

a) Posterior mean and 95% interval for mean and standard deviation for each school

	Posterior Mean	95% Interval-Mean	95% Interval-SD
School 1	9.300369	(7.83145, 10.74498)	(3.007565, 5.110134)
School 2	6.96606	(5.262186, 8.648512)	(3.367144, 5.940036)
School 3	7.869304	(6.260536, 9.350922)	(2.782333, 5.106006)

b) Posterior probability that $\theta_i < \theta_j < \theta_k$ for all the 6 permutation

$\theta_1 < 2 < 3$	$\theta_1 < 3 < 2$	$\theta_2 < 1 < 3$	$\theta_2 < 3 < 1$	$\theta_3 < 1 < 2$	$\theta_3 < 2 < 1$	θ sum p
0.00467	0.00282	0.07995	0.68493	0.0124	0.22763	≈ 1

c) Posterior probability that $y_i < y_j < y_k$ for all the 6 permutation

$y_1 < 2 < 3$	$y_1 < 3 < 2$	$y_2 < 1 < 3$	$y_2 < 3 < 1$	$y_3 < 1 < 2$	$y_3 < 2 < 1$	y sum p
0.10631	0.10199	0.18533	0.26816	0.14031	0.19777	≈ 1

d) Compute the probability that $y_1 > y_2$ & $y_1 > y_3$, and the probability that $\theta_1 > \theta_2$ & $\theta_1 > \theta_3$

$$p(\theta_1 > \theta_2 \text{ \& } \theta_1 > \theta_3) = p(\theta_1 > \theta_2 > \theta_3) + p(\theta_1 > \theta_3 > \theta_2) = 0.90176$$

$$p(y_1 > y_2 \text{ \& } y_1 > y_3) = p(y_1 > y_2 > y_3) + p(y_1 > y_3 > y_2) = 0.46606$$

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#####Code#####
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```
##### (a) #####
```

```
S=100000
data1<-read.table("http://www.stat.washington.edu/~hoff/courses/564-
2010/Data/HwData/school1.dat")
data2<-read.table("http://www.stat.washington.edu/~hoff/courses/564-
2010/Data/HwData/school2.dat")
data3<-read.table("http://www.stat.washington.edu/~hoff/courses/564-
2010/Data/HwData/school3.dat")

school1 <- data1
mu.0=5;sigma.0.square=4;k.0=1;v.0=2;
n.1=length(t(school1))
k.1=k.0+n.1
mu.1=(sum(school1)+k.0*mu.0)/k.1
v.1=v.0+n.1
sigma.1.square=1/v.1*(v.0*sigma.0.square+var(school1)*(n.1-1)+(n.1*k.0/k.1*(mean(t(school1))-
mu.0)^2))

mean.1=rnorm(S,mu.1,sqrt(sigma.1.square/k.1))
sd.1=sqrt(1/rgamma(S,v.1/2,v.1*sigma.1.square/2))
posterior.1<- rnorm(S,mean.1,sd.1)

mean.credible.lower=quantile(mean.1,0.025)
mean.credible.higher=quantile(mean.1,0.975)
sd.credible.lower=quantile(sd.1,0.025)
sd.credible.higher=quantile(sd.1,0.975)
mean.posterior.1=1/S*sum(posterior.1)

print(mean.credible.lower)
print(mean.credible.higher)
print(sd.credible.lower)
print(sd.credible.higher)
print(mean.posterior.1)
```

```
##### (b) #####
```

```
S=100000;
data1<-read.table("http://www.stat.washington.edu/~hoff/courses/564-
2010/Data/HwData/school1.dat")
data2<-read.table("http://www.stat.washington.edu/~hoff/courses/564-
2010/Data/HwData/school2.dat")
data3<-read.table("http://www.stat.washington.edu/~hoff/courses/564-
2010/Data/HwData/school3.dat")

school1 <- data1
mu.0=5;sigma.0.square=4;k.0=1;v.0=2;
```

```

n.1=length(t(school1))
k.1=k.0+n.1
mu.1=(sum(school1)+k.0*mu.0)/k.1
v.1=v.0+n.1
sigma.1.square=1/v.1*(v.0*sigma.0.square+var(school1)*(n.1-1)+(n.1*k.0/k.1*(mean(t(school1))-mu.0)^2))

```

```

mean.1=rnorm(S,mu.1,sqrt(sigma.1.square/k.1))
sd.1=sqrt(1/rgamma(S,v.1/2,v.1*sigma.1.square/2))
posterior.1<- rnorm(S,mean.1,sd.1)

```

```

school1 <- data2
mu.0=5;sigma.0.square=4;k.0=1;v.0=2;
n.1=length(t(school1))
k.1=k.0+n.1
mu.1=(sum(school1)+k.0*mu.0)/k.1
v.1=v.0+n.1
sigma.1.square=1/v.1*(v.0*sigma.0.square+var(school1)*(n.1-1)+(n.1*k.0/k.1*(mean(t(school1))-mu.0)^2))

```

```

mean.2=rnorm(S,mu.1,sqrt(sigma.1.square/k.1))
sd.2=sqrt(1/rgamma(S,v.1/2,v.1*sigma.1.square/2))
posterior.2<- rnorm(S,mean.2,sd.2)

```

```

school1 <- data3
mu.0=5;sigma.0.square=4;k.0=1;v.0=2;
n.1=length(t(school1))
k.1=k.0+n.1
mu.1=(sum(school1)+k.0*mu.0)/k.1
v.1=v.0+n.1
sigma.1.square=1/v.1*(v.0*sigma.0.square+var(school1)*(n.1-1)+(n.1*k.0/k.1*(mean(t(school1))-mu.0)^2))

```

```

mean.3=rnorm(S,mu.1,sqrt(sigma.1.square/k.1))
sd.3=sqrt(1/rgamma(S,v.1/2,v.1*sigma.1.square/2))
posterior.3<- rnorm(S,mean.3,sd.3)

```

```

p.1=1/S*sum(as.numeric((mean.1<mean.2)&(mean.2<mean.3)))
p.2=1/S*sum(as.numeric((mean.1<mean.3)&(mean.3<mean.2)))
p.3=1/S*sum(as.numeric((mean.2<mean.1)&(mean.1<mean.3)))
p.4=1/S*sum(as.numeric((mean.2<mean.3)&(mean.3<mean.1)))
p.5=1/S*sum(as.numeric((mean.3<mean.1)&(mean.1<mean.2)))
p.6=1/S*sum(as.numeric((mean.3<mean.2)&(mean.2<mean.1)))
print(p.1);print(p.2);print(p.3);print(p.4);print(p.5);print(p.6)

```

```
##### (c) #####
```

```

p.1=1/S*sum(as.numeric((posterior.1<posterior.2)&(posterior.2<posterior.3)))
p.2=1/S*sum(as.numeric((posterior.1<posterior.3)&(posterior.3<posterior.2)))

```

```
p.3=1/S*sum(as.numeric((posterior.2<posterior.1)&(posterior.1<posterior.3)))  
p.4=1/S*sum(as.numeric((posterior.2<posterior.3)&(posterior.3<posterior.1)))  
p.5=1/S*sum(as.numeric((posterior.3<posterior.1)&(posterior.1<posterior.2)))  
p.6=1/S*sum(as.numeric((posterior.3<posterior.2)&(posterior.2<posterior.1)))
```

```
#####(d)#####
```

```
p.1=1/S*sum(as.numeric((mean.1>mean.2)&(mean.1>mean.3)))  
p.2=1/S*sum(as.numeric((posterior.1>posterior.2)&(posterior.1>posterior.3)))
```