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Rule	Example
$(cf)' = cf'$ , where $c \in \mathbb{R}$	$(2 \sin(x))' = 2 \cos(x)$
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# Example

# Example

$\gamma$ :

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$$\gamma(t)$$

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Can we find a parametrization of the circle

## Example

## Making the “speed” 1

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Can we find a parametrization of the circle to ensure the speed is 1?

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