Exercise sheet 2

Curves and Surfaces, MTH201

1. For $\mathbf{v} : (\alpha, \beta) \to \mathbf{R}^2$ and $\mathbf{w} : (\alpha, \beta) \to \mathbf{R}^2$, show that $(\mathbf{v}(t).\mathbf{w}(t))' = \mathbf{v}'(t).\mathbf{w}(t) + \mathbf{v}(t).\mathbf{w}'(t)$.
2. If $\mathbf{n} : (\alpha, \beta) \to \mathbf{R}^2$ is such that $||\mathbf{n}(t)||$ is constant, then prove that $\dot{\mathbf{n}}(t)$ is either 0 or perpendicular to $\mathbf{n}(t)$.
3. For $\mathbf{v} : (\alpha, \beta) \to \mathbf{R}^2$ and $\mathbf{w} : (\alpha, \beta) \to \mathbf{R}^2$, show that $(\mathbf{v}(t).\mathbf{w}(t))' = \mathbf{v}'(t).\mathbf{w}(t) + \mathbf{v}(t).\mathbf{w}'(t)$ (Assume that all the functions are smooth).
4. $$s_\alpha(t) := \int_{t_\alpha}^{t} ||\dot{\gamma}(u)|| \mathrm{d}u$$
   $$s_\beta(t) := \int_{t_\beta}^{t} ||\dot{\gamma}(u)|| \mathrm{d}u$$
   Prove that $s_\beta(t) - s_\alpha(t)$ is a constant.

To be updated...