Exercise sheet 6

Curves and Surfaces, MTH201

1. If $\mathbf{v}_1(t), \mathbf{v}_2(t)$, and $\mathbf{v}_3(t)$, unit vector fields which are *not necessarily* $\mathbf{T}(t), \mathbf{N}(t), \mathbf{B}(t)$, but nevertheless satisfy the same equations:

$$\dot{\mathbf{v}}_1 = \kappa(t)\mathbf{v}_2(t)$$
$$\dot{\mathbf{v}}_2 = -\kappa(t)\mathbf{v}_1(t) + \tau(t)\mathbf{v}_3(t)$$
$$\dot{\mathbf{v}}_3 = -\tau(t)\mathbf{v}_2(t)$$

   This exercise will show that if $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ are equal to $\mathbf{T}$, $\mathbf{N}$, and $\mathbf{B}$, respectively, for one t_0, then they are equal for all t, but in a slightly less straightforward manner than you may expect to work out.

   (a) Show that $f(t) = \mathbf{v}_1(t).\mathbf{T}(t) + \mathbf{v}_2(t).\mathbf{N}(t) + \mathbf{v}_3(t).\mathbf{B}(t)$ is constant. Note that we are not saying that the individual terms themselves are constant, so this exercise is not saying, for instance, that the the dot products, $\mathbf{v}_1(t).\mathbf{T}(t)$, $\mathbf{v}_2(t).\mathbf{N}(t)$, or $\mathbf{v}_3(t).\mathbf{B}(t)$, are individually constant; it is only when we take their sum that we can ensure the resulting sum is constant. Luckily, this will be sufficient to prove the next part.

   (b) Show that if $\mathbf{v}_1(t_0) = \mathbf{T}(t_0)$, $\mathbf{v}_2(t_0) = \mathbf{N}(t_0)$, or $\mathbf{v}_3(t_0) = \mathbf{B}(t_0)$, for some t_0, then the equalities hold for all t. (*Hint:* What is an equivalent condition, in terms of dot products, for two vectors being equal? Remember, these are unit vectors! Note that the previous part was only useful because we had equality of the corresponding vectors at t_0.)