Exercise sheet 10

Probability and Statistics, MTH102

1. Compute the moment generating functions of the Poisson distribution, binomial distribution, and the normal distribution.
2. Use the moment generating functions to show that the sum of two Poisson distributions is also a Poisson distribution.
3. If you assume a coin is fair, and you toss it 1000 times, use the central limit theorem to estimate the probability that you get more than 70% heads. Compare it with the answer that you will get if you were to use the the law of large numbers.
4. Show that following properties of $\mathrm{Cov}(X, Y)$
   (a) $\mathrm{Cov}(X, X) = \mathrm{Var}(X)$
   (b) $\mathrm{Cov}(X, Y) = \mathrm{Cov}(X, Y)$
   (c) $\mathrm{Cov}(aX, Y) = a\mathrm{Cov}(X, Y)$
   (d) $\mathrm{Cov}(X_1 + Y_2, Y_1 + Y_2) = \mathrm{Cov}(X_1, Y_1) + \mathrm{Cov}(X_1, Y_2) + \mathrm{Cov}(X_2, Y_1) + \mathrm{Cov}(X_2, Y_2)$
   (e) $\mathrm{Var}(X + Y) = \mathrm{Var}(X) + \mathrm{Var}(Y) + \mathrm{Cov}(X, Y)$.