Exercise sheet 1

Manifolds, MTH406

1. Prove that any open subset of a smooth manifold is smooth.
2. Prove that the product of smooth manifolds is smooth.
3. Prove that any chart $\phi : U \to \mathbb{R}^n$ on a smooth manifold is a smooth map.
4. Prove that if $F : M \to N$ and $G : N \to P$ are smooth maps between manifolds, then the composition $G \circ F$ is also smooth.
5. Prove that if X_p is a derivation at $p \in M$, then $X_p(c) = 0$ for any constant function c.
6. Given a point p on a smooth manifold M, let F_p denote the ideal of germs that vanish at p. Prove that the dual of the vector space F_p/F_p^2 is isomorphic to $T_p(M)$.
7. Let $[\gamma]_p$ denote the equivalence class of paths under the equivalence relation $\gamma_1 \gamma_2$ if and only if