

Exercise sheet 1

1. Prove that homotopy is an equivalence relation
2. Prove that if Y is a convex subset of \mathbb{R}^n , then any two continuous functions from X to Y are homotopic. What if Y is star convex?
3. Prove that the relation of homotopy equivalence between two *spaces* is an equivalence relation.
4. Prove that the space $\mathbb{R}^n \setminus \{0\}$ is homotopically equivalent to S^{n-1} .
5. Prove that $\{1, 2, \dots\} \cup \{0\}$ can never be homotopically equivalent to $\{0, 1, 2, \dots\}$ (both spaces are subsets of \mathbb{R} and are given the subspace topology.)

to be updated