Lab Assignment 6: Gibbs Sampling

Your friend, "Sloppy" Jo(e) is an aspiring pollster, and she (he) agrees to conduct a poll for you, free of charge. You give the following instructions: "Please ask about 25 people whether they are in favor of more gun control, and report back to me the number who are in favor." After a few days Sloppy returns with the poll results: there were $y = 20$ in favor. "And how many people did you ask?" you inquire. "Ummm, I dunno. You didn't ask me to record that. All I know is that it was about 25."

Assume $(y|N, \beta) \sim Binomial(N, \beta)$. Furthermore, assume a uniform prior on β and a Poisson prior on N. Do the following:

1. Derive the joint distribution of (y, N, β).
2. Derive full conditionals $(N|\beta, y)$ and $\beta|N, y)$.
3. Use these to sample (using Gibbs sampling) from the joint posterior $(\beta, N|y)$ using a starting value of $(\beta^{(1)}, N^{(1)})$=(0.05,50).
4. Show trace plots for β and N.
5. Show the 2D trace plot for the first 10 draws of the Gibbs sampler, $(\beta^{(1:10)}, N^{(1:10)})$. Show both the points and the connecting lines.
6. Give the central 90% posterior credible interval for β, accurate to (and rounded to) the nearest 1% for both upper and lower limits.
7. What is the probability that exactly 20 people were polled? Base your answer on at least 10,000 draws (post-burn-in), and round to the nearest one tenth of 1%.

Note: When finding the full conditional for N, you may want to find the distribution of $N - y$ and add this to the given value for y.

Please submit a report that includes answers to the above questions as well as your R/Matlab code.