Exercise sheet 5

Probability and Statistics, MTH102

1. Prove that the addition and multiplication operations are well defined for integers, rationals, and reals, and are also commutative and associative.
2. Prove that the order defined on integers, rationals, and reals are transitive. Prove that it is a total ordering.
3. Prove that the integers, rationals, and reals are not well ordered.
4. Prove that there is a rational number between any two rational numbers.
5. Suppose $p(k)$ is a formula and there exists an integer n_0 so that $p(n) \to p(n+1)$ for all $n \geq n_0$, and that $p(n_0)$ holds, then prove that $p(k)$ holds for all integers $k \geq n_0$.