Exercise sheet 1

1. Verify the following using truth tables:
   a) $P \vee P \wedge Q = P$
   b) $P \wedge (P \vee Q) = P$
   c) $P \vee (\neg P \wedge Q) = P \vee Q$
2. Find a boolean expression for the following truth table in terms of $\wedge$, $\vee$, and $\neg$ and then simplify it:

P	Q	R	
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

4. Define the NOR operator by the rule, P NOR Q is true if and only if P and Q are both false.
   a) Write a truth table for NOR.
   b) Find an expression for P NOR Q in terms of $\wedge$, $\vee$, and $\neg$.
   c) Prove that $\neg P$ can be defined completely in terms of NOR *(Hint: since* NOR *takes two arguments but $\neg P$ involves just one variable, there is only one thing you can do!).*
   d) Prove that $P \vee Q$ can be expressed using only the NOR operator. *(Hint: $\vee$ is the negation of* NOR *and part c. shows how to express negation in terms of* NOR*).*
   e) Prove that $P \wedge Q$ can be expressed using only the NOR operator, and therefore, by c. and d. you can express any boolean function using only the NOR operator. *(Hint: compare the truth tables of $\wedge$ and* NOR*. How do you get one from the other?)*
5. Simplify the following boolean expressions:
   a) $(\neg P \vee \neg Q) \wedge (\neg P \vee Q)$
   b) $\neg P \wedge \neg(P \vee Q)$

To be completed...