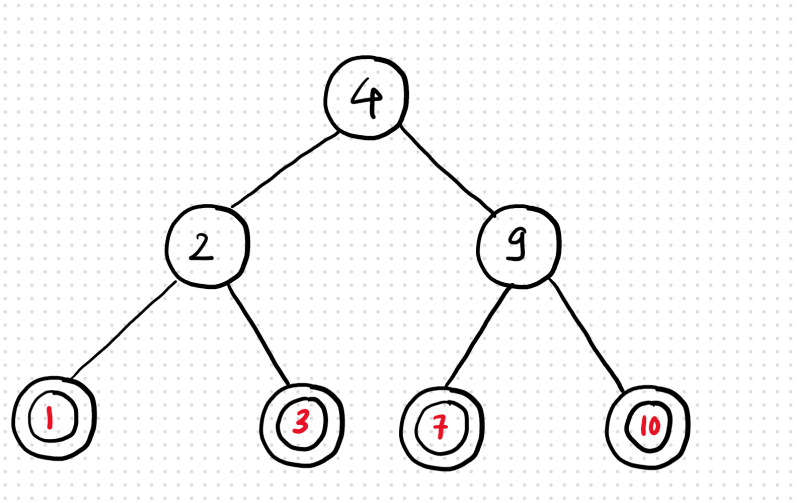
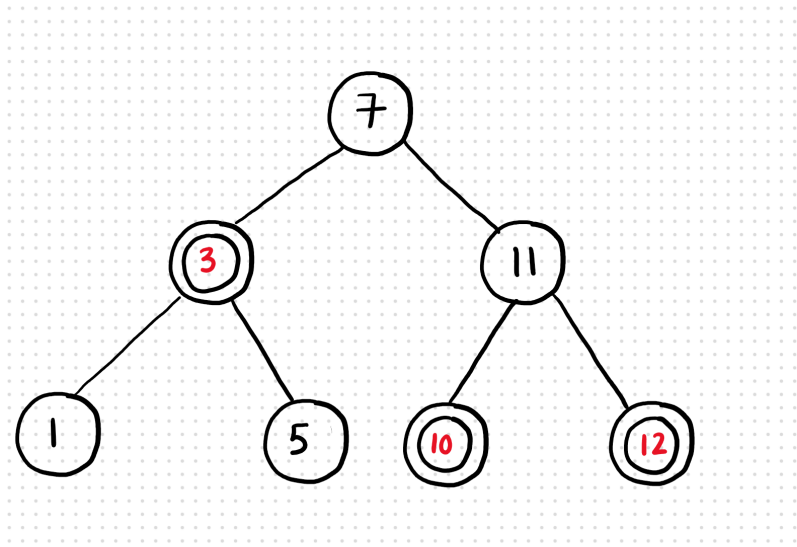
Written Homework Assignment 4: Red-black Trees, B+ Trees

ANSWERS :

1.

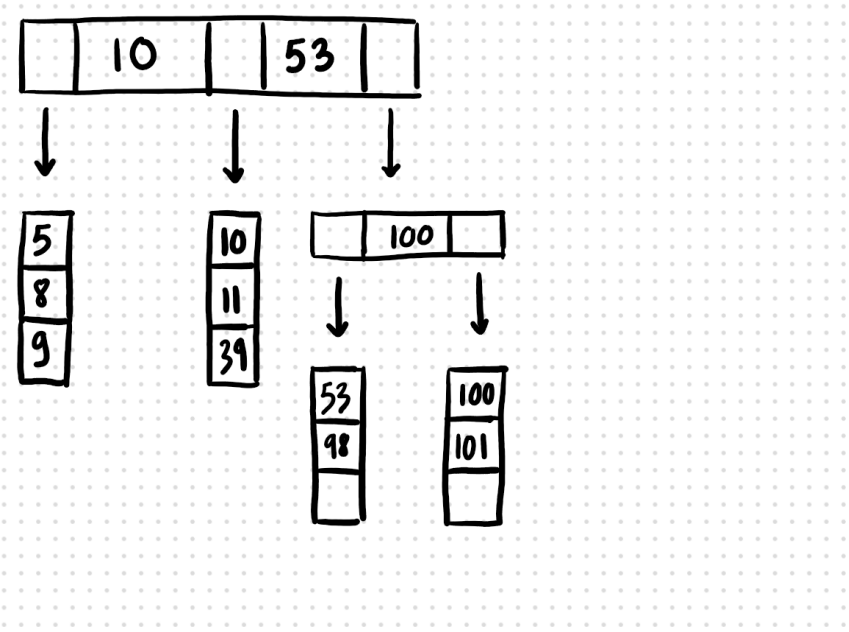


2.

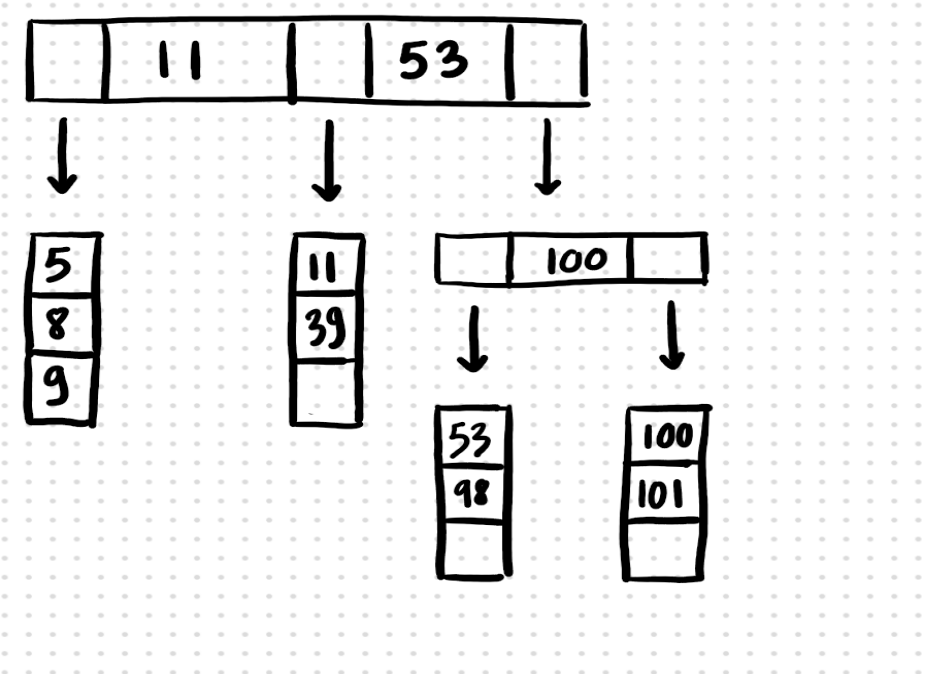


3.

a.


10
53
5
8
9
10
11
39
100
53
98
100
101

b.


11
53
5
8
9
11
39
100
53
98
100
101

4.

a.

Since the size of a pointer is 8 bytes, M child pointers will need space :

8 * M bytes

M - 1 key entries will take up (M - 1)* K space.

Therefore, B = 8 * M + (M - 1) * K

Solving for M :

M = floor[(B + K) / (8 + K)]

Here,

B = 8192 bytes

K = 8 bytes

M = floor[(8192 + 8) / (8 + 8)] = floor(8200 / 16) = floor(512.5)

M = 512

Hence the maximum number of children in each internal node is 512.

b.

Let us find the size of a data element, denoted by D.

D = 64 bits + 32 bytes + 32 bits

Let us convert all the bits to bytes.

1 bit = 0.125 bytes

D =64*0.125 + 32 + 32*0.125 = 8 + 32 + 4

D = 44 bytes

L = floor[B / D] = floor[8192 / 44] = floor[186.1818]

$L = 186$

c.

The height of a B+ tree is given by :

$\log_M X$, where $X = \text{ceil}(N/L)$

Let us denote the height by H,

$H = \log_M X = \log_M[\ \text{ceil}(N/L)\ ]$

M = 512 and L = 186

$H = \log_{512}[\ \text{ceil}(N / 186)\ ]$

d.

N = 30,000

$H = \log_{512}[\ \text{ceil}(N / 186)\ ] = \log_{512}[\text{ceil}(\ 30000 / 186)]$

$H = \log_{512}(162)$

e.

N = 2,500,000

$H = \log_{512}[\ \text{ceil}(N / 186)\ ] = \log_{512}[\text{ceil}(\ 2500000 / 186)]$

$H = \log_{512}(13441)$