

# Exercise sheet 1

Manifolds, MTH406

1. Prove that any open subset of a smooth manifold is smooth.
2. Prove that the product of smooth manifolds is smooth.
3. Prove that any chart  $\phi : U \rightarrow \mathbb{R}^n$  on a smooth manifold is a smooth map.
4. Prove that if  $F : M \rightarrow N$  and  $G : N \rightarrow P$  are smooth maps between manifolds, then the composition  $G \circ F$  is also smooth.
5. Prove that if  $X_p$  is a derivation at  $p \in M$ , then  $X_p(c) = 0$  for any constant function  $c$ .
6. Given a point  $p$  on a smooth manifold  $M$ , let  $F_p$  denote the ideal of germs that vanish at  $p$ . Prove that the dual of the vector space  $F_p/F_p^2$  is isomorphic to  $T_p(M)$ .
7. Let  $\gamma_p]$  denote the equivalence class of paths under the equivalence relation  $\gamma_1 \sim \gamma_2$  if and only if