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$(f(g(x)))' = f'(g(x))g'(x)$	$(\sin(x^2))' =$

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$(f/g)' = \frac{f'g - fg'}{g^2}$	$(\frac{\sin(x)}{x^2})' = \frac{\cos(x)x^2 - \sin(x)2x}{x^4}$
$(f(g(x)))' = f'(g(x))g'(x)$	$(\sin(x^2))' = \cos(x^2)2x$

Derivative facts

1. $c' = 0$
2. $(x^n)' = nx^{n-1}$
3. $(\sin(x))' = \cos(x)$
4. $(\cos(x))' = -\sin(x)$
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7. $(\mathrm{e}^x)' = \mathrm{e}^x$

Example.

$$(x^2\sin(x^3) + \cos(x))' \; =$$

Rule	**Example**
$(cf)' = cf'$, where $c \in \mathbb{R}$	$(2\sin(x))' = 2\cos(x)$
$(f+g)' = f' + g'$	$(\sin(x) + x^3)' = \cos(x) + 3x^2$
$(f-g)' = f' - g'$	$(\sin(x) - x^3)' = \cos(x) - 3x^2$
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Example.

$$(x^2 \sin(x^3) + \cos(x))' \;=\; 2x\sin(x^3) + x^2(\sin(x^3))'$$

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Example

Example

γ :

Example

$\gamma : (-\pi,$

Example

$\gamma : (-\pi, \pi)$

Example

$\gamma : (-\pi, \pi) \to \mathbb{R}^2$

Example

$\gamma : (-\pi, \pi) \to \mathbb{R}^2$
$\gamma(t)$

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$\gamma : (-\pi, \pi) \to \mathbb{R}^2$
$\gamma(t) := (r\cos(t), r\sin(t))$

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$\dot{\gamma}(t)$

Example

$\gamma : (-\pi, \pi) \to \mathbb{R}^2$
$\gamma(t) := (r\cos(t), r\sin(t))$
$\dot{\gamma}(t) := (-r\sin(t), r\cos(t))$

Example

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$$\gamma(t) := (r\cos(t), r\sin(t))$$
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The "speed" at time t is defined as

$$\sqrt{v_1(t)^2 + v_2(t)^2}$$

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$$\sqrt{v_1(t)^2 + v_2(t)^2} \;=\; \sqrt{(-r\sin(t))^2 + (r\cos(t))^2}$$

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Can we find a parametrization of the circle

Example

Making the "speed" 1

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Can we find a parametrization of the circle to ensure the speed is 1?

Example

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Making the "speed" 1

$\gamma :$

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Can we find a parametrization of the circle to ensure the speed is 1?

Making the "speed" 1

$\gamma : (-r\pi,$

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Can we find a parametrization of the circle to ensure the speed is 1?

Making the "speed" 1

$\gamma : (-r\pi, r\pi)$

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Making the "speed" 1

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Can we find a parametrization of the circle to ensure the speed is 1?

Making the "speed" 1

$\gamma : (-r\pi, r\pi) \to \mathbb{R}^2$
$\gamma(t)$

Example

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Making the "speed" 1

$\gamma : (-r\pi, r\pi) \to \mathbb{R}^2$
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The "speed" is

$$\sqrt{v_1(t)^2 + v_2(t)^2}$$

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The "speed" is

$$\sqrt{v_1(t)^2 + v_2(t)^2} = \sqrt{(-\sin(t))^2 + (\cos(t))^2}$$

Example

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Can we find a parametrization of the circle to ensure the speed is 1?

Making the "speed" 1

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