

Stress Drop Reading Group

Week 2

August 7th, 2024

Corner frequency (f_c) and stress drop ($\Delta\sigma$)

- (Moment-based) stress drop \leftarrow source dimension (radius)
- Most popular approach is to estimate radius (a) from corner frequency (f_c).

$$\Delta\sigma = C(a, b, \nu) \frac{M_0}{bS} \quad (\text{Eshelby, 1957}) - (1)$$

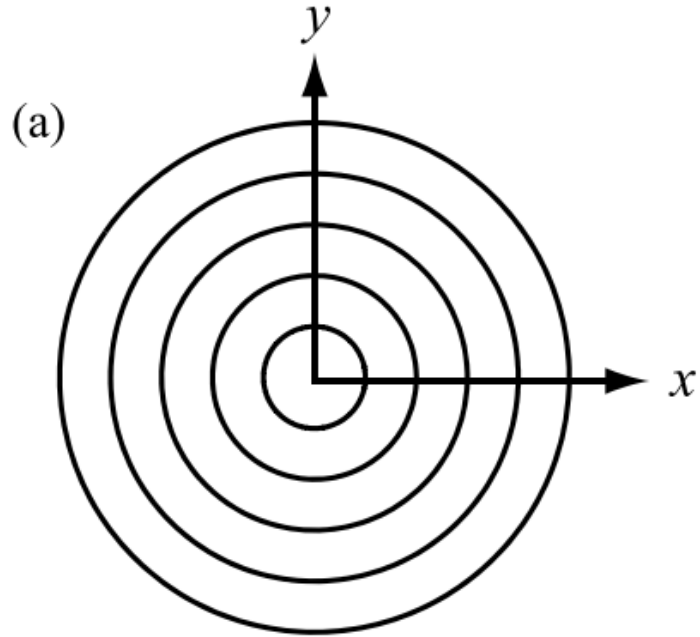
$$\Delta\sigma = \frac{7}{16} \frac{M_0}{a^3} \quad (a = b; \nu = 0.25) - (2)$$

$$\bar{f}_c = k \frac{\beta}{a} \quad (\text{Brune, 1970; Madariaga, 1976}) - (3)$$

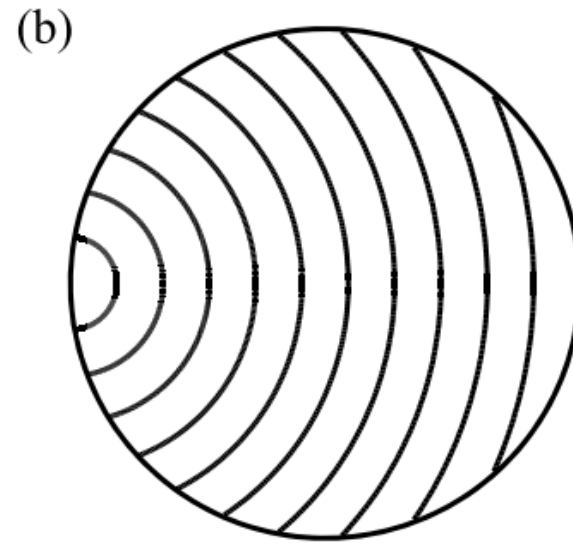
$$(2) \ \& \ (3) \rightarrow \Delta\sigma = \frac{7}{16} \left(\frac{\bar{f}_c}{k\beta} \right)^3 M_0 - (4)$$

- Q) How is k affected by different assumptions about seismic source?

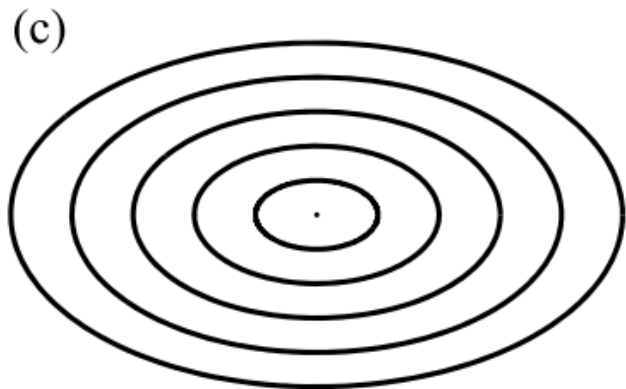
Different source scenarios



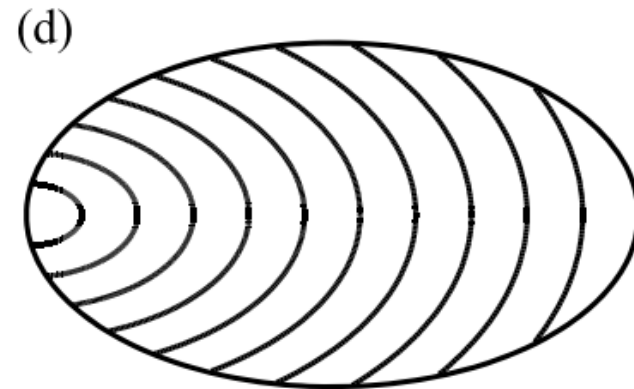
Symmetrical circular rupture



Asymmetrical circular rupture



Symmetrical elliptical rupture



Asymmetrical elliptical rupture

- Symmetry
- Geometry
- Rupture speed
- Rupture mode

Numerical scheme (Kaneko & Shearer, 2014, 2015)

- Infinite homogeneous, isotropic elastic medium

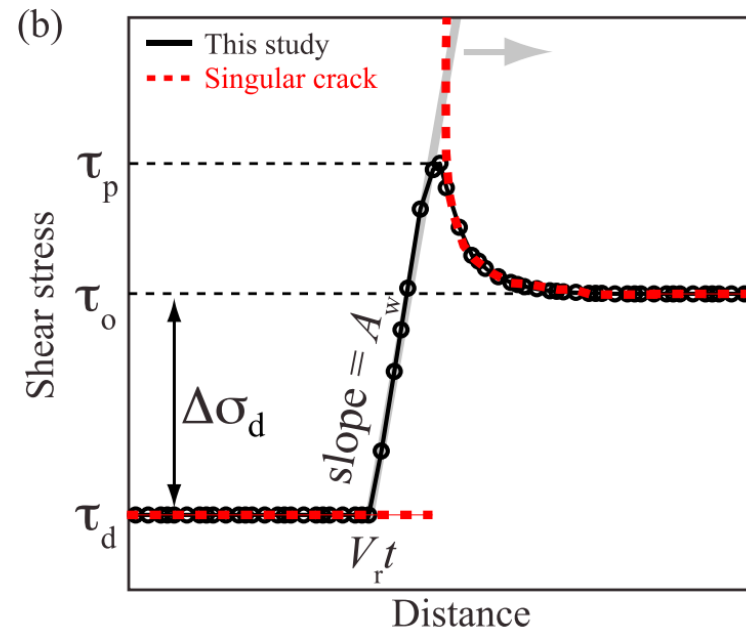
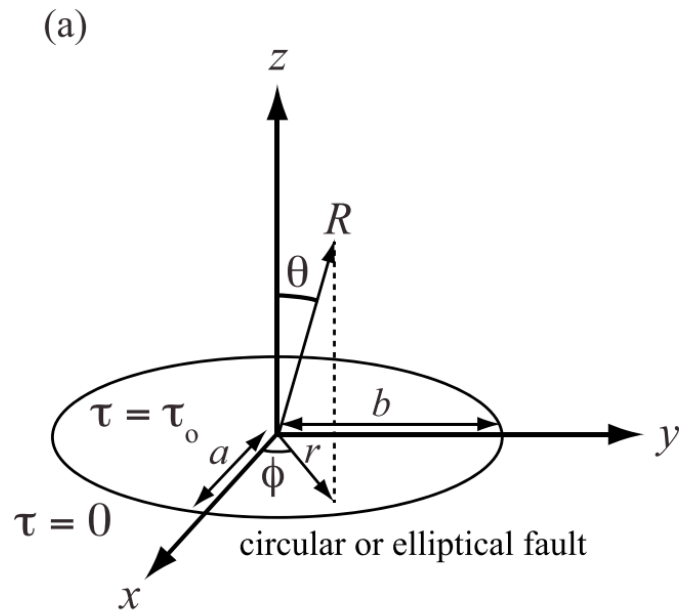
- **Cohesive zone model**

$$\tau_{st} = \max\{\tau_d, \tau_d + A_w(|\vec{r} - V_r t|)\} \text{ (Andrews, 1985; Dunham and Bhat, 2008)}$$

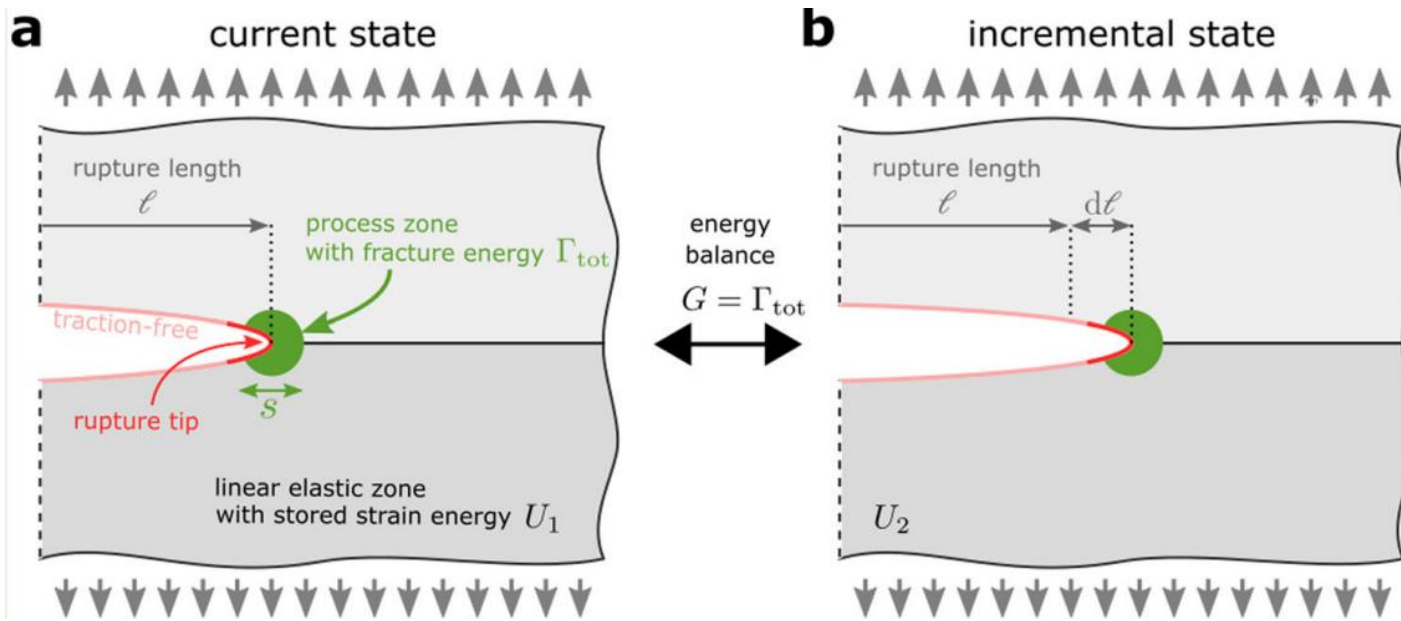
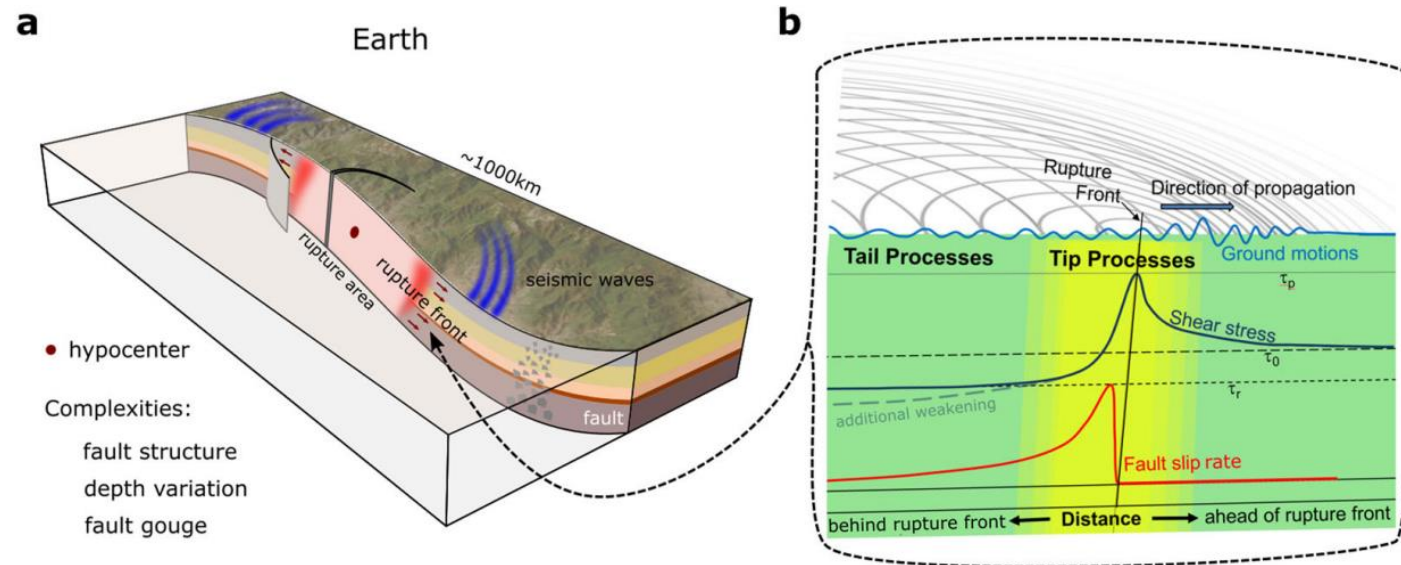
$$V_r = V_x V_y \left(\frac{x^2 + y^2}{V_y^2 x^2 + V_x^2 y^2} \right)^{1/2} \text{ (} V_x \text{: major axis, } V_y \text{: minor axis)}$$

τ_p : determined by A_w & τ_d

- Fixed rupture speeds (not spontaneous)



Cohesive zone model (CZM)



Linear Elastic Fracture Mechanics (LEFM)

G_0 : Static energy release rate

Γ_{tot} : Total fracture energy

$$G_0 = -\frac{d\Pi}{dl} \text{ where } d\Pi = (U_2 - U_1) - F$$

To avoid infinite stresses at the crack tip, the dissipation is often smeared out in “*process zone*” of size $s \ll l$, commonly implemented via *cohesive zone models*.

LEFM predicts the crack growth speed through an energy balance

$$\rightarrow G = \Gamma_{tot} \text{ where } G \leq G_0$$

(G : dynamic energy release rate)

$$G = \left(1 - \frac{C_f}{C_R}\right) G_0 \text{ where } C_f = dl/dt$$

C_R : Rayleigh wave speed

Case I Symmetrical circular rupture

- Nondimensional variables (normalized)

Length $r' = r/\sqrt{ab}$

Time $t' = t\beta/\sqrt{ab}$

Stress $\sigma'_{ij} = \sigma_{ij}/\Delta\sigma_d$

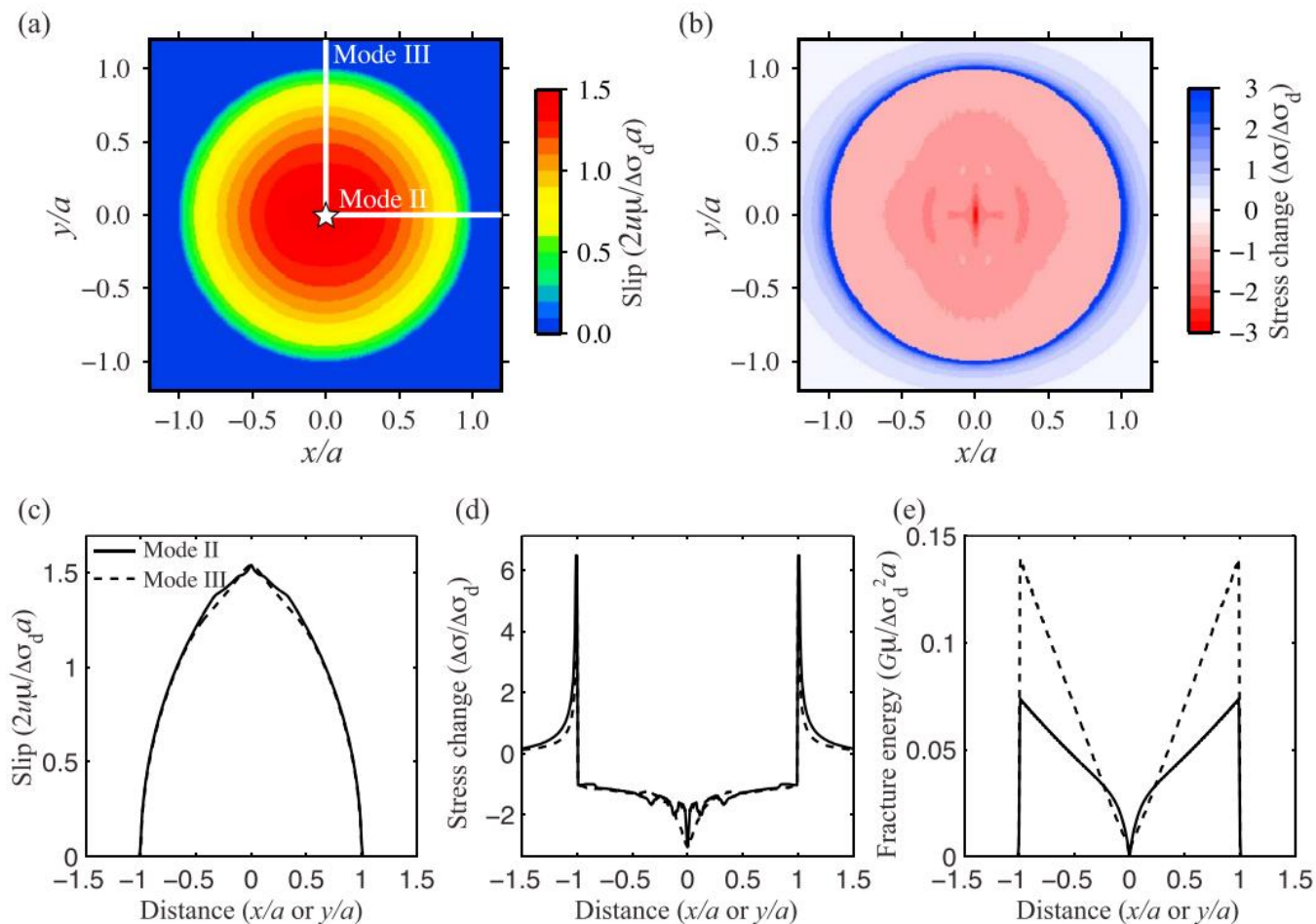
Displacement $u'_i = u_i\mu/(\Delta\sigma_d\sqrt{ab})$

Weakening rate $A'_w = A_w\sqrt{ab}/\Delta\sigma_d$

Seismic moment $M'_0 = M_0/[\Delta\sigma_d^2(ab)^{3/2}]$

Fracture energy density $G' = G\mu/(\Delta\sigma_d^2\sqrt{ab})$

Radiated energy $E'_r = E\mu/[\Delta\sigma_d^2(ab)^{3/2}]$



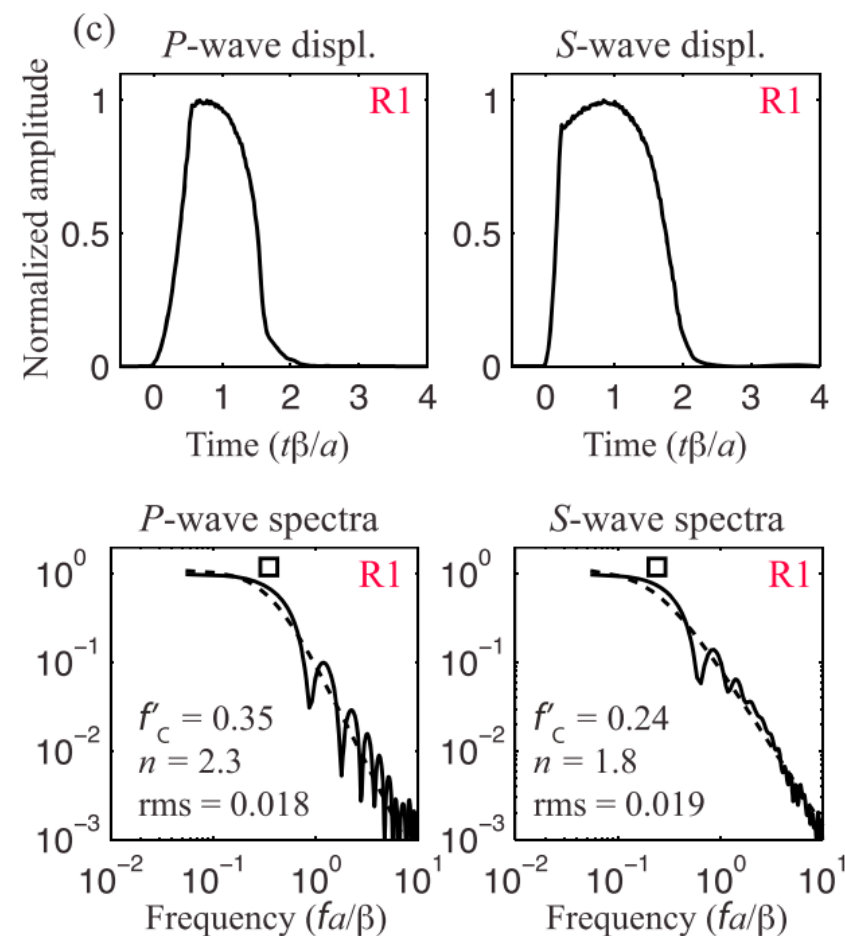
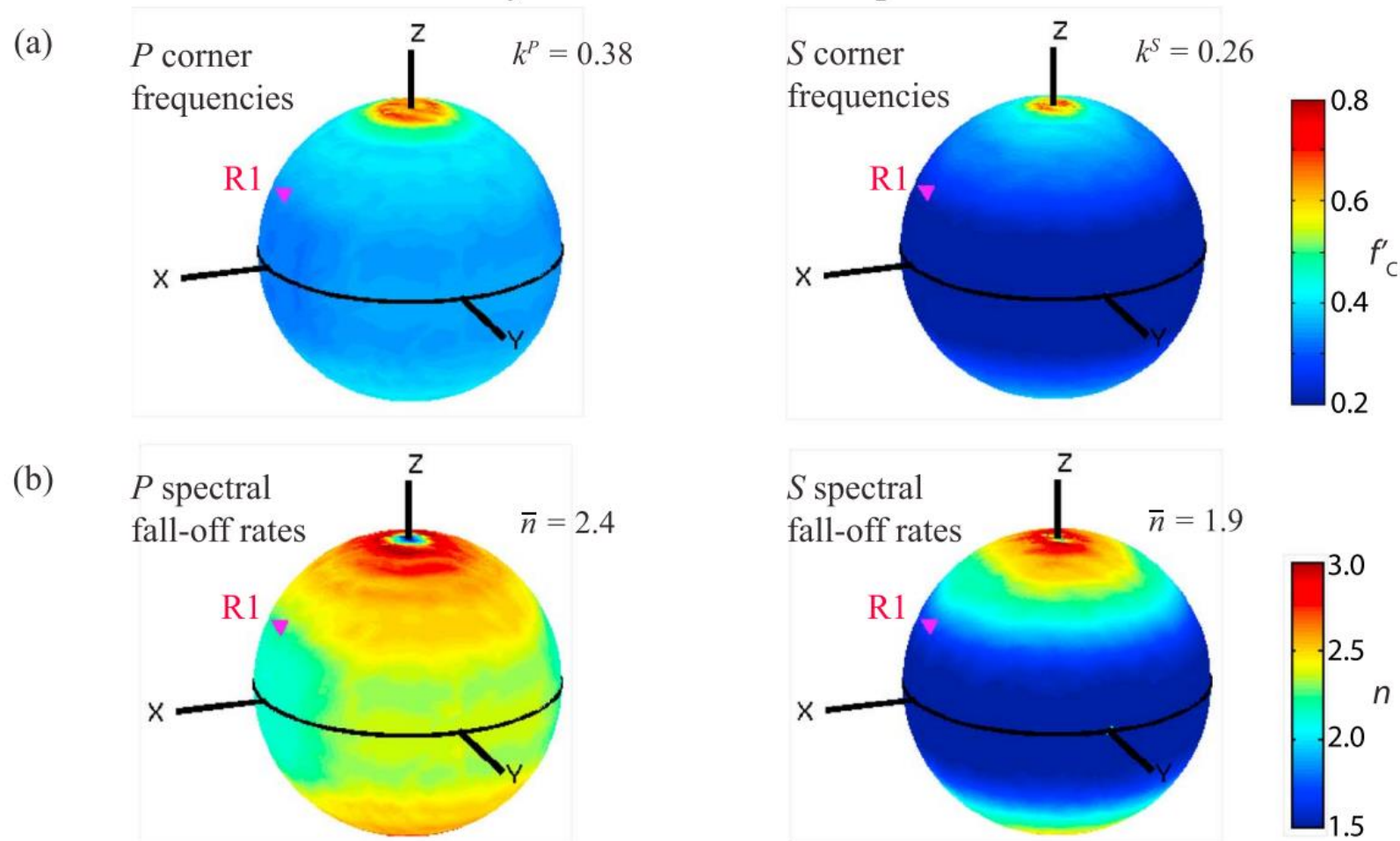
$$A'_w = 84, V_r/\beta = 0.9$$

Case I Symmetrical circular rupture

$$\bar{f}_c^P = k^P \frac{\beta}{a} = 0.38 \frac{\beta}{a}$$

$$\bar{f}_c^S = k^S \frac{\beta}{a} = 0.26 \frac{\beta}{a}$$

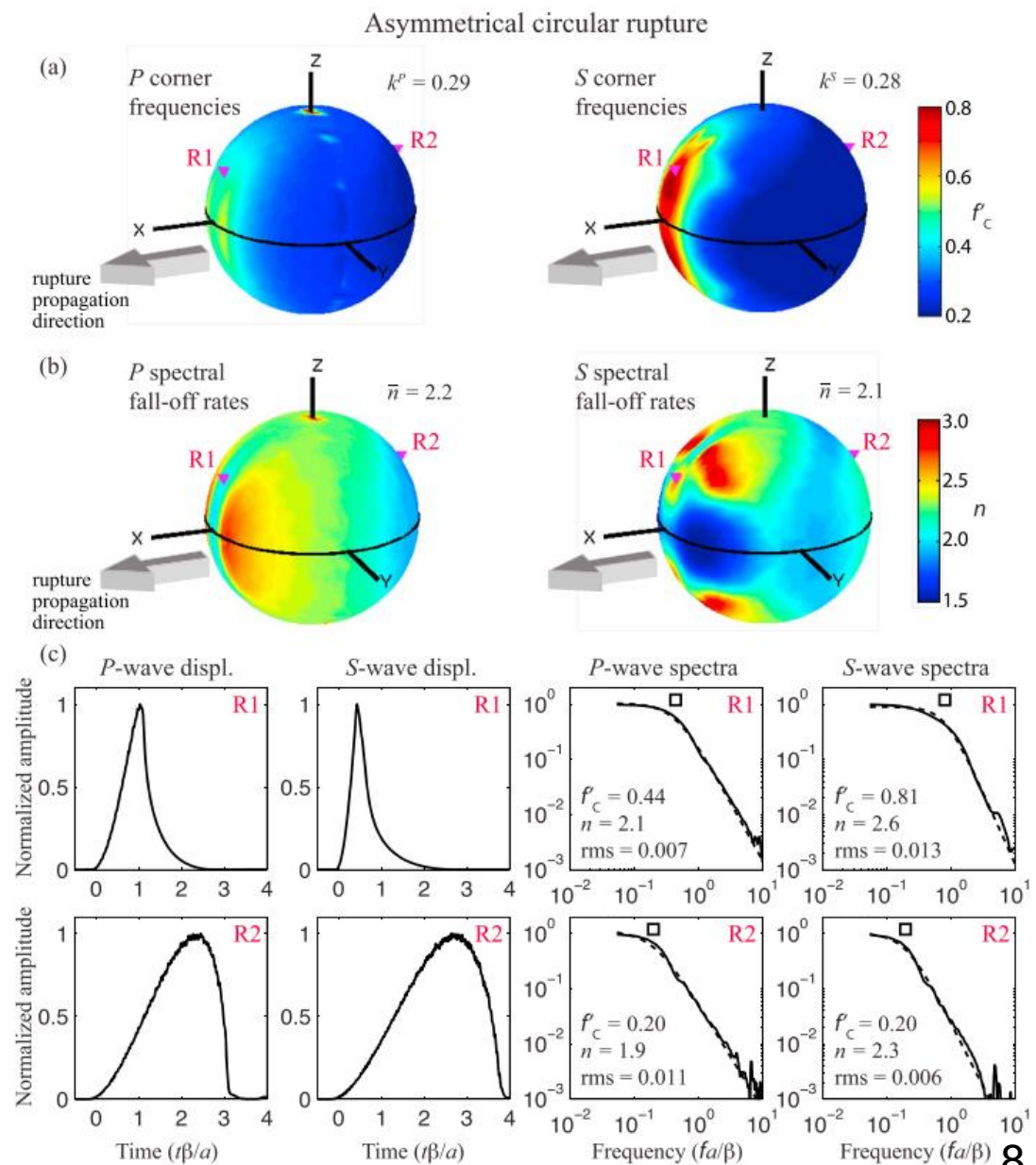
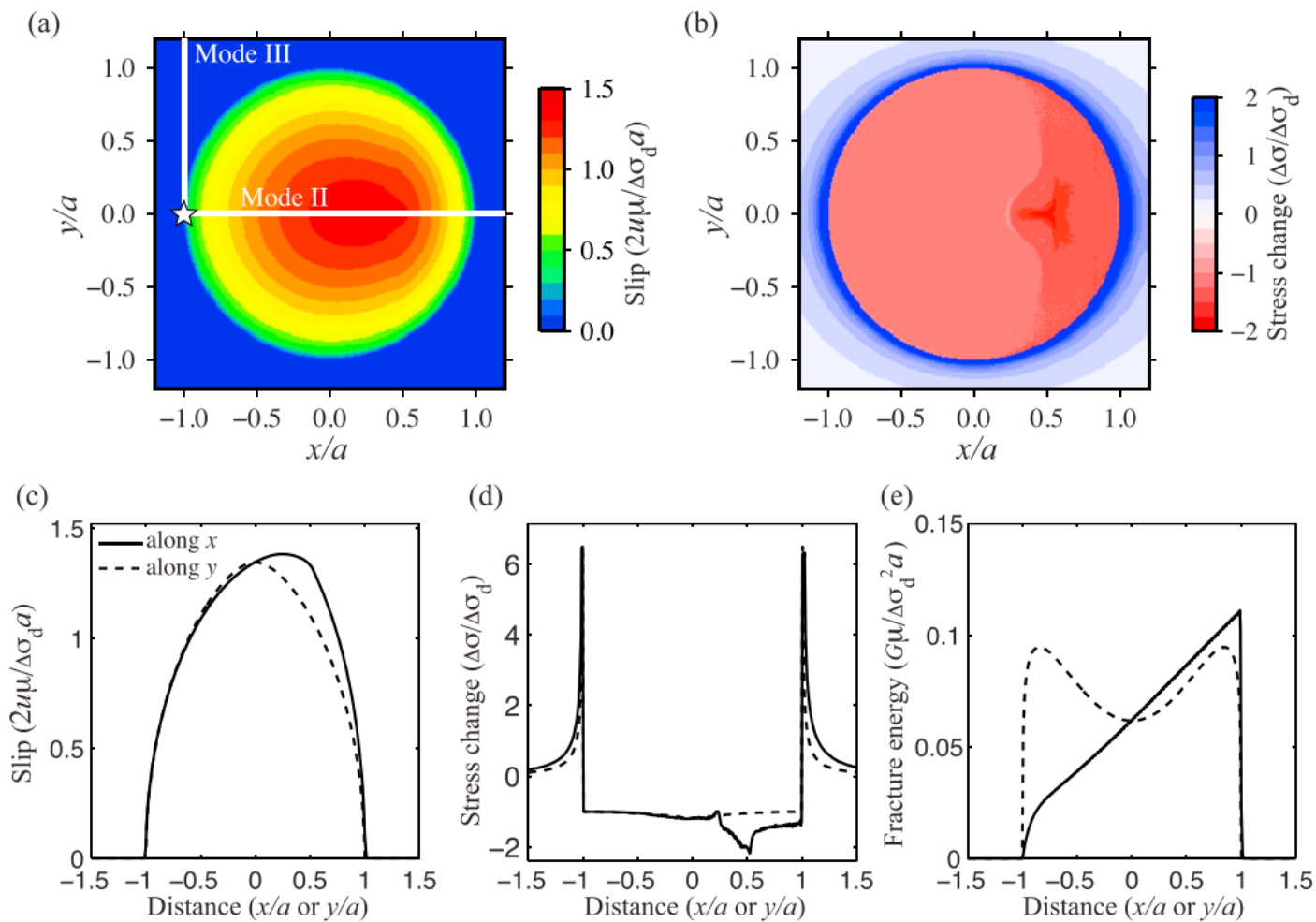
Symmetrical circular rupture



Case II Asymmetrical circular rupture

$$\bar{f}_c^P = k^P \frac{\beta}{a} = 0.29 \frac{\beta}{a}$$

$$\bar{f}_c^S = k^S \frac{\beta}{a} = 0.28 \frac{\beta}{a}$$

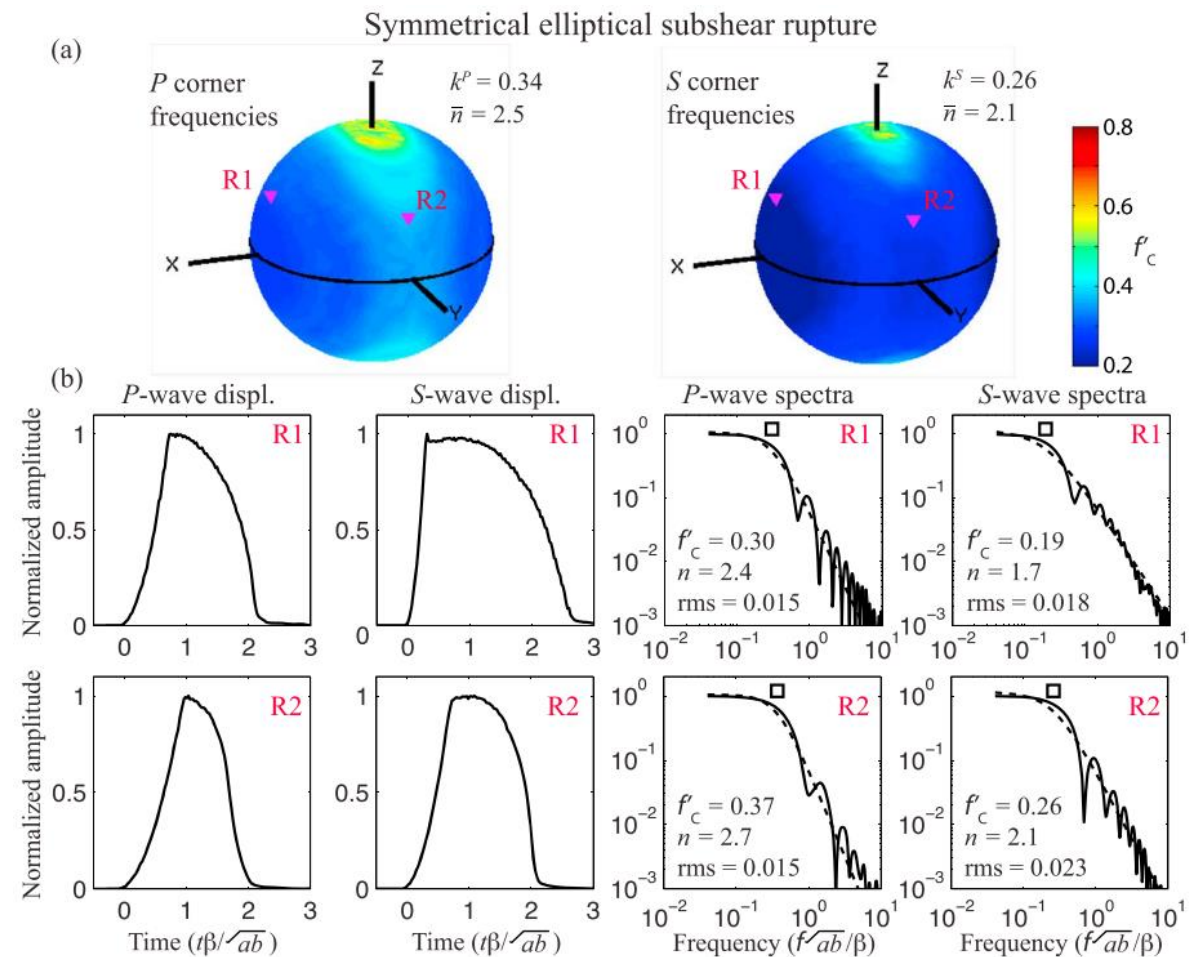
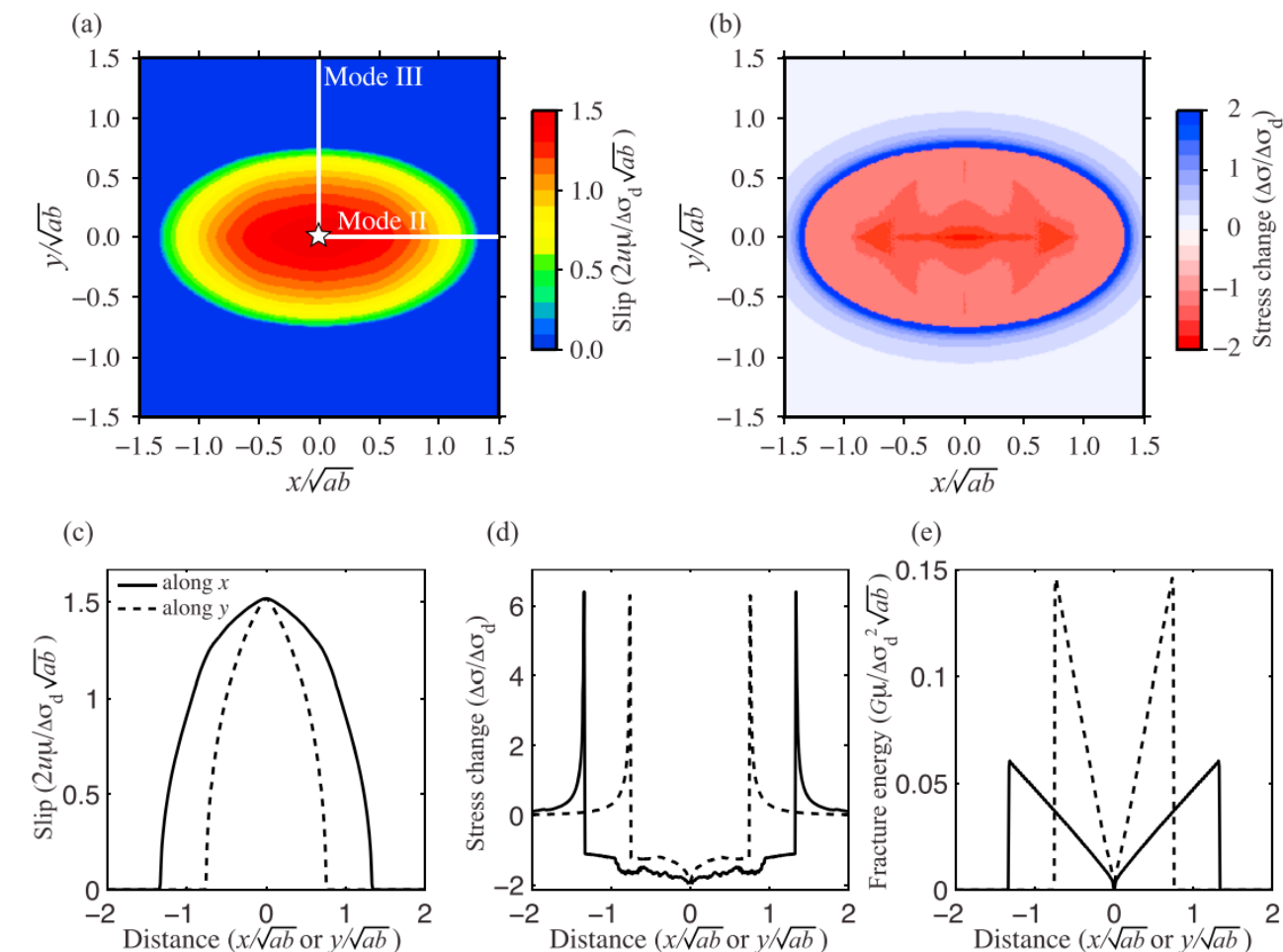


Case III Symmetrical elliptical rupture

$$\bar{f}_c^P = k^P \frac{\beta}{\sqrt{ab}} = 0.34 \frac{\beta}{\sqrt{ab}}$$

(subshear)

$$\bar{f}_c^S = k^S \frac{\beta}{\sqrt{ab}} = 0.26 \frac{\beta}{\sqrt{ab}}$$



Case III Symmetrical elliptical rupture

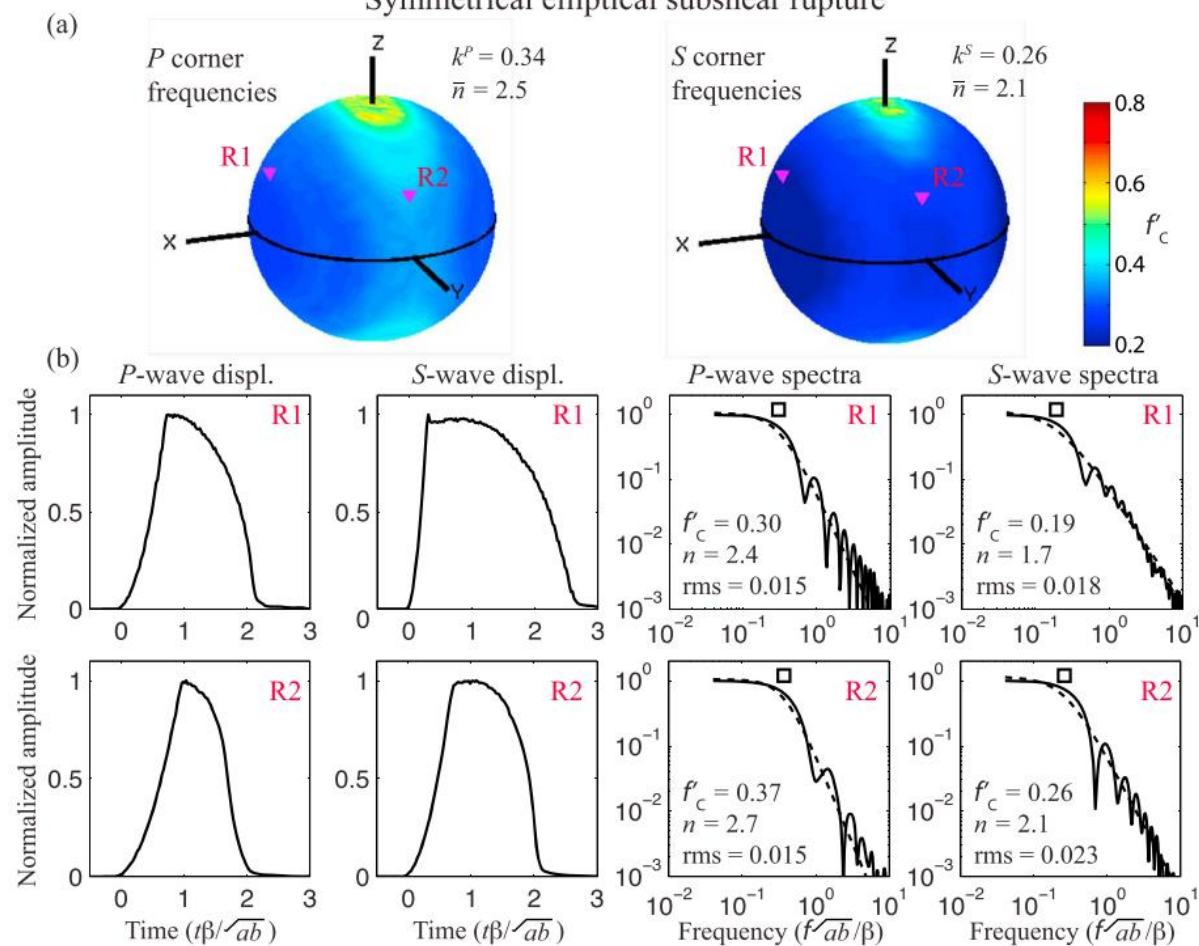
$$\bar{f}_c^P = k^P \frac{\beta}{\sqrt{ab}} = 0.34 \frac{\beta}{\sqrt{ab}} \quad (\text{subshear})$$

$$\bar{f}_c^S = k^S \frac{\beta}{\sqrt{ab}} = 0.26 \frac{\beta}{\sqrt{ab}}$$

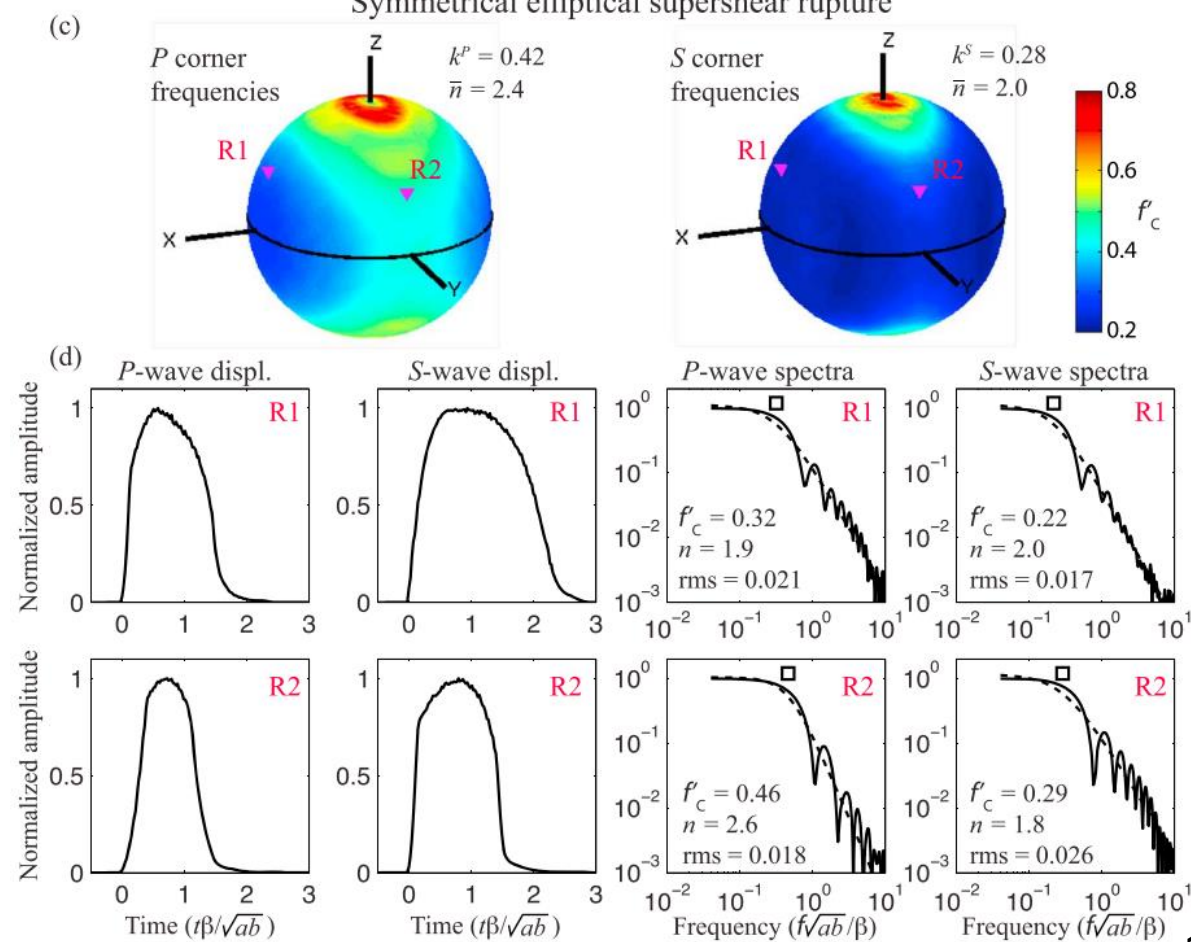
$$\bar{f}_c^P = k^P \frac{\beta}{\sqrt{ab}} = 0.42 \frac{\beta}{\sqrt{ab}} \quad (\text{supershear})$$

$$\bar{f}_c^S = k^S \frac{\beta}{\sqrt{ab}} = 0.28 \frac{\beta}{\sqrt{ab}}$$

Symmetrical elliptical subshear rupture



Symmetrical elliptical supershear rupture

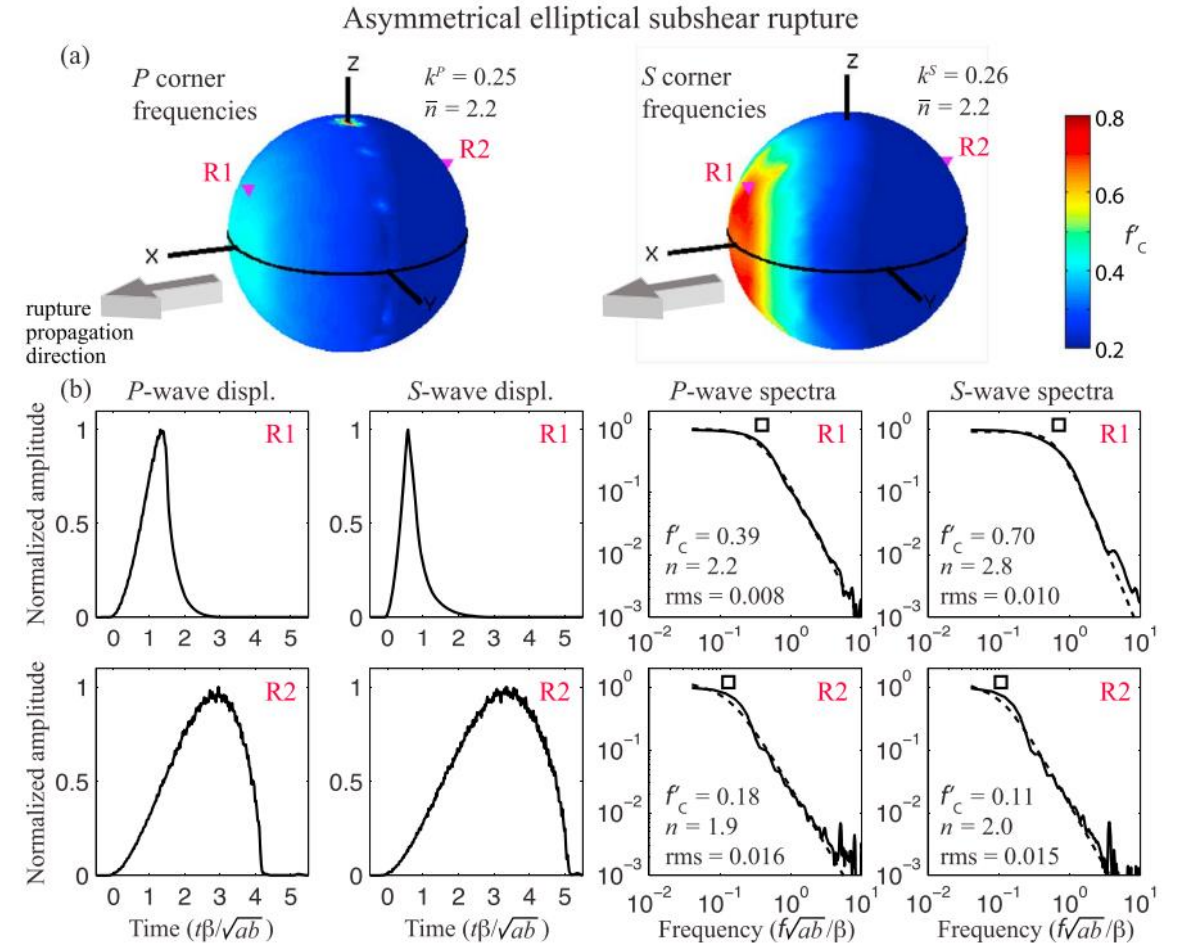
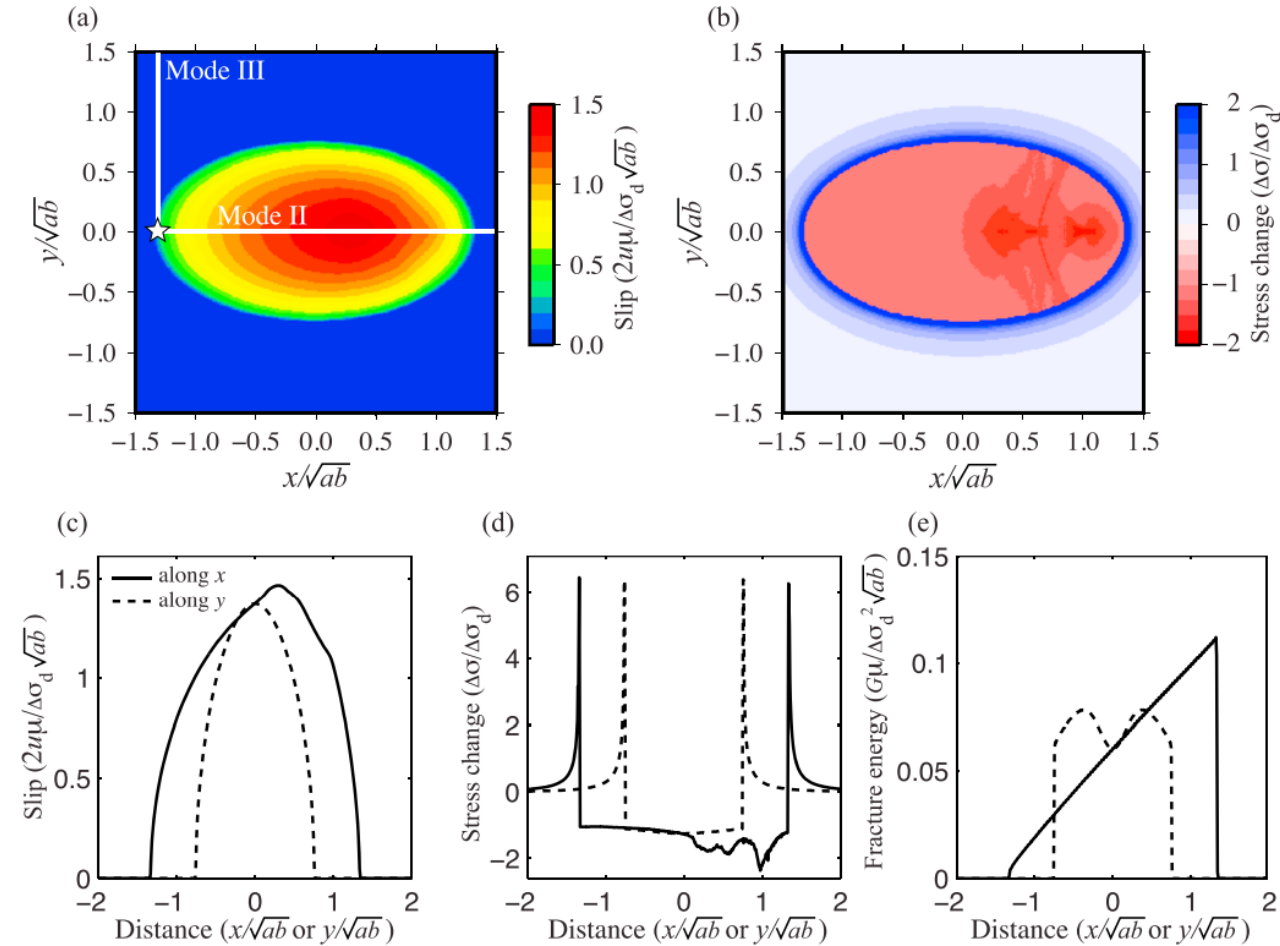


Case IV Asymmetrical elliptical rupture

$$\bar{f}_c^P = k^P \frac{\beta}{\sqrt{ab}} = 0.25 \frac{\beta}{\sqrt{ab}}$$

(subshear)

$$\bar{f}_c^S = k^S \frac{\beta}{\sqrt{ab}} = 0.26 \frac{\beta}{\sqrt{ab}}$$



Case IV Asymmetrical elliptical rupture

$$\bar{f}_c^P = k^P \frac{\beta}{\sqrt{ab}} = 0.25 \frac{\beta}{\sqrt{ab}}$$

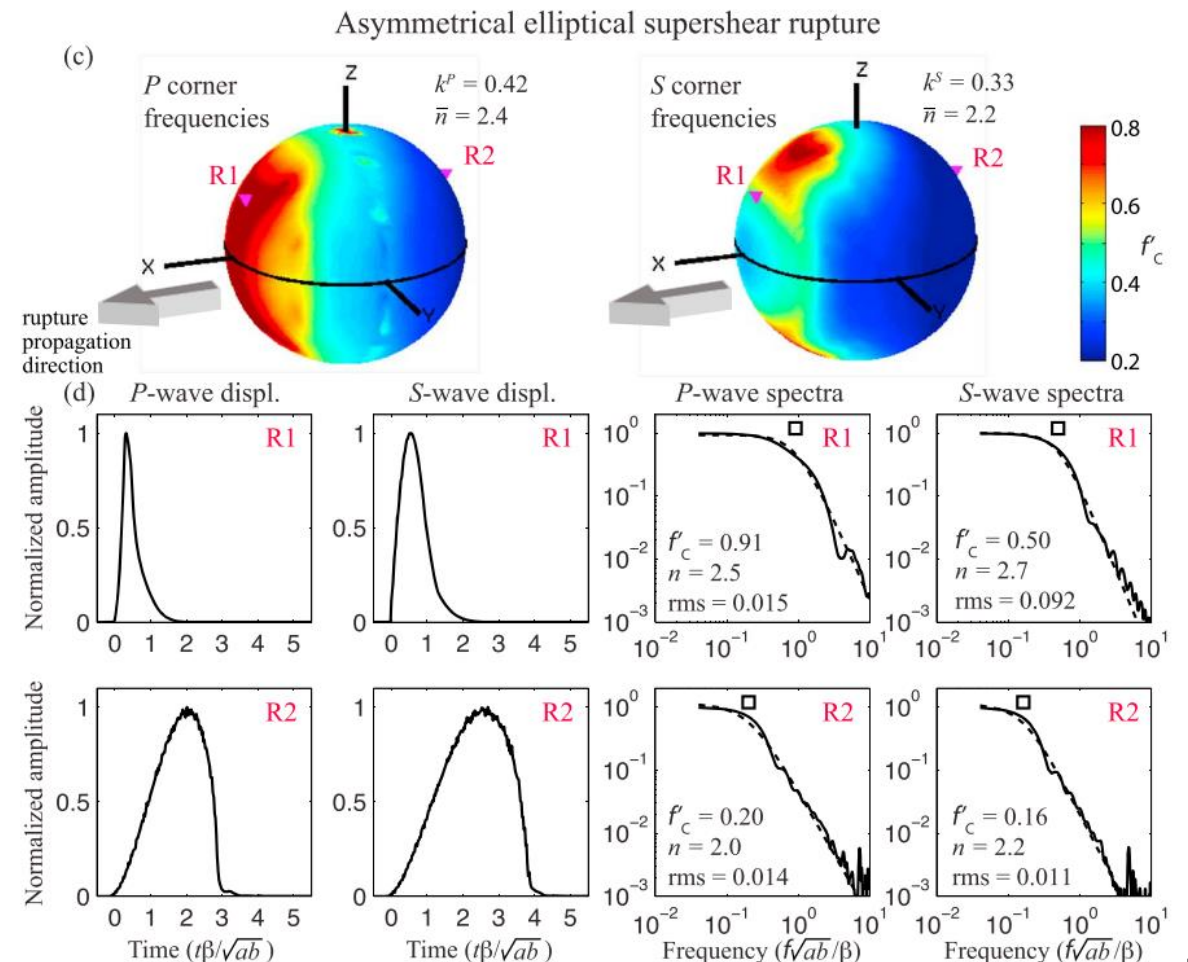
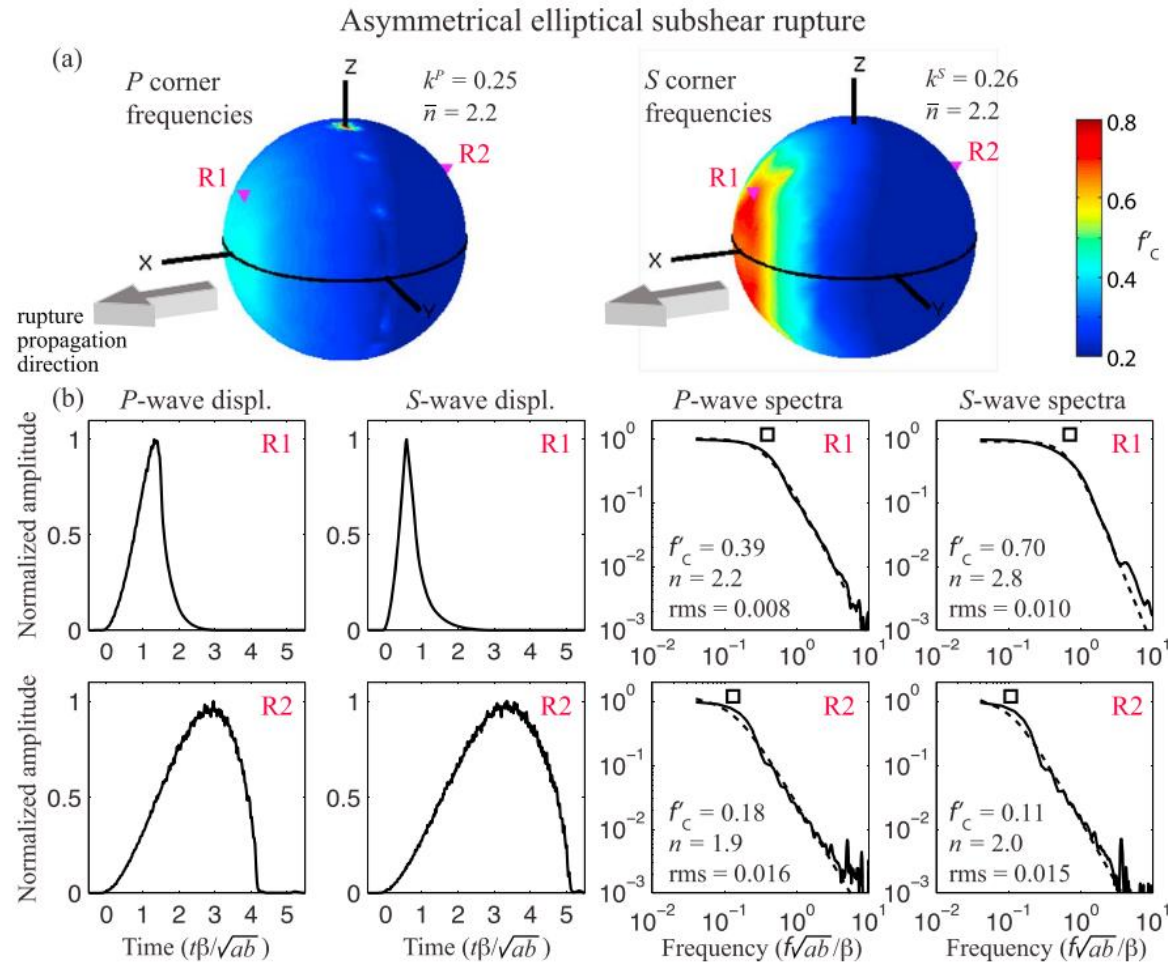
$$\bar{f}_c^S = k^S \frac{\beta}{\sqrt{ab}} = 0.26 \frac{\beta}{\sqrt{ab}}$$

(subshear)

$$\bar{f}_c^P = k^P \frac{\beta}{\sqrt{ab}} = 0.42 \frac{\beta}{\sqrt{ab}}$$

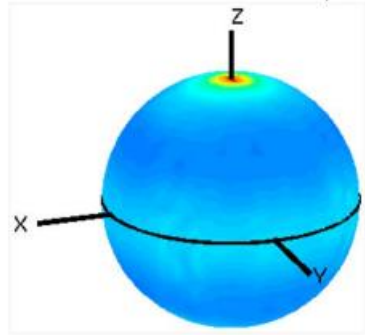
$$\bar{f}_c^S = k^S \frac{\beta}{\sqrt{ab}} = 0.33 \frac{\beta}{\sqrt{ab}}$$

(supershear)

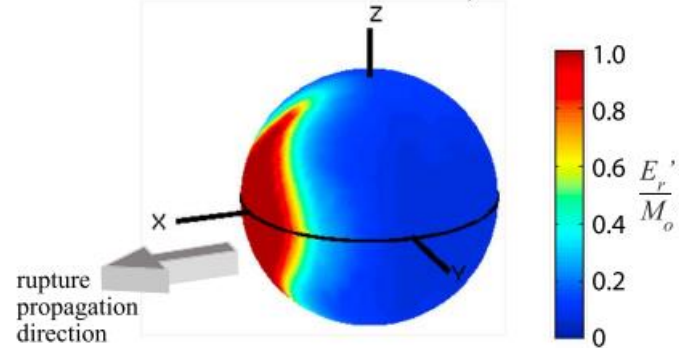


Radiated energy

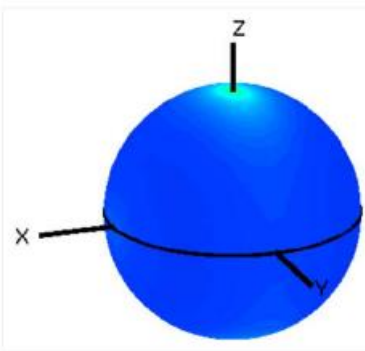
(a) Sym. circular rupture, $V_r = 0.9\beta$



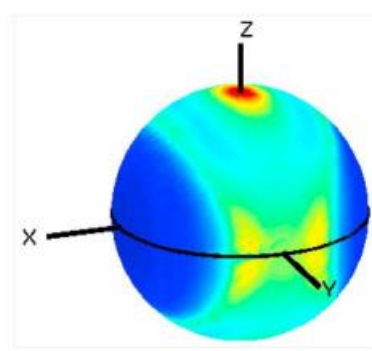
(b) Asym. circular rupture, $V_r = 0.9\beta$



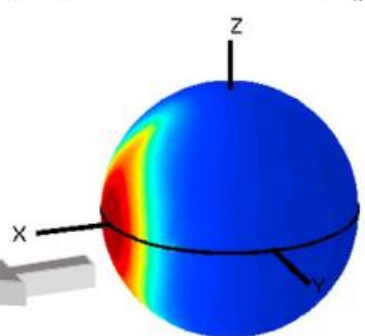
(c) Sym. elliptical rupture, $V_x = 0.9\beta$



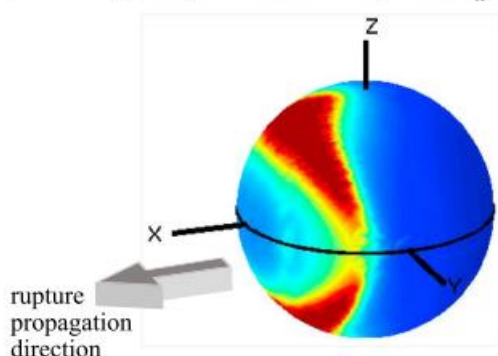
(d) Sym. elliptical rupture, $V_x = 1.6\beta$



(e) Asym. elliptical rupture, $V_x = 0.9\beta$



(f) Asym. elliptical rupture, $V_x = 1.6\beta$



$$E_r = \int_{\Sigma} \frac{\tau^o(\xi) + \tau^f(\xi)}{2} D(\xi) d\Sigma - \int_0^{\infty} \int_{\Sigma} \tau(\xi, t) V(\xi, t) d\Sigma dt$$

Far-field estimation

$$E_r = E_r^P + E_r^S = 2\rho \int_{\Gamma} \int_0^{\infty} [\alpha(\dot{u}^P)^2 + \beta(\dot{u}^S)^2] df d\Gamma$$

(frequency domain)

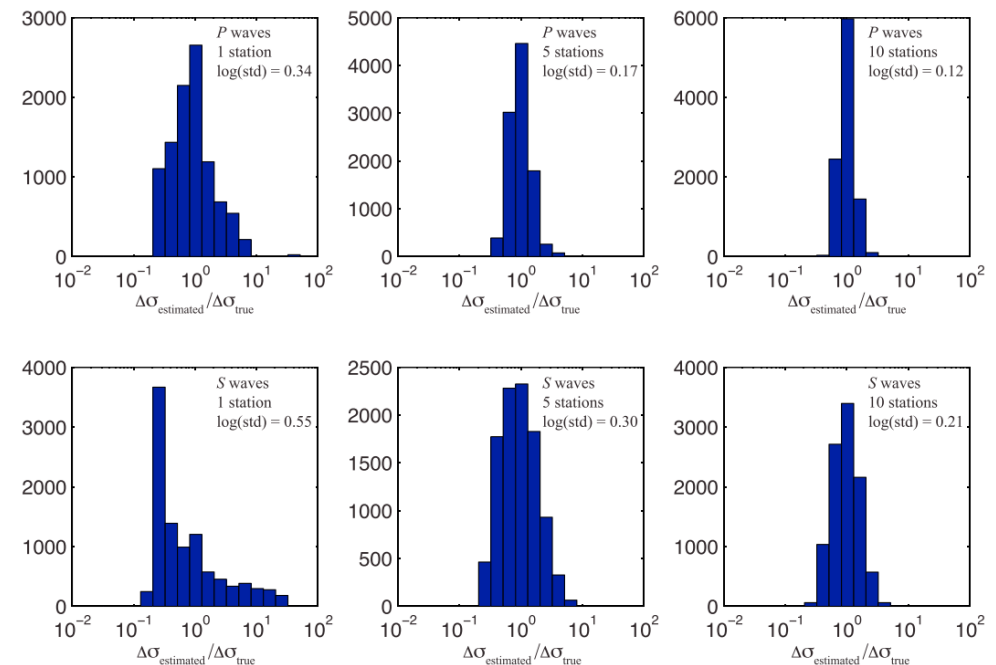
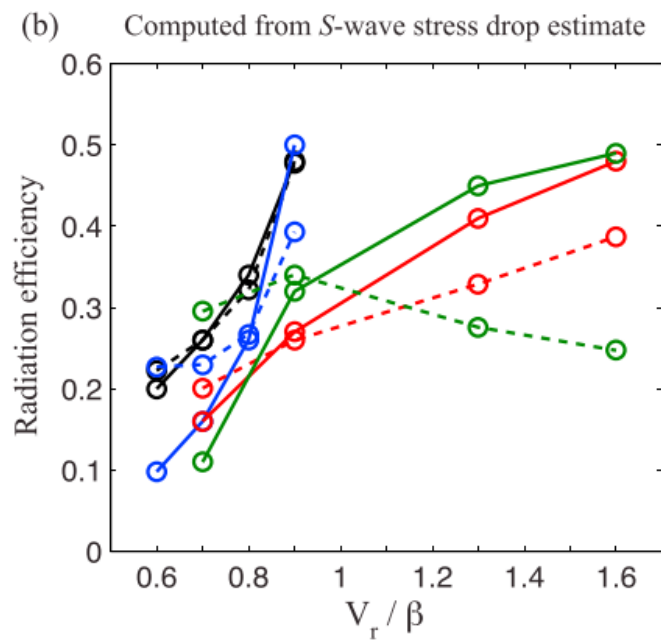
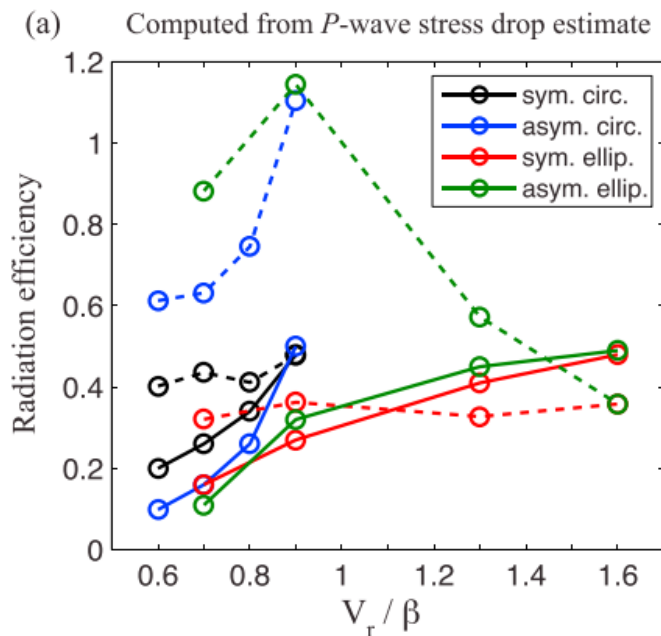
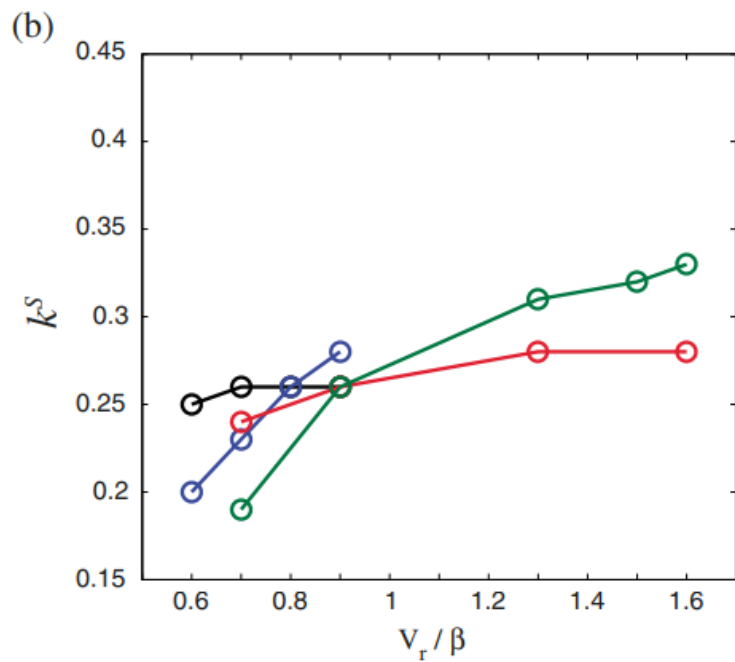
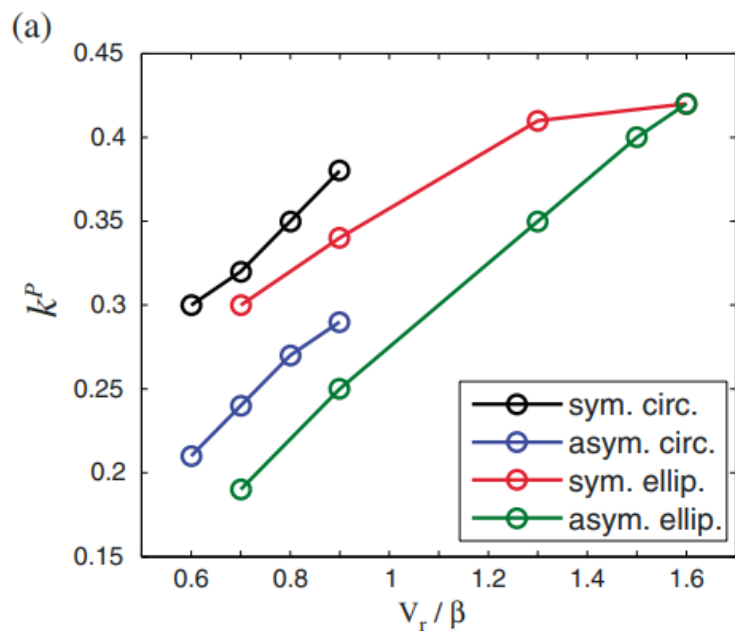
$$E_r = E_r^P + E_r^S = \rho \int_{\Gamma} \int_0^{\infty} [\alpha(\dot{u}^P)^2 + \beta(\dot{u}^S)^2] dt d\Gamma$$

(time domain)

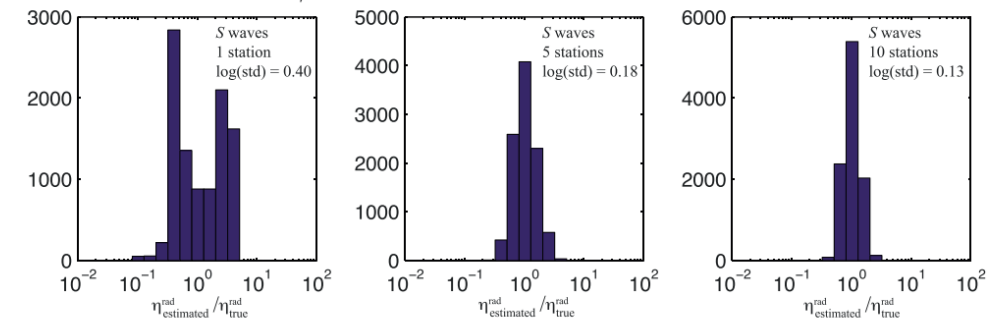
Point source: $E_r^S / E_r^P = 23$ ($\nu = 0.25$)

$$E^r / M_0 \approx E_r^S / M_0 \quad (E_r^S \gg E_r^P)$$

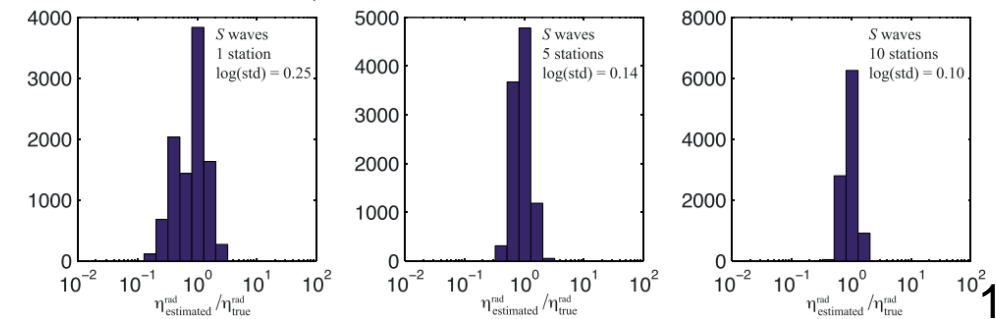
Implications?



(a) Sym. circular rupture, $V_r = 0.9\beta$

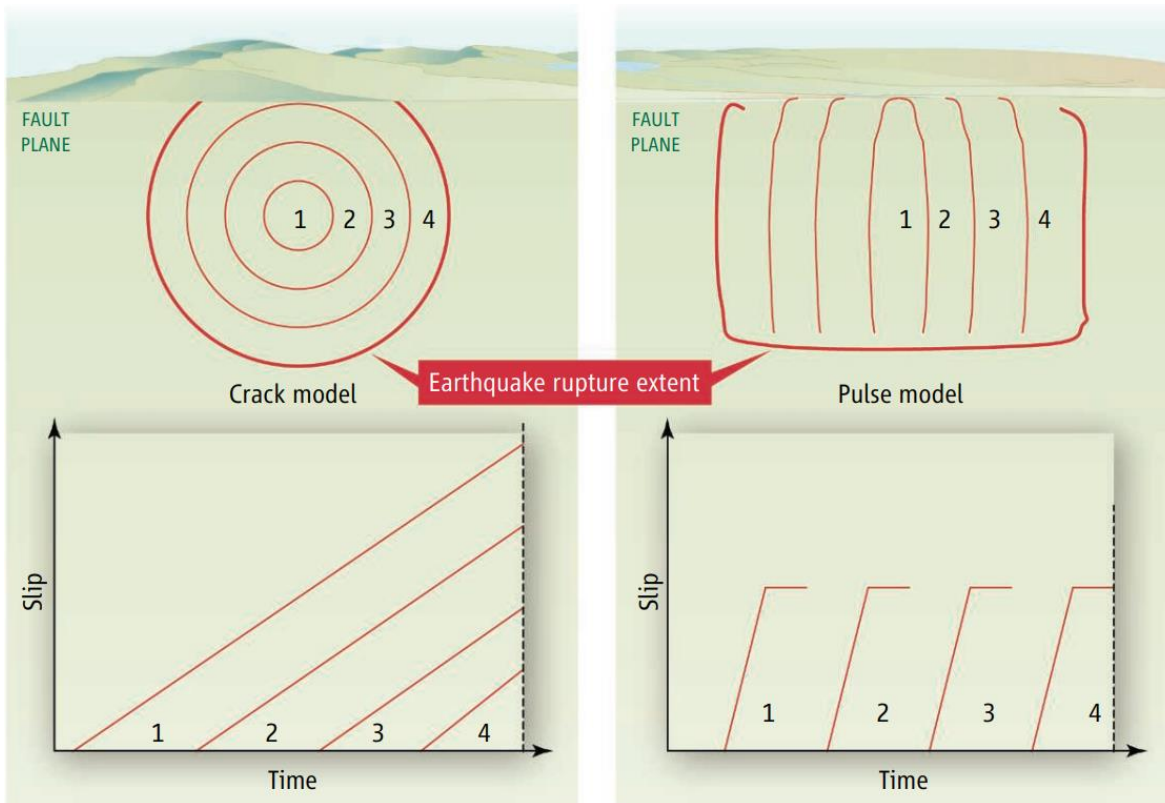


(b) Asym. circular rupture, $V_r = 0.9\beta$

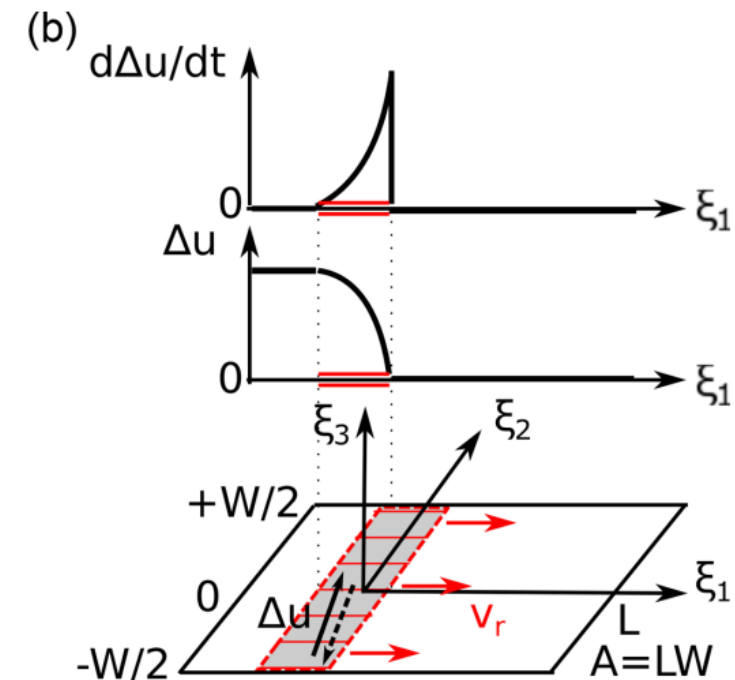
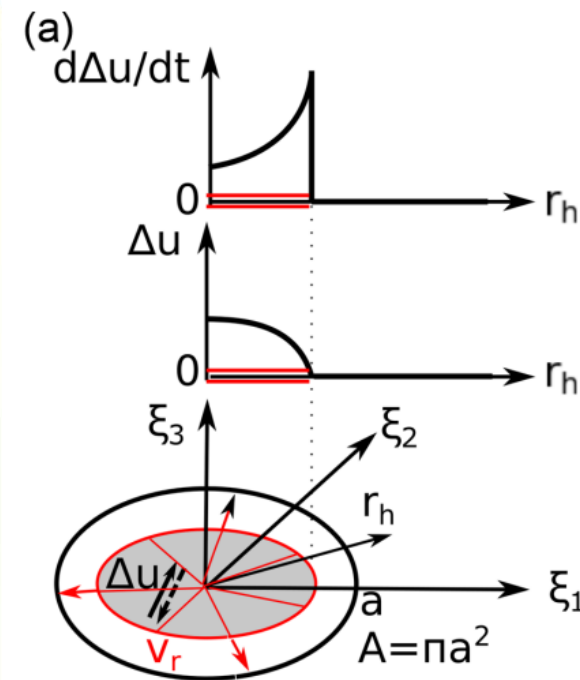


Crack-like vs Pulse-like rupture

- Crack-like rupture: local slip duration \approx rupture duration (e.g., Madariaga, 1976)
- Pulse-like rupture: local slip duration \ll rupture duration (e.g., Haskell, 1964; Heaton, 1990)
- Exact mechanism behind pulse-like rupture is still under debate.



Marone & Richardson (2006)

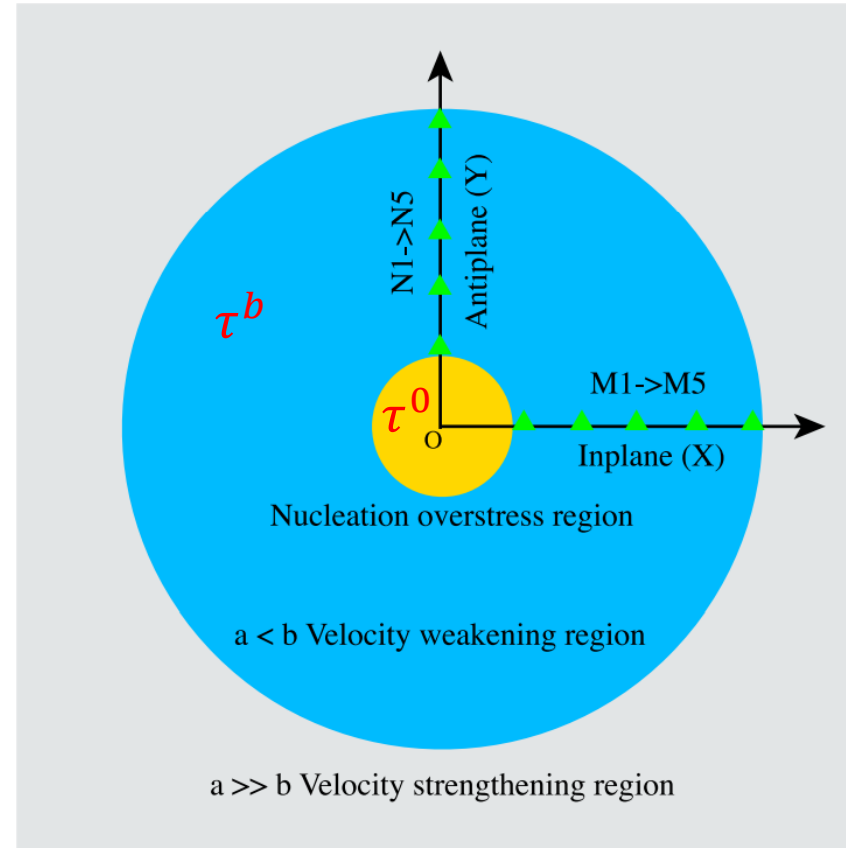
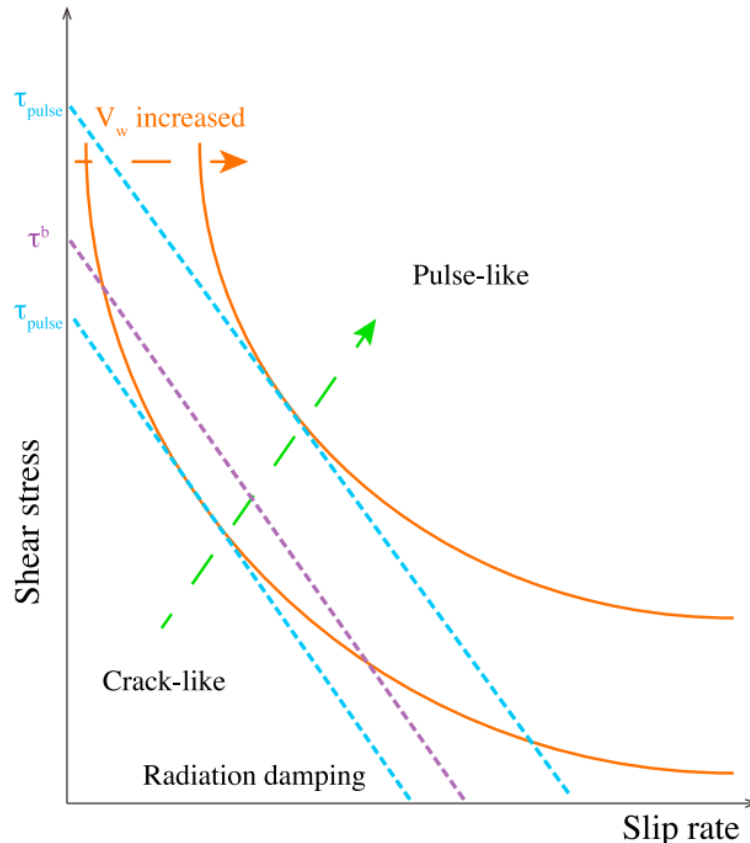


Dahm et al. (2021)

Numerical scheme (Wang & Day, 2017)

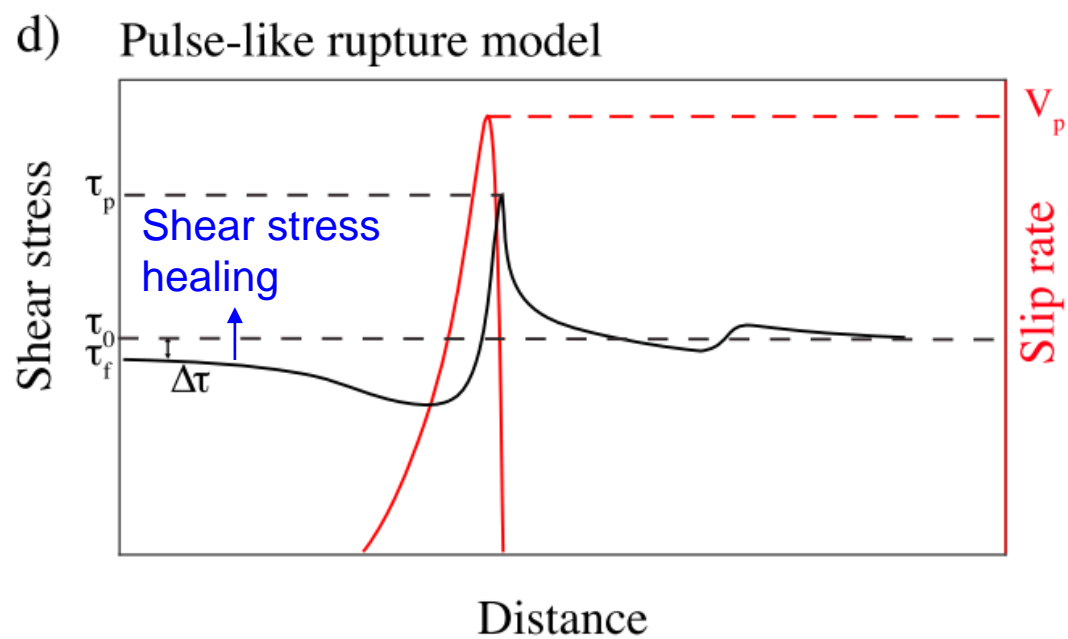
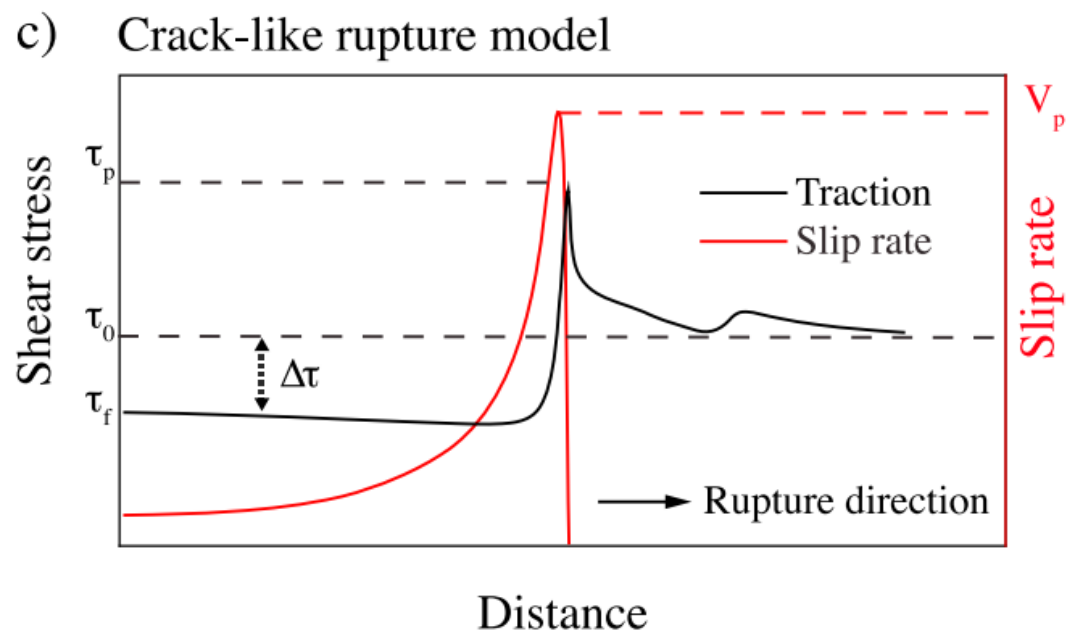
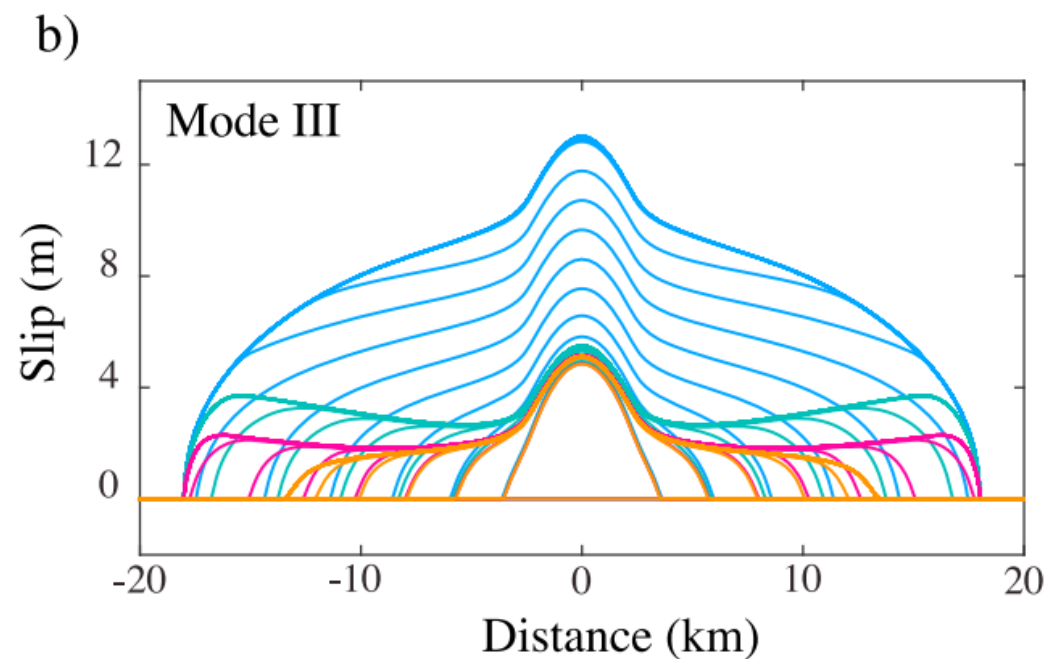
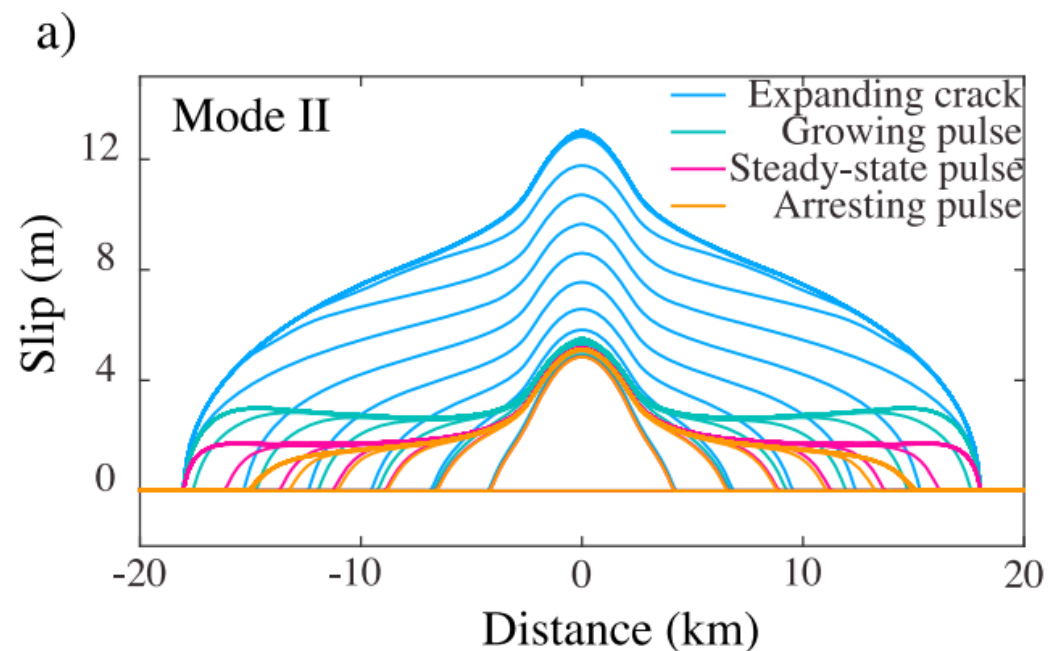
- Slip velocity-dependent friction (rate- and state-dependent friction)
- $f(V, \psi) = a \sinh^{-1} \left[\frac{V}{2V_0} \exp \left(\frac{\psi}{a} \right) \right]$ where $\dot{\psi} = -\frac{V}{L} [\psi - \psi_{ss}(V)]$, $\psi_{ss}(V) = a \ln \left\{ \frac{2V_0}{V} \sinh \left[\frac{f_{ss}(V)}{a} \right] \right\}$,

$$f_{ss}(V) = f_w + \frac{f_{lv} - f_w}{[1 + (V/V_w)^8]^{1/8}}, \text{ and } f_{lv}(V) = f_0 - (b - a) \ln(V/V_0)$$



- Crack \rightarrow Pulse
 - 1) Decrease τ_b
 - 2) Increase V_w
- Spontaneous rupture
 - \rightarrow Not fixed V_r

Simulation results



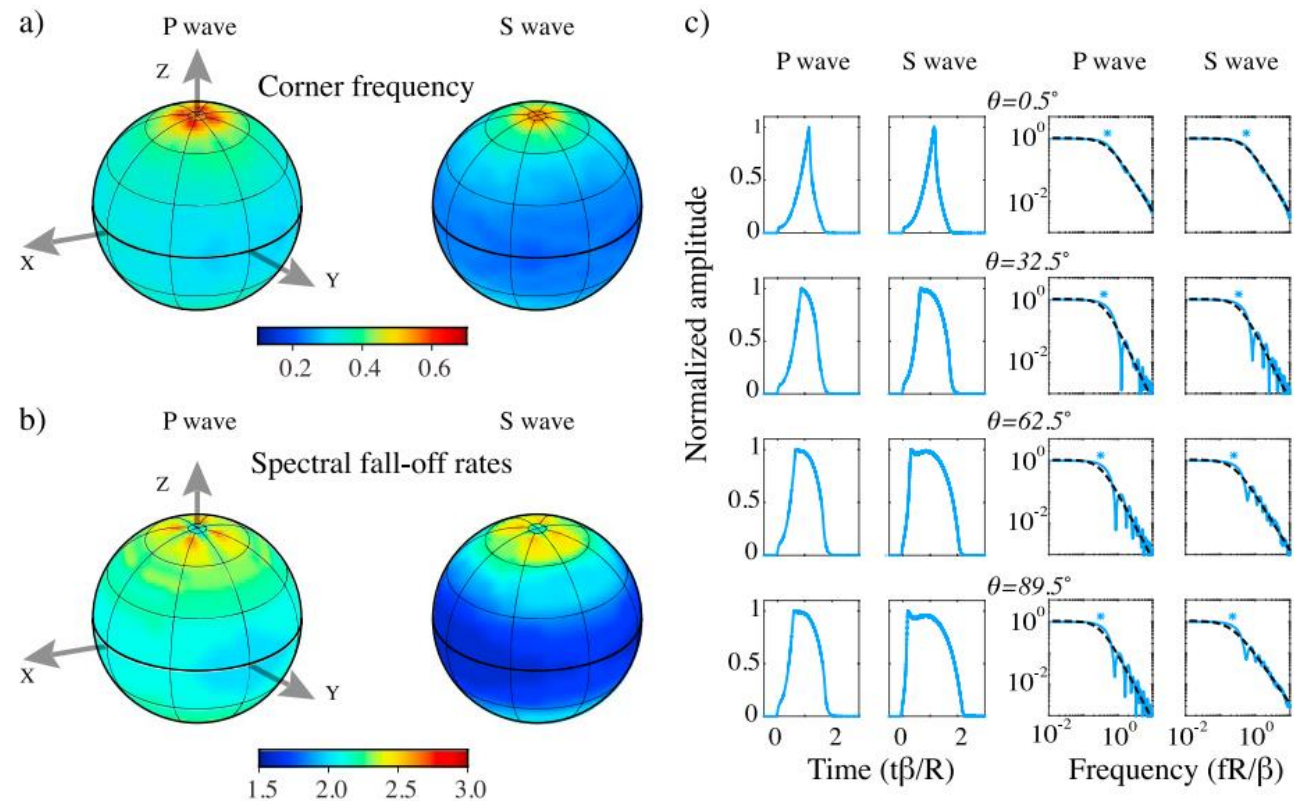
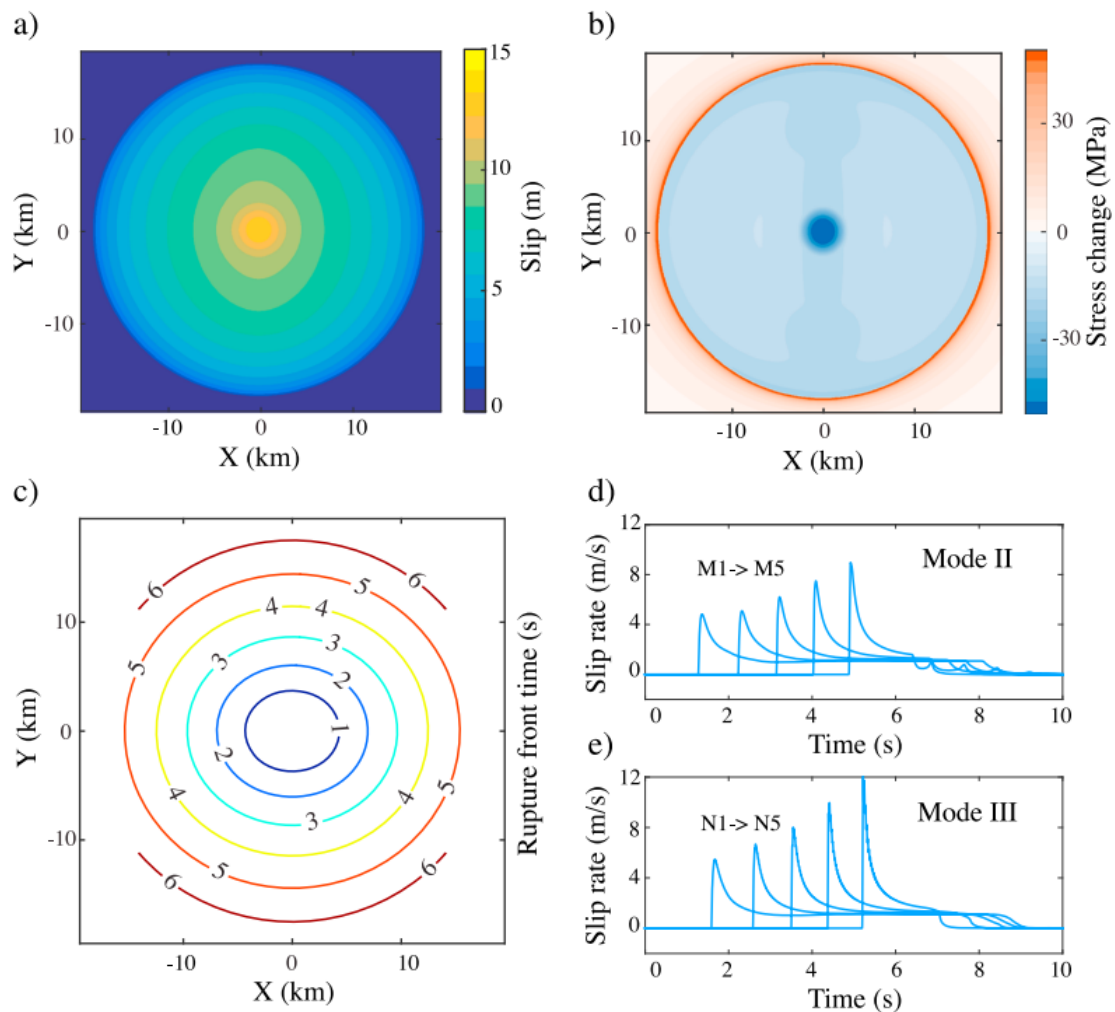
Case I Expanding crack

$$\bar{f}_c^P = k^P \frac{\beta}{a} = 0.35 \frac{\beta}{a}$$

$$V_r^2 = 0.88\beta$$

$$\bar{f}_c^S = k^S \frac{\beta}{a} = 0.27 \frac{\beta}{a}$$

$$V_r^3 = 0.84\beta$$



Four-lobes of high falloff rate not present in fixed rupture velocity model (Kaneko & Shearer, 2014)

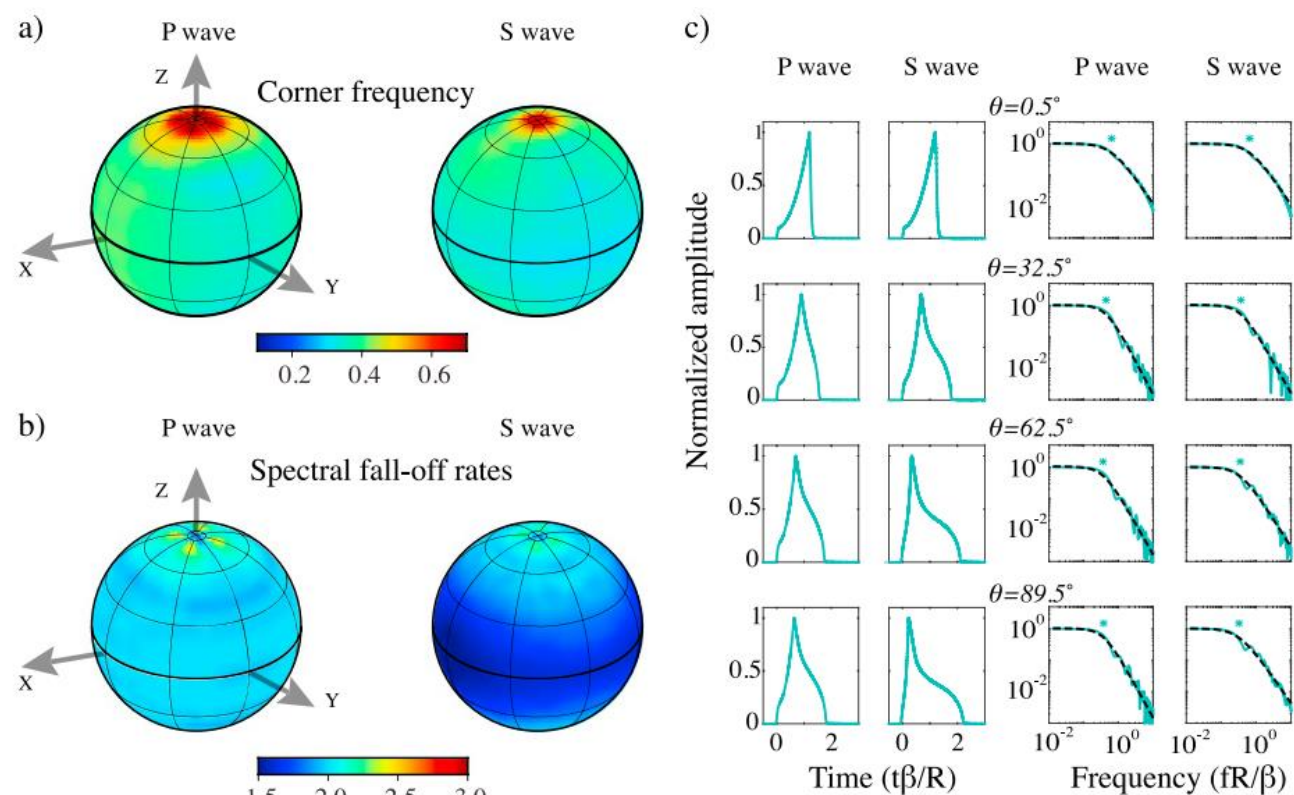
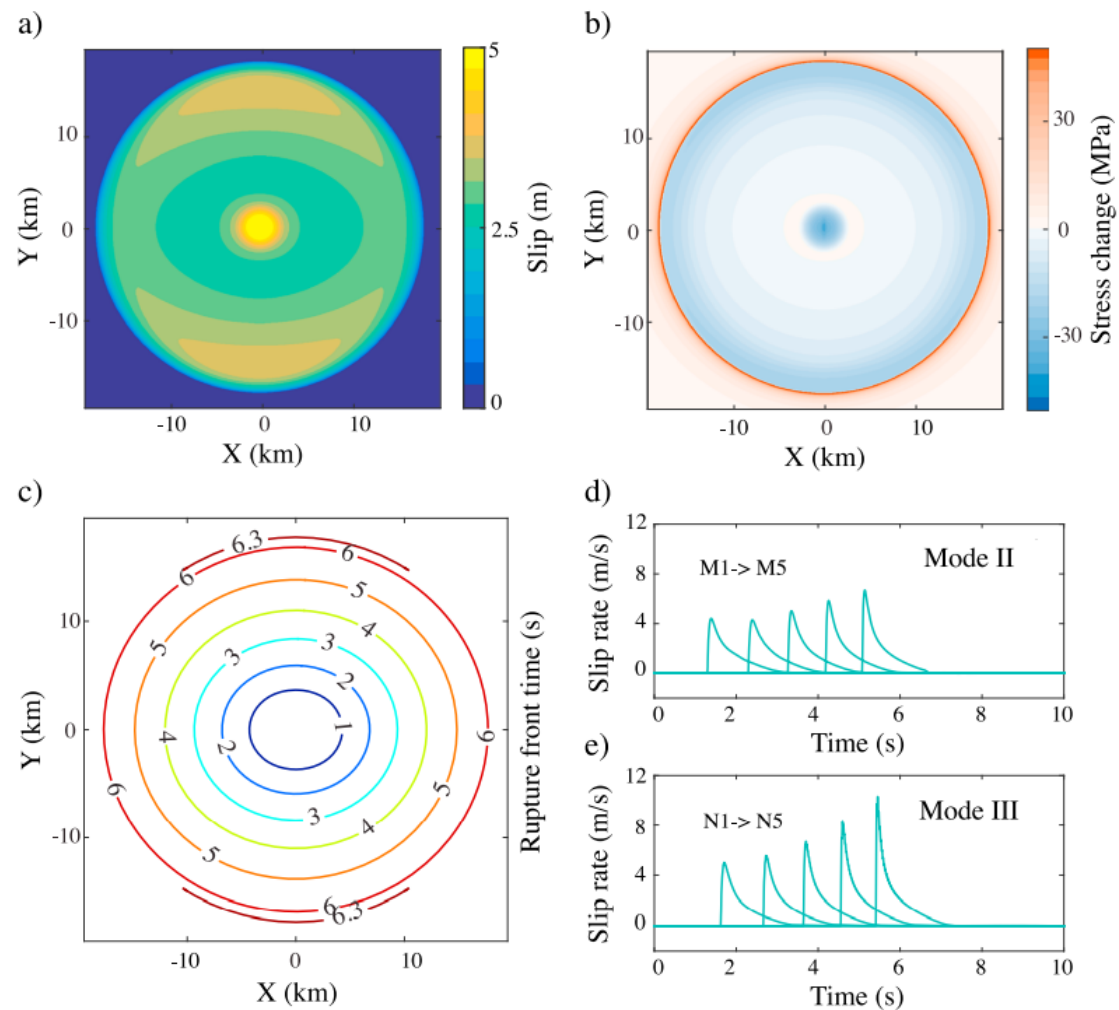
Case II Growing pulse

$$\bar{f}_c^P = k^P \frac{\beta}{a} = 0.40 \frac{\beta}{a}$$

$$V_r^2 = 0.85\beta$$

$$\bar{f}_c^S = k^S \frac{\beta}{a} = 0.36 \frac{\beta}{a}$$

$$V_r^3 = 0.81\beta$$



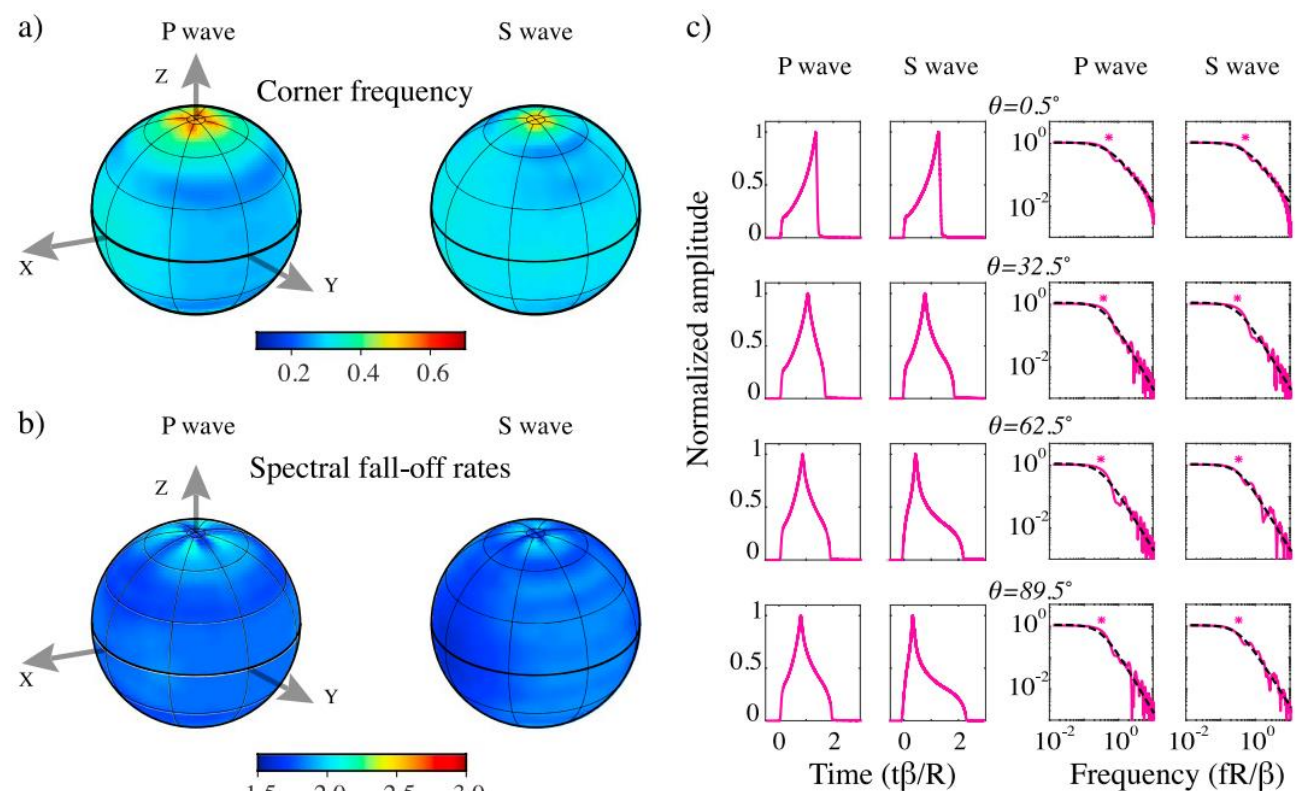
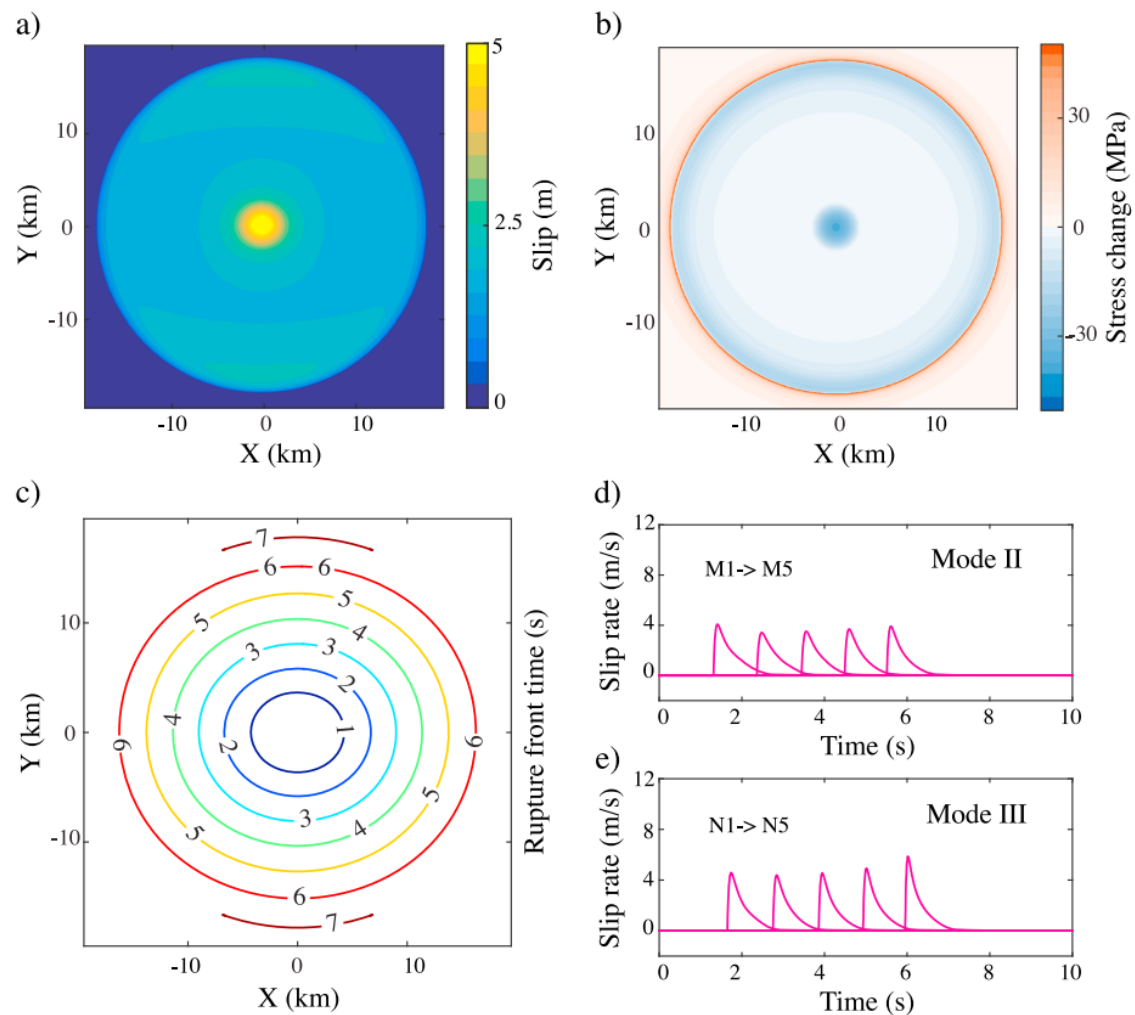
Case II Steady state pulse

$$\bar{f}_c^P = k^P \frac{\beta}{a} = 0.31 \frac{\beta}{a}$$

$$V_r^2 = 0.78\beta$$

$$\bar{f}_c^S = k^S \frac{\beta}{a} = 0.31 \frac{\beta}{a}$$

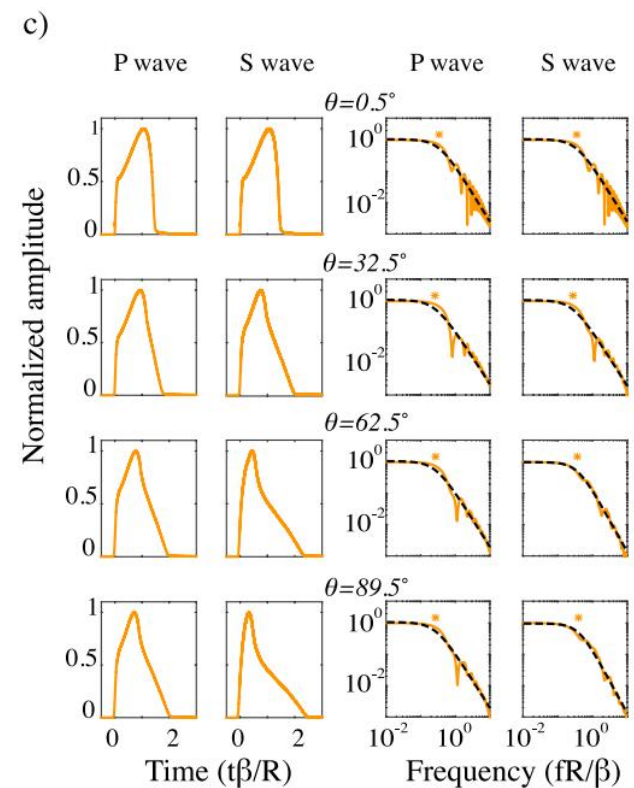
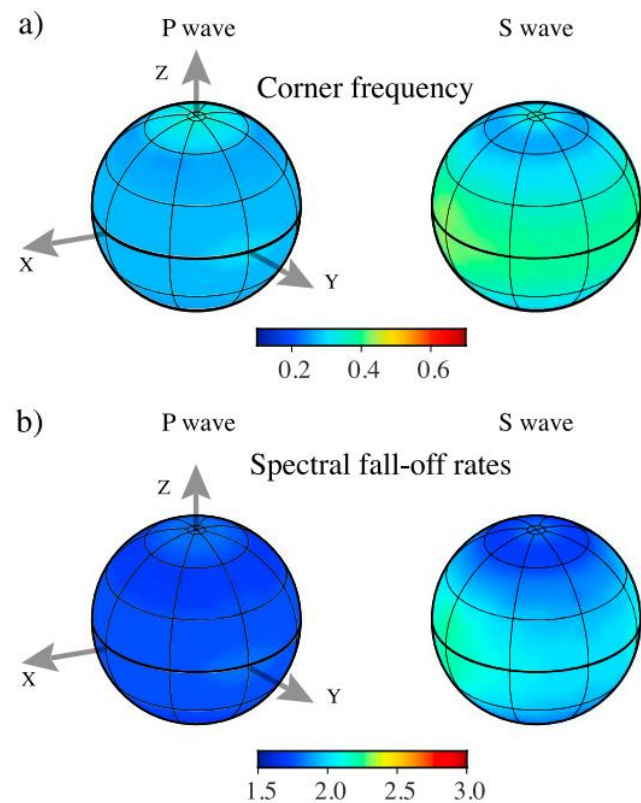
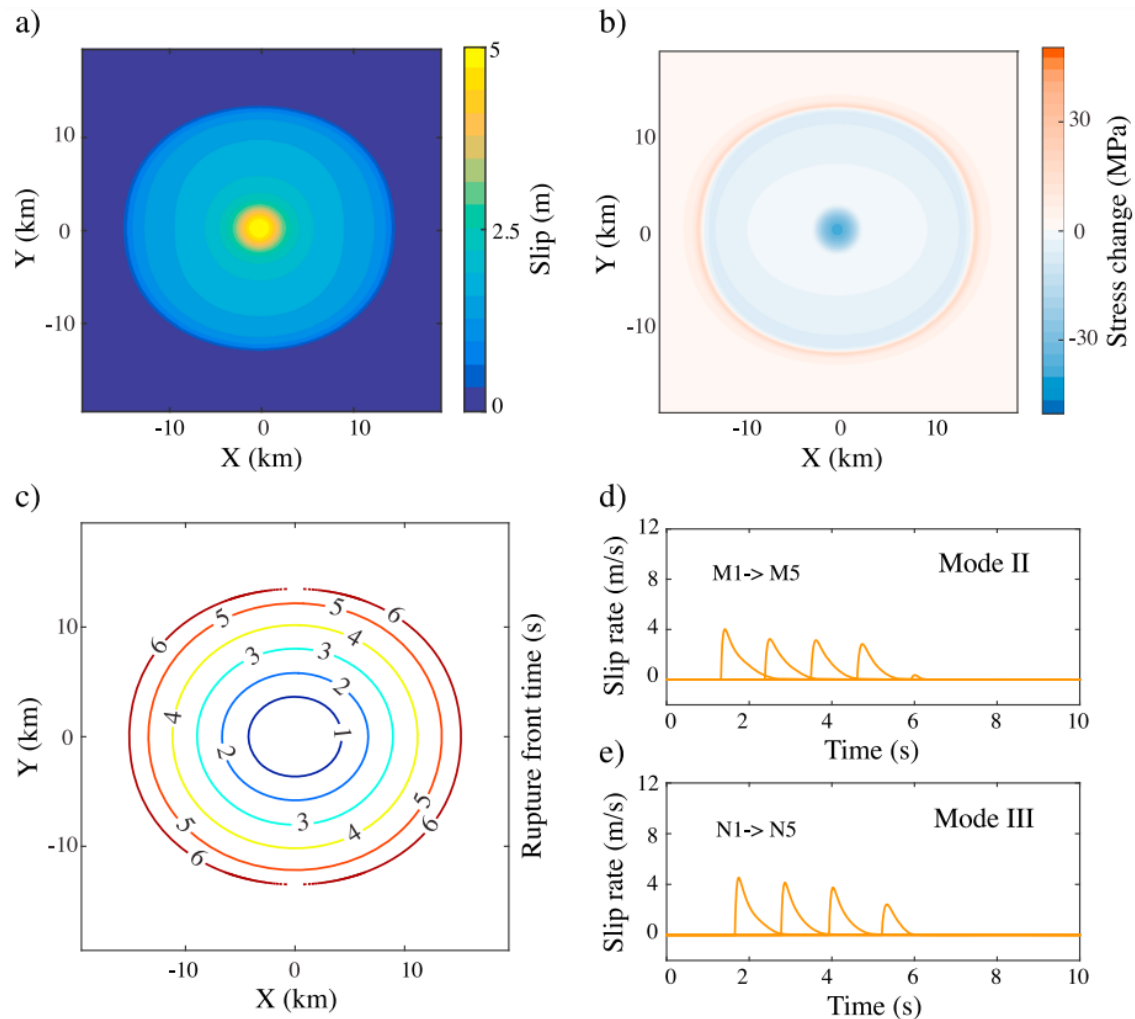
$$V_r^3 = 0.74\beta$$



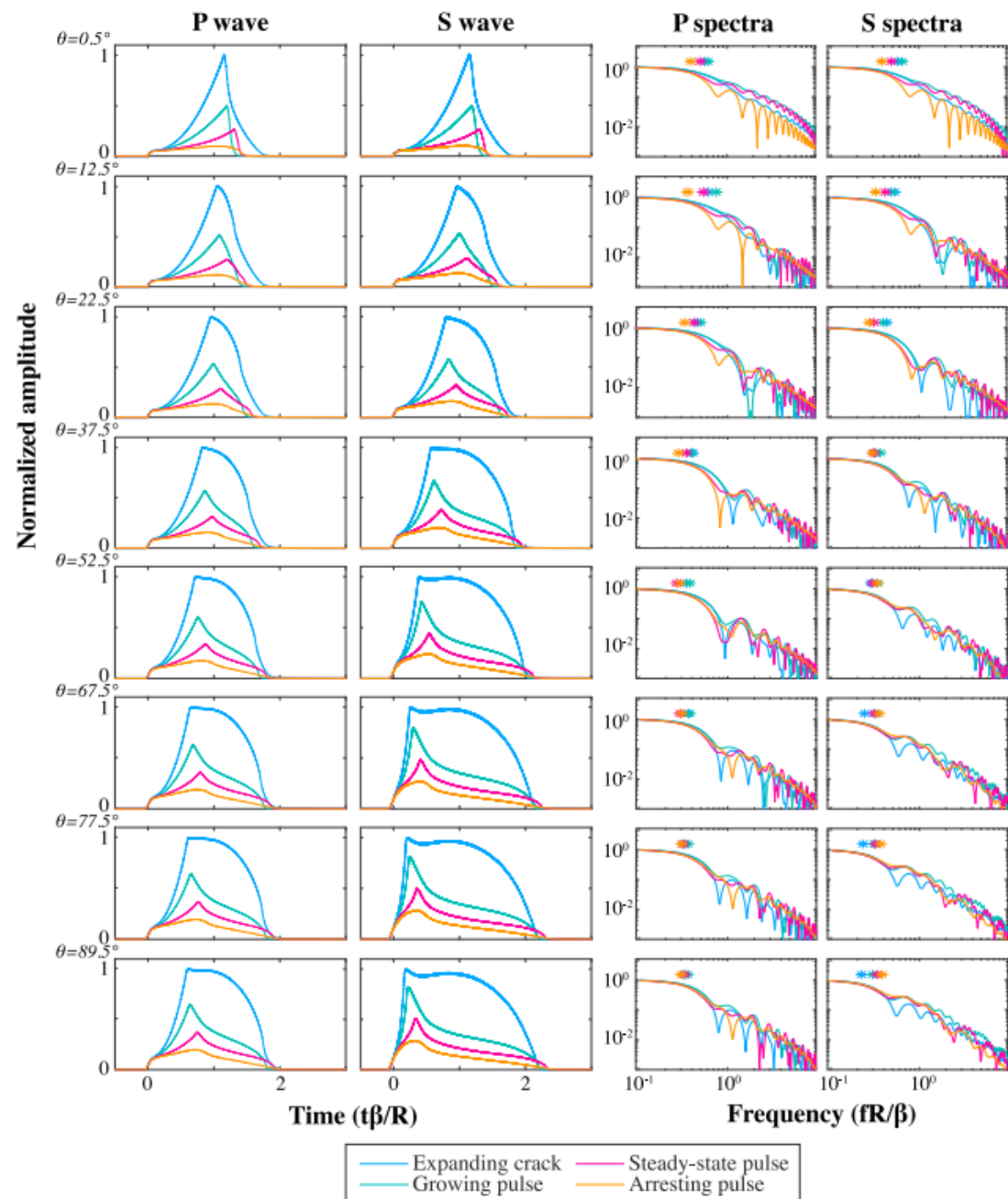
Case IV Arresting pulse

$$\bar{f}_c^P = k^P \frac{\beta}{\sqrt{ab}} = 0.28 \frac{\beta}{\sqrt{ab}} \quad V_r^2 = 0.72\beta$$

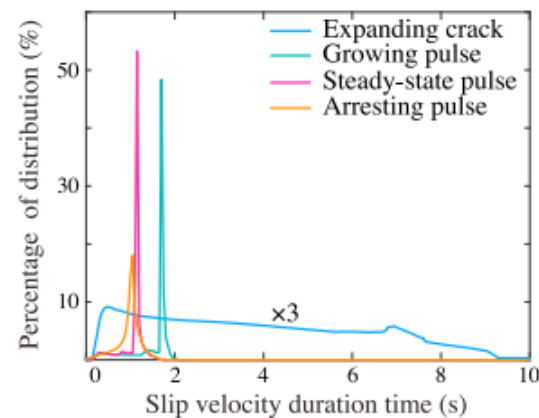
$$\bar{f}_c^S = k^S \frac{\beta}{\sqrt{ab}} = 0.34 \frac{\beta}{\sqrt{ab}} \quad V_r^3 = 0.66\beta$$



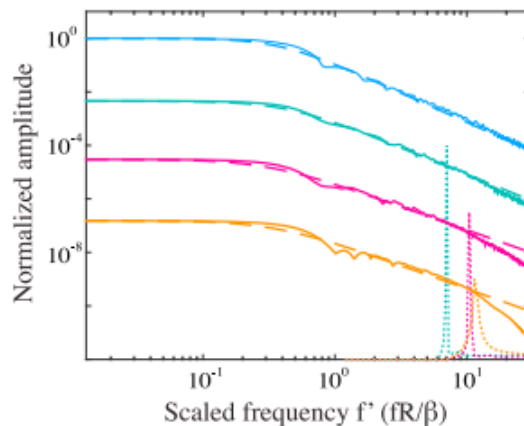
Far-field displacements/spectra



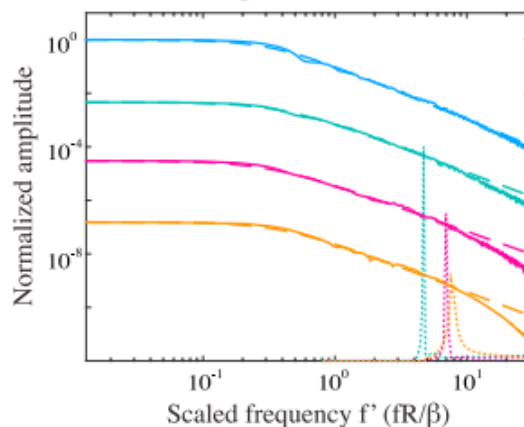
a) Slip duration distribution



b) Stacked P-wave spectra



c) Stacked S-wave spectra



Secondary corner frequency for pulse-like models

$$f_c^{2nd} = \frac{K}{\bar{T}} \quad (\bar{T}: \text{mean risetime})$$

Radiated energy / Stress drop

