Exercise sheet 9

1. Design a Turing machine to add two numbers in binary. More specifically, givan an input of the form `101+10`, the tape should finally have `101+10=111` written on it. You could try to do this on a multi-tape Turing machine since any one can always then build a single tape one that will simulate it.
2. Show that if we allow a Turing machine's head to stay where it is, i.e. the set of possible directions for the head are $\{L, R, S\}$, then any computation that it can perform, can also be performed by an ordinary Turing machine.
3. Show that this language is undecidable:

   $$\{\langle T \rangle \mid \text{T is a Turing machine that outputs 1011}\}$$

4. Show that this language and its complement are both undecidable:

   $$\{\langle T_1, T_2 \rangle \mid T_1, T_2 \text{ are Turing machines and } L(T_1) = L(T_2)\}$$

5. Exercise 5.16 from the third edition of Sipser's book (6.28 from the second edition) on Rice's theorem.