

Lab Assignment 3

Suppose $x|(\tau^2) \sim N(0, 1/(\tau^2))$ and $\tau^2 \sim \text{Gamma}(\text{shape}=\nu/2, \text{rate}=\nu/2)$. Find the marginal distribution of x : $p(x)$. Show ALL work.

Let $\nu=1$. Get a sample of 10,000 from marginal distr. of x by drawing 10,000 τ^2 's and then 10,000 x 's given the τ^2 's. Plot sample (either histogram or density is fine). Give two names for the actual marginal distribution $p(x)$ when $\nu=1$. Also, compute 2.5% & 97.5% percentile points of the distribution using the random samples and compare them to the theoretical values.

Use Kolmogorov-Smirnov test (`ks.test` in R) to test whether your observed distribution is equal to a $t(\text{df}=1)$. Report p-value. What is the conclusion of the test?

Does the Central Limit Theorem hold for the mean of a sample from $p(x)$ when $\nu=1$? What about $\nu=2$? $\nu=3$? Why or why not? A quick explanation will do; an involved proof is NOT required.