

Exercise sheet 8

Probability and Statistics, MTH102

1. Let X denote a normal random variable with mean μ and variance σ^2 . Remember that $\Phi(x)$ is the cumulative function for the *standard* normal distribution. Prove that $F_X(x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$
2. Let X denote a normal random variable with mean μ and variance σ^2 . Show that
 - (a) $P\{X > \mu + \sigma\} = 1 - \Phi(1)$
 - (b) $P\{\mu < X < \mu + \sigma\} = \Phi(1) - \Phi(0)$
 - (c) $P\{\mu - \sigma < X < \mu\} = \Phi(0) - \Phi(-1)$
 - (d) $P\{\mu - 2\sigma < X < \mu - \sigma\} = \Phi(1) - \Phi(2)$
 - (e) $P\{X < \mu - 2\sigma\} = \Phi(-2)$
3. Let X denote the uniform distribution on the interval $(0, 1)$. Let Y denote the random variable satisfying $Y = X^3$. Find a probability density function for Y . (*Hint:* You will need to use the cumulative function and its interpretation as a probability)