Exercise sheet 2

Probability and Statistics, MTH102

1. Show that if the probability of $P(E|F) = P(E|F^c)$ then E is independent of F. In other words, if the probability of E remains unchanged whether F occurs or does not occur, then E must be independent of F.

2. Show that if $P(E|F^c) = 0$, then $P(F|E) = 1$. Can you give a real life interpetation of this?

3. Show that $P(A \cap B \cap C) = P(A)P(B|A)P(C|A \cap B)$

4. Show that $P(E_1 \cap E_2 \cap E_3 \cap E_4) = P(E_1)P(E_2|E_1)P(E_3|E_1 \cap E_2)P(E_4|E_1 \cap E_2 \cap E_3)$

5. If a deck of playing cards is divided into 4 piles of 13 cards each then what is the probability that each pile has at least one ace.

6. If a pair of dice is rolled and we know that the numbers on it are different, what is the probability that one of them is a 4?

7. Suppose 90% of all the students who attend the lectures regularly pass a certain course. While, only 30% of students who do not attend the lectures regularly pass the course. If I know that a student passed the course, what is the probability that that student attended regularly? Assume that half the students attended the course regularly.

8. Prove that if E and F are independent events, then so is E and F^c.

9. If three cards are in a box so that one card has both sides black, another has has both sides white, and the remaining card has one side black and one side white. If a card is picked at random and is found to have one side white, what is the probability that the other side is black?

10. A scientist has four competing hypothesis, say H_1, H_2, H_3, and H_4 for a particular phenomenon. Initially, the scientist assumes that H_1 is very likely and believes the probability to be around 0.7. However, the scientist believes that the other 3 hypotheses are equally likely. To test this the scientist performs an experiment and obtains some outcome E. Unfortunately, the experiment is not foolproof. If H_1 were true, then the probability of the experimental outcome being E is 0.1. If H_2 were true, then E willstill occur with probability 0.8. If H_3 were true, then the probability of the experimental outcome being E is 0.2, and if H_4 were true, then the probability of the experimental outcome being E is 0.1. How should the scientiest update the probabilities of H_1, H_2, H_3, and H_4,

now that the experiment has been performed and resulted in outcome E. Which hypothesis does the scientist now believe is most probable? Which hypothesis does the scientist now believe is most probable?