Ben Radford **Stat 601 - Lab Homework 1** **January 18, 2013**

```
test <- function(N,n)
{
  draws <- rbinom(N, n, 0.8)
  phat <- draws/n
  se <- sqrt(phat*(1-phat)/n)
  fCI.upper <- phat + 1.96*se
  fCI.lower <- phat - 1.96*se
  fi.length <- mean(fCI.upper-fCI.lower)
  fi.covprop <- sum(as.numeric(0.8>=fCI.lower & 0.8<=fCI.upper))/N
  b1.lower <- qbeta(.025,1+draws,1+n-draws)
  b1.upper <- qbeta(0.975,1+draws,1+n-draws)
  b1.covprop <- sum(as.numeric(0.8>=b1.lower & 0.8<=b1.upper))/N
  b2.lower <- qbeta(.025,8+draws,2+n-draws)
  b2.upper <- qbeta(.975,8+draws,2+n-draws)
  b2.covprop <- sum(as.numeric(0.8>=b2.lower & 0.8<=b2.upper))/N
  print(paste("Average Interval Length, Freq:",fi.length))
  print(paste("Average Interval Length, Bayes 1: ",mean(b1.upper-b1.lower)))
  print(paste("Average Interval Length, Bayes 2:",mean(b2.upper-b2.lower)))
  print(paste("Coverage Proportion, Freq:",fi.covprop))
  print(paste("Coverage Proportion, Bayes 1:",b1.covprop))
  print(paste("Coverage Proportion, Bayes 2:",b2.covprop))
}
test(10000,30)
test(10000,5)
```

Method	Prior	N	n	Avg. Interval Length	Coverage Proportion
Frequentist	NA	10,000	30	0.278	0.9473
Bayesian	$\beta(1,1)$	10,000	30	0.272	0.968
Bayesian	$\beta(8,2)$	10,000	30	0.238	0.982
Frequentist	NA	10,000	5	0.508	0.663
Bayesian	$\beta(1,1)$	10,000	5	0.566	0.944
Bayesian	$\beta(8,2)$	10,000	5	0.374	$\sim$1.000

At high n (n=30), Bayesian and frequentist methods both perform well. The informed prior helps by somewhat reducing the average interval length and slightly increasing the coverage when compared to both the uninformed prior and the frequentist approach. The naive Bayesian approach (prior = $\beta(1,1)$) just barely outperforms the frequentist approach at n=30.

At low n (n=5), Bayesian methods also outperform frequentist methods with one caveat. While the coverage proportion of the uninformed Bayesian prior is greater than the frequentist coverage, the average interval length derived from the uninformed prior is a bit larger than that from the frequentist approach. The informed prior results in greatly improved coverage and reduced average interval length. Larger n generally performs better than lower n.