Hints / Solutions to Exercise sheet 4

Curves and Surfaces, MTH201

Question 1: Show that the curvature at any point of a line segment is always 0.

Solution 1: A line segment is parametrized by $\gamma(t) = p + \mathbf{v}t$. Note that $\dot{\gamma}(t) = v$ and so $\|\dot{\gamma}(t)\| = \|v\|$. So a unit speed reparametrization is $\tilde{\gamma}(\tilde{t}) = p + \frac{v}{\|v\|}\tilde{t}$. Now $\ddot{\gamma}(\tilde{t}) = 0$.

Question 3: Given *any* smooth parametrization, $\gamma : (\alpha, \beta) \to \mathbb{R}^2$, is the curvature function $\kappa(t)$ always smooth? Do you need to add some condition? What is it?

Hint 3: The definition of curvature involves a norm, which involves taking a square root. $\sqrt{f(t)}$ is smooth if and only if $f(t) > 0$. So it is smooth only if the acceleration is not 0, which is itself equivalent to the curvature being non-zero.

Question 5: If $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ parametrizes a curve, compute the curvature of the curve parametrized by $\tilde{\gamma}(t) = \gamma(-t)$ in terms of the curvature of γ. What about the relation between the signed curvatures of γ and $\tilde{\gamma}$?

Hint 5: After computing, you will realize that they differ only by a sign. Intuitively, if you move along a curve in the opposite direction, then you "turn" in the other direction.