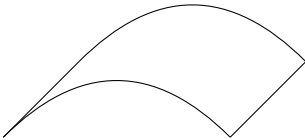


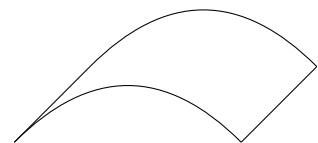
Surfaces



Definition (Surface patch).

$$\sigma :$$

Surfaces

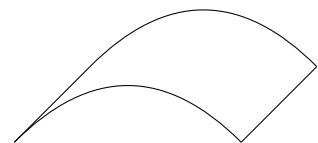


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Surfaces

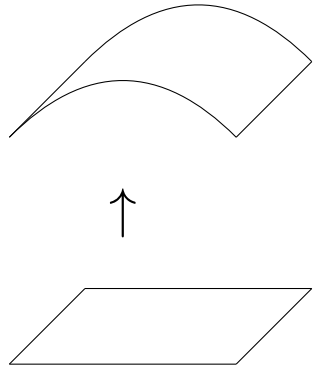


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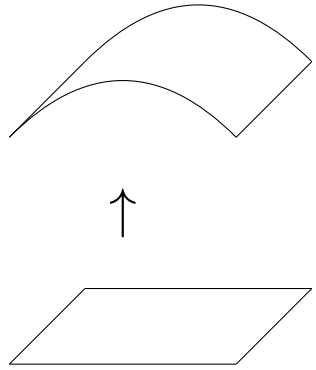
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Coordinate transformation

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Importance of partial derivatives

$$f$$

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$$\gamma(t)$$

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$$\gamma(t) = (x(t),y(t))$$

$$f \circ \gamma : (\alpha,\beta) \rightarrow \mathbb{R}$$

$$\begin{aligned} (f \circ \gamma)'(t_0) &= f_x(x(t_0),y(t_0))x'(t_0) + \\ f_y(x(t_0),y(t_0))y'(t_0) &= (f_x(x,y),f_y(x,y)).\dot{\gamma}(t_0) \end{aligned}$$

Coordinate transformation

$$\sigma : U \rightarrow \mathbb{R}^3$$

$$\tilde{\sigma} : \tilde{U} \rightarrow \mathbb{R}^3$$

$$\Phi : \tilde{U} \rightarrow U \text{ smooth, invertible, and inverse smooth}$$

$$\tilde{\sigma}(x, y) = \sigma(\Phi(x, y))$$

Importance of partial derivatives

$$f : \mathbb{R}^2 \rightarrow \mathbb{R}$$

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$$f \circ \gamma : (\alpha, \beta) \rightarrow \mathbb{R}$$

$$(f \circ \gamma)'(t_0) = f_x(x(t_0), y(t_0))x'(t_0) + f_y(x(t_0), y(t_0))y'(t_0) = \nabla(f)(x(t_0), y(t_0)) \cdot \dot{\gamma}(t_0),$$

$$\text{where } \nabla(f)(x, y) = (f_x(x, y), f_y(x, y)),$$

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where $\nabla(f)(x,y) = (f_x(x,y),f_y(x,y))$,

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$$\boxed{f_{\mathbf{v}}(x(t_0), y(t_0)) := (f \circ \gamma)'(t_0) = \nabla(f)(x(t_0), y(t_0)) \cdot \dot{\gamma}(t_0)},$$

$$\text{where } \nabla(f)(x, y) = (f_x(x, y), f_y(x, y)),$$

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$f_{\mathbf{v}}(x(t_0),y(t_0)) := (f \circ \gamma)'(t_0) = \nabla(f)(x(t_0),y(t_0)).\mathbf{v},$

where $\nabla(f)(x,y) = (f_x(x,y),f_y(x,y)),$

$$\mathbf{v} = \dot{\gamma}(t_0),$$

Coordinate transformation

$$\begin{aligned}\sigma &: U \rightarrow \mathbb{R}^3 \\ \tilde{\sigma} &: \tilde{U} \rightarrow \mathbb{R}^3 \\ \Phi &: \tilde{U} \rightarrow U \text{ smooth, invertible, and inverse smooth} \\ \tilde{\sigma}(x,y) &= \sigma(\Phi(x,y))\end{aligned}$$

Importance of partial derivatives

$$\begin{aligned}f &: \mathbb{R}^2 \rightarrow \mathbb{R} \\ \gamma &: (\alpha,\beta) \rightarrow \mathbb{R}^2 \\ \gamma(t) &= (x(t),y(t)) \\ f \circ \gamma &: (\alpha,\beta) \rightarrow \mathbb{R}\end{aligned}$$

$f_{\mathbf{v}}(x(t_0),y(t_0)) := (f \circ \gamma)'(t_0) = \nabla(f)(p).\mathbf{v},$

where $\nabla(f)(x,y) = (f_x(x,y),f_y(x,y)),$

$\mathbf{v} = \dot{\gamma}(t_0),$

and $p = (x(t_0),y(t_0))$