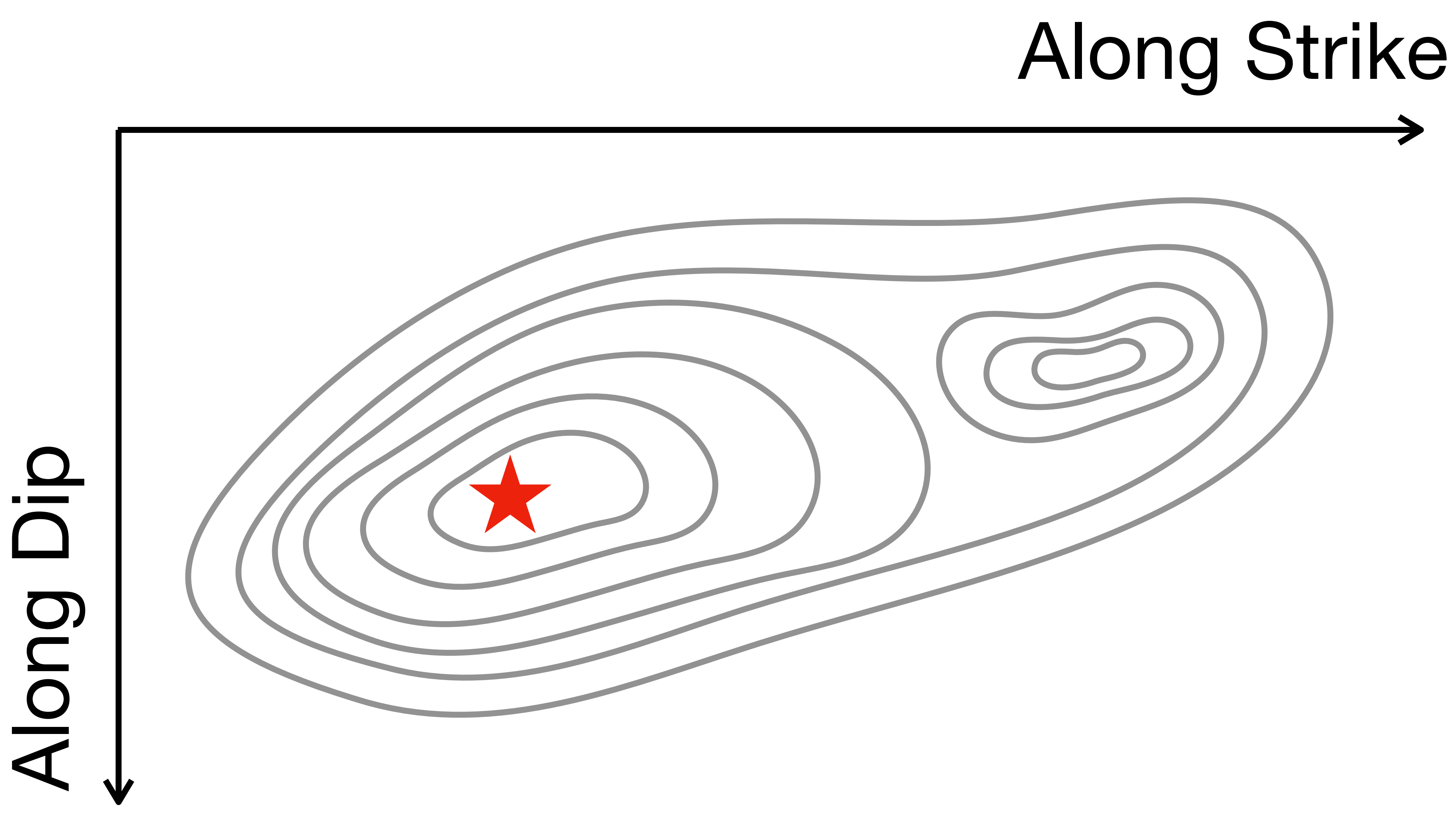
Rupture dimension

Spatial moment: 3 by 3 tensor

$$\underline{\underline{\widehat{\mu}}}^{(2,0)} = \int\int \dot{f}(\underline{r}, t)(\underline{r} - \underline{r}_0)^{\mathrm{T}} (\underline{r} - \underline{r}_0)\mathrm{d}V\mathrm{d}t$$

Characteristic Rupture length $\underline{L_c}$ and width W_c

$$\boxed{x_c(\underline{\widehat{n}}) = 2\sqrt{\underline{\widehat{n}}^T \widehat{\mu}^{(2,0)} \underline{\widehat{n}}}}$$



$\underline{L_c}$ W_c

McGuire, 2014 & 2017

Rupture dimension

Spatial moment: 3 by 3 tensor

$$\underline{\underline{\widehat{\mu}}}^{(2,0)} = \int\int \dot{f}(\underline{r}, t)(\underline{r} - \underline{r}_0)^{\mathrm{T}} (\underline{r} - \underline{r}_0)\mathrm{d}V\mathrm{d}t$$

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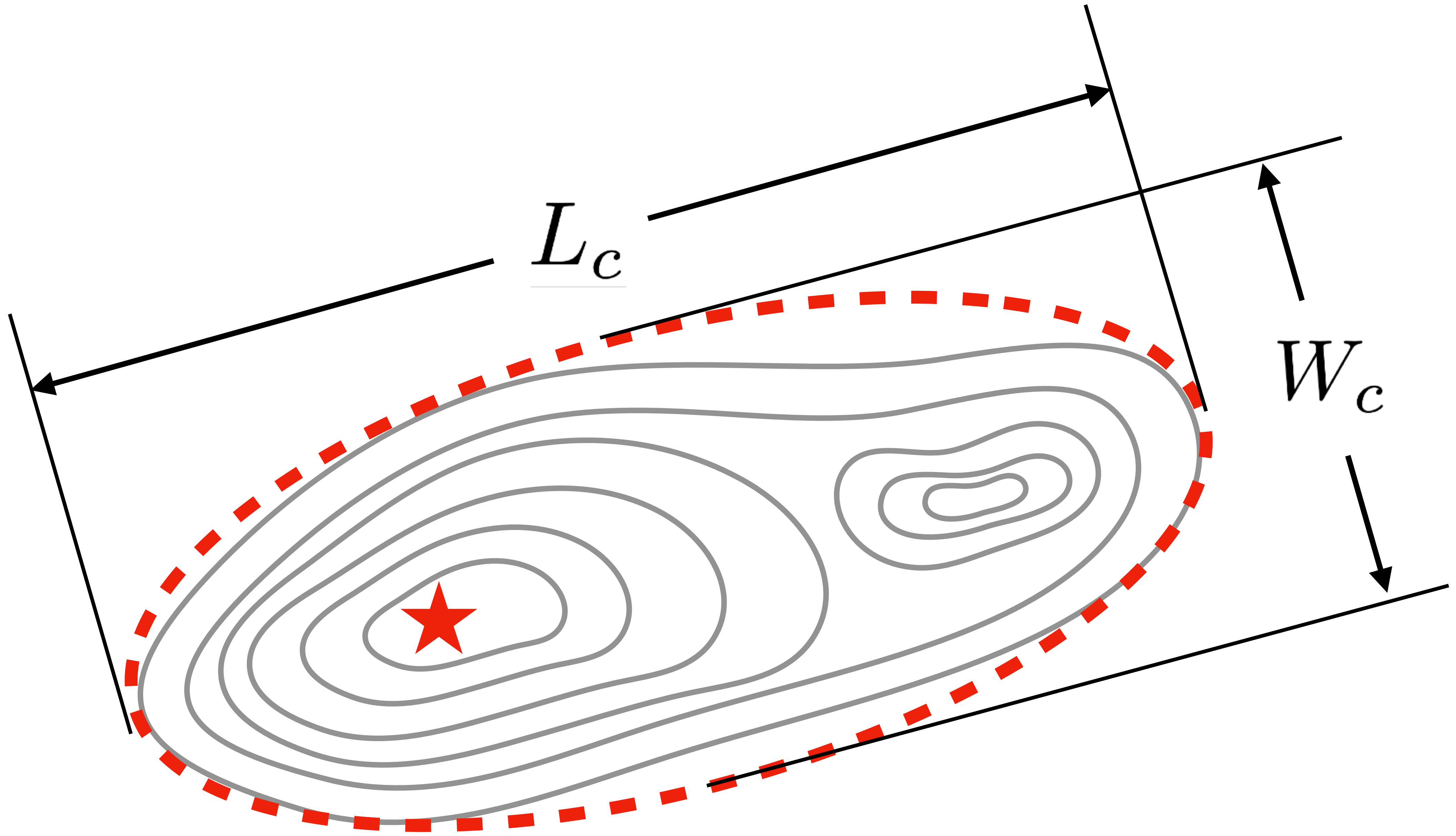


$\underline{L_c}$ W_c

McGuire, 2014 & 2017

Rupture duration

Temporal moment: scalar

$$\widehat{\mu}^{(0,2)} = \int\int \dot{f}(\underline{r},t)(t-t_0)\,(t-t_0)\,\mathrm{d}V\,\mathrm{d}t$$

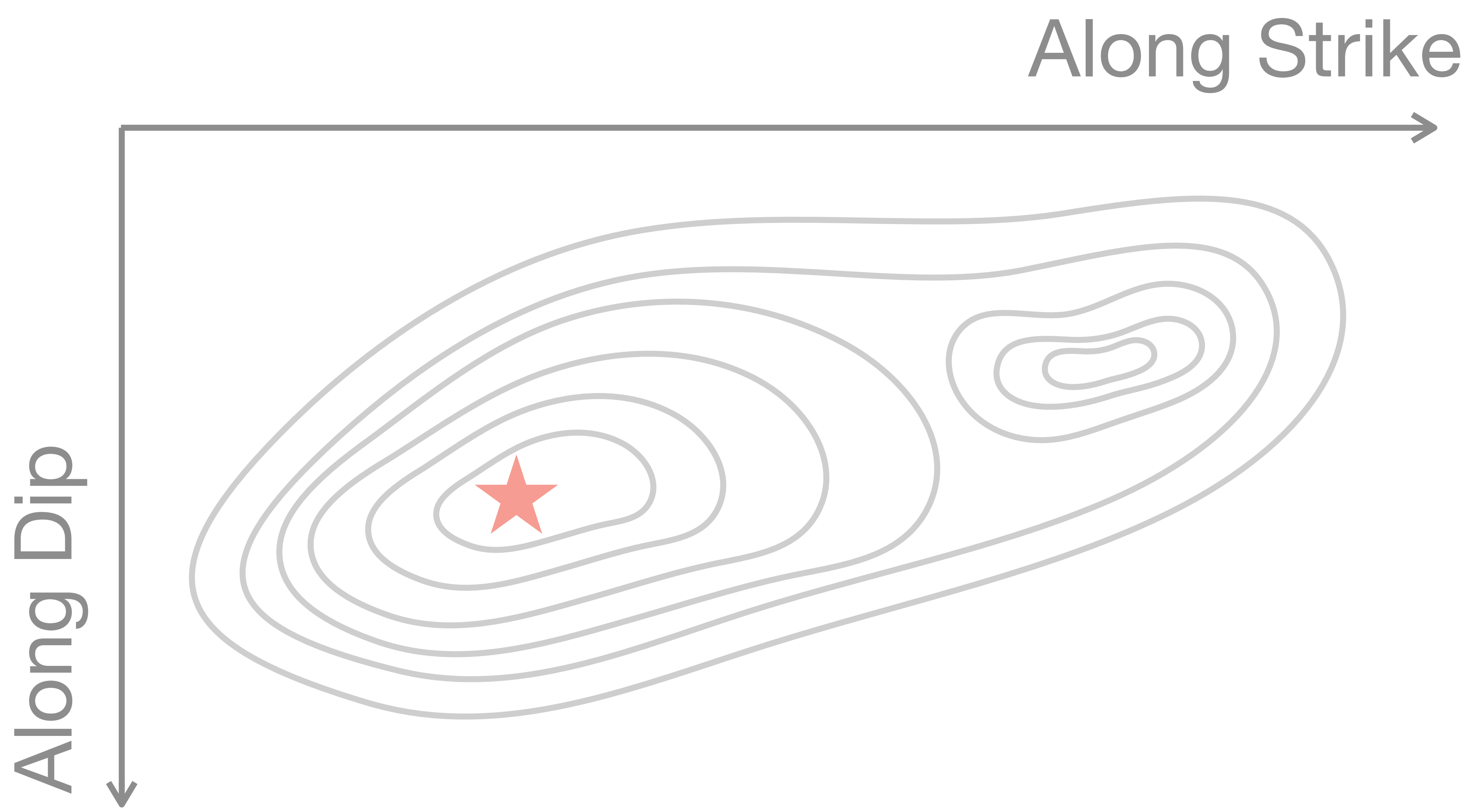
Characteristic duration

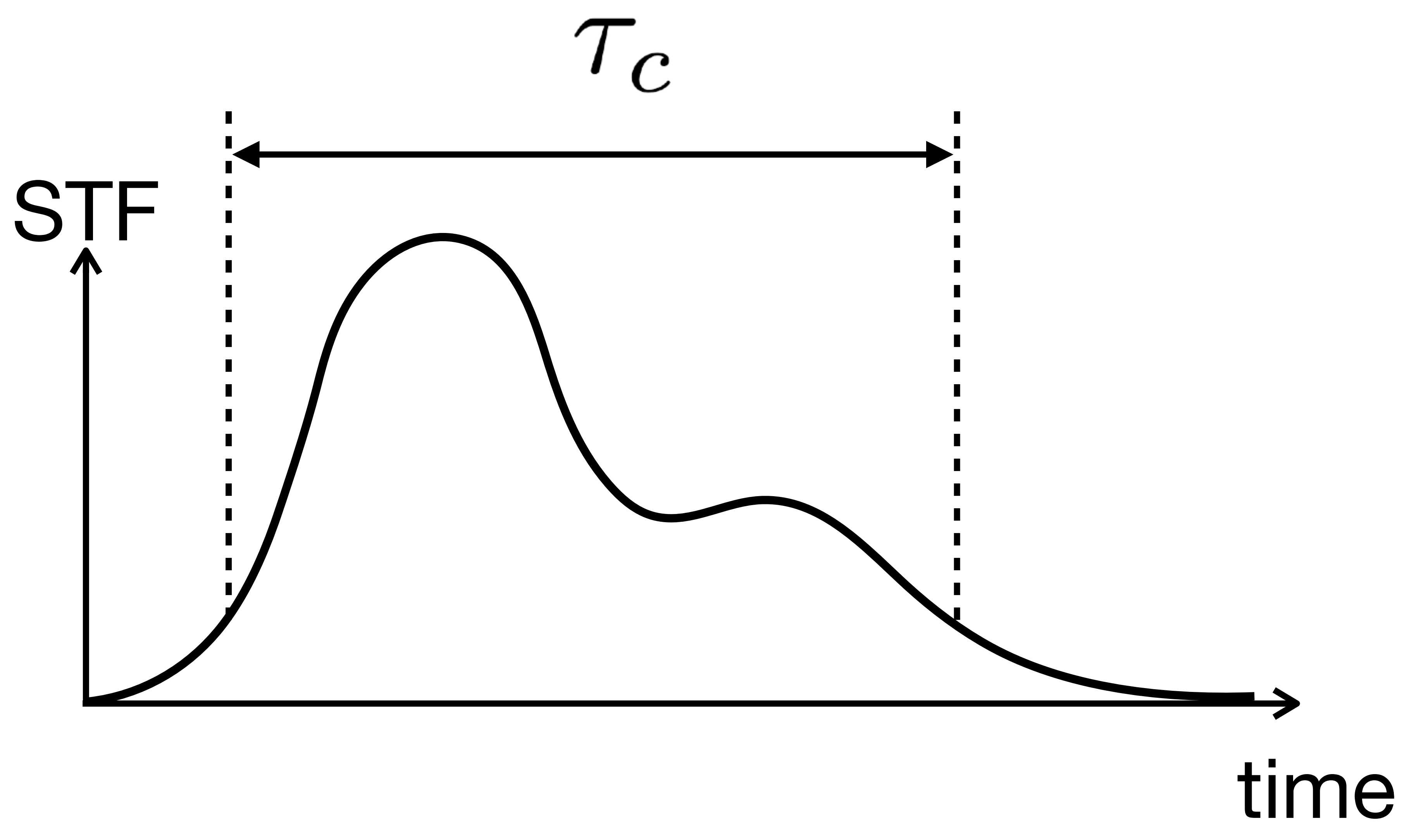
$$\boxed{\tau_c = 2\sqrt{\widehat{\mu}^{(0,2)}}}$$

$$\boxed{\int \dot{f}(\underline{r},t)\mathrm{d}V = \mathrm{STF}(t)}$$

$L_c \quad W_c \quad \tau_c$

McGuire, 2014 & 2017





Rupture propagation

Spatiotemporal moment: 3 by 1 vector

$$\underline{\widehat{\mu}}^{(1,1)} = \iint \dot{f}(\underline{r}, t)(\underline{r} - \underline{r}_0)\,(t - t_0)\,\mathrm{d}V\,\mathrm{d}t$$

Temporal moment: scalar

$$\widehat{\mu}^{(0,2)} = \iint \dot{f}(\underline{r}, t)(t - t_0)\,(t - t_0)\,\mathrm{d}V\,\mathrm{d}t$$

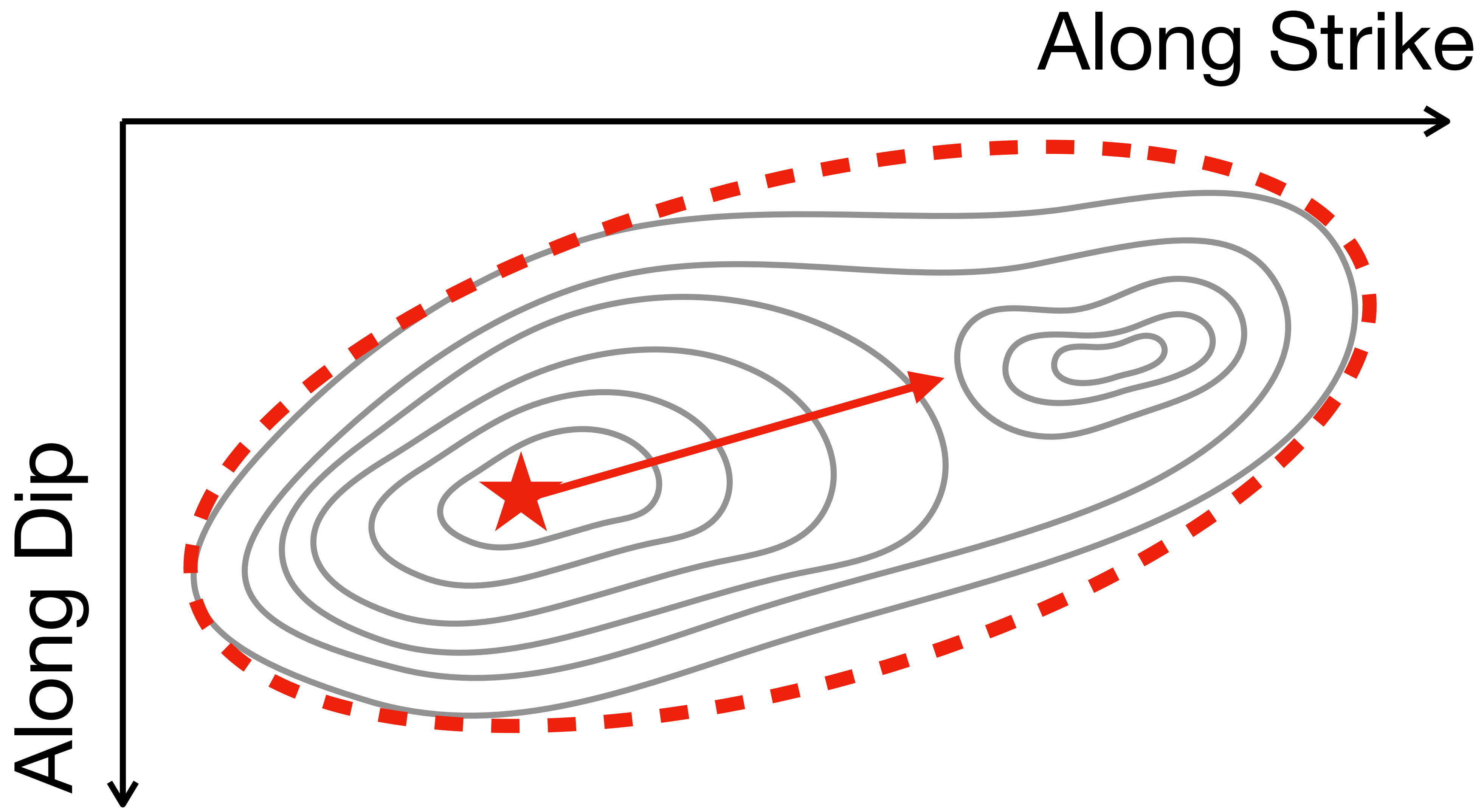
Centroid rupture velocity

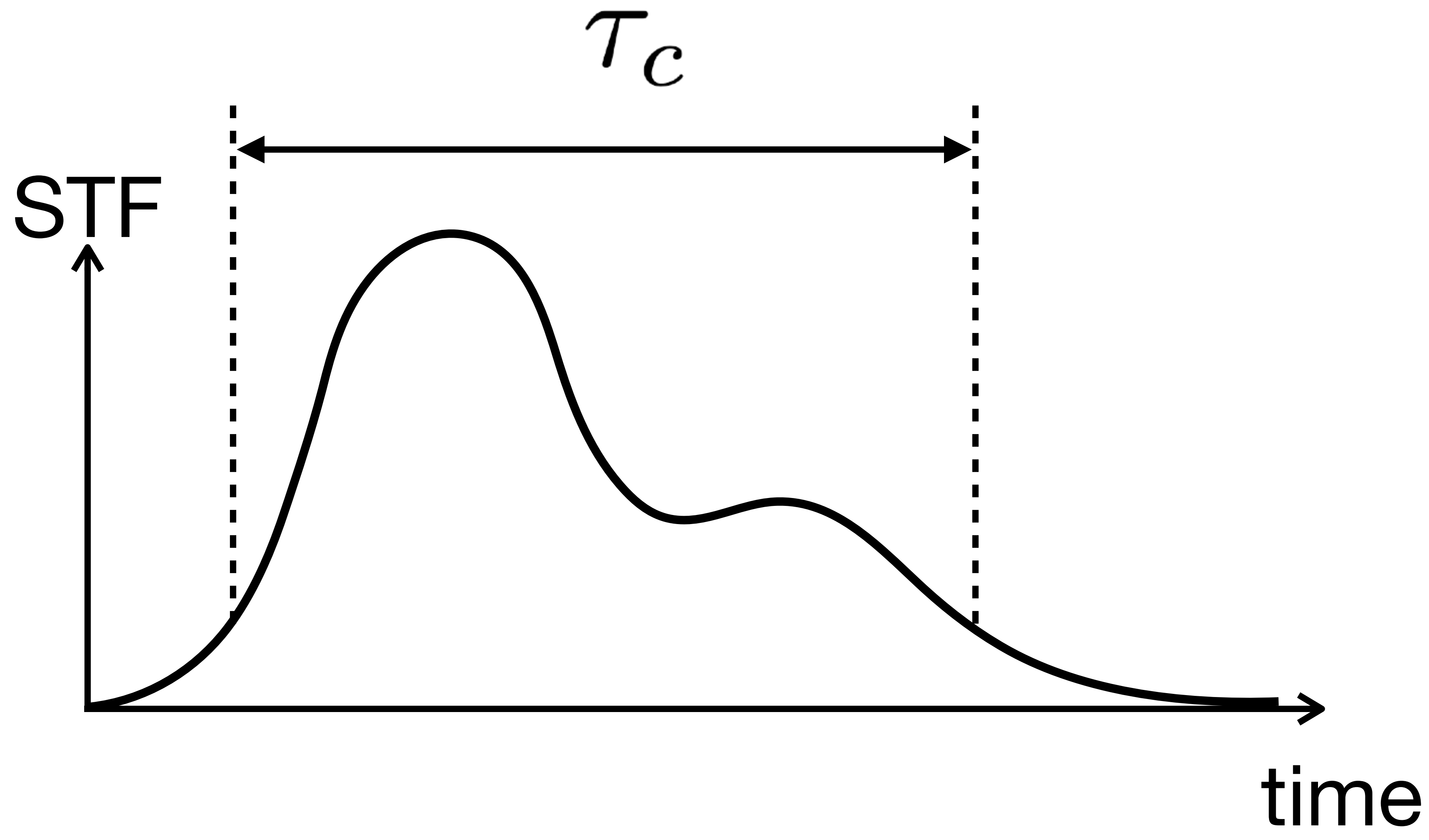
$$\boxed{\underline{v}_0 = \frac{\underline{\widehat{\mu}}^{(1,1)}}{\widehat{\mu}^{(0,2)}}}$$

$\underline{v}_0 = 0$ for perfect symmetric rupture

$\leq$ the propagating velocity of rupture front

$L_c \quad W_c \quad \tau_c \quad \underline{v}_0$





McGuire, 2014 & 2017

Rupture propagation

Spatiotemporal moment: 3 by 1 vector

$$\underline{\widehat{\mu}}^{(1,1)} = \int\int \dot{f}(\underline{r}, t)(\underline{r} - \underline{r}_0)\,(t - t_0)\mathrm{d}V\mathrm{d}t$$

Temporal moment: scalar

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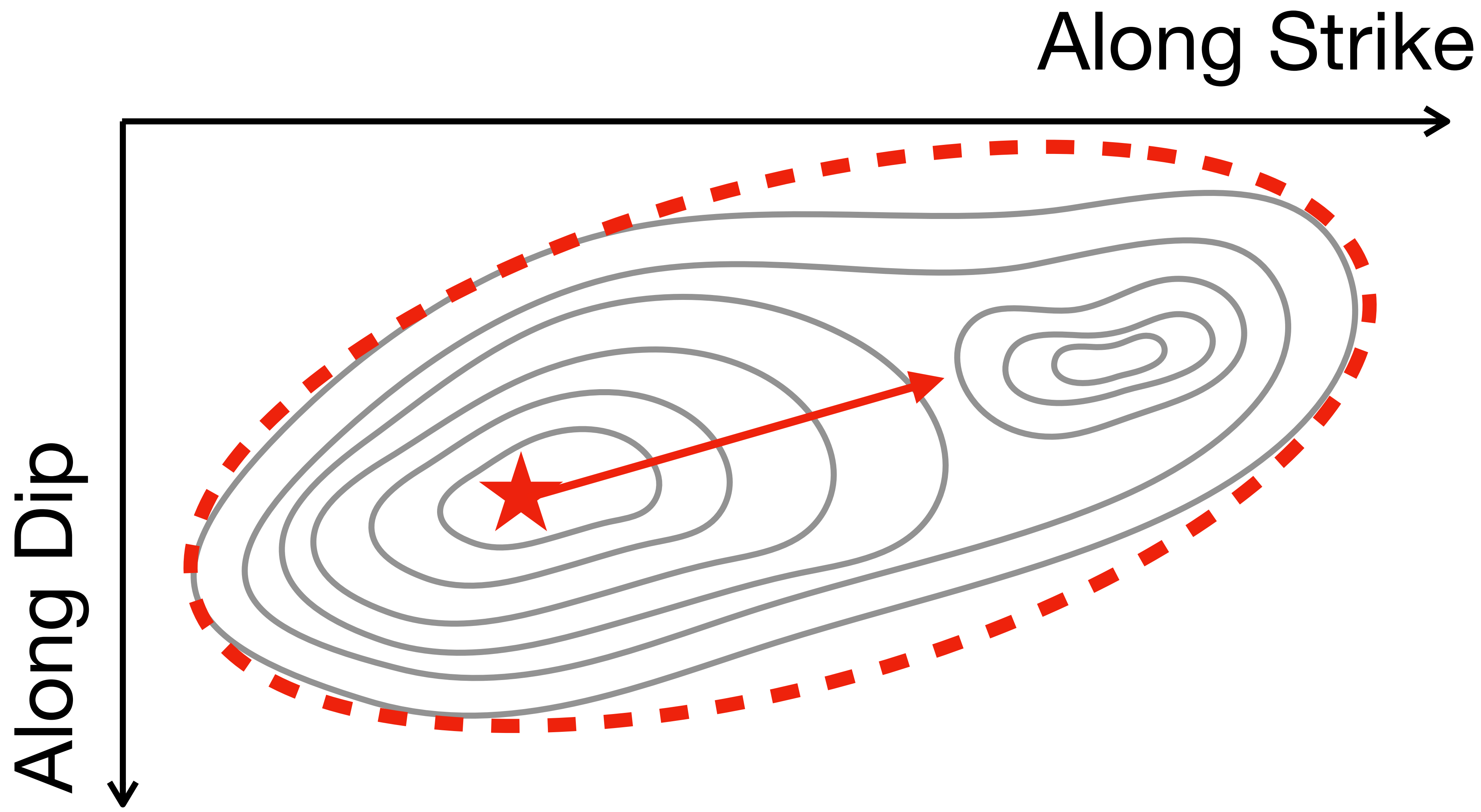
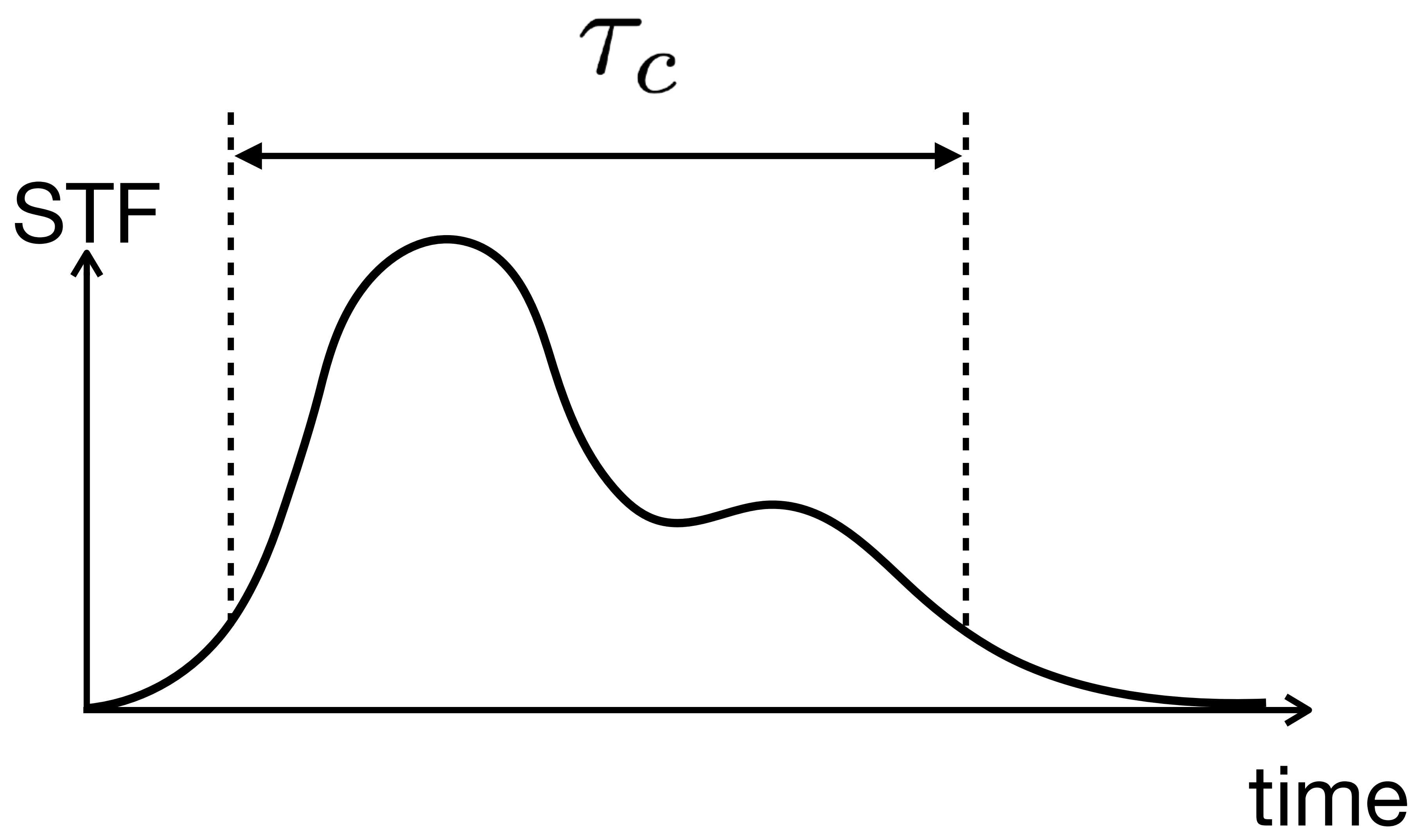
$L_c \quad W_c \quad \tau_c \quad \underline{v}_0$





McGuire, 2014 & 2017

Rupture propagation

Spatiotemporal moment: 3 by 1 vector

$$\widehat{\mu}^{(1,1)} = \iint \dot{f}(\underline{r}, t)(\underline{r} - \underline{r}_0)\,(t - t_0)\,\mathrm{d}V\,\mathrm{d}t$$

Temporal moment: scalar

$$\widehat{\mu}^{(0,2)} = \iint \dot{f}(\underline{r}, t)(t - t_0)\,(t - t_0)\,\mathrm{d}V\,\mathrm{d}t$$

Centroid rupture velocity

$$\boxed{\underline{v}_0 = \frac{\widehat{\underline{\mu}}^{(1,1)}}{\widehat{\mu}^{(0,2)}}}$$

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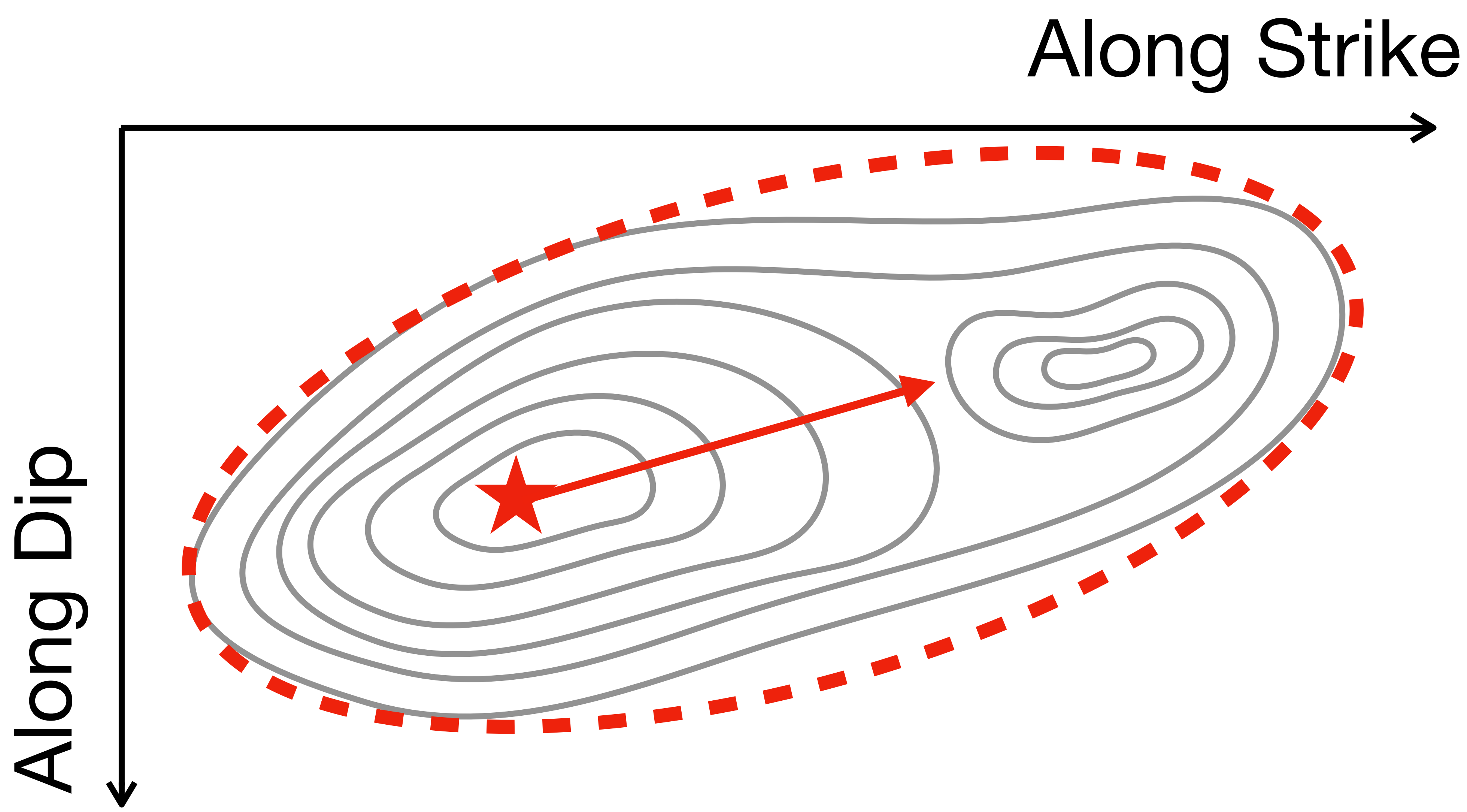
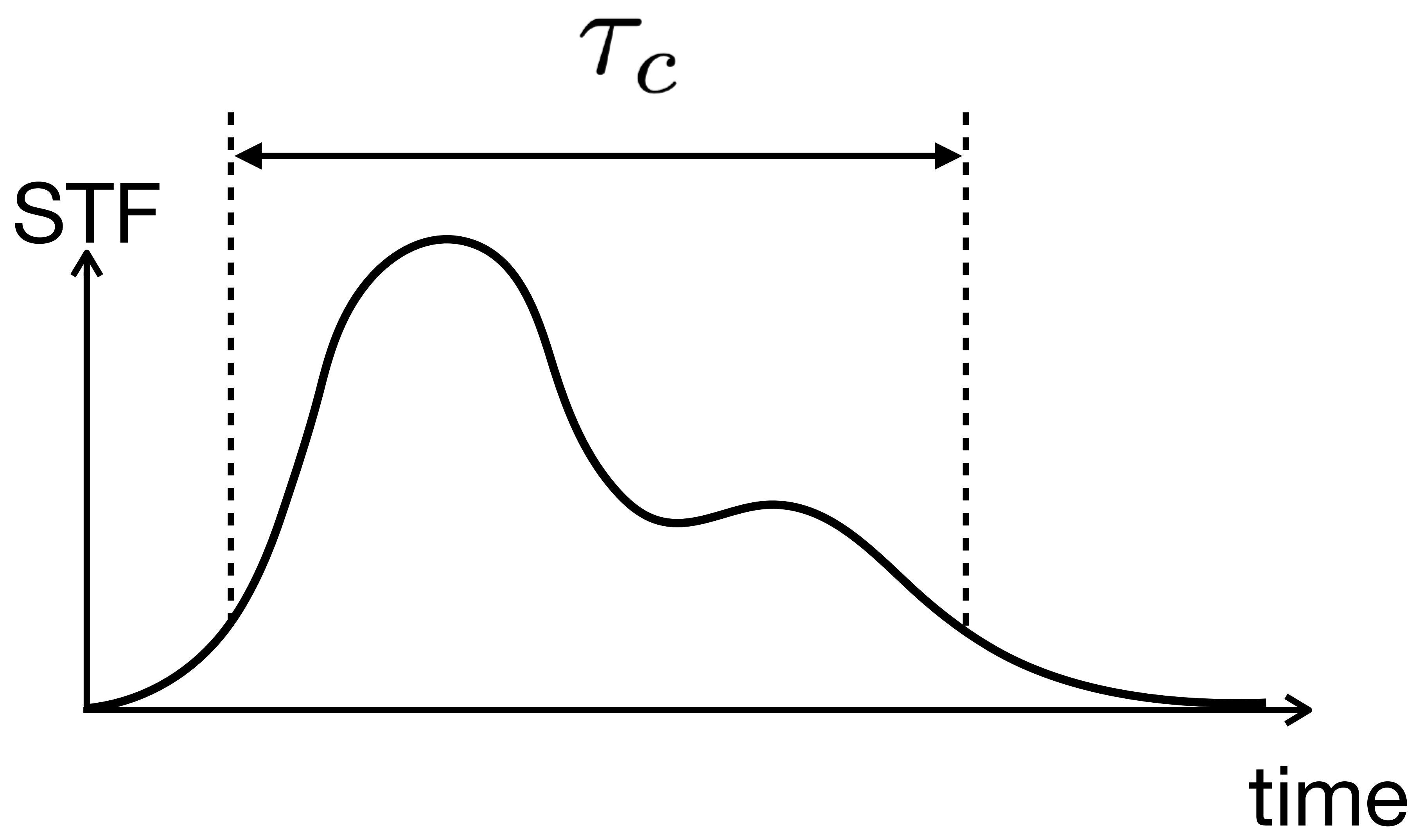
$\leq$ the propagating velocity of rupture front

$\underline{L}_c \quad W_c \quad \tau_c \quad \underline{v}_0$





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Rupture propagation

Apparent characteristic rupture velocity

$$\boxed{v_c = \frac{L_c}{\tau_c}}$$

≥ the centroid rupture velocity
≥ actual rupture front velocity

Directivity ratio

$$\boxed{dir = \frac{|\underline{v_0}|}{v_c}}$$

	Perfect symmetric bilateral rupture	Uniform slip unilateral rupture
v_c	2*Vr	Vr
dir	0	1

$\underline{L_c}$ W_c τ_c $\underline{v_0}$ v_c dir

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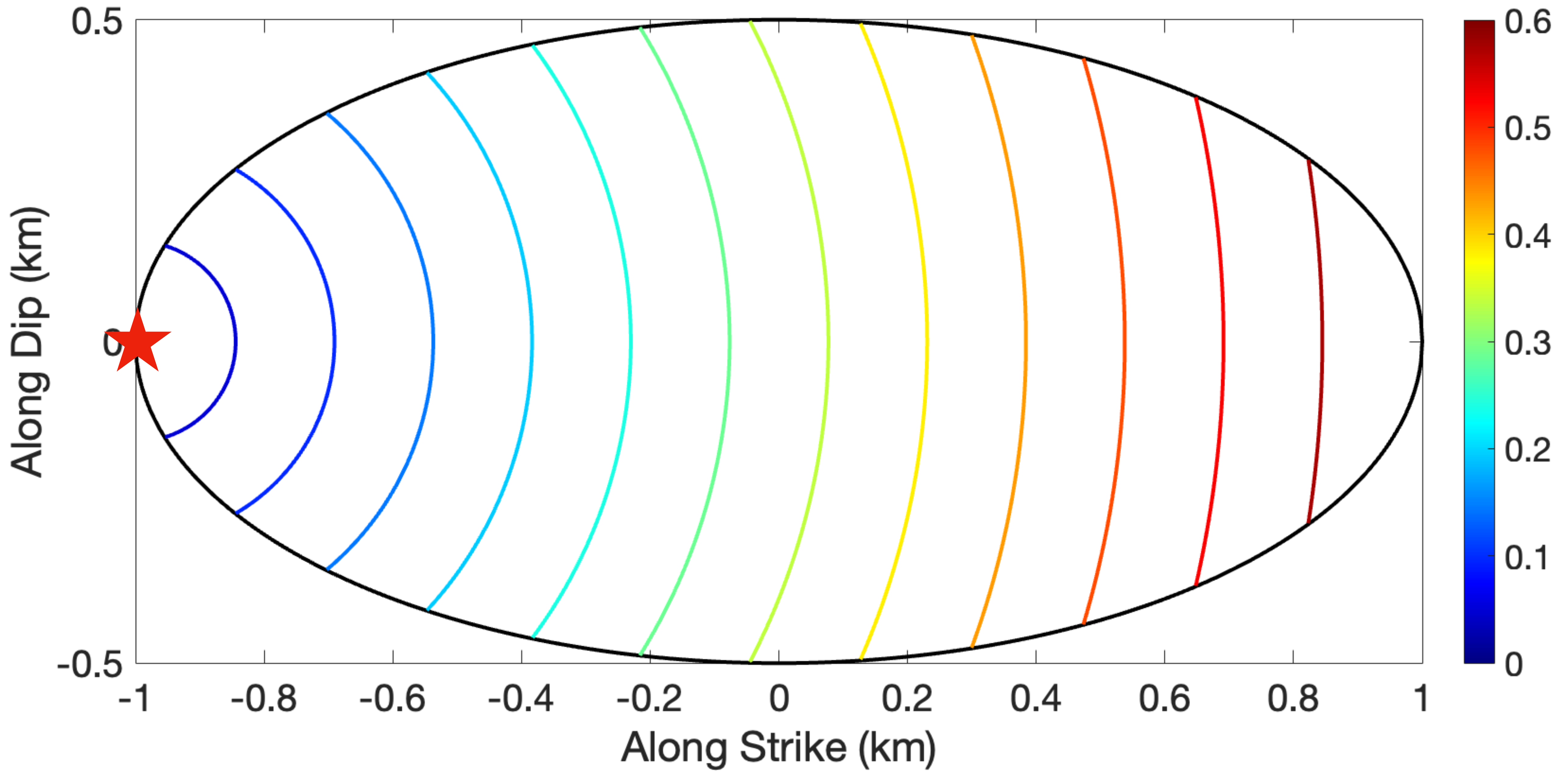
2D example

- Strike slip earthquake
  - Elliptical rupture area
    - Semi-major axis: 1km
    - Semi-minor axis: 0.5 km
  - Unilateral rupture:
    - Starting from the left corner
    - Vr = 3.2 km/s
  - Uniform slip: slip = 1m



Matlab code

```
clear; clc; close all;

strike = 0; dip = 90;

Lc = 1; % km along strike
Wc = Lc/2; % km along dip
Vr = 3.5613*0.9; % rupture velocity
```

2D example

- Strike slip earthquake
  - Elliptical rupture area
    - Semi-major axis: 1km
    - Semi-minor axis: 0.5 km
  - Unilateral rupture:
    - Starting from the left corner
    - Vr = 3.2 km/s
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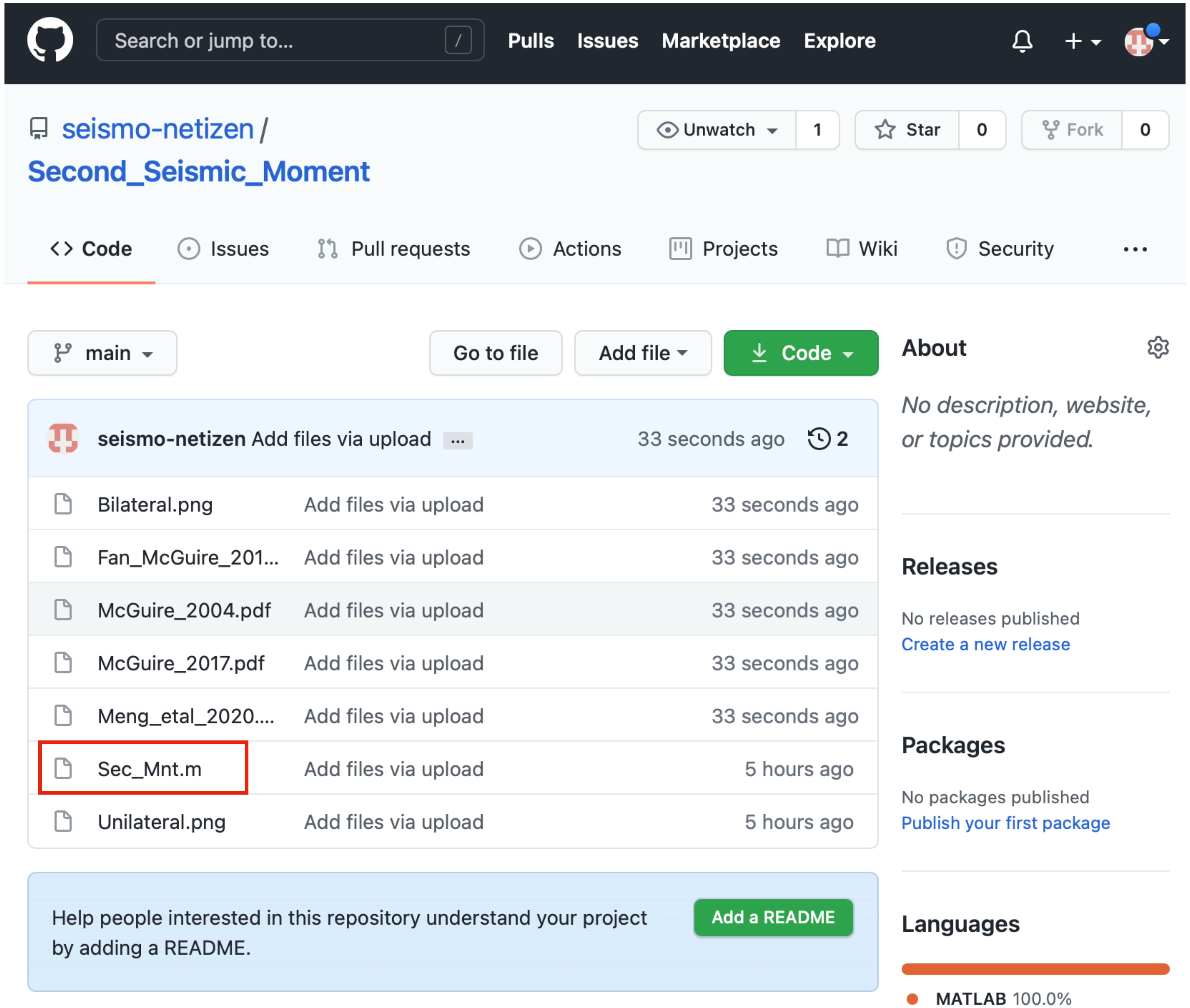
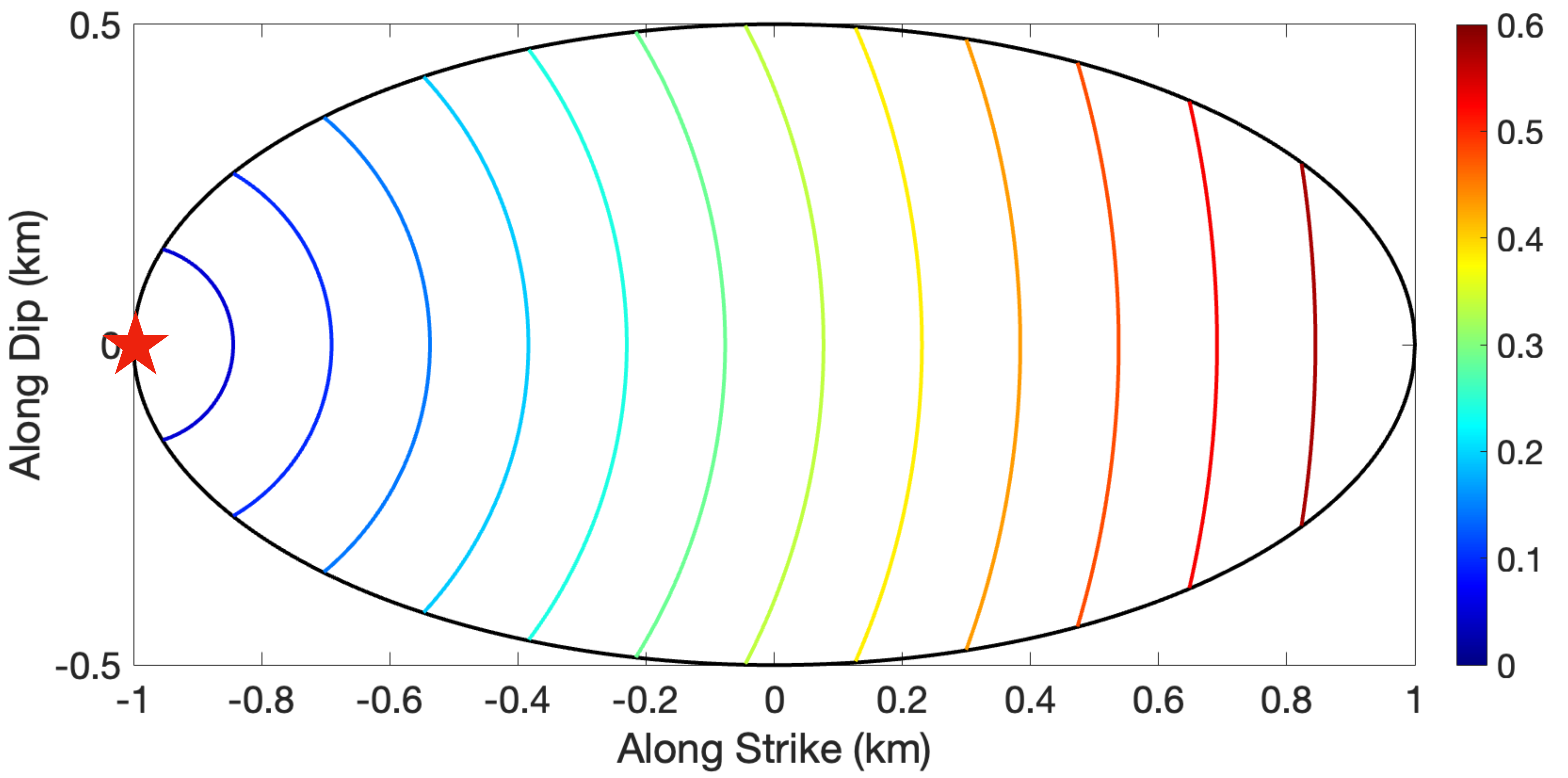


https://github.com/seismo-netizen/Second_Seismic_Moment.git

Centroid

- Compute first seismic moments
  - Centroid location
  - Centroid time



Centroid location $\underline{r}_0 = \int \dot{f}(\underline{r}, t)\underline{r}\,\mathrm{d}V\mathrm{d}t$

$= (0.000, 0.000)$ km

Centroid time $t_0 = \int \dot{f}(\underline{r}, t)t\,\mathrm{d}V\mathrm{d}t$

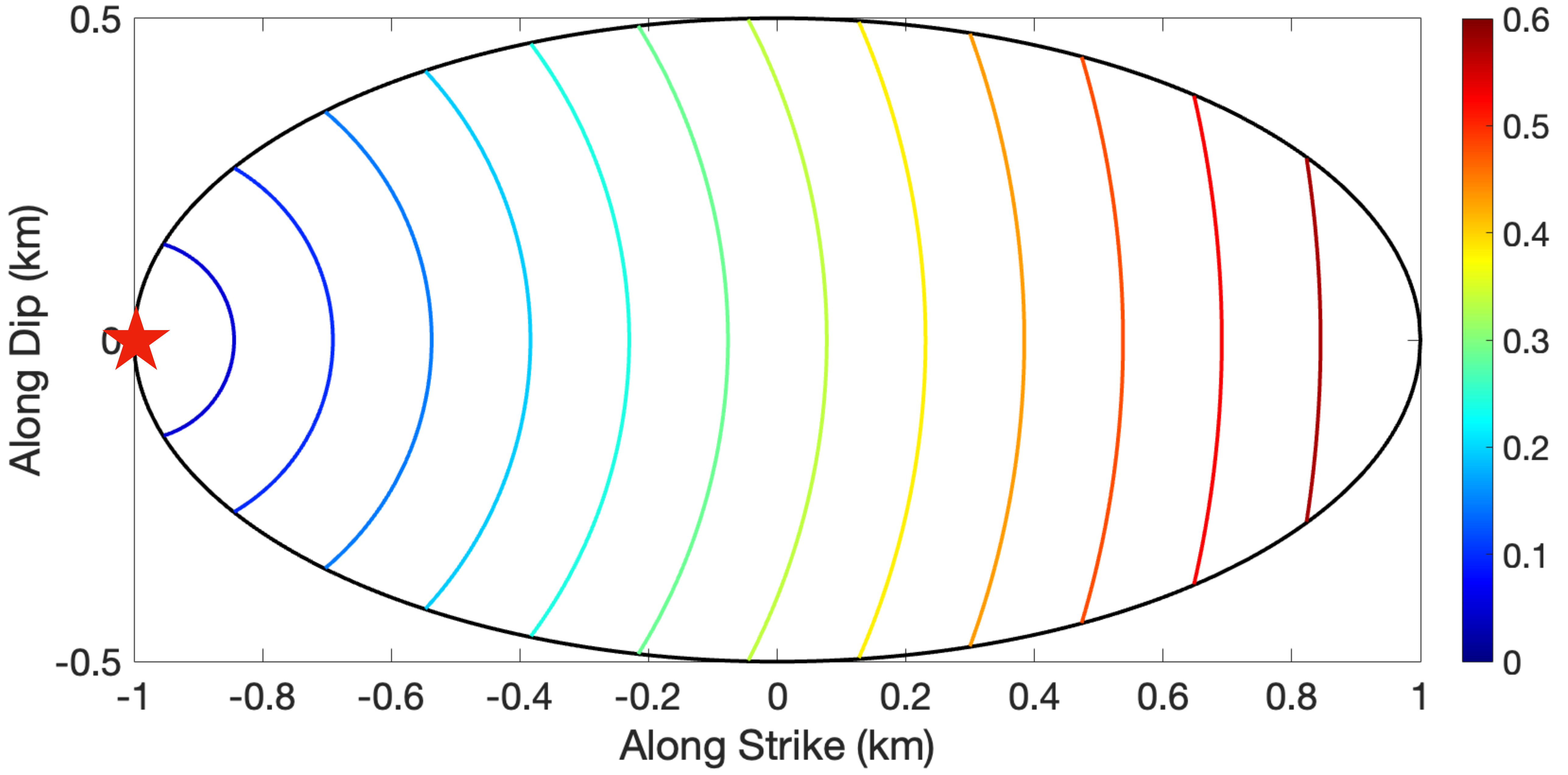
$= 0.324$ s

Matlab code

```
%% centroid
x0 = 0; y0 = 0; t0 = 0;
for i = 1:length(x)
    for j = 1:length(y)
        if W(i,j) == 1
            x0 = x0 + x(i)*dx*dy/sum_f;
            y0 = y0 + y(j)*dx*dy/sum_f;
            dist = sqrt((x(i) - xc)^2 + (y(j) - yc)^2);
            t0 = t0 + (dist/Vr)*dx*dy/sum_f;
        end
    end
end
```

Centroid

- Compute first seismic moments
  - Centroid location
  - Centroid time



Centroid location $$\underline{r}_0 = \int \dot{f}(\underline{r}, t)\underline{r}\mathrm{d}V\mathrm{d}t$$

= (0.000, 0.000) km

Centroid time $$t_0 = \int \dot{f}(\underline{r}, t)t\mathrm{d}V\mathrm{d}t$$

= 0.324 s

Matlab code

```
%% centroid
x0 = 0; y0 = 0; t0 = 0;
for i = 1:length(x)
    for j = 1:length(y)
        if W(i,j) == 1
            x0 = x0 + x(i)*dx*dy/sum_f;
            y0 = y0 + y(j)*dx*dy/sum_f;
            dist = sqrt((x(i) - xc)^2 + (y(j) - yc)^2);
            t0 = t0 + (dist/Vr)*dx*dy/sum_f;
        end
    end
end
```

Spatial moment

Spatial moment: 2 by 2 tensor

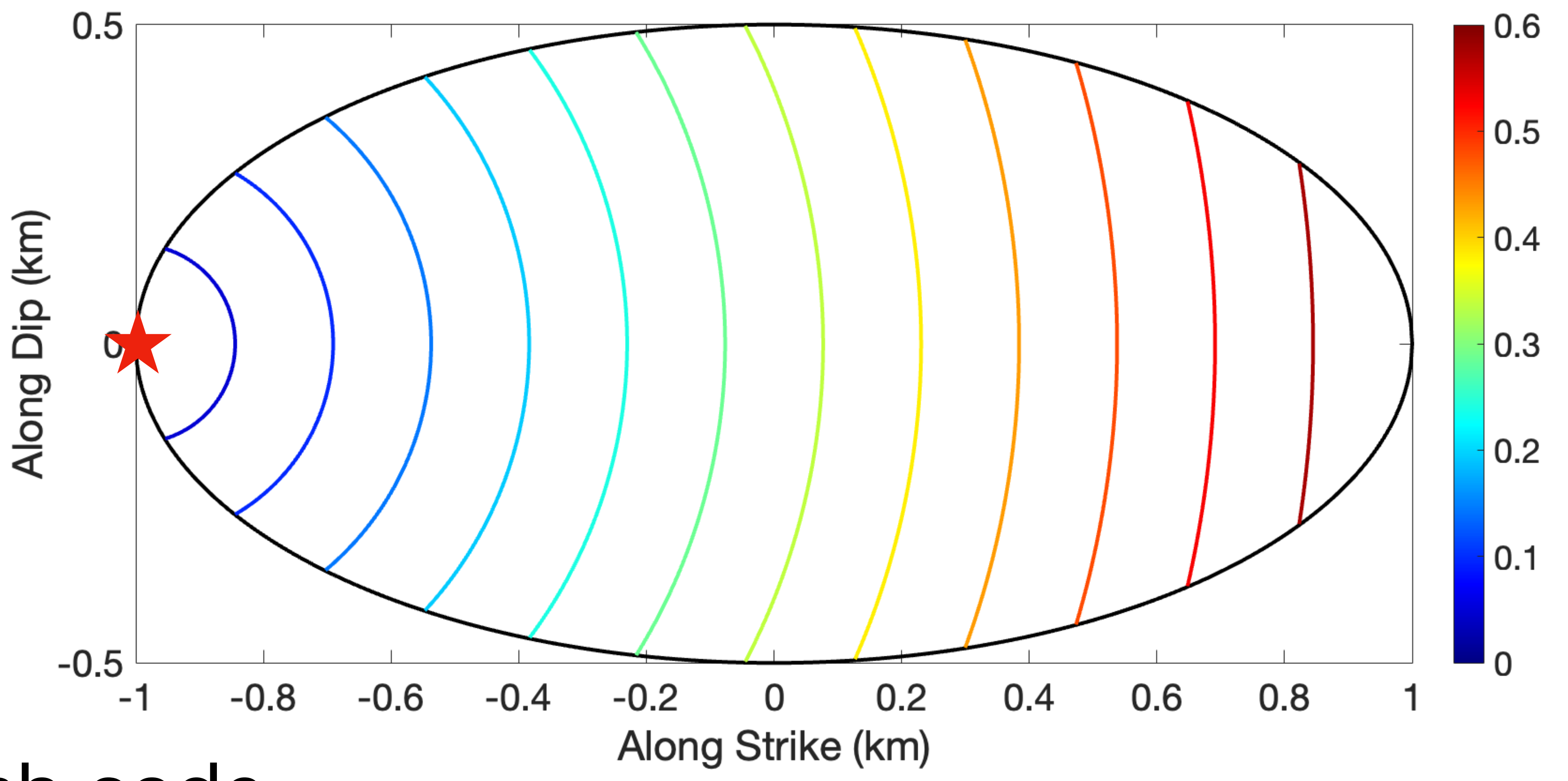
$$\underline{\underline{\widehat{\mu}}}^{(2,0)} = \iint \dot{f}(\underline{r},t)\,(\underline{r}-\underline{r}_0)^{\mathrm{T}}\,(\underline{r}-\underline{r}_0)\,\mathrm{d}V\,\mathrm{d}t$$

$$= \begin{vmatrix} 0.2500 & 0.0000 \\ 0.0000 & 0.0625 \end{vmatrix} \text{ (km^2)}$$

Characteristic Rupture length $\underline{L_c}$ and width W_c

$\underline{L_c}$ = 2*sqrt(0.25) = 1.000 (km)

W_c = 2*sqrt(0.0625) = 0.500 (km)



Matlab code

```matlab
%% 2nd moment
xx = 0; xy = 0; yy = 0;
xt = 0; yt = 0;
tt = 0;
for i = 1:length(x)
    for j = 1:length(y)
        if W(i,j) == 1

            xx = xx + (x(i)-x0)^2*dx*dy/sum_f;
            yy = yy + (y(j)-y0)^2*dx*dy/sum_f;
            xy = xy + (x(i)-x0)*(y(j)-y0)*dx*dy/sum_f;

            dist = sqrt((x(i) - xc)^2 + (y(j) - yc)^2);

            xt = xt + (x(i)-x0)*(dist/Vr-t0)*dx*dy/sum_f;
            yt = yt + (y(j)-y0)*(dist/Vr-t0)*dx*dy/sum_f;

            tt = tt + (dist/Vr-t0)^2*dx*dy/sum_f;
        end
    end
end
```

temporal moment

Temporal moment: scalar

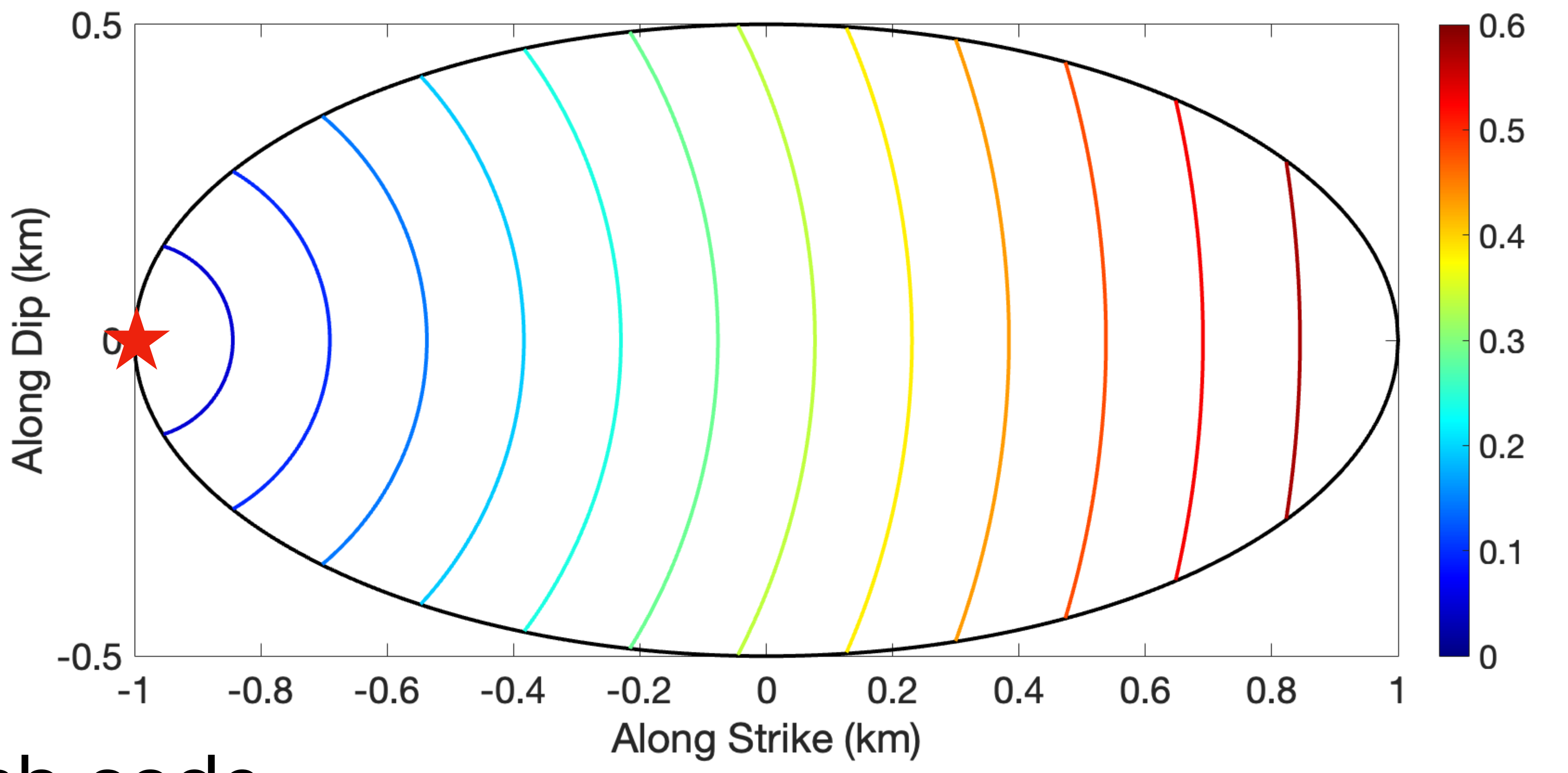
$$\widehat{\mu}^{(0,2)} = \int\int \dot{f}(\underline{r}, t)\,{\color{red}(t - t_0)}\,{\color{red}(t - t_0)}\,\mathrm{d}V\,\mathrm{d}t$$

= 0.0229 (s^2)

Characteristic duration

$$\boxed{\tau_c = 2\sqrt{\widehat{\mu}^{(0,2)}}}$$

= 0.303 (s)



Matlab code

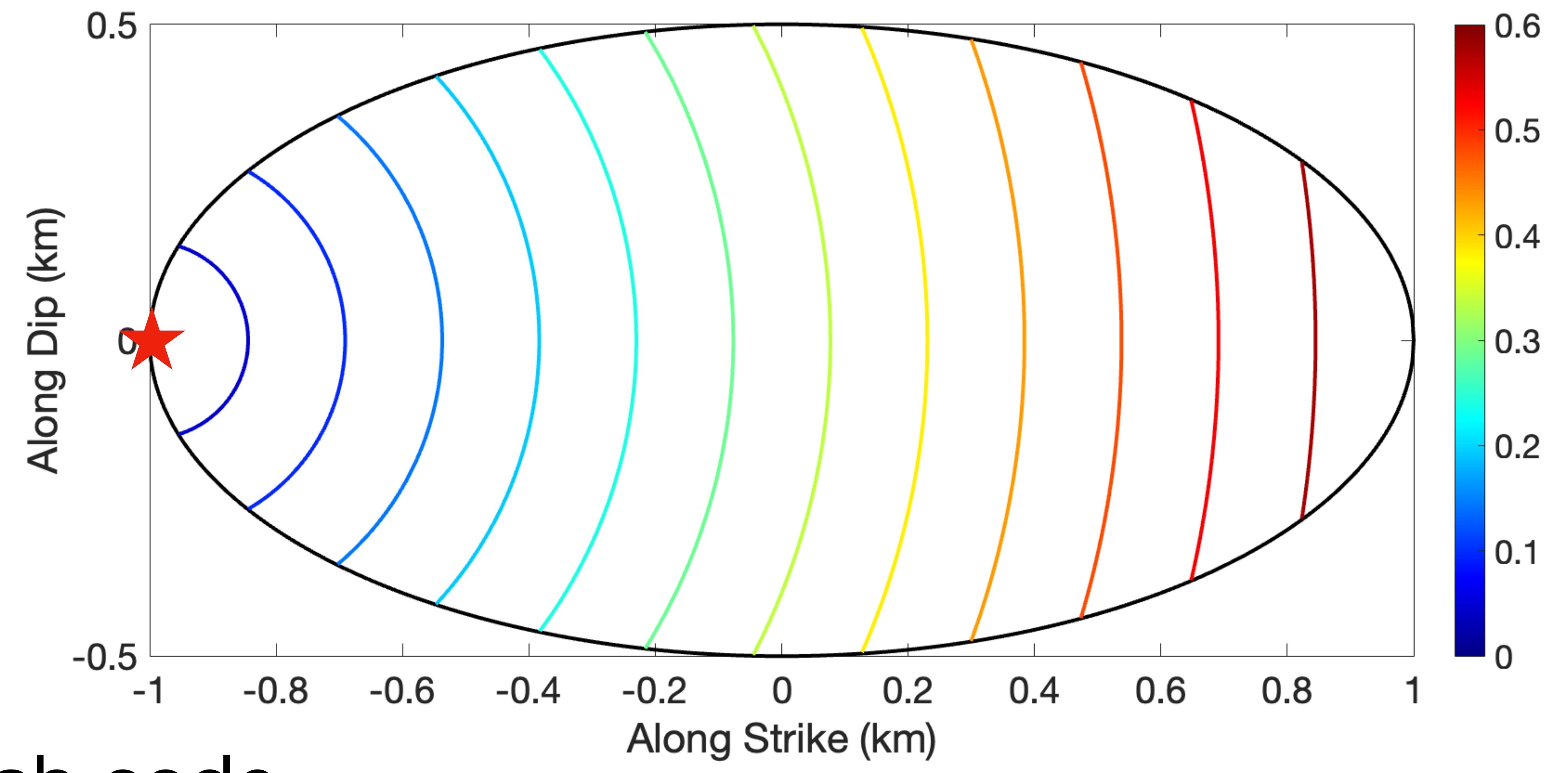
```matlab
%% 2nd moment
xx = 0; xy = 0; yy = 0;
xt = 0; yt = 0;
tt = 0;
for i = 1:length(x)
    for j = 1:length(y)
        if W(i,j) == 1

            xx = xx + (x(i)-x0)^2*dx*dy/sum_f;
            yy = yy + (y(j)-y0)^2*dx*dy/sum_f;
            xy = xy + (x(i)-x0)*(y(j)-y0)*dx*dy/sum_f;

            dist = sqrt((x(i) - xc)^2 + (y(j) - yc)^2);

            xt = xt + (x(i)-x0)*(dist/Vr-t0)*dx*dy/sum_f;
            yt = yt + (y(j)-y0)*(dist/Vr-t0)*dx*dy/sum_f;

            tt = tt + (dist/Vr-t0)^2*dx*dy/sum_f;
        end
    end
end
```

Spatiotemporal

Spatiotemporal moment: 2 by 1 vector

$$\underline{\widehat{\mu}}^{(1,1)} = \int\int \dot{f}(\underline{r},t)(\underline{r}-\underline{r}_0)\,(t-t_0)\,\mathrm{d}V\,\mathrm{d}t$$

$$= \begin{vmatrix} 0.0754 \\ 0.0000 \end{vmatrix} \text{ (km*s)}$$



Matlab code

```
%% 2nd moment
xx = 0; xy = 0; yy = 0;
xt = 0; yt = 0;
tt = 0;
for i = 1:length(x)
    for j = 1:length(y)
        if W(i,j) == 1

            xx = xx + (x(i)-x0)^2*dx*dy/sum_f;
            yy = yy + (y(j)-y0)^2*dx*dy/sum_f;
            xy = xy + (x(i)-x0)*(y(j)-y0)*dx*dy/sum_f;

            dist = sqrt((x(i) - xc)^2 + (y(j) - yc)^2);

            xt = xt + (x(i)-x0)*(dist/Vr-t0)*dx*dy/sum_f;
            yt = yt + (y(j)-y0)*(dist/Vr-t0)*dx*dy/sum_f;

            tt = tt + (dist/Vr-t0)^2*dx*dy/sum_f;
        end
    end
end
```

Temporal moment: scalar

$$\widehat{\mu}^{(0,2)} = 0.0229 \text{ (s^2)}$$

Centroid rupture velocity

$$\boxed{\underline{v}_0 = \frac{\underline{\widehat{\mu}}^{(1,1)}}{\widehat{\mu}^{(0,2)}}} = \begin{vmatrix} 3.29 \\ 0.00 \end{vmatrix} \text{ (km/s)}$$

Spatiotemporal

Spatiotemporal moment: 2 by 1 vector

$$\underline{\widehat{\mu}}^{(1,1)} = \iint \dot{f}(\underline{r},t)(\underline{r}-\underline{r}_0)\,(t-t_0)\,\mathrm{d}V\mathrm{d}t$$

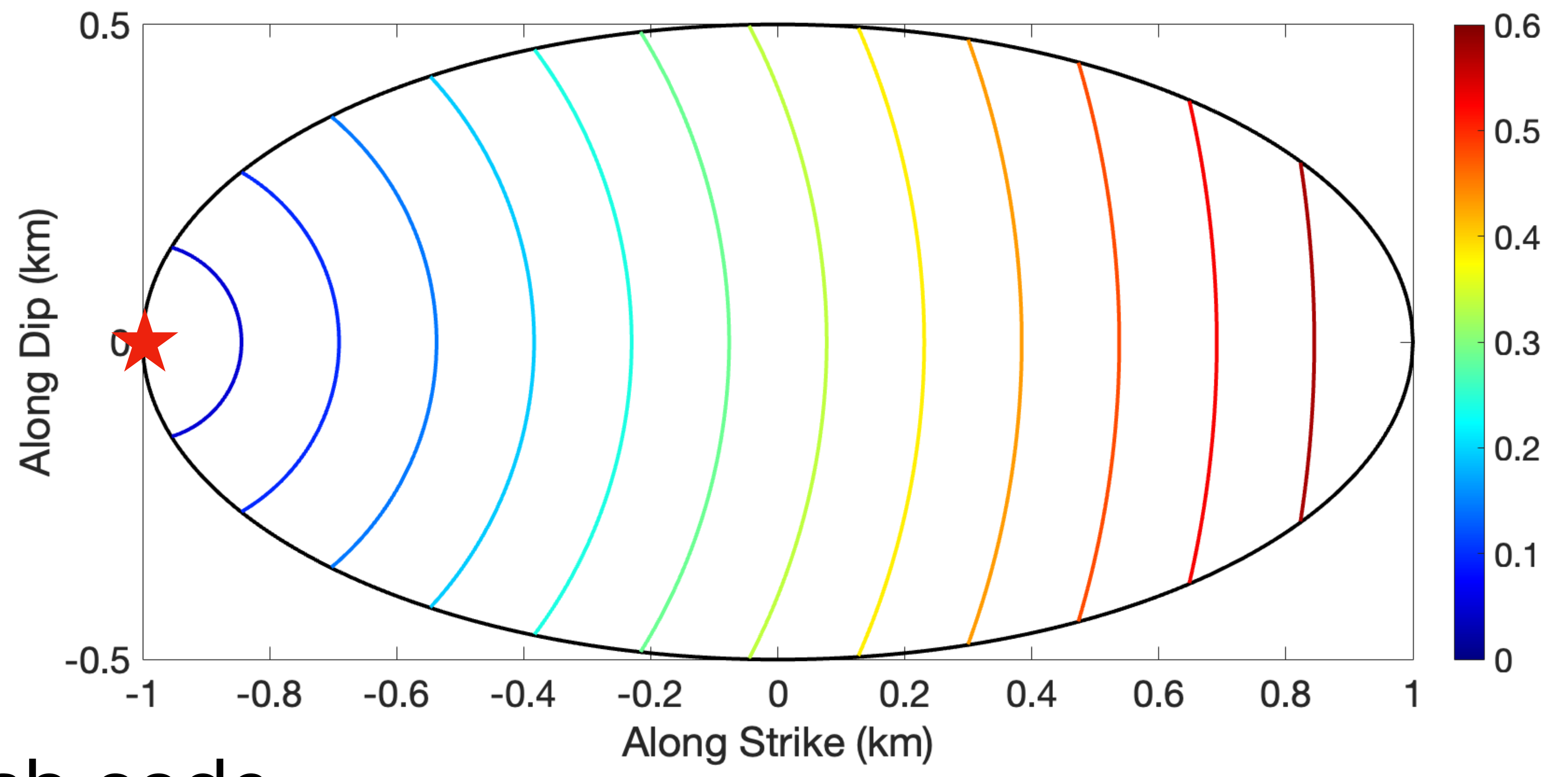
$$= \begin{vmatrix} 0.0754 \\ 0.0000 \end{vmatrix} \text{ (km*s)}$$

Temporal moment: scalar

$$\widehat{\mu}^{(0,2)} = 0.0229 \text{ (s^2)}$$

Centroid rupture velocity

$$\boxed{\underline{v}_0 = \frac{\underline{\widehat{\mu}}^{(1,1)}}{\widehat{\mu}^{(0,2)}}} = \begin{vmatrix} 3.29 \\ 0.00 \end{vmatrix} \text{ (km/s)}$$



Matlab code

```
%% 2nd moment
xx = 0; xy = 0; yy = 0;
xt = 0; yt = 0;
tt = 0;
for i = 1:length(x)
    for j = 1:length(y)
        if W(i,j) == 1

            xx = xx + (x(i)-x0)^2*dx*dy/sum_f;
            yy = yy + (y(j)-y0)^2*dx*dy/sum_f;
            xy = xy + (x(i)-x0)*(y(j)-y0)*dx*dy/sum_f;

            dist = sqrt((x(i) - xc)^2 + (y(j) - yc)^2);

            xt = xt + (x(i)-x0)*(dist/Vr-t0)*dx*dy/sum_f;
            yt = yt + (y(j)-y0)*(dist/Vr-t0)*dx*dy/sum_f;

            tt = tt + (dist/Vr-t0)^2*dx*dy/sum_f;
        end
    end
end
```

Finite source attributes

Apparent rupture velocity

$$\boxed{v_c = \frac{L_c}{\tau_c}} = 3.29 \text{ km/s}$$

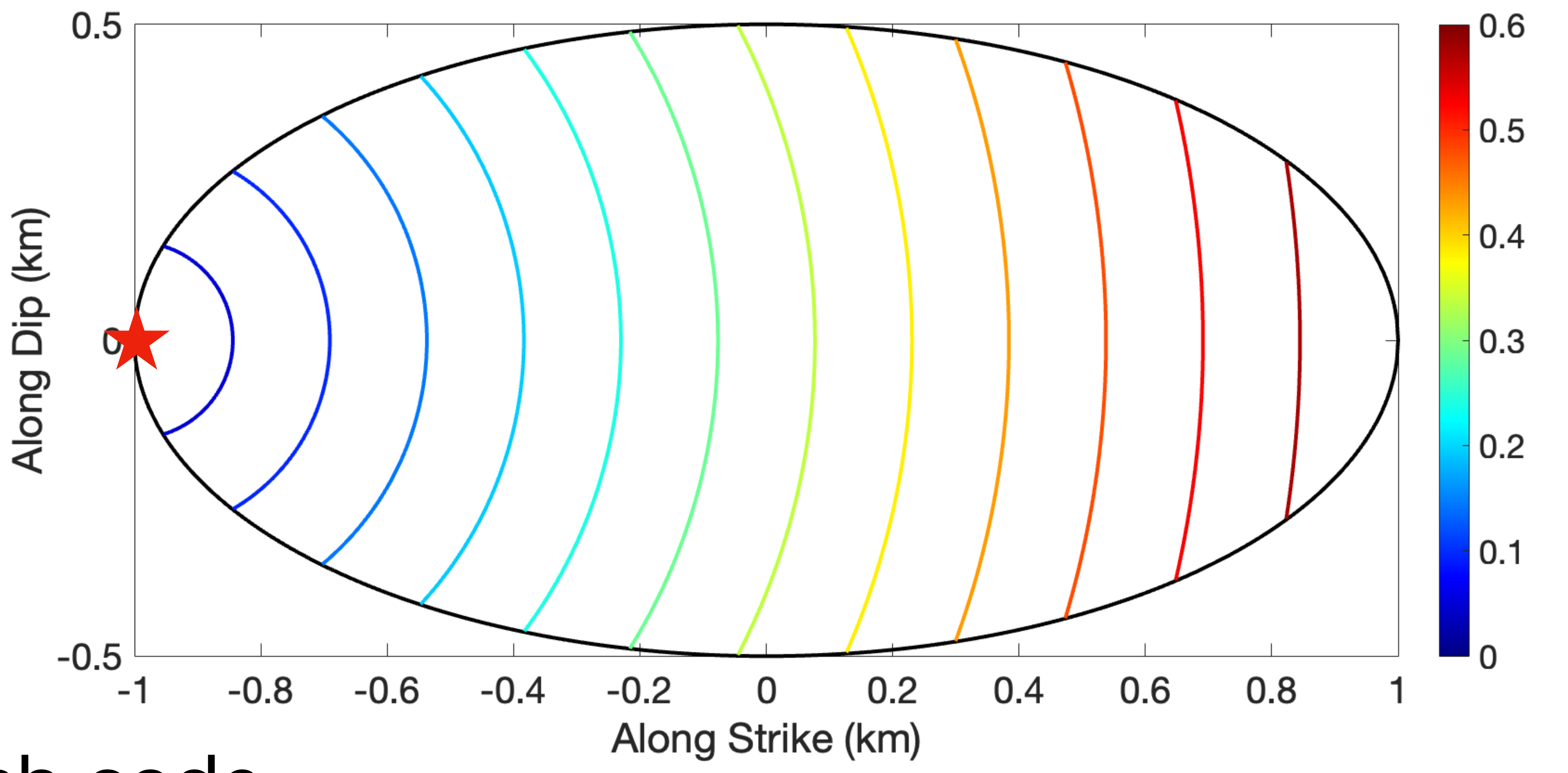
Directivity ratio

$$\boxed{dir = \frac{|v_0|}{v_c}} = 1.00$$

$\underline{L_c}$ = 1.00 km $\quad$ $\underline{v_0}$ = 3.29 km

W_c = 0.50 km $\quad$ v_c = 3.29 km

τ_c = 0.30 s $\quad$ dir =1.00

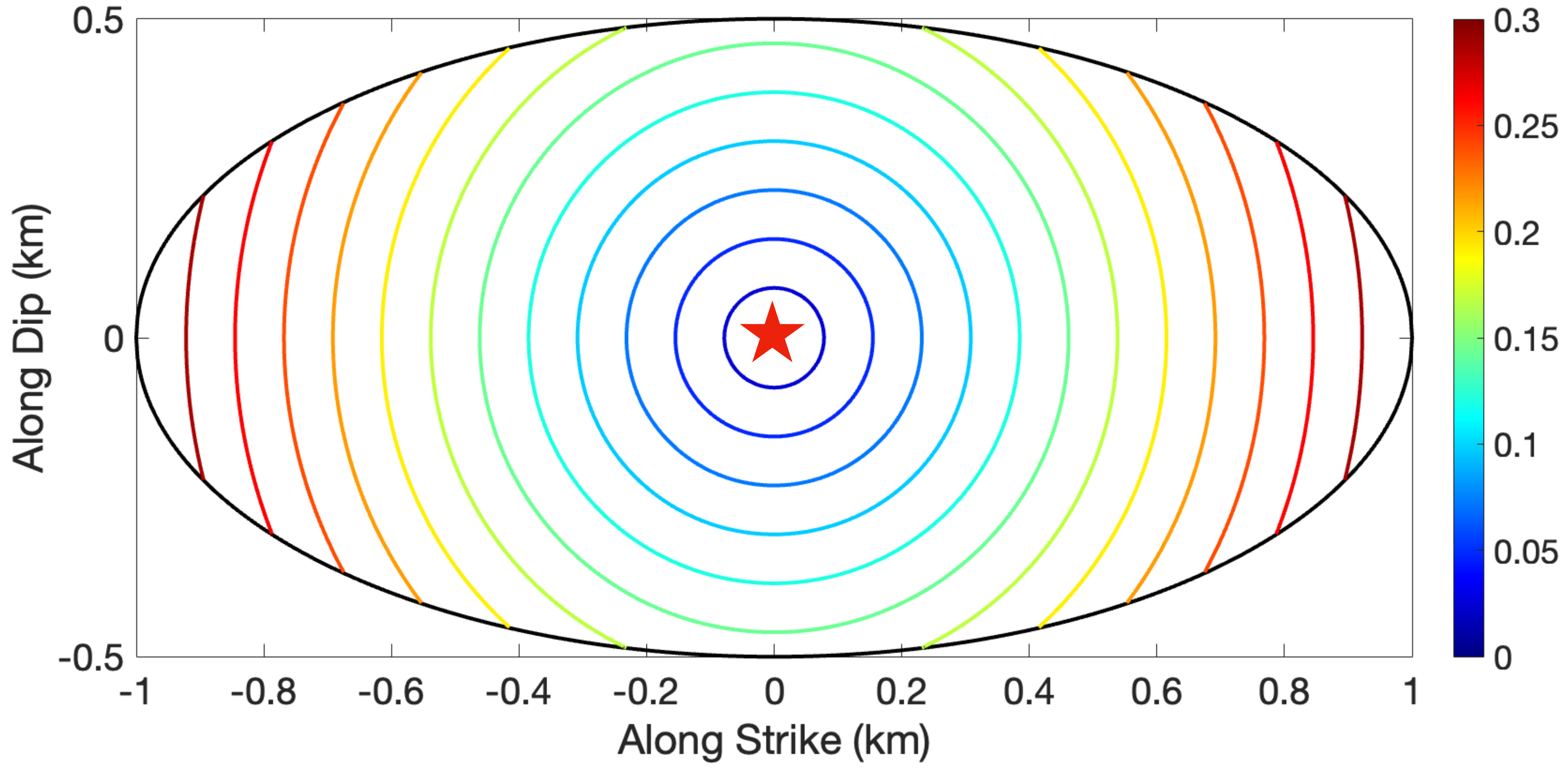


Matlab code

```
%% 2nd moment
xx = 0; xy = 0; yy = 0;
xt = 0; yt = 0;
tt = 0;
for i = 1:length(x)
    for j = 1:length(y)
        if W(i,j) == 1

            xx = xx + (x(i)-x0)^2*dx*dy/sum_f;
            yy = yy + (y(j)-y0)^2*dx*dy/sum_f;
            xy = xy + (x(i)-x0)*(y(j)-y0)*dx*dy/sum_f;

            dist = sqrt((x(i) - xc)^2 + (y(j) - yc)^2);

            xt = xt + (x(i)-x0)*(dist/Vr-t0)*dx*dy/sum_f;
            yt = yt + (y(j)-y0)*(dist/Vr-t0)*dx*dy/sum_f;

            tt = tt + (dist/Vr-t0)^2*dx*dy/sum_f;
        end
    end
end
```

Bilateral rupture

- Strike slip earthquake
  - Elliptical rupture area
    - Semi-major axis: 1km
    - Semi-minor axis: 0.5 km
  - bilateral rupture:
    - Starting from the center
    - Vr = 3.2 km/s
  - Uniform slip: slip = 1m

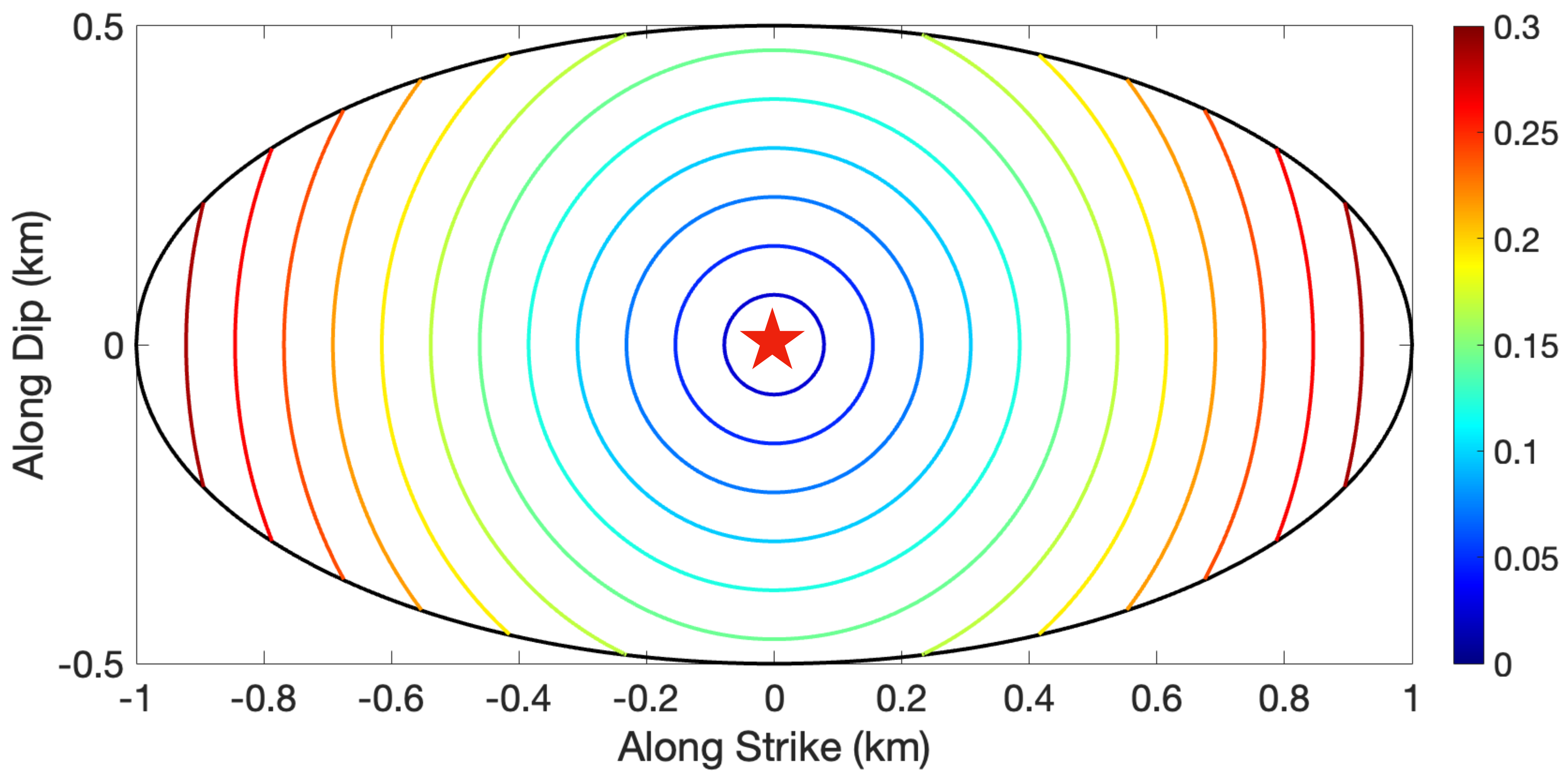


Matlab code

```
t = linspace(0,2*Lc/Vr,nn);
dt = t(2) - t(1);


%% starting point
xc = -Lc; yc = 0;
%xc = 0; yc = 0;

```

Bilateral rupture

- Strike slip earthquake
  - Elliptical rupture area
    - Semi-major axis: 1km
    - Semi-minor axis: 0.5 km
  - bilateral rupture:
    - Starting from the center
    - Vr = 3.2 km/s
  - Uniform slip: slip = 1m



$\underline{L_c}$ = 1.00 km $\quad \underline{v_0}$ = 0.00 km

W_c = 0.50 km $\quad v_c$ = 6.58 km

τ_c = 0.14 s $\quad dir$ = 0.00

Summary I

Spatial moment: tensor $\underline{L_c}$ W_c

$$\widehat{\underline{\underline{\mu}}}^{(2,0)} = \iint \dot{f}(\underline{r},t)(\underline{r}-\underline{r}_0)^{\mathrm{T}}(\underline{r}-\underline{r}_0)\mathrm{d}V\mathrm{d}t$$

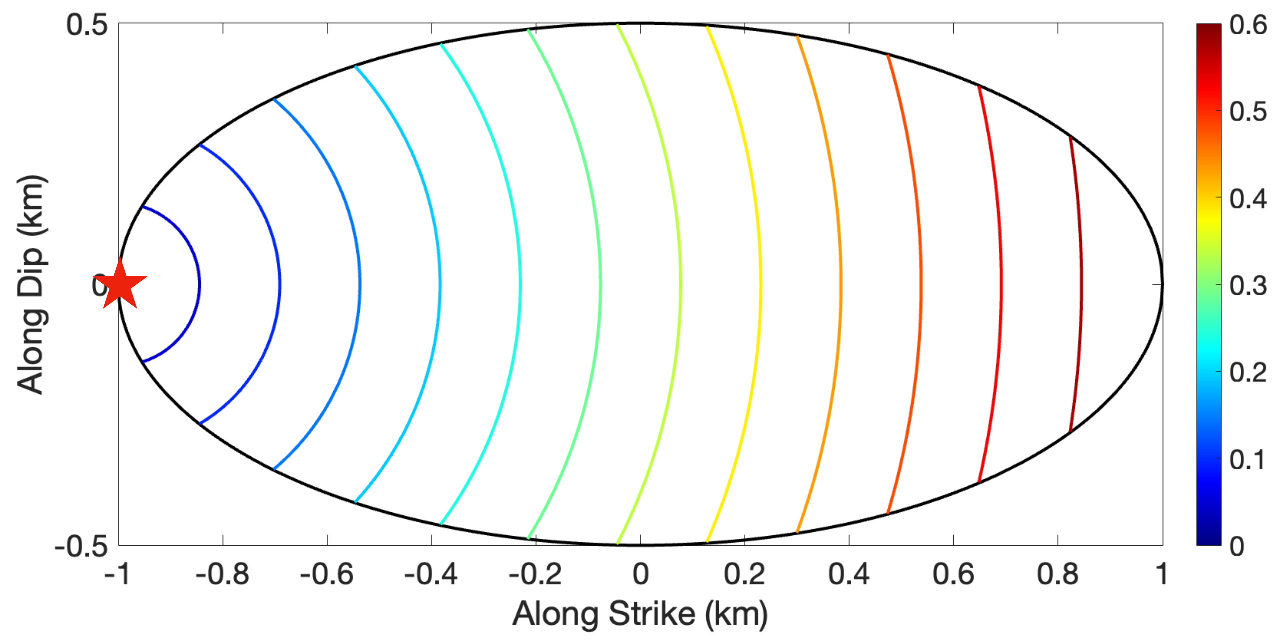
Temporal moment: scalar τ_c

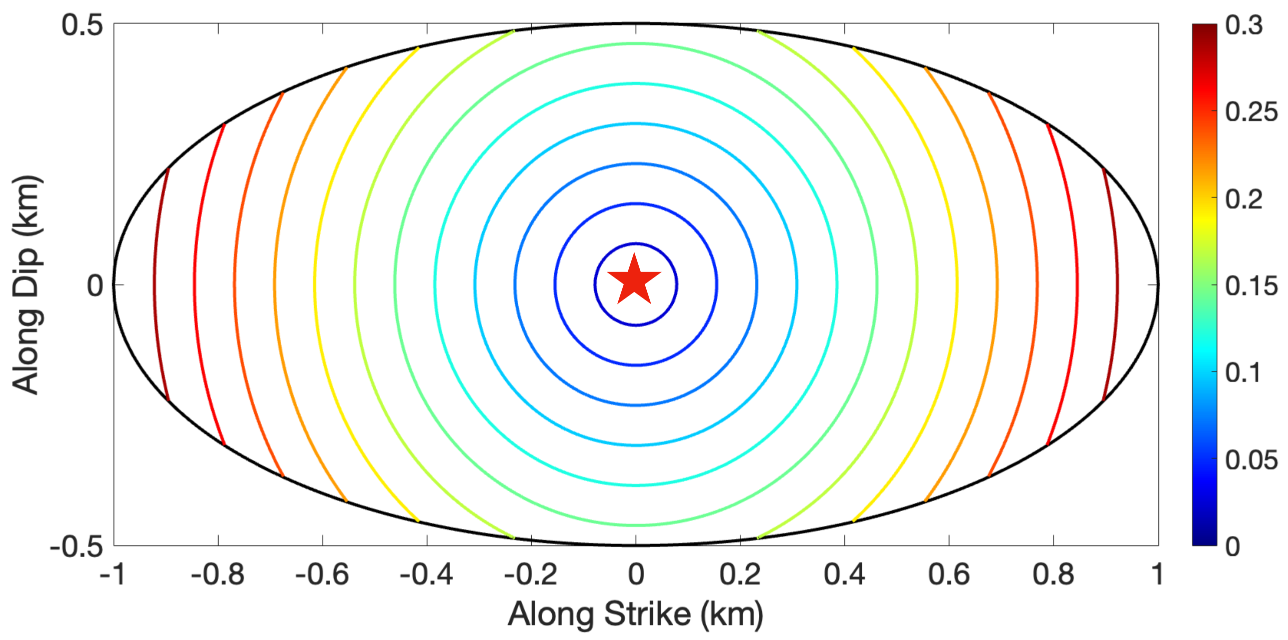
$$\widehat{\mu}^{(0,2)} = \iint \dot{f}(\underline{r},t)(t-t_0)(t-t_0)\mathrm{d}V\mathrm{d}t$$

Spatiotemporal moment: vector

$$\widehat{\underline{\mu}}^{(1,1)} = \iint \dot{f}(\underline{r},t)(\underline{r}-\underline{r}_0)(t-t_0)\mathrm{d}V\mathrm{d}t$$

$\underline{L_c}$ W_c τ_c $\underline{v_0}$ v_c dir

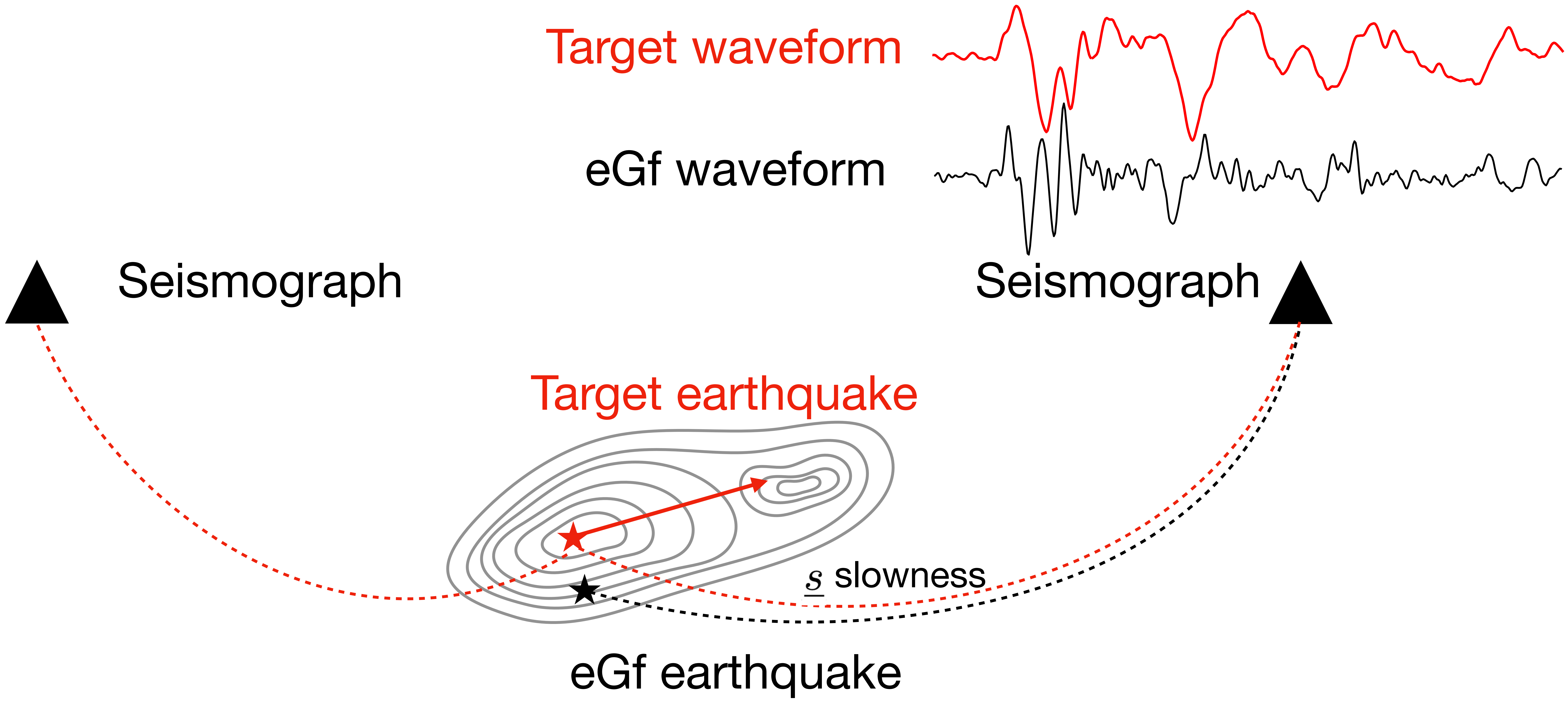




Unilateral rupture	Bilateral rupture
$\underline{L_c}$ = 1.00 km	$\underline{L_c}$ = 1.00 km
W_c = 0.50 km	W_c = 0.50 km
τ_c = 0.30 km	τ_c = 0.14 km
$\underline{v_0}$ = 3.29 km	$\underline{v_0}$ = 0.00 km
v_c = 3.29 km	v_c = 6.58 km
dir = 1.00	dir = 0.00

How to observe second seismic moments?

Observing Second Moments



What is an empirical Green's function

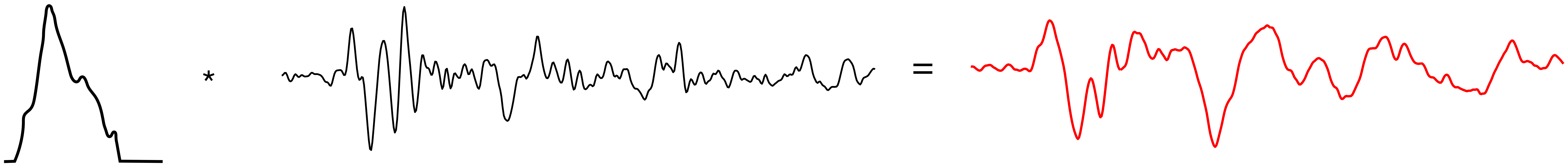
- Green's function:
  - Impulsive response of the media
  - Source time function: Dirac delta
- Empirical Green's function
  - Smaller earthquake: short duration
  - Colocated with target earthquake
  - Similar focal mechanism

STF, eGf, and target waveform

ASTF

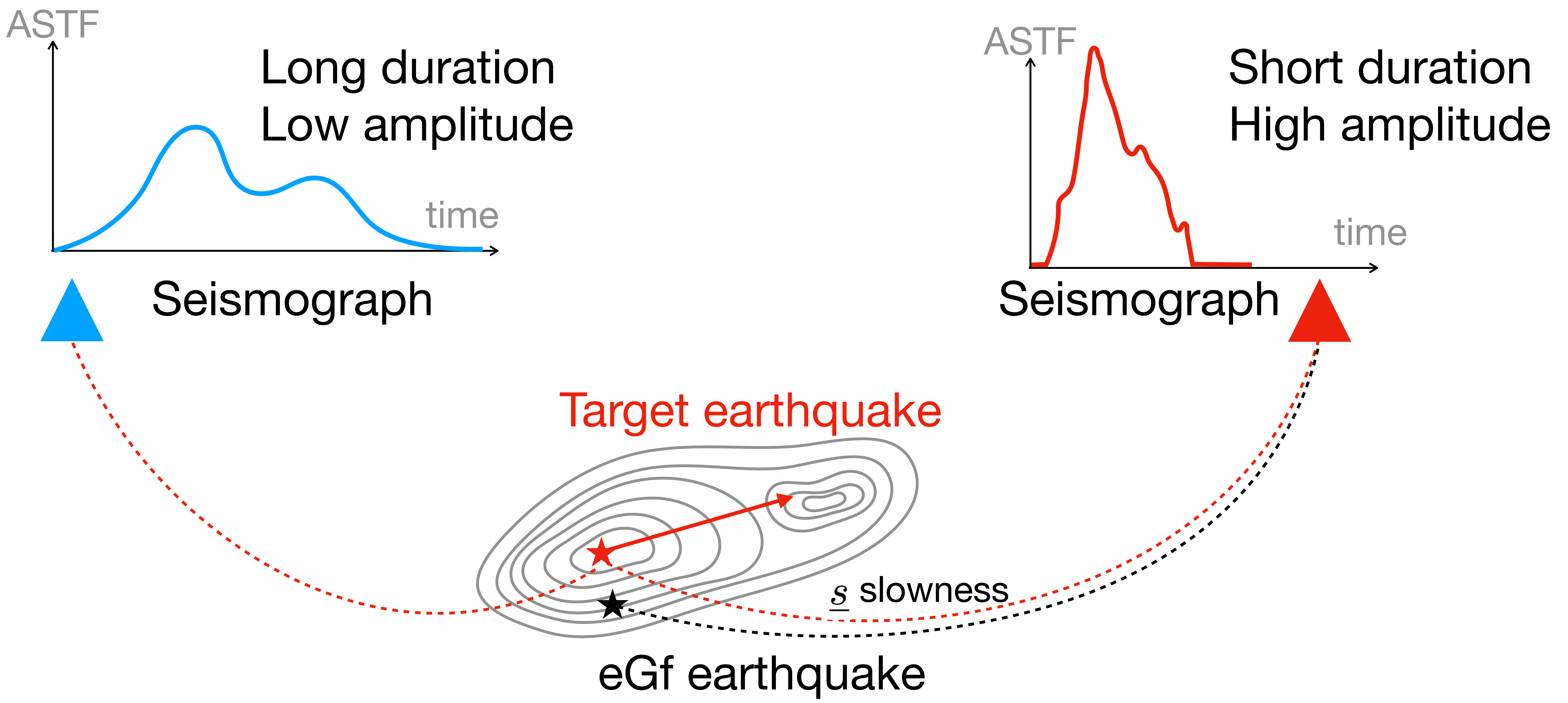
eGf waveform

Target waveform

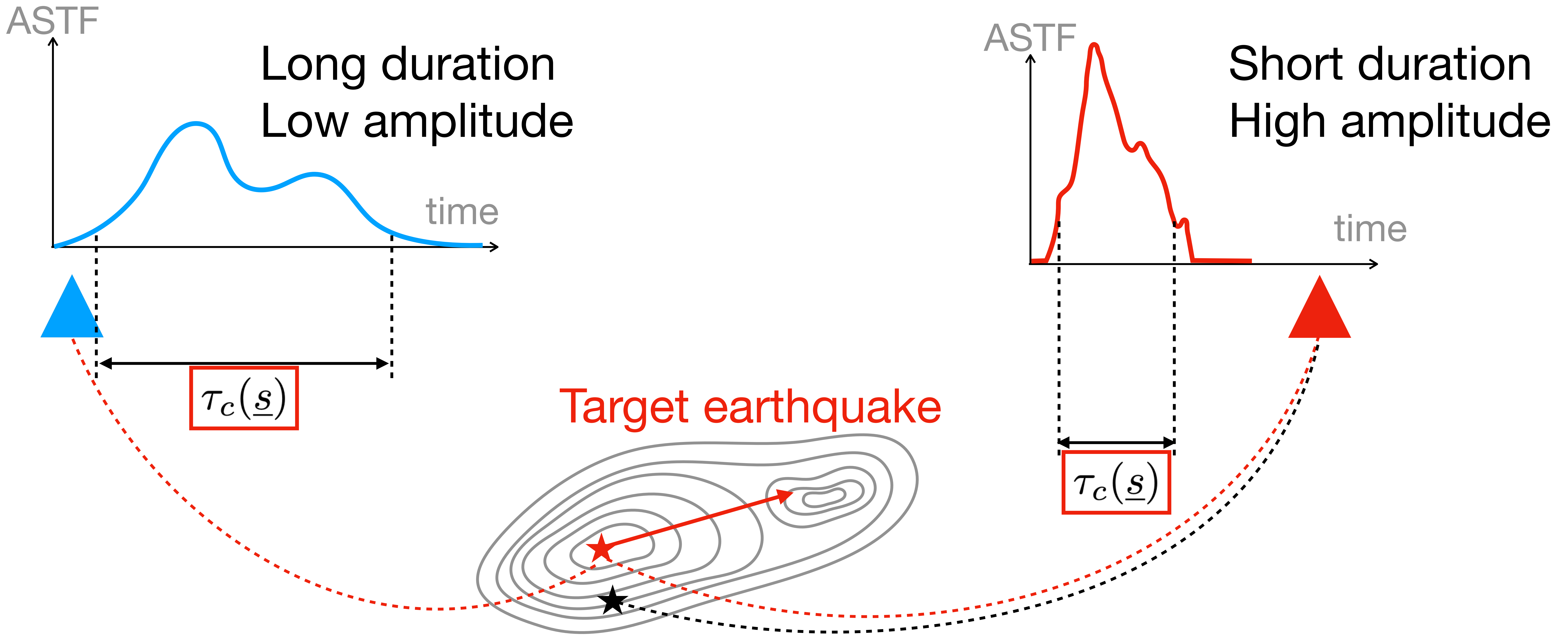


Perform deconvolution to get ASTF.

Apparent Source Time Function (ASTF)



Apparent Source Time Function (ASTF)



Apparent temporal moment

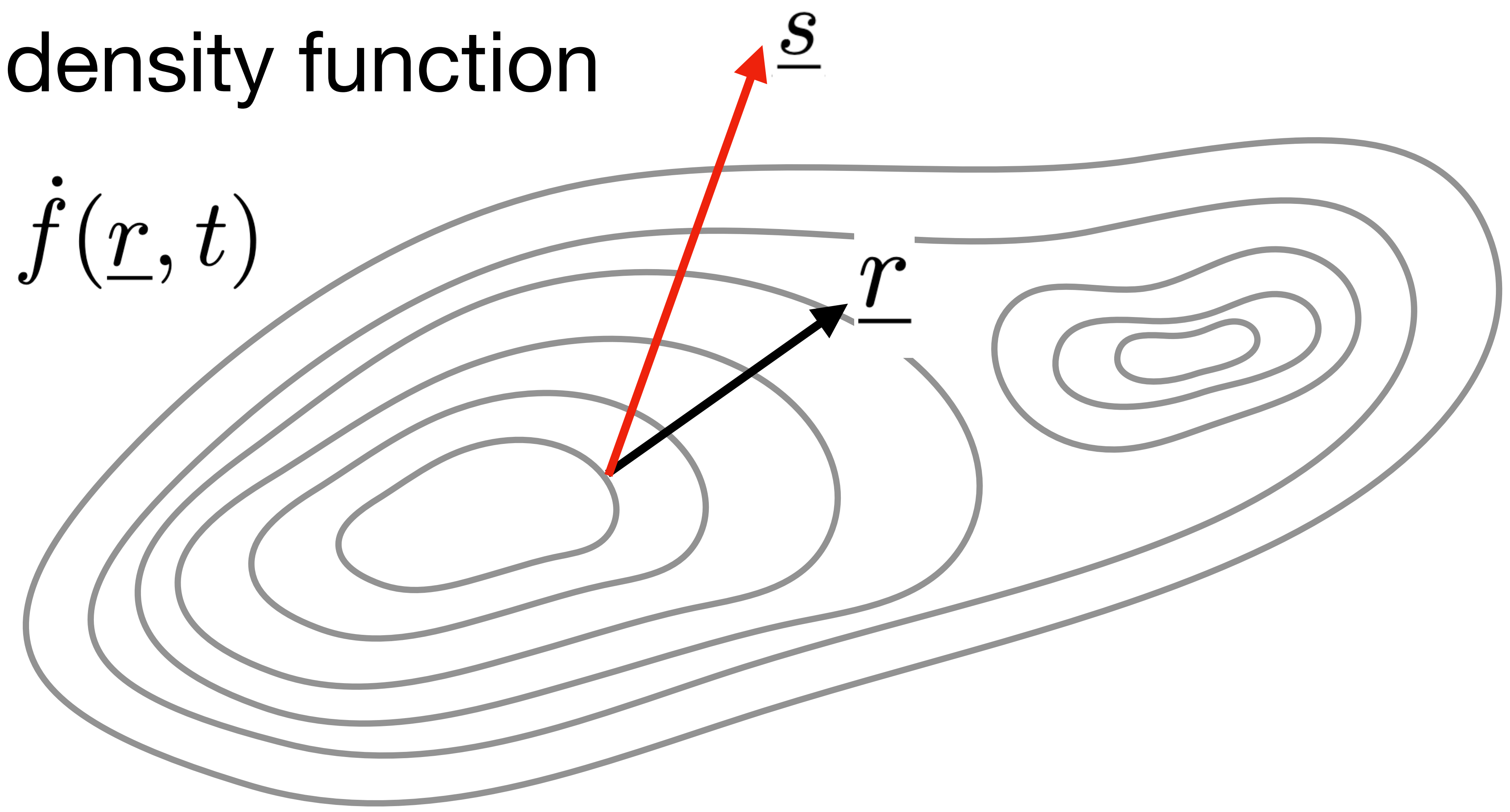
$$\widehat{\mu}^{(0,2)}(\underline{s}) = \int \mathrm{ASTF}(t)(t-t_0)\,(t-t_0)\mathrm{d}t$$

$$\tau_c(\underline{s}) = 2\sqrt{\widehat{\mu}^{(0,2)}(\underline{s})}$$

Apparent Source Time Function (ASTF)

Normalized moment rate density function

$\dot{f}(\underline{r}, t)$



Centroid $\underline{r}_0 = 0 \quad t_0 = 0$

ASTF $\boxed{\mathrm{ASTF}(t) = \int \dot{f}(\underline{r}, t + \underline{r} \cdot \underline{s}) \mathrm{d}V}$

P-, S-wave ASTFs are different.

$$\hat{\mu}^{(0,2)}(\underline{s}) = \int \boxed{\mathrm{ASTF}(t)} (t - t_0)^2 \mathrm{d}t$$

$$= \iint \dot{f}(\underline{r}, \boxed{t + \underline{r} \cdot \underline{s}}) t^2 \mathrm{d}V \mathrm{d}t$$

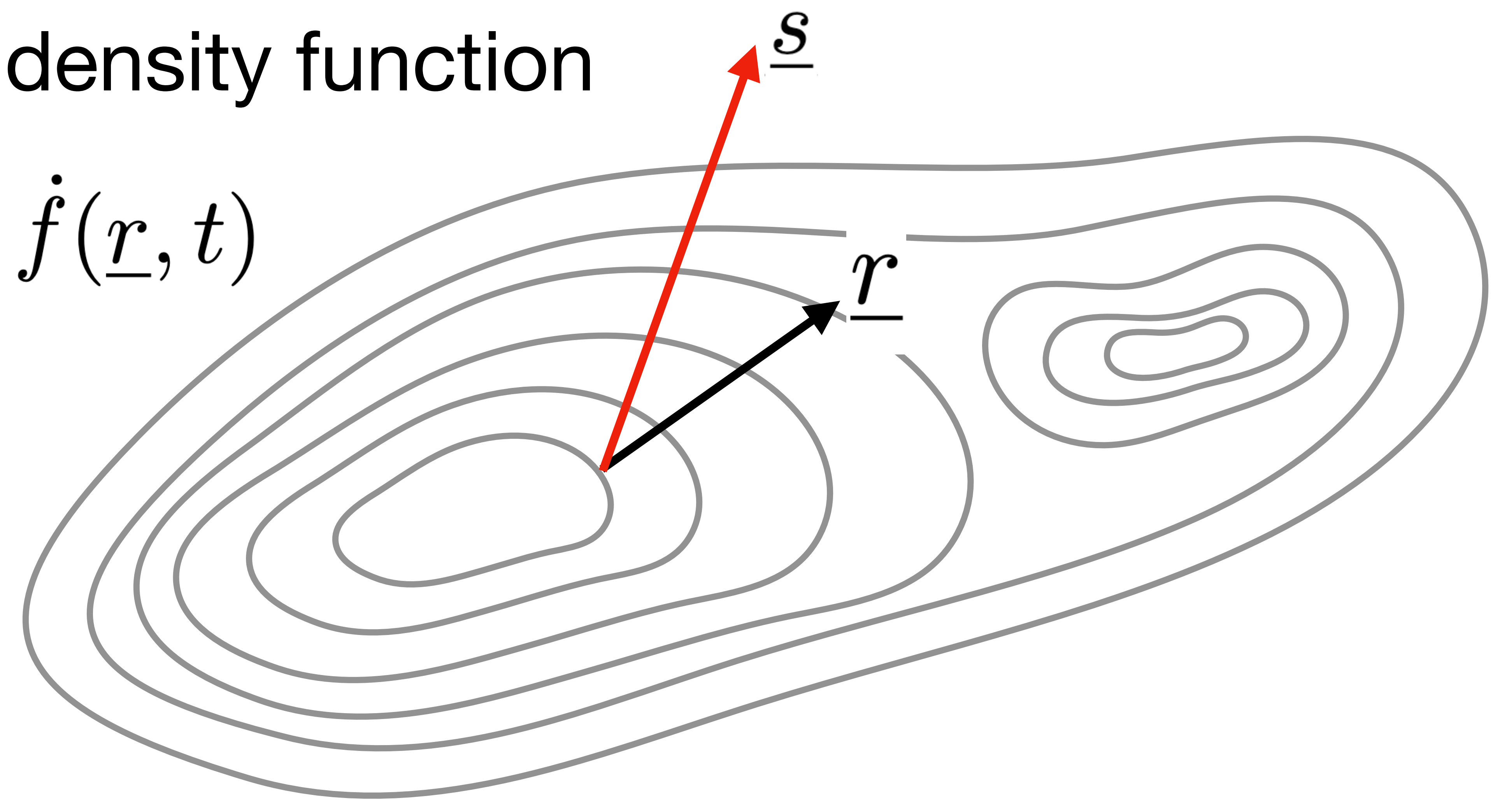
$$\boxed{\tau = t + \underline{r} \cdot \underline{s}}$$

$$\hat{\mu}^{(0,2)}(\underline{s}) = \iint \dot{f}(\underline{r}, \tau)(\tau - \underline{r} \cdot \underline{s})^2 \mathrm{d}V \mathrm{d}\tau$$

$$= \iint \dot{f}(\underline{r}, \tau)(\tau^2 - 2\underline{s} \cdot \underline{r}\tau + \underline{s}^T \cdot \underline{r}^T \underline{r} \cdot \underline{s}) \mathrm{d}V \mathrm{d}\tau$$

Apparent Source Time Function (ASTF)

Normalized moment rate density function



Centroid $\underline{r}_0 = 0 \quad t_0 = 0$

ASTF $\boxed{\mathrm{ASTF}(t) = \int \dot{f}(\underline{r}, t + \underline{r} \cdot \underline{s}) \mathrm{d}V}$

P-, S-wave ASTFs are different.

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$$= \iint \dot{f}(\underline{r}, \boxed{t + \underline{r} \cdot \underline{s}}) t^2 \mathrm{d}V \mathrm{d}t$$

$$\boxed{\tau = t + \underline{r} \cdot \underline{s}}$$

$\underline{s}$ is unchanged in the source region.

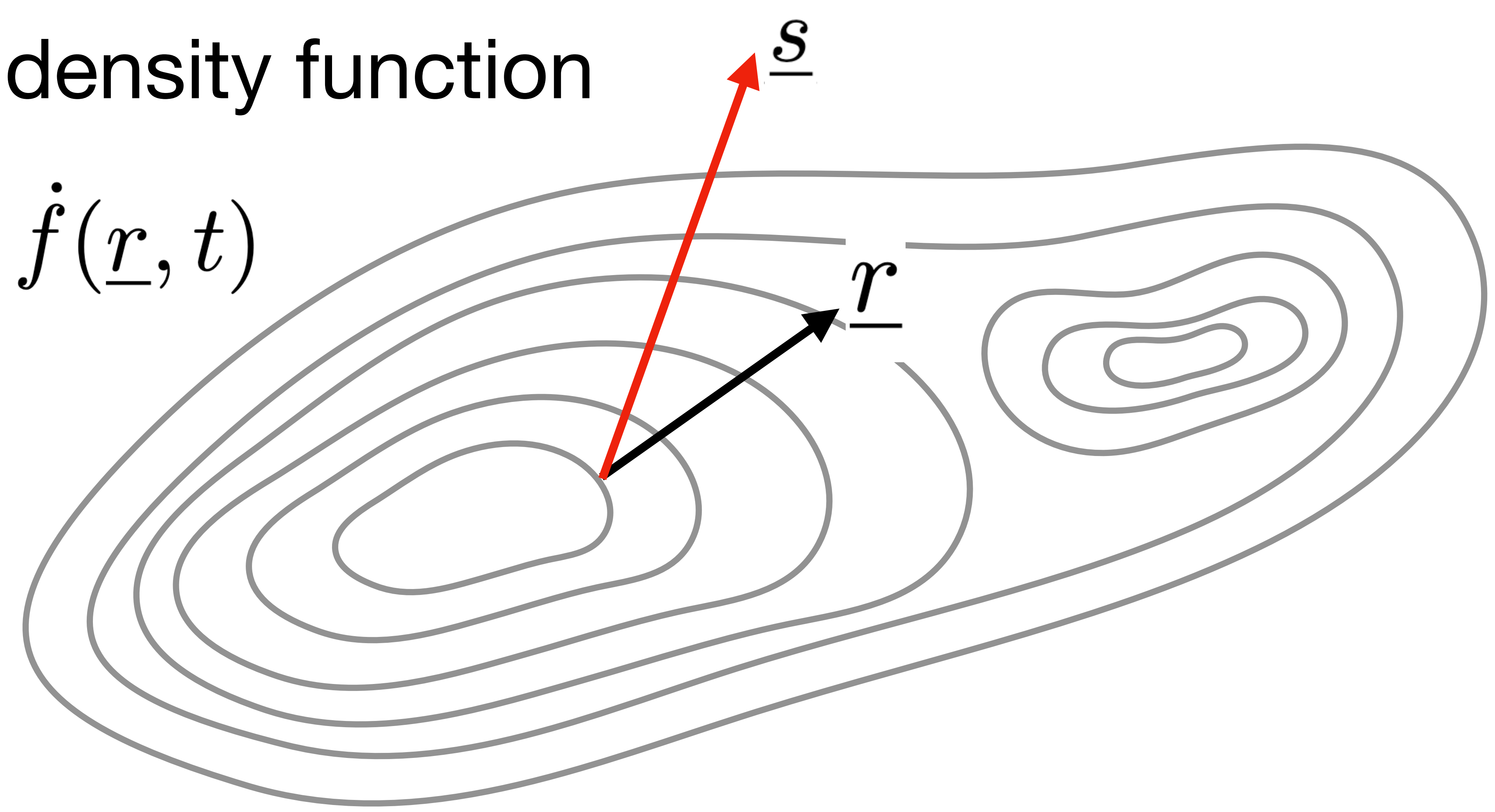
$$\hat{\mu}^{(0,2)}(\underline{s}) = \iint \dot{f}(\underline{r}, \tau)(\tau - \underline{r} \cdot \underline{s})^2 \mathrm{d}V \mathrm{d}\tau$$

$$= \iint \dot{f}(\underline{r}, \tau)(\tau^2 - 2\underline{s} \cdot \underline{r}\tau + \underline{s}^T \cdot \underline{r}^T \underline{r} \cdot \underline{s}) \mathrm{d}V \mathrm{d}\tau$$

Apparent Source Time Function (ASTF)

Normalized moment rate density function

$\dot{f}(\underline{r}, t)$



Centroid $\underline{r}_0 = 0 \quad t_0 = 0$

ASTF $\boxed{\mathrm{ASTF}(t) = \int \dot{f}(\underline{r}, t + \underline{r} \cdot \underline{s}) \mathrm{d}V}$

P-, S-wave ASTFs are different.

$$\hat{\mu}^{(0,2)}(\underline{s}) = \int \boxed{\mathrm{ASTF}(t)} (t - t_0)^2 \mathrm{d}t$$

$$= \iint \dot{f}(\underline{r}, \boxed{t + \underline{r} \cdot \underline{s}}) t^2 \mathrm{d}V \mathrm{d}t$$

$$\boxed{\tau = t + \underline{r} \cdot \underline{s}}$$

$\underline{s}$ is unchanged in the source region.

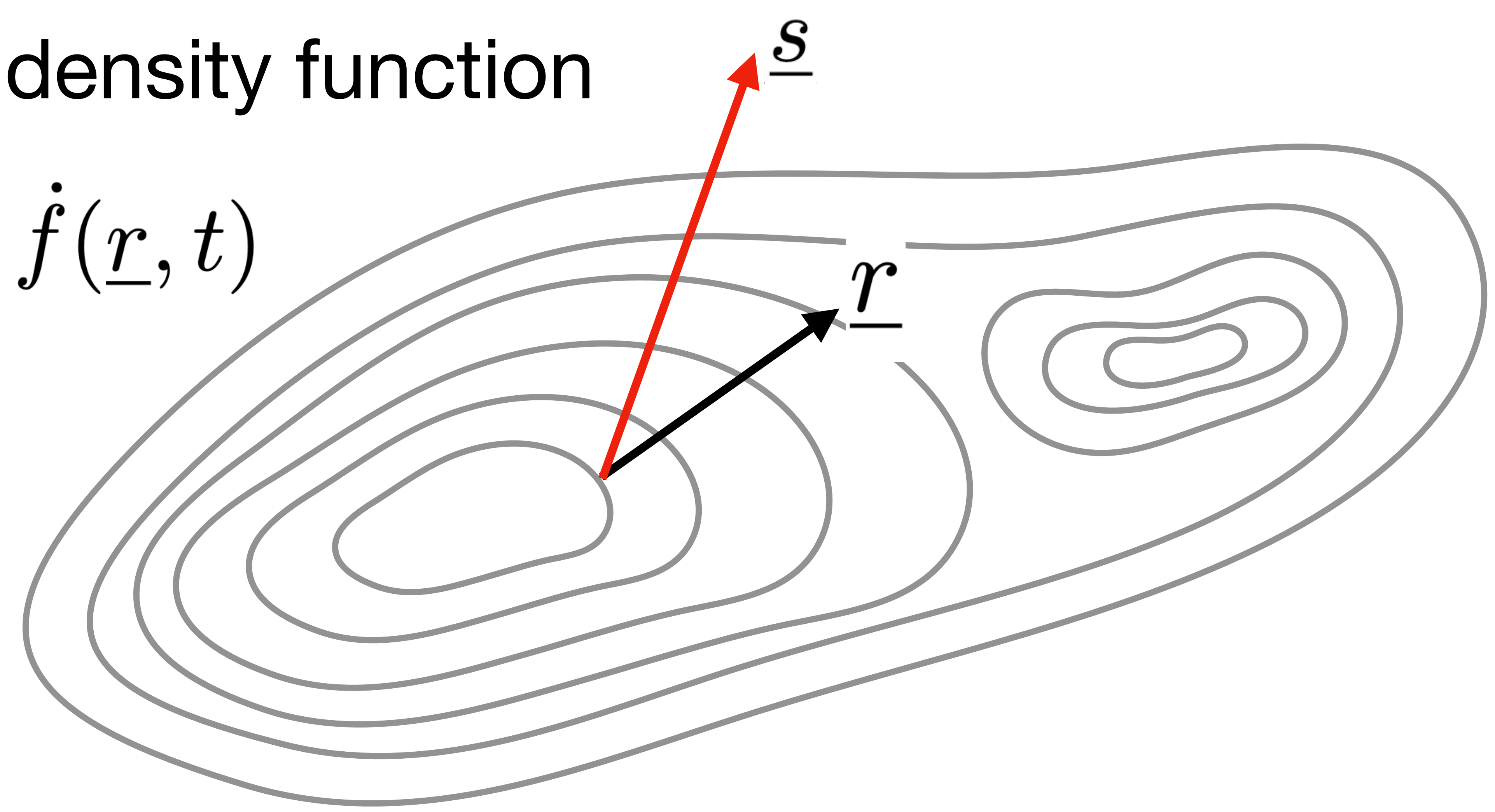
$$\hat{\mu}^{(0,2)}(\underline{s}) = \iint \dot{f}(\underline{r}, \tau)(\tau - \underline{r} \cdot \underline{s})^2 \mathrm{d}V \mathrm{d}\tau$$

$$= \iint \dot{f}(\underline{r}, \tau)(\tau^2 - 2\underline{s} \cdot \underline{r}\tau + \underline{s}^T \cdot \underline{r}^T \underline{r} \cdot \underline{s}) \mathrm{d}V \mathrm{d}\tau$$

Observing 2nd Moments

Normalized moment rate density function

$\dot{f}(\underline{r}, t)$



Temporal moment: scalar

$$\widehat{\mu}^{(0,2)} = \iint \dot{f}(\underline{r}, t)(t - t_0)\,(t - t_0)\mathrm{d}V\mathrm{d}t$$

$$\hat{\mu}^{(0,2)}(\underline{s}) = \int \mathrm{ASTF}(t)\,(t - t_0)^2 \mathrm{d}t$$

$$= \iint \dot{f}(\underline{r}, \tau)(\tau^2 - 2\underline{s} \cdot \underline{r}\tau + \underline{s}^T \cdot \underline{r}^T \underline{r} \cdot \underline{s})\mathrm{d}V\mathrm{d}\tau$$

$$= \boxed{\iint \dot{f}(\underline{r}, \tau)\tau^2 \mathrm{d}V\mathrm{d}\tau} - 2\underline{s} \cdot \iint \dot{f}(\underline{r}, \tau)\underline{r}\tau \mathrm{d}V\mathrm{d}\tau$$

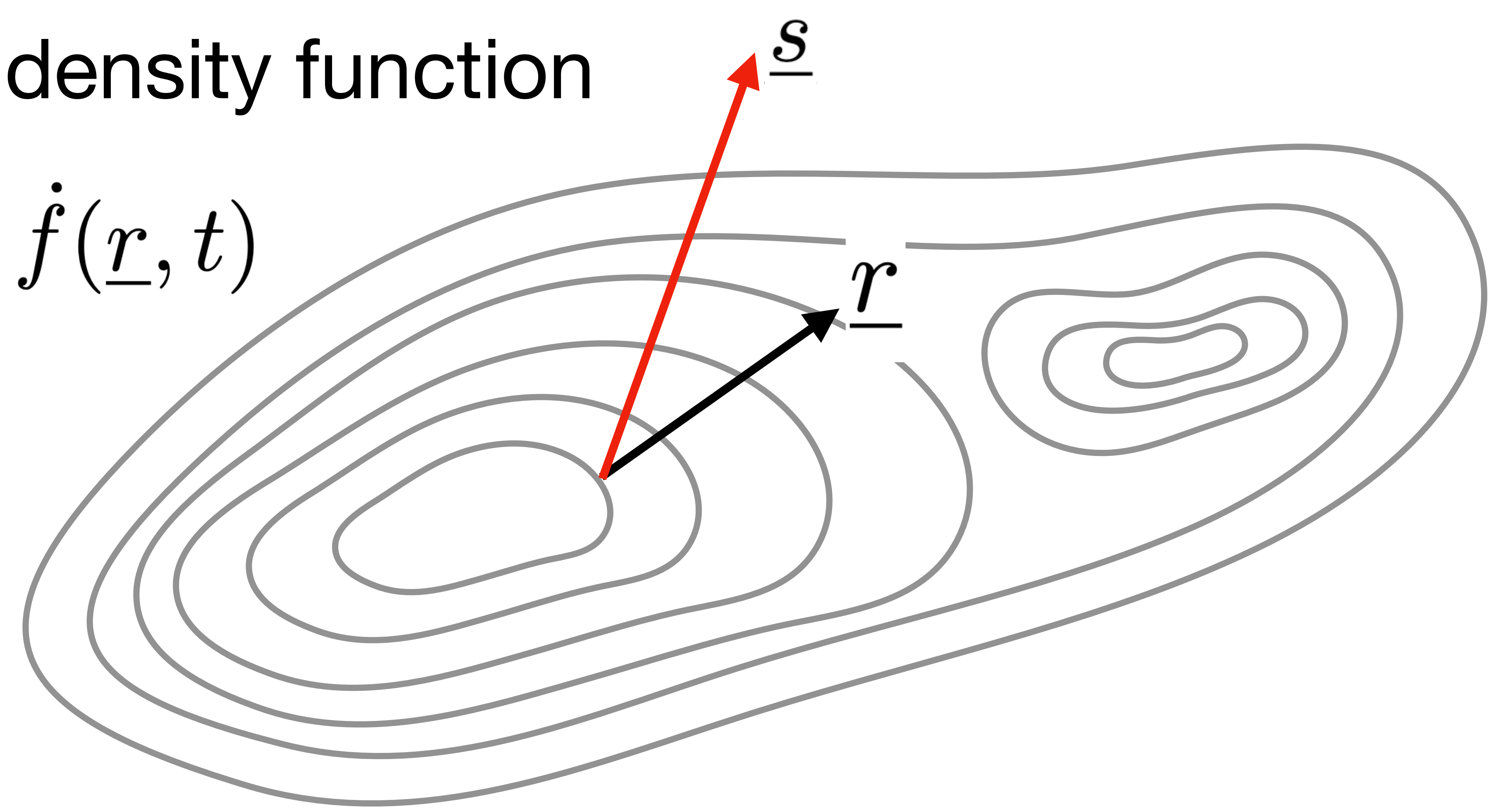
$$+ \underline{s}^T \cdot \iint \dot{f}(\underline{r}, \tau)\underline{r}^T \underline{r} \mathrm{d}V\mathrm{d}\tau \cdot \underline{s}$$

$$\hat{\mu}^{(0,2)}(\underline{s}) = \boxed{\hat{\mu}^{(0,2)}} - 2\underline{s} \cdot \underline{\hat{\mu}}^{(1,1)} + \underline{s}^T \cdot \underline{\underline{\hat{\mu}}}^{(2,0)} \cdot \underline{s}$$

Observing 2nd Moments

Normalized moment rate density function

$\dot{f}(\underline{r}, t)$



Spatiotemporal moment: vector

$$\underline{\widehat{\mu}}^{(1,1)} = \iint \dot{f}(\underline{r}, t)(\underline{r} - \underline{r}_0)\,(t - t_0)\,\mathrm{d}V\,\mathrm{d}t$$

$$\hat{\mu}^{(0,2)}(\underline{s}) = \int \mathrm{ASTF}(t)\,(t - t_0)^2\,\mathrm{d}t$$

$$= \iint \dot{f}(\underline{r}, \tau)(\tau^2 - 2\underline{s} \cdot \underline{r}\tau + \underline{s}^T \cdot \underline{r}^T\underline{r} \cdot \underline{s})\,\mathrm{d}V\,\mathrm{d}\tau$$

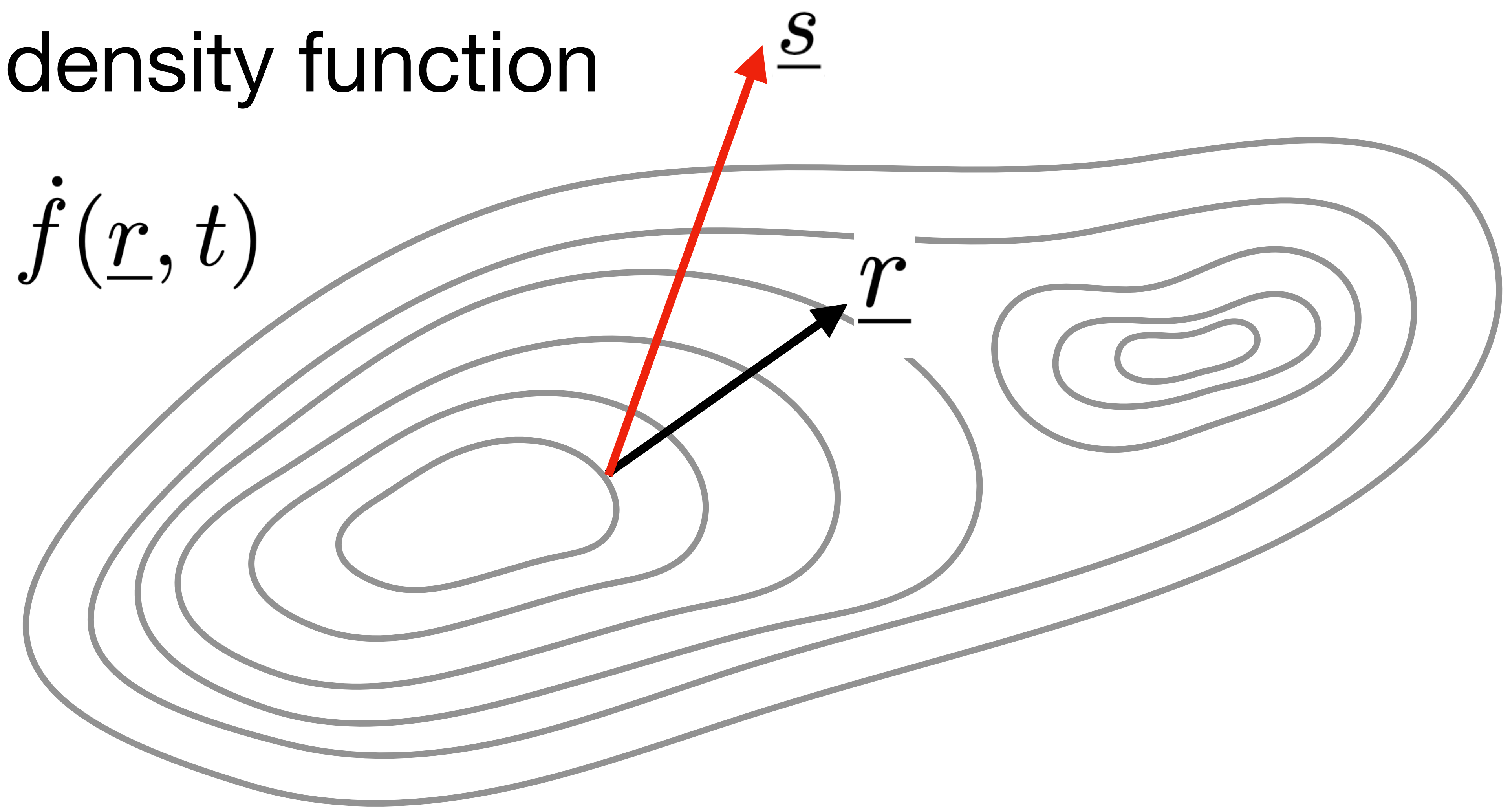
$$= \iint \dot{f}(\underline{r}, \tau)\tau^2\,\mathrm{d}V\,\mathrm{d}\tau - 2\underline{s} \cdot \boxed{\iint \dot{f}(\underline{r}, \tau)\underline{r}\tau\,\mathrm{d}V\,\mathrm{d}\tau}$$

$$+ \underline{s}^T \cdot \iint \dot{f}(\underline{r}, \tau)\underline{r}^T\underline{r}\,\mathrm{d}V\,\mathrm{d}\tau \cdot \underline{s}$$

$$\hat{\mu}^{(0,2)}(\underline{s}) = \hat{\mu}^{(0,2)} - 2\underline{s} \cdot \boxed{\underline{\hat{\mu}}^{(1,1)}} + \underline{s}^T \cdot \underline{\underline{\hat{\mu}}}^{(2,0)} \cdot \underline{s}$$

Observing 2nd Moments

Normalized moment rate density function



Spatial moment: tensor

$$\underline{\underline{\hat{\mu}}}^{(2,0)} = \iint \dot{f}(\underline{r}, t)\,(\underline{r} - \underline{r}_0)^{\mathrm{T}}\,(\underline{r} - \underline{r}_0)\,\mathrm{d}V\,\mathrm{d}t$$

$$\hat{\mu}^{(0,2)}(\underline{s}) = \int \mathrm{ASTF}(t)\,(t - t_0)^2\,\mathrm{d}t$$

$$= \iint \dot{f}(\underline{r}, \tau)(\tau^2 - 2\underline{s} \cdot \underline{r}\tau + \underline{s}^T \cdot \underline{r}^T \underline{r} \cdot \underline{s})\,\mathrm{d}V\mathrm{d}\tau$$

$$= \iint \dot{f}(\underline{r}, \tau)\tau^2 \mathrm{d}V\mathrm{d}\tau - 2\underline{s} \cdot \iint \dot{f}(\underline{r}, \tau)\underline{r}\tau\,\mathrm{d}V\mathrm{d}\tau$$

$$+ \underline{s}^T \cdot \boxed{\iint \dot{f}(\underline{r}, \tau)\underline{r}^T \underline{r}\,\mathrm{d}V\mathrm{d}\tau} \cdot \underline{s}$$

$$\hat{\mu}^{(0,2)}(\underline{s}) = \hat{\mu}^{(0,2)} - 2\underline{s} \cdot \underline{\hat{\mu}}^{(1,1)} + \underline{s}^T \cdot \boxed{\underline{\underline{\hat{\mu}}}^{(2,0)}} \cdot \underline{s}$$

Observing Second Moments

Temporal

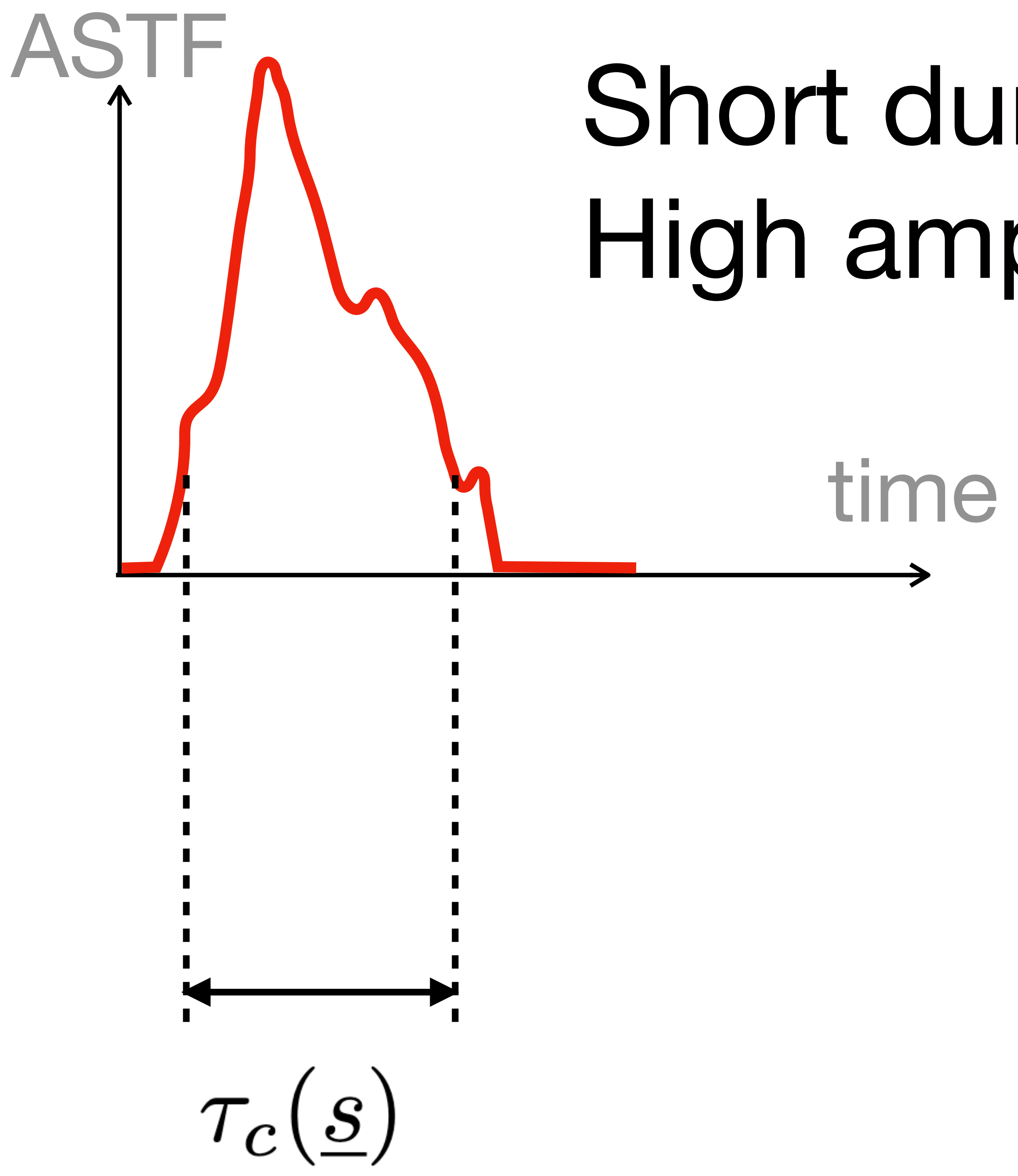
Spatiotemporal

Spatial

$$\boxed{\widehat{\mu}^{(0,2)}(\underline{s}) = \widehat{\mu}^{(0,2)} - 2\underline{s}\cdot\underline{\widehat{\mu}}^{(1,1)} + \underline{s}\cdot\underline{\underline{\widehat{\mu}}}^{(2,0)}\cdot\underline{s}}$$

Observable

Slowness vector



Short duration
High amplitude

Apparent temporal moment

$$\widehat{\mu}^{(0,2)}(\underline{s}) = \int \mathrm{ASTF}(t)(t-t_0)\,(t-t_0)\mathrm{d}t$$

$$\tau_c(\underline{s}) = 2\sqrt{\widehat{\mu}^{(0,2)}(\underline{s})}$$

$L_c \quad W_c \quad \tau_c \quad \underline{v_0} \quad v_c \quad dir$

Second moments inversion

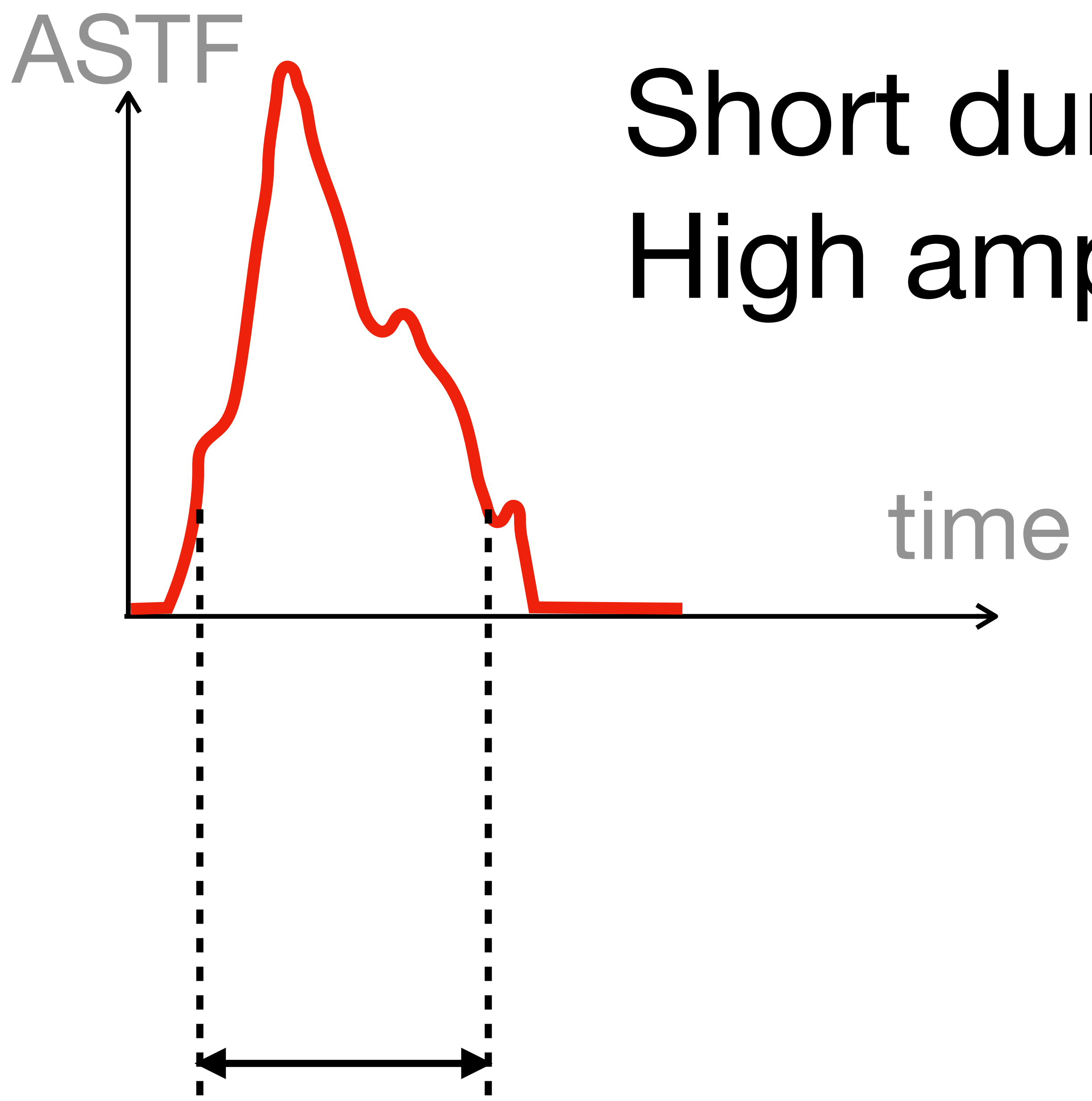
Expanding

$$\widehat{\mu}^{(0,2)}(\underline{s}) = \widehat{\mu}^{(0,2)} - 2\underline{s}\cdot\underline{\widehat{\mu}}^{(1,1)} + \underline{s}\cdot\underline{\underline{\widehat{\mu}}}^{(2,0)}\cdot\underline{s}$$

Project to fault plane using a focal mechanism

$$\widehat{\mu}^{(0,2)}(\underline{s}) = \begin{bmatrix} s_1^2 & 2s_1s_2 & s_2^2 & -2s_1 & -2s_2 & 1 \end{bmatrix}\cdot\underline{x},$$
$$\underline{x} = \begin{bmatrix} \widehat{\mu}_{11}^{(2,0)} & \widehat{\mu}_{12}^{(2,0)} & \widehat{\mu}_{22}^{(2,0)} & \widehat{\mu}_{1}^{(1,1)} & \widehat{\mu}_{2}^{(1,1)} & \widehat{\mu}^{(0,2)} \end{bmatrix}$$

$$\boxed{\underline{b} = \underline{\underline{A}}\cdot\underline{x}}$$



$$\boxed{\tau_c(\underline{s})}$$

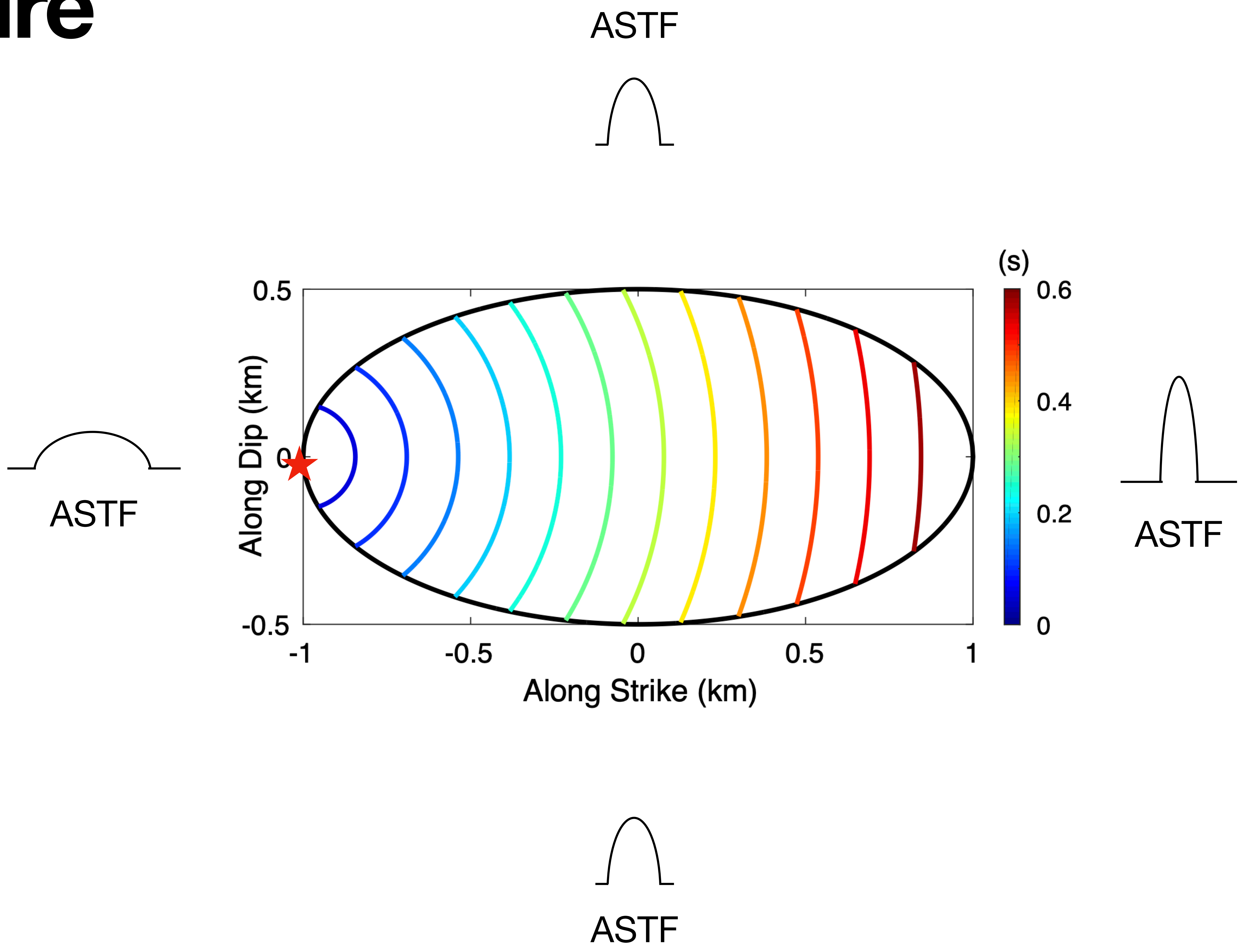
Short duration
High amplitude

$$\boxed{\widehat{\mu}^{(0,2)}(\underline{s})} = \int \mathrm{ASTF}(t)(t-t_0)\,(t-t_0)\mathrm{d}t$$
$$\tau_c(\underline{s}) = 2\sqrt{\widehat{\mu}^{(0,2)}(\underline{s})}$$

L_c W_c τ_c $\underline{v_0}$ v_c dir

Unilateral rupture

- Strike slip earthquake
  - Elliptical rupture area
    - Semi-major axis: 1km
    - Semi-minor axis: 0.5 km
  - Unilateral rupture:
    - Starting from the left corner
    - Vr = 3.2 km/s
  - Uniform slip: slip = 1m



Unilateral rupture

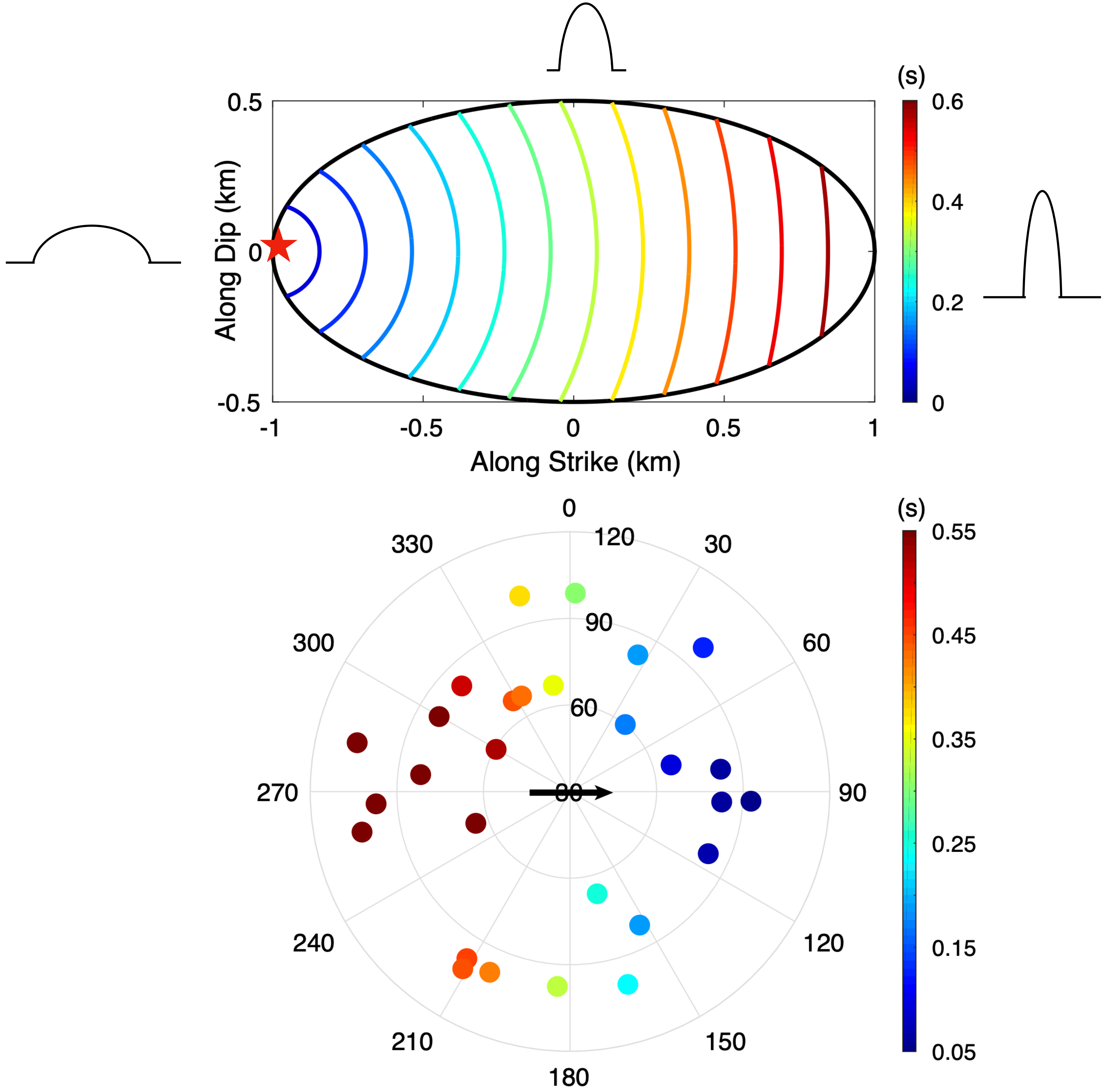
- Apparent Source time functions

$$\boxed{\mathrm{ASTF}(t) = \int \dot{f}(\underline{r}, t + \underline{r} \cdot \underline{s})\mathrm{d}V}$$

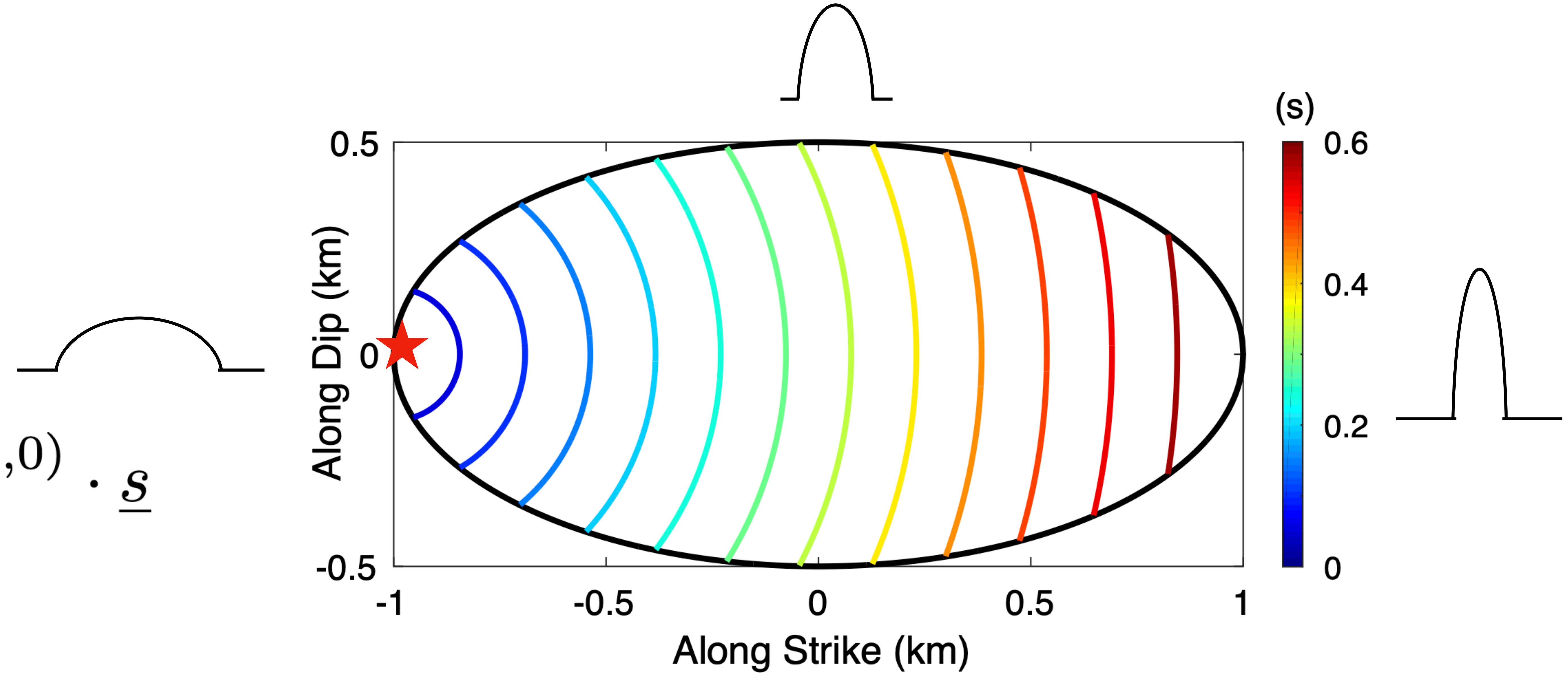
  - Sharper & shorter at the propagation direction
  - Wider & smoother in other directions

- Apparent temporal moments at multiple stations

$$\hat{\mu}^{(0,2)}(\underline{s}) = \int \mathrm{ASTF}(t)\,(t - t_0)^2 \mathrm{d}t$$



Inversion



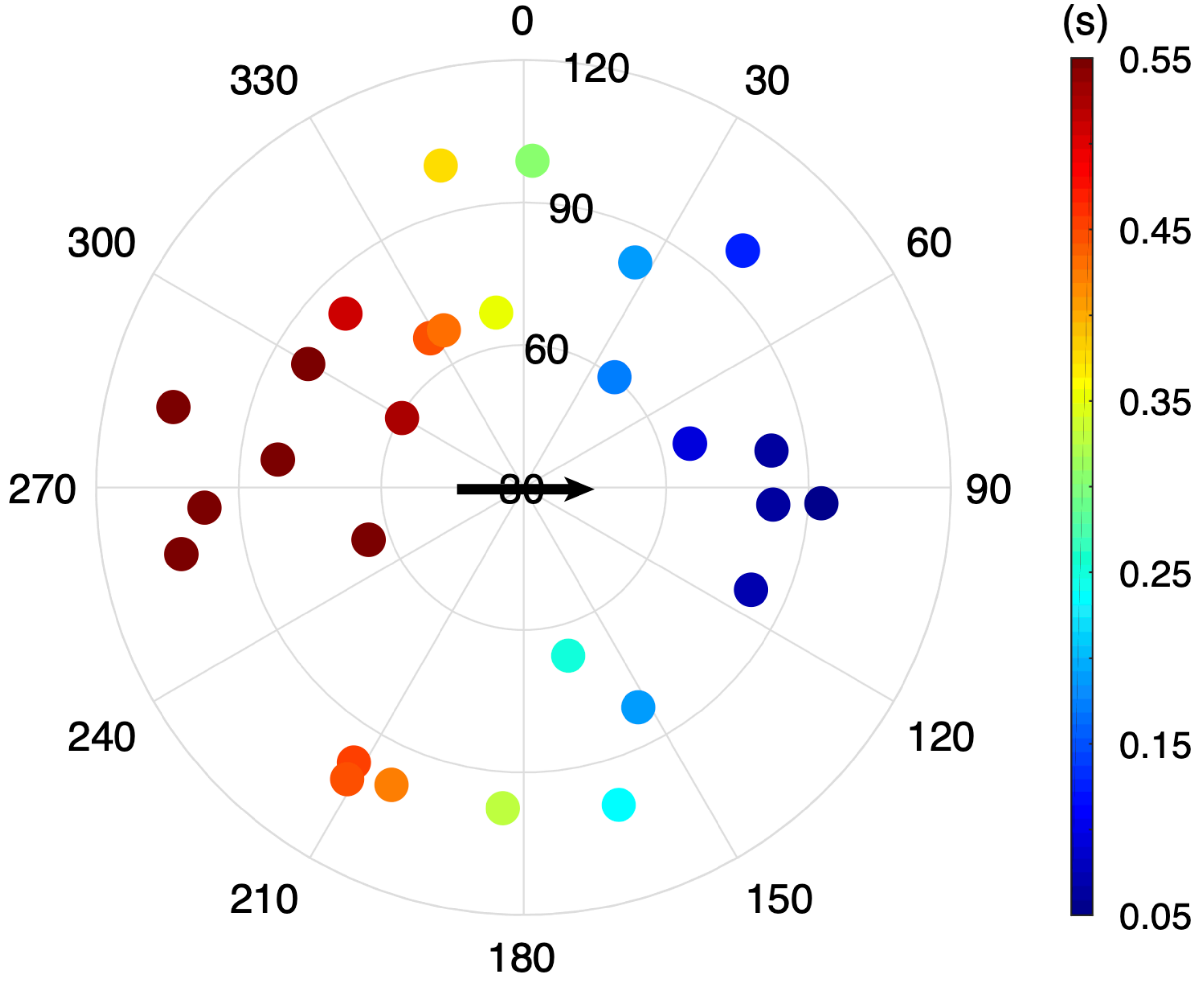
$$\widehat{\mu}^{(0,2)}(\underline{s}) = \widehat{\mu}^{(0,2)} - 2\underline{s}\cdot\underline{\widehat{\mu}}^{(1,1)} + \underline{s}\cdot\underline{\underline{\widehat{\mu}}}^{(2,0)}\cdot\underline{s}$$

- With good station coverage, inversion results are almost identical to the modeled parameters

$\underline{L_c}$ = 1.00 km $\qquad W_c$ = 0.50 km

τ_c = 0.30 km $\qquad \underline{v_0}$ = 3.29 km

dir = 1.00 $\qquad v_c$ = 3.29 km



Bilateral rupture

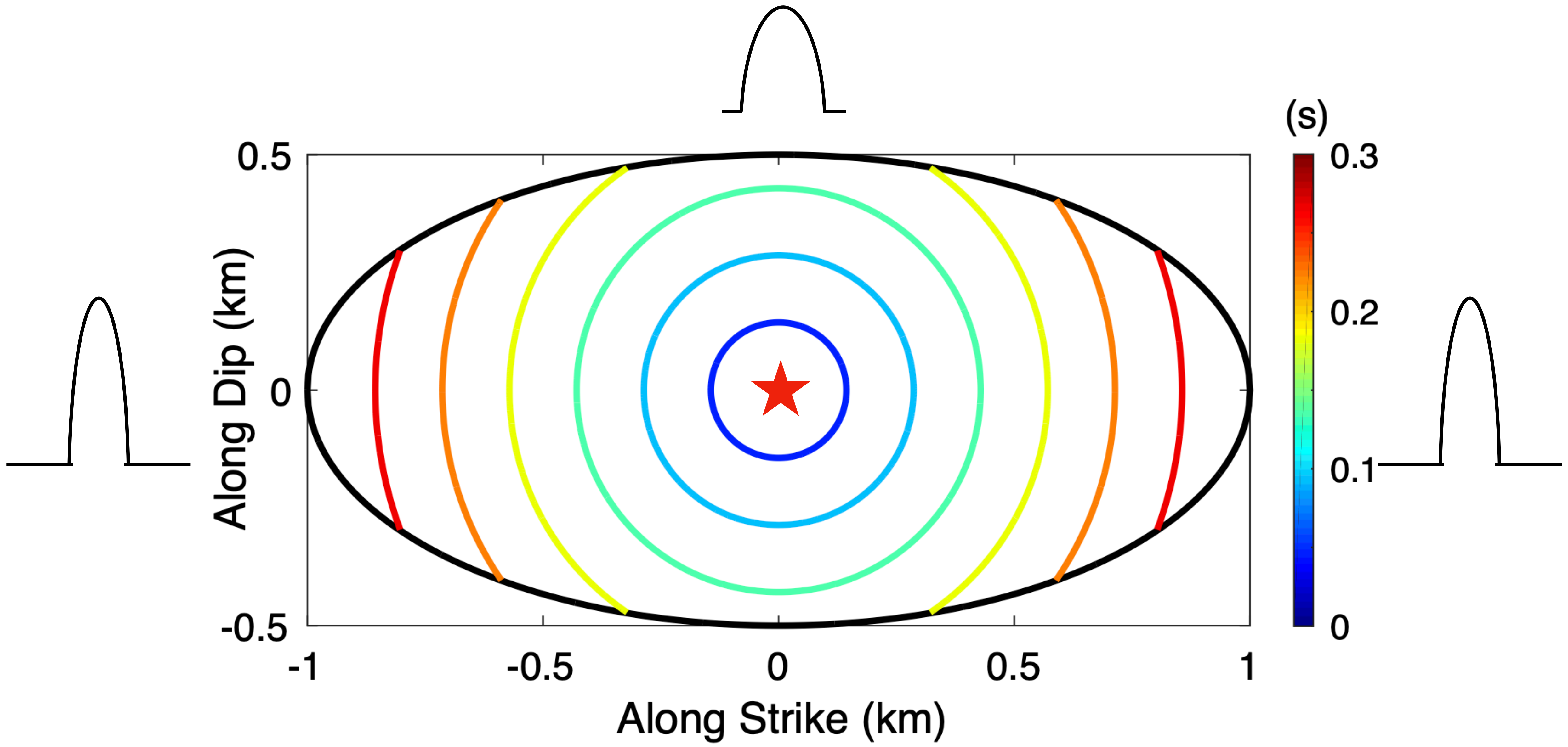
$$\widehat{\mu}^{(0,2)}(\underline{s}) = \widehat{\mu}^{(0,2)} - 2\underline{s}\cdot\underline{\widehat{\mu}}^{(1,1)} + \underline{s}\cdot\underline{\underline{\widehat{\mu}}}^{(2,0)}\cdot\underline{s}$$

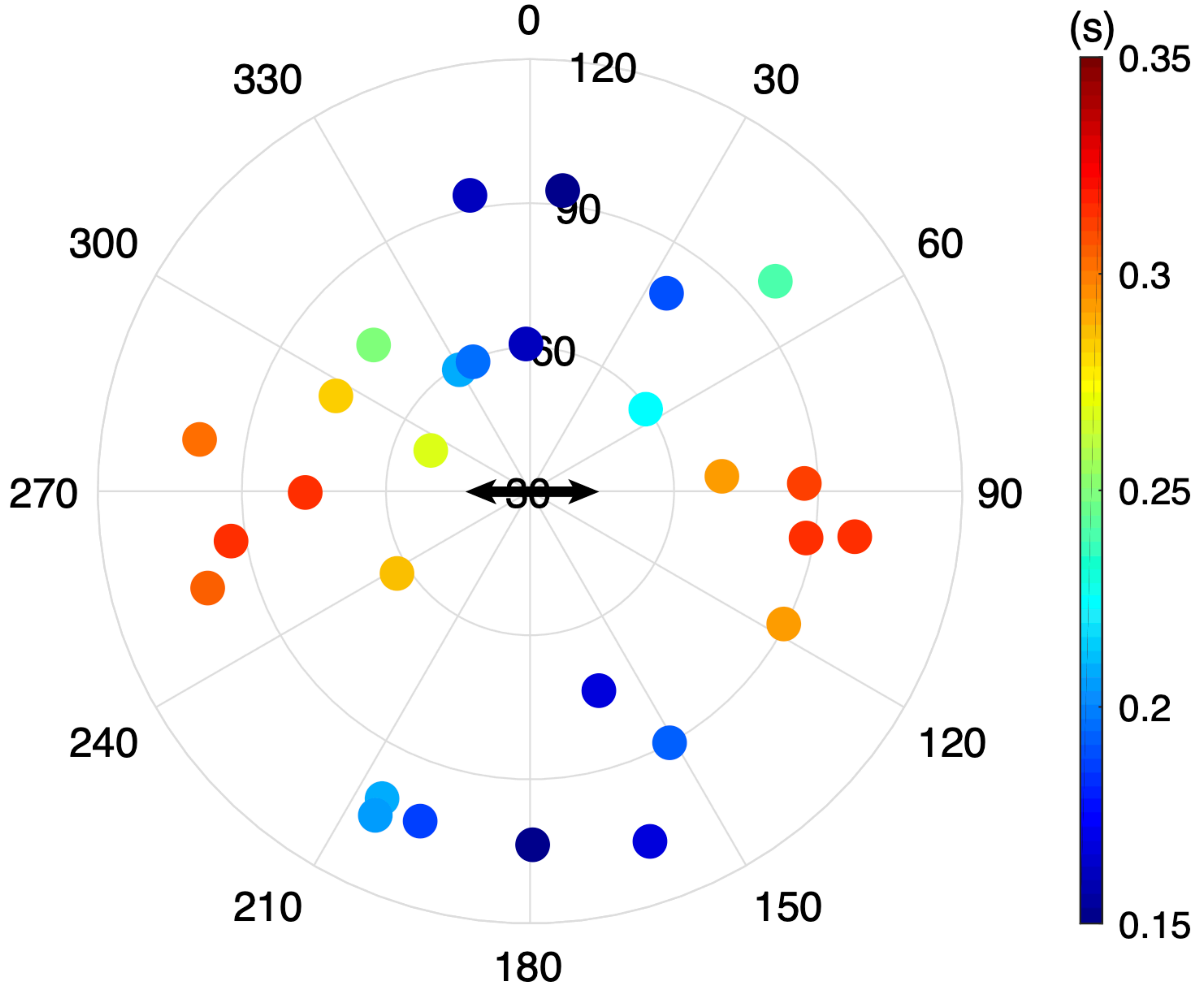
- With good station coverage, inversion results are almost identical to the modeled parameters

$\underline{L_c}$ = 1.00 km W_c = 0.50 km

τ_c = 0.13 km $\underline{v_0}$ = 0.00 km

dir = 0.00 v_c = 6.58 km





Summary II

- Find good eGf events
- Perform deconvolution to get ASTFs
- Measure the apparent temporal moments

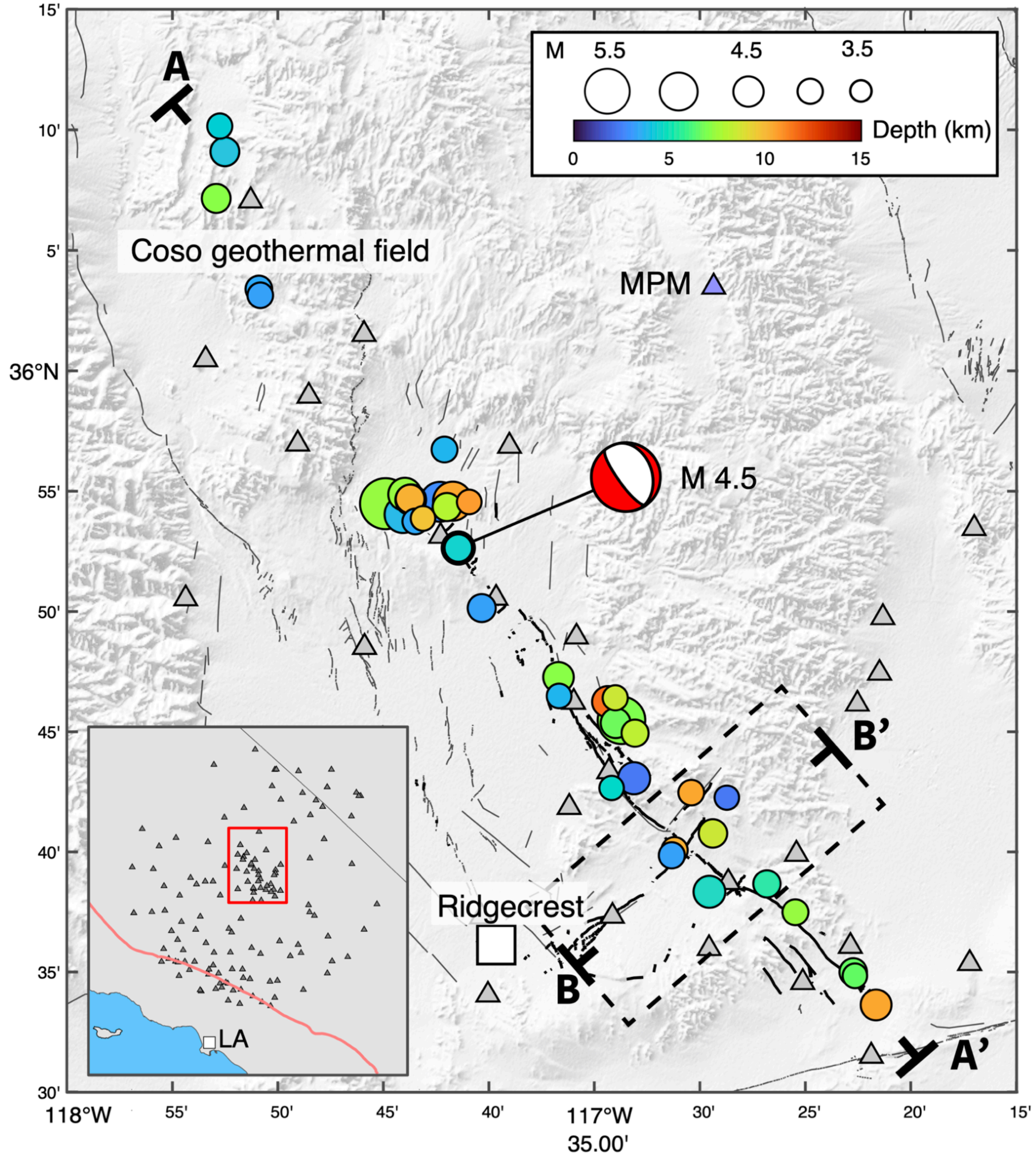
$$\hat{\mu}^{(0,2)}(\underline{s}) = \int \mathrm{ASTF}(t)\,(t - t_0)^2 \mathrm{d}t$$

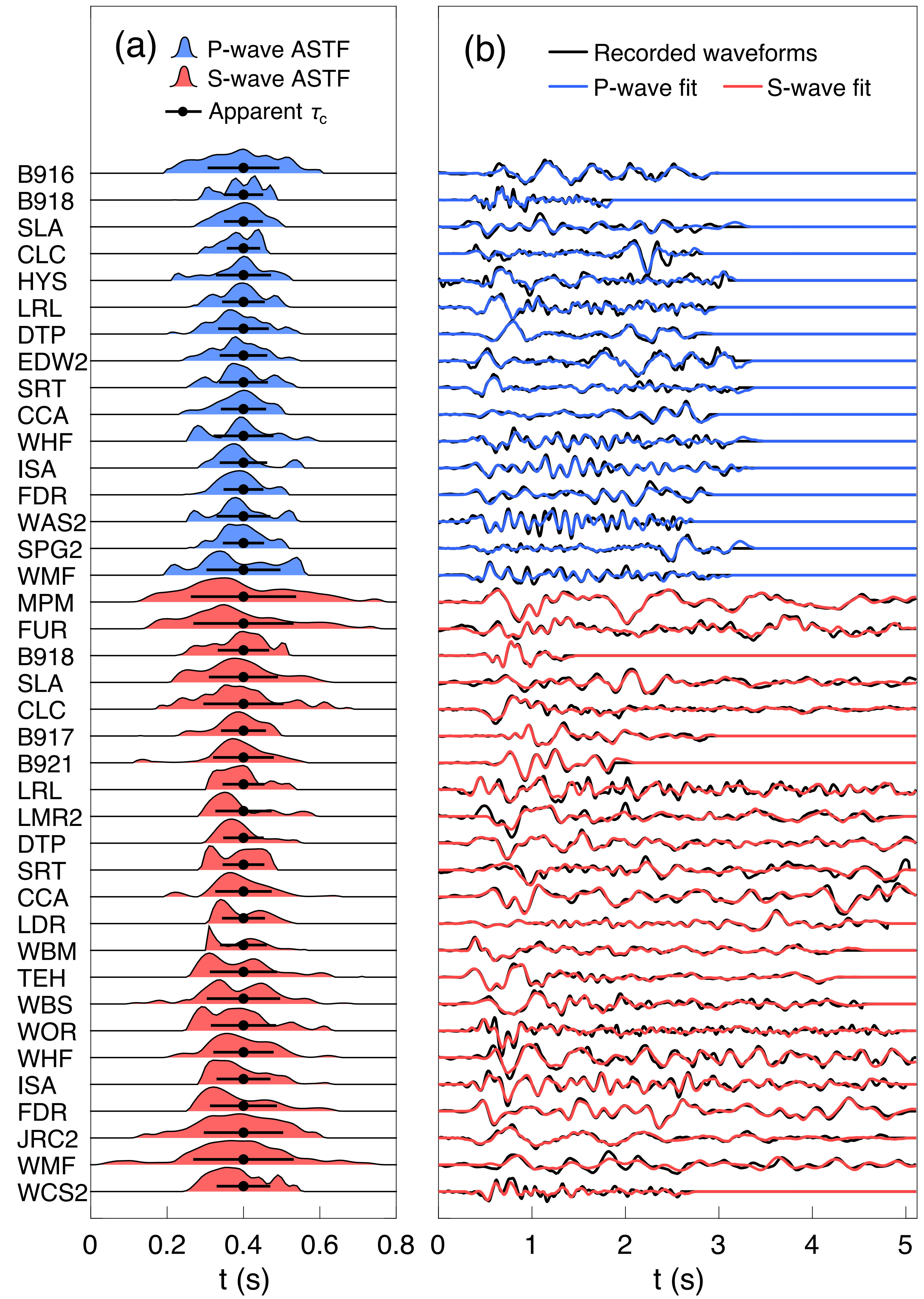
- Compute the slowness vector at the source region using ray tracing
- Perform second moment inversion

$$\widehat{\mu}^{(0,2)}(\underline{s}) = \widehat{\mu}^{(0,2)} - 2\underline{s} \cdot \underline{\widehat{\mu}}^{(1,1)} + \underline{s} \cdot \underline{\underline{\widehat{\mu}}}^{(2,0)} \cdot \underline{s}$$

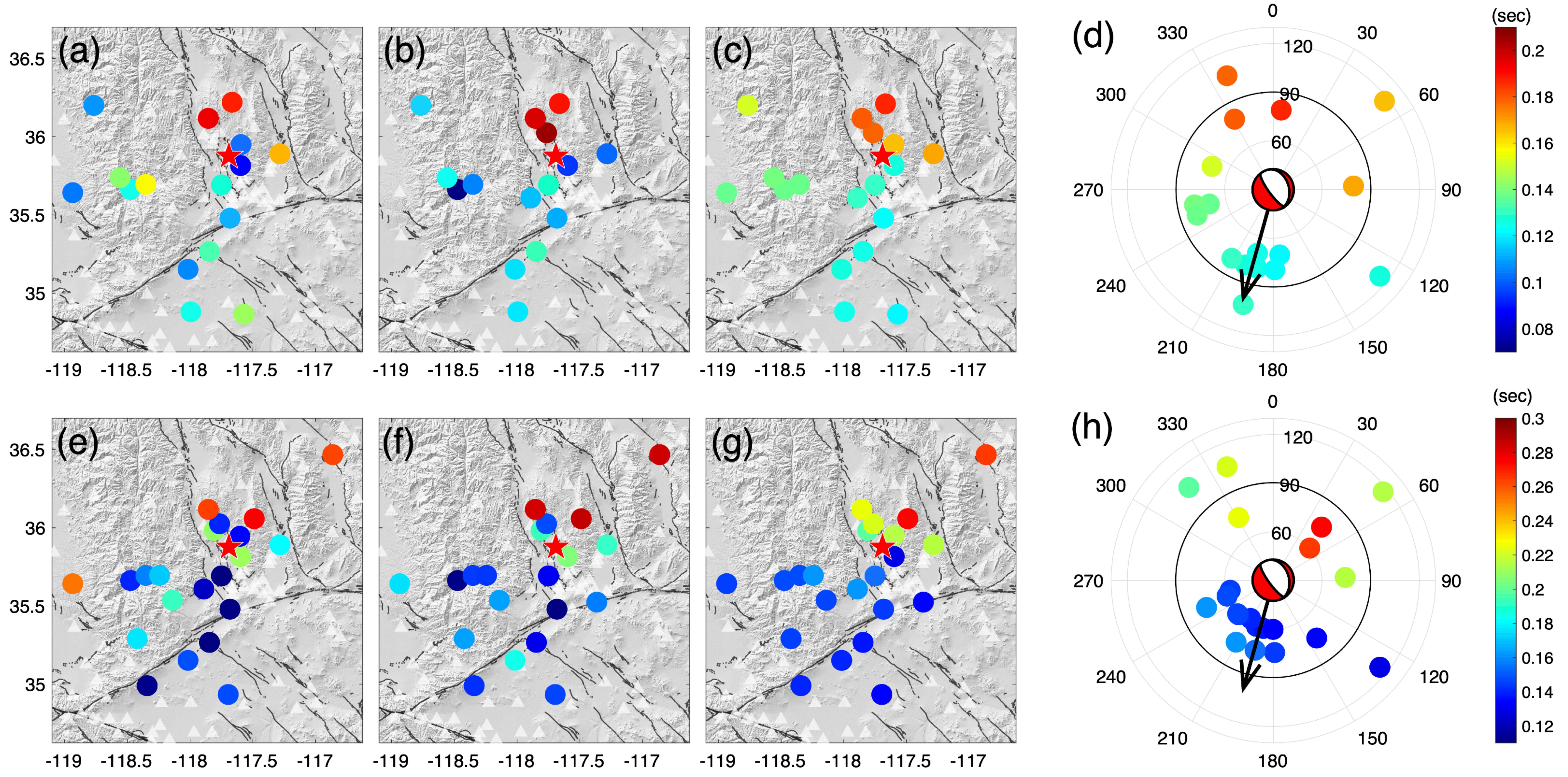
- Compute finite fault attributes $\underline{L_c}$ W_c τ_c $\underline{v_0}$ dir

M 4.5 earthquake in Ridgecrest, California





Real example of a M 4.5 earthquake



L_c 0.58 km

W_c 0.31 km

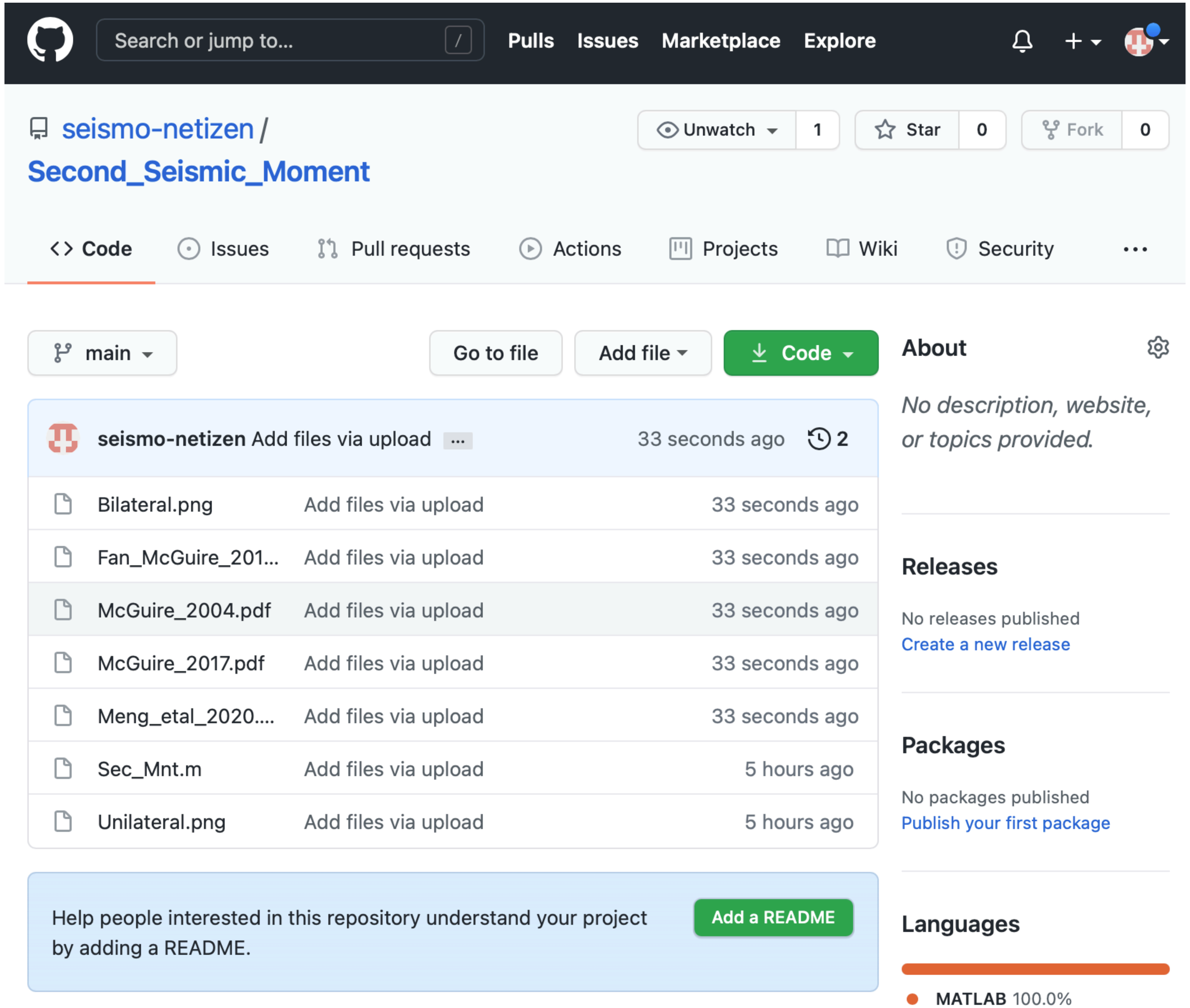
τ_c 0.14 sec

dir 0.42

v_0 = 1.82 km

Reading list

- **McGuire, Jeffrey J.** "Estimating finite source properties of small earthquake ruptures." *Bulletin of the Seismological Society of America* 94.2 (2004): 377-393.
- **McGuire, Jeffrey J.** "A MATLAB toolbox for estimating the second moments of earthquake ruptures." *Seismological Research Letters* 88.2A (2017): 371-378.
- **Fan, Wenyuan, and Jeffrey J. McGuire.** "Investigating microearthquake finite source attributes with IRIS Community Wavefield Demonstration Experiment in Oklahoma." *Geophysical Journal International* 214.2 (2018): 1072-1087.
- **Meng, Haoran, Jeffrey J. McGuire, and Yehuda Ben-Zion**. "Semiautomated estimates of directivity and related source properties of small to moderate southern California earthquakes using second seismic moments." *Journal of Geophysical Research: Solid Earth* 125.4 (2020): e2019JB018566.



https://github.com/seismo-netizen/Second_Seismic_Moment.git

Q & A

Haoran Meng
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- Which deconvolution method will you choose to get the ASTFs?
- Are there other methodology to estimate finite fault attributes of small earthquakes?
- Can we estimate stress drop from second seismic moments?

Q & A

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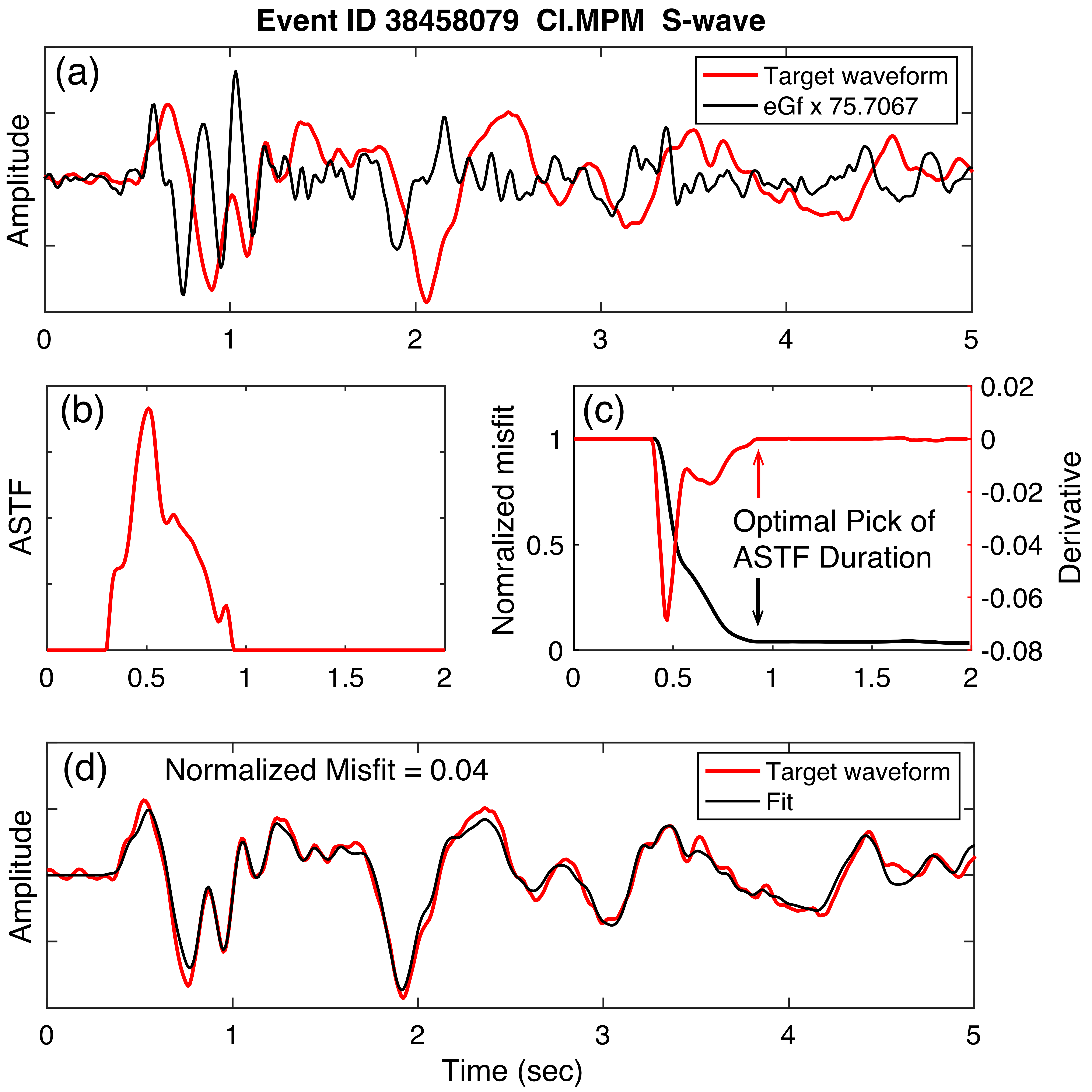
Deconvolution

- Projected Land-weber deconvolution
- Bertero et al.,1997
- Lanza et al.,1999



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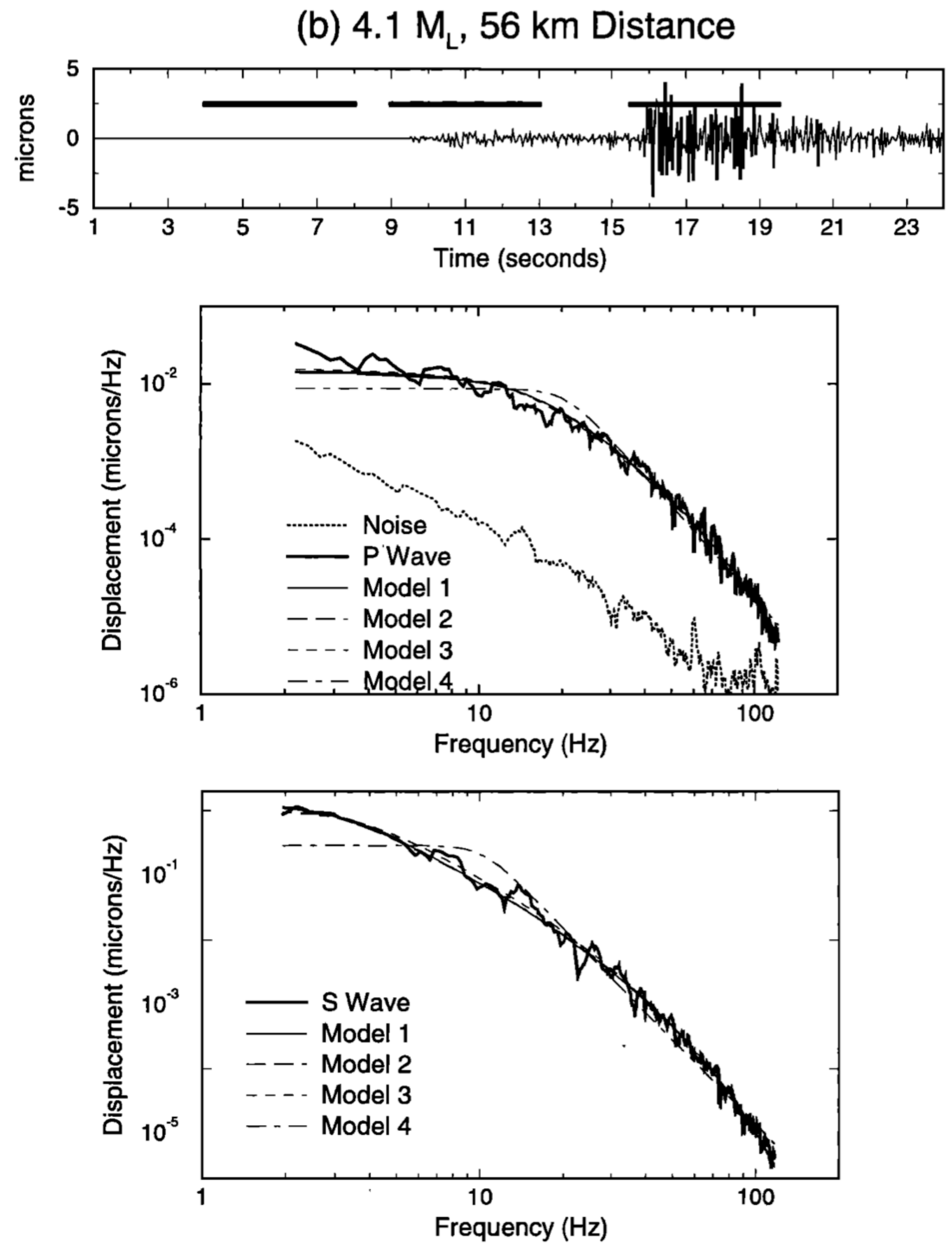
Frequency domain

- Corner frequency of spectral ratio
  - Savage (1972)
  - Madariaga (1976)
  - Abercrombie (1995)



Abercrombie 1995

Q & A

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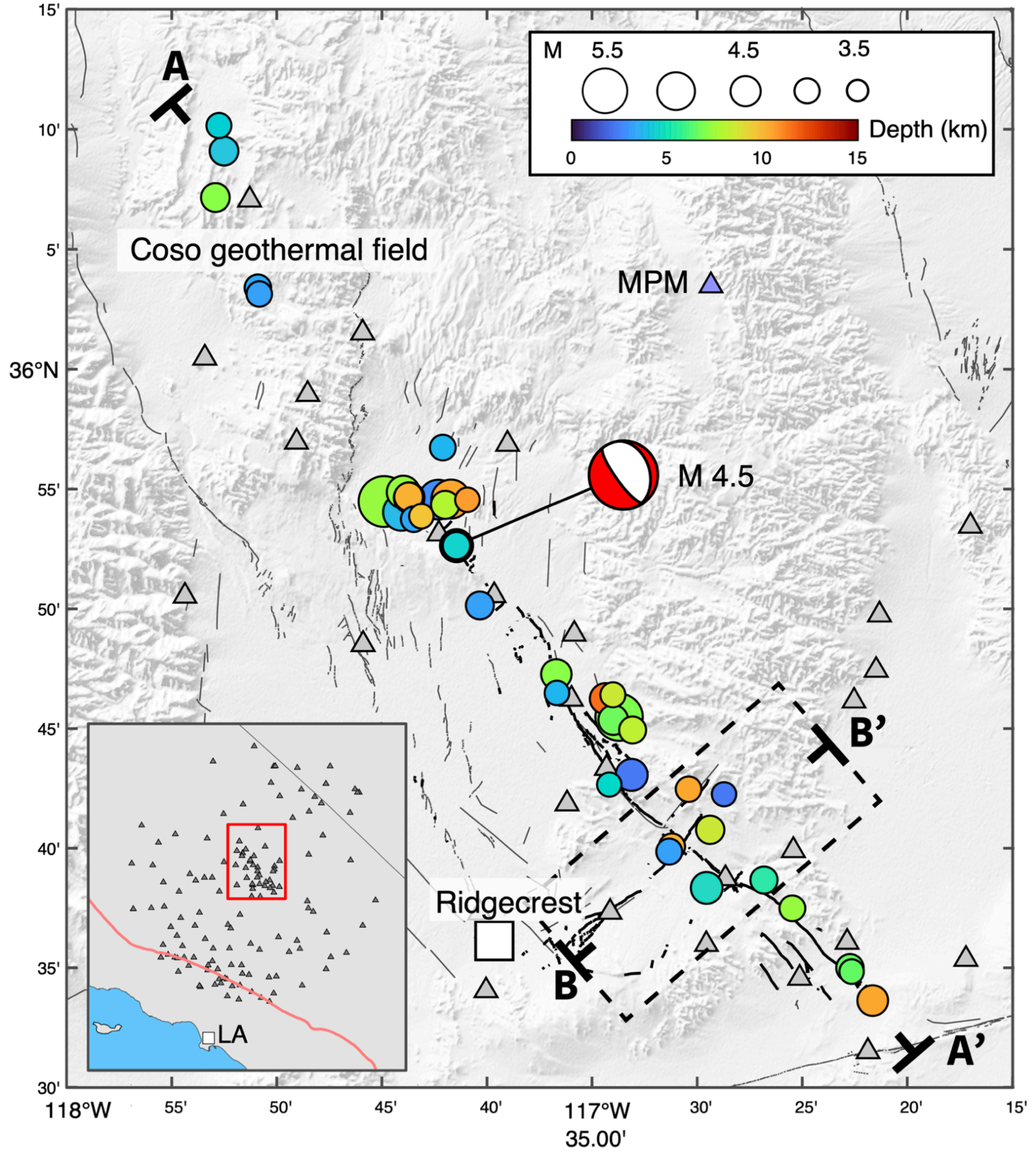
Stress Drop Estimates



- Elliptical crack, Length Lc, width Wc
- 37.8 MPa (Eshelby, 1957)

$$\Delta\sigma = C(L_c, W_c, \nu)\frac{M_0}{W_c S}$$

L_c 0.58 km

W_c 0.31 km

Q & A

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- Which deconvolution method will you choose to get the ASTFs?
  - Projected Land-weber deconvolution
- Are there other methodology to estimate finite fault attributes of small earthquakes?
  - Frequency domain method
- Can we estimate stress drop from second seismic moments?
  - Yes, using Eshelby (1957)