Exercise sheet 1

Probability and Statistics, MTH102

1. Show from the axioms that $P(\emptyset) = 0$.
2. If a coin is tossed 3 times, compute and observe the probability of getting
   (a) 0 heads
   (b) 1 head
   (c) 2 heads
   (d) 3 heads
   (e) What do you think the sum of the probabilities that you computed above should be?
3. If I roll a pair of dice, and add the sum of the numbers on the dice, which number has the highest probability of occuring and why?
4. If you feel that the probability that it will rain tomorrow is 0.3 and you feel that probability that I will be absent tomorrow is 0.4. Let us assume that you feel that the probability that it rains tomorrow and that I will be absent is 0.2. What is the probability that...
   (a) I will be present tomorrow and it will rain tomorrow?
   (b) I will be absent tomorrow and that it will rain tomorrow?
   (c) I will be present tomorrow or that it will not rain tomorrow?
5. Consider a coin tossed n times.
   (a) What is the probability of getting k tails?
   (b) Use this to show that ${}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n = 2^n$
   (c) If n is even, what is the probability of getting heads in exactly half the number tosses?
   (d) If n is odd, use the previous part to show that ${}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_{\lfloor n/2 \rfloor} = 2^{n-1}$
6. Recall that we defined the "conditional probability" $P(A|B)$ which denotes "probability of A given that we know that B has occurred" by $P(A|B) = \frac{P(A \cap B)}{P(B)}$. If two events are independent, it means that the probability of A does not depend on B, i.e. $P(A|B) = P(A)$.
   (a) Show that if the A and B are independent then $P(A \cap B) = P(A)P(B)$
   (b) Show that if $P(A|B) = P(A)$, then $P(B|A) = P(B)$.

(c) If a coin is tossed twice, let A_1 denote the event of getting a head in the first toss, and A_2 denote the event of getting a head in the second toss. We may assume that they are independent events. Then use the above discussions to show that the probability of getting a head in both tosses is $1/4$.

(d) If a coin is tossed n times, use the above observations to compute the probability of getting all heads.

(e) Derive a formula for $P(A|B)$ in terms of $P(B|A)$, $P(A)$, and $P(B)$. (*Hint:* eliminate the $P(A \cap B)$ in the definitions of the conditional probabilities).