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Signals - Part II

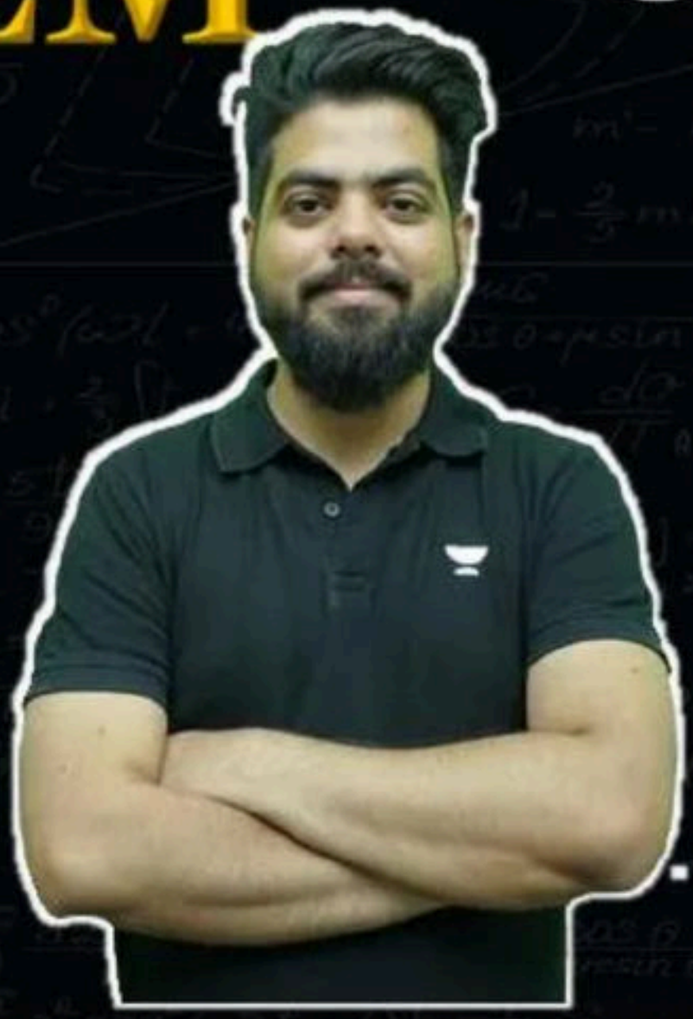
Comprehensive Course on SIGNAL SYSTEM ECE/EE/IN

Vishal Soni • Lesson 2 • Feb 22, 2025

SIGNAL SYSTEM

Basics of Signals

Lecture-02



Vishal Soni Sir
(ESE AIR 25)

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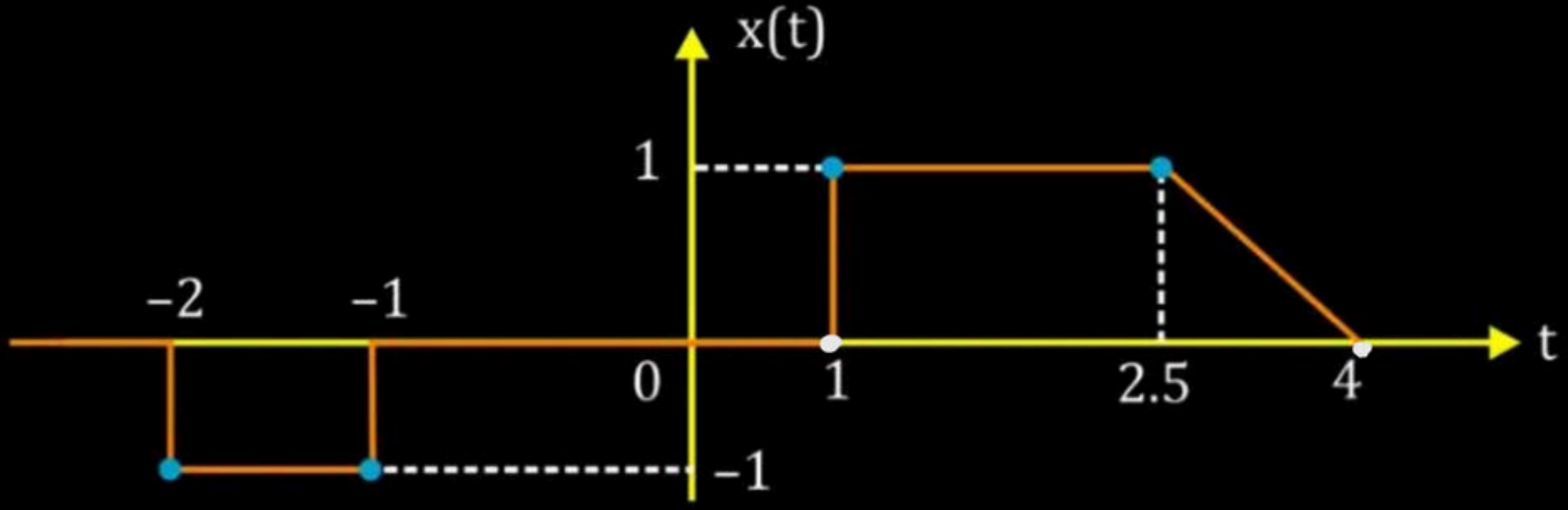
Q. PLOT. $x\left(\frac{1-2t}{3}\right) = ?$



$-2 - 1/3 = -7/3$

$1 - 1/3 = 2/3$

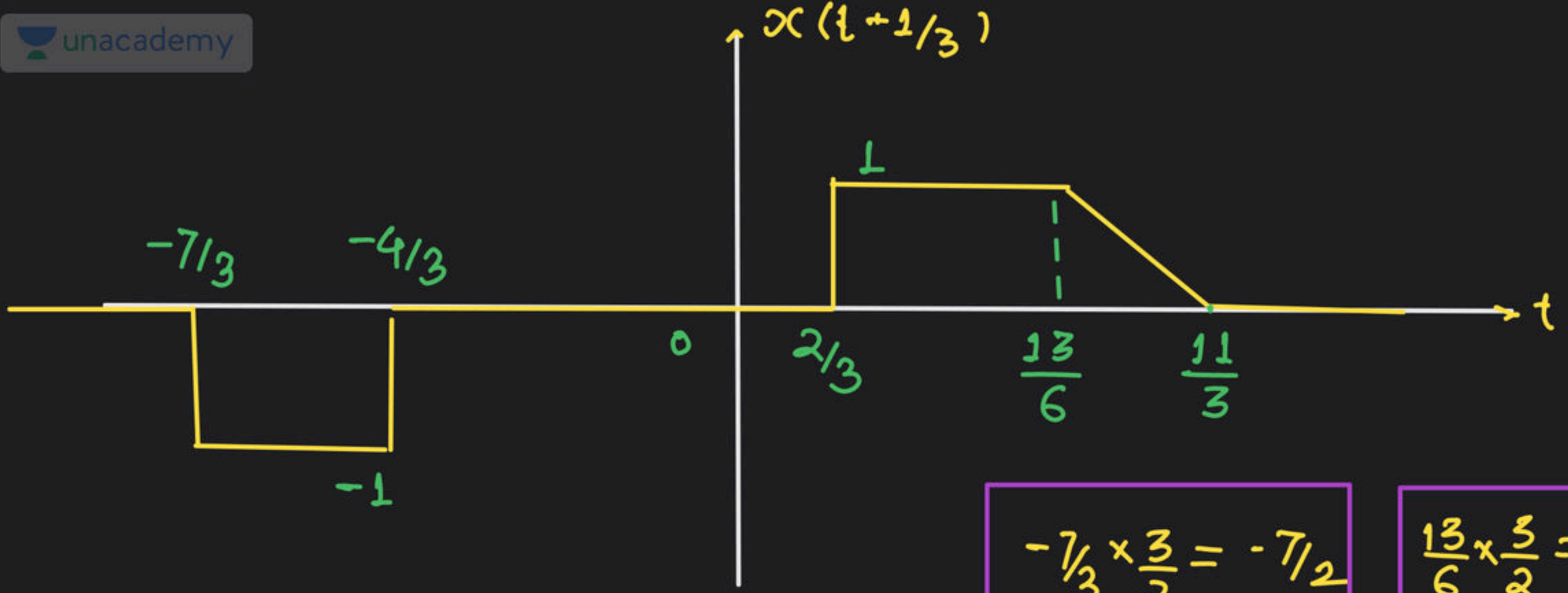
$4 - 1/3 = 11/3$

$-1 - 1/3 = -4/3$

$\frac{5}{2} - \frac{1}{3} = \frac{13}{6}$

$$x\left(\frac{1-2t}{3}\right) = x\left(\frac{-2t+1}{3}\right) = x\left(\frac{-2}{3}t + \frac{1}{3}\right)$$

$x(t) \xrightarrow{\text{SHIFT}} x\left(t+\frac{1}{3}\right) \xrightarrow{\text{SCALE}} x\left(\frac{2t}{3}+\frac{1}{3}\right) \xrightarrow{\text{REVERSAL}} x\left(\frac{-2t}{3}+\frac{1}{3}\right)$



Horizontal axis → multiply $3/2$

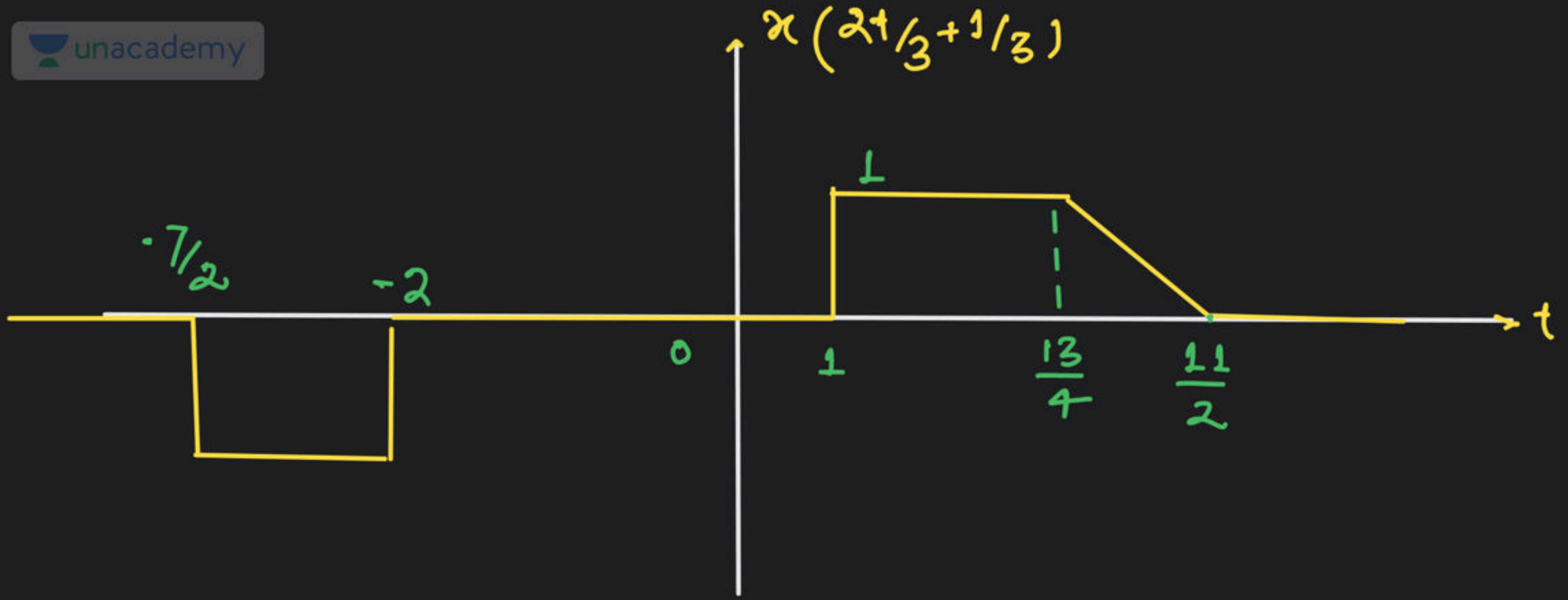
$-7/3 \times \frac{3}{2} = -7/2$

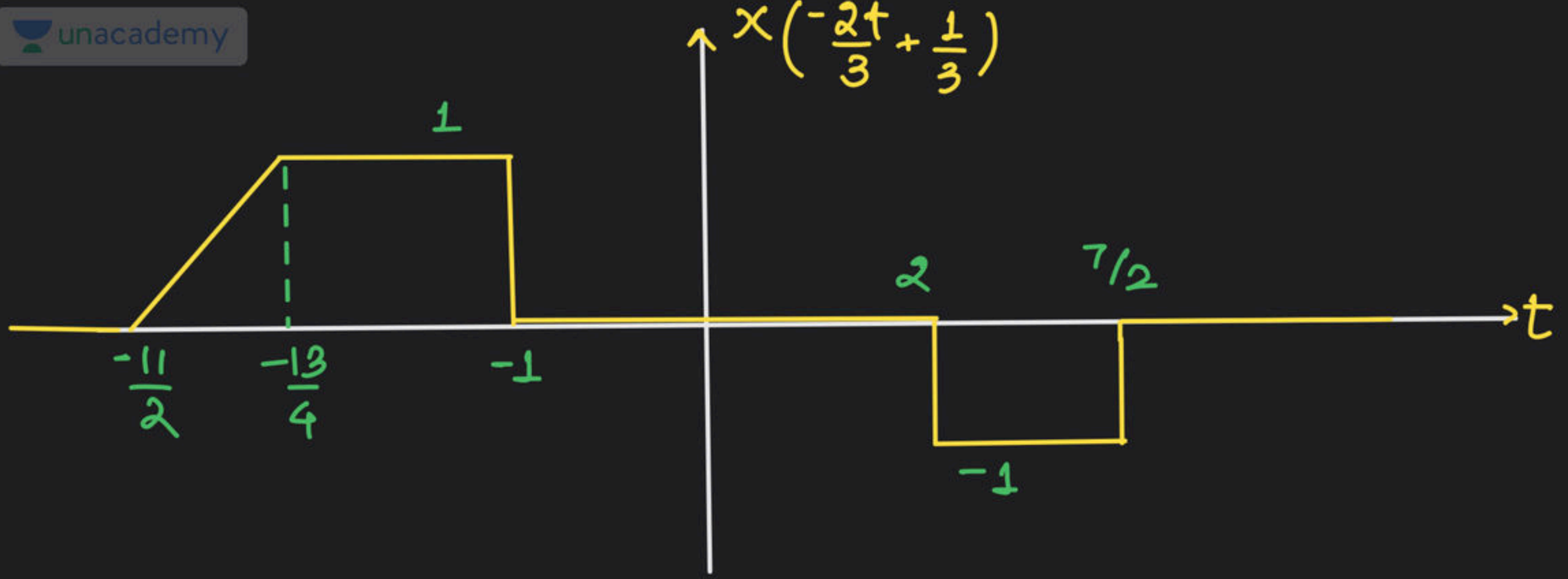
$\frac{-4}{3} \times \frac{3}{2} = -2$

$2/3 \times 3/2 = 1$

$\frac{13}{6} \times \frac{3}{2} = \frac{13}{4}$

$\frac{11}{3} \times \frac{3}{2} = \frac{11}{2}$


x(2t/3+1/3)
1
-7/2
-2
0
1
13/4
11/2
t


x(-2t/3 + 1/3)
1
-11/2
-13/4
-1
2
7/2
-1
t

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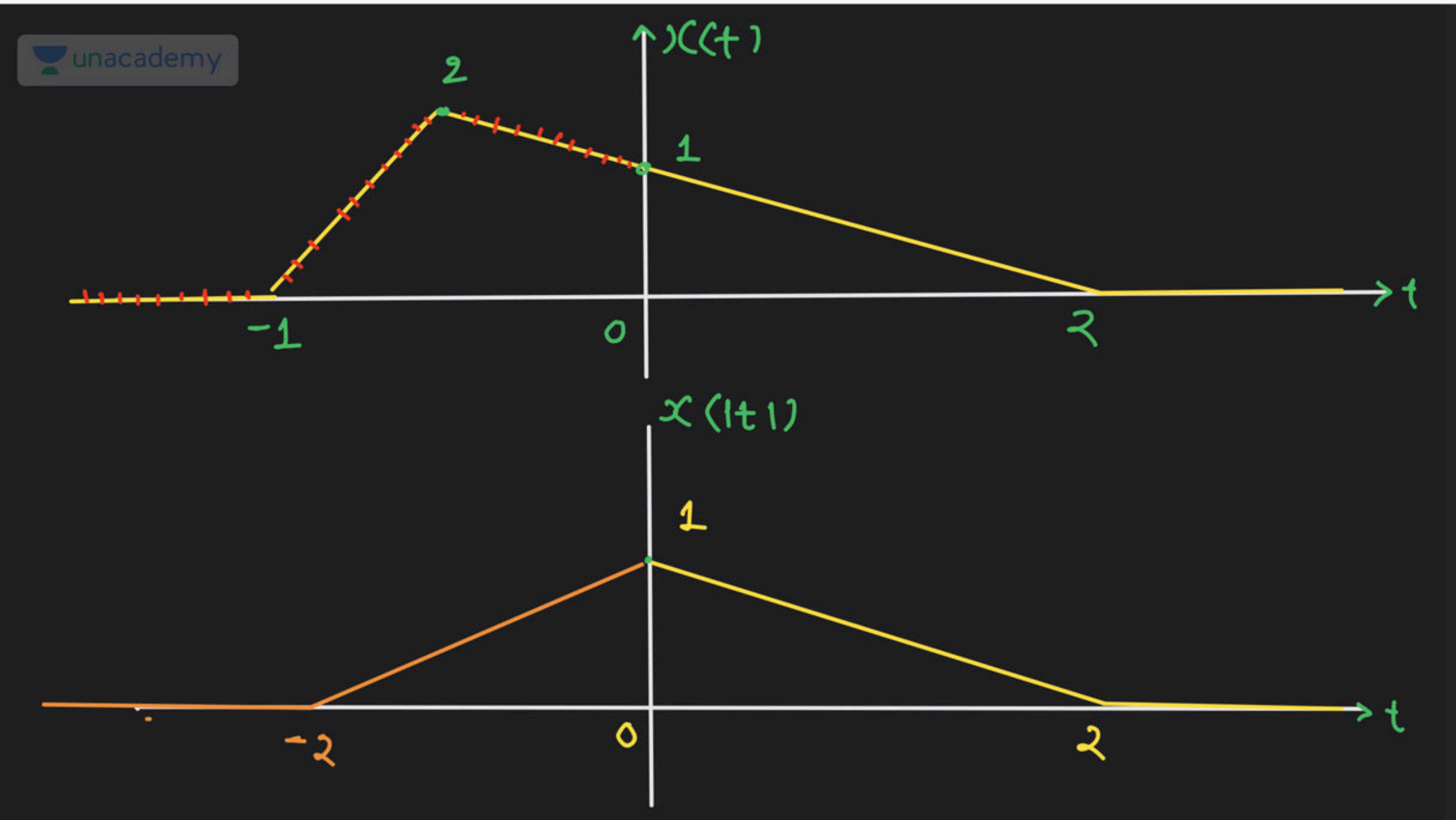
4. Given x(t) vs t

Plot x(|t|) vs t

PROCESS

- Remove left Half of GRAPH x(t)
- Take mirror image of Right Half of GRAPH x(t) on to left Half side w.r.t. vertical axis.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x(t)
2
1
-1
0
2
t
x(1+t)
1
-2
0
2
t

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Important Observation

1. Argument: $x(-2t+3)$

$ARG = -2t+3$

2. Break Point: $ARG = 0$

$-2t+3=0$

$t = 3/2$ → B.P.

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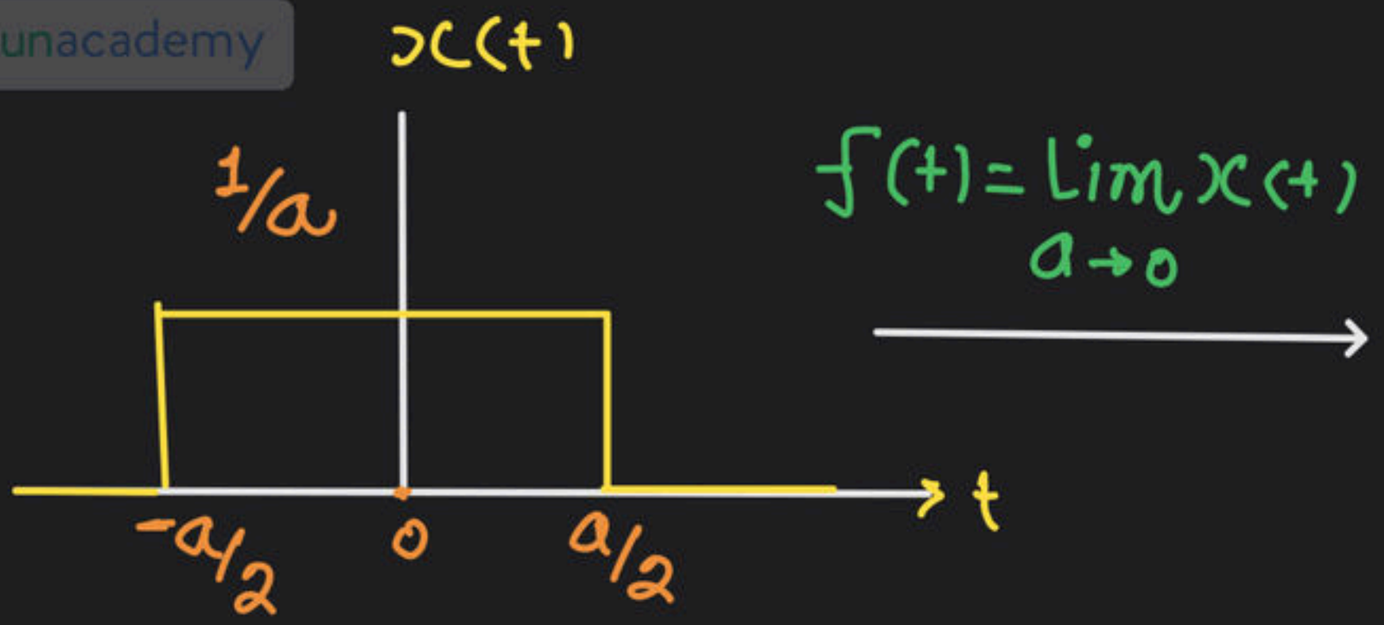
3. Approximation of signal

$$\lim_{t \to a} x(t) = f(t)$$

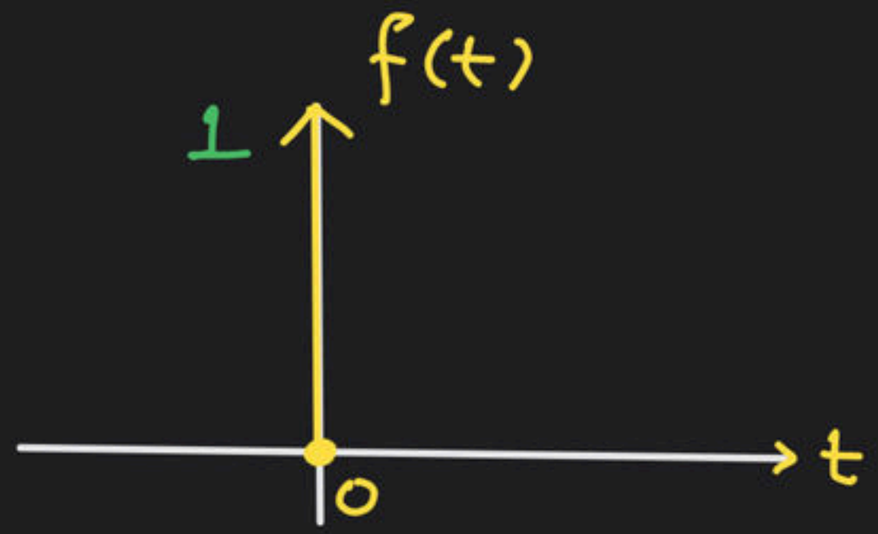
$$\int_{-\infty}^{\infty} x(t)\,dt = A_1$$

$$\int_{-\infty}^{\infty} f(t)\,dt = A_2$$

$\rightarrow A_1 = A_2$

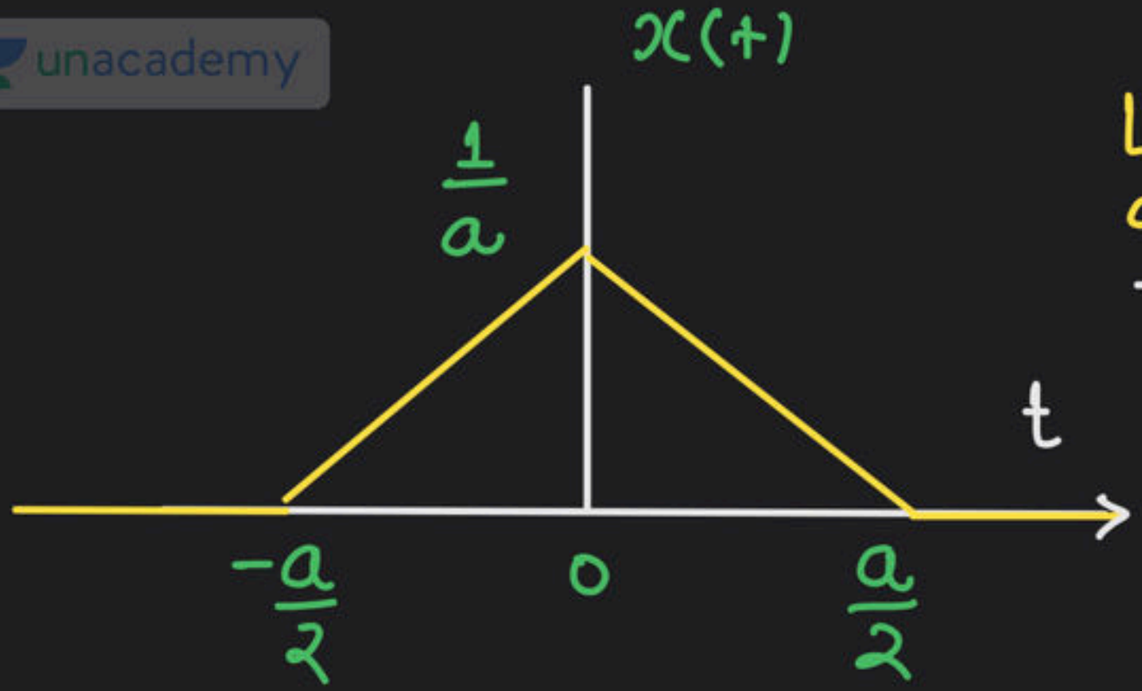


$$f(t) = \lim_{a \to 0} x(t)$$

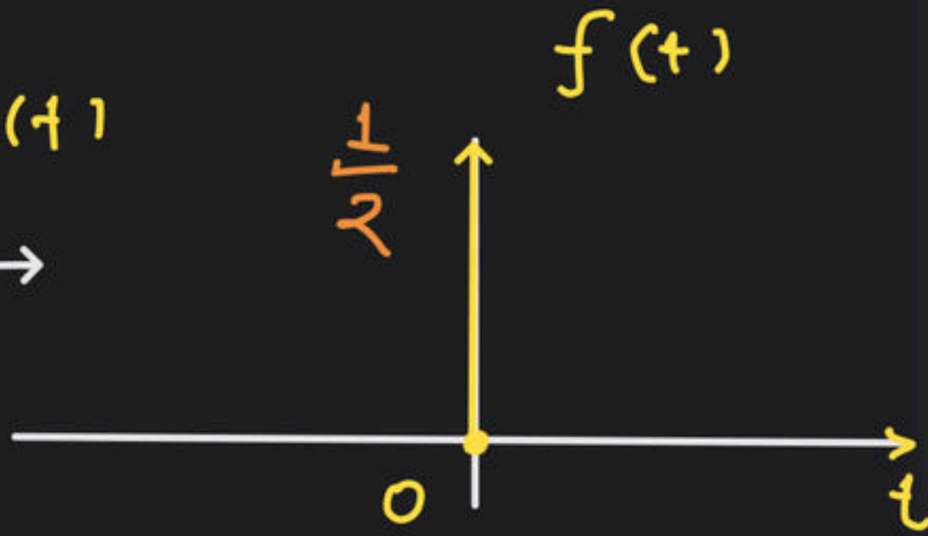


$$\int_{-\infty}^{\infty} x(t)\,dt = \text{AREA} = 1$$

$$\int_{-\infty}^{\infty} f(t)\,dt = \text{AREA} = 1$$



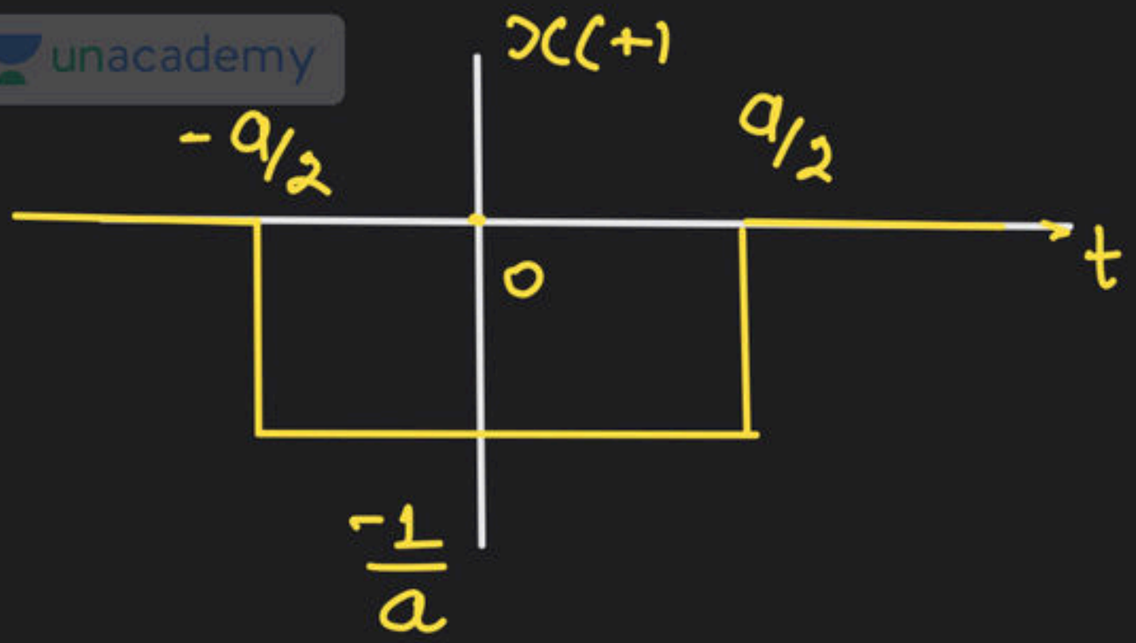
$$\lim_{a \to 0} x(t) = f(t)$$



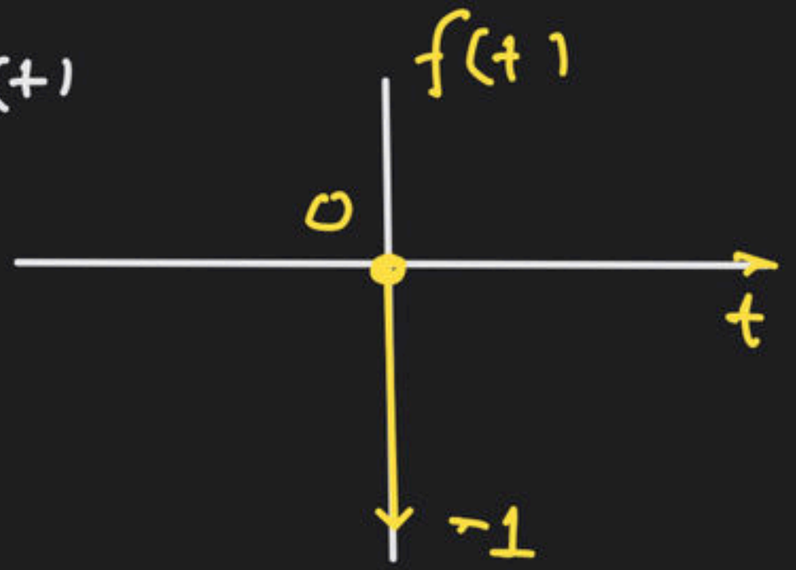
$$\int_{-\infty}^{\infty} x(t)\,dt = \frac{1}{2} \times a \times \frac{1}{a}$$

$$= 1/2$$

$$\int_{-\infty}^{\infty} f(t)\,dt = 1/2$$



$$\lim_{a \to 0} x(t) = f(t)$$



$$\int_{-\infty}^{\infty} x(t)\,dt = -1$$

$$\int_{-\infty}^{\infty} f(t)\,dt = -1$$

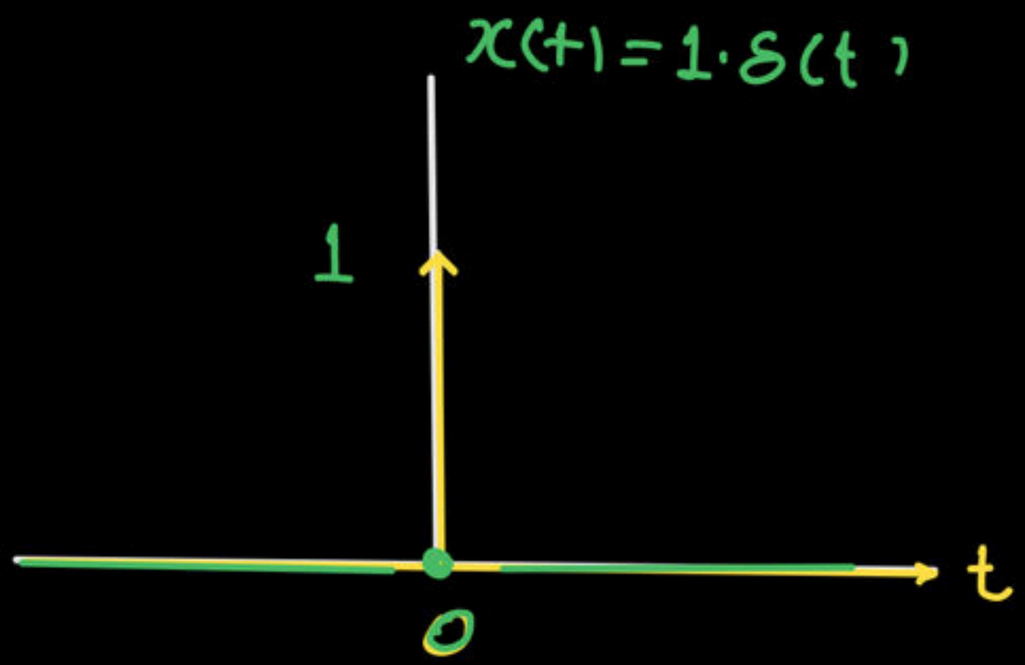
STANDARD C.T.S.

1. UNIT IMPULSE SIGNAL:

Mathematical Expression

$$x(t) = 1 \cdot \delta(t)$$

GRAPH



→ Impulse signal is approximation of certain parent signal

→ Parent signal is not unique

Mathematical Definition

$$\delta(t) = 0 \quad : t \neq 0$$

$$\delta(t) \to \infty \quad : t = 0$$

$$\int_{-\infty}^{\infty} \delta(t)\,dt = \int_{0^-}^{0^+} \delta(t)\,dt = 1$$

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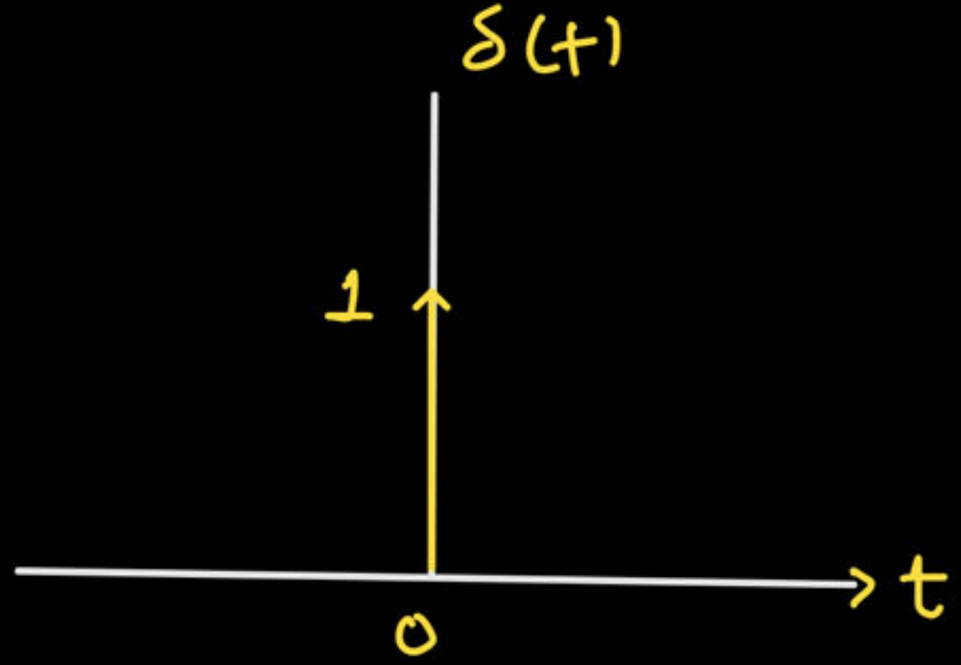
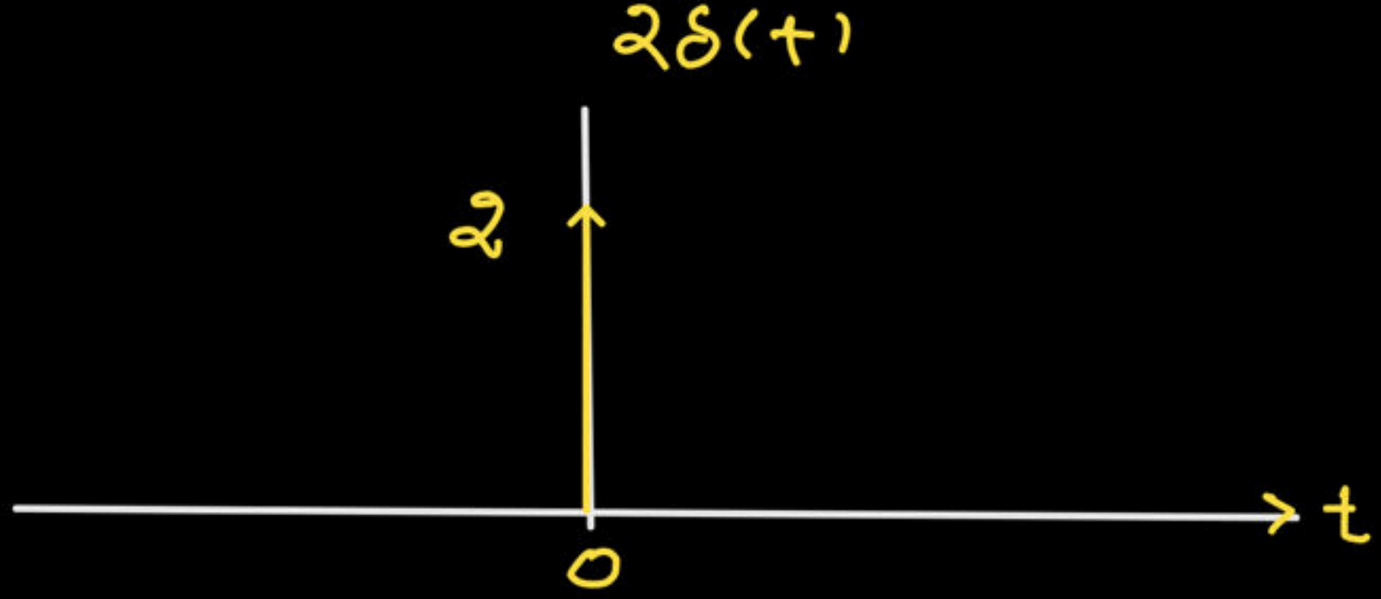
Basic Operations

$\delta(t) \longrightarrow$



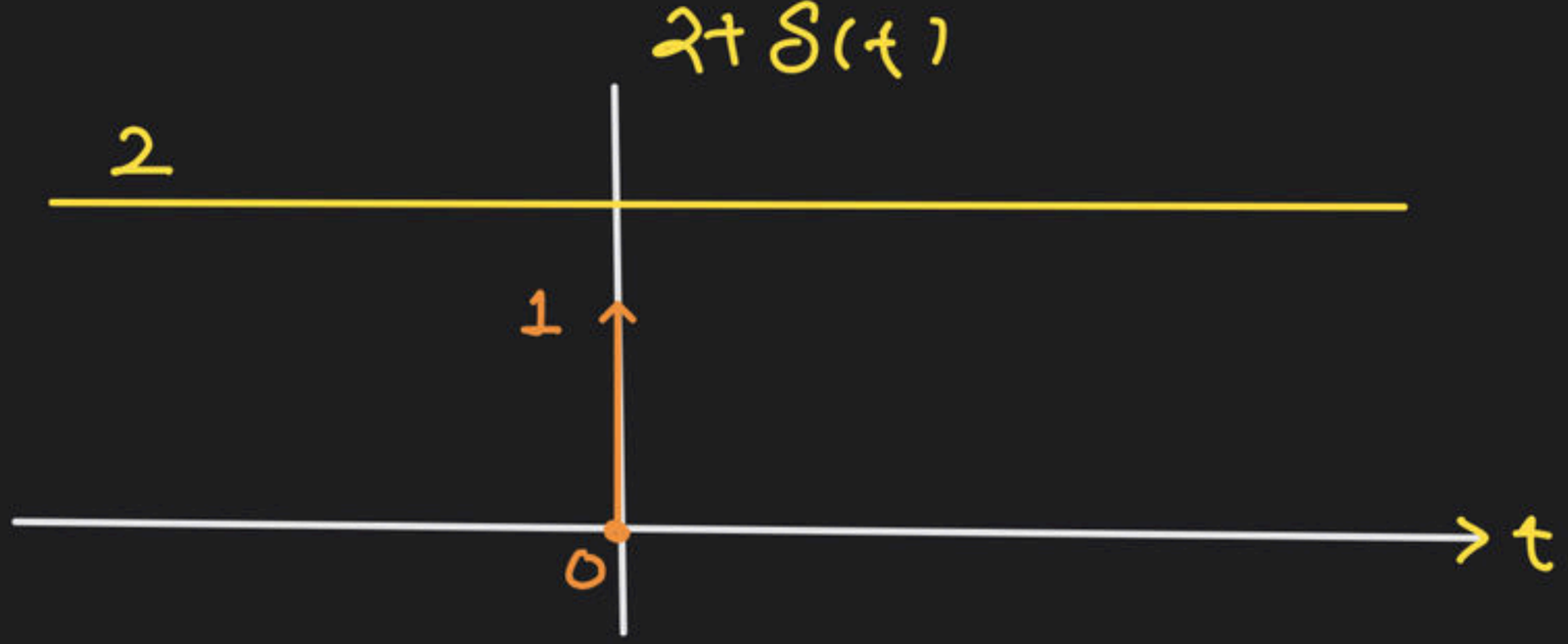
1 $2\delta(t) \longrightarrow$



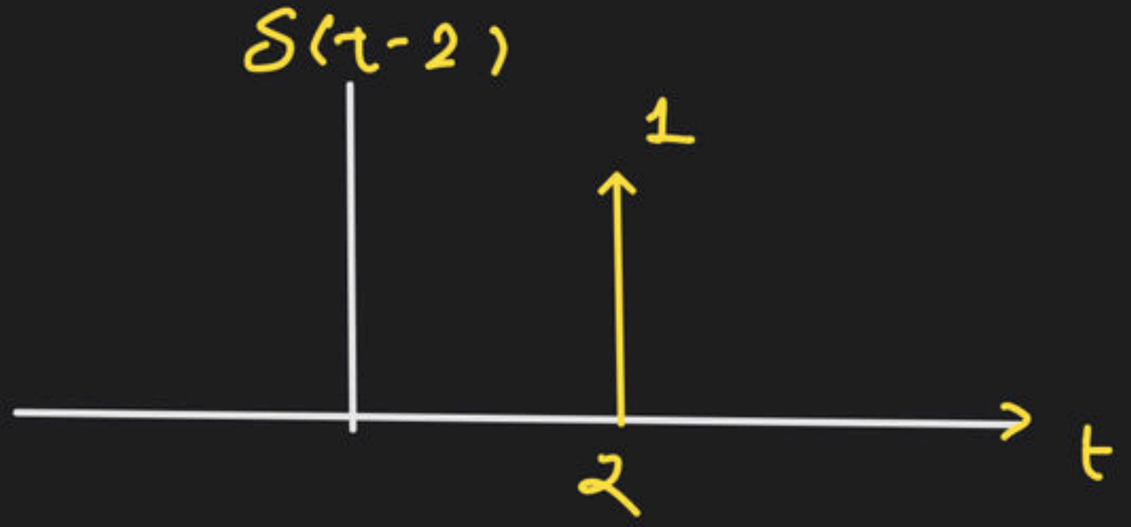
2. $-\delta(t)$ ⟶

**

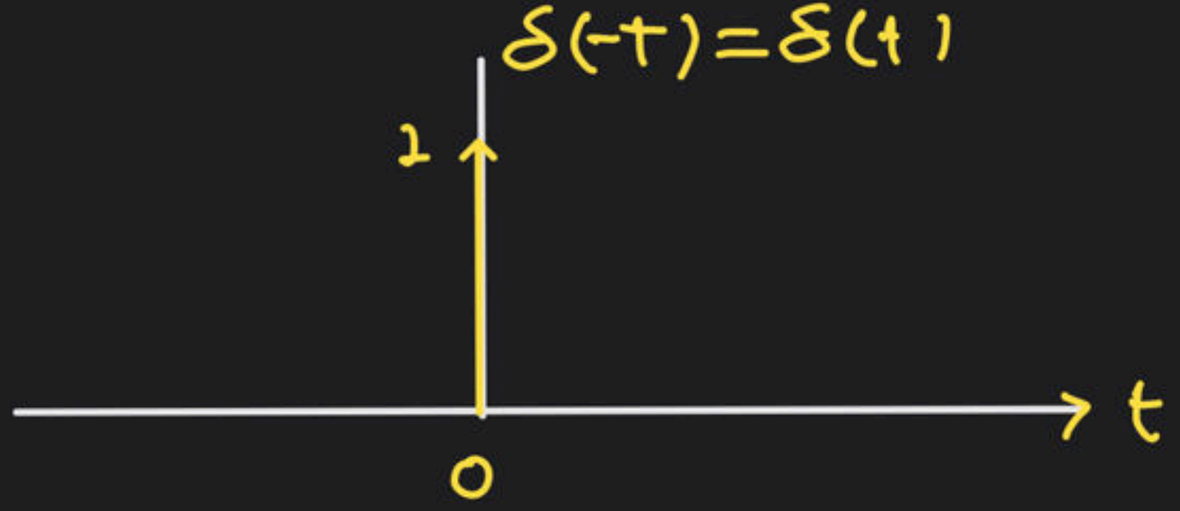
3. $2+\delta(t)$ ⟶



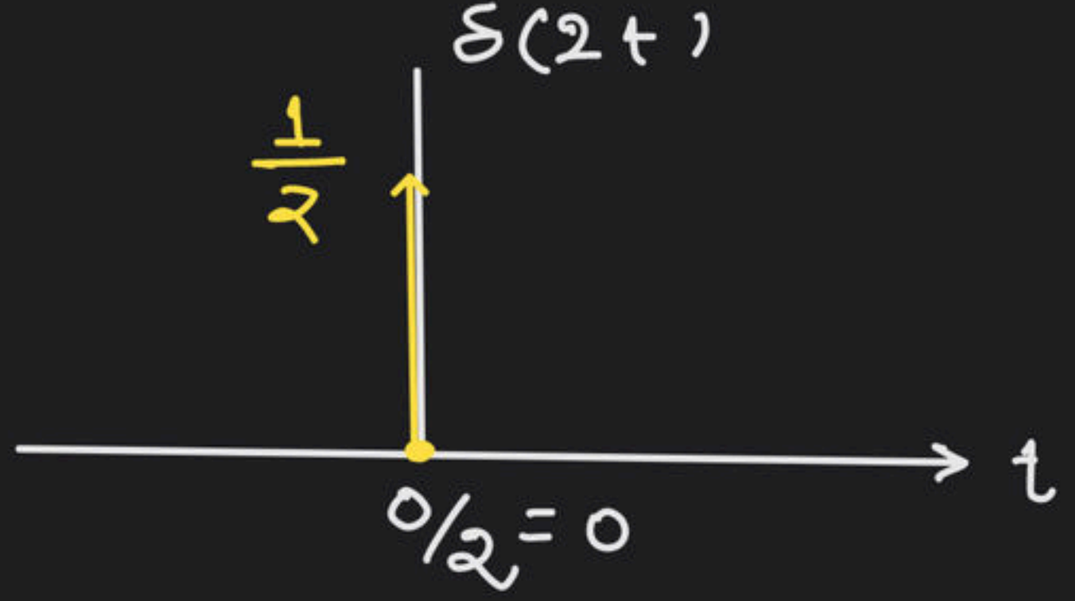
4 $\delta(t-2)$ ⟶



5 $\delta(-t)$ ⟶



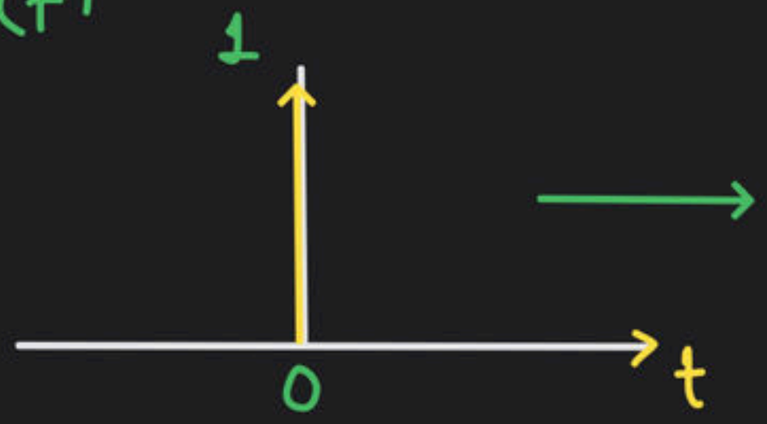
6 $\delta(2t)$ ⟶



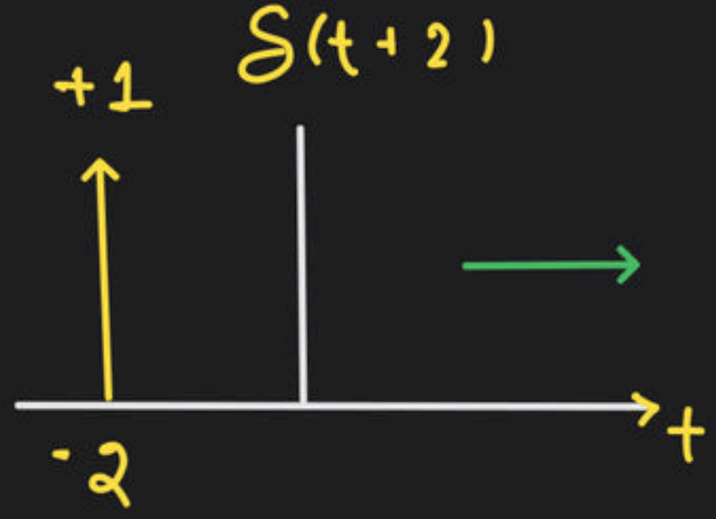
$$\delta(2t) = \delta(2(t)) = \frac{1}{2}\delta(t)$$

7 $\delta(-3t+2)$

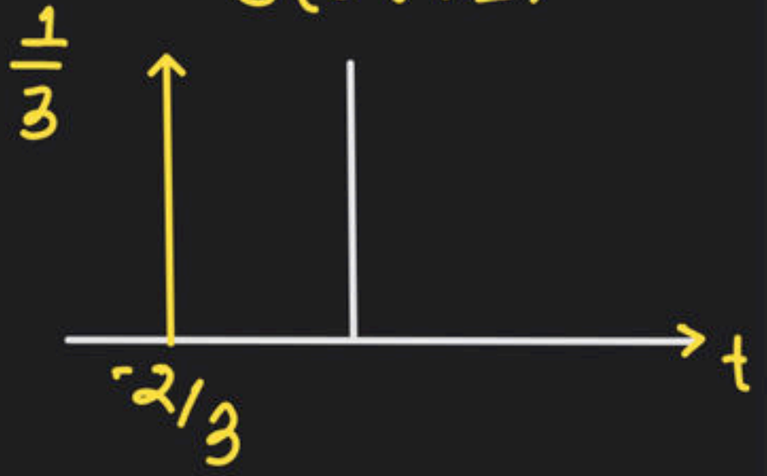
$\delta(t)$



$\delta(t+2)$



$\delta(3t+2)$



$\delta(-3t+2)$

$\frac{1}{3}\delta(t-\frac{2}{3})$

$$\delta(-3t+2) = \delta\left(-3\left(t-\frac{2}{3}\right)\right) = \frac{1}{|-3|}\delta(t-2/3)$$

$$= \frac{1}{3}\delta(t-2/3)$$

**

PROPERTIES

1. Impulse Signal at $t=0$ is EVEN SIGNAL.

$$\delta(-t) = \delta(t)$$

2. $$\delta(at) = \frac{1}{|a|}\delta(t)$$

3. $$\delta(-at) = \frac{1}{|-a|}\delta(t)$$

4 $\delta(at+b) = \delta\left(a\left(t+\frac{b}{a}\right)\right) = \frac{1}{|a|}\delta\left(t+\frac{b}{a}\right)$

5 $\delta(-at+b) = \delta\left(-a\left(t-\frac{b}{a}\right)\right) = \frac{1}{|-a|}\delta\left(t-\frac{b}{a}\right)$

6 $\delta(-at-b) = \delta\left(-a\left(t+\frac{b}{a}\right)\right) = \frac{1}{|-a|}\delta\left(t+\frac{b}{a}\right)$

7 $\delta(at-b) = \delta\left(a\left(t-\frac{b}{a}\right)\right) = \frac{1}{|a|}\delta\left(t-\frac{b}{a}\right)$

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Q. $\delta(-t-1)$

$$= \delta(-(t+1)) = \frac{1}{|-1|}\,\delta(t+1) = \delta(t+1)$$

Q. $\delta\left(\frac{1-2t}{3}\right)$

$$\delta\left(\frac{-2t}{3}+\frac{1}{3}\right) = \delta\left(\frac{-2}{3}\left(t-\frac{1}{2}\right)\right)$$

$$= \frac{1}{\left|\frac{-2}{3}\right|}\,\delta\left(t-\frac{1}{2}\right) = \frac{3}{2}\,\delta\left(t-\frac{1}{2}\right)$$

8 $\delta[g(t)]$

S·1 SOLVE : $g(t)=0$ & CALCULATE ROOTS $t=t_1, t_2, t_3$

S·2 $g'(t) \longrightarrow g'(t_1)\,,\ g'(t_2)\,,\ g'(t_3)$

S·3
$$\delta[g(t)] = \frac{\delta(t-t_1)}{|g'(t_1)|} + \frac{\delta(t-t_2)}{|g'(t_2)|} + \frac{\delta(t-t_3)}{|g'(t_3)|}$$

$$\delta[g(t)] = \frac{\sum_i \delta(t - t_i)}{|g'(t_i)|}$$

Q. $\delta(t^2-3t+2) = \delta[g(t)]$

S·1 $g(t) = t^2-3t+2 = 0 \longrightarrow t=1,\ t=2$

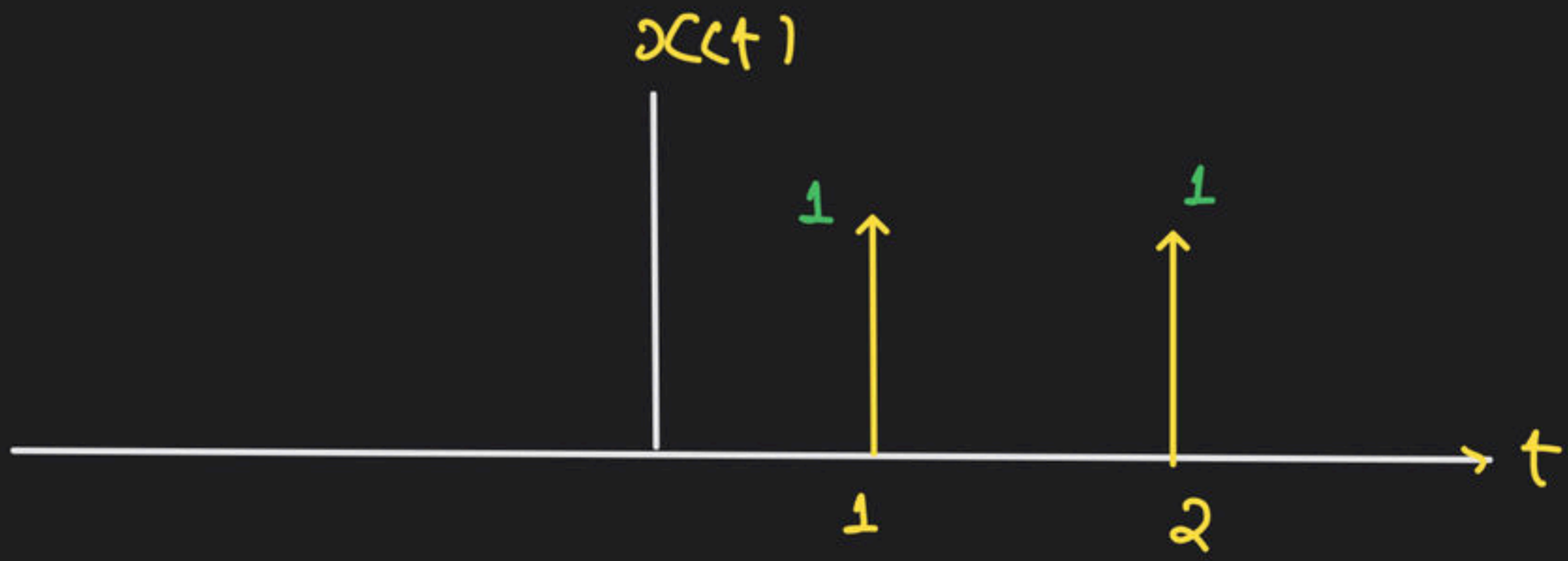
S·2 $g'(t) = 2t-3$

$\rightarrow g'(t)|_{t=1} = -1$

$\rightarrow g'(t)|_{t=2} = 1$

S·3 $\delta(t^2-3t+2) = \dfrac{\delta(t-1)}{|g'(1)|} + \dfrac{\delta(t-2)}{|g'(2)|} = \delta(t-1)+\delta(t-2)$

$x(t) = \delta(t^2 - 3t + 2)$



Q. $\delta[\sin t] = \delta[g(t)]$

S·1 $g(t) = 0 \longrightarrow \sin t = 0 \longrightarrow t = n\pi$

$t = 0,\ t = -\pi,\ t = \pi$

$t = -2\pi,\ t = 2\pi$

S·2 $g'(t) = \cos t \longrightarrow g'(n\pi) = \cos n\pi = (-1)^n$

$g'(0) = \cos 0 = 1$

$g'(-\pi) = \cos(-\pi) = -1$

$g'(\pi) = \cos \pi = -1$

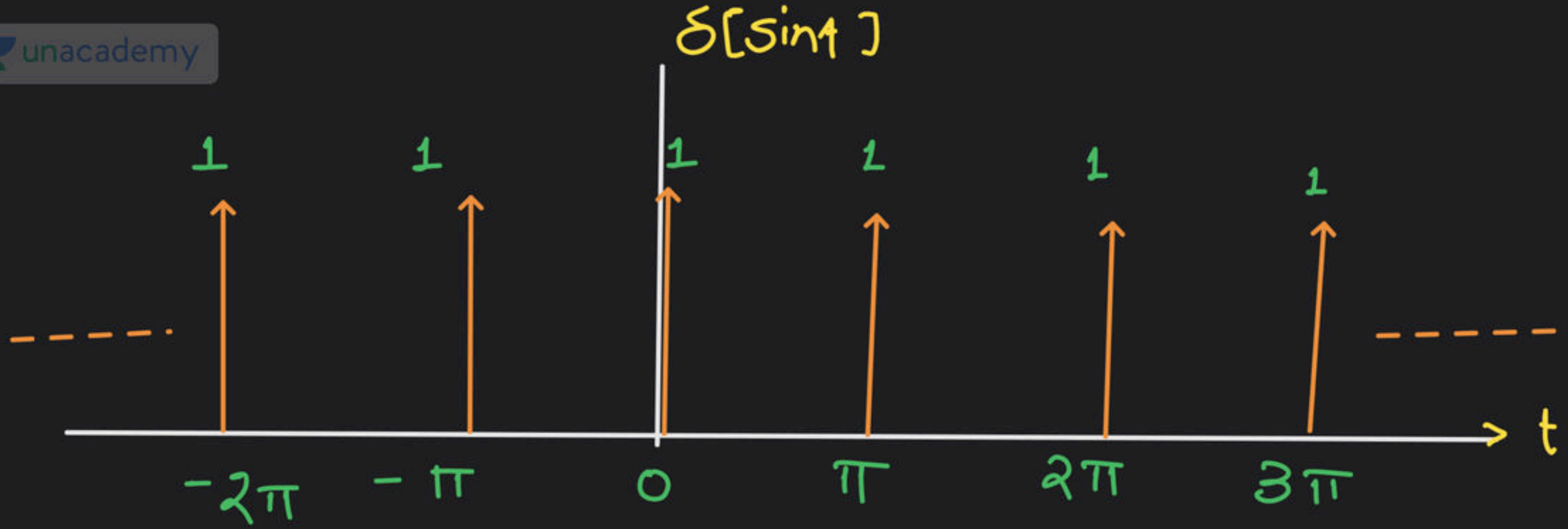
$g'(2\pi) = \cos 2\pi = 1$

S.3 $\delta[\sin t] =$

$$+ \cdots + \frac{\delta(t+2\pi)}{|g'(-2\pi)|} + \frac{\delta(t+\pi)}{|g'(-\pi)|} + \frac{\delta(t-0)}{|g'(0)|}$$

$$+ \frac{\delta(t-\pi)}{|g'(\pi)|} + \frac{\delta(t-2\pi)}{|g'(2\pi)|} + \cdots$$

$$\delta[\sin t] = \sum_{K \in I} \delta(t - \pi K)$$



PERIODIC Impulse Train

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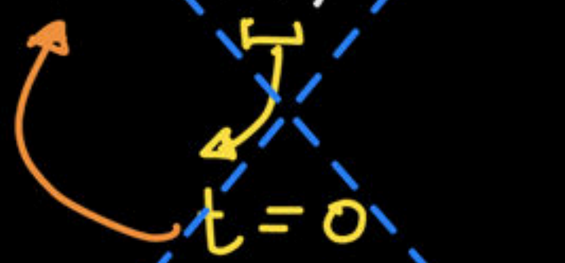
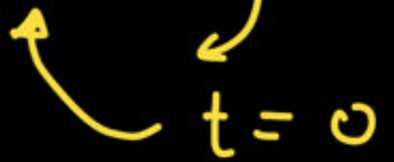
Q. $\delta[\cos t] = ?$

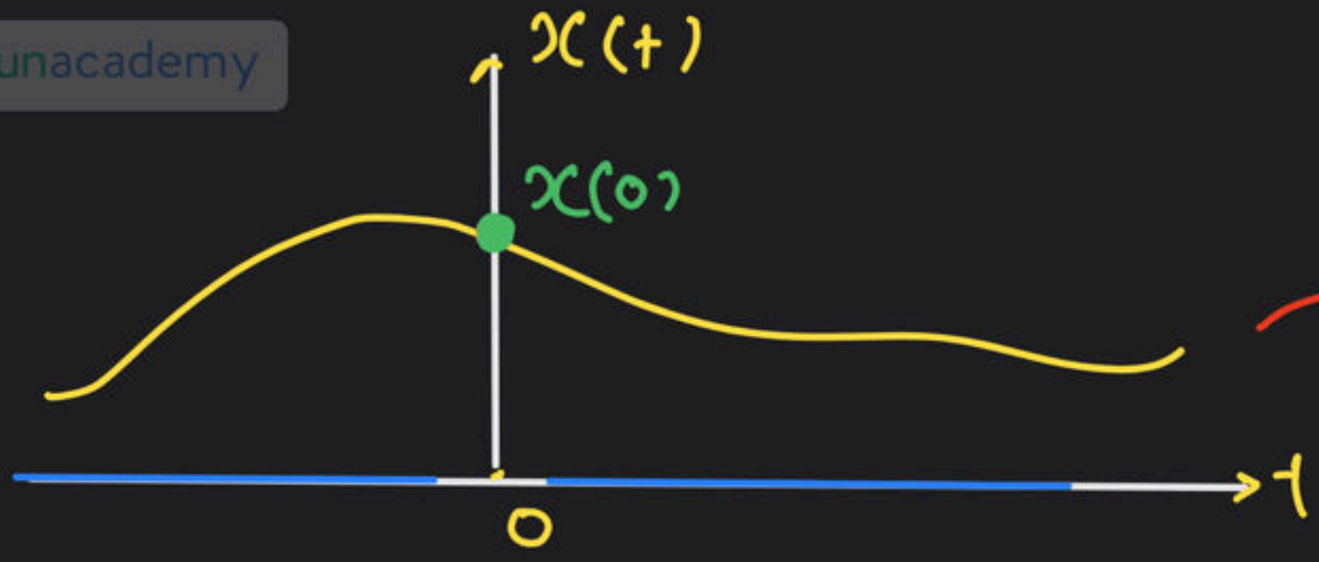
Sampling or Multiplication Property:

$$x(t)\delta(t) = x(0)$$

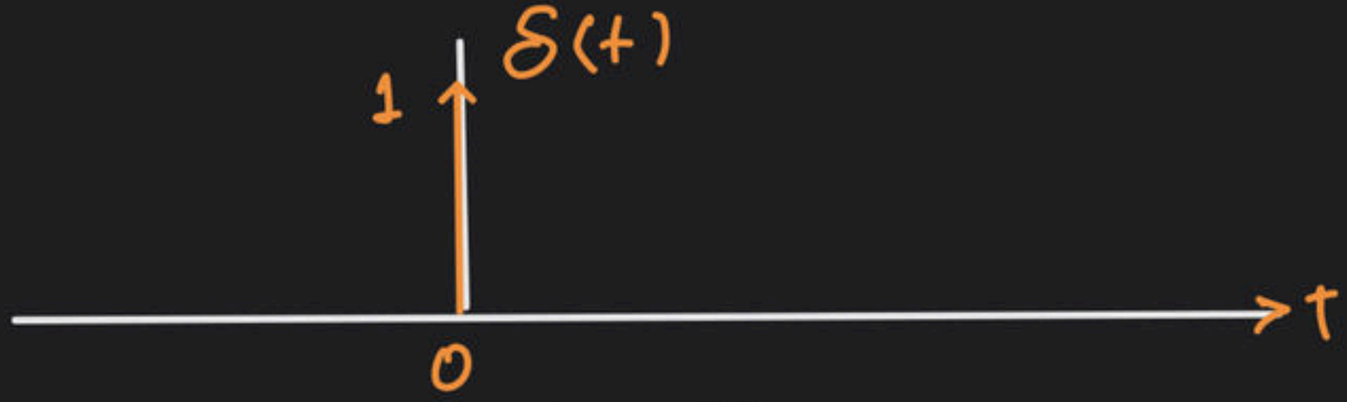


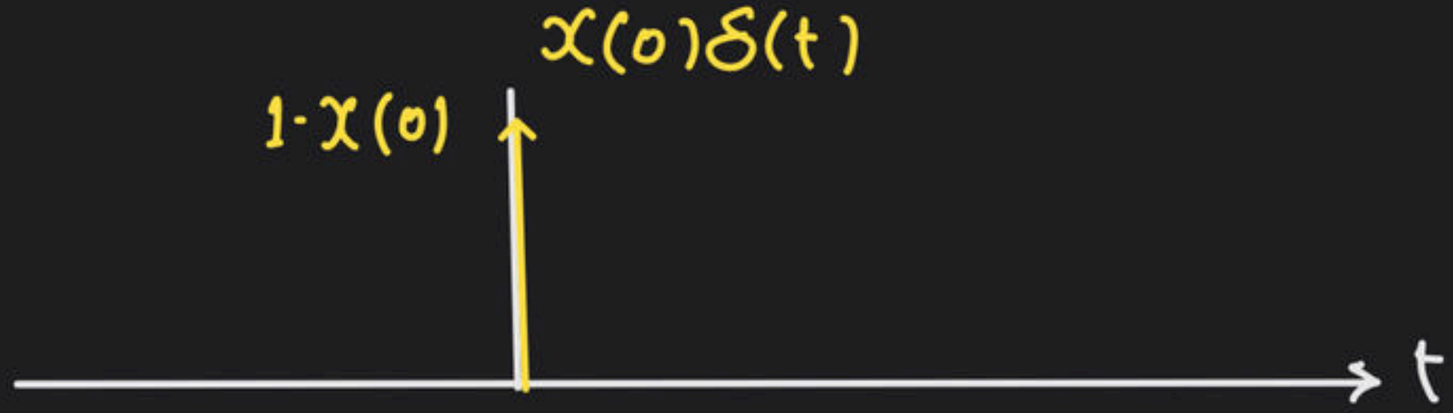
$$x(t)\delta(t) = x(0)\delta(t)$$





$$x(t)\delta(t) = x(0)\delta(t)$$





$x(t)\,\delta(-at+b)$

$x(t)\,\delta\left(-a\left(t-\frac{b}{a}\right)\right)$

$$\frac{1}{|-a|}\,x(t)\,\delta\left(t-\frac{b}{a}\right) = \frac{1}{|-a|}\,x\left(\frac{b}{a}\right)\delta\left(t-\frac{b}{a}\right)$$

$t=\frac{b}{a}$

Q. $x(t)\delta(2t)$

$$x(t)\frac{1}{2}\delta(t) = \frac{1}{2}x(t)\delta(t) = \frac{1}{2}x(0)\delta(t)$$

$t = 0$

Q. $x(t)\delta(-2t+3)$

$$x(t)\,\delta\left(-2\left(t-\frac{3}{2}\right)\right)$$

$$\frac{1}{|-2|}\,x(t)\,\delta\left(t-\frac{3}{2}\right) = \frac{1}{2}x\left(\frac{3}{2}\right)\delta\left(t-\frac{3}{2}\right)$$

$t = 3/2$

Q. $x(t) = \sin t\, \delta(3t - \pi)$

$$x(t) = \sin t\, \delta\left(3\left(t - \frac{\pi}{3}\right)\right)$$

$$x(t) = \frac{1}{3} \sin t\, \delta\left(t - \frac{\pi}{3}\right) = \frac{1}{3} \sin\frac{\pi}{3}\, \delta(t - \pi/3)$$

$t = \pi/3$

$$= \frac{1}{2\sqrt{3}}\, \delta\left(t - \frac{\pi}{3}\right)$$

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Q. $x(t) = e^{-2t} \sin t \, \delta(-2t + \pi)$

$$x(t) = e^{-2t} \sin t \, \delta(-2(t - \pi/2))$$

$$x(t) = \frac{1}{2} e^{-2t} \sin t \, \delta(t - \pi/2)$$

$t = \pi/2$

$$x(t) = \frac{1}{2} e^{-2\pi/2} \sin\left(\frac{\pi}{2}\right) \delta(t - \pi/2)$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$e^{-j\theta} = \cos\theta - j\sin\theta$$

$$e^{-j\pi/2} = \cos\pi/2 - j\sin\frac{\pi}{2} = -j$$

$$x(t) = \frac{1}{2} e^{-j\frac{\pi}{2}} \sin\left(\frac{\pi}{2}\right) \delta(t - \pi/2)$$

$$x(t) = \frac{-j}{2} \delta\left(t - \frac{\pi}{2}\right)$$

INTEGRATION PROPERTY:

$$I = \int_{t_1}^{t_2} x(t)\,\delta(-at+b)\,dt$$

$$I = \int_{t_1}^{t_2} x(t)\,\delta\left(-a\left(t-\frac{b}{a}\right)\right)dt$$

$$I = \frac{1}{|1-a|} \int_{t_1}^{t_2} x(t)\, \delta\left(t - \frac{b}{a}\right) dt$$

$t = b/a$

$$I = \frac{1}{|1-a|} \int_{t_1}^{t_2} x\left(\frac{b}{a}\right) \delta\left(t - \frac{b}{a}\right) dt$$

$$I = \frac{1}{|1-a|}\, x\left(\frac{b}{a}\right) \int_{t_1}^{t_2} \delta\left(t - \frac{b}{a}\right) dt$$

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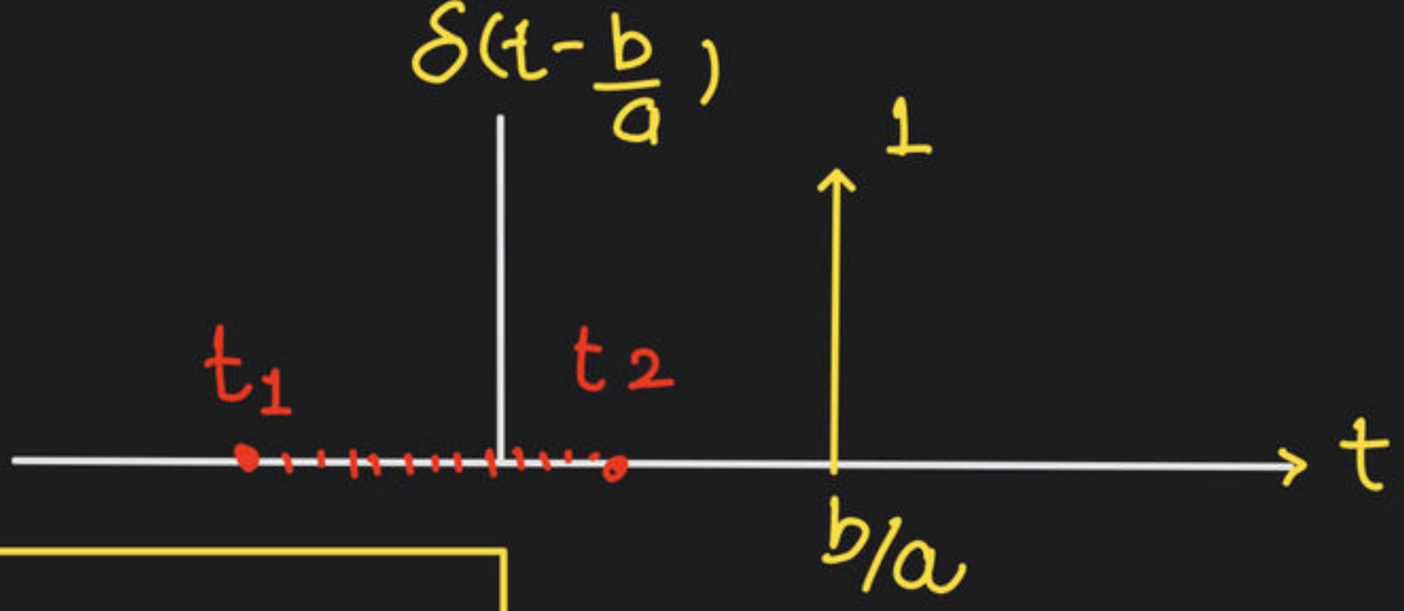
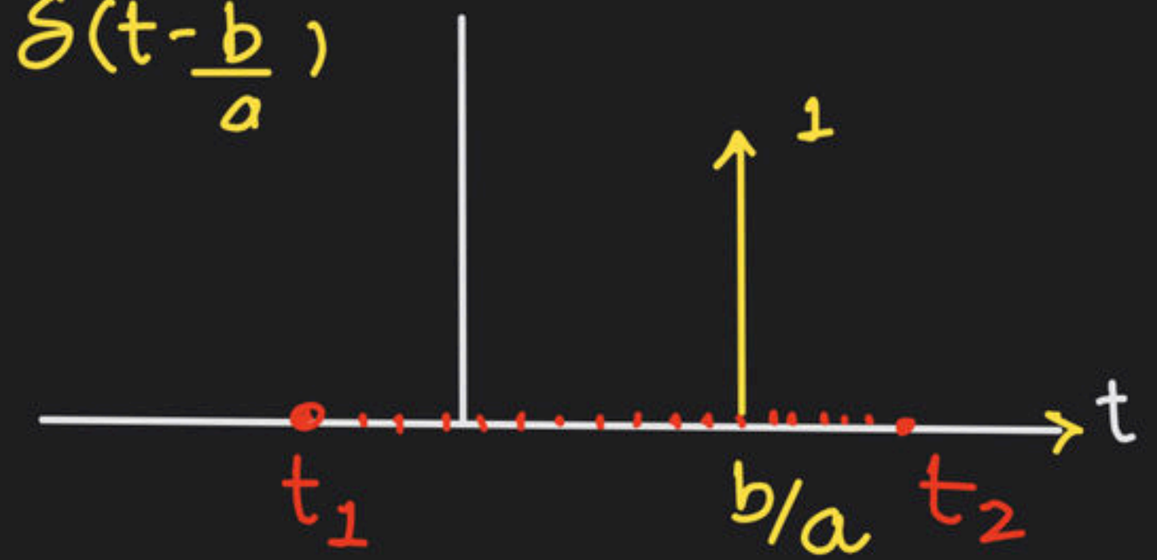
$$I = \frac{1}{|1-a|} x\left(\frac{b}{a}\right) \int_{t_1}^{t_2} \delta\left(t - \frac{b}{a}\right) dt$$

CASE 1



$$\int_{t_1}^{t_2} \delta\left(t - \frac{b}{a}\right) dt = 0 \longrightarrow I = 0$$

CASE 2



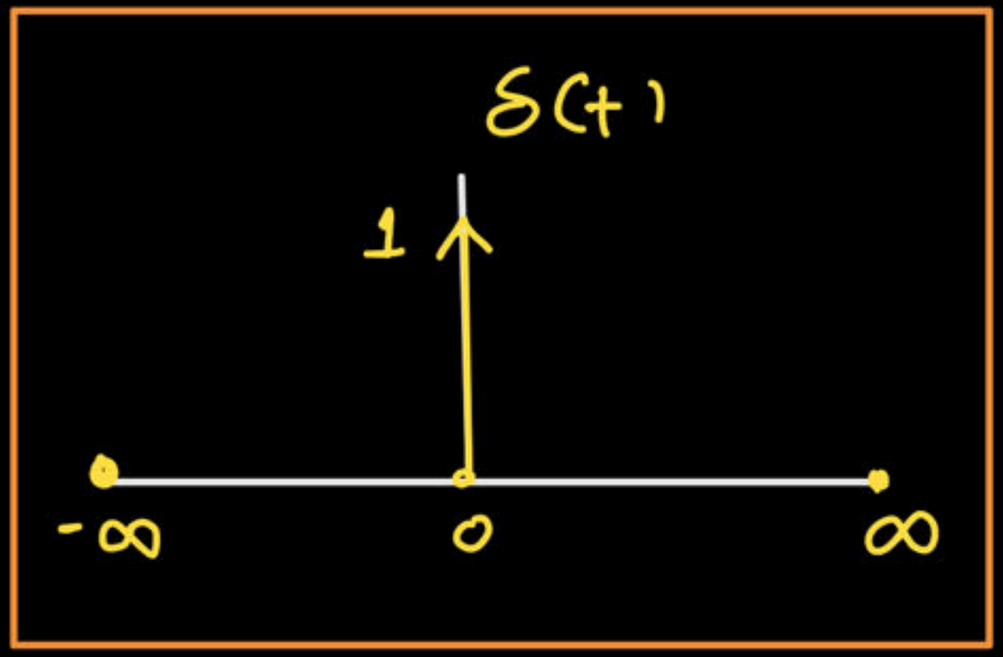
$$\int_{t_1}^{t_2} \delta\left(t-\frac{b}{a}\right) dt = 1 \longrightarrow$$

$$I = \frac{1}{|a|} x\left(\frac{b}{a}\right) \times 1$$

Q. $$I = \int_{-\infty}^{\infty} x(t)\delta(t)\,dt$$

$\delta(t)$ is non-zero only at $t = 0$

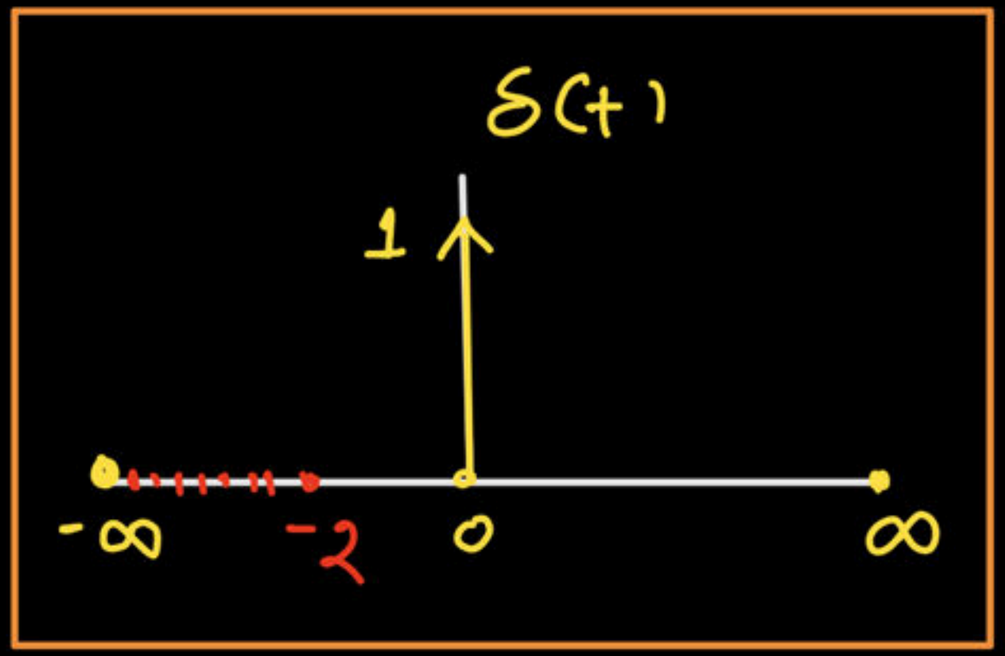
$$I = \int_{-\infty}^{\infty} x(0)\delta(t)\,dt$$



$$I = x(0)\int_{-\infty}^{\infty}\delta(t)dt = x(0)\times 1 = x(0)$$

Q. $$I = \int_{-\infty}^{-2} x(t)\,\delta(t)\,dt$$

$\delta(t)$: $t = 0$

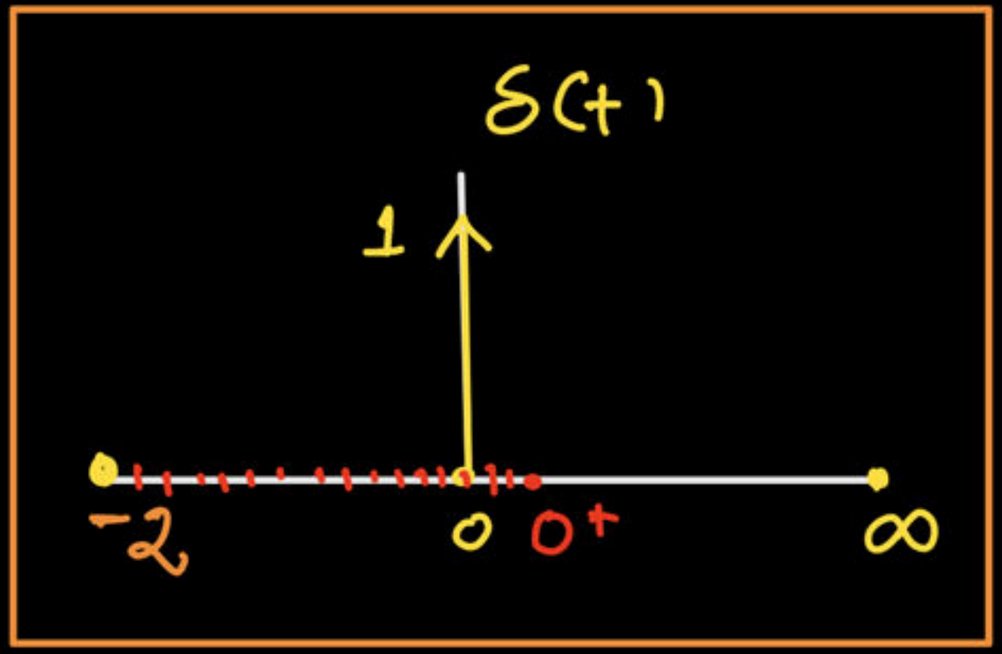


$$I = \int_{-\infty}^{-2} x(0)\,\delta(t)\,dt$$

$$I = x(0) \int_{-\infty}^{-2} \delta(t)\,dt = x(0) \times 0 = 0$$

Q. $I = \int_{-2}^{0^+} x(t)\delta(t)\,dt$

$t = 0$



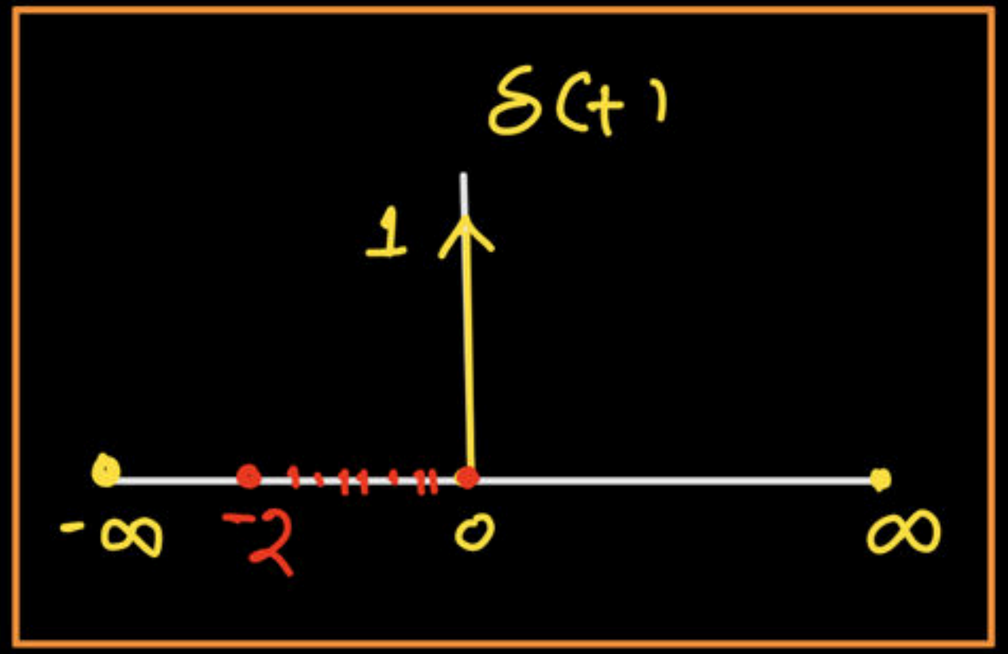
$$I = \int_{-2}^{0^+} x(0)\,\delta(t)\,dt$$

$$I = x(0)\int_{-2}^{0^+} \delta(t)\,dt = x(0) \times 1 = x(0)$$

Q. $I = \int_{-2}^{0} x(t)\,\delta(t)\,dt$

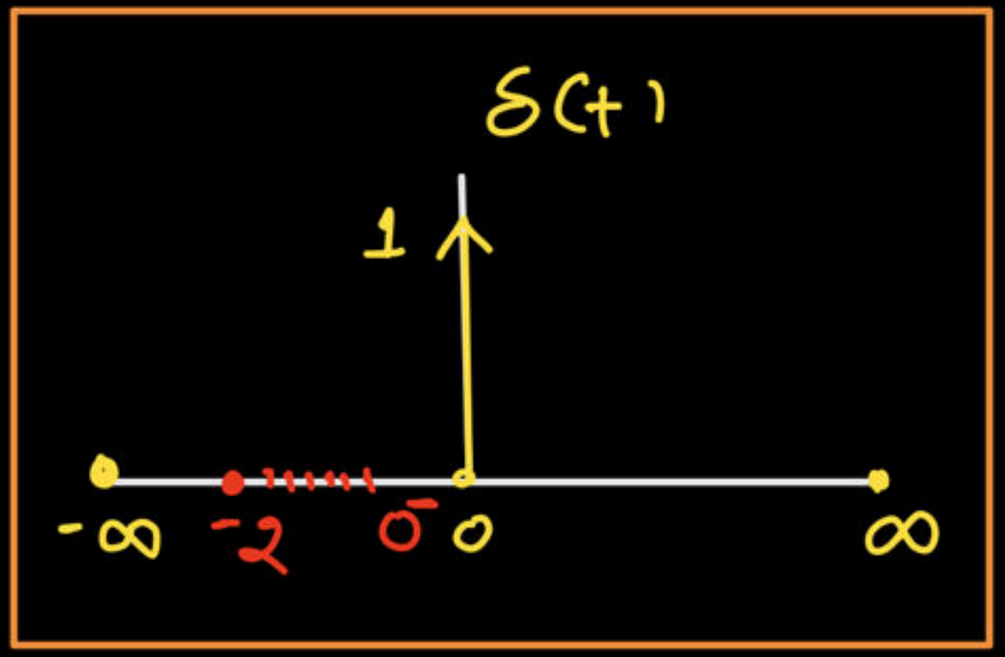
$t = 0$

$$I = x(0)\int_{-2}^{0} \delta(t)\,dt$$



ABSURD CASE

Q. $$I = \int_{-2}^{0^-} x(t)\delta(t)\,dt$$



$$I = x(0)\int_{-2}^{0^-} \delta(t)\,dt$$

$$x(0) \times 0 = 0$$

Q. $I = \int_{-\infty}^{\infty} e^{-t^2}\,\delta(-2t+1)\,dt$

$$I = \int_{-\infty}^{\infty} e^{-t^2}\,\delta\left(-2\left(t-\frac{1}{2}\right)\right)dt$$

$$I = \frac{1}{2}\int_{-\infty}^{\infty} e^{-t^2}\,\delta\left(t-\frac{1}{2}\right)dt = \frac{1}{2}\int_{-\infty}^{\infty} e^{-1/4}\,\delta\left(t-\frac{1}{2}\right)dt$$

$t = 1/2$

$$= \frac{1}{2}\int_{-\infty}^{\infty} e^{-1/4}\,\delta\left(t-\frac{1}{2}\right)dt$$

$$= \frac{e^{-1/4}}{2}\int_{-\infty}^{\infty}\delta\left(t-\frac{1}{2}\right)dt = \frac{e^{-1/4}}{2}\times 1 = \frac{e^{-1/4}}{2}$$

Q. $$I = \int_{-\pi/6}^{\pi/6} \sin\left(t - \frac{\pi}{2}\right)\delta(3t - \pi)\,dt$$

$$I = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sin\left(t - \frac{\pi}{2}\right)\delta\left(3\left(t - \frac{\pi}{3}\right)\right)dt$$

$$I = \frac{1}{3}\int_{-\pi/6}^{\pi/6} \sin\left(t - \frac{\pi}{2}\right)\delta\left(t - \frac{\pi}{3}\right)dt$$

$$I = \frac{1}{3}\int_{-\pi/6}^{\pi/6} \sin\left(t-\frac{\pi}{2}\right)\delta\left(t-\frac{\pi}{3}\right)dt$$

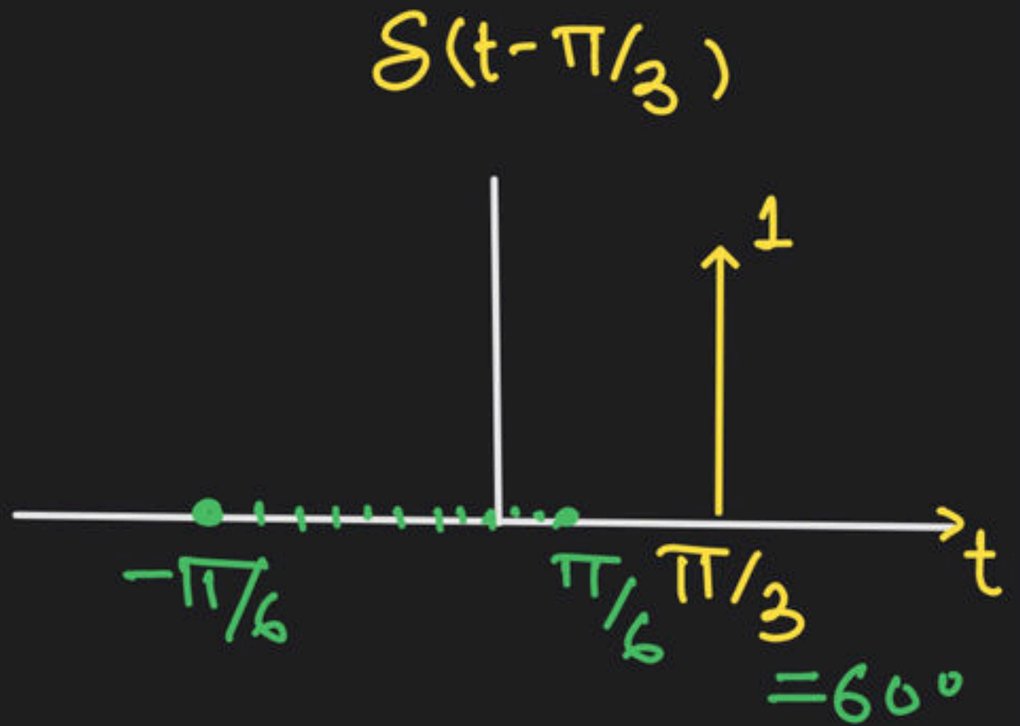
$t = \pi/3$

$$I = \frac{1}{3}\int_{\frac{-\pi}{6}}^{\pi/6} \sin\left(\frac{\pi}{3}-\frac{\pi}{2}\right)\delta\left(t-\frac{\pi}{3}\right)dt$$

$$I = \frac{1}{3}\sin\left(\frac{-\pi}{6}\right)\int_{-\pi/6}^{\pi/6} \delta\left(t-\frac{\pi}{3}\right)dt$$

$$I = \frac{1}{3}\sin\left(\frac{-\pi}{6}\right)\int_{-\pi/6}^{\pi/6} \delta\left(t-\frac{\pi}{3}\right)dt$$

$$I = \frac{1}{3}\sin\left(\frac{-\pi}{6}\right) \times 0 = 0$$



Q. $$I = \int_{-\pi/2}^{\pi/2} \sin\left(t - \frac{\pi}{2}\right)\delta(3t - \pi)\,dt$$

$$I = \int_{-\pi/2}^{\pi/2} \frac{1}{3}\sin\left(t - \frac{\pi}{2}\right)\delta\left(t - \frac{\pi}{3}\right)dt$$

$t = \pi/3$

$$I = \frac{1}{3}\sin\left(\frac{-\pi}{6}\right)\int_{-\pi/2}^{\pi/2} \delta(t - \pi/3)\,dt$$

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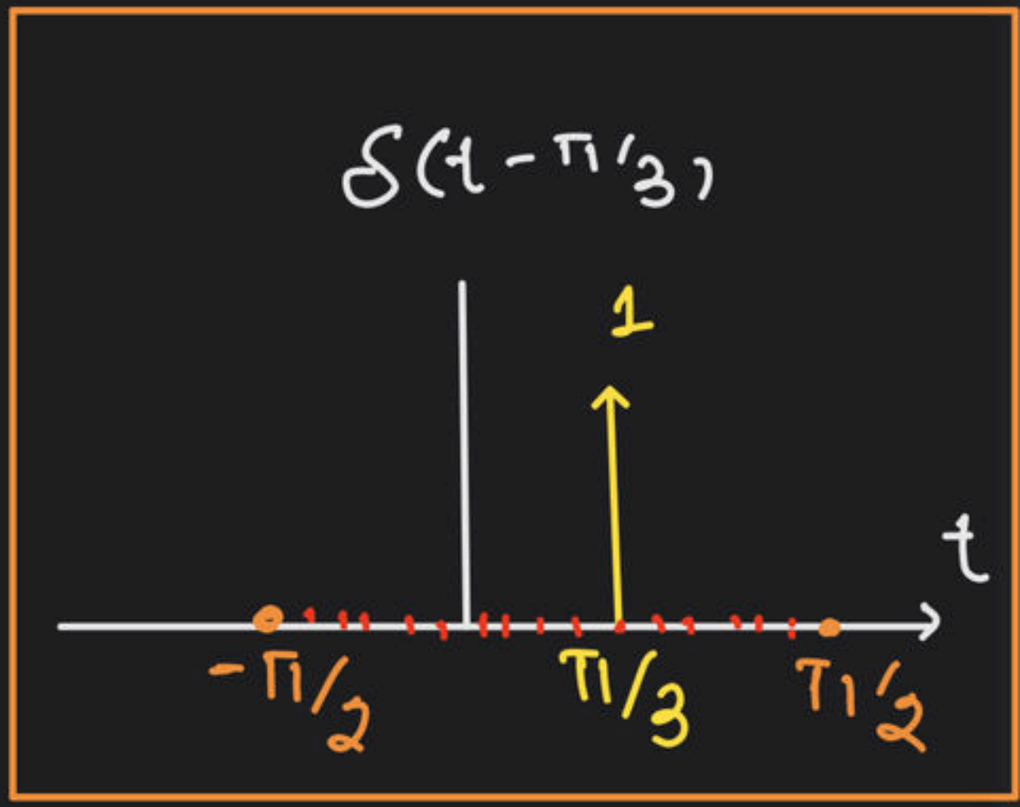
$$I = \frac{1}{3}\sin\left(\frac{-\pi}{6}\right)\int_{-\pi/2}^{\pi/2}\delta(t-\pi/3)\,dt$$



$$I = \frac{1}{3}\times\frac{-1}{2}\times 1 = -\frac{1}{6}$$

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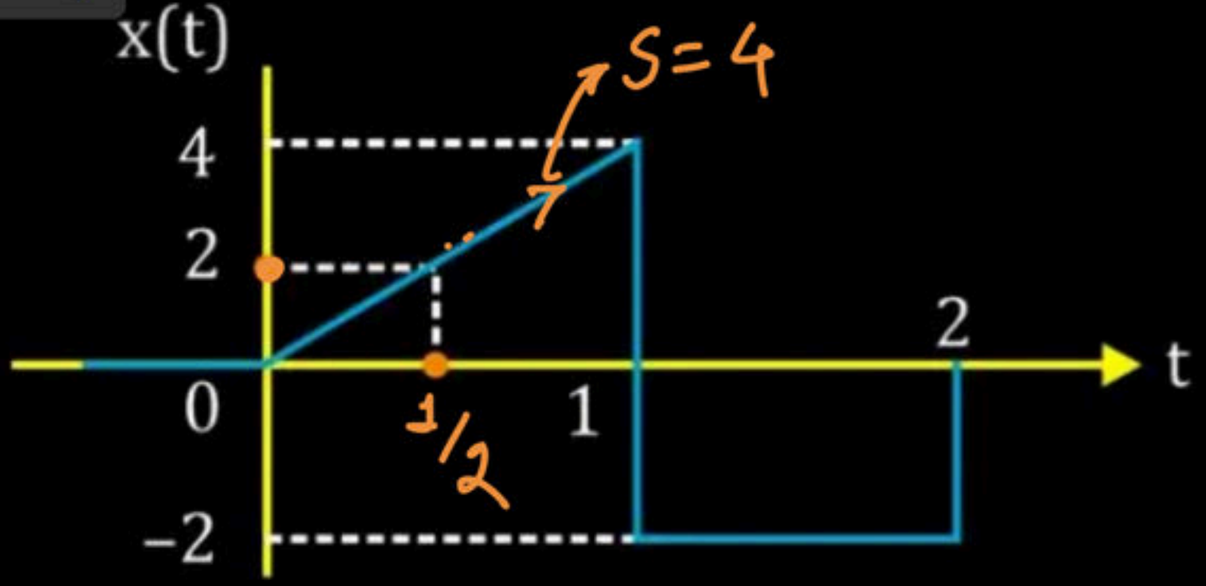
Q.



$$\Delta y = m \Delta x$$

$$\Delta y = 4 \times 1/2 = 2$$

$$I = \int_{-4}^{6} x(t)\,\delta(-2t+1)\,dt$$

$$I = \int_{-4}^{6} x(t)\,\delta(-2t+1)\,dt = \int_{-4}^{6} x(t)\,\delta\left(-2\left(t-\frac{1}{2}\right)\right)dt$$

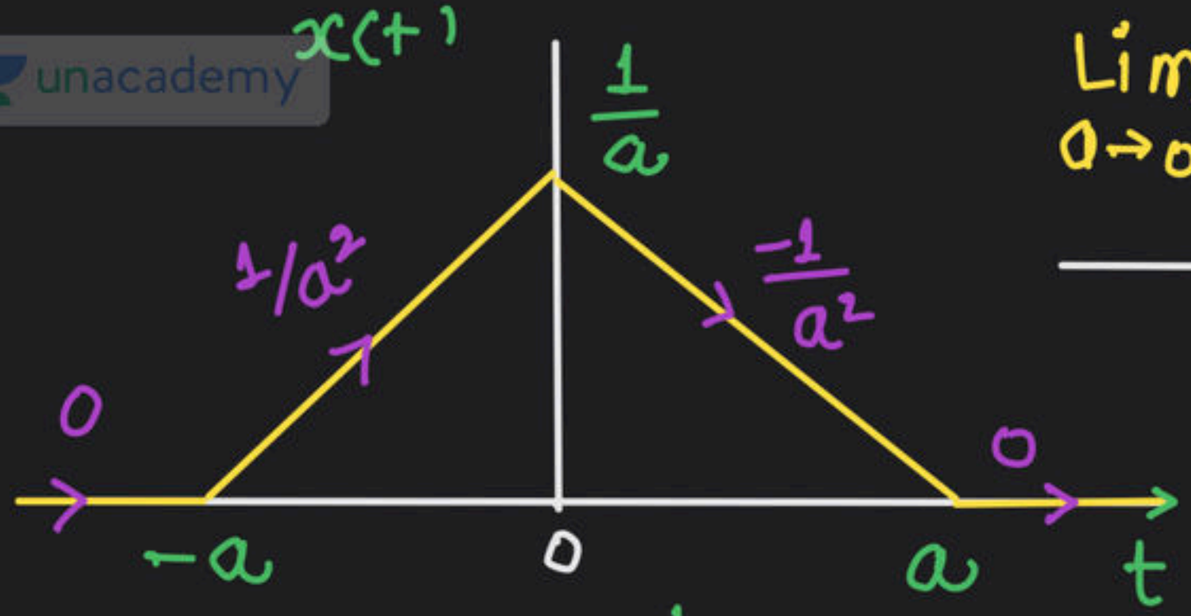
$$= \frac{1}{2}\int_{-4}^{6} x(t)\,\delta\left(t-\frac{1}{2}\right)dt$$

$t = 1/2$

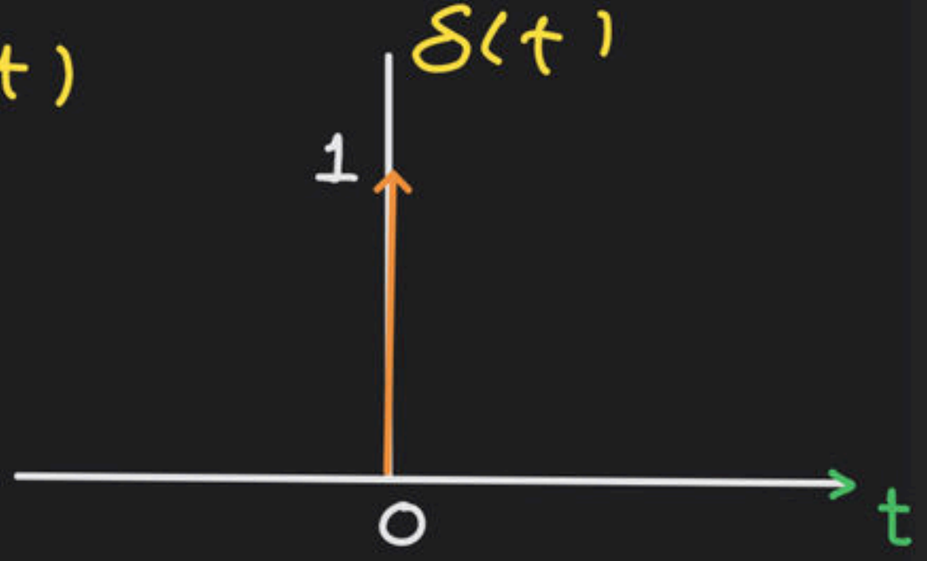
$$= \frac{1}{2}x\left(\frac{1}{2}\right)\int_{-4}^{6}\delta\left(t-\frac{1}{2}\right)dt = \frac{1}{2}x\left(\frac{1}{2}\right)$$

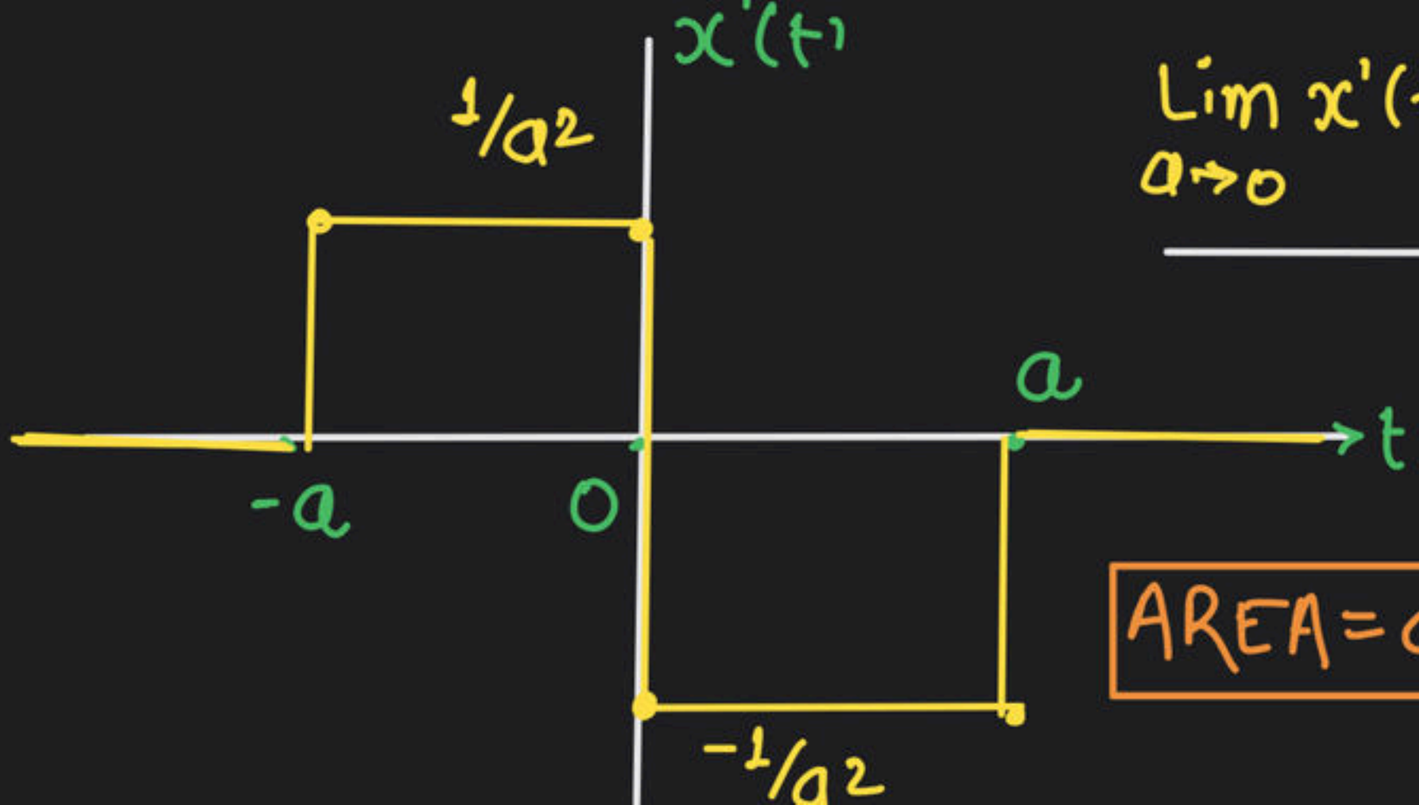
$$= 1/2 \times 2 = 1$$

IMPORTANT DISCUSSION

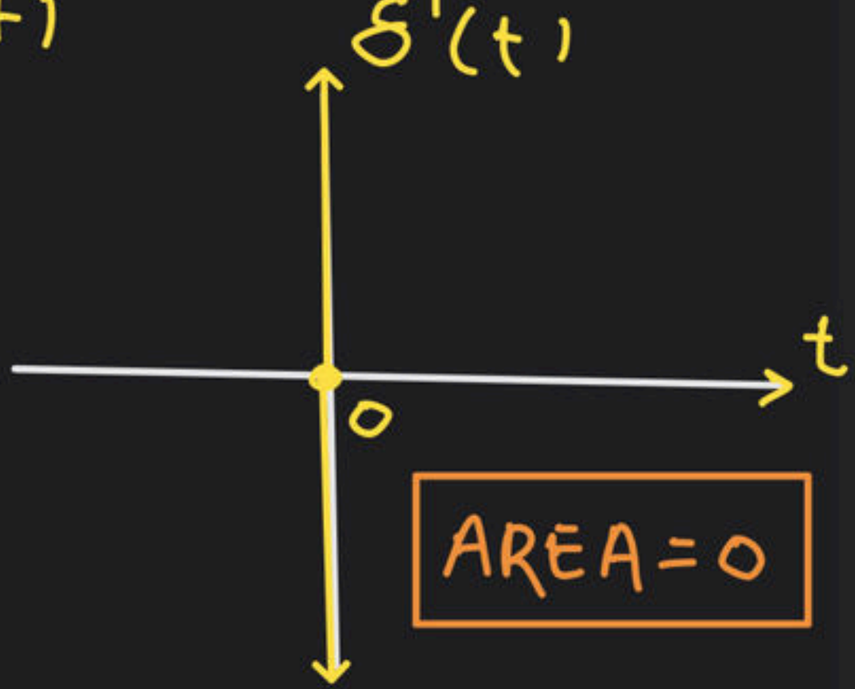


$$\lim_{a \to 0} x(t) = \delta(t)$$





$$\lim_{a \to 0} x'(t) = \delta'(t)$$



AREA = 0

AREA = 0

DOUBLET FUNCTION

$$\delta'(t) = 0 \; : \; t \neq 0$$

$$\delta'(t) = \pm\infty \; : \; t = 0$$

$$\int_{-\infty}^{\infty} \delta'(t)\,dt = 0$$

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PROPERTIES:

1. $\delta(t)$: EVEN $\longrightarrow$ $\delta'(t)$: ODD

2. $$\int_{-\infty}^{\infty} \delta'(t)\,dt = 0$$

3. $$x(t)\,\delta'(t) = x(0)\,\delta'(t) - x'(0)\,\delta(t)$$

   $t = 0$

4 $x(t)\,\delta'(t-t_0) = x(t_0)\,\delta'(t-t_0) - x'(t_0)\,\delta(t-t_0)$

$t = t_0$

5 $t\,\delta'(t) = 0\,\delta'(t) - 1\,\delta(t) = -\delta(t)$

$t = 0$

$$t\,\delta'(t) = -\delta(t)$$

Q. $x(t) = t \sin\left(t + \frac{\pi}{2}\right) \delta'(t)$

$$x(t) = \sin\left(t + \frac{\pi}{2}\right)(t\delta'(t))$$

$$x(t) = \sin\left(t + \frac{\pi}{2}\right)(-\delta(t))$$

$t = 0$

$$x(t) = -\sin\left(0 + \frac{\pi}{2}\right)\delta(t) = -\delta(t)$$

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Q. $x(t)=\left[t+t^2+\sin t+\cos t\right]\left[t+t^2+t^3\right]\delta'(t)$

$$x(t)=(t+t^2+\sin t+\cos t)(1+t+t^2)[t\delta'(t)]$$

$$x(t)=(t+t^2+\sin t+\cos t)(1+t+t^2)(-\delta(t))$$

$t=0$

$$x(t)=(1)(1)(-\delta(t)=-\delta(t)$$

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Q. $x(t) = \underbrace{\sin 2t}_{g(t)}\,\delta'(t)$

$$x(t) = g(t)\delta'(t) = g(0)\delta'(t) - g'(0)\delta(t)$$

($t = 0$)

$g(t) = \sin 2t \longrightarrow g(0) = 0$

$g'(t) = 2\cos 2t \longrightarrow g'(0) = 2$

$$x(t) = -2\delta(t)$$

6

$$\int_{-\infty}^{\infty} x(t)\,\delta'(t)\,dt = -\left\{\frac{dx(t)}{dt}\right\}_{t=0}$$

7

$$\int_{-\infty}^{\infty} x(t)\,\delta'(t-t_0)\,dt = -\left\{\frac{dx(t)}{dt}\right\}_{t=t_0}$$

8. $$\int_{-\infty}^{\infty} x(t)\,\delta^{k}(t)\,dt = (-1)^{k}\left\{\frac{d^{k}x(t)}{dt^{k}}\right\}_{t=0}$$

9. $$\int_{-\infty}^{\infty} x(t)\,\delta^{k}(t-t_0)\,dt = (-1)^{k}\left\{\frac{d^{k}x(t)}{dt^{k}}\right\}_{t=t_0}$$

10 $\delta(at+b) = \frac{1}{|a|}\delta\left(t+\frac{b}{a}\right)$

$a\,\delta'(at+b) = \frac{1}{|a|}\delta'\left(t+\frac{b}{a}\right)$

**

$$\delta'(at+b) = \frac{1}{a|a|}\delta'\left(t+\frac{b}{a}\right)$$

I $\delta'(at+b) = \delta'(a(t+\frac{b}{a})) = \frac{1}{a|a|}\delta'(t+\frac{b}{a})$

II $\delta'(-at+b) = \delta'(-a(t-\frac{b}{a})) = \frac{1}{-a|-a|}\delta'(t-\frac{b}{a})$

III $\delta'(-at-b) = \delta'(-a(t+\frac{b}{a})) = \frac{1}{-a|-a|}\delta'(t+\frac{b}{a})$

IV $\delta'(at-b) = \delta'(a(t-\frac{b}{a})) = \frac{1}{a|a|}\delta'(t-\frac{b}{a})$

Q. $\delta'(-2t)$

$$= \delta'(-2(t)) = \frac{1}{-2|-2|}\,\delta'(t) = -\frac{1}{4}\,\delta'(t)$$

Q. $\delta'(-3t+2)$

$$\delta'(-3(t-\frac{2}{3}))=\frac{1}{-3|-3|}\delta'(t-\frac{2}{3})=-\frac{1}{9}\delta'(t-2/3)$$

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Q. $x(t) = \left(e^{-t} + t^2\right) \delta'(-2t)$

$$x(t) = \frac{1}{-4} \underbrace{\left(e^{-t} + t^2\right)}_{g(t)} \delta'(t)$$

$$x(t) = -\frac{1}{4} \left[g(0)\delta'(t) - g'(0)\delta(t) \right]$$

$$x(t) = -\frac{1}{4} \left[2\delta'(t) + \delta(t) \right]$$

Q. $$I = \int_{-2\pi}^{2\pi} \sin 2t \, \underbrace{\delta'(t)}_{t=0} dt = -\left\{\frac{d}{dt} \sin 2t\right\}_{t=0}$$

$$= -2$$

Q. $I = \int_{-\pi}^{\pi} \sin 2t\, \delta''\left(t - \frac{\pi}{4}\right) dt = \left\{(-1)^2 \frac{d^2}{dt^2} \sin 2t\right\}_{t=\pi/4}$

$t = \pi/4$

$= (-4 \sin 2t)_{t=\pi/4}$

$= -4 \sin \frac{\pi}{2} = -4$

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Q. $$I=\int_{-\infty}^{\infty}\sin\left(\frac{t^2+\pi}{4}\right)\left(t^2+t\right)\delta'(t)\,dt$$

$$I=\int_{-\infty}^{\infty}\sin\left(\frac{t^2+\pi}{4}\right)(t+1)\left(t\,\delta'(t)\right)dt$$

$$I=\int_{-\infty}^{\infty}\sin\left(\frac{t^2+\pi}{4}\right)(t+1)\left(-\delta(t)\right)dt=\frac{-1}{\sqrt{2}}$$

$t=0$

SIGNAL SYSTEM

IMPULSE SIGNAL

DPP-2

Vishal Soni Sir
(ESE AIR 25)

Q1. Consider the integral

$$y(t) = \frac{1}{2}\int_{-\infty}^{\infty} x(\tau)[\delta(\tau - 2) + \delta(\tau + 2)]d\tau,$$

$y(t)$ can be simplified to

A $x(2)$

B $x(-2)$

C $\frac{1}{2}[x(2) - x(-2)]$

D $\frac{1}{2}[x(2) + x(-2)]$

Q2. Consider the integral,

$$I = \int_{-\infty}^{\infty} \delta(at - b)\sin^2(t - 4)dt, \text{ Where } a > 0.$$

If 'I' is simplified to $\frac{1}{P}\sin^2\left[\frac{Q}{R} - S\right]$, then P, Q, R and S are respectively given by

A. b, a, b and 4

B. a, b, a and 4

C. b, 4, a and b

D. 4, a, b and a

Q3. The value of the integral $\int_{-\infty}^{t-5} \delta(\tau - 1)d\tau$ is

A 0

B $u(t-5)$

C $u(t-6)$

D $u(t+4)$

Q4. Let

$$I_1 = \int_{-\infty}^{(\pi/6)} \cos(t)u(t)dt, \qquad I_2 = \int_{-\infty}^{(\pi/6)} \cos(t)\delta(t)dt$$

$$I_3 = \int_{-\infty}^{(\pi/6)} \cos(t)(t-1)\delta(t)dt \text{ and } I_4 = \int_{0}^{2\pi} t\sin\left(\frac{t}{2}\right)\delta(\pi - t)dt$$

Then these integrals satisfy

A $I_2 < I_4 < I_3 < I_1$

B $I_3 < I_1 < I_2 < I_4$

C $I_4 < I_1 < I_3 < I_2$

D $I_1 < I_3 < I_4 < I_2$

Q5. For a unit impulse function $\delta(t)$, which of the following is true?

A $\delta[a(t-t_0)] = \frac{1}{|a|}\delta(t)$

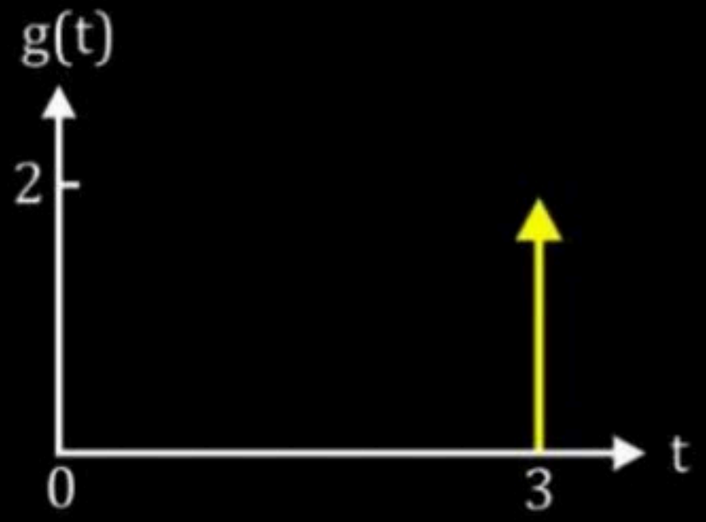
B $\delta[a(t-t_0)] = |a|\delta(t-t_0)$

C $\delta[a(t-t_0)] = \frac{1}{|a|}\delta(t-t_0)$

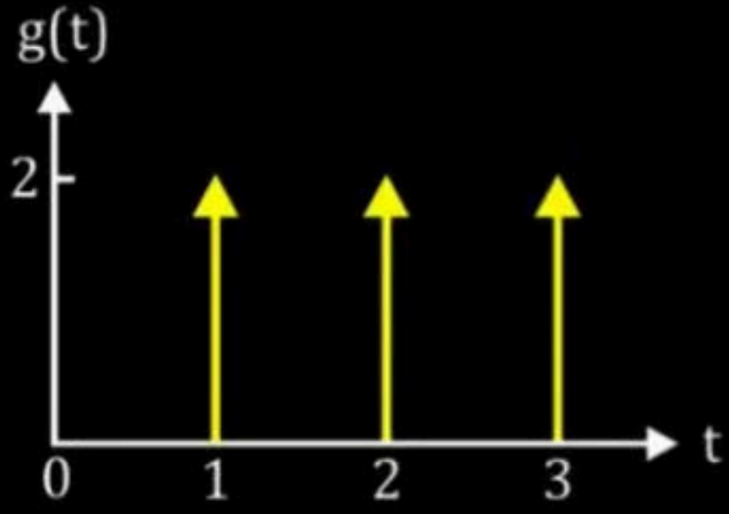
D $\delta[a(t-t_0] = |a|\delta(t)$

Q6. If $\delta(t)$ is an unit impulse function then which of the following waveform represents a signal $g(t) = 6\delta(3t + 9)$?

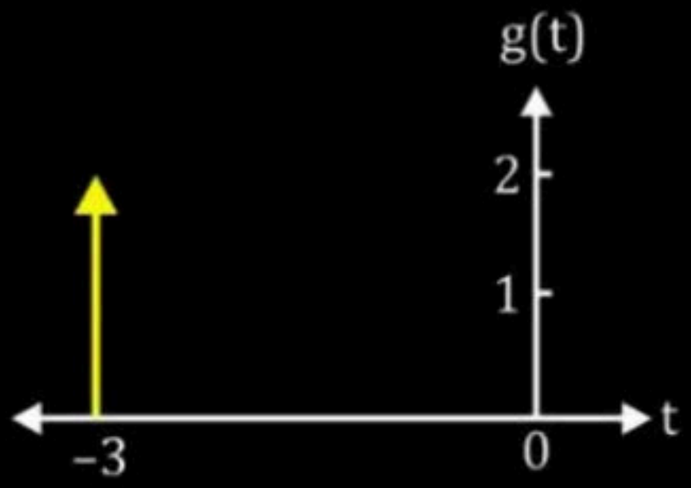
A



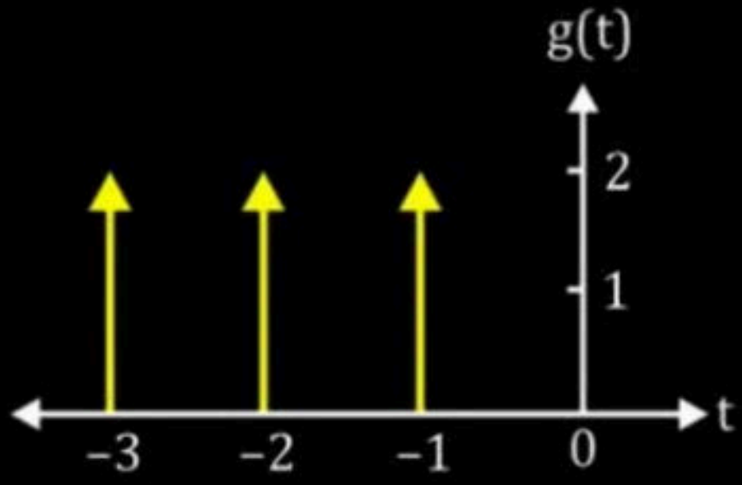
B



C

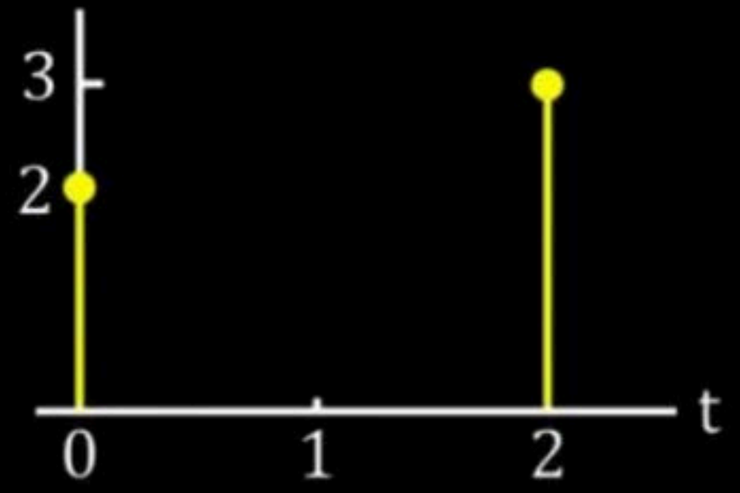


D

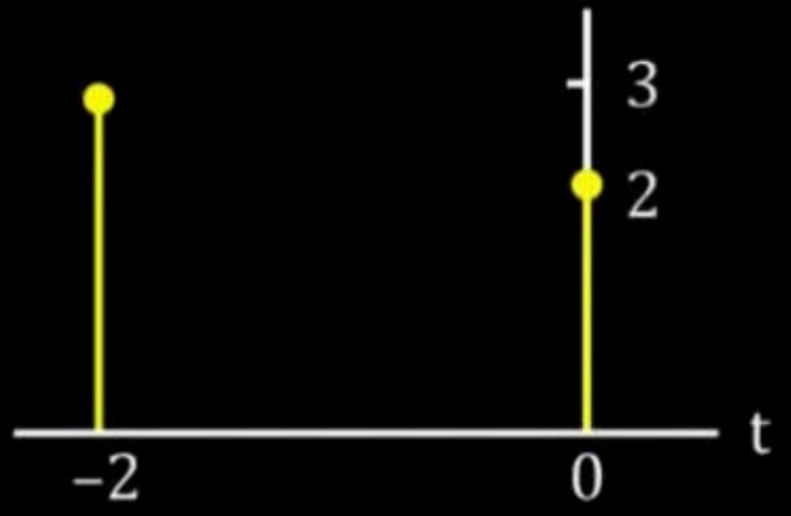


Q7. The graph of function $x(t) = 2\delta(2t) + 6\delta(3(t-2))$ is

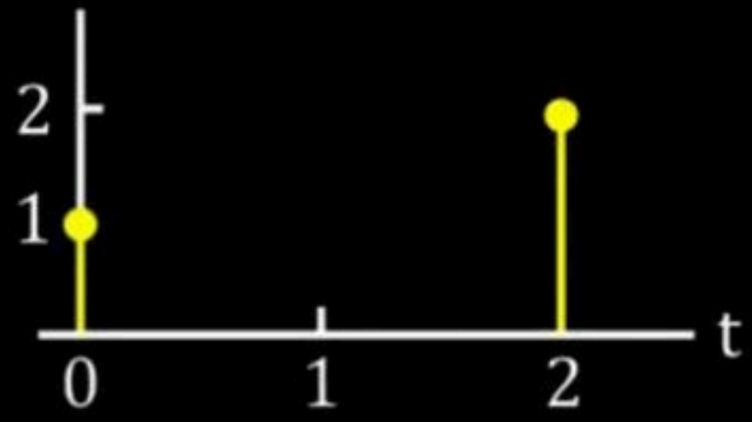
A



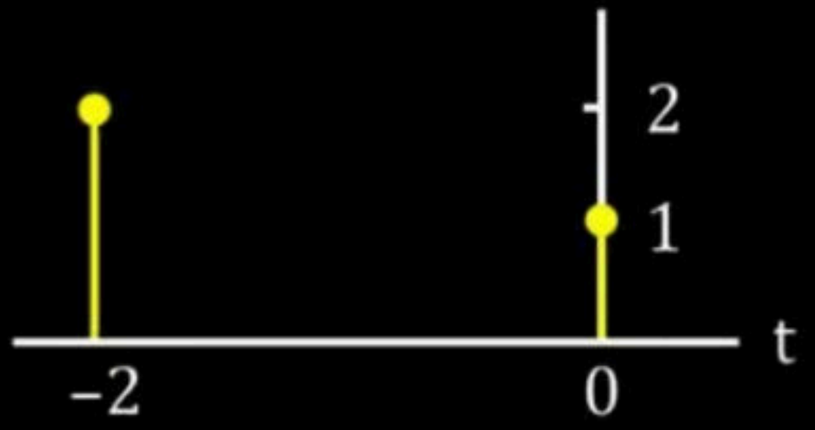
B



C



D



Q8. The function $\int_{-\infty}^{\infty} x(\tau)[\delta(\tau-2)+\delta(\tau+2)]d\tau$ is equal to

A $x(2)+x(-2)$

B $\frac{x(2)+x(-2)}{2}$

C $2x(2)+2x(-2)$

D None of these

Q9. The value of the function $\int_{-\infty}^{\infty} \delta(at - b)\sin^2(t - 4)dt$ where $a > 0$, is

A 1

B $\frac{\sin^2\left(\frac{a}{b} - 4\right)}{b}$

C 0

D $\frac{\sin^2\left(\frac{b}{a} - 4\right)}{a}$

Q10. If $\delta(t)$ is an unit impulse function, then the value of integral $\int_{-\infty}^{\infty} e^{-t}\delta(2t-2)dt$ equals to _____

Q11. What is the numerical value of the following integral

$$x(t) = \int_{-\infty}^{\infty} \delta(t+5)\cos(\pi t)dt$$

Q12. Let the CT unit impulse function be defined by

$$\delta(x) = \lim_{\alpha \to 0} \left(\frac{1}{\alpha}\right) \text{tri}\left(\frac{x}{\alpha}\right), a > 0$$

The function $\delta(x)$ has an area of one regardless the value of α

What is the area of the function $\delta(4x)$?

Q13. Let the CT unit impulse function be defined by

$$\delta(x) = \lim_{\alpha \to 0} \left(\frac{1}{\alpha}\right) \text{tri}\left(\frac{x}{\alpha}\right), a > 0$$

The function $\delta(x)$ has an area of one regardless the value of α

What is the area of the function $\delta(-6x)$?

Q14. Let the CT unit impulse function be defined by

$$\delta(x) = \lim_{\alpha \to 0} \left(\frac{1}{\alpha}\right) \text{tri}\left(\frac{x}{\alpha}\right), a > 0$$

The function $\delta(x)$ has an area of one regardless the value of α

A signal $x(t)$ is defined as $x(t) = 2\,\text{tri}[2(t-1)] + 6\text{rect}\left(\frac{t}{4}\right)$. The value of $x\left(\frac{3}{2}\right)$ is _____.

Q15. A signal x(t) and its transformed signal y(t) are shown in figure (A) and figure (B) respectively. If $y(t) = x(at + b)$, then value of b is _____ .

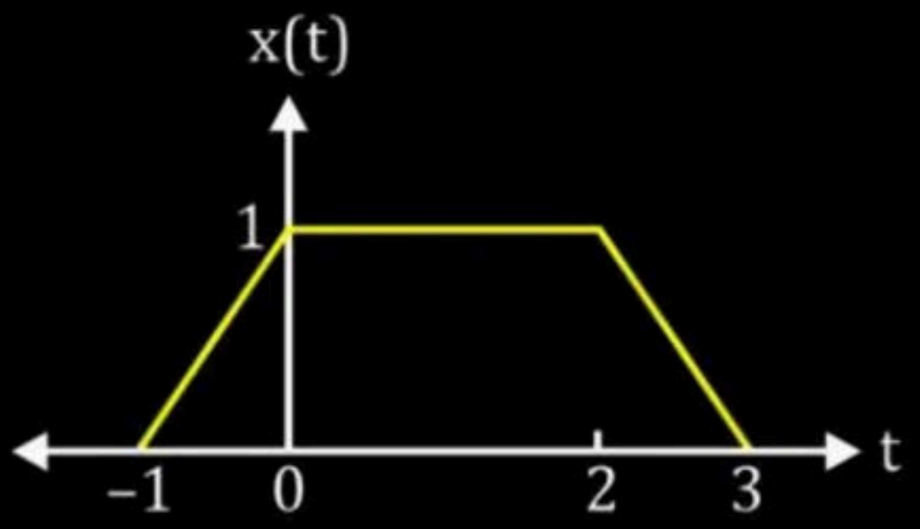


Fig. (A)

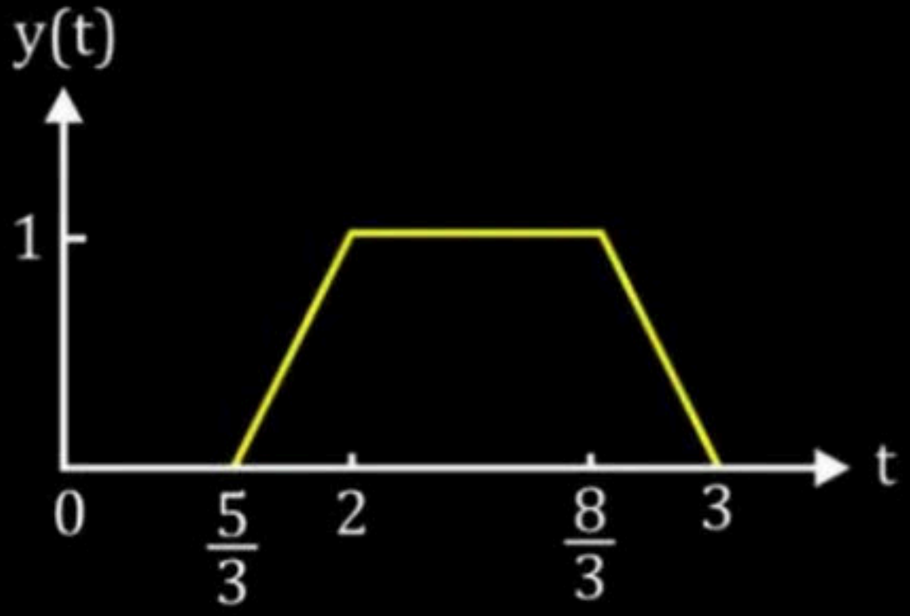
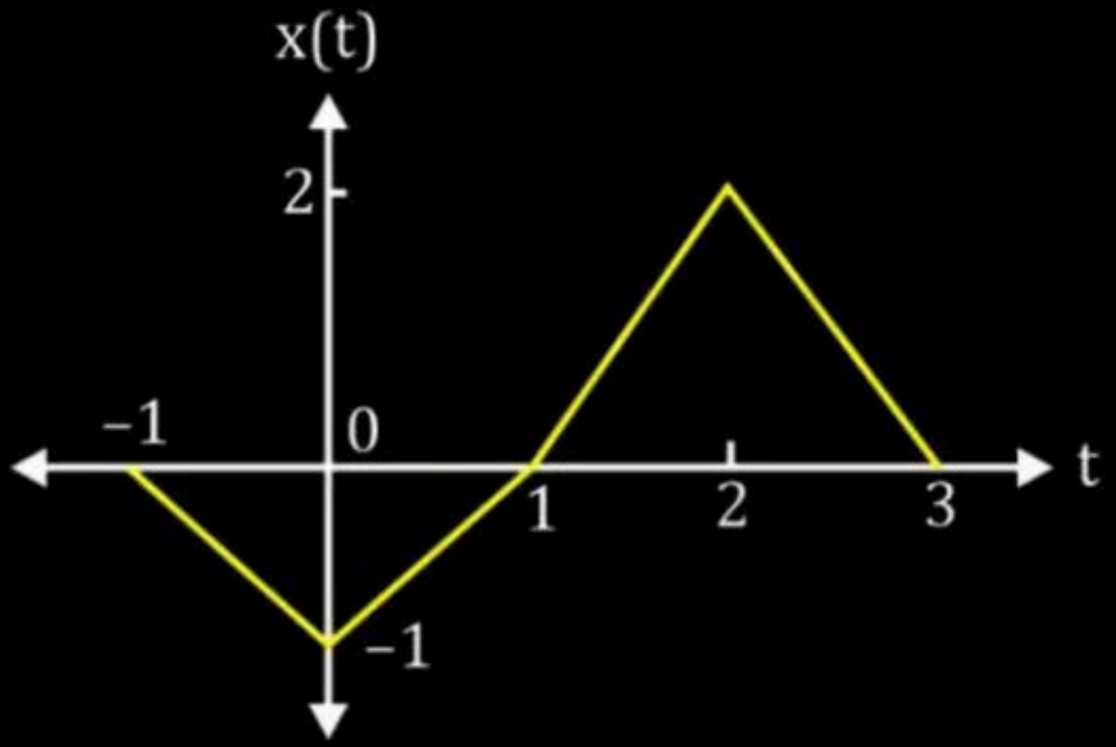
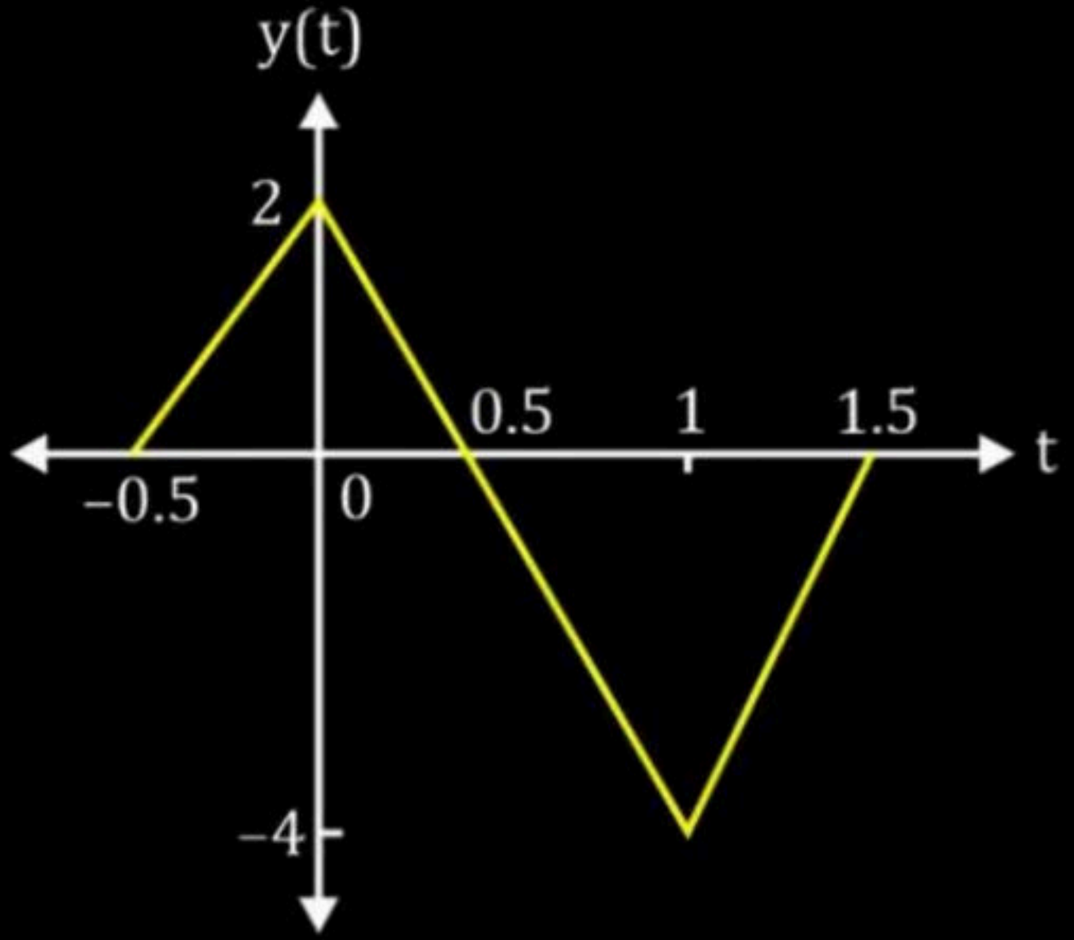


Fig. (B)

Q16. Consider two signals x(t) and y(t) shown in figure below.
If $y(t) = -kx(kt)$ then, the value of k is _____





Q17. The value of $\int_{-\pi}^{\pi} 2\cos \omega t \delta(\omega) d\omega$ is

A 0

B $\pi/2$

C 1

D 2

Q18. If $\delta(t)$ is the unit impulse function, then $\int_{-\infty}^{\infty} x(t)\delta(t)\,dt$ equals to

A $x(t)$

B $x(0)$

C $x(\infty)$

D $x(1)$

Q19. For unit impulse function $\delta(t)$, which of the following relation holds true?

A $\delta(-t) = \delta\left(\frac{t}{2}\right)$

B $\delta(-t) = \delta(t^2)$

C $\delta(-t) = \delta(t)$

D $\delta(-t) = \delta^2(t)$

Q20. The function $f(t) = t\delta(t)$ will be equal to

A t

B ∞

C 1

D 0

Q21. The unit impulse is defined as,

A $\delta(t) = \infty, t = 0$

B $\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases}$

C $\delta(t) = \infty, t = 0$ and $\int_{-\infty}^{+\infty} \delta(t)\, dt = 0$

D $\delta(t) = \begin{cases} \infty, t = 0 \\ 0, \ t \neq 0 \end{cases}$ and $\int_{-\infty}^{+\infty} \delta(t) dt = 1$

Q22. If $x(t)$ is a continuous time signal and $\delta(t)$ is a unit impulse signal then value of integral $\int_{-\infty}^{\infty} x(t)\delta(t - t_0)$ is equal to

A $x(t)$

B $x(t_0)$

C $\delta(t)$

D 1

Q23. A weighted impulse function $\delta(at)$ has

A unit area and unit amplitude

B infinite area and finite amplitude

C finite area and infinite amplitude

D infinite area and infinite amplitude

Q24. Which one of the following relations is not correct?

A $f(t)\delta(t) = f(0)\delta(t)$

B $\int_{-\infty}^{\infty} f(t)\delta(\tau)d\tau = 1$

C $\int_{-\infty}^{\infty} \delta(\tau)d\tau = 1$

D $f(t)\delta(t-\tau) = f(\tau)\delta(t-\tau)$

Q25. The Dirac delta function $\delta(t)$ is defined as

(A) $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases}$

(B) $\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & \text{otherwise} \end{cases}$

(C) $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty} \delta(t)dt = 1$

(D) $\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty} \delta(t)dt = 1$

Q26. Let $\delta(t)$ denote the delta function. The value of the integral $\int_{-\infty}^{\infty} \delta(t)\cos\left(\frac{3t}{2}\right) dt$ is

A 1

B -1

C 0

D $\frac{\pi}{2}$

Q27. The Integral $\int_{-\infty}^{\infty} \delta\left(t-\frac{\pi}{6}\right) 6 \sin(t) dt$ evaluates to

A 6

B 3

C 1.5

D 0

Q28. The integral $\frac{1}{2\pi}\int_{-\infty}^{\infty} t^2 e^{-t^2/2}\delta(1-2t)dt$ is equal to

A $\frac{1}{8\sqrt{2\pi}}e^{-1/8}$

B $\frac{1}{4\sqrt{2\pi}}e^{-1/8}$

C $\frac{1}{\sqrt{2\pi}}e^{-1/2}$

D 1

▲ 1 • Asked by Swagato

Sir i had purchased the pyq book of gate academy ee shall i purchase the pyq book of ece too?

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▲ 1 • Asked by Abdul

sir evening m bhi 10min break ho sakta h kya

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▲ 1 • Asked by Komal

sir jis question me dono grapgh given hota hai usme dono me relation har option ko varify karke dekhna hai ya koi tarika hai

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▲ 1 • Asked by Vedant

sir aisa koi books hai kya jisme ee ece in ke merge pyq ho

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▲ 1 • Asked by Deepak

Sir abhi itne sare exam h psu or ese confused ho rha hun kaise manage kru please help me sir phle psu exam ki tyari kru ya ese ki

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▲ 1 • Asked by Ritu

Sir mera barc ka exam 8 march ko apaka signal recorded kiya hu leki adha adha adhura hai abhi live class kare apaki ya fir 8 march ke bad bhi kare baki subject theek se taiyar bhi

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▲ 1 • Asked by Rishi

Sir i have already studied signal and system, from another Educator but I'm not fluent with Fourier series, dft ,fft dfts, yeh topics aap se kr lu?

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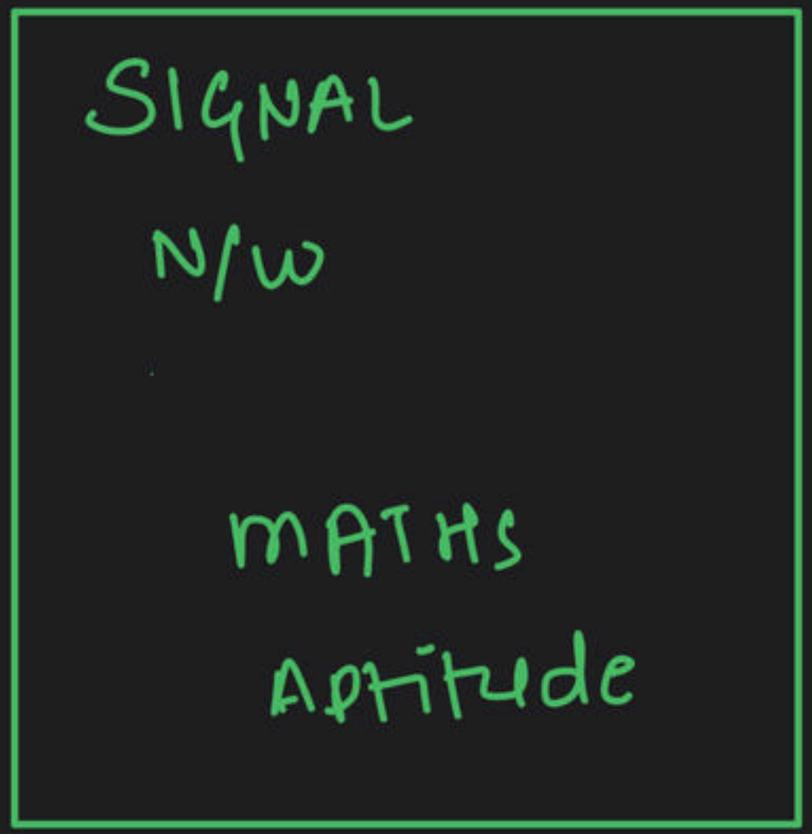
▲ 1 • Asked by G Mudassir

sir 2026 ke liye prepare karrahahun, mera EMT aur Control hogaya, next subject kya karun. din me ek subject 2-3 lec sunna acha hai ya 2 subjects karna acha hai??

SIGNAL



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Sir EDC aap abaki bar le lejiye kyuki EDC samajh nahi aata

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▲ 1 • Asked by Arjun

Sir I have done signal and system from your you tube lecture should i start from now or later