Exercise sheet 5

1. Prove that if $f_i : X \to Y$, $i = 1, 2$ are covering maps, then so is $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$.
2. Prove that if $f : X \to Y$ is a covering map, and A is a subspace of Y, then $f : f^{-1}(A) \to A$ is a covering map.
3. If $f : X \to Y$ is a covering, then the set $f^{-1}(y)$ is called the fibre at y. Prove that if Y is connected, and the fibre at one point is finite, then all fibres have the same number of elements.
4. A covering map f is said to be finite sheeted if all its fibres are finite. Prove that if f is a finite sheeted covering and g is another covering map, then $f \circ g$ is also a covering map.
5. Given an example of a cover of S^1 whose fibre has n points for some given n.
6. Find an example of a local homeomorphism $f : X \to Y$ which is not a covering map.