Exercise sheet 9

Probability and Statistics, MTH102

1. Let $f(x, y)$ be a joint density function for the random variables X and Y. Express $P\{X > a, Y < b\}$ and $P\{X > a, Y > b\}$ in terms of the density function and the marginal density functions.

2. If X and Y are random variables then show that $f_{X,Y}(x, y) = f_X(x)g_Y(y)$ if and only if X and y are independent.

3. Consider a uniform random variable defined on an interval (a, b). For what values of c and d will the random variable $Y = cX + d$ be a uniform random variable on $(0, 1)$?

4. Let X and Y denote the uniform random variables, uniform on the interval $(0, 1)$. Compute the probability density function of the sum $X + Y$.

5. If two people plan to meet and their probability of arriving between say 9am and 10am is uniformly and independently distributed, compute the probability that one of them will have to wait for at least 5 minutes.

6. Show that the sum of two binomial distributions with parameters (m, p) and (n, p) is also a binomial distribution with parameters $(m + n, p)$. Use the following identity from combinatorics:

$$^{m+n}C_k = \sum_{i=0}^{n} {}^{n}C_i \, {}^{m}C_{k-i}$$