$\tilde{\gamma}$:

$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta})$

$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to \mathbb{R}^2$

$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to \mathbb{R}^2$ is a *unit speed*

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Therefore,

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Therefore, $\mathbf{T}(\tilde{t})$

$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to \mathbb{R}^2$ is a *unit speed* parametrization.
Therefore, $\mathbf{T}(\tilde{t}) = \dot{\tilde{\gamma}}(\tilde{t})$

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Therefore, $\mathbf{T}(\tilde{t}) = \dot{\tilde{\gamma}}(\tilde{t})$
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$\mathbf{N}_s(\tilde{t})$

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Therefore, $\mathbf{T}(\tilde{t}) = \dot{\tilde{\gamma}}(\tilde{t})$
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Therefore, $\dot{\mathbf{T}}(\tilde{t}) = \ddot{\tilde{\gamma}}(\tilde{t}) =??\mathbf{N}_s(\tilde{t})$

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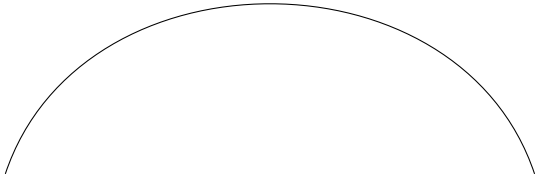

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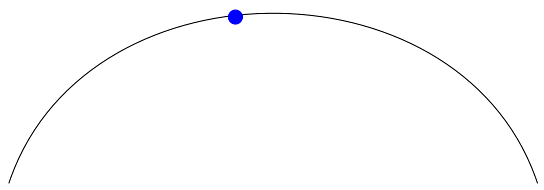

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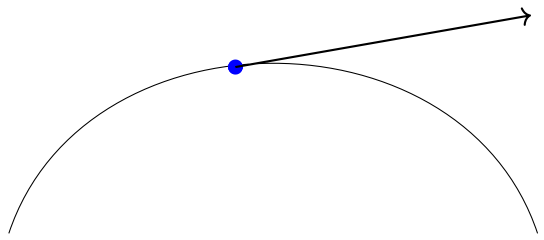

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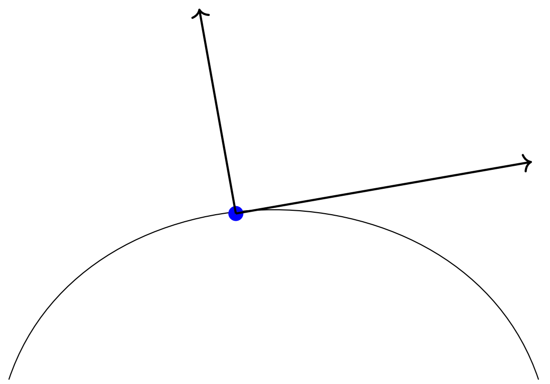

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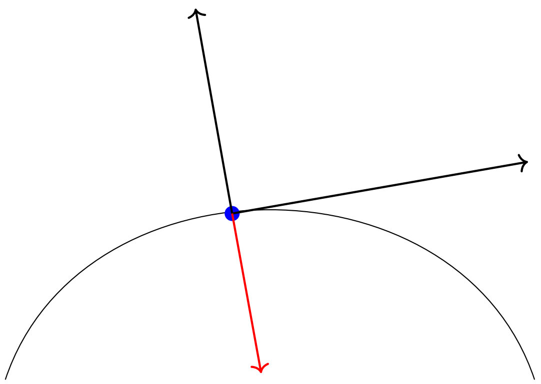

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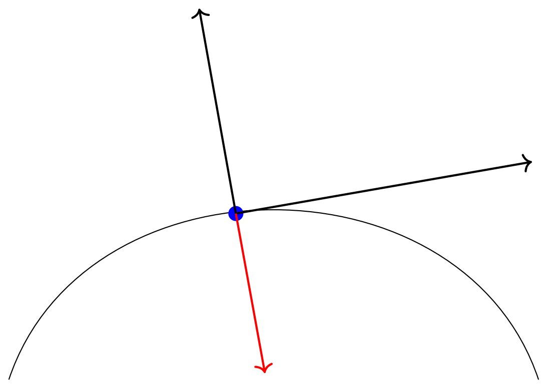

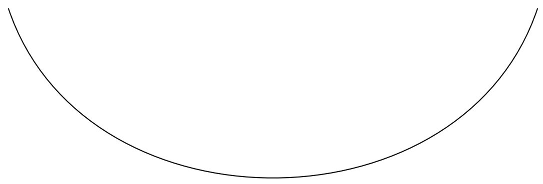

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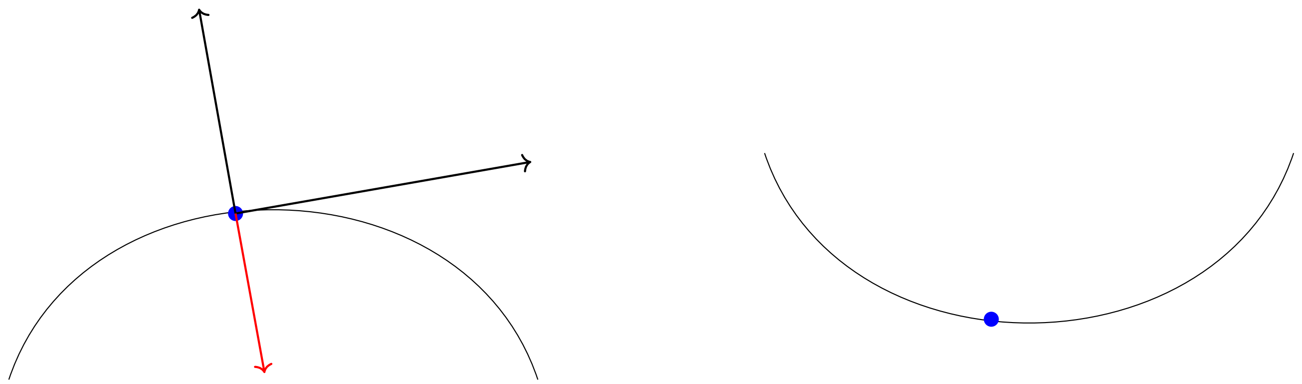

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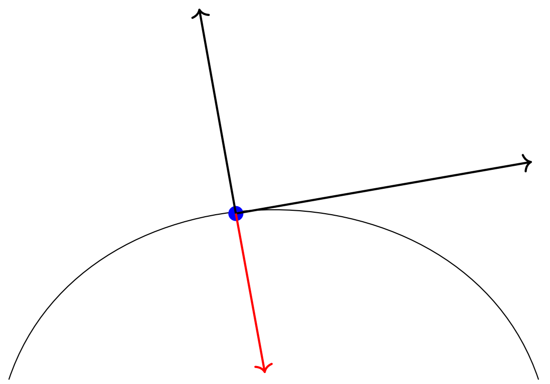

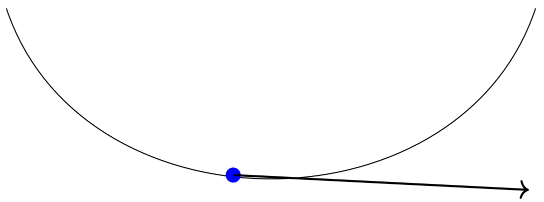

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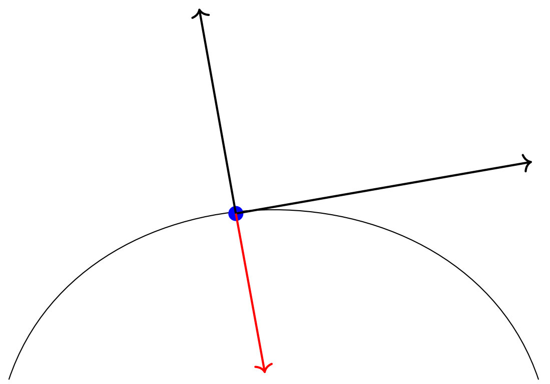

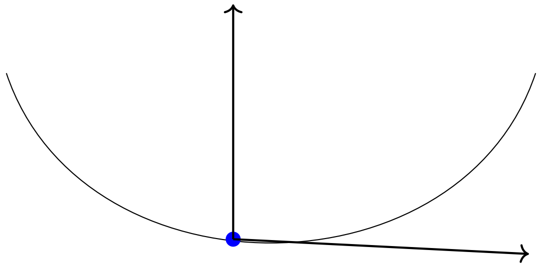

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Therefore, $\mathbf{T}(\tilde{t}) = \dot{\tilde{\gamma}}(\tilde{t})$

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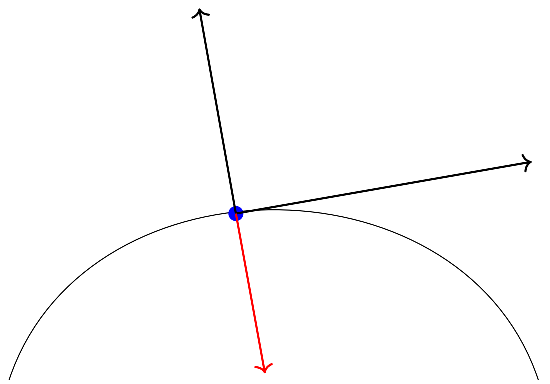

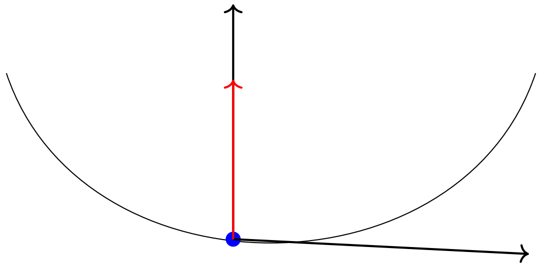

$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to \mathbb{R}^2$ is a *unit speed* parametrization. γ :

Therefore, $\mathbf{T}(\tilde{t}) = \dot{\tilde{\gamma}}(\tilde{t})$

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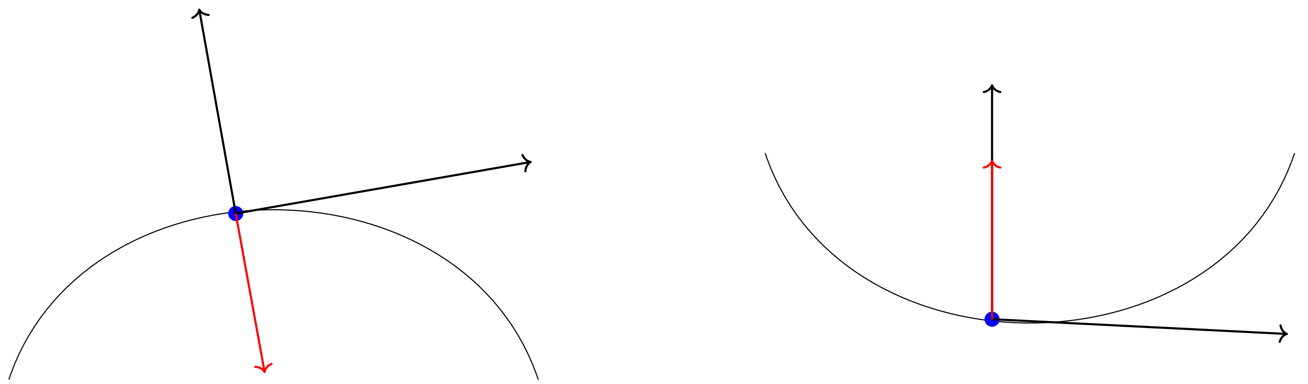

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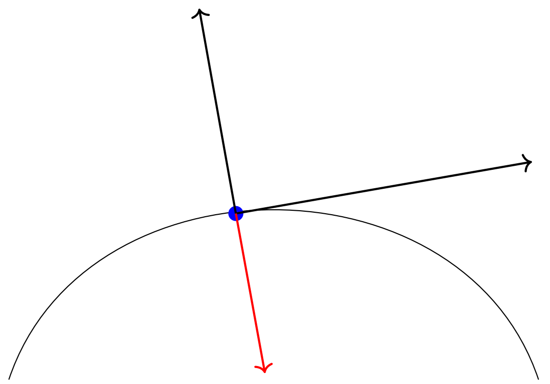

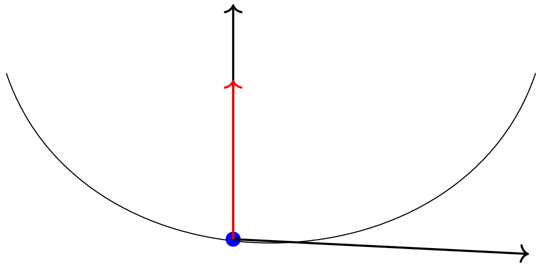

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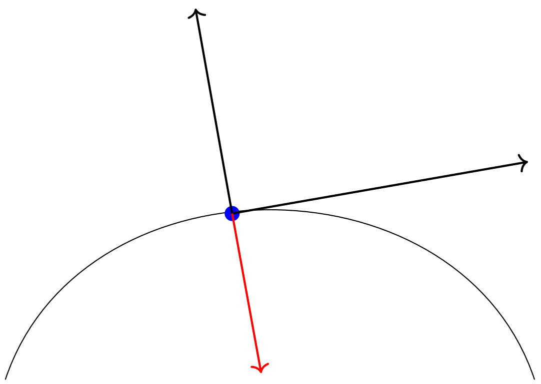

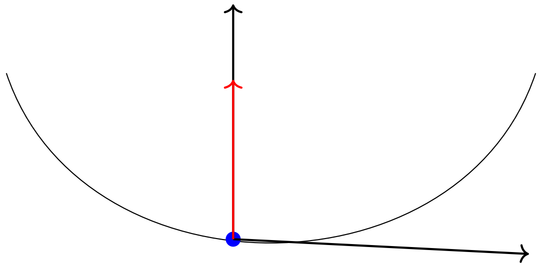

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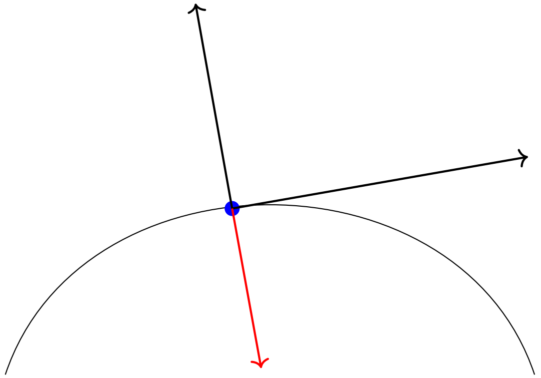

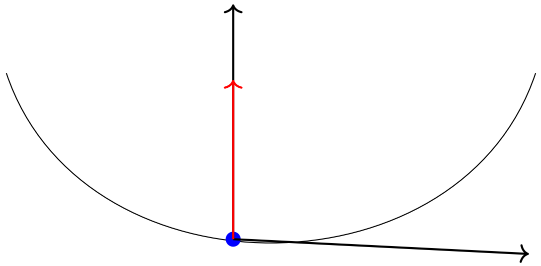

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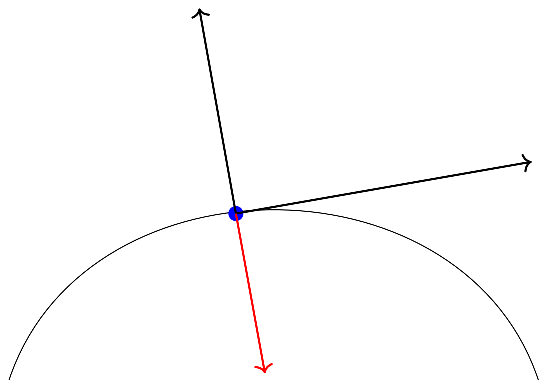

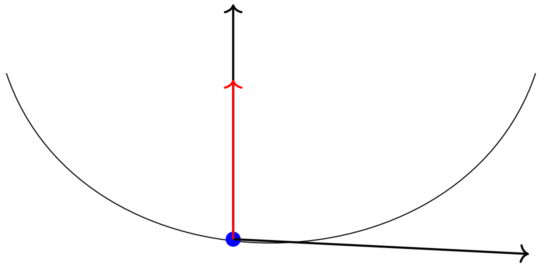

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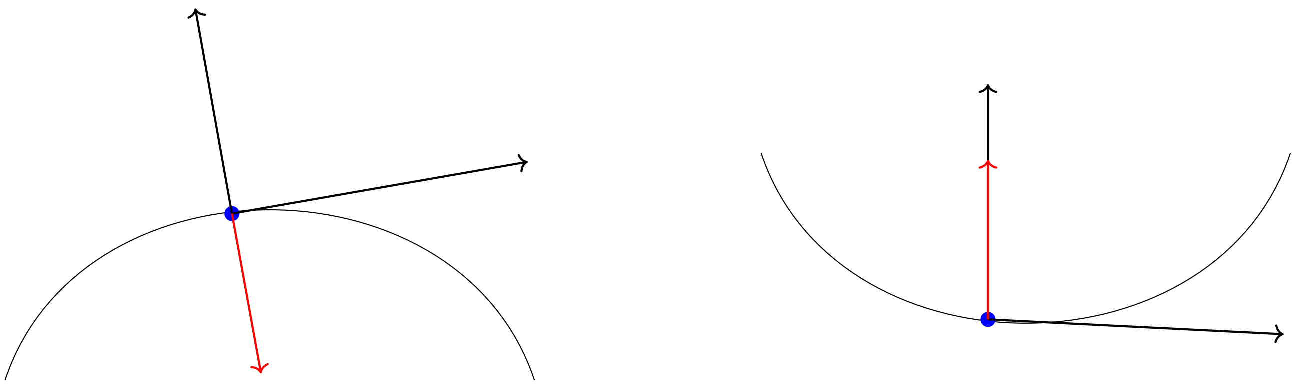

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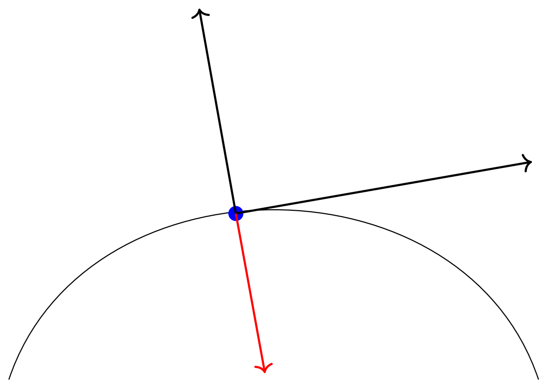

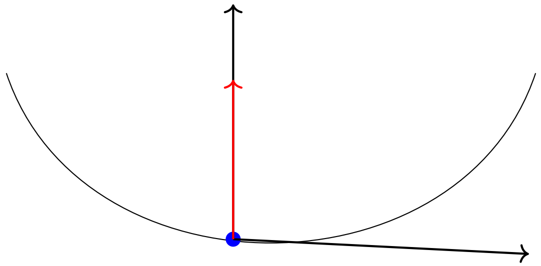

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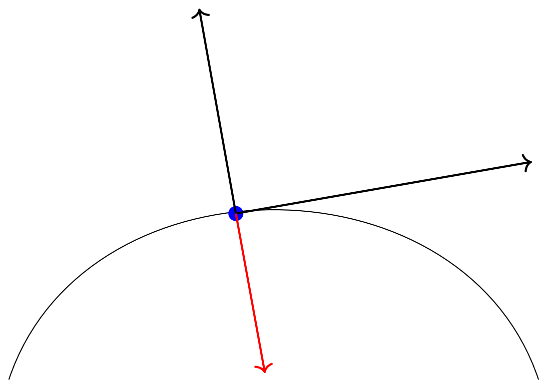

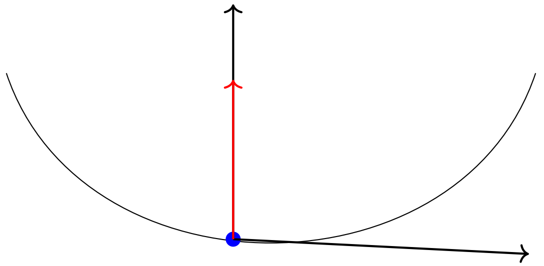

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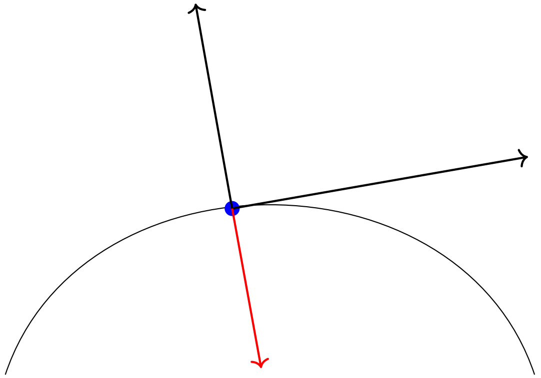

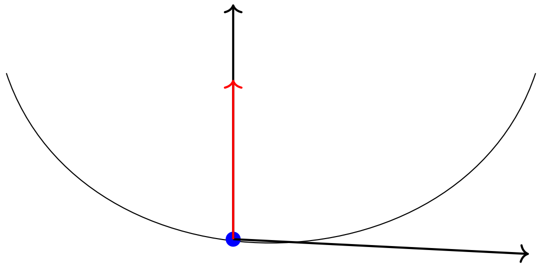

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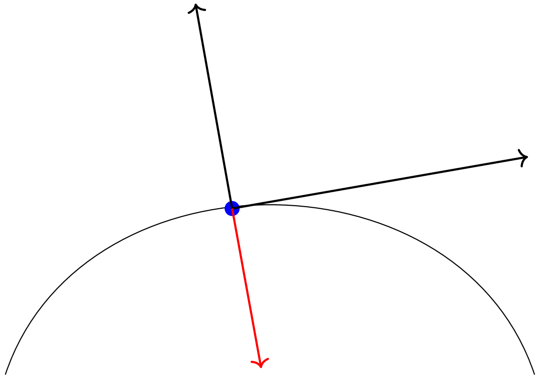

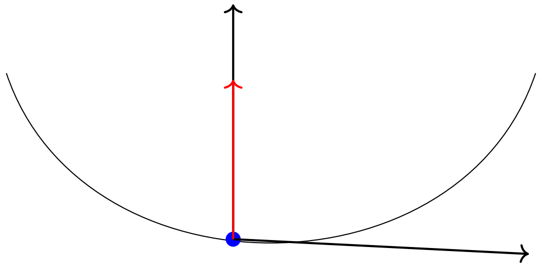

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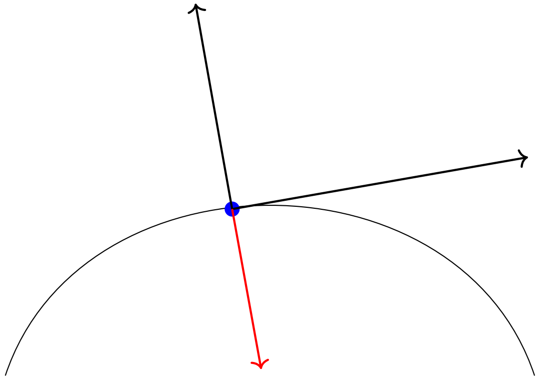

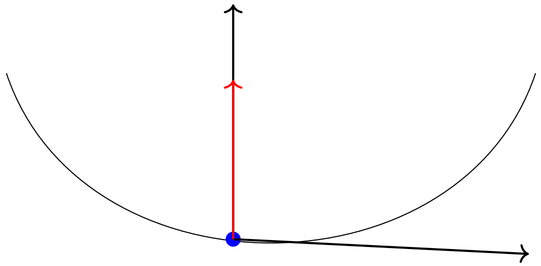

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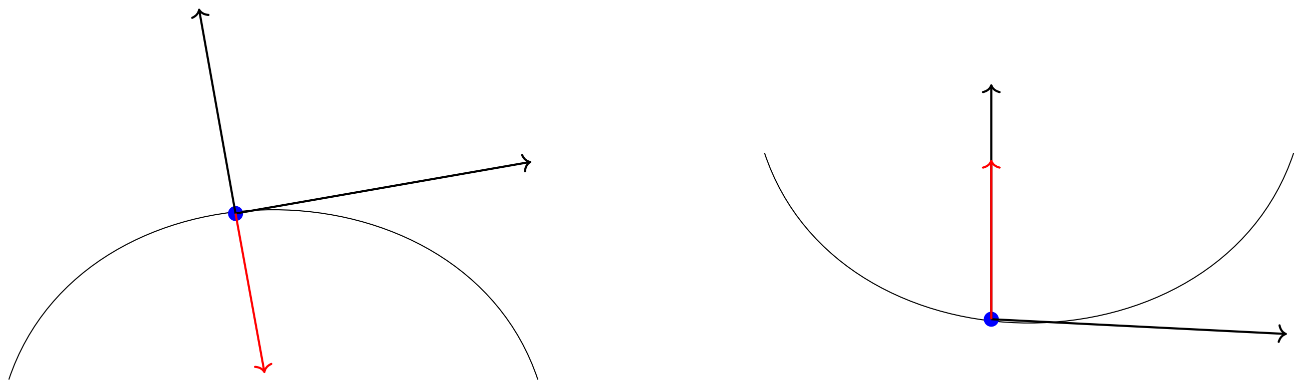

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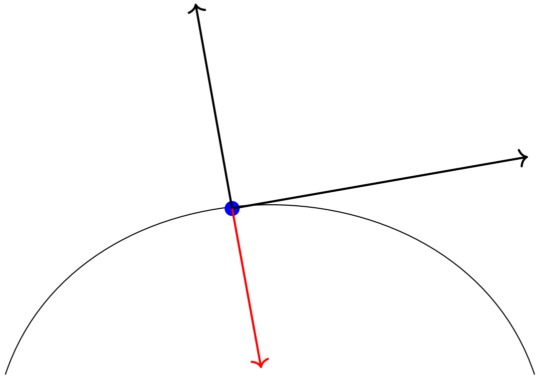

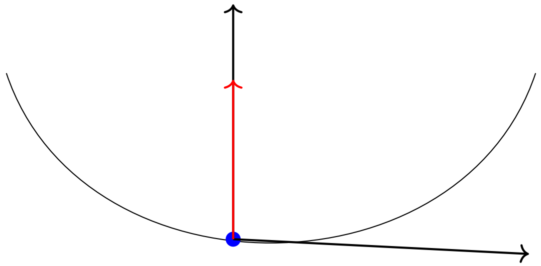

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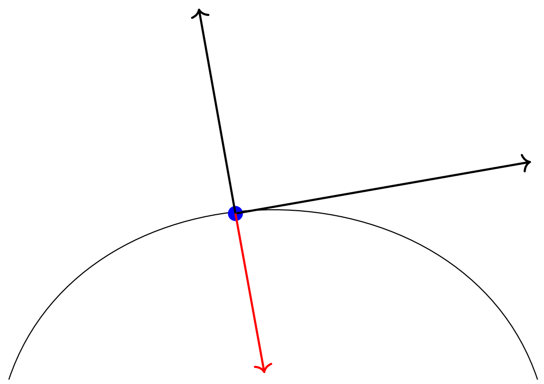

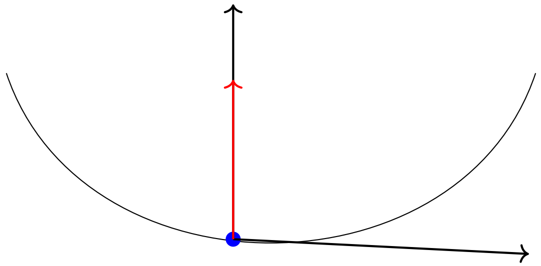

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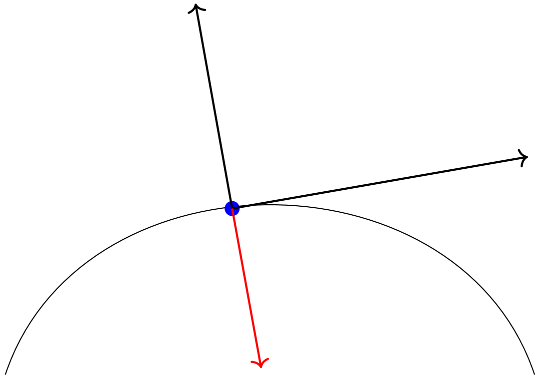

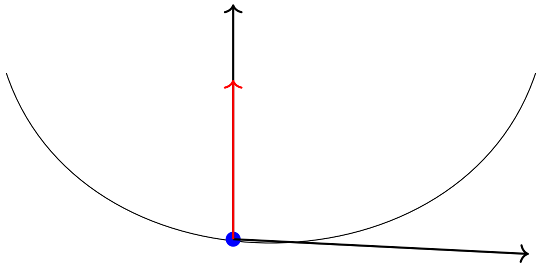

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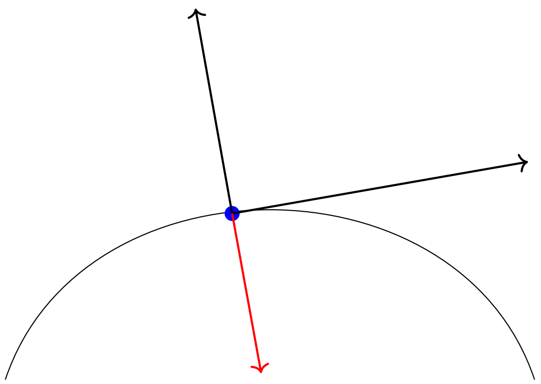
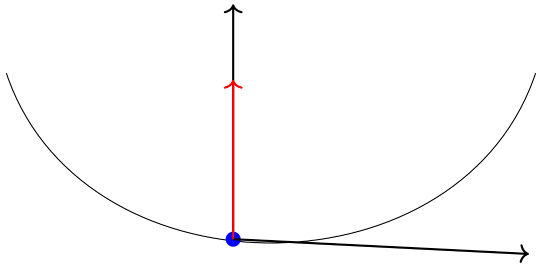




$\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is **not** a unit speed parametrization.
Then, $\gamma(t) = \tilde{\gamma}(s(t))$

$$\begin{aligned} \dot{\gamma}(t) &= \dot{\tilde{\gamma}}(s(t))s'(t) \\ &= \mathbf{T}(s(t))\|\dot{\gamma}(t)\| \end{aligned}$$

$$\begin{aligned} \left(\frac{\dot{\gamma}(t)}{\|\dot{\gamma}(t)\|}\right)' &= (\mathbf{T}(s(t)))' \\ &= \dot{\mathbf{T}}(s(t))s'(t) \\ &= \kappa_s(s(t))s'(t)\mathbf{N}_s(s(t)) \\ &= \kappa_s(s(t))\|\gamma(t)\|\mathbf{N}_s(s(t)) \end{aligned}$$

$$\kappa_s(s(t))\mathbf{N}_s(s(t)) = \frac{1}{\|\dot{\gamma}(t)\|}\left(\frac{\dot{\gamma}(t)}{\|\dot{\gamma}(t)\|}\right)'$$

$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to \mathbb{R}^2$ is a *unit speed* parametrization.

Therefore, $\mathbf{T}(\tilde{t}) = \dot{\tilde{\gamma}}(\tilde{t})$

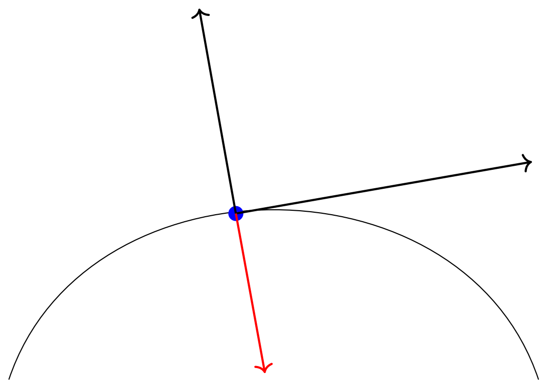
Therefore, $\dot{\mathbf{T}}(\tilde{t}) = \ddot{\tilde{\gamma}}(\tilde{t})$ is perpendicular to $\mathbf{T}(\tilde{t}) = \dot{\tilde{\gamma}}(\tilde{t})$

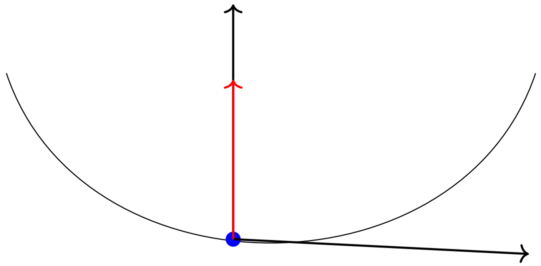
$\mathbf{N}_s(\tilde{t})$ denote the *unit* vector obtained by rotating $\mathbf{T}(\tilde{t})$ *anticlockwise*.

Therefore, $\dot{\mathbf{T}}(\tilde{t}) = \ddot{\tilde{\gamma}}(\tilde{t}) = \kappa_s(\tilde{t})\mathbf{N}_s(\tilde{t})$

$\kappa_s(\tilde{t})$ called the **signed curvature**.

$\kappa(\tilde{t}) = \|\ddot{\tilde{\gamma}}(\tilde{t})\| = \|\kappa_s \mathbf{N}_s(\tilde{t})\| = |\kappa_s|\|\mathbf{N}_s(\tilde{t})\| = |\kappa_s(\tilde{t})|$





$\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is **not** a unit speed parametrization.

Then, $\gamma(t) = \tilde{\gamma}(s(t))$

$$
\begin{aligned}
\dot{\gamma}(t) &= \dot{\tilde{\gamma}}(s(t))s'(t) \\
&= \mathbf{T}(s(t))\|\dot{\gamma}(t)\|
\end{aligned}
$$

$$
\begin{aligned}
\left(\frac{\dot{\gamma}(t)}{\|\dot{\gamma}(t)\|}\right)' &= (\mathbf{T}(s(t)))' \\
&= \dot{\mathbf{T}}(s(t))s'(t) \\
&= \kappa_s(s(t))s'(t)\mathbf{N}_s(s(t)) \\
&= \kappa_s(s(t))\|\gamma(t)\|\mathbf{N}_s(s(t))
\end{aligned}
$$

$$
\begin{aligned}
\kappa_s(s(t))\mathbf{N}_s(s(t)) &= \frac{1}{\|\dot{\gamma}(t)\|}\left(\frac{\dot{\gamma}(t)}{\|\dot{\gamma}(t)\|}\right)' \\
&= \frac{1}{\|\dot{\gamma}(t)\|}\frac{\|\dot{\gamma}(t)\|\ddot{\gamma}(t) - \dot{\gamma}(t)\frac{\dot{\gamma}(t).\ddot{\gamma}(t)}{\|\dot{\gamma}(t)\|}}{\|\dot{\gamma}(t)\|^2}
\end{aligned}
$$