

$\tilde{\gamma} :$

$$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta})$$

$$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \rightarrow \mathbb{R}^2$$

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Therefore,

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Therefore,  $\mathbf{T}(\tilde{t})$

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Therefore,  $\mathbf{T}(\tilde{t}) = \dot{\tilde{\gamma}}(\tilde{t})$



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$\mathbf{N}_s(\tilde{t})$  denote the *unit* vector obtained by rotating  $\mathbf{T}(\tilde{t})$  *anticlockwise*.

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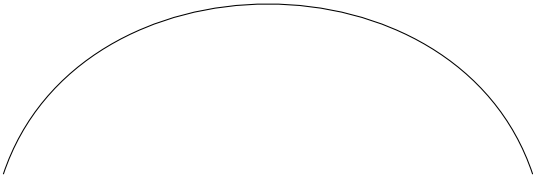
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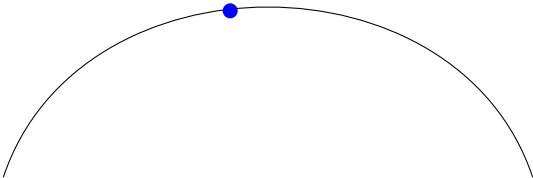
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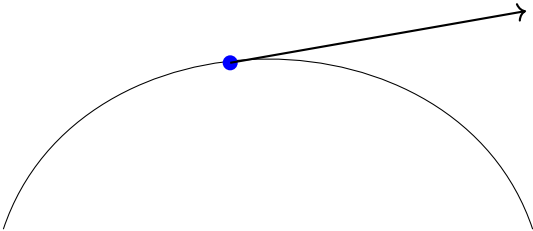
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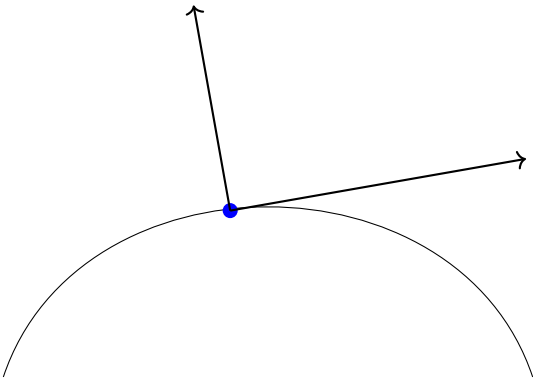
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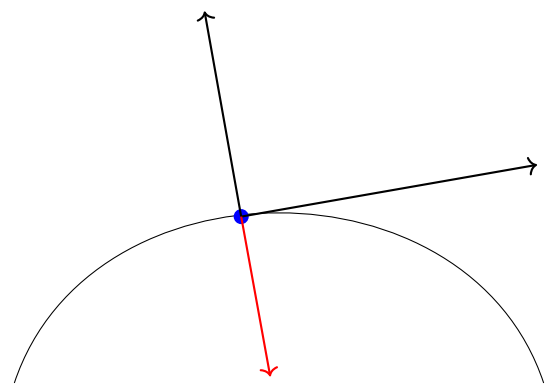
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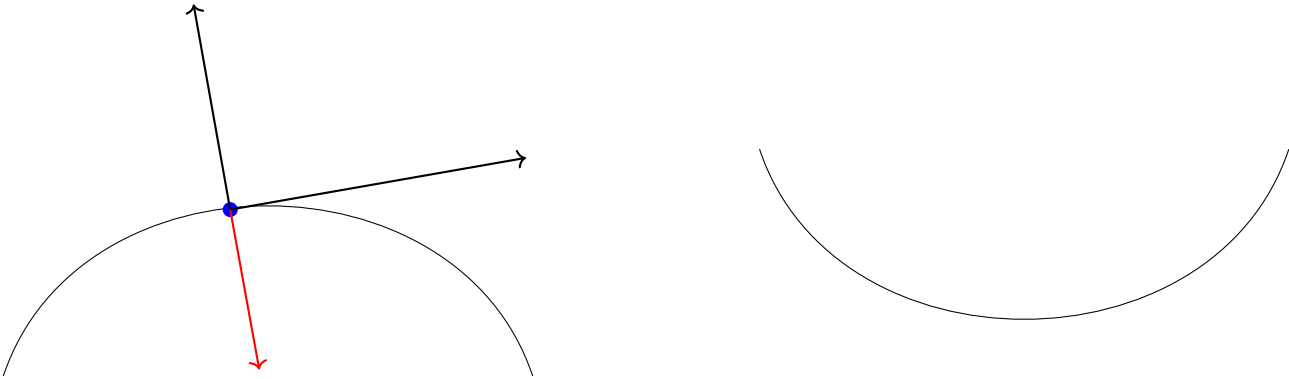
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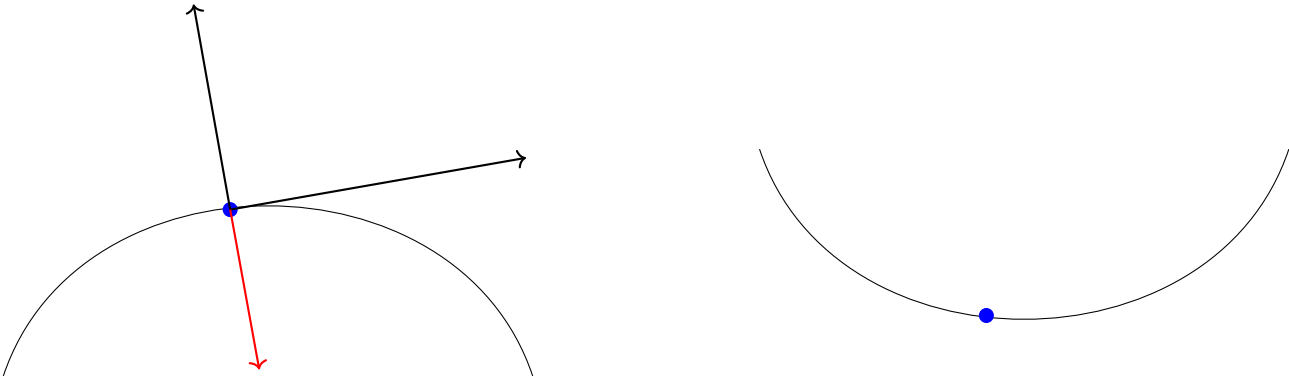
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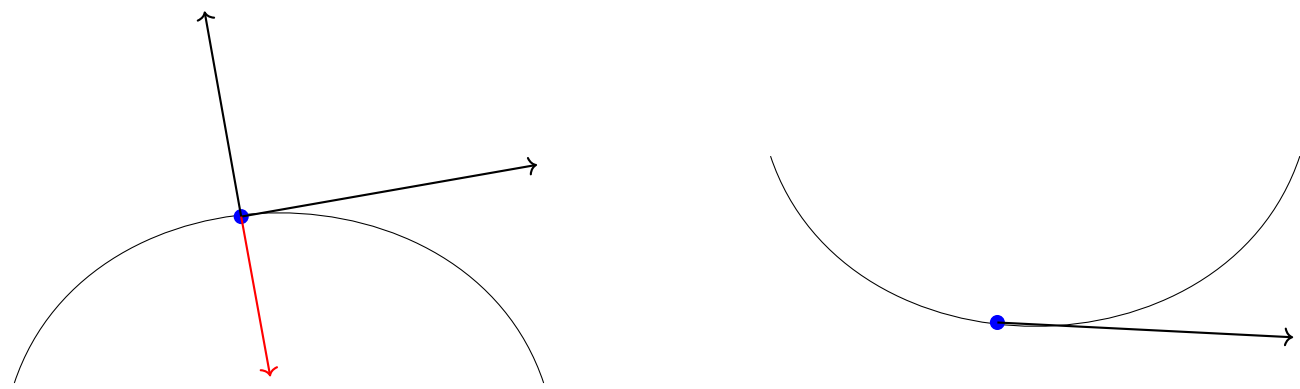
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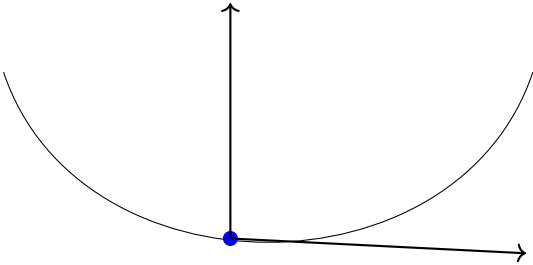
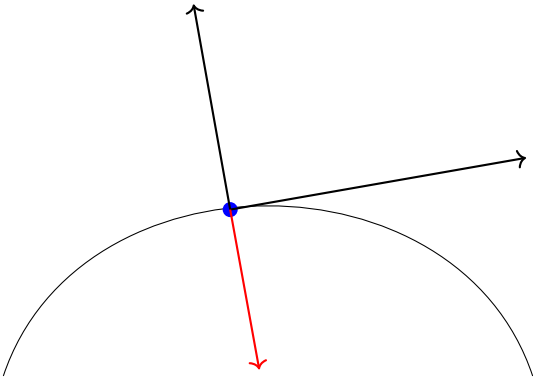
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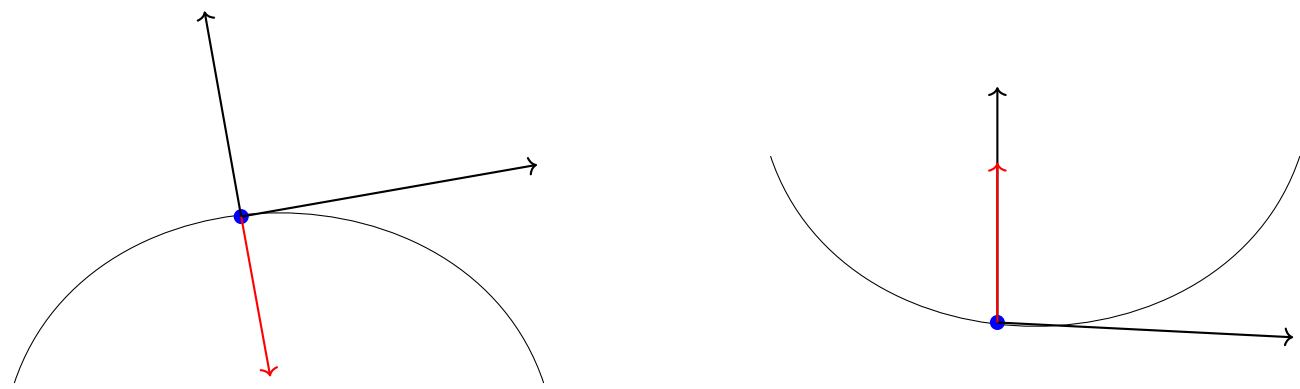
Therefore,  $\mathbf{T}(\tilde{t}) = \dot{\tilde{\gamma}}(\tilde{t})$

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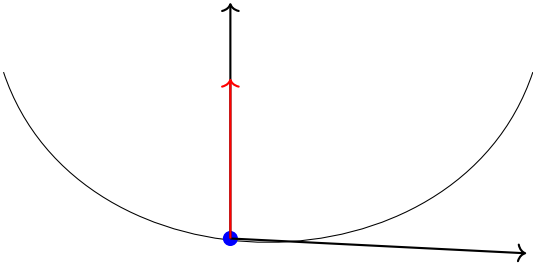
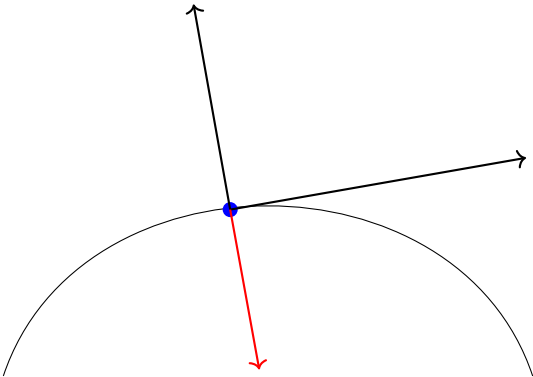
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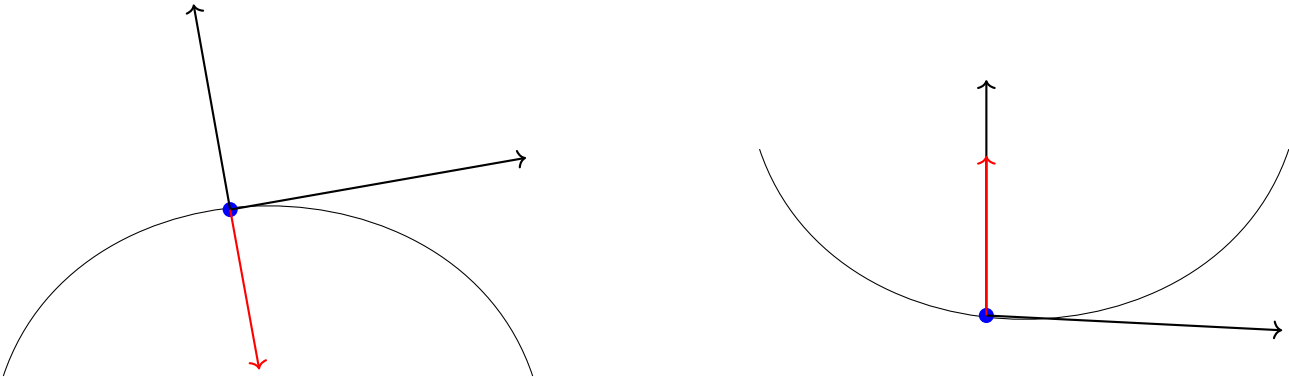
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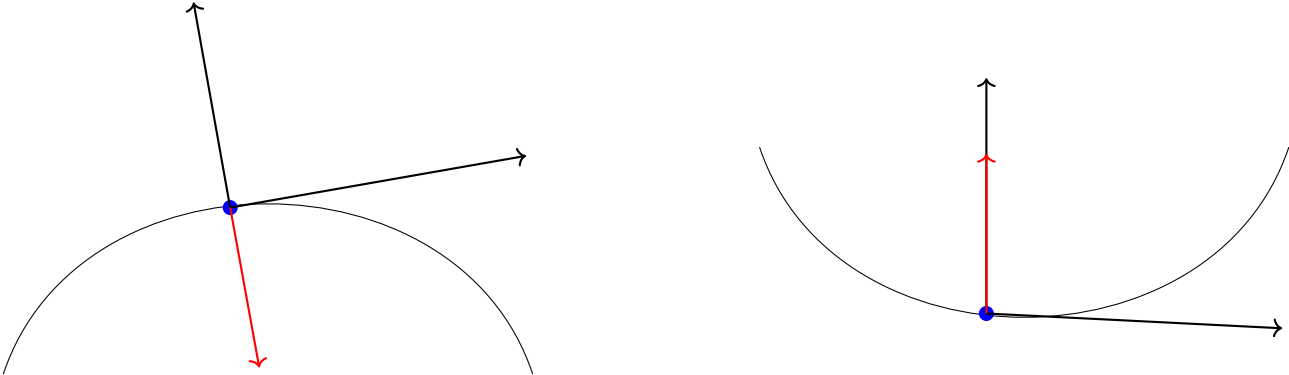
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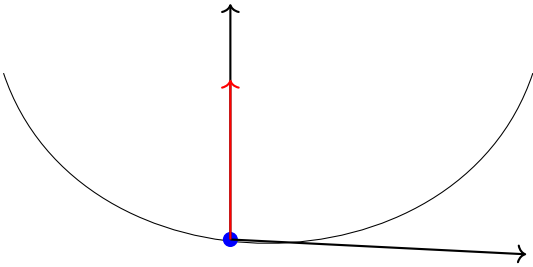
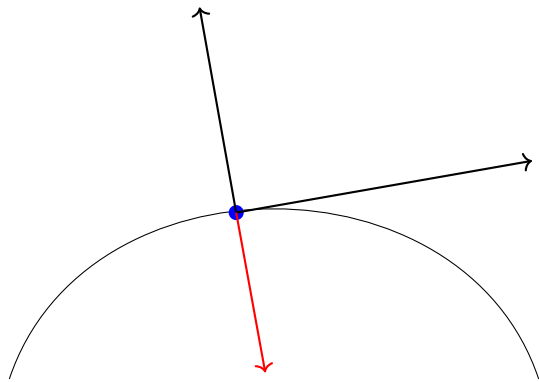
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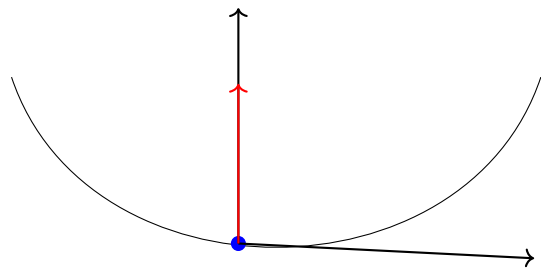
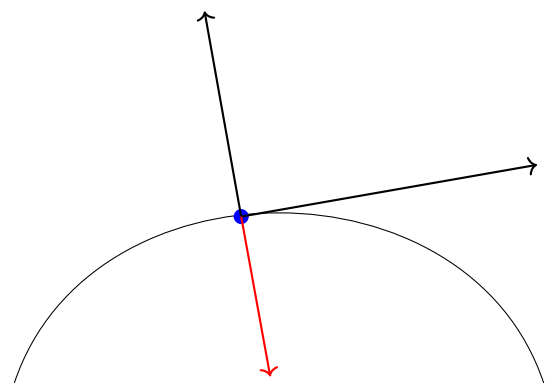
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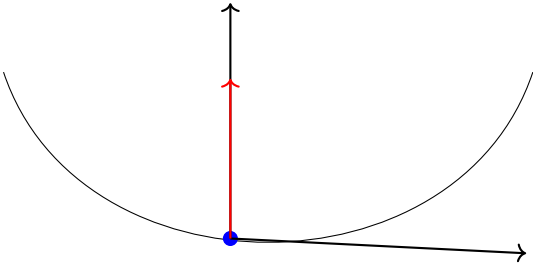
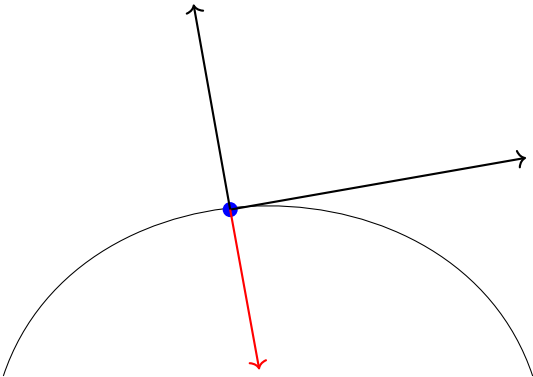
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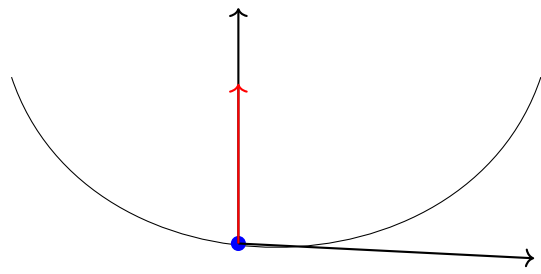
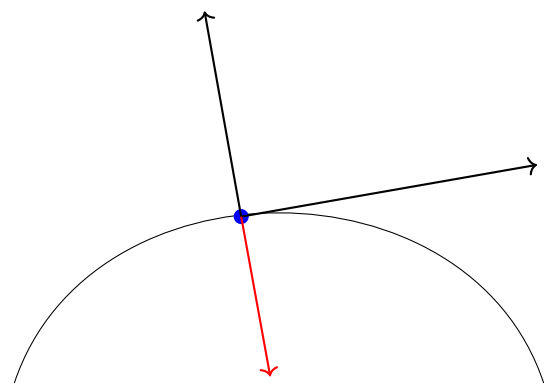
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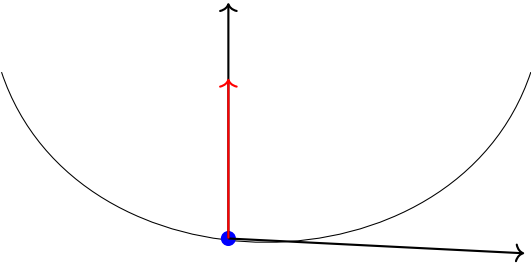
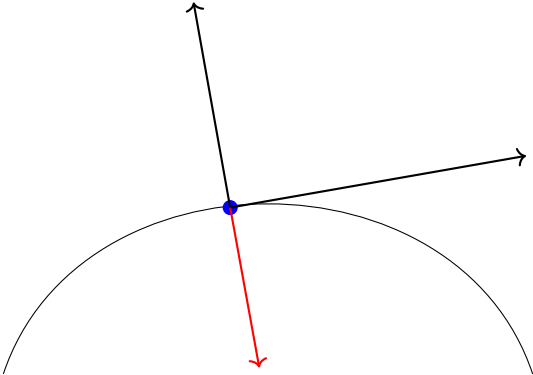
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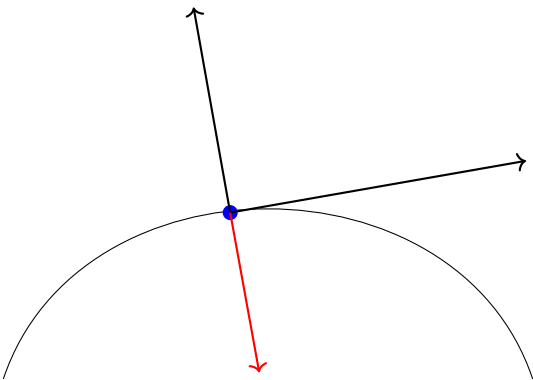
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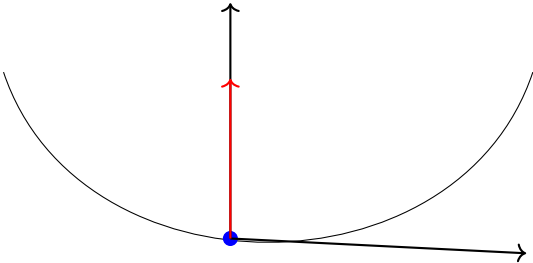
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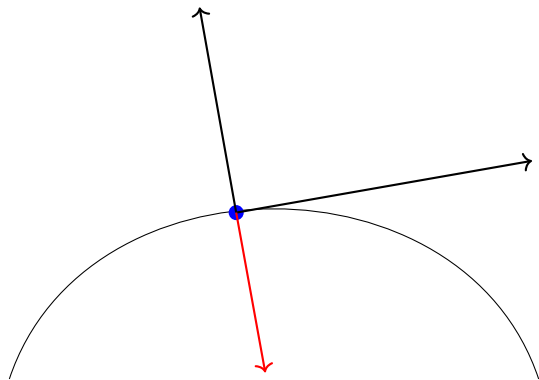
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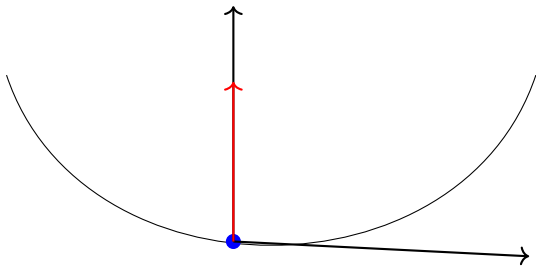
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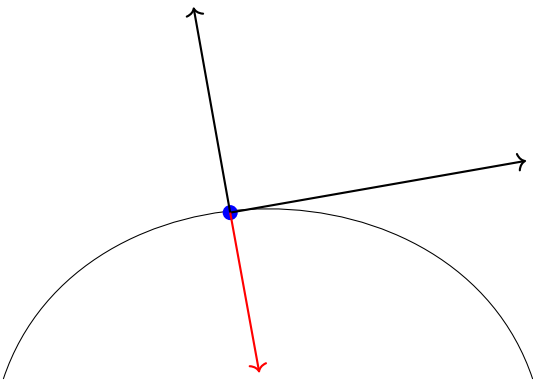
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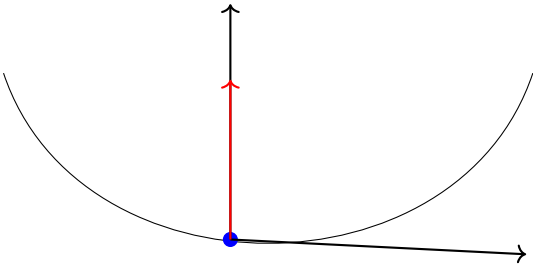


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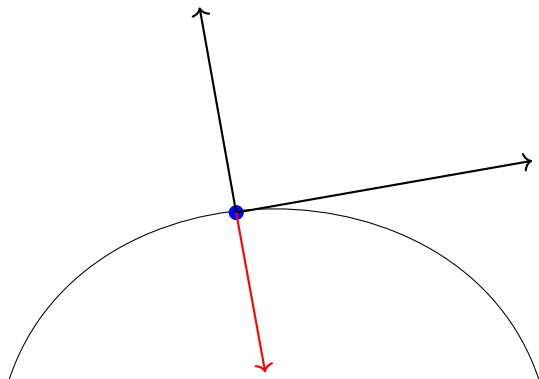
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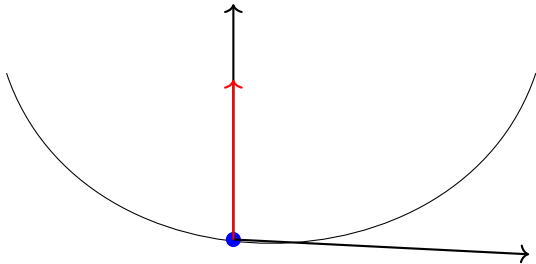


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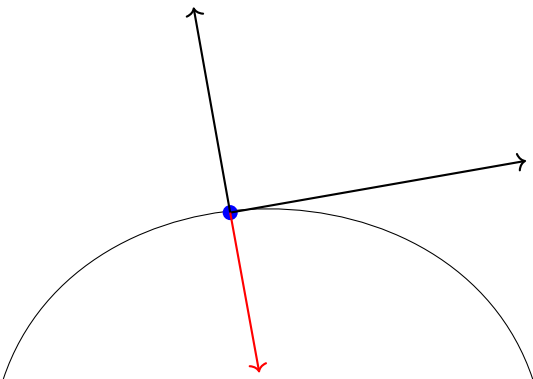
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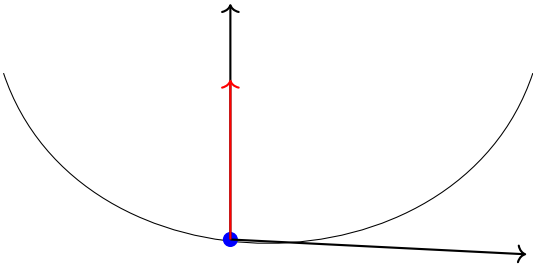


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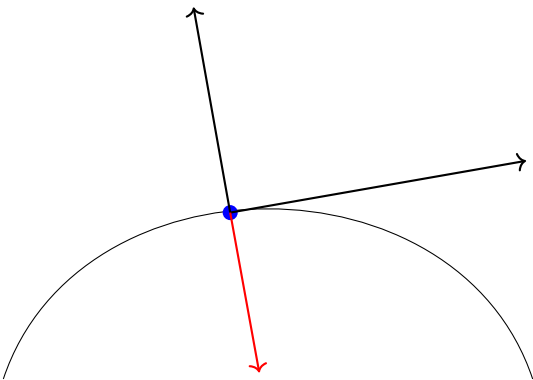
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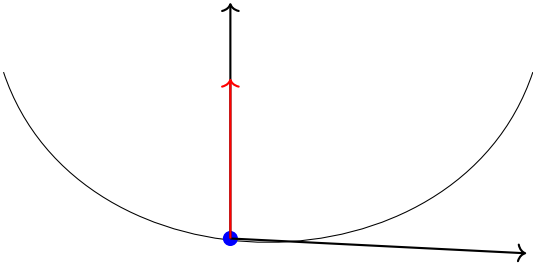


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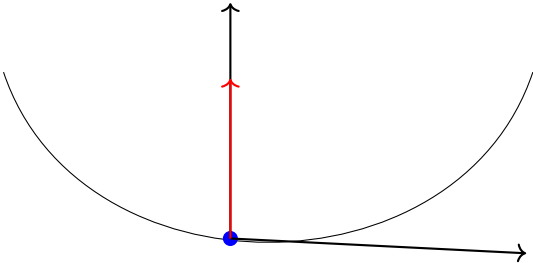
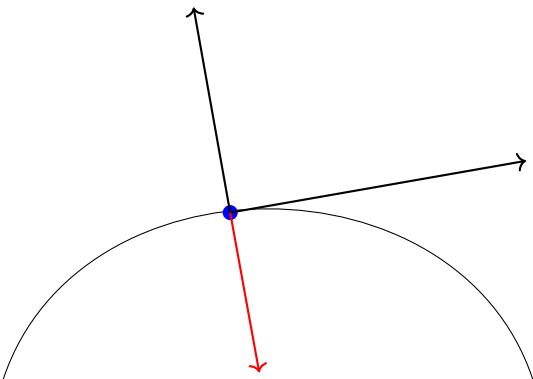
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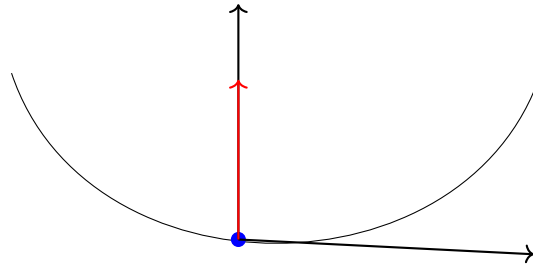
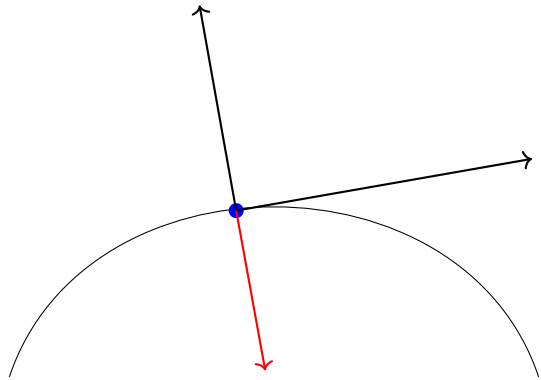
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$$\begin{aligned} \left( \frac{\dot{\gamma}(t)}{\|\dot{\gamma}(t)\|} \right)' &= (\mathbf{T}(s(t)))' \\ &= \dot{\mathbf{T}}(s(t))s'(t) \\ &= \kappa_s(s(t))s'(t)\mathbf{N}_s(s(t)) \\ &= \kappa_s(s(t))\|\dot{\gamma}(t)\|\mathbf{N}_s(s(t)) \end{aligned}$$

$$\kappa_s(s(t))\mathbf{N}_s(s(t)) = \frac{1}{\|\dot{\gamma}(t)\|} \left( \frac{\dot{\gamma}(t)}{\|\dot{\gamma}(t)\|} \right)'$$

$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \rightarrow \mathbb{R}^2$  is a *unit speed* parametrization.

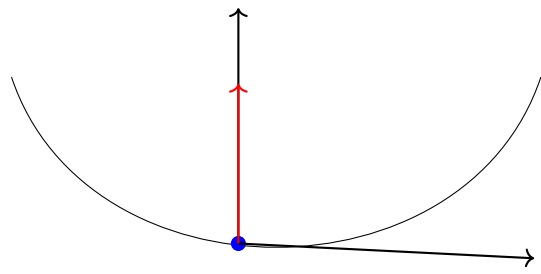
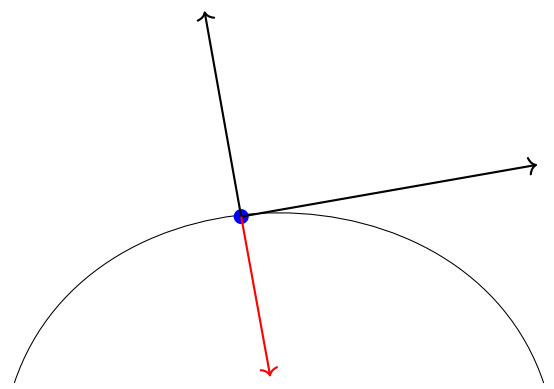
Therefore,  $\mathbf{T}(\tilde{t}) = \dot{\tilde{\gamma}}(\tilde{t})$

Therefore,  $\dot{\mathbf{T}}(\tilde{t}) = \ddot{\tilde{\gamma}}(\tilde{t})$  is perpendicular to  $\mathbf{T}(\tilde{t}) = \dot{\tilde{\gamma}}(\tilde{t})$   
 $\mathbf{N}_s(\tilde{t})$  denote the *unit* vector obtained by rotating  $\mathbf{T}(\tilde{t})$   
*anticlockwise*.

Therefore,  $\dot{\mathbf{T}}(\tilde{t}) = \ddot{\tilde{\gamma}}(\tilde{t}) = \kappa_s(\tilde{t})\mathbf{N}_s(\tilde{t})$

$\kappa_s(\tilde{t})$  called the **signed curvature**.

$$\kappa(\tilde{t}) = \|\ddot{\tilde{\gamma}}(\tilde{t})\| = \|\kappa_s \mathbf{N}_s(\tilde{t})\| = |\kappa_s| \|\mathbf{N}_s(\tilde{t})\| = |\kappa_s(\tilde{t})|$$



$\gamma : (\alpha, \beta) \rightarrow \mathbb{R}^2$  is **not** a unit speed parametrization.

Then,  $\gamma(t) = \tilde{\gamma}(s(t))$

$$\begin{aligned} \dot{\gamma}(t) &= \dot{\tilde{\gamma}}(s(t))s'(t) \\ &= \mathbf{T}(s(t))\|\dot{\gamma}(t)\| \end{aligned}$$

$$\begin{aligned} \left( \frac{\dot{\gamma}(t)}{\|\dot{\gamma}(t)\|} \right)' &= (\mathbf{T}(s(t)))' \\ &= \dot{\mathbf{T}}(s(t))s'(t) \\ &= \kappa_s(s(t))s'(t)\mathbf{N}_s(s(t)) \\ &= \kappa_s(s(t))\|\dot{\gamma}(t)\|\mathbf{N}_s(s(t)) \end{aligned}$$

$$\begin{aligned} \kappa_s(s(t))\mathbf{N}_s(s(t)) &= \frac{1}{\|\dot{\gamma}(t)\|} \left( \frac{\dot{\gamma}(t)}{\|\dot{\gamma}(t)\|} \right)' \\ &= \frac{1}{\|\dot{\gamma}(t)\|} \frac{\|\dot{\gamma}(t)\|\ddot{\gamma}(t) - \dot{\gamma}(t)\frac{\dot{\gamma}(t) \cdot \ddot{\gamma}(t)}{\|\dot{\gamma}(t)\|}}{\|\dot{\gamma}(t)\|^2} \end{aligned}$$