

Exercise sheet 6

1. Prove that every map from a simply connected space to S^1 is null homotopic.
2. Prove that every map from \mathbb{RP}^2 to S^1 is null homotopic.
3. Prove that if X has a simply connected cover, then given any point in $x \in X$, there exists a neighbourhood U of x so that $i_* : \pi_1(U, x) \rightarrow \pi_1(X, x)$ is trivial (here, i_* is the homomorphism induced by the inclusion map $i : U \rightarrow X$).
4. Prove that any two universal coverings are equivalent.
5. Prove that the “Hawaiian earring” is not semi-locally simply connected.
6. Find all path-connected coverings of S^1 and $S^1 \times S^1$ up to equivalence.