Exercise sheet 5

Probability and Statistics, MTH102

1. For some random variable X, and some real numbers a and b, show that $\mathrm{Var}(aX + b) = a^2\mathrm{Var}(X)$.
2. Consider a random variable for which $E[X] = 1$ and $\mathrm{Var}(X) = 5$. Then compute $E[(2 + X)^2]$ and $\mathrm{Var}(4 + 3X)$.
3. Let X denote the random variable which is the number of times a coin lands on heads if it is tossed n times. Show that $P\{X = k + 1\} = \frac{p}{1-p}\frac{n-k}{k+1}P\{X = k\}$
4. Use the above to compute the approximate probability of getting k heads, for each k, for an unbiased coin which is tossed 5 times.
5. A multiple choice exam has 20 questions and each question has exactly 4 choices out of which only one is correct.
   (a) If a student guesses all the answers, what is the probability that the person will get all the answers correct?
   (b) How many points should we assign a correct answer or a wrong answer so that the expected score is 0 for a student who guesses all the answers.
6. Show that $E[X + Y] = E[X] + E[Y]$ for any two random variables X and Y.
7. If X and Y are independent, then show that $E[X \times Y] = E[X]E[Y]$.
8. If X and Y are independent, then show that $\mathrm{Var}(X + Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$.