Quantifiers

Existential quantifier ∃ -

There exists some x in the domain of discourse where $P(x) = \text{True}$

$\exists\ x\ P(x)$

Universal Quantifier ∀ -

For every x in domain in discourse, $P(x) = \text{True}$

$\forall\ xP(x)$

Negation of Quantifiers ¬

$\neg(\ \forall x\ P(x)\ ) \equiv \exists x\ (\ \neg P(x)\ )\ \neg(\ \exists x\ P(x)\ ) \equiv \forall x\ (\ \neg P(x)\ )$

Example

What is a limit?

Suppose $an \rightarrow l$

$\equiv \exists\ l\ (\ \forall\epsilon>0\ \exists N \in \mathbb{N}\ (\ n>N \Longrightarrow |an - l|<\epsilon\ )\ )$

$\equiv \exists\ l\ (\ \forall\epsilon\in\mathbb{R}\ \exists N \in \mathbb{N}\ (\ n>N \Lsh\!\!\wedge\ \epsilon>0 \Longrightarrow |an - l|<\epsilon\ )\ )$

$P(n) = \text{“}n > N\text{”}$

$Q(\epsilon) = \text{“}\epsilon > 0\text{”}$

$R(l,\epsilon,n) = |an - l|<\epsilon$

Hence -

$\exists l(\forall\epsilon\in\mathbb{R}\ \exists\ N\in\mathbb{N}\ \forall\ n(\ P(n) \wedge Q(\epsilon) \Longrightarrow R(l,\epsilon,n)\ ))$

Negation of Above

If an does not converge,

$\neg(\ \exists l\ (\ \forall\epsilon\in\mathbb{R}\ \exists N\in\mathbb{N}\ \forall n\ (\ P(n) \wedge Q(\epsilon) \Longrightarrow R(l,\epsilon,n)\ )\ )\ )$

$\forall l\ \neg(\ \forall\epsilon\in\mathbb{R}\ \exists N\in\mathbb{N}\ \forall n\in\mathbb{N}\ (\ P(n) \wedge Q(\epsilon) \Longrightarrow R(l,\epsilon,n)\ )\ )$

$\forall l\ \exists\epsilon\in\mathbb{R}\ \neg(\ \exists N\in\mathbb{N}\ \forall n\in\mathbb{N}\ (\ P(n) \wedge Q(\epsilon) \Longrightarrow R(l,\epsilon,n)\ )\ )$

$\forall l\ \exists\epsilon\in\mathbb{R}\ \forall N\in\mathbb{N}\ \neg(\ \forall n\in\mathbb{N}\ (\ P(n) \wedge Q(\epsilon) \Longrightarrow R(l,\epsilon,n)\ )\ )$

$\forall l\ \exists\epsilon\in\mathbb{R}\ \forall N\in\mathbb{N}\ \exists n\in\mathbb{N}\ \neg(\ P(n) \wedge Q(\epsilon) \Longrightarrow R(l,\epsilon,n)\ )\ )$

$\forall l\ \exists\epsilon\in\mathbb{R}\ \forall N\in\mathbb{N}\ \exists n\in\mathbb{N}\ (\ P(n) \wedge Q(\epsilon) \wedge \neg R(l,\epsilon,n)\ )\ )$

In English, for all l, there exists ϵ in $\mathbb{R}$ and there exists n and N in $\mathbb{N}$ such that n>N and $\epsilon > 0$ and | an - l | NOT < ϵ

Proofs

General Domains

- $\forall x P(x)$ is easy to disprove. Just find on x. Very hard to prove.
- $\exists x P(x)$ is easy to prove. Just find on x. Very hard to disprove.

Finite Domains

Both are easy. You could just run through all.

Enumeratable Domains

$\forall$ is easy to show by induction.

Contradiction

If either $\forall$ or $\exists$ is difficult to show, or if $\neg P(x)$ is more well known, you can disprove the negation of the predicate.

Full Adder

Stupid way

Make a truth table for an nbit input

Better way

Truth Table -

in1	in2	out	carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Hence out is the XOR $(\neg P \wedge Q) \vee (\neg Q \wedge P)$

Carry is the And $P \wedge Q$