MTH 201, Curves and surfaces

Practice problem set 2

1. Which of the following parametrizations are regular? If a parametrization fails to be regular, is it because it is not smooth somewhere or because the derivative is 0? In either case, can you think of an alternative which will be regular? For each regular curve, find a unit speed reparametrization.

   a) $\gamma(t) = (a\,\cos(t), a\,\sin(t), bt)$ for some fixed real numbers $a > 0$ and $b > 0$, $t \in \mathbb{R}$
   b) $\gamma(t) = (a\,\cos(t^3), a\,\sin(t^3), bt^3)$ for some fixed real numbers $a > 0$ and $b > 0$, $t \in \mathbb{R}$
   c) $\gamma(t) = (t, t^2)$, $t \in \mathbb{R}$
   d) $\gamma(t) = (t^2, t^3)$, $t \in \mathbb{R}$
   e) $\gamma(t) = (t, t^2, t^3)$, $t \in \mathbb{R}$
   f) $\gamma(t) = (t^2, |t|)$, $t \in \mathbb{R}$
   g) $\gamma(t) = (\cos^2(t), \sin^2(t))$, $t \in \mathbb{R}$
   h) $\gamma(t) = (t, \cosh(t))$, $t \in \mathbb{R}$ (Remember that $\cosh(t) = \frac{e^t + e^{-t}}{2}$. You can easily check that its derivative, denoted by $\sinh(t)$, satisfies the equation $\cosh^2(t) - \sinh^2(t) = 1$).

2. For what values of m and n will the curve parametrized by $\gamma(t) = (t^m, t^n)$, $t \in \mathbb{R}$, be regular?

3. Show that any unit speed parametrization $\gamma : (z, b) \to \mathbb{R}^3$, satisfies the equation $\dot{\gamma}(t) \cdot \ddot{\gamma}(t) = 0$. Here, $\ddot{\gamma}(t)$ denotes the second derivative. Can you account for this intuitively?

4. Consider the line in $\mathbb{R}^3$ containing two different points $p = (x_1, x_2, x_3)$ and $q = (x_1, x_2, x_3)$

   a) Find a parametrization γ for the entire line in terms of x_i and y_i.
   b) Calculate $\ddot{\gamma}(t)$.

5. Can you characterize curves that can be parametrized by γ so that $\ddot{\gamma}(t) = 0$?