$\tilde{\gamma}$:

$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta})$

$$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to \mathbb{R}^2$$

$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to \mathbb{R}^2$ is a *unit speed*

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Therefore,

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Therefore, $\mathbf{T}(\tilde{t})$

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Therefore, $\mathbf{T}(\tilde{t}) = \dot{\tilde{\gamma}}(\tilde{t})$

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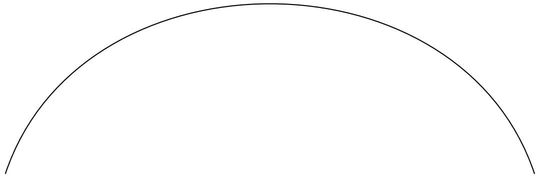

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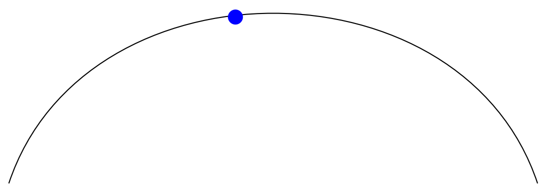

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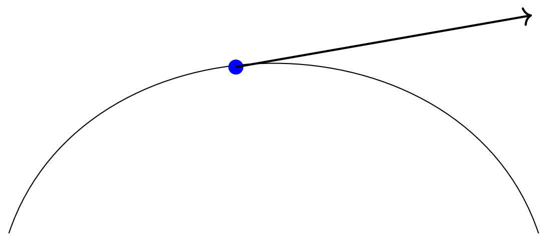

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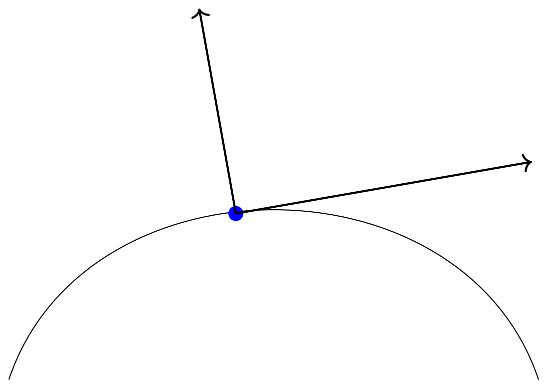

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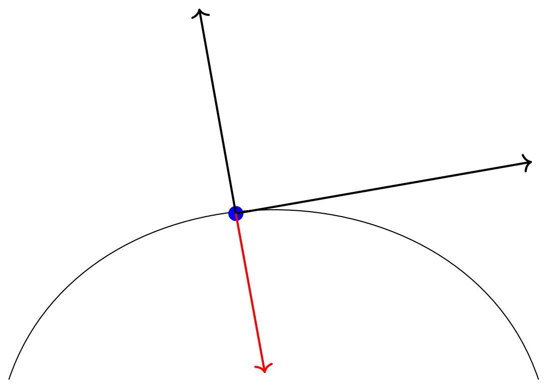

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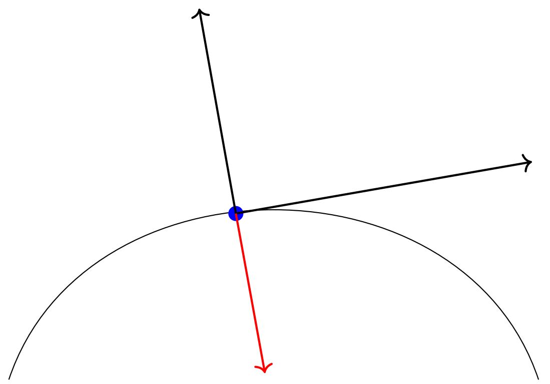

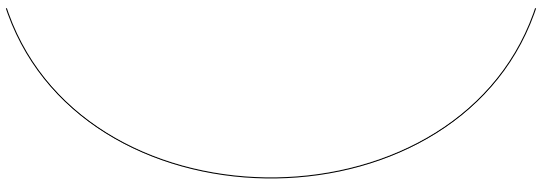

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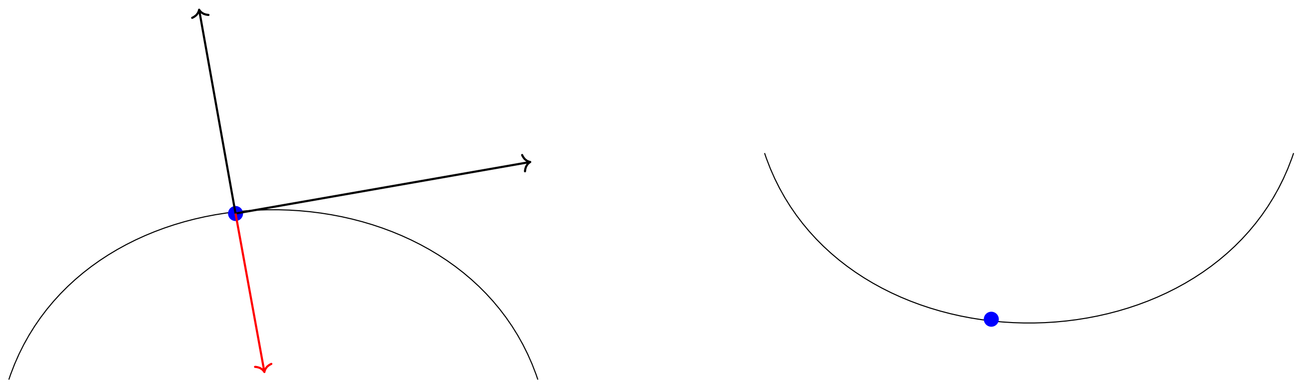

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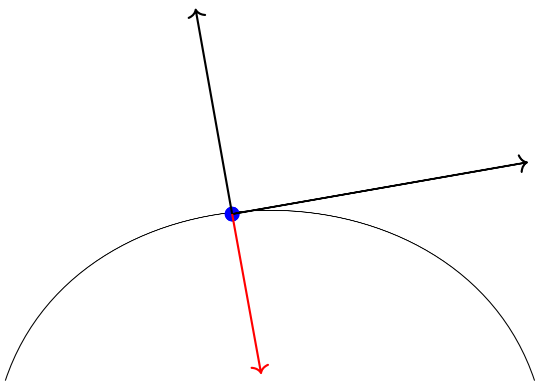

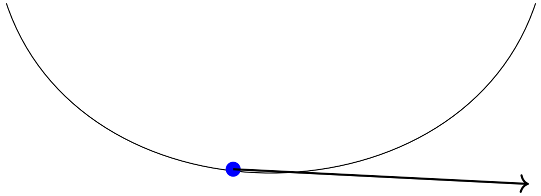

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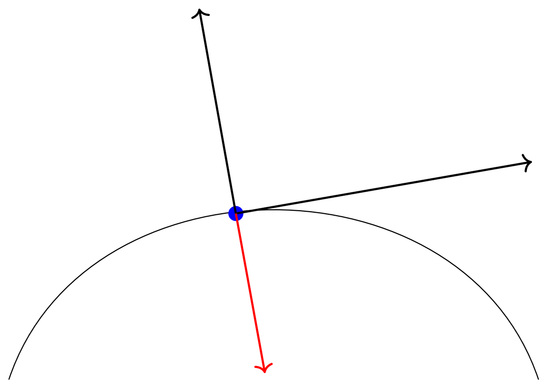

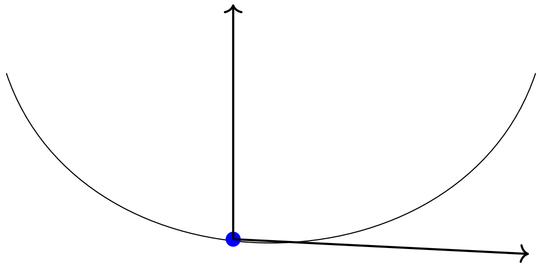

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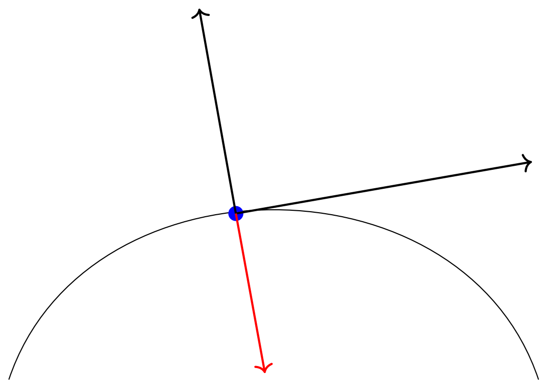

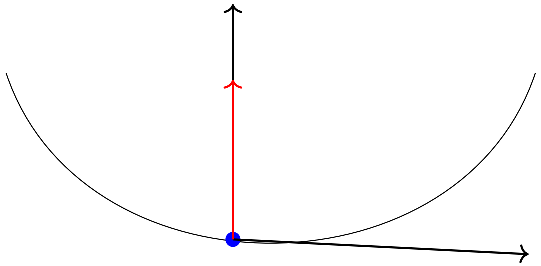

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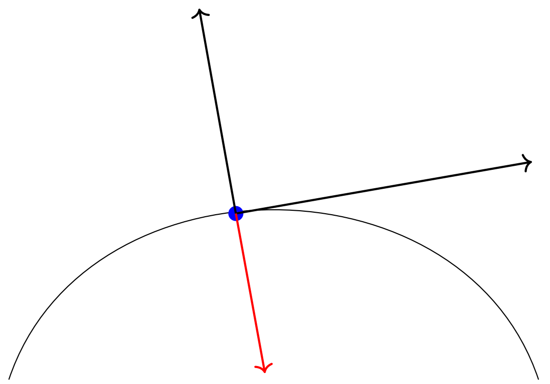

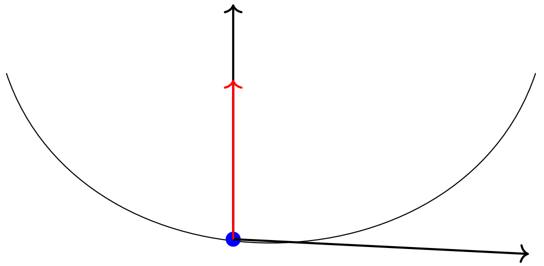

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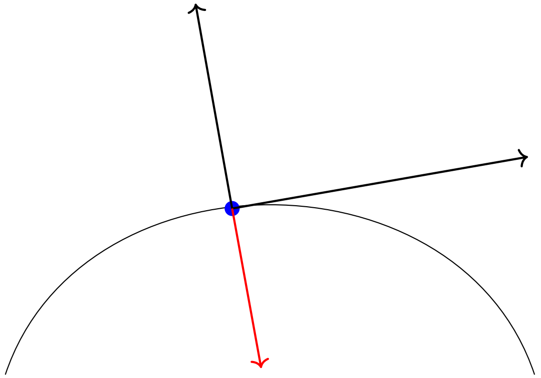

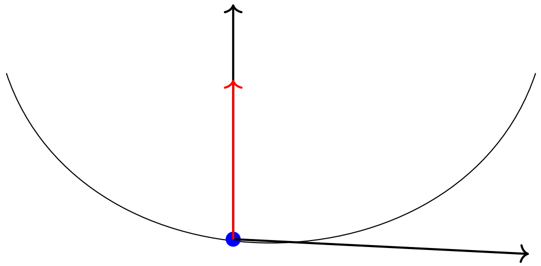

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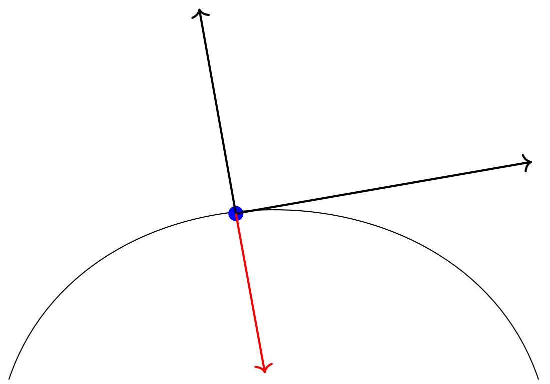

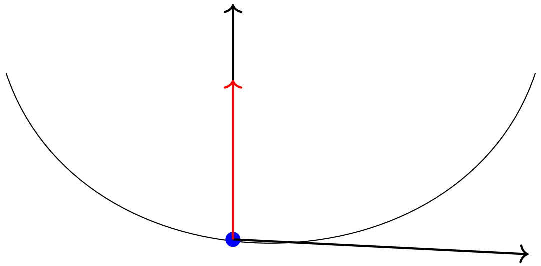

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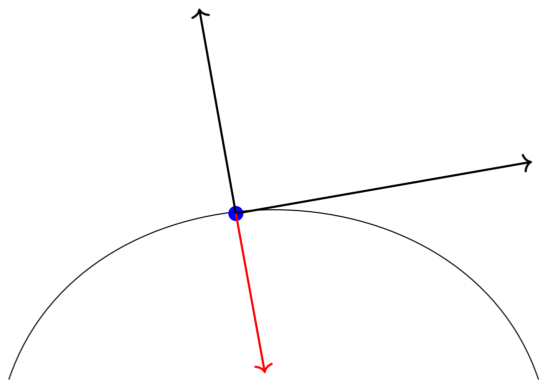

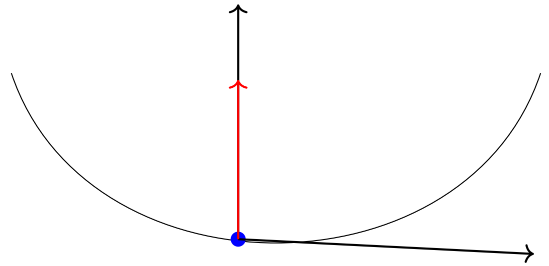

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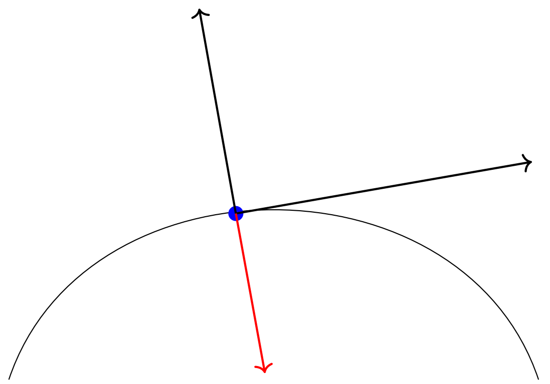

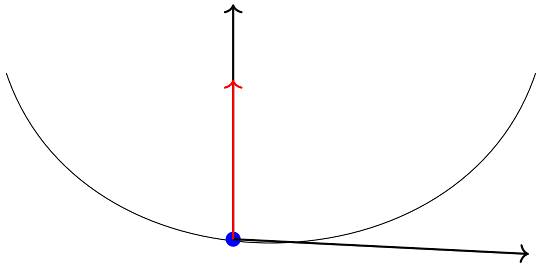

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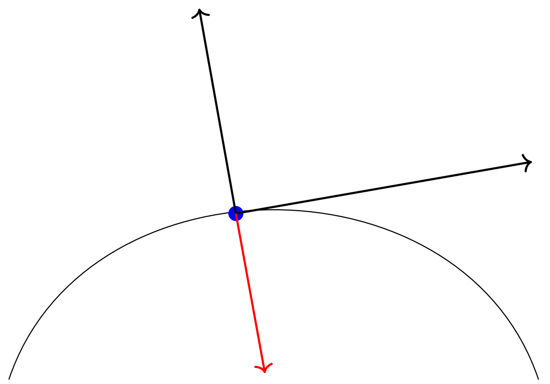

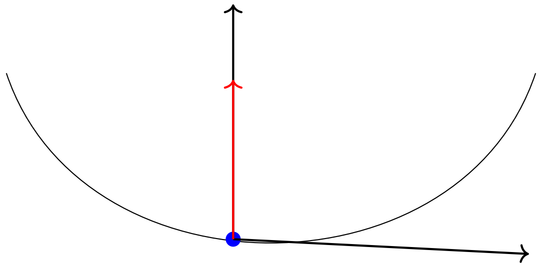

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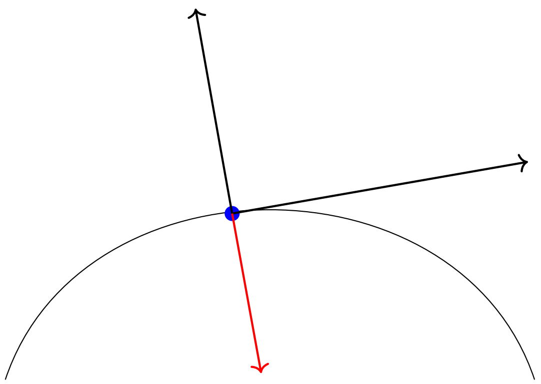

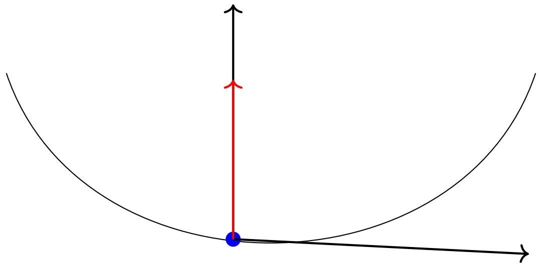

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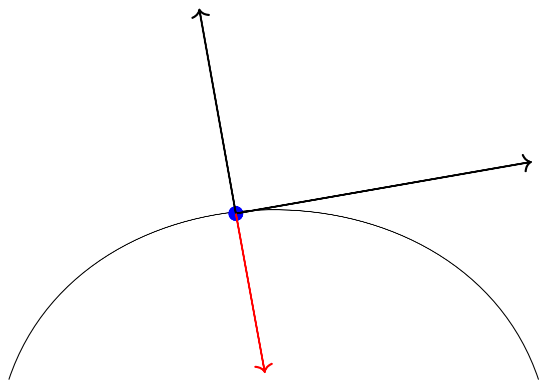

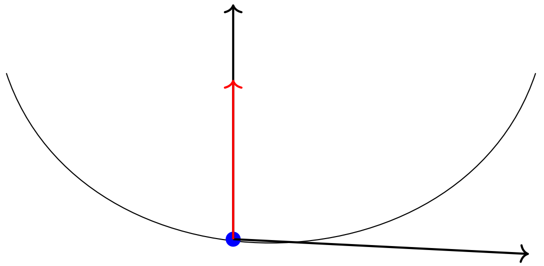

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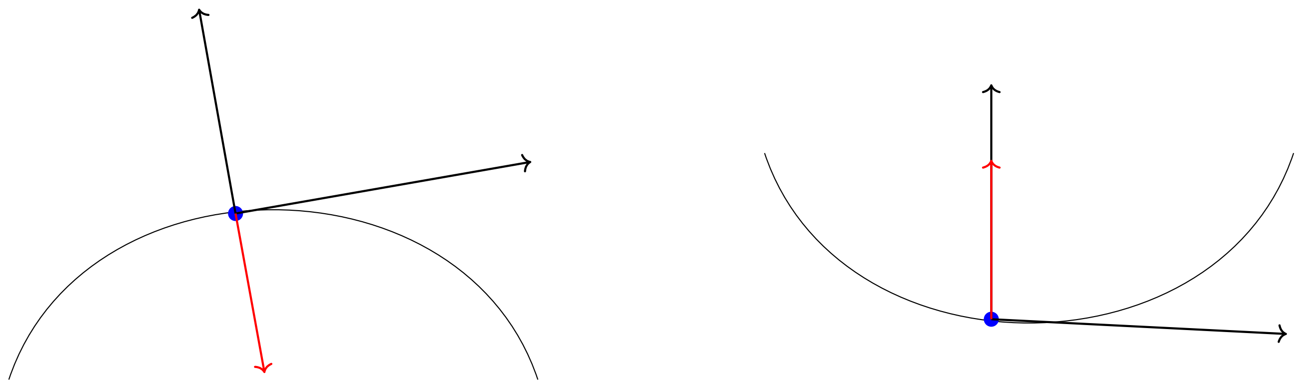

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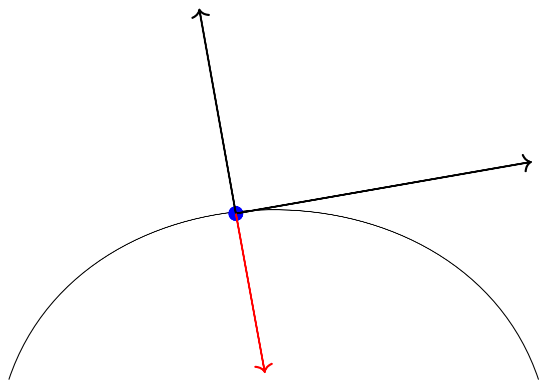

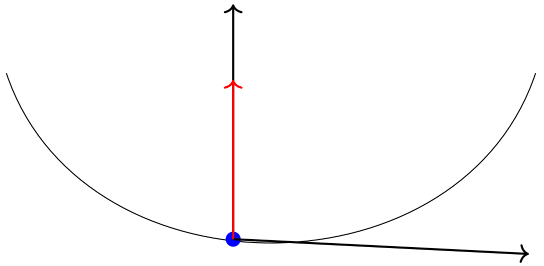

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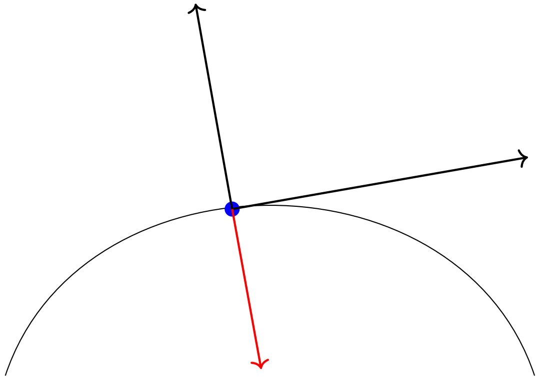

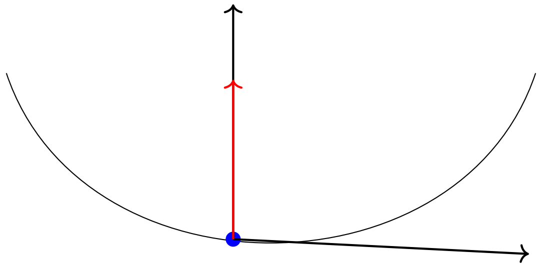

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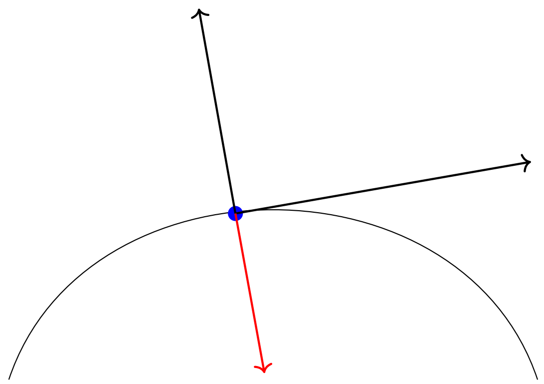

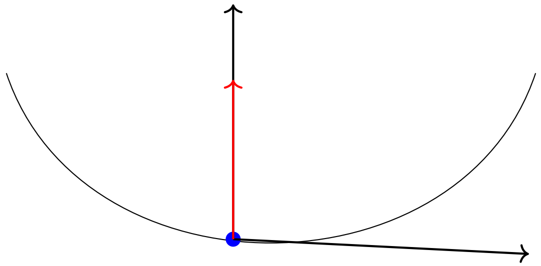

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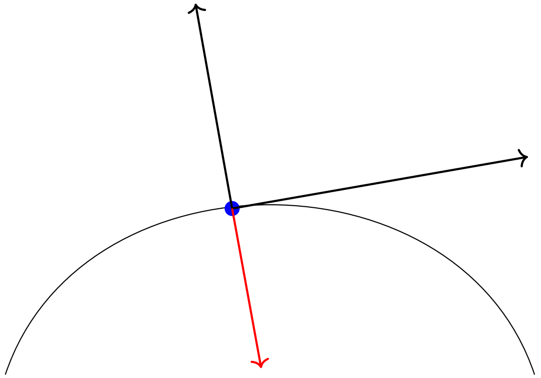

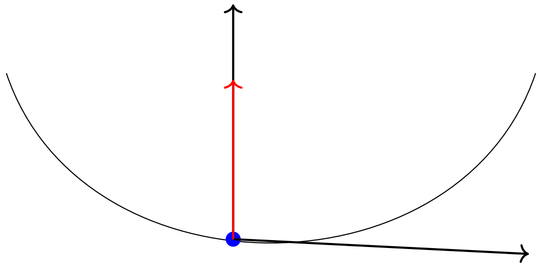

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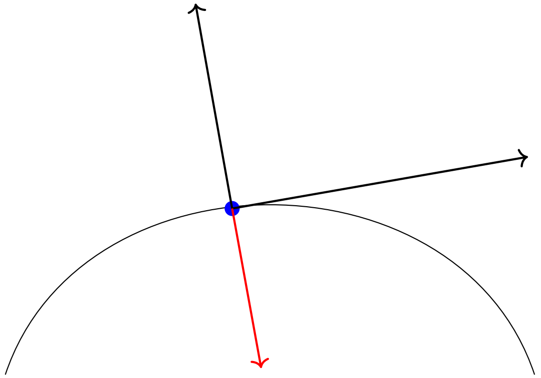

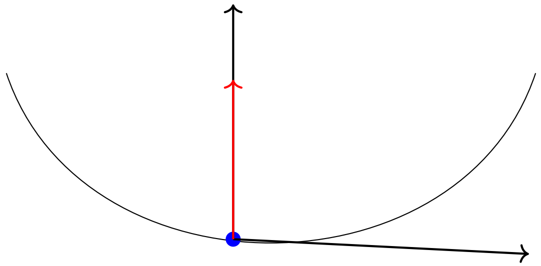

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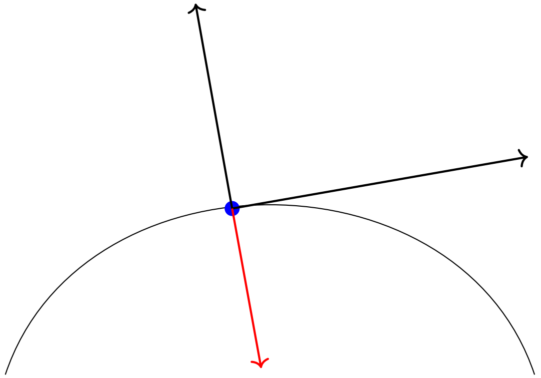

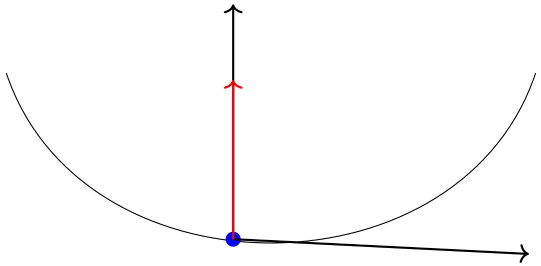

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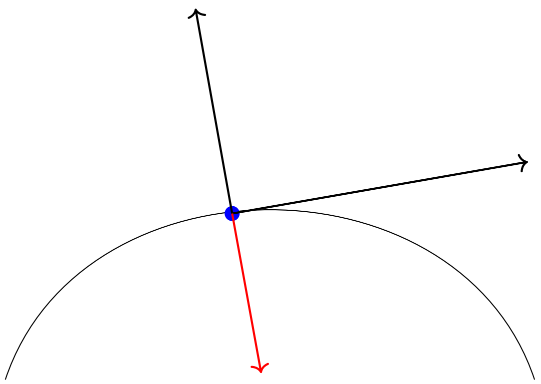
Therefore, $\dot{\mathbf{T}}(\tilde t) = \ddot{\tilde\gamma}(\tilde t)$ is perpendicular to $\mathbf{T}(\tilde t) = \dot{\tilde\gamma}(\tilde t)$

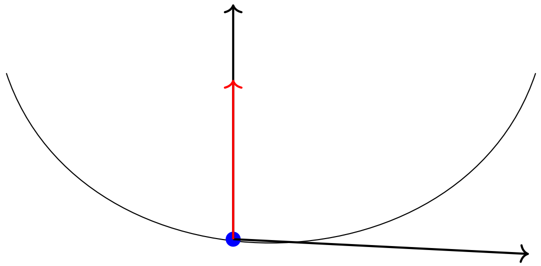
$\mathbf{N}_s(\tilde t)$ denote the *unit* vector obtained by rotating $\mathbf{T}(\tilde t)$ *anticlockwise*.

Therefore, $\dot{\mathbf{T}}(\tilde t) = \ddot{\tilde\gamma}(\tilde t) = \kappa_s(\tilde t)\mathbf{N}_s(\tilde t)$

$\kappa_s(\tilde t)$ called the **signed curvature**.

$\kappa(\tilde t) = \|\ddot{\tilde\gamma}(\tilde t)\| = \|\kappa_s \mathbf{N}_s(\tilde t)\| = |\kappa_s|\|\mathbf{N}_s(\tilde t)\| = |\kappa_s(\tilde t)|$





$\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is **not** a unit speed parametrization.

Then, $\gamma(t) = \tilde\gamma(s(t))$

$$\begin{aligned}\dot\gamma(t) &= \dot{\tilde\gamma}(s(t))s'(t)\\ &= \mathbf{T}(s(t))\|\dot\gamma(t)\|\end{aligned}$$

$$\begin{aligned}\left(\frac{\dot\gamma(t)}{\|\dot\gamma(t)\|}\right)' &= (\mathbf{T}(s(t)))'\\ &= \dot{\mathbf{T}}(s(t))s'(t)\\ &= \kappa_s(s(t))s'(t)\mathbf{N}_s(s(t))\\ &= \kappa_s(s(t))\|\gamma(t)\|\mathbf{N}_s(s(t))\end{aligned}$$

$$\kappa_s(s(t))\mathbf{N}_s(s(t)) = \frac{1}{\|\dot\gamma(t)\|}\left(\frac{\dot\gamma(t)}{\|\dot\gamma(t)\|}\right)'$$

$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to \mathbb{R}^2$ is a *unit speed* parametrization.

Therefore, $\mathbf{T}(\tilde{t}) = \dot{\tilde{\gamma}}(\tilde{t})$

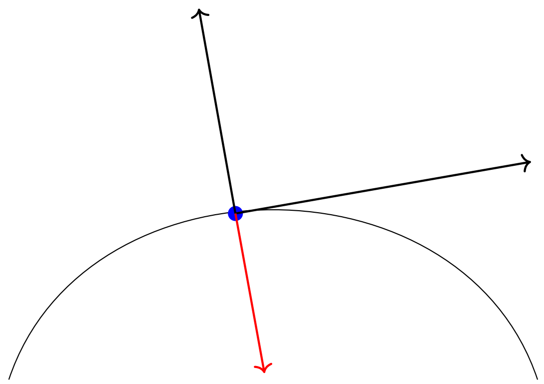
Therefore, $\dot{\mathbf{T}}(\tilde{t}) = \ddot{\tilde{\gamma}}(\tilde{t})$ is perpendicular to $\mathbf{T}(\tilde{t}) = \dot{\tilde{\gamma}}(\tilde{t})$

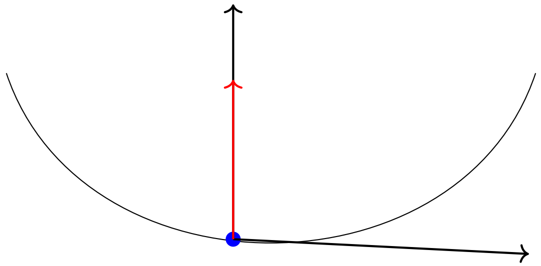
$\mathbf{N}_s(\tilde{t})$ denote the *unit* vector obtained by rotating $\mathbf{T}(\tilde{t})$ *anticlockwise*.

Therefore, $\dot{\mathbf{T}}(\tilde{t}) = \ddot{\tilde{\gamma}}(\tilde{t}) = \kappa_s(\tilde{t})\mathbf{N}_s(\tilde{t})$

$\kappa_s(\tilde{t})$ called the **signed curvature**.

$\kappa(\tilde{t}) = \|\ddot{\tilde{\gamma}}(\tilde{t})\| = \|\kappa_s \mathbf{N}_s(\tilde{t})\| = |\kappa_s|\|\mathbf{N}_s(\tilde{t})\| = |\kappa_s(\tilde{t})|$





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$$\begin{aligned}\kappa_s(s(t))\mathbf{N}_s(s(t)) &= \frac{1}{\|\dot{\gamma}(t)\|}\left(\frac{\dot{\gamma}(t)}{\|\dot{\gamma}(t)\|}\right)'\\ &= \frac{1}{\|\dot{\gamma}(t)\|}\frac{\|\dot{\gamma}(t)\|\ddot{\gamma}(t) - \dot{\gamma}(t)\frac{\dot{\gamma}(t).\ddot{\gamma}(t)}{\|\dot{\gamma}(t)\|}}{\|\dot{\gamma}(t)\|^2}\end{aligned}$$