Exercise sheet 7

Probability and Statistics, MTH102

1. The average number of typos in a certain newspaper is 0.3. What is the probability that the newspaper has more than 3 typos?

2. The probability of winning in a particular game is 0.01. Use the Poisson approximation to compute the probability of winning in 5 out of 10000 games.

3. If X is a Poisson random variable with parameter λ, then show the following relations on the moments: $E[X^n] = \lambda E[(X+1)^{n-1}]$.

4. If X is a Poisson random variable with parameter λ, then $p(i)$ increases and then decreases. Compute $p(i+1)/p(i)$ and use that to find out for which value of i the $p(i)$ will attain its maximum value.

5. If X is a continuous random variable with density function $f_X(x)$ defined so that $f_X(x) = c$ as long as $a < x < b$, and 0 otherwise, then what should the value of c be so that it is a valid density function?

6. If X and Y are continuous random variables, prove the following counterparts of discrete random variables:

   (a) $\mathrm{E}[aX + b] = a\mathrm{E}[X] + b$ for any numbers a and b

   (b) $\mathrm{E}[X + Y] = \mathrm{E}[X] + \mathrm{E}[Y]$

   (c) $\mathrm{Var}(aX + b) = a^2\mathrm{Var}(X)$

7. Let X be a *continuous* random variable that cannot take negative values. Show the Markov's inequality, i.e. $P\{X \geq k\} \leq \frac{E[X]}{k}$. Why does the Weak Law of large numbers hold for continious random variables too?