

Lab Assignment 6: Metropolis-Hastings

Let

$$\pi(\theta) \propto \exp\{-.5\theta^2\} + \frac{1}{2}\exp\{-.5(\theta - 3)^2\}$$

be a normal mixture with modes at $\theta = 0$ and $\theta = 3$. Find the normalizing constant analytically.

Now, implement a Metropolis-Hastings sampler with proposal density

$$p(\theta'|\theta) = N(\theta, \sigma_{cand}).$$

Use a starting value of $\theta_0 = 3$ and no burn-in (this is OK because we are starting in high-density region). Choose $\sigma_{cand} \in \mathbb{N}$ such that the acceptance probability is very close to 45%. Which value did you choose?

Plot the analytic and sampled densities on the same plot; do not use any histograms.

In theory, what's wrong with using $\sigma_{cand} = .05$? Is there anything wrong in practice? Do your answers change for $\sigma_{cand} = 8$? What about for $\sigma_{cand} = 100$? Please be specific.