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$(f/g)' = \frac{f'g - fg'}{g^2}$	$(\frac{\sin(x)}{x^2})' = \frac{\cos(x)x^2 - \sin(x)2x}{x^4}$

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Rule	Example
$(cf)' = cf'$ , where $c \in \mathbb{R}$	$(2 \sin(x))' = 2 \cos(x)$
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$\gamma$ :

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Can we find a parametrization of the circle

## Example

## Making the “speed” 1

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