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The arc-length,

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The arc-length, s from $t = a$ to $t = b$

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The arc-length, s from $t = a$ to $t = b$ is approximated by,
$f(t_1)(t_2 - t_1) +$

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The arc-length, s from $t = a$ to $t = b$ is approximated by,
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The arc-length, s from $t = a$ to $t = b$ is approximated by,
$||\dot{\gamma}(t_1)||(t_2 - t_1) + ||\dot{\gamma}(t_2)||(t_3 - t_2) + \ldots + ||\dot{\gamma}(t_{n-1})||(t_n - t_{n-1})$

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The arc-length, s from $t = a$ to $t = b$ is approximated by,
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Better and better approximations "converge".

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$$||\dot{\gamma}(t)||\mathrm{d}t$$

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$$\int ||\dot{\gamma}(t)|| \mathrm{d}t$$

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Better and better approximations "converge". Denoted,

$$\int_{t=a} ||\dot{\gamma}(t)|| \mathrm{d}t$$

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Better and better approximations "converge". Denoted,

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Arc length

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Better and better approximations "converge". Denoted,

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Definition.
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a smooth and regular (of course!) parametrization.

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Exercise.

$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$f(t) = ||\dot{\gamma}(t)||$
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The arc-length, s from $t = a$ to $t = b$ is approximated by,
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Exercise.

$$s_\alpha(t)$$

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Exercise.

$$s_\alpha(t) := \int_{t_\alpha}^{t} ||\dot{\gamma}(u)|| \mathrm{d}u$$

$$s_\beta(t)$$

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$$s_\alpha(t) := \int_{t_\alpha}^{t} ||\dot{\gamma}(u)||\mathrm{d}u$$

$$s_\beta(t) := \int_{t_\beta}^{t} ||\dot{\gamma}(u)||\mathrm{d}u$$

Prove that $s_\beta(t) - s_\alpha(t)$ is a constant.

Theorem (First Fundamental theorem of calculus)**.**
f

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$f : [\alpha, \beta] \to \mathbb{R}$ *continuous*

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$$F(t)$$

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Corollary. $\gamma : (\alpha, \beta) \to \mathbb{R}^2$

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$s'(t) = ||\dot{\gamma}(t)||$

Proof.

$$s(t)$$

□

Theorem (First Fundamental theorem of calculus)**.**
$f : [\alpha, \beta] \to \mathbb{R}$ continuous

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Corollary. *$\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a smooth and regular parametrization.*
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Proof.

$$s(t) := \int_{t_0}^{t} ||\dot{\gamma}(u)||\mathrm{d}u$$

$\square$

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Corollary. *$\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a smooth and regular parametrization.*
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Theorem (First Fundamental theorem of calculus). *$f : [\alpha, \beta] \to \mathbb{R}$ continuous*

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Corollary. *The arc length function*

Theorem (First Fundamental theorem of calculus)**.** $f : [\alpha, \beta] \to \mathbb{R}$ *continuous*

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Corollary. $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ *is a smooth and regular parametrization.*
and $s(t)$ *its arc-length function beginning at* t_0 *then,* $s'(t) = ||\dot{\gamma}(t)||$

Proof.

$$s(t) := \int_{t_0}^{t} ||\dot{\gamma}(u)||\mathrm{d}u$$

by the First Fundamental Theorem of Calculus, $s'(t) = ||\dot{\gamma}(t)||$ $\square$

Corollary. *The arc length function* $s(t)$

Theorem (First Fundamental theorem of calculus)**.** $f : [\alpha, \beta] \to \mathbb{R}$ *continuous*

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and $s(t)$ *its arc-length function beginning at* t_0 *then,* $s'(t) = ||\dot{\gamma}(t)||$

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$$s(t) := \int_{t_0}^{t} ||\dot{\gamma}(u)||\mathrm{d}u$$

by the First Fundamental Theorem of Calculus, $s'(t) = ||\dot{\gamma}(t)||$ □

Corollary. *The arc length function* $s(t)$ *is smooth.*

Proof. □

Theorem (First Fundamental theorem of calculus)**.** *$f : [\alpha, \beta] \to \mathbb{R}$ continuous*

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then, $F'(t) = f(t)$

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$\tilde{\gamma}(\tilde{t})$

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If $\phi(t) = s^{-1}(t)$,

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$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$
$\tilde{\gamma}'(\tilde{t}) = \gamma'(\phi(\tilde{t}))\phi'(\tilde{t})$

If $\phi(t) = s^{-1}(t)$,
$\tilde{\gamma}'(\tilde{t}) = \gamma'(s^{-1}(\tilde{t}))(s^{-1})'(\tilde{t}) = \gamma'(s^{-1}(\tilde{t}))\frac{1}{||\gamma'(s^{-1}(\tilde{t}))||}$

$||\tilde{\gamma}'(\tilde{t})|| = ||\gamma'(s^{-1}(\tilde{t}))||\frac{1}{||\gamma'(s^{-1}(\tilde{t}))||} = 1$

Proved that,

Theorem.
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$ *a regular smooth parametrization*

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