STA601

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1.

$$p(x) = \int_{-\infty}^{+\infty} p(x,\tau^2)d\tau^2$$

$$= \int_{-\infty}^{+\infty} p(x|\tau^2)p(\tau^2)d\tau^2$$

$$= \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}}(\tau^2)^{1/2}\exp\{-\frac{1}{2}\tau^2x^2\}\frac{1}{\Gamma\left(\frac{\nu}{2}\right)\left(\frac{2}{\nu}\right)^{\frac{\nu}{2}}}(\tau^2)^{\frac{\nu}{2}-1}\exp\{-\frac{\nu}{2}\tau^2\}d\tau^2$$

$$= \frac{1}{\sqrt{2\pi}} \times \frac{1}{\Gamma\left(\frac{\nu}{2}\right)\left(\frac{2}{\nu}\right)^{\frac{\nu}{2}}}\int_{-\infty}^{+\infty}(\tau^2)^{1/2}\exp\{-\frac{1}{2}\tau^2x^2\}(\tau^2)^{\frac{\nu}{2}-1}\exp\{-\frac{\nu}{2}\tau^2\}d\tau^2$$

$$= \frac{1}{\sqrt{2\pi}} \times \frac{1}{\Gamma\left(\frac{\nu}{2}\right)\left(\frac{2}{\nu}\right)^{\frac{\nu}{2}}}\int_{-\infty}^{+\infty}(\tau^2)^{\frac{\nu+1}{2}-1}\exp\{-(\frac{1}{2}x^2+\frac{\nu}{2})\tau^2\}d\tau^2$$

$$= \frac{1}{\sqrt{2\pi}} \times \frac{1}{\Gamma\left(\frac{\nu}{2}\right)\left(\frac{2}{\nu}\right)^{\frac{\nu}{2}}}\Gamma\left(\frac{\nu+1}{2}\right)\left(\frac{1}{\frac{1}{2}x^2+\frac{\nu}{2}}\right)^{\frac{\nu+1}{2}}$$

$$= \frac{\left(\frac{2}{\nu}\right)^{\frac{1}{2}}}{\sqrt{2\pi}} \times \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)}\frac{\left(\frac{1}{\frac{1}{2}x^2+\frac{\nu}{2}}\right)^{\frac{\nu+1}{2}}}{\left(\frac{2}{\nu}\right)^{\frac{\nu+1}{2}}}$$

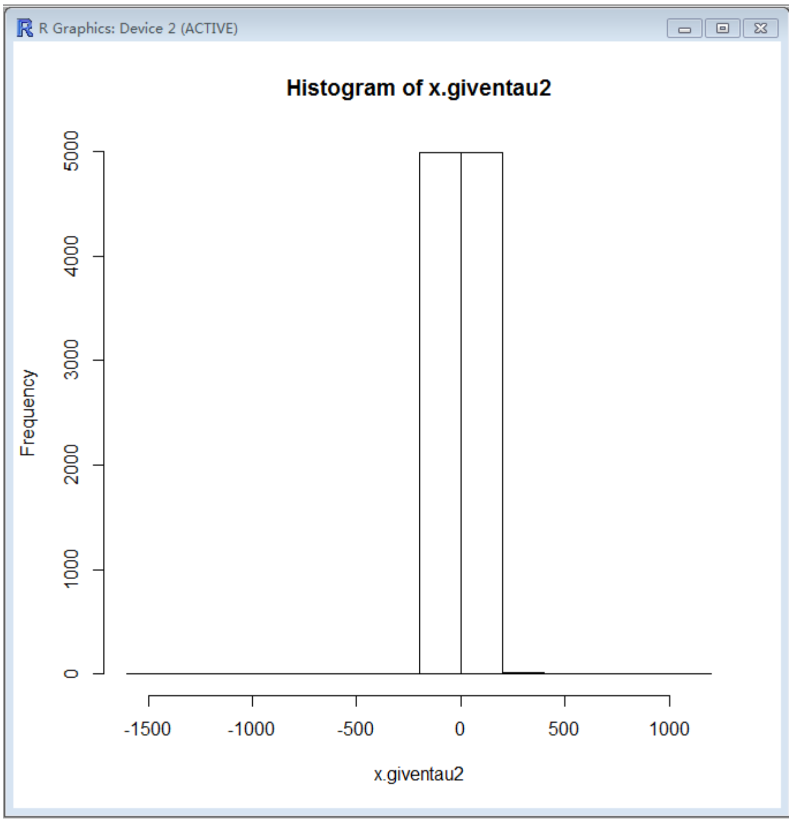
$$= \frac{1}{\sqrt{\nu\pi}} \times \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)}\left(\frac{x^2}{\nu}+1\right)^{-\frac{\nu+1}{2}}$$

2. When nu=1,

$$f(x) = \frac{1}{\sqrt{\pi}} \times \frac{\Gamma(1)}{\Gamma\left(\frac{1}{2}\right)}(x^2+1)^{-1} \sim \mathrm{t}_1$$

or $f(x) = \frac{1}{\pi(x^2+1)} \sim Cauchy\ Dis.$

The figure is as below.



Compute 2.5% &97.5% percentile points of the distribution:

The theoretical values: -12.7062 , 12.7062

The random samples values: -12.71138, 11.33432

They are very close to each other.

3. p-value for ks test is 0.3592. It denotes that x may have a same distribution as t(1) distribution.

4.

The central limit theory does not hold for x when ν =1, for the distribution of x will be a Cauchy Distribution, whose first and second moment don't converge.

When ν=2, The central limit theory does not hold either. Because the variance of t_2 distribution does not converge.

The theory holds when ν=3.

#######################R Code#####################

#2#

```
tau2<-rgamma(10000,shape=.5,rate=.5)

x.giventau2<-rnorm(10000,0,1/sqrt(tau2))

hist( x.giventau2)


qt(c(0.025,0.975),1)


quantile(x.giventau2,probs=0.0250)

quantile(x.giventau2,probs=0.0251)

(quantile(x.giventau2,probs=0.975)+quantile(x.giventau2,probs=
0.9751))/2


quantile(x.giventau2,probs=0.9750)

quantile(x.giventau2,probs=0.9751)
```

```
(quantile(x.giventau2,probs=0.0250)+quantile(x.giventau2,probs
=0.0251))/2
```

#3#

```
ks.test(c(x.giventau2),"pt",1)
```