Stress Drop Reading Group

Week 2

August 7th, 2024

Corner frequency (f_c) and stress drop ($\Delta\sigma$)

- (Moment-based) stress drop ← source dimension (radius)
- Most popular approach is to estimate radius (a) from corner frequency (f_c).

$\Delta\sigma = C(a, b, v)\frac{M_0}{bS}$ (Eshelby, 1957) – (1)

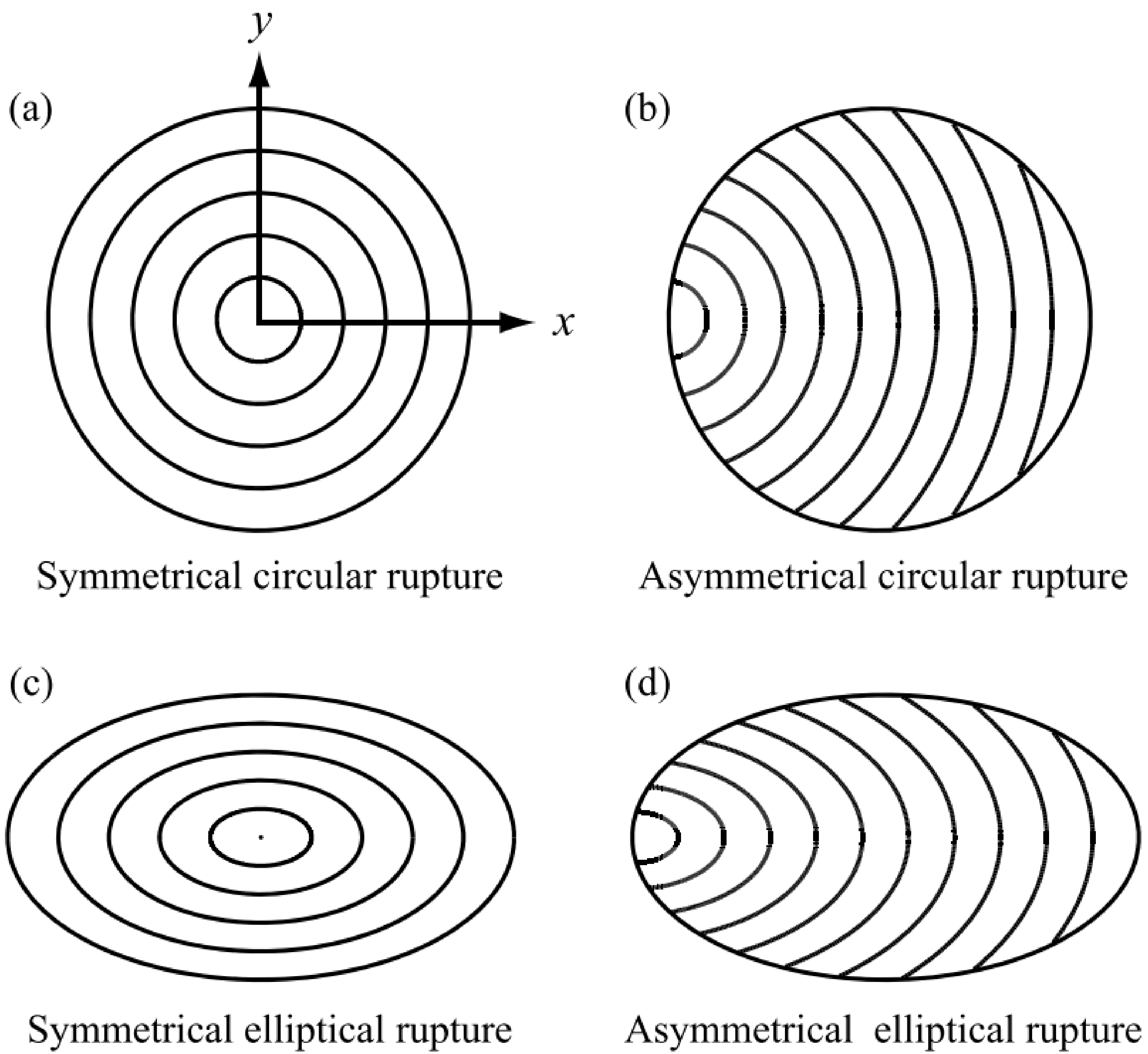
$\Delta\sigma = \frac{7}{16}\frac{M_0}{a^3}$ ($a = b;\ \upsilon = 0.25$) – (2)

$\overline{f_c} = k\frac{\beta}{a}$ (Brune, 1970; Madariaga, 1976) – (3)

(2) & (3) → $\Delta\sigma = \frac{7}{16}\left(\frac{\overline{f_c}}{k\beta}\right)^3 M_0$ – (4)

- Q) How is k affected by different assumptions about seismic source?

Different source scenarios



- Symmetry
- Geometry
- Rupture speed
- Rupture mode

Numerical scheme (Kaneko & Shearer, 2014, 2015)

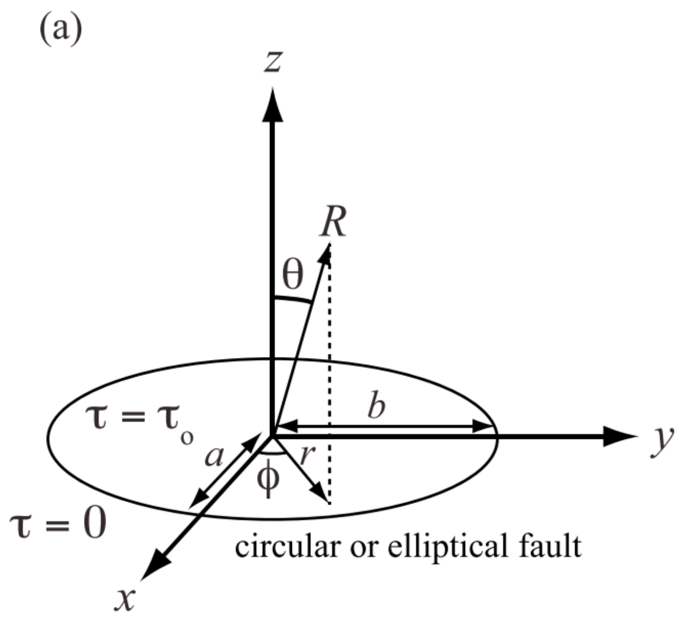
- Infinite homogeneous, isotropic elastic medium
- **Cohesive zone model**

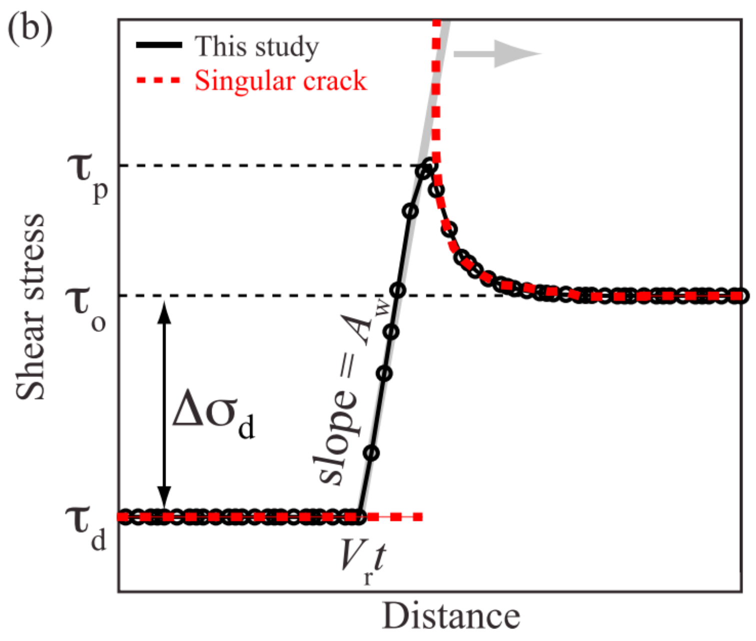
  $\tau_{st} = \max\{\tau_d, \tau_d + A_w(|\vec{r} - V_r t|)\}$ (Andrews, 1985; Dunham and Bhat, 2008)

  $V_r = V_x V_y \left(\frac{x^2+y^2}{V_y^2 x^2 + V_x^2 y^2}\right)^{1/2}$ (V_x: major axis, V_y: minor axis)

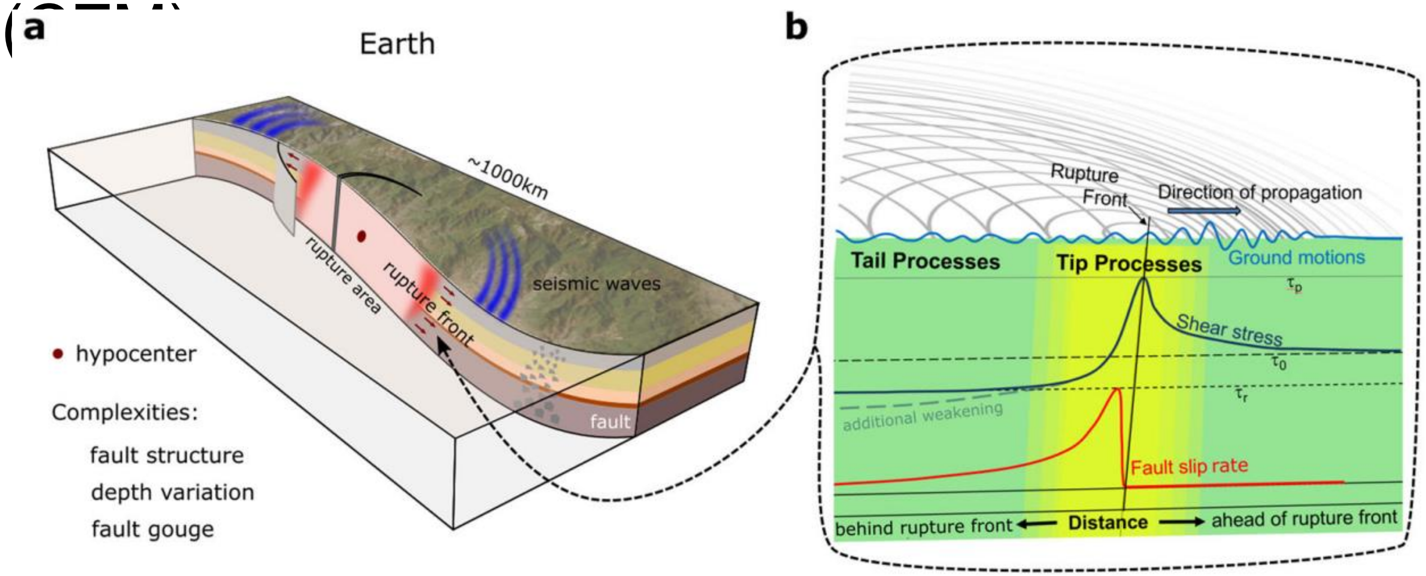
  τ_p: determined by A_w & τ_d

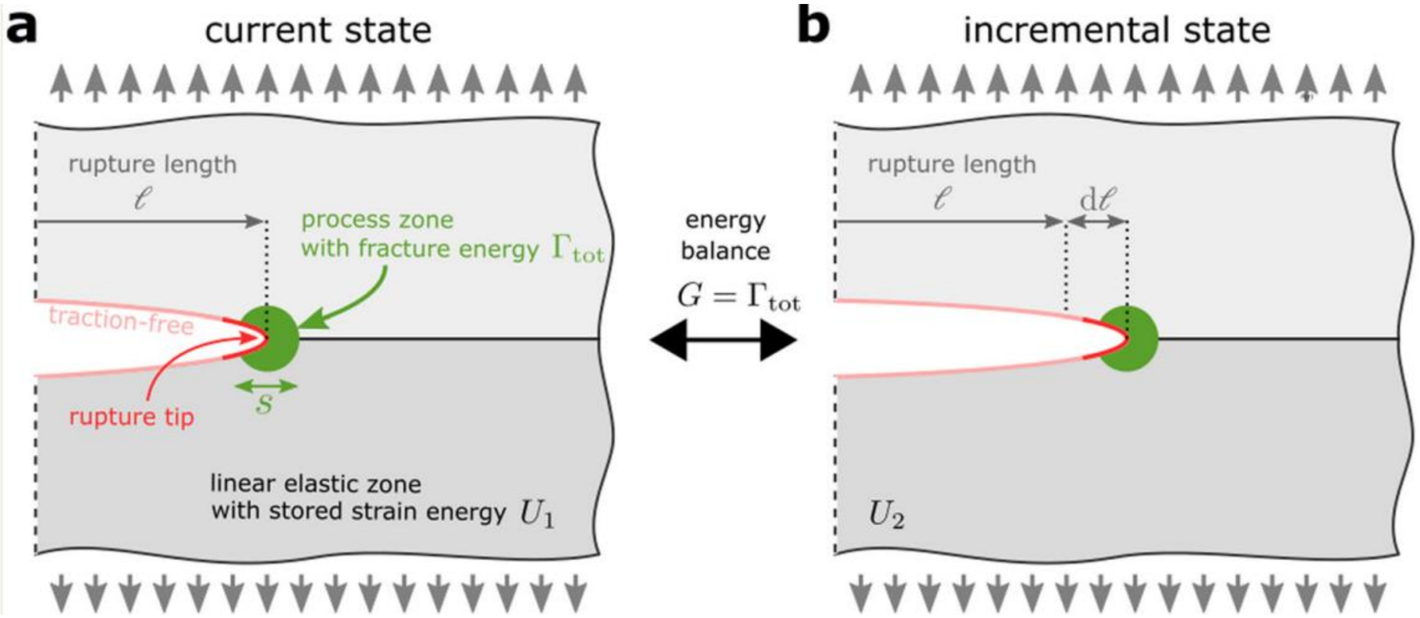
- Fixed rupture speeds (not spontaneous)





Cohesive zone model (CZM)





Kammer et al., (2024)

Linear Elastic Fracture Mechanics (LEFM)

G_0: Static energy release rate

Γ_{tot}: Total fracture energy

$G_0 = -\frac{d\Pi}{dl}$ where $d\Pi = (U_2 - U_1) - F$

To avoid infinite stresses at the crack tip, the dissipation is often smeared out in *"process zone"* of size $s \ll l$, commonly implemented via *cohesive zone models*.

LEFM predicts the crack growth speed through an energy balance

→ $G = \Gamma_{tot}$ where $G \leq G_0$
(G: dynamic energy release rate)

$G = \left(1 - \frac{C_f}{C_R}\right) G_0$ where $C_f = dl/dt$

C_R: Rayleigh wave speed

Case I Symmetrical circular rupture

- Nondimensional variables (normalized)

  Length $r' = r/\sqrt{ab}$

  Time $t' = t\beta/\sqrt{ab}$

  Stress $\sigma'_{ij} = \sigma_{ij}/\Delta\sigma_d$

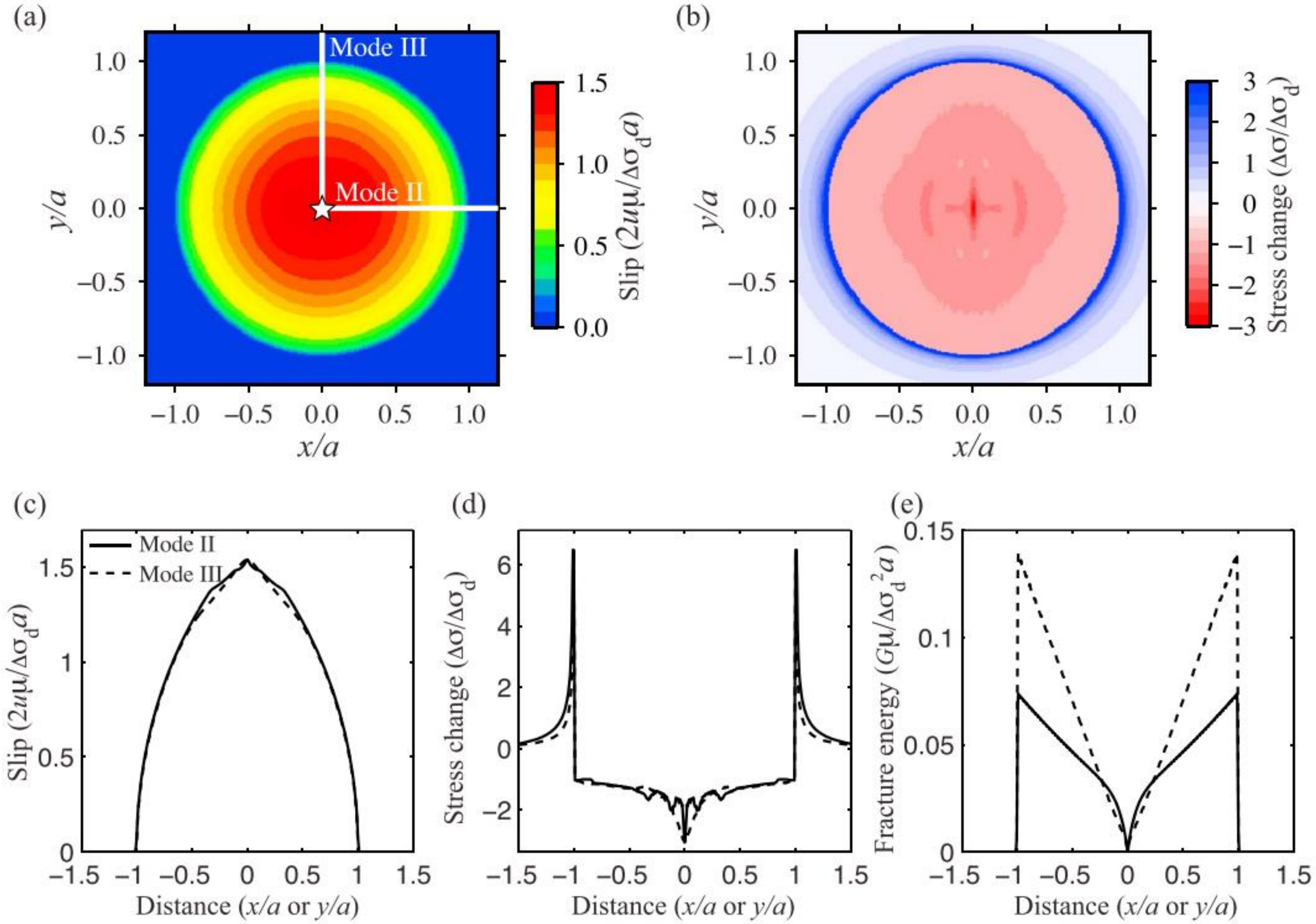
  Displacement $u'_i = u_i\mu/(\Delta\sigma_d\sqrt{ab})$

  Weakening rate $A'_w = A_w\sqrt{ab}/\Delta\sigma_d$

  Seismic moment $M'_0 = M_0/[\Delta\sigma_d^2(ab)^{3/2}]$

  Fracture energy density $G' = G\mu/(\Delta\sigma_d^2\sqrt{ab})$

  Radiated energy $E'_r = E\mu/[\Delta\sigma_d^2(ab)^{3/2}]$



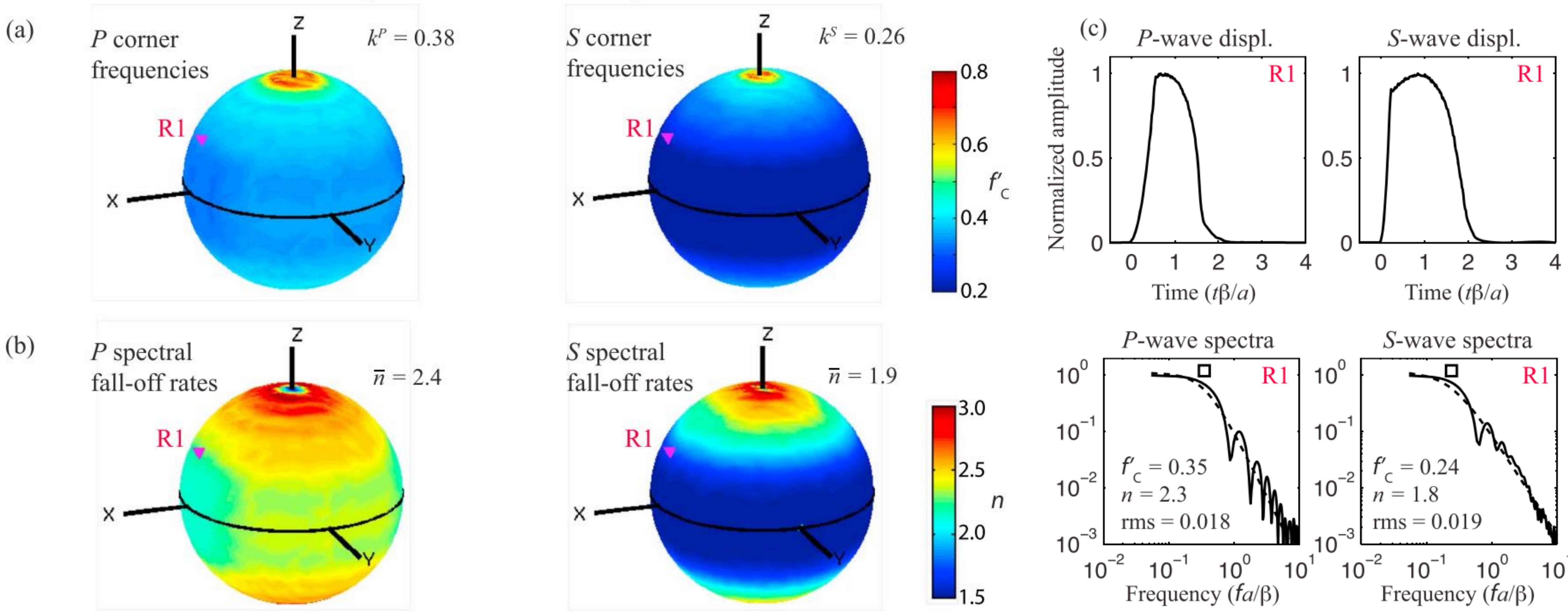
$A'_w = 84,\ V_r/\beta = 0.9$

Case I Symmetrical circular rupture

$$\overline{f_c}^P = k^P \frac{\beta}{a} = 0.38 \frac{\beta}{a}$$

$$\overline{f_c}^S = k^S \frac{\beta}{a} = 0.26 \frac{\beta}{a}$$

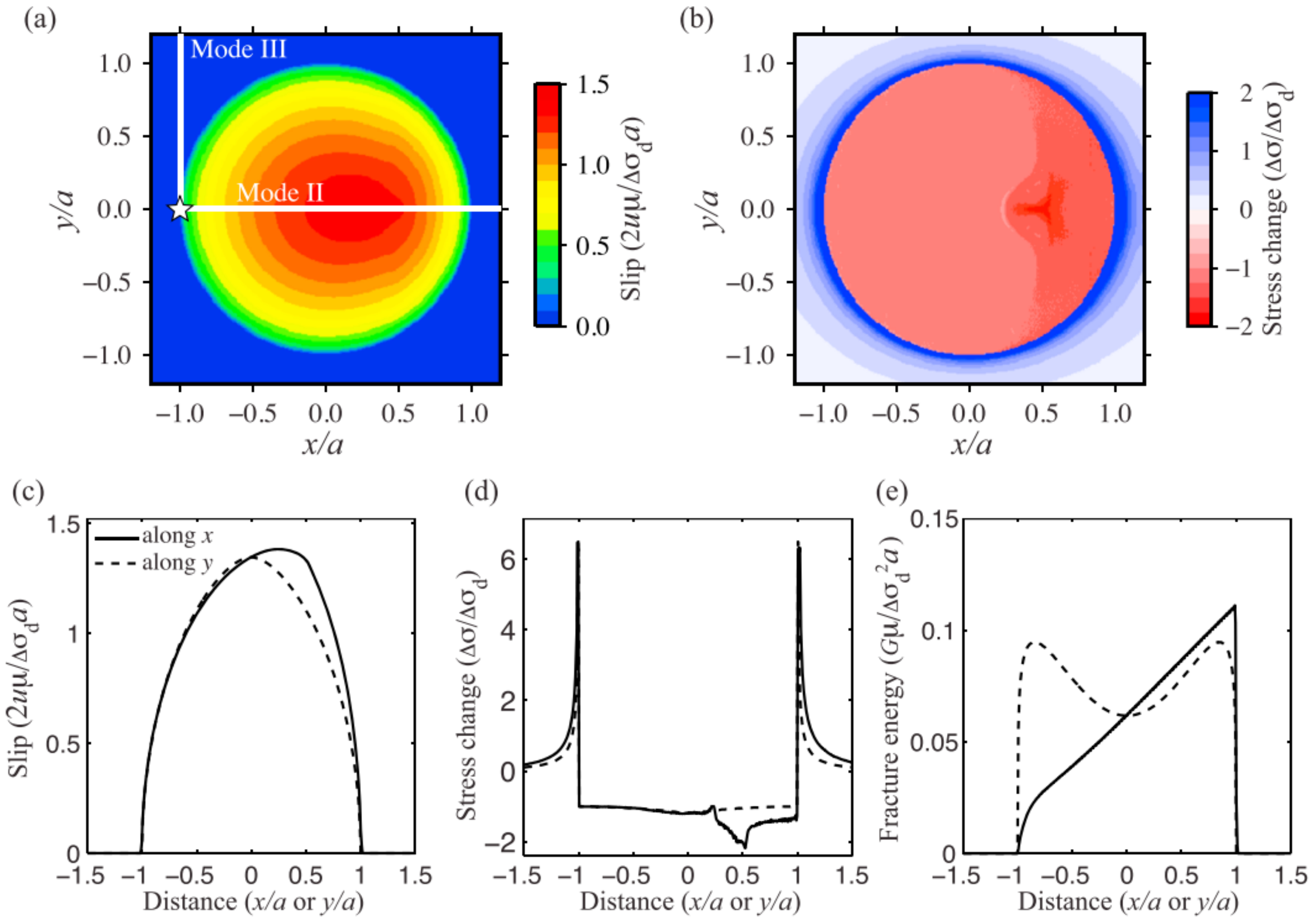
Symmetrical circular rupture

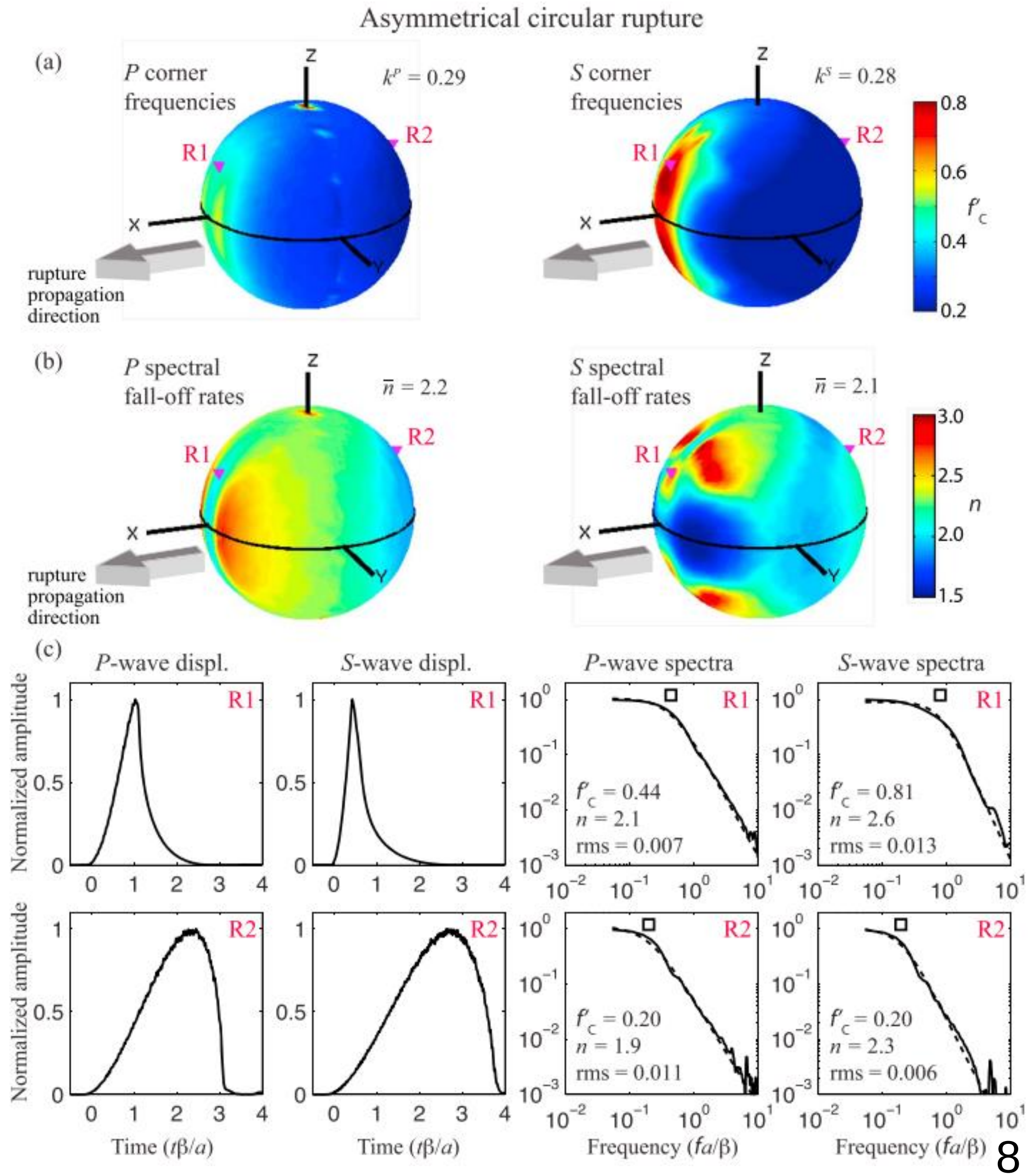


Case II Asymmetrical circular rupture

$$\overline{f_c}^P = k^P \frac{\beta}{a} = 0.29 \frac{\beta}{a}$$

$$\overline{f_c}^S = k^S \frac{\beta}{a} = 0.28 \frac{\beta}{a}$$



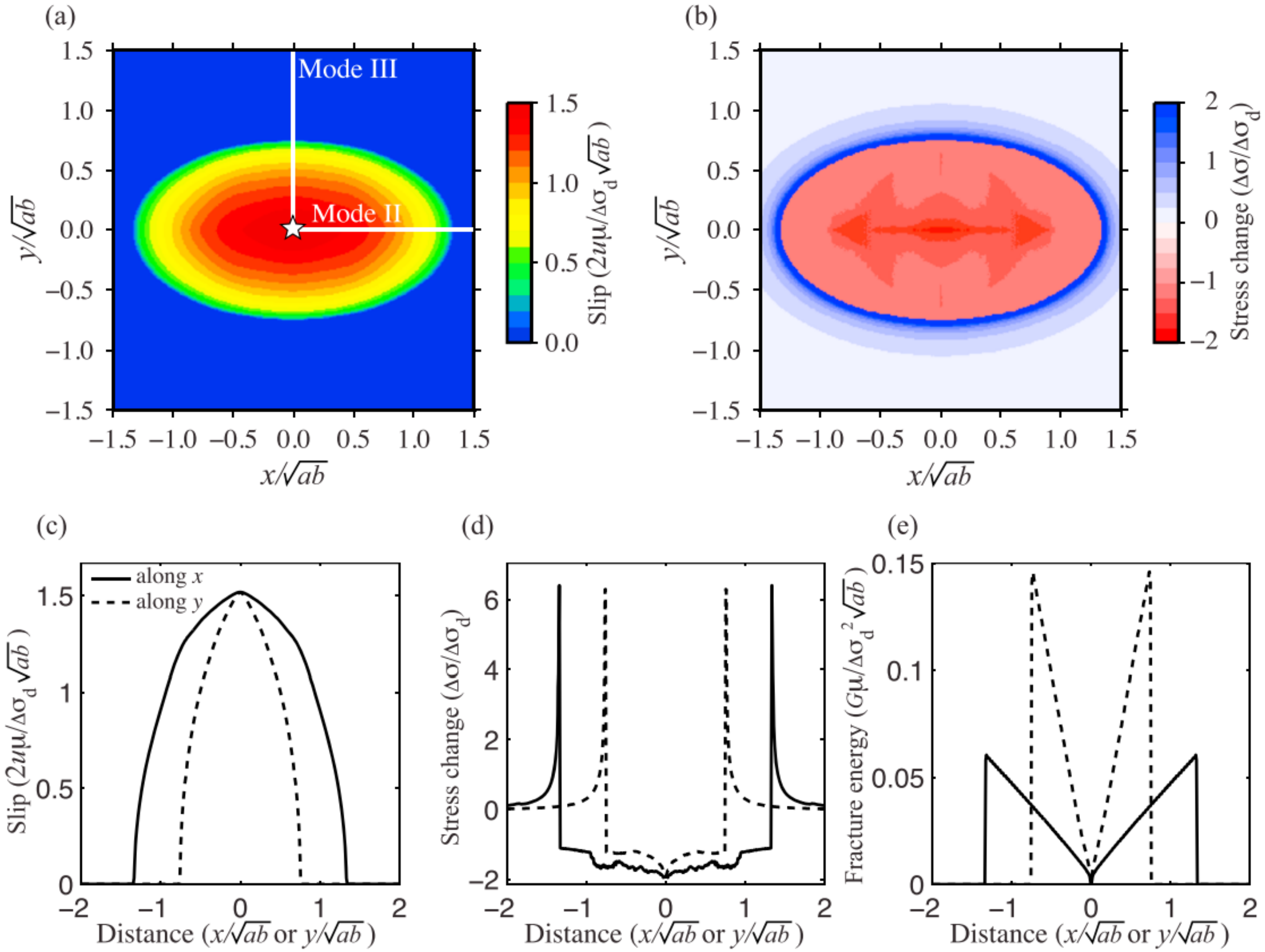


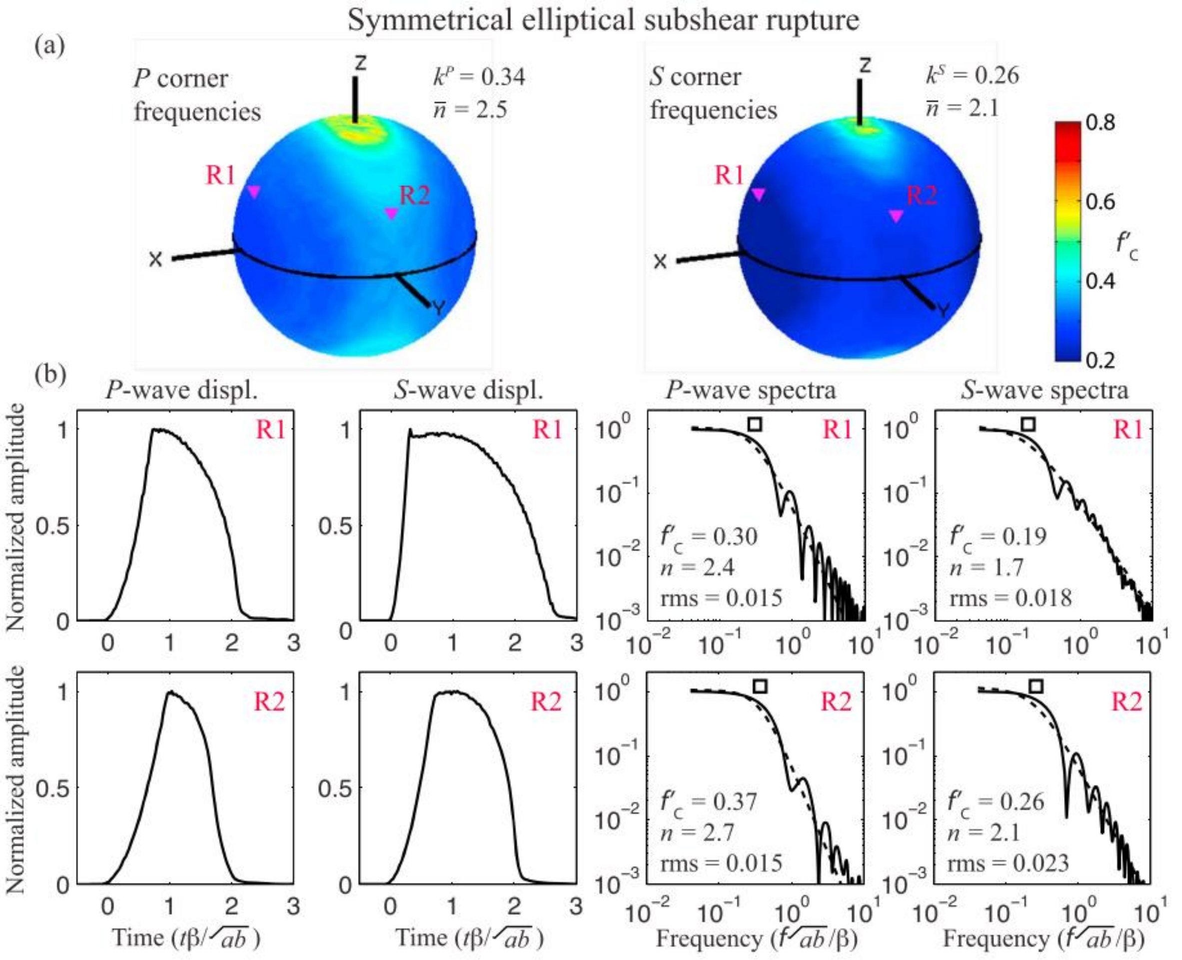
Case III Symmetrical elliptical rupture

$$\overline{f_c}^P = k^P \frac{\beta}{\sqrt{ab}} = 0.34 \frac{\beta}{\sqrt{ab}}$$

(subshear)

$$\overline{f_c}^S = k^S \frac{\beta}{\sqrt{ab}} = 0.26 \frac{\beta}{\sqrt{ab}}$$





Case III Symmetrical elliptical rupture

$$\bar{f_c}^P = k^P \frac{\beta}{\sqrt{ab}} = 0.34 \frac{\beta}{\sqrt{ab}}$$

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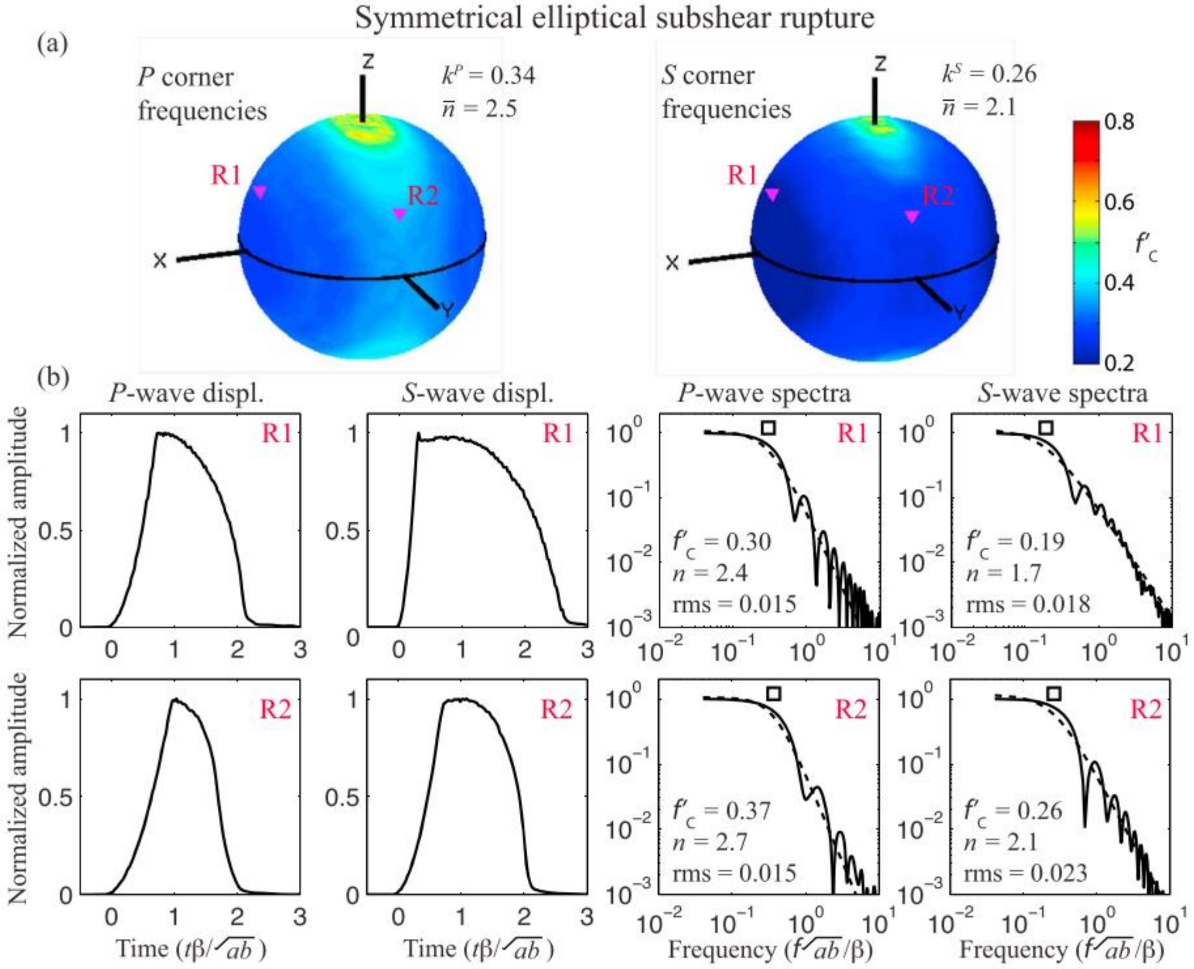
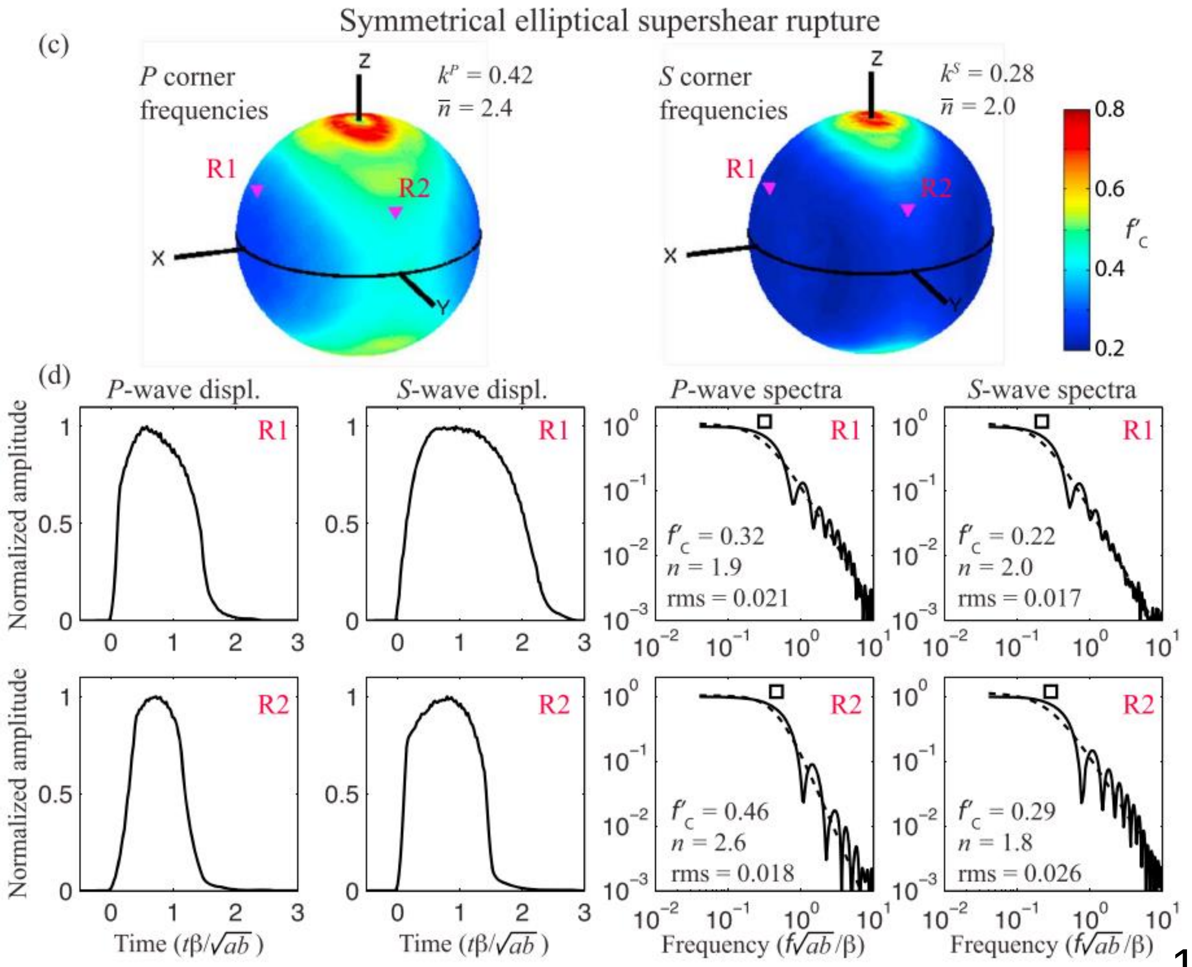
(subshear)

$$\bar{f_c}^P = k^P \frac{\beta}{\sqrt{ab}} = 0.42 \frac{\beta}{\sqrt{ab}}$$

$$\bar{f_c}^S = k^S \frac{\beta}{\sqrt{ab}} = 0.28 \frac{\beta}{\sqrt{ab}}$$

(supershear)



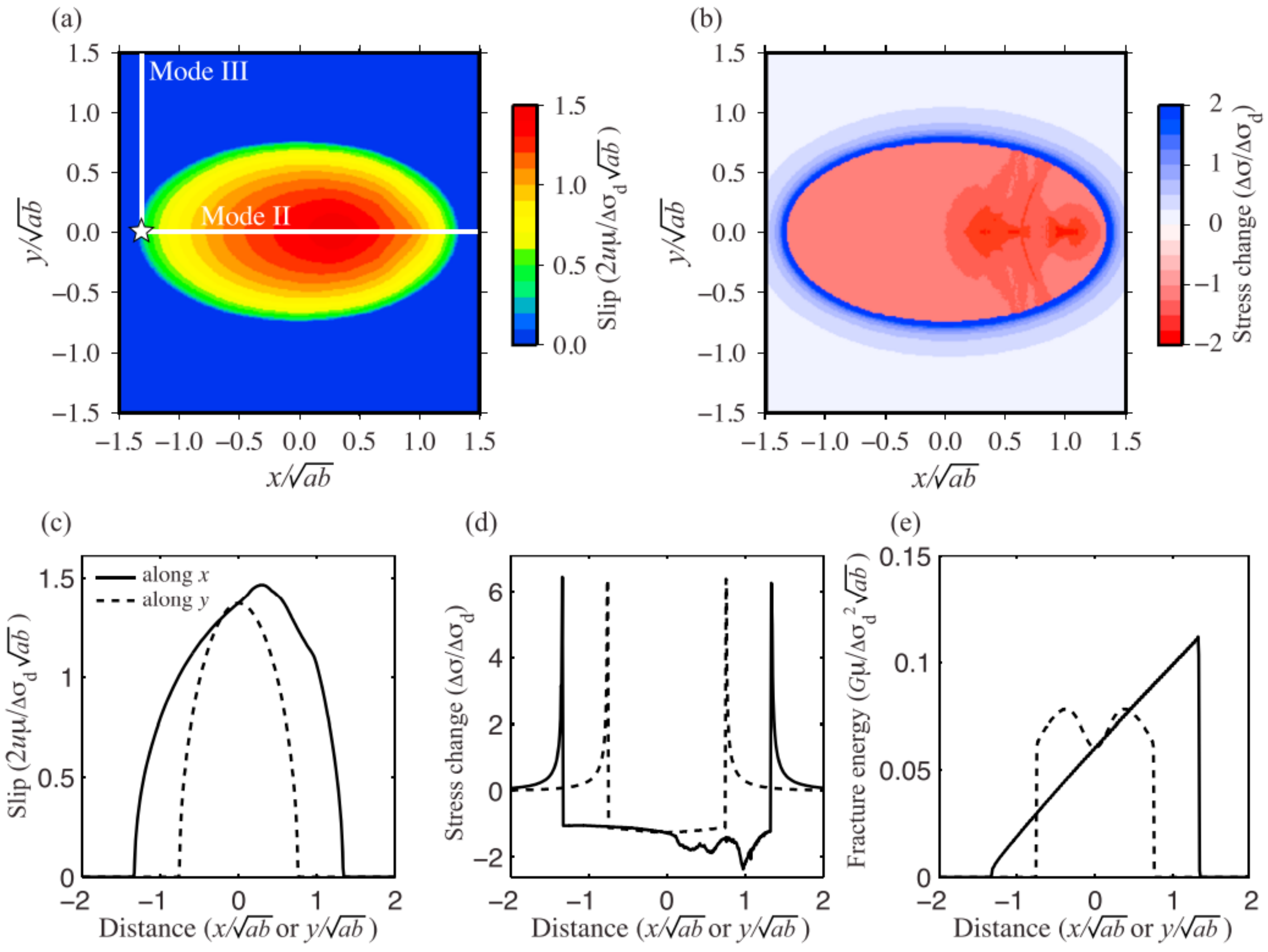


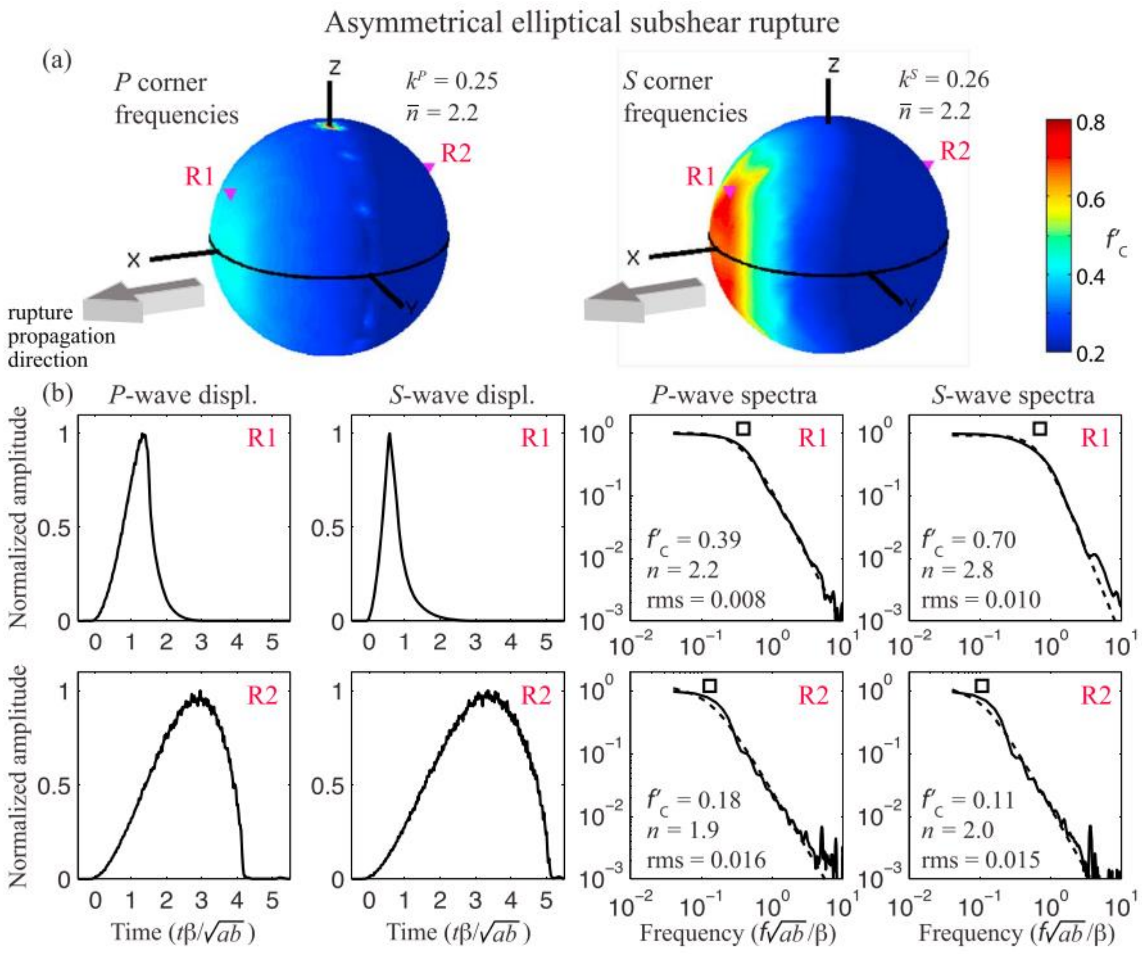
Case IV Asymmetrical elliptical rupture

$$\overline{f_c}^{P} = k^{P}\frac{\beta}{\sqrt{ab}} = 0.25\frac{\beta}{\sqrt{ab}}$$

$$\overline{f_c}^{S} = k^{S}\frac{\beta}{\sqrt{ab}} = 0.26\frac{\beta}{\sqrt{ab}}$$

(subshear)





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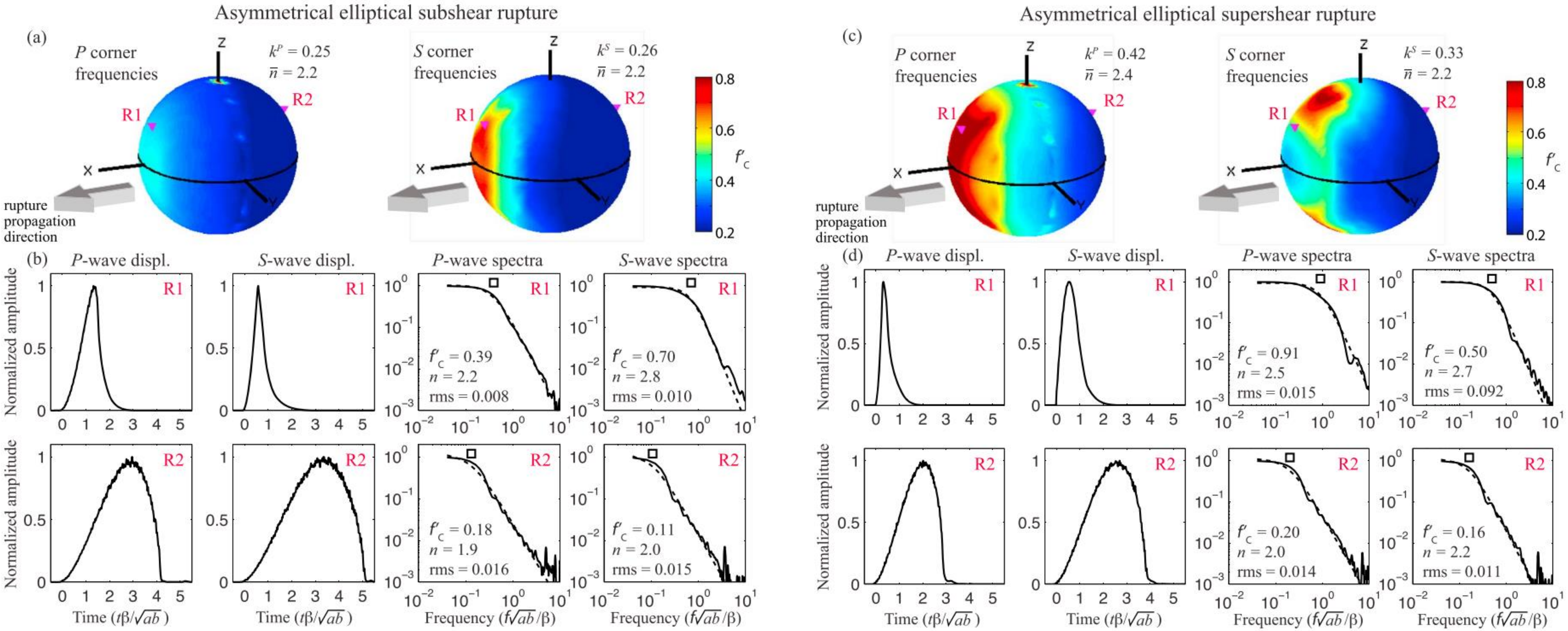
(subshear
)

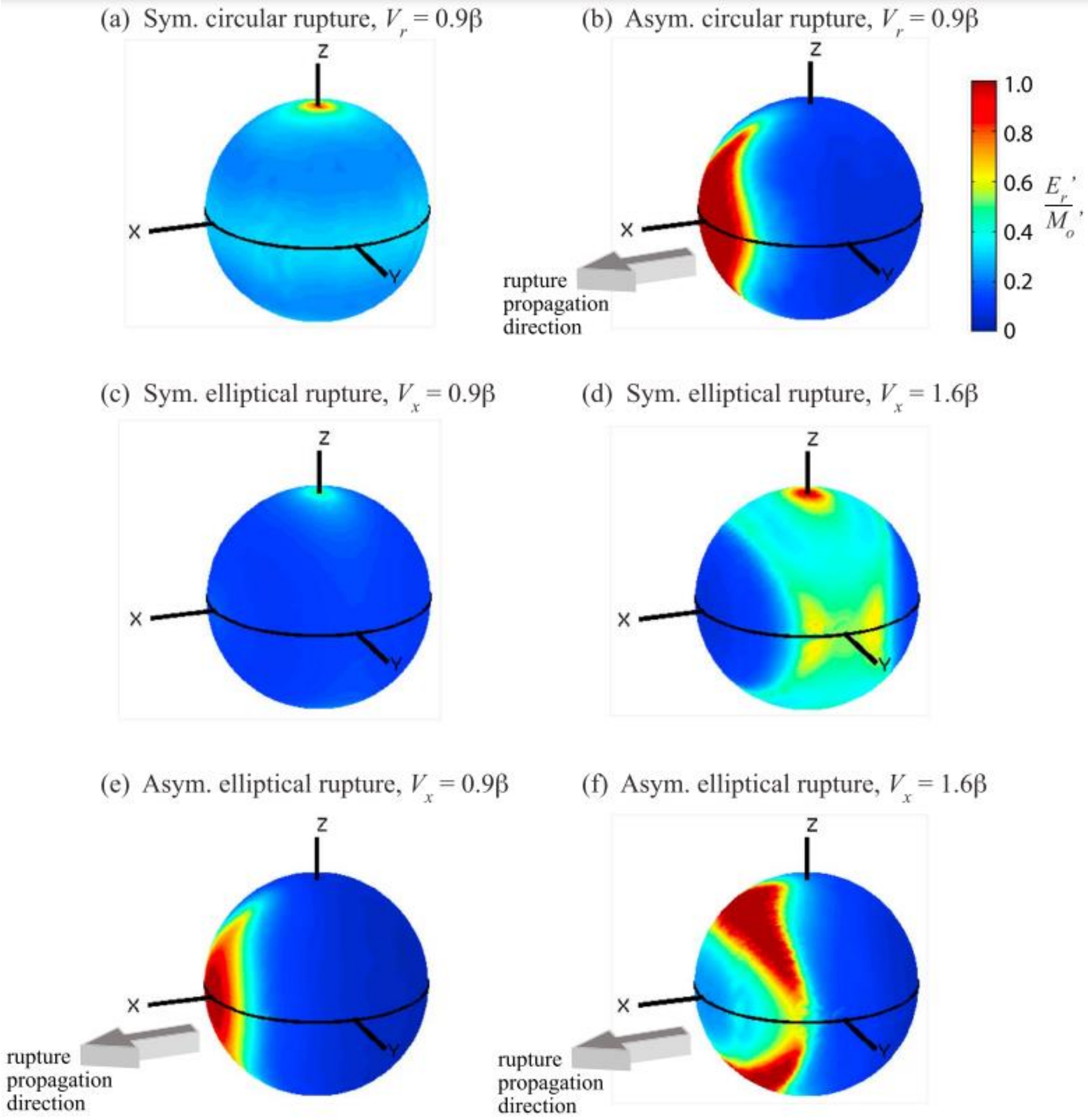
$$\bar{f_c}^P = k^P \frac{\beta}{\sqrt{ab}} = 0.42 \frac{\beta}{\sqrt{ab}}$$

$$\bar{f_c}^S = k^S \frac{\beta}{\sqrt{ab}} = 0.33 \frac{\beta}{\sqrt{ab}}$$

(supershear)



Radiated energy



$$E_r = \int_\Sigma \frac{\tau^o(\xi) + \tau^f(\xi)}{2} D(\xi) d\Sigma - \int_0^\infty \int_\Sigma \tau(\xi,t) V(\xi,t) d\Sigma dt$$

Far-field estimation

$$E_r = E_r^P + E_r^S = 2\rho \int_\Gamma \int_0^\infty [\alpha(\dot{u}^P)^2 + \beta(\dot{u}^S)^2] df d\Gamma$$

(frequency domain)

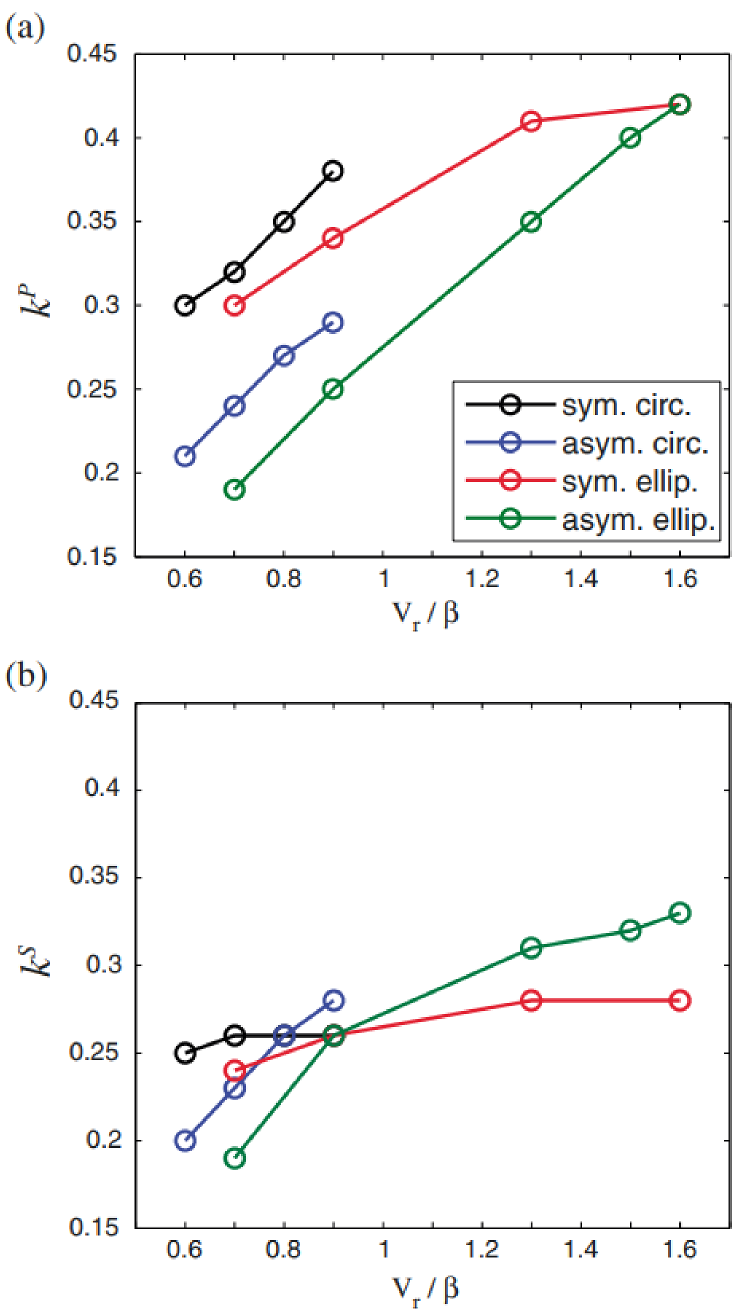
$$E_r = E_r^P + E_r^S = \rho \int_\Gamma \int_0^\infty [\alpha(\dot{u}^P)^2 + \beta(\dot{u}^S)^2] dt d\Gamma$$

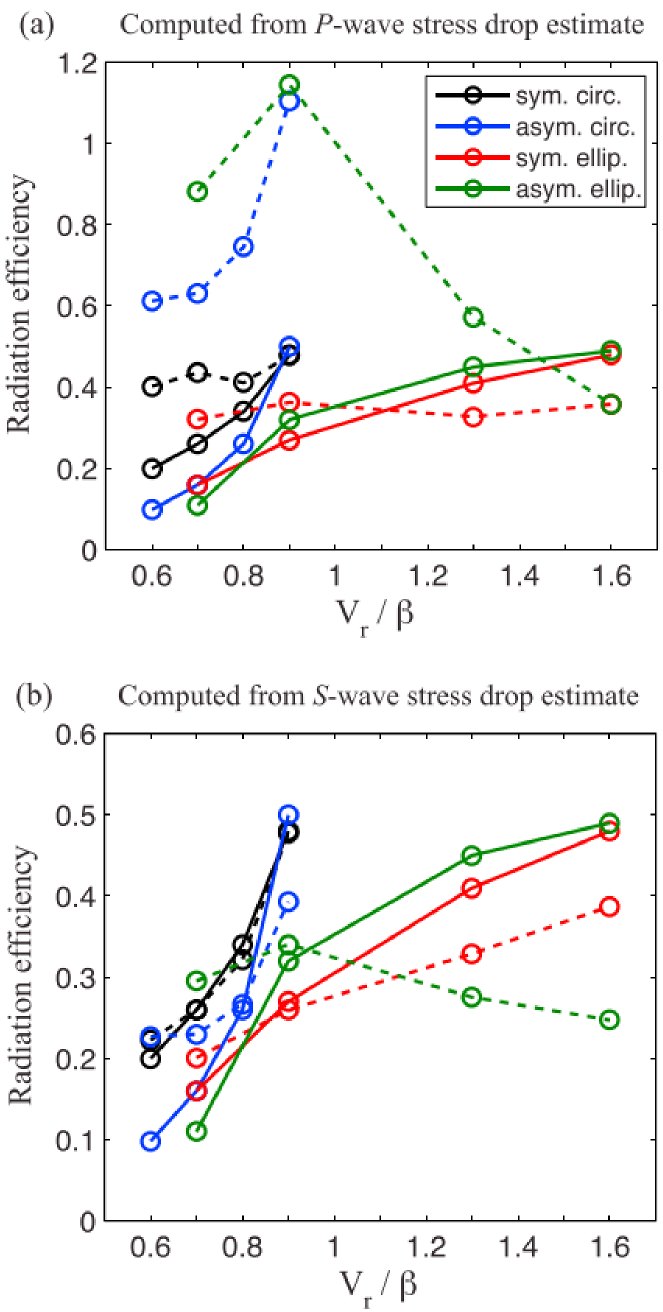
(time domain)

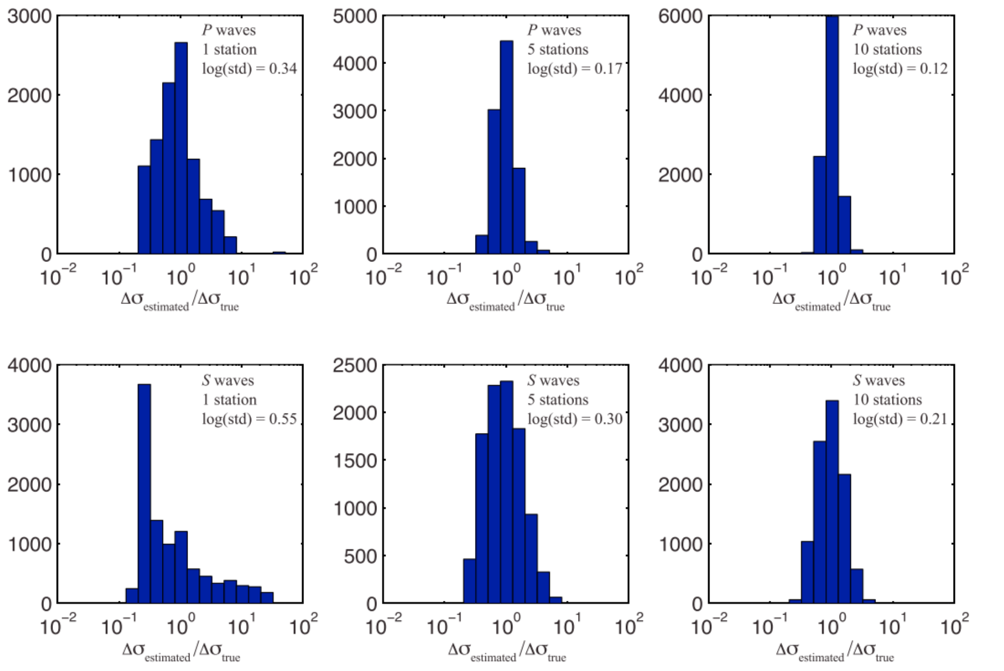
Point source: $E_r^S / E_r^P = 23$ ($\upsilon = 0.25$)

$E^r / M_0 \approx E_r^S / M_0$ ($E_r^S \gg E_r^P$)

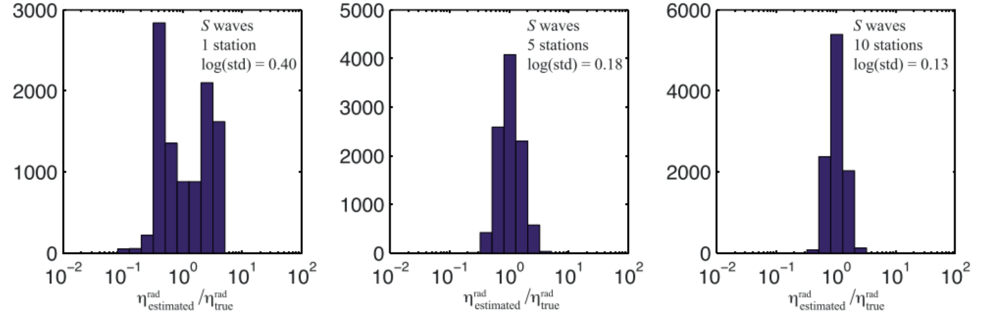
Implications?



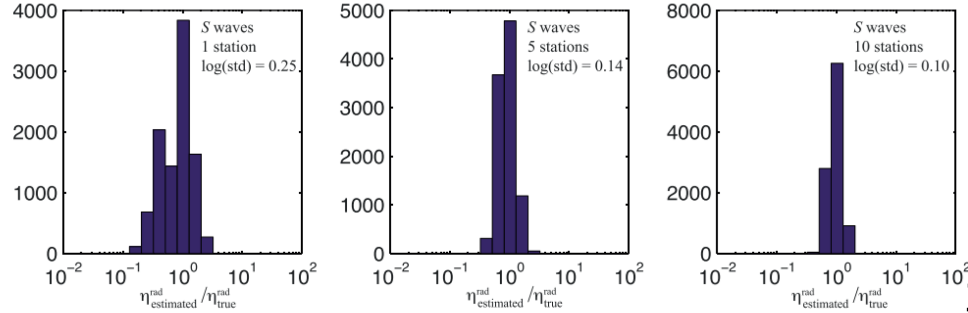




(a) Sym. circular rupture, $V_r = 0.9\beta$

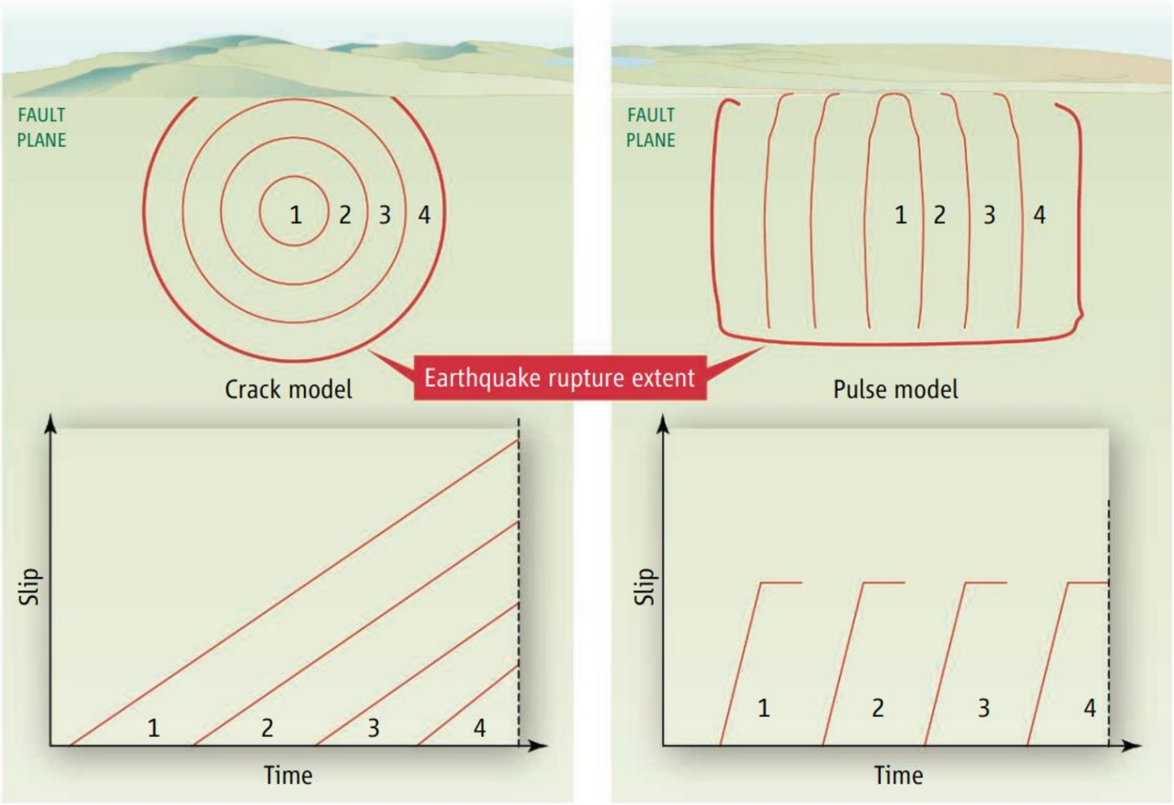


(b) Asym. circular rupture, $V_r = 0.9\beta$

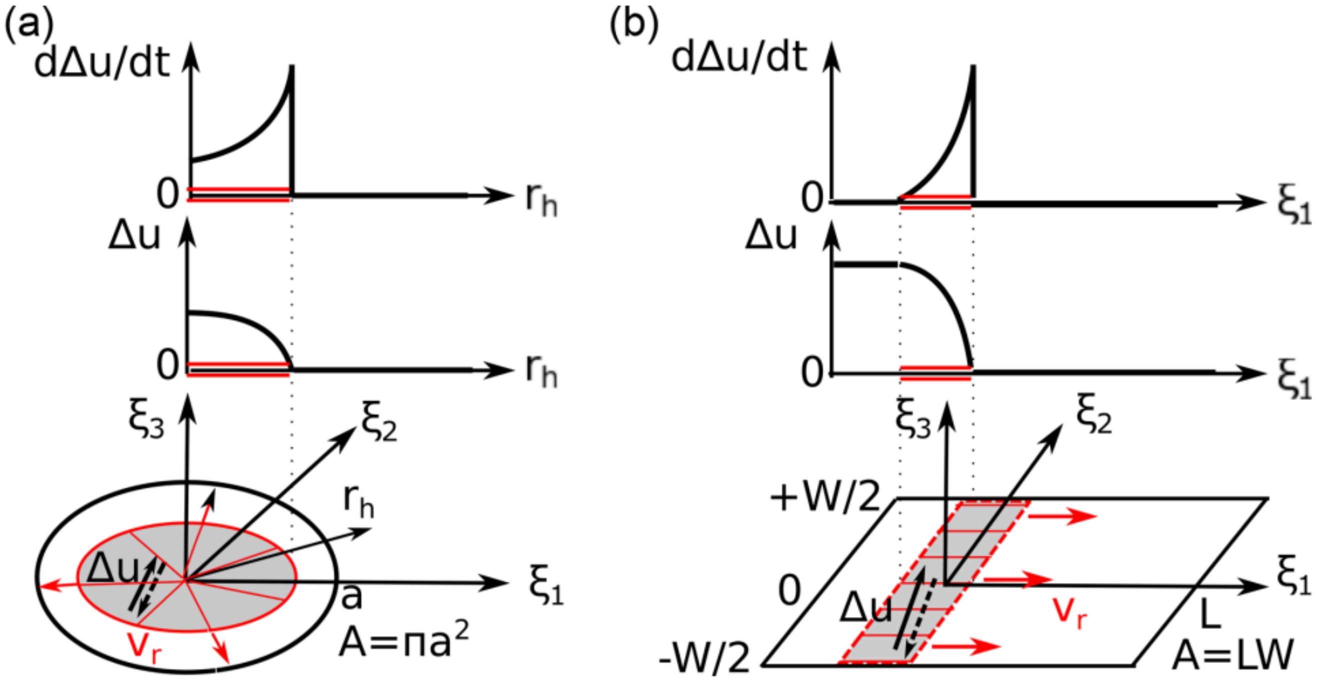


Crack-like vs Pulse-like rupture

- Crack-like rupture: local slip duration ≈ rupture duration (e.g., Madariaga, 1976)
- Pulse-like rupture: local slip duration ≪ rupture duration (e.g., Haskell, 1964; Heaton, 1990)
- Exact mechanism behind pulse-like rupture is still under debate.



Marone & Richardson (2006)



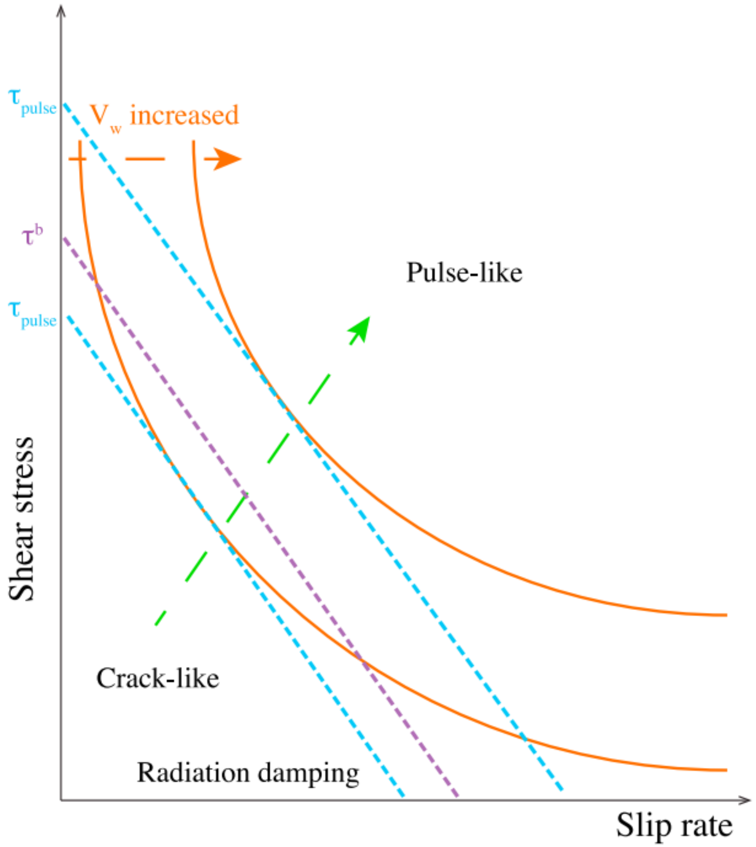
Dahm et al. (2021)

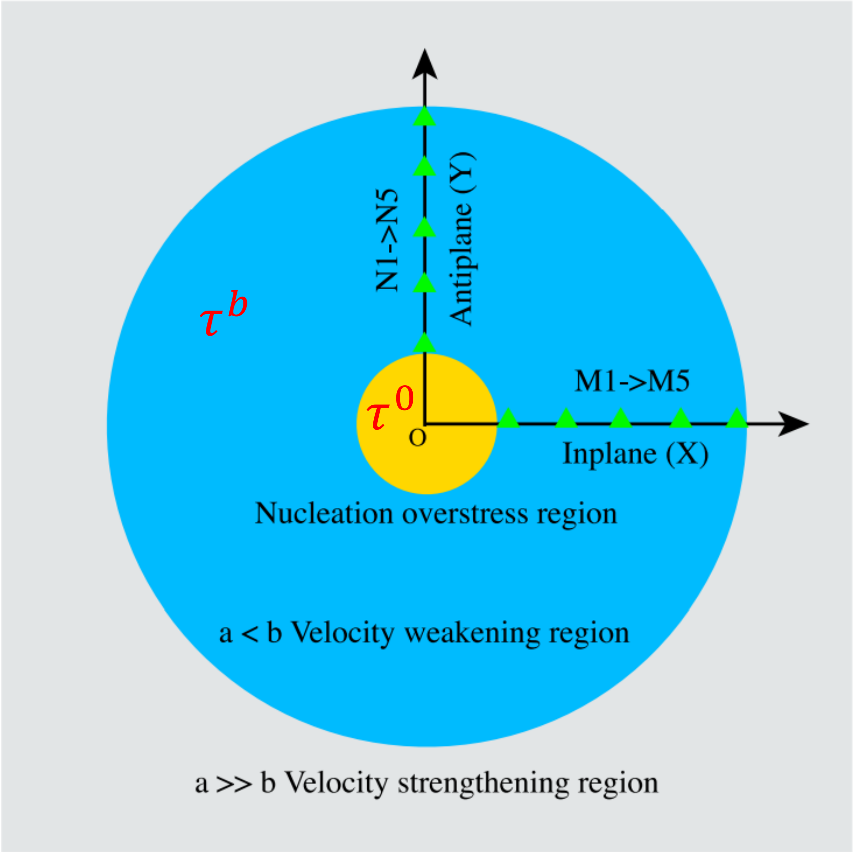
Numerical scheme (Wang & Day, 2017)

- Slip velocity-dependent friction (rate- and state-dependent friction)

- $f(V,\psi) = a\sinh^{-1}\left[\frac{V}{2V_0}\exp\left(\frac{\psi}{a}\right)\right]$ where $\dot{\psi} = -\frac{V}{L}[\psi - \psi_{ss}(V)]$, $\psi_{ss}(V) = a\ln\left\{\frac{2V_0}{V}\sinh\left[\frac{f_{ss}(V)}{a}\right]\right\}$,

  $f_{ss}(V) = f_w + \frac{f_{lv}-f_w}{[1+(V/V_w)^8]^{1/8}}$, and $f_{lv}(V) = f_0 - (b-a)\ln(V/V_0)$





- Crack → Pulse

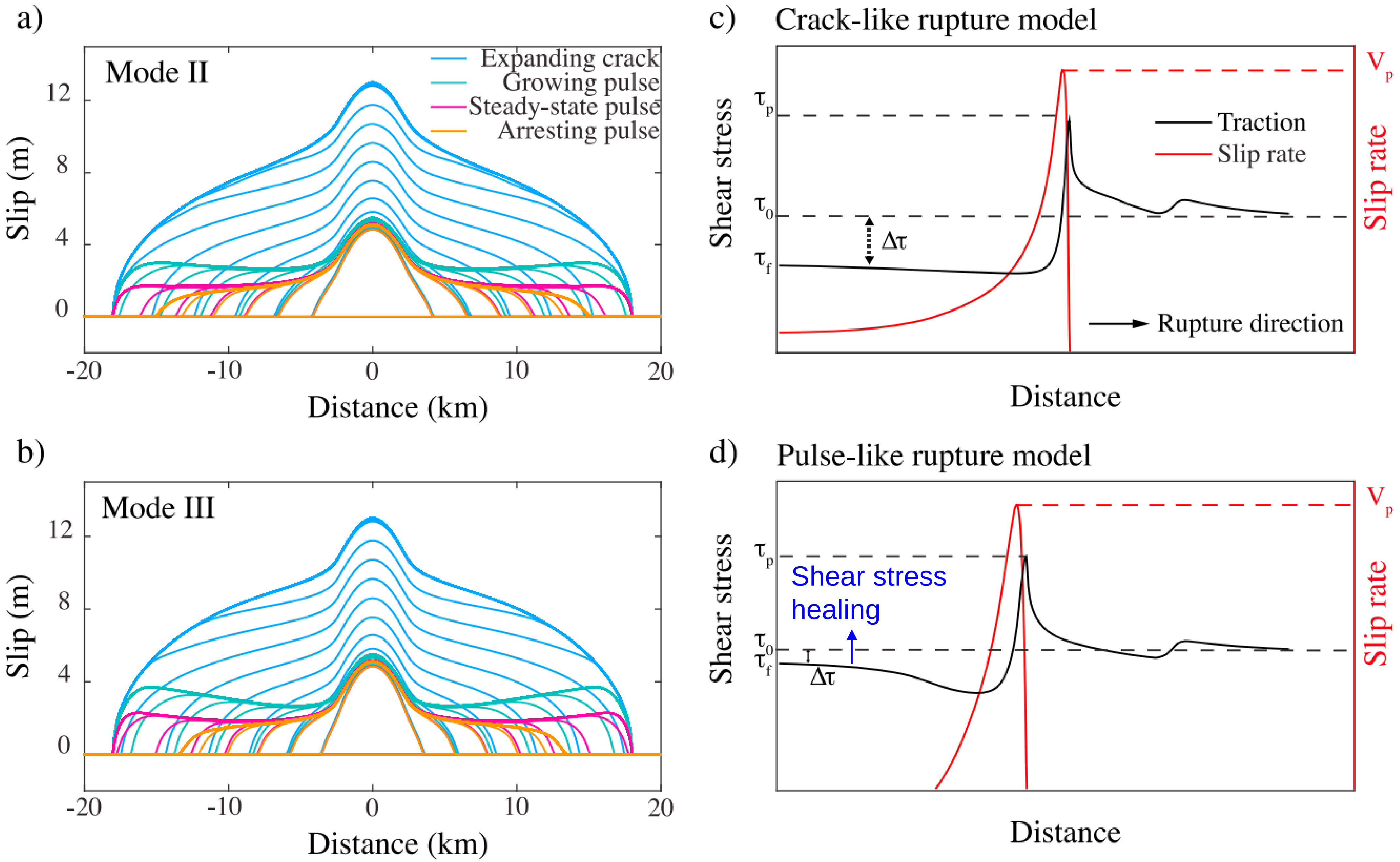
  1) Decrease τ_b

  2) Increase V_w

- Spontaneous rupture

  → Not fixed V_r

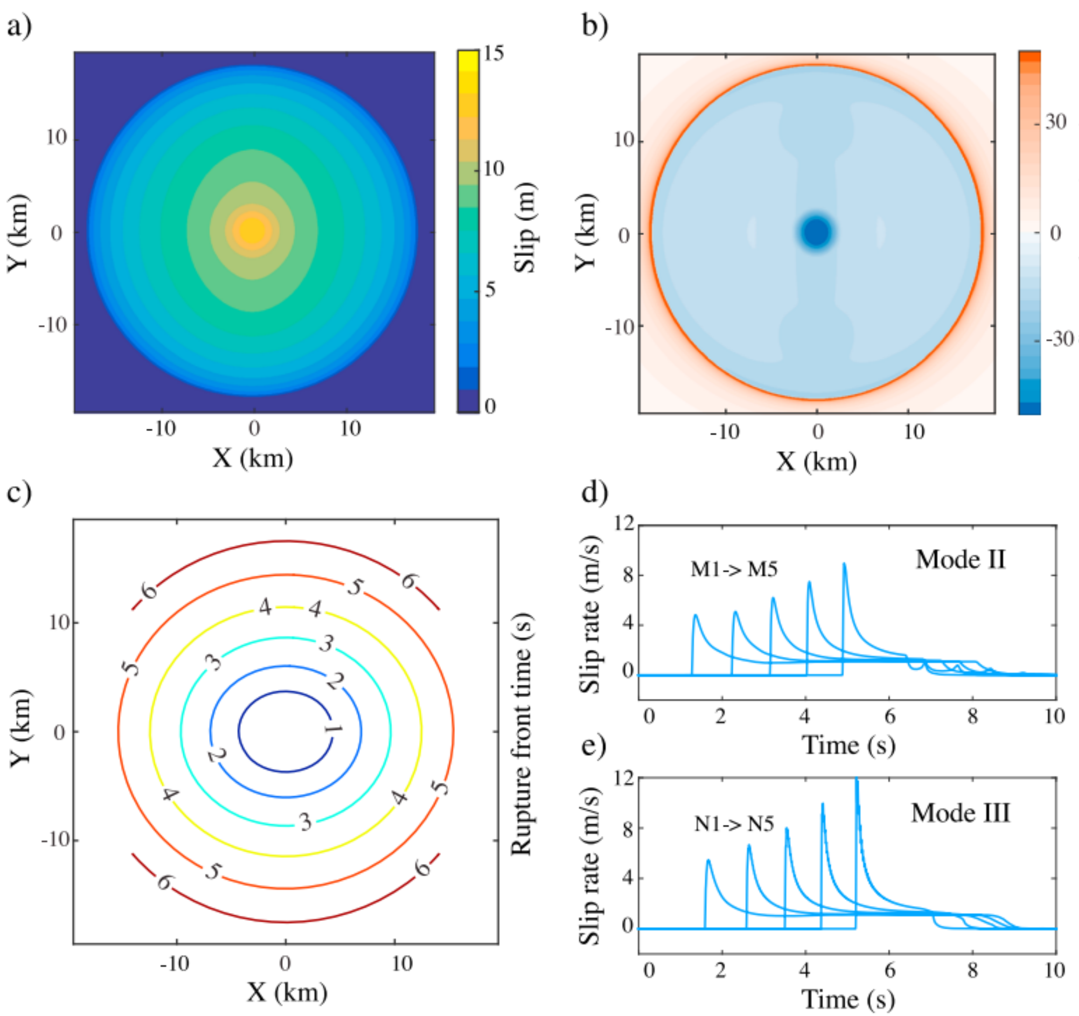
Simulation results

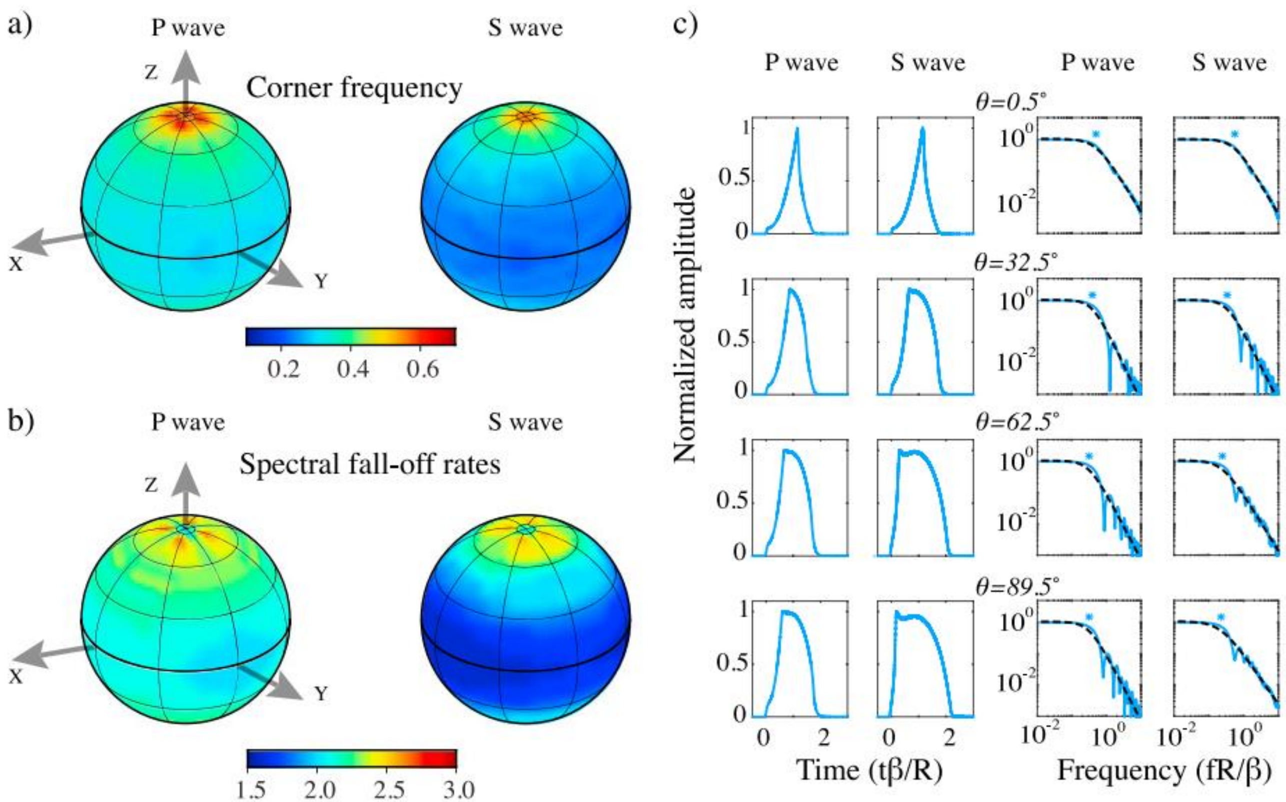


Case I Expanding crack

$$\overline{f_c}^{P} = k^{P}\frac{\beta}{a} = 0.35\frac{\beta}{a} \qquad V_r^2 = 0.88\beta$$

$$\overline{f_c}^{S} = k^{S}\frac{\beta}{a} = 0.27\frac{\beta}{a} \qquad V_r^3 = 0.84\beta$$



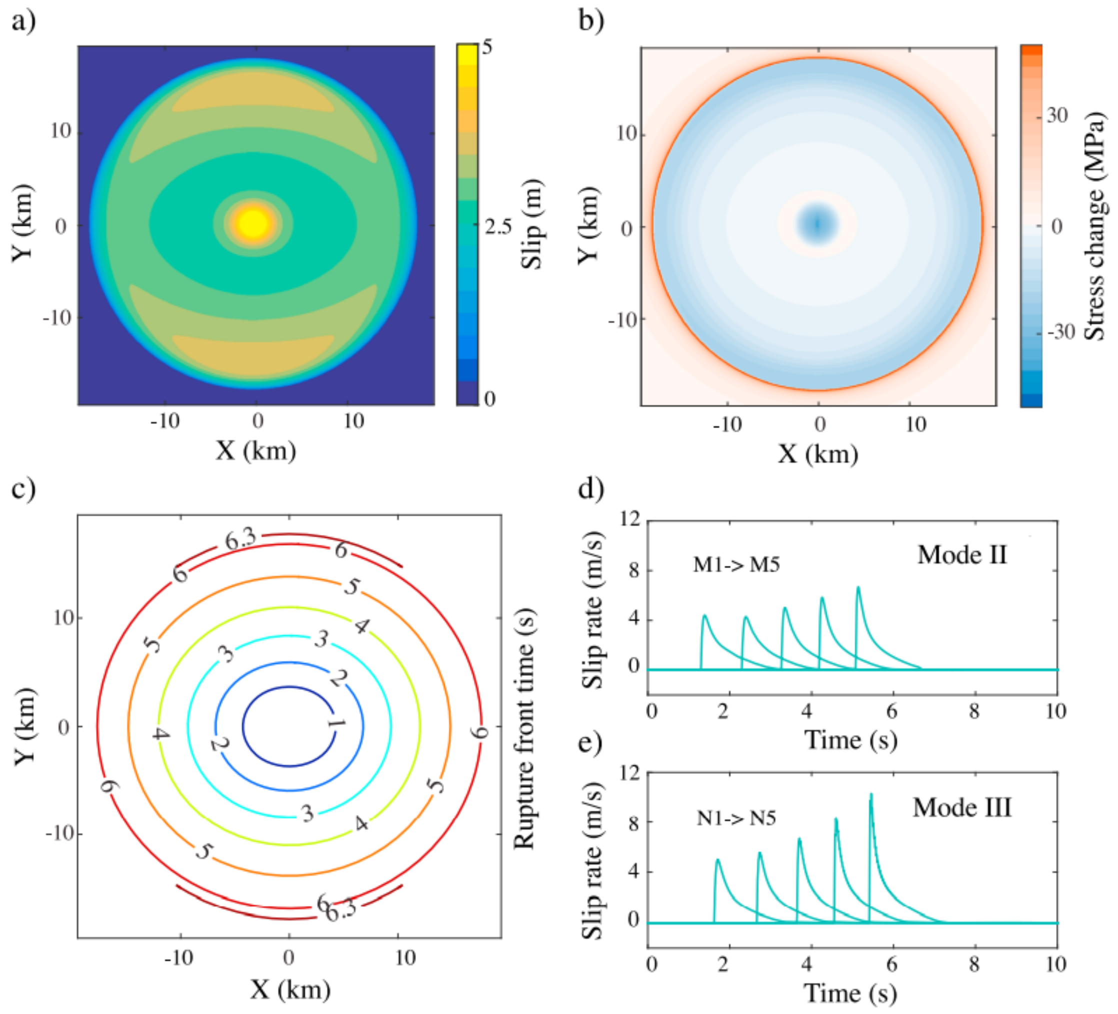


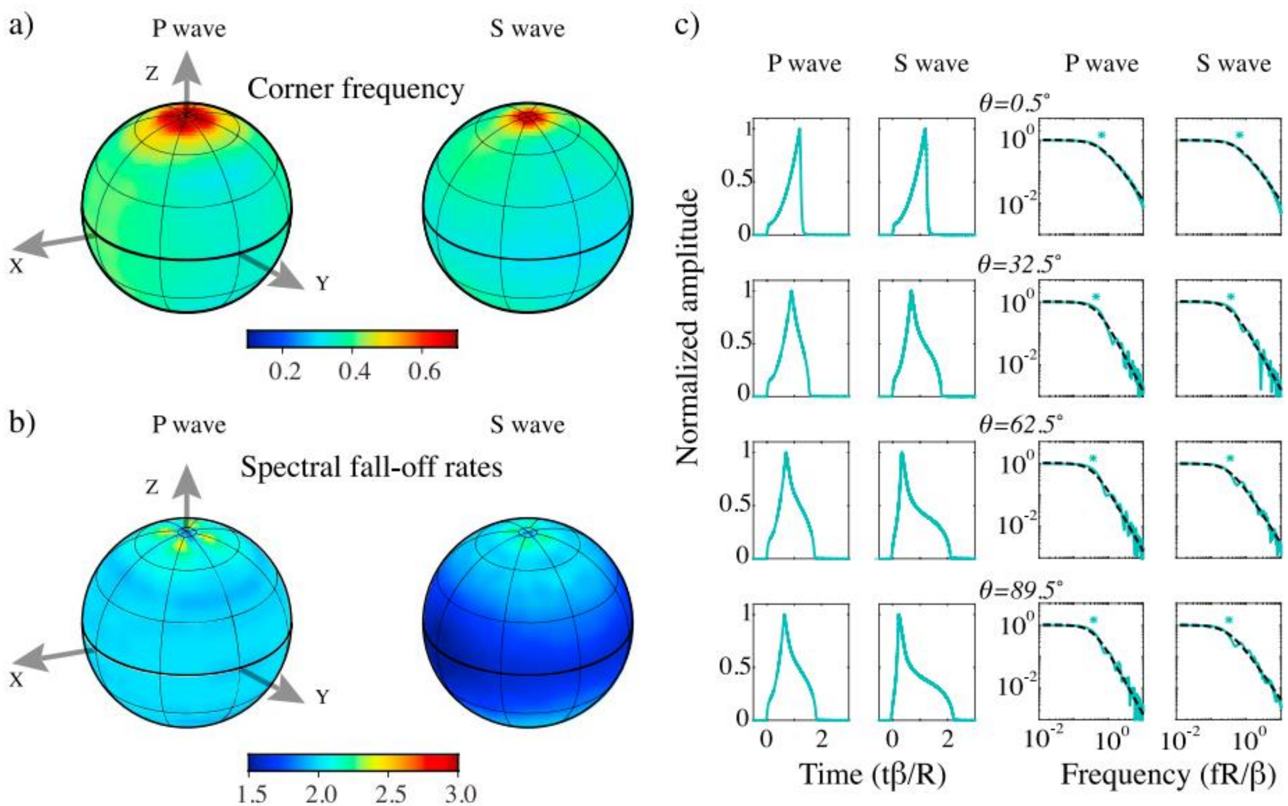
Four-lobes of high falloff rate not present in fixed rupture velocity model (Kaneko & Shearer, 2014)

Case II Growing pulse

$$\overline{f_c}^P = k^P \frac{\beta}{a} = 0.40 \frac{\beta}{a} \qquad V_r^2 = 0.85\beta$$

$$\overline{f_c}^S = k^S \frac{\beta}{a} = 0.36 \frac{\beta}{a} \qquad V_r^3 = 0.81\beta$$

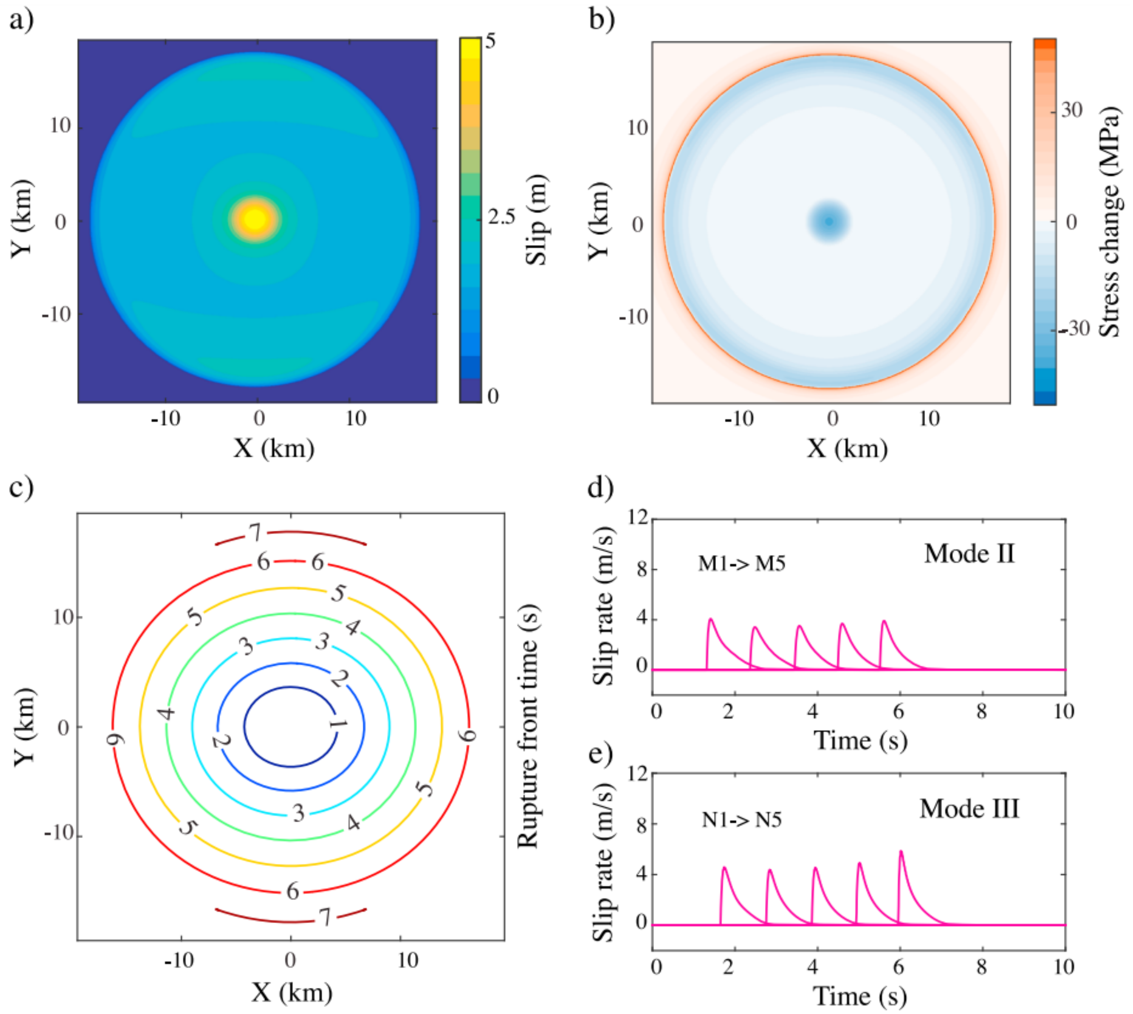


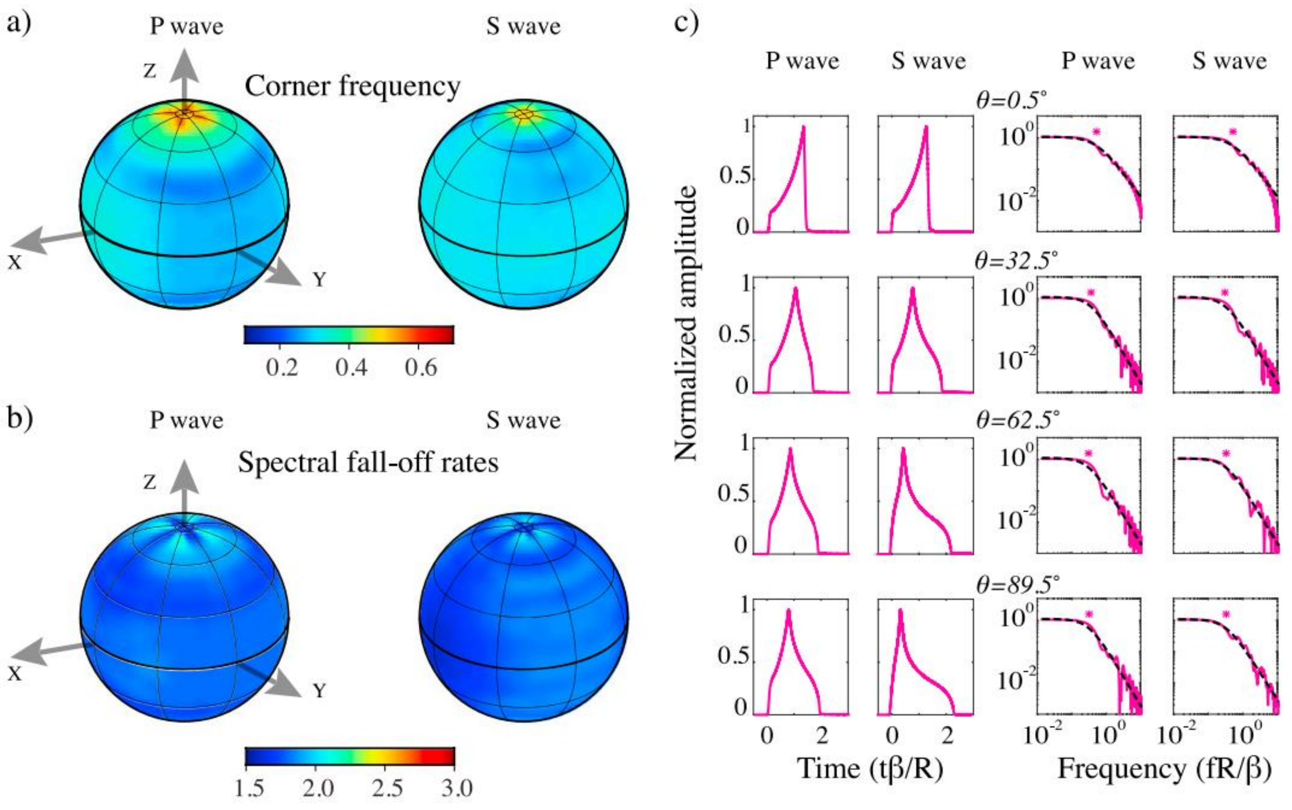


Case II Steady state pulse

$$\overline{f_c}^P = k^P \frac{\beta}{a} = 0.31 \frac{\beta}{a} \qquad V_r^2 = 0.78\beta$$

$$\overline{f_c}^S = k^S \frac{\beta}{a} = 0.31 \frac{\beta}{a} \qquad V_r^3 = 0.74\beta$$

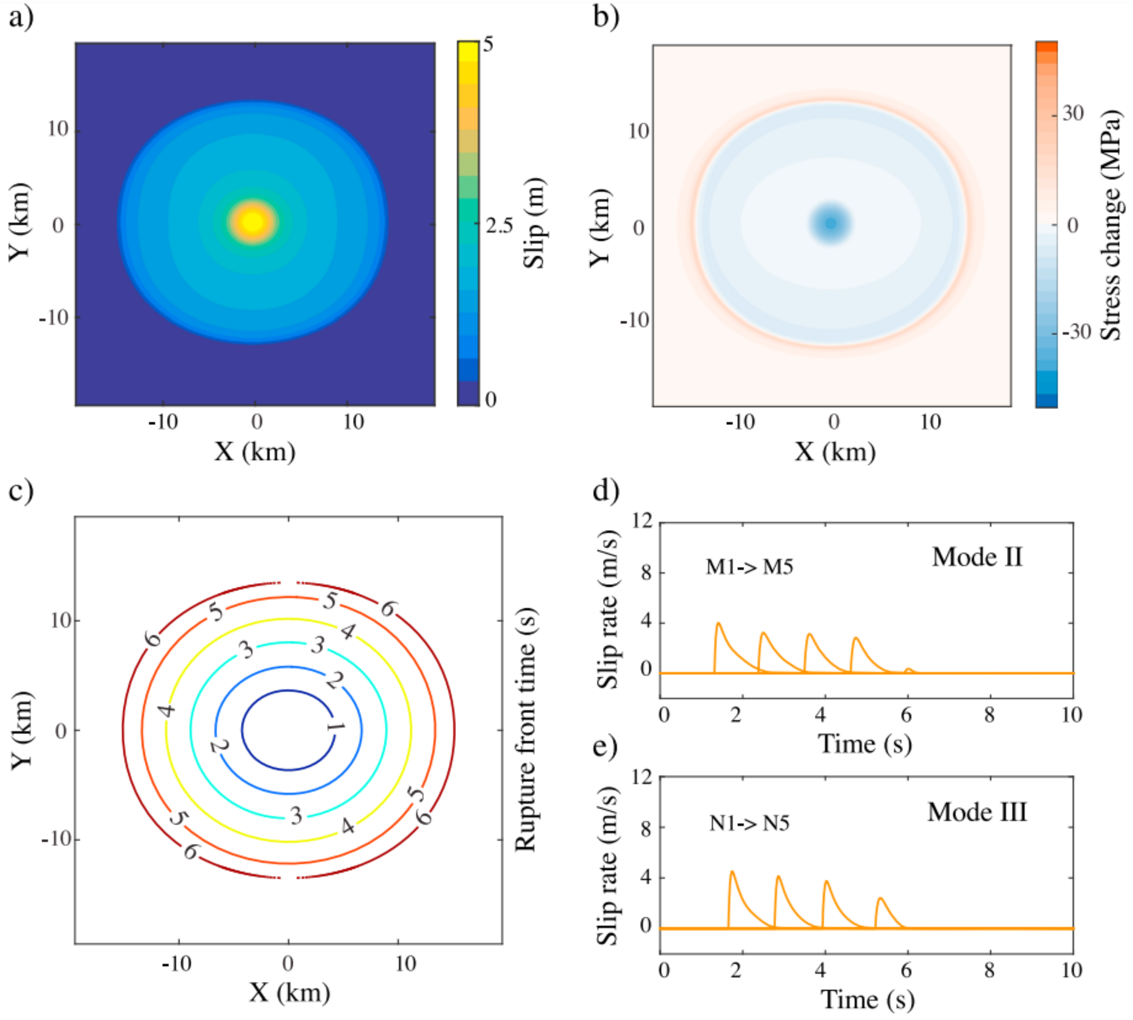


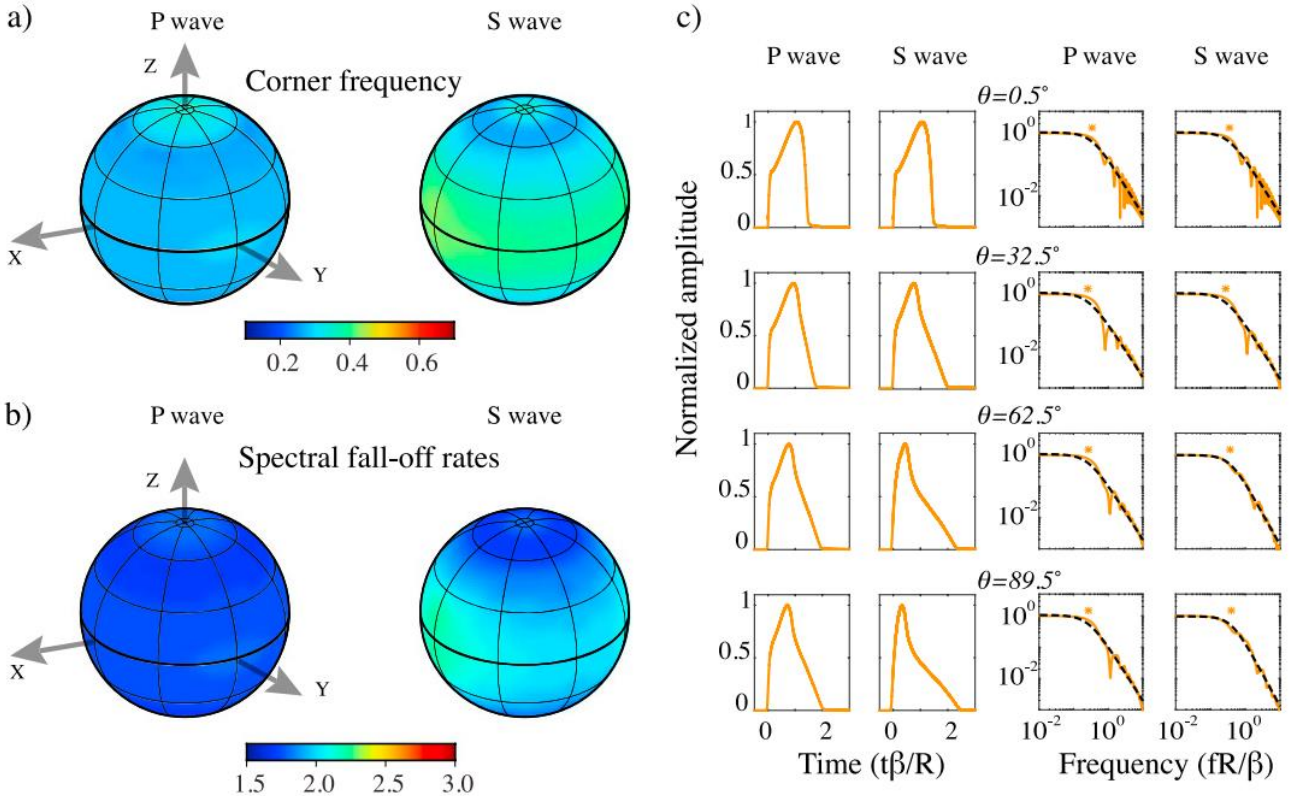


Case IV Arresting pulse

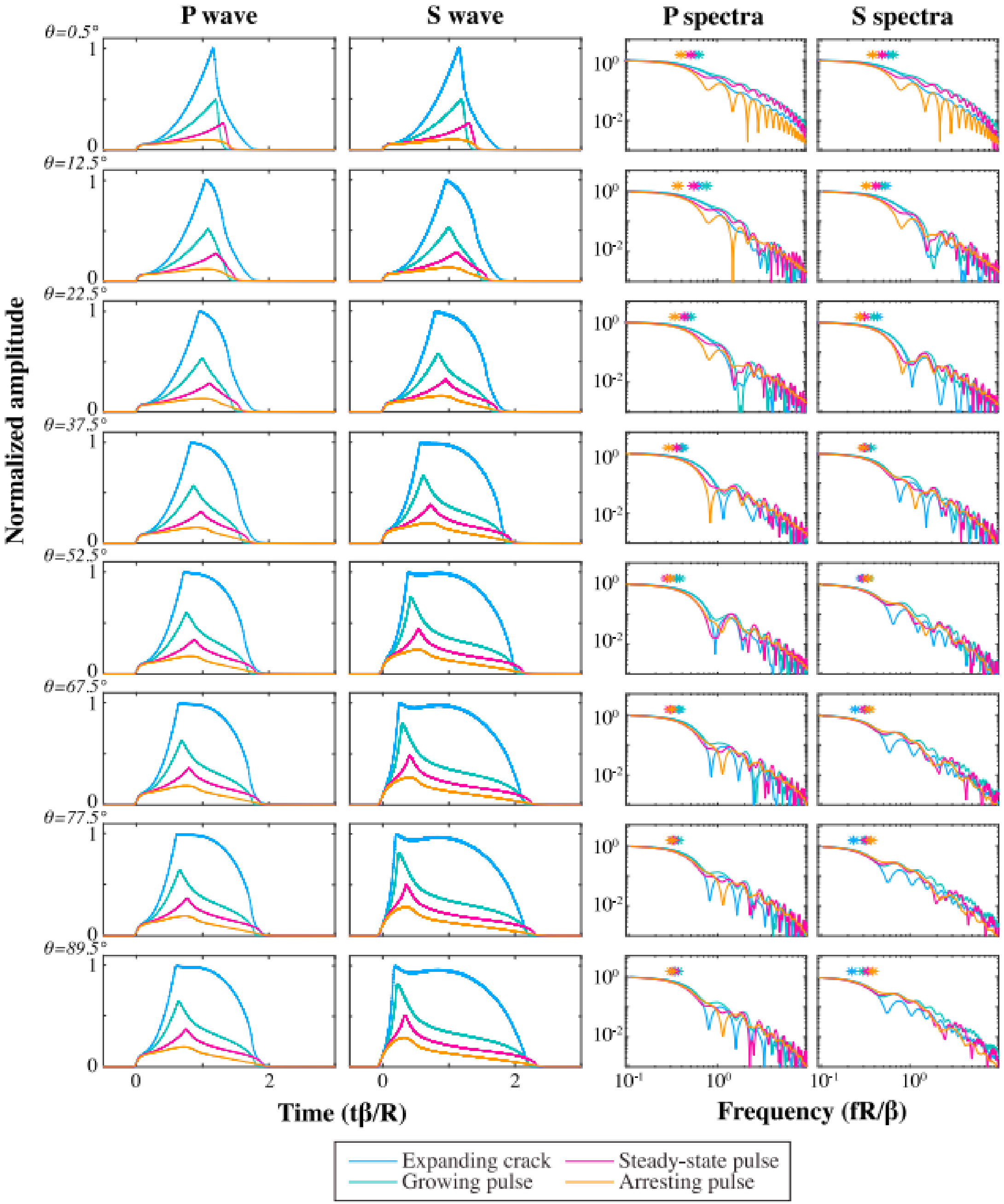
$$\overline{f_c}^{P} = k^P \frac{\beta}{\sqrt{ab}} = 0.28 \frac{\beta}{\sqrt{ab}} \qquad V_r^2 = 0.72\beta$$

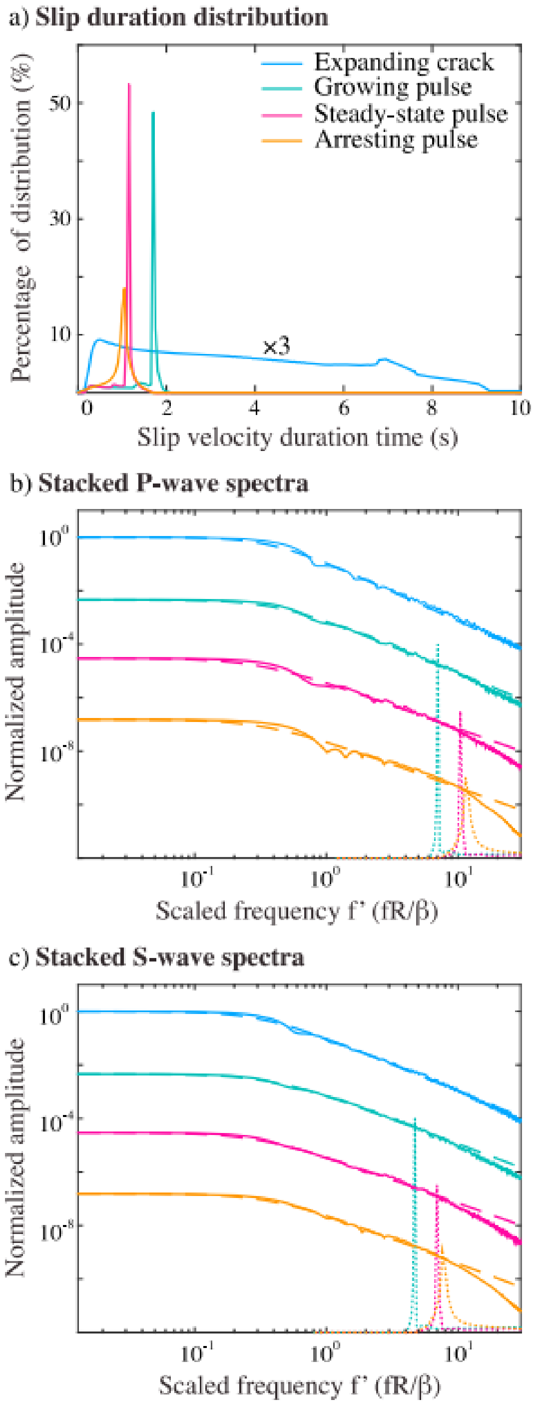
$$\overline{f_c}^{S} = k^S \frac{\beta}{\sqrt{ab}} = 0.34 \frac{\beta}{\sqrt{ab}} \qquad V_r^3 = 0.66\beta$$





Far-field displacements/spectra

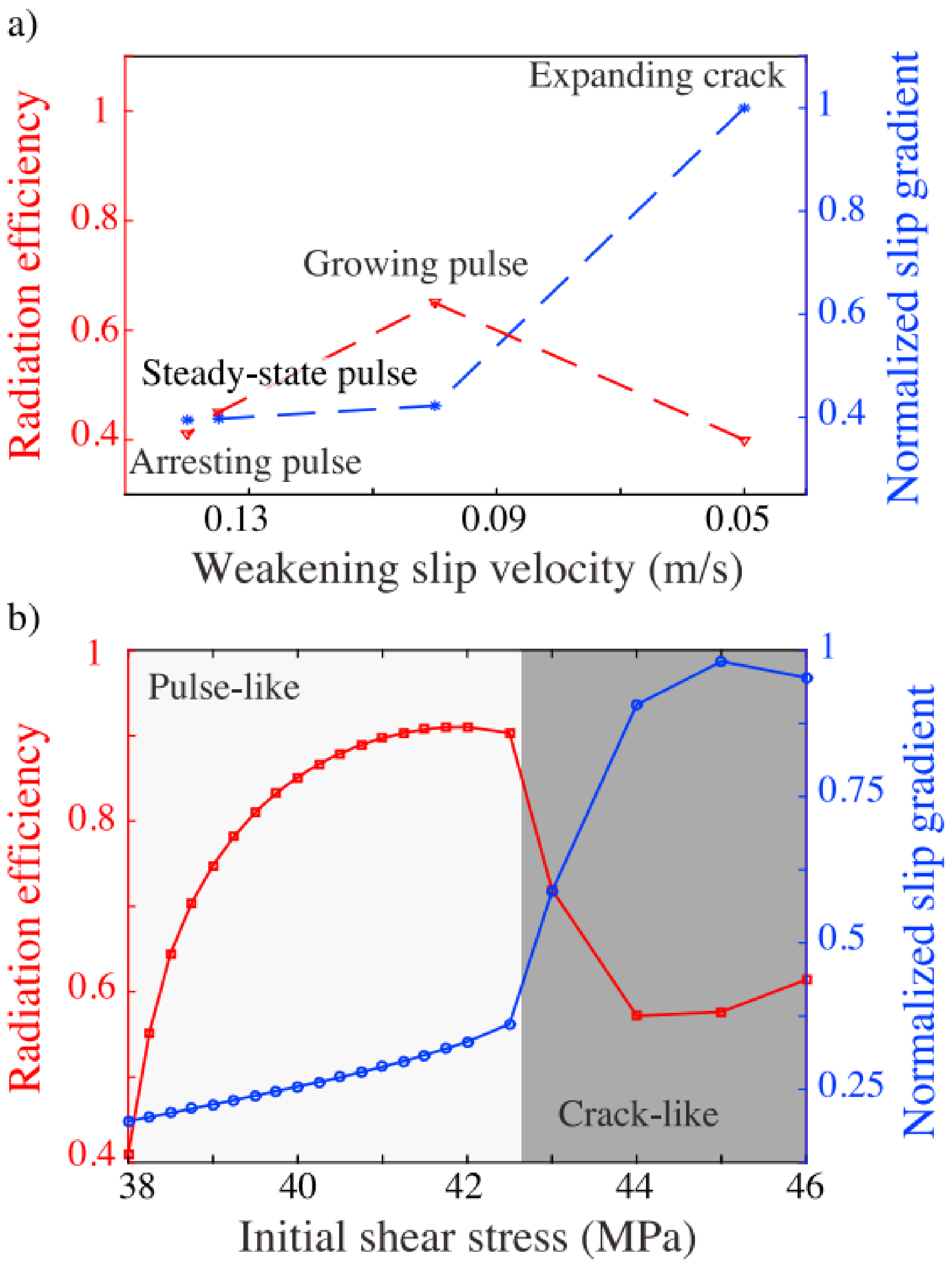


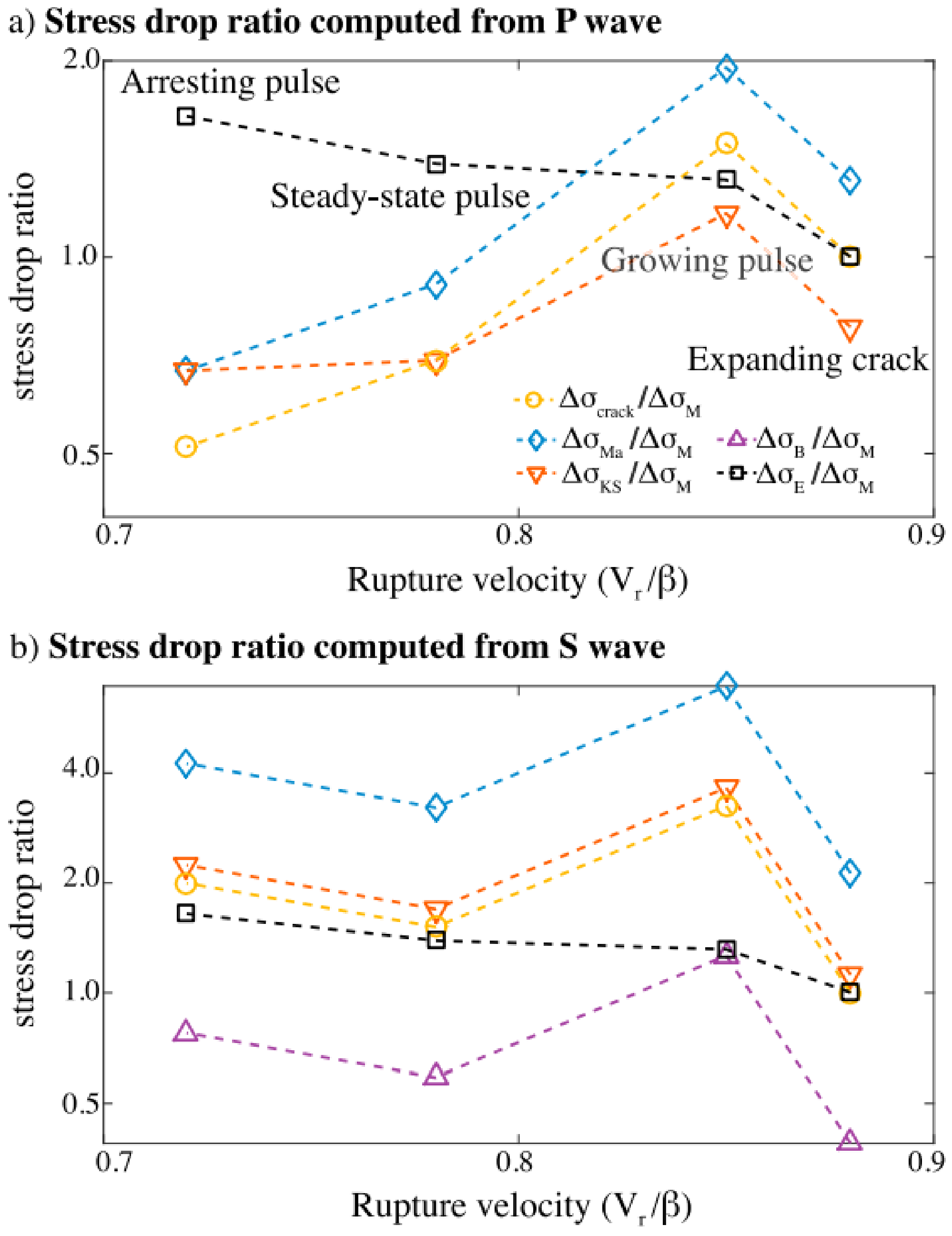


Secondary corner frequency for pulse-like models

$$f_c^{2nd} = \frac{K}{\bar{T}} \ (\bar{T}\text{: mean risetime})$$

Radiated energy / Stress drop





Questions

1. Can we learn anything meaningful about kinematics/dynamics of earthquakes by measuring the ratio of average P and S corner frequencies $\left(\overline{f_c}^P / \overline{f_c}^S\right)$?

2. What does corner frequency represent? It seems that it is not a simply inverse of pulse duration even for ruptures with simple geometry.

3. Regarding actual measurements, station coverage seems to be “much” important than model-dependent $k_{P,S}$. Can you agree with this?