

## Quizzes

1. Consider a curve parametrized by some  $\gamma : (-1, 1) \rightarrow \mathbb{R}^2$ . For what values of  $t$  in the interval  $(-1, 1)$  will  $\dot{\gamma}(t)$  and  $\ddot{\gamma}(t)$  be orthogonal if its arc length function, written in terms of the parametrization  $\gamma$ , is
  - a)  $s(t) = t$
  - b)  $s(t) = t^2$
2. Consider a curve parametrized by  $\gamma : (-1, 1) \rightarrow \mathbb{R}^2$  which has the property that  $\ddot{\gamma}(t_0) = c\dot{\gamma}(t_0)$ , for some real number  $c$  and some  $t_0 \in (-1, 1)$ . What is the curvature of the curve at the point  $\gamma(t_0)$ ? Justify your answer.
3. If all the tangent lines to a space curve pass through a fixed point  $c$ , then prove that the curve lies on a straight line.
4.
  - a) Consider the function  $F(x, y, z) = 2x^2 + y^2 - 3z^2 - 1$ . Compute the partial derivatives  $\frac{\partial F}{\partial x}$ ,  $\frac{\partial F}{\partial y}$ , and  $\frac{\partial F}{\partial z}$ .
  - b) For the same  $F$  as above, compute  $\nabla F$ . Why is  $S := \{(x, y, z) \mid F(x, y, z) = 0\}$  a surface?
5. Given a surface patch  $\sigma(x, y) = (x, y, \sqrt{1 - x^2 - y^2})$ , compute the standard unit normal at the point  $(0, 0, 1)$ .
6. If for every point on a surface patch  $\sigma : U \rightarrow S$  of a surface,  $S$ , the two principle curvatures are equal, then prove that the principle curvature is constant.