

## Exercise sheet 6

1. Prove that every map from a simply connected space to  $S^1$  is null homotopic.
2. Prove that every map from  $\mathbb{RP}^2$  to  $S^1$  is null homotopic.
3. Prove that if  $X$  has a simply connected cover, then given any point in  $x \in X$ , there exists a neighbourhood  $U$  of  $x$  so that  $i_* : \pi_1(U, x) \rightarrow \pi_1(X, x)$  is trivial (here,  $i_*$  is the homomorphism induced by the inclusion map  $i : U \rightarrow X$ ).
4. Prove that any two universal coverings are equivalent.
5. Prove that the “Hawaiian earring” is not semi-locally simply connected.
6. Find all path-connected coverings of  $S^1$  and  $S^1 \times S^1$  up to equivalence.

**To be updated**