Exercise sheet 3

Probability and Statistics, MTH102

1. If on rolling a pair of dice it is known that the sum of the numbers that appear on the them is 8, what is the probability that at least one of them was a 3?

2. There are three rounds of admission tests to a particular institute. A person is allowed to take the test for the next round only if they clear the previous rounds. The probability of a person passing the first round is 0.5. The probability of passing the second round if it is known that the person passed the first, is 0.3, and the pobability of passing the third round, if it is known that the person passed the first two rounds, is 0.1.

   (a) What is the probability that a person passes all three rounds?

   (b) If it is known that a person did not get admission, what is the probability that that person failed the second round?

3. Fix an event E. Show that $P(.|E)$ satisfies the axioms of probability.

4. Let E denote an event and F_1, F_2, ..., F_n denote mutually exclusive events such that $F_1 \cup F_2 \cup \ldots \cup F_n = \Omega$, i.e. the entire sample space. In other words, exactly one of the events F_1, F_2, ..., F_n has to occur. Show that

$$P(F_i|E) = \frac{P(E|F_i)}{P(E|F_1)P(F_1) + P(E|F_2)P(F_2) + \cdots + P(E|F_n)P(F_n)}$$

5. If a drone is missing and is initially thought to be equally likely to be found in either one of three regions: the first region has dense trees, and the probability of overlooking it if the drone were there is high and estimated to be around 0.7. The second region has has fewer trees and the probability of overlooking the drone there is 0.5. The third region has very few trees and so the probability of overlooking the drone there is very small at 0.2.

   (a) Let N_3 denote the event that the drone was *not* found in region 3. Let R_1 denote the event that the drone is in region 1, R_2 denote the event that the drone is in region 2, and R_3 denote the event that the drone is in region 3. What are $P(R_1)$, $P(R_2)$, $P(R_3)$, $P(N_3|R_1)$, $P(N_3|R_2)$, and $P(N_3|R_3)$?

   (b) If the drone was not found in region 3, how will you update the probabilities of it being found in each of the three regions. Note which ones have increased and which ones have decreased.

6. In a box there are 2 biased coins and 8 unbiased coins. Both biased coins have a 70 percent chance of landing on a head when tossed.

   (a) You toss the coin and get a head. What is the probability that you picked a biased coin?

   (b) You toss the coin 1000 times and get a head 700 times. What is the probability that you picked a head?

7. Assume that a coin is biased so that the probability of getting a head is 0.1. How many times must the coin be tossed so that the probability of getting at least one head is more than half?