Exercise sheet 1

Probability and Statistics, MTH102

1. Show from the axioms that $P(\emptyset) = 0$.
2. If a coin is tossed 3 times, compute and observe the probability of getting
   (a) 0 heads
   (b) 1 head
   (c) 2 heads
   (d) 3 heads
   (e) What do you think the sum of the probabilities that you computed above should be?
3. If I roll a pair of dice, and add the sum of the numbers on the dice, which number has the highest probability of occuring and why?
4. If you feel that the probability that it will rain tomorrow is 0.3 and you feel that probability that I will be absent tomorrow is 0.4. Let us assume that you feel that the probability that it rains tomorrow and that I will be absent is 0.2. What is the probability that...
   (a) I will be present tomorrow even though it will rain tomorrow?
   (b) I will be absent tomorrow and that it will rain tomorrow?
   (c) I will be present tomorrow or that it will not rain tomorrow?
5. Consider a coin tossed n times.
   (a) What is the probability of getting k tails?
   (b) Use this to show that ${}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n = 2^n$
   (c) If n is even, what is the probability of getting heads in half the tosses and tails in the rest?
   (d) If n is even, use the previous part to show that ${}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_{n/2} = 2^{n-1}$