Exercise sheet 6

Probability and Statistics, MTH102

1. For a random variable X which only takes positive values, show that $aP\{X \geq a\} \leq E[X]$ (*Hint*: why is $E[X] = \sum x_i p(x_i) \geq \sum_{i \text{ so that } x_i \geq a} ap(x_i)$? How do you use that?)

2. Note that if $|X - E[X]| \geq k$ then $(X - E[X])^2 \geq k^2$.

   (a) Use this observation and the previous exercise to show that $P\{|X - E[X]| \geq k\} \leq \frac{\mathrm{Var}(X)}{k^2}$

   (b) Show that if the variance is 0, then the probability that X is exactly the expectation is 1. (*Hint*: Show that the probability that the $|X - E[X]|$ is bigger than any non-zero number (however small!) is 0.)

3. Let X_1, X_2, $\ldots X_n$ denote independent and identical distributions. Since they are identical, $E[X_1] = E[X_2] = \cdots = E[X_n]$, and let us denote this common expectation by μ. Similarly, $\mathrm{Var}(X_1) = \mathrm{Var}(X_2) = \cdots = \mathrm{Var}(X_n)$, and we denote the common variance by σ^2. Use the above to show that $P\{|\frac{X_1+X_2+\cdots+X_n}{n} - \mu| \geq k\} \leq \frac{\sigma^2}{k^2 n}$