By all accounts, Shady Jim's Used Autos has a strange commission policy. If you sell

$$y = \sum_{i=1}^{5} x_i \tag{1}$$

cars during the 5-day work week, then you receive a weekly bonus of

$$b = (2\pi y)^{\sqrt{2}} \tag{2}$$

dollars. You are a brand new employee at Shady Jim's. Your friend, Ronald Aylmer, has been working here for just two weeks and he has already sold 10 cars, for an average of 1/day. You think you are about as good a salesman as he is.

Report a good choice for a single-parameter likelihood for the number of cars you sell in one day (x_i), and an informative conjugate prior for the parameter. As there are infinitely many priors that meet this requirement I'll help you narrow the choice to just one by requiring that the variance of the prior distribution be 1.

After one week, you sell 4 cars. Report the posterior distribution of your parameter.

Use the posterior predictive distribution to calculate (or simulate, either is OK) the probability that you will match or exceed Ronald Aylmer's 2-week performance. Report this probability (rounded to nearest percent). Assume that your ability to sell cars (and the customers' willingness to buy them) remains constant throughout your employment.

What is the probability that your bonus after your second week of work, b_2, will exceed \$100, given that you sold 4 cars in week 1? Report this to the nearest percent. Plot the distribution of b_2 as a histogram, not a density.

Hint for this lab: Sums of Poisson random variables are Poisson-distributed.