

# MTH 201, Curves and surfaces

## Practice problem set 6

1. Which of the following subsets of  $\mathbb{R}^3$  are open in  $\mathbb{R}^3$ ?
  - a)  $\{(x, y, z) \mid x^2 + y^2 + z^2 = 1\}$
  - b)  $\{(x, y, z) \mid x^2 + y^2 + z^2 < 1\}$
  - c)  $\{(x, y, 0) \mid x^2 + y^2 < 1\}$
  - d)  $\{(x, y, z) \mid x^2 + y^2 + z^2 \leq 1\}$
  - e)  $\{(x, y, 0) \mid x^2 + y^2 \leq 1\}$
  - f)  $\{(x, y, z) \mid 1 < x^2 + y^2 + z^2 < 2\}$
  - g)  $\{(x, y, z) \mid 1 \leq x^2 + y^2 + z^2 < 2\}$
  - h)  $\{(0, y, z) \mid 1 < y^2 + z^2 < 2\}$
  - i)  $\{(x, y, z) \mid |x| < 1\}$
  - j)  $\{(x, y, 0) \mid |x| < 1, |y| < 1\}$
  - k) A finite set
2. A function,  $f : X \rightarrow Y$ , from any subset  $X$  of  $\mathbb{R}^m$  to any subset  $Y$  of  $\mathbb{R}^n$  is said to be continuous at  $p$  if given any real number  $\epsilon > 0$  (however small, but strictly positive), there is a real number  $\delta > 0$ , so that for any point  $x$ , where  $\|x - p\| < \delta$ ,  $\|f(x) - f(p)\| < \epsilon$ . On which points of their domain are the following functions continuous?
  - a)  $f : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ ,  $f(x, y) = (x^3, x + y, x)$
  - b)  $f : D \rightarrow \mathbb{R}^3$ ,  $f(x, y) = (x^3, x + y, 1/(x-6))$  where  $D := \{(x, y) \mid x^2 + y^2 < 1\}$
  - c)  $f : D \rightarrow \mathbb{R}^3$ ,  $f(x, y) = (x^3, x + y, 1/x)$  where  $D := \{(x, y) \mid x^2 + y^2 = 1\}$
  - d)  $D := \{(x, y) \mid x^2 + y^2 < 1\}$  and  $f : D \rightarrow \mathbb{R}^3$ , where  $f(x, y) = (x^3, x + y, 1/x)$  for  $(x, y) \neq (0, 0)$  and  $f(0, 0) = (0, 0, 0)$ .
  - e) Any function where the domain is finite