Space curves

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$\mathbf{v} = \alpha_1\mathbf{e_1} + \alpha_2\mathbf{e_2} + \alpha_3\mathbf{e_3}$
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$$\mathbf{v} \times \mathbf{w} \ = \ (\alpha_1\mathbf{e_1} + \alpha_2\mathbf{e_2} + \alpha_3\mathbf{e_3}) \times (\beta_1\mathbf{e_1} + \beta_2\mathbf{e_2} + \beta_3\mathbf{e_3})$$

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$$\begin{aligned} \mathbf{v} \times \mathbf{w} &= (\alpha_1\mathbf{e_1} + \alpha_2\mathbf{e_2} + \alpha_3\mathbf{e_3}) \times (\beta_1\mathbf{e_1} + \beta_2\mathbf{e_2} + \beta_3\mathbf{e_3}) \\ &= (\beta_2\alpha_3 - \beta_3\alpha_2)\mathbf{e_1} + \cdots \end{aligned}$$

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$\mathbf{v}(t) = \alpha_1(t)\mathbf{e_1} + \alpha_2(t)\mathbf{e_2} + \alpha_3(t)\mathbf{e_3}$
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$$\begin{aligned}\mathbf{v}(t) \times \mathbf{w}(t) &= (\alpha_1(t)\mathbf{e_1} + \alpha_2(t)\mathbf{e_2} + \alpha_3(t)\mathbf{e_3}) \times (\beta_1(t)\mathbf{e_1} + \beta_2(t)\mathbf{e_2} + \beta_3(t)\mathbf{e_3}) \\ &= (\beta_2(t)\alpha_3(t) - \beta_3(t)\alpha_2(t))\mathbf{e_1} + \cdots\end{aligned}$$

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&= (\beta_2(t)\alpha_3(t) - \beta_3(t)\alpha_2(t))\mathbf{e_1} + \cdots
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So, if $\mathbf{v}(t)$ and $\mathbf{w}(t)$ are smooth,

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So, if $\mathbf{v}(t)$ and $\mathbf{w}(t)$ are smooth, then $\mathbf{v}(t) \times \mathbf{w}(t)$ is smooth.

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So, any vector field,

$$\mathbf{v}(t) = x(t)\mathbf{T}(t) + y(t)\mathbf{N}(t) + z(t)\mathbf{B}(t)$$

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Space curves

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\dot{\mathbf{v}}(t) &= \dot{x}(t)\mathbf{T}(t) + x(t)\dot{\mathbf{T}}(t) \\
&+ \dot{y}(t)\mathbf{N}(t) + y(t)\dot{\mathbf{N}}(t) \\
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What are $\dot{\mathbf{T}}(t)$,

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So, $x : (\alpha, \beta) \to \mathbb{R}$, $y : (\alpha, \beta) \to \mathbb{R}$, and $z : (\alpha, \beta) \to \mathbb{R}$ are functions

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$$\begin{aligned}
\dot{\mathbf{v}}(t) &= \dot{x}(t)\mathbf{T}(t) + x(t)\dot{\mathbf{T}}(t) \\
&+ \dot{y}(t)\mathbf{N}(t) + y(t)\dot{\mathbf{N}}(t) \\
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What are $\dot{\mathbf{T}}(t)$, $\dot{\mathbf{N}}(t)$,

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So, $x : (\alpha, \beta) \to \mathbb{R}$, $y : (\alpha, \beta) \to \mathbb{R}$, and $z : (\alpha, \beta) \to \mathbb{R}$ are functions

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What are $\dot{\mathbf{T}}(t)$, $\dot{\mathbf{N}}(t)$, and $\dot{\mathbf{B}}(t)$

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$$\mathbf{v}(t) = x(t)\mathbf{T}(t) + y(t)\mathbf{N}(t) + z(t)\mathbf{B}(t)$$

for some (unique!) $x(t), y(t), z(t) \in \mathbb{R}$
So, $x : (\alpha, \beta) \to \mathbb{R}$, $y : (\alpha, \beta) \to \mathbb{R}$, and $z : (\alpha, \beta) \to \mathbb{R}$ are functions

$x(t) = \mathbf{v}(t).\mathbf{T}(t)$
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What are $\dot{\mathbf{T}}(t)$, $\dot{\mathbf{N}}(t)$, and $\dot{\mathbf{B}}(t)$ in terms of the basis?

Space curves

Given $\gamma : (\alpha, \beta) \to \mathbb{R}^3$ **unit speed** parametrization, $\kappa(t) \neq 0$
$\mathbf{T}(t) := \dot{\gamma}(t)$, (unit) vector in direction of velocity
$\mathbf{N}(t) := \frac{1}{\kappa(t)}\ddot{\gamma}(t)$, (unit) vector perpendicular to $\mathbf{T}(t)$
$\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$, (unit) vector perpendicular to $\mathbf{T}(t)$ and $\mathbf{N}(t)$.

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What are $\dot{\mathbf{T}}(t)$, $\dot{\mathbf{N}}(t)$, and $\dot{\mathbf{B}}(t)$ in terms of the basis?

$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$

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\dot{\mathbf{v}}(t) &= \dot{x}(t)\mathbf{T}(t) + x(t)\dot{\mathbf{T}}(t) \\
&+ \dot{y}(t)\mathbf{N}(t) + y(t)\dot{\mathbf{N}}(t) \\
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\end{aligned}$$

What are $\dot{\mathbf{T}}(t)$, $\dot{\mathbf{N}}(t)$, and $\dot{\mathbf{B}}(t)$ in terms of the basis?

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$\dot{\mathbf{N}}(t) = ??$

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$\dot{\mathbf{N}}(t).\mathbf{T}(t)$

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$$\dot{\mathbf{N}}(t).\mathbf{T}(t) + \mathbf{N}(t).\dot{\mathbf{T}}(t)$$

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What are $\dot{\mathbf{T}}(t)$, $\dot{\mathbf{N}}(t)$, and $\dot{\mathbf{B}}(t)$ in terms of the basis?

$\dot{\mathbf{T}}(t) = 0\mathbf{T}(t) + \kappa(t)\mathbf{N}(t) + 0\mathbf{B}(t)$
$\dot{\mathbf{N}}(t) =??$

$$\dot{\mathbf{N}}(t).\mathbf{T}(t) + \mathbf{N}(t).\dot{\mathbf{T}}(t) = (\mathbf{N}(t).\mathbf{T}(t))'$$

Space curves

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$\dot{\mathbf{N}}(t) =??$

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$z(t) = \mathbf{v}(t).\mathbf{B}(t)$

$$\begin{aligned}\dot{\mathbf{v}}(t) &= \dot{x}(t)\mathbf{T}(t) + x(t)\dot{\mathbf{T}}(t)\\ &+ \dot{y}(t)\mathbf{N}(t) + y(t)\dot{\mathbf{N}}(t)\\ &+ \dot{z}(t)\mathbf{B}(t) + z(t)\dot{\mathbf{B}}(t)\end{aligned}$$

What are $\dot{\mathbf{T}}(t)$, $\dot{\mathbf{N}}(t)$, and $\dot{\mathbf{B}}(t)$ in terms of the basis?

$\dot{\mathbf{T}}(t) = 0\mathbf{T}(t) + \kappa(t)\mathbf{N}(t) + 0\mathbf{B}(t)$

$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + 0\mathbf{N}(t) + ??\mathbf{B}(t)$

$$\dot{\mathbf{N}}(t).\mathbf{T}(t) + \underbrace{\mathbf{N}(t).\dot{\mathbf{T}}(t)}_{\kappa(t)} = \underbrace{(\mathbf{N}(t).\mathbf{T}(t))'}_{0}$$

Space curves

Given $\gamma : (\alpha, \beta) \to \mathbb{R}^3$ **unit speed** parametrization, $\kappa(t) \neq 0$
$\mathbf{T}(t) := \dot{\gamma}(t)$, (unit) vector in direction of velocity
$\mathbf{N}(t) := \frac{1}{\kappa(t)}\ddot{\gamma}(t)$, (unit) vector perpendicular to $\mathbf{T}(t)$
$\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$, (unit) vector perpendicular to $\mathbf{T}(t)$ and $\mathbf{N}(t)$.

$\{\mathbf{T}(t), \mathbf{N}(t), \mathbf{B}(t)\}$ form an orthonormal basis for each $t \in (\alpha, \beta)$.

So, any vector field,

$$\mathbf{v}(t) = x(t)\mathbf{T}(t) + y(t)\mathbf{N}(t) + z(t)\mathbf{B}(t)$$

for some (unique!) $x(t), y(t), z(t) \in \mathbb{R}$
So, $x : (\alpha, \beta) \to \mathbb{R}$, $y : (\alpha, \beta) \to \mathbb{R}$, and $z : (\alpha, \beta) \to \mathbb{R}$ are functions

$x(t) = \mathbf{v}(t).\mathbf{T}(t)$
$y(t) = \mathbf{v}(t).\mathbf{N}(t)$
$z(t) = \mathbf{v}(t).\mathbf{B}(t)$

$$\begin{aligned}\dot{\mathbf{v}}(t) &= \dot{x}(t)\mathbf{T}(t) + x(t)\dot{\mathbf{T}}(t) \\ &+ \dot{y}(t)\mathbf{N}(t) + y(t)\dot{\mathbf{N}}(t) \\ &+ \dot{z}(t)\mathbf{B}(t) + z(t)\dot{\mathbf{B}}(t)\end{aligned}$$

What are $\dot{\mathbf{T}}(t)$, $\dot{\mathbf{N}}(t)$, and $\dot{\mathbf{B}}(t)$ in terms of the basis?

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Space curves

Given $\gamma : (\alpha, \beta) \to \mathbb{R}^3$ **unit speed** parametrization, $\kappa(t) \neq 0$

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$$\begin{aligned} \dot{\mathbf{v}}(t) &= \dot{x}(t)\mathbf{T}(t) + x(t)\dot{\mathbf{T}}(t) \\ &+ \dot{y}(t)\mathbf{N}(t) + y(t)\dot{\mathbf{N}}(t) \\ &+ \dot{z}(t)\mathbf{B}(t) + z(t)\dot{\mathbf{B}}(t) \end{aligned}$$

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$\dot{\mathbf{N}}(t).\mathbf{B}(t)$

Space curves

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\dot{\mathbf{v}}(t) &= \dot{x}(t)\mathbf{T}(t) + x(t)\dot{\mathbf{T}}(t) \\
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$$\dot{\mathbf{N}}(t).\mathbf{B}(t) + \mathbf{N}(t).\dot{\mathbf{B}}(t)$$

Space curves

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$$\dot{\mathbf{N}}(t).\mathbf{T}(t) + \underbrace{\mathbf{N}(t).\dot{\mathbf{T}}(t)}_{\kappa(t)} = \underbrace{(\mathbf{N}(t).\mathbf{T}(t))'}_{0}$$

$$\dot{\mathbf{N}}(t).\mathbf{B}(t) + \mathbf{N}(t).\dot{\mathbf{B}}(t) = (\mathbf{N}(t).\mathbf{B}(t))'$$

Space curves

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\end{aligned}$$

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$$\dot{\mathbf{N}}(t).\mathbf{T}(t) + \underbrace{\mathbf{N}(t).\dot{\mathbf{T}}(t)}_{\kappa(t)} = \underbrace{(\mathbf{N}(t).\mathbf{T}(t))'}_{0}$$

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Space curves

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What are $\dot{\mathbf{T}}(t)$, $\dot{\mathbf{N}}(t)$, and $\dot{\mathbf{B}}(t)$ in terms of the basis?

$\dot{\mathbf{T}}(t) = 0\mathbf{T}(t) + \kappa(t)\mathbf{N}(t) + 0\mathbf{B}(t)$

$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + 0\mathbf{N}(t) + ??\mathbf{B}(t)$

$\dot{\mathbf{B}}(t) =$

$$\dot{\mathbf{N}}(t).\mathbf{T}(t) + \underbrace{\mathbf{N}(t).\dot{\mathbf{T}}(t)}_{\kappa(t)} = \underbrace{(\mathbf{N}(t).\mathbf{T}(t))'}_{0}$$

$$\dot{\mathbf{N}}(t).\mathbf{B}(t) + \mathbf{N}(t).\dot{\mathbf{B}}(t) = \underbrace{(\mathbf{N}(t).\mathbf{B}(t))'}_{0}$$

Space curves

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So, $x : (\alpha, \beta) \to \mathbb{R}$, $y : (\alpha, \beta) \to \mathbb{R}$, and $z : (\alpha, \beta) \to \mathbb{R}$ are functions

$x(t) = \mathbf{v}(t).\mathbf{T}(t)$

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$z(t) = \mathbf{v}(t).\mathbf{B}(t)$

$$\begin{aligned}\dot{\mathbf{v}}(t) &= \dot{x}(t)\mathbf{T}(t) + x(t)\dot{\mathbf{T}}(t)\\ &+ \dot{y}(t)\mathbf{N}(t) + y(t)\dot{\mathbf{N}}(t)\\ &+ \dot{z}(t)\mathbf{B}(t) + z(t)\dot{\mathbf{B}}(t)\end{aligned}$$

What are $\dot{\mathbf{T}}(t)$, $\dot{\mathbf{N}}(t)$, and $\dot{\mathbf{B}}(t)$ in terms of the basis?

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$$\dot{\mathbf{N}}(t).\mathbf{T}(t) + \underbrace{\mathbf{N}(t).\dot{\mathbf{T}}(t)}_{\kappa(t)} = \underbrace{(\mathbf{N}(t).\mathbf{T}(t))'}_{0}$$

$$\dot{\mathbf{N}}(t).\mathbf{B}(t) + \mathbf{N}(t).\dot{\mathbf{B}}(t) = \underbrace{(\mathbf{N}(t).\mathbf{B}(t))'}_{0}$$

Space curves

Given $\gamma : (\alpha, \beta) \to \mathbb{R}^3$ **unit speed** parametrization, $\kappa(t) \neq 0$
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$$\begin{aligned}
\dot{\mathbf{v}}(t) &= \dot{x}(t)\mathbf{T}(t) + x(t)\dot{\mathbf{T}}(t) \\
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\end{aligned}$$

What are $\dot{\mathbf{T}}(t)$, $\dot{\mathbf{N}}(t)$, and $\dot{\mathbf{B}}(t)$ in terms of the basis?

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$\dot{\mathbf{B}}(t).\mathbf{T}(t)$

Space curves

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$$\begin{aligned} \dot{\mathbf{v}}(t) &= \dot{x}(t)\mathbf{T}(t) + x(t)\dot{\mathbf{T}}(t) \\ &+ \dot{y}(t)\mathbf{N}(t) + y(t)\dot{\mathbf{N}}(t) \\ &+ \dot{z}(t)\mathbf{B}(t) + z(t)\dot{\mathbf{B}}(t) \end{aligned}$$

What are $\dot{\mathbf{T}}(t)$, $\dot{\mathbf{N}}(t)$, and $\dot{\mathbf{B}}(t)$ in terms of the basis?

$\dot{\mathbf{T}}(t) = 0\mathbf{T}(t) + \kappa(t)\mathbf{N}(t) + 0\mathbf{B}(t)$
$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + 0\mathbf{N}(t) + ??\mathbf{B}(t)$
$\dot{\mathbf{B}}(t) =$

$$\dot{\mathbf{N}}(t).\mathbf{T}(t) + \underbrace{\mathbf{N}(t).\dot{\mathbf{T}}(t)}_{\kappa(t)} = \underbrace{(\mathbf{N}(t).\mathbf{T}(t))'}_{0}$$

$$\dot{\mathbf{N}}(t).\mathbf{B}(t) + \mathbf{N}(t).\dot{\mathbf{B}}(t) = \underbrace{(\mathbf{N}(t).\mathbf{B}(t))'}_{0}$$

$$\dot{\mathbf{B}}(t).\mathbf{T}(t) + \mathbf{B}(t).\dot{\mathbf{T}}(t) = \underbrace{(\mathbf{B}(t).\mathbf{B}(t))'}_{0}$$

Space curves

Given $\gamma : (\alpha, \beta) \to \mathbb{R}^3$ **unit speed** parametrization, $\kappa(t) \neq 0$
$\mathbf{T}(t) := \dot{\gamma}(t)$, (unit) vector in direction of velocity
$\mathbf{N}(t) := \frac{1}{\kappa(t)}\ddot{\gamma}(t)$, (unit) vector perpendicular to $\mathbf{T}(t)$
$\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$, (unit) vector perpendicular to $\mathbf{T}(t)$ and $\mathbf{N}(t)$.

$\{\mathbf{T}(t), \mathbf{N}(t), \mathbf{B}(t)\}$ form an orthonormal basis for each $t \in (\alpha, \beta)$.

So, any vector field,

$$\mathbf{v}(t) = x(t)\mathbf{T}(t) + y(t)\mathbf{N}(t) + z(t)\mathbf{B}(t)$$

for some (unique!) $x(t), y(t), z(t) \in \mathbb{R}$
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$$\dot{\mathbf{N}}(t).\mathbf{T}(t) + \underbrace{\mathbf{N}(t).\dot{\mathbf{T}}(t)}_{\kappa(t)} = \underbrace{(\mathbf{N}(t).\mathbf{T}(t))'}_{0}$$

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$\dot{\mathbf{B}}(t) = 0\mathbf{T}(t) + \cdots$

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$\dot{\mathbf{B}}(t) = 0\mathbf{T}(t) + \cdots + 0\mathbf{B}(t)$

$$\dot{\mathbf{N}}(t).\mathbf{T}(t) + \underbrace{\mathbf{N}(t).\dot{\mathbf{T}}(t)}_{\kappa(t)} = \underbrace{(\mathbf{N}(t).\mathbf{T}(t))'}_{0}$$

$$\dot{\mathbf{N}}(t).\mathbf{B}(t) + \mathbf{N}(t).\dot{\mathbf{B}}(t) = \underbrace{(\mathbf{N}(t).\mathbf{B}(t))'}_{0}$$

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Space curves

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$\dot{\mathbf{B}}(t) = 0\mathbf{T}(t) - \tau(t)\mathbf{N}(t) + 0\mathbf{B}(t)$

$$\dot{\mathbf{N}}(t).\mathbf{T}(t) + \underbrace{\mathbf{N}(t).\dot{\mathbf{T}}(t)}_{\kappa(t)} = \underbrace{(\mathbf{N}(t).\mathbf{T}(t))'}_{0}$$

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$$\dot{\mathbf{N}}(t).\mathbf{T}(t) + \underbrace{\mathbf{N}(t).\dot{\mathbf{T}}(t)}_{\kappa(t)} = \underbrace{(\mathbf{N}(t).\mathbf{T}(t))'}_{0}$$

$$\dot{\mathbf{N}}(t).\mathbf{B}(t) + \underbrace{\mathbf{N}(t).\dot{\mathbf{B}}(t)}_{-\tau(t)} = \underbrace{(\mathbf{N}(t).\mathbf{B}(t))'}_{0}$$

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$\dot{\mathbf{B}}(t) = 0\mathbf{T}(t) - \tau(t)\mathbf{N}(t) + 0\mathbf{B}(t)$

$$\dot{\mathbf{N}}(t).\mathbf{T}(t) + \underbrace{\mathbf{N}(t).\dot{\mathbf{T}}(t)}_{\kappa(t)} = \underbrace{(\mathbf{N}(t).\mathbf{T}(t))'}_{0}$$

$$\dot{\mathbf{N}}(t).\mathbf{B}(t) + \underbrace{\mathbf{N}(t).\dot{\mathbf{B}}(t)}_{-\tau(t)} = \underbrace{(\mathbf{N}(t).\mathbf{B}(t))'}_{0}$$

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$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$

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Frenet-Serret equations

Frenet-Serret equations

$$\dot{\mathbf{T}}(t) = 0\mathbf{T}(t) + \kappa(t)\mathbf{N}(t) + 0\mathbf{B}(t)$$
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Frenet-Serret equations

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Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$

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Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion

Frenet-Serret equations

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$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$
$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ

Frenet-Serret equations

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$
$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t.

Planes

$$P = \{(x, y, z) \in \mathbb{R}^3 \mid a(x-x_0)+b(y-y_0)+c(z-z_0) = 0\}$$

Frenet-Serret equations

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$
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Frenet-Serret equations

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$\gamma :$

Frenet-Serret equations

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
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Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t.

Planes

$$P = \{(x, y, z) \in \mathbb{R}^3 \mid a(x-x_0)+b(y-y_0)+c(z-z_0) = 0\}$$
$$P = \{\mathbf{v} \in \mathbb{R}^3 \mid \mathbf{n}.\mathbf{v} = 0\}$$

$\gamma : (\alpha, \beta)$

Frenet-Serret equations

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
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$$P = \{\mathbf{v} \in \mathbb{R}^3 \mid \mathbf{n}.\mathbf{v} = 0\}$$

$$\gamma : (\alpha, \beta) \to \mathbb{R}^3$$

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$$P = \{\mathbf{v} \in \mathbb{R}^3 \mid \mathbf{n}.\mathbf{v} = 0\}$$

$\gamma : (\alpha, \beta) \to \mathbb{R}^3$ parametrizes a curve

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Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t.

Planes

$$P = \{(x, y, z) \in \mathbb{R}^3 \,|\, a(x-x_0)+b(y-y_0)+c(z-z_0) = 0\}$$
$$P = \{\mathbf{v} \in \mathbb{R}^3 \mid \mathbf{n}.\mathbf{v} = 0\}$$

$\gamma : (\alpha, \beta) \to \mathbb{R}^3$ parametrizes a curve that lies on the plane, P,

Frenet-Serret equations

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$
$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

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$\gamma : (\alpha, \beta) \to \mathbb{R}^3$ parametrizes a curve that lies on the plane, P, if and only if

$$\mathbf{n}.(\gamma(t) - \gamma(t_0)) = 0$$

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Planes

$$P = \{(x, y, z) \in \mathbb{R}^3 \mid a(x-x_0)+b(y-y_0)+c(z-z_0) = 0\}$$
$$P = \{\mathbf{v} \in \mathbb{R}^3 \mid \mathbf{n}.\mathbf{v} = 0\}$$

$\gamma : (\alpha, \beta) \to \mathbb{R}^3$ parametrizes a curve that lies on the plane, P, if and only if

$$\mathbf{n}.(\gamma(t) - \gamma(t_0)) = 0$$

for all t

Frenet-Serret equations

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
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$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t.

Planes

$$P = \{(x, y, z) \in \mathbb{R}^3 \mid a(x-x_0)+b(y-y_0)+c(z-z_0) = 0\}$$
$$P = \{\mathbf{v} \in \mathbb{R}^3 \mid \mathbf{n}.\mathbf{v} = 0\}$$

$\gamma : (\alpha, \beta) \to \mathbb{R}^3$ parametrizes a curve that lies on the plane, P, if and only if

$$\mathbf{n}.(\gamma(t) - \gamma(t_0)) = 0$$

for all $t \in (\alpha, \beta)$.

$$
\begin{aligned}
\dot{\mathbf{T}}(t) &= \kappa(t)\mathbf{N}(t) \\
\dot{\mathbf{N}}(t) &= -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t) \\
\dot{\mathbf{B}}(t) &= -\tau(t)\mathbf{N}(t)
\end{aligned}
$$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t.

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$
$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t.

If $\tau(t) = 0$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$
$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t.

If $\tau(t) = 0$ for all t

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$
$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t.

If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$
$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t.

If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,
then $\dot{\mathbf{B}}(t)$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
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Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t.

If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,
then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t)$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$
$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t.

If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,
then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$
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Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t.

If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,
then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$
So, $\mathbf{B}(t)$ is constant,

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$
$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t.

If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,
then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$
So, $\mathbf{B}(t)$ is constant, say, $\mathbf{B}$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
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If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,
then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$
So, $\mathbf{B}(t)$ is constant, say, $\mathbf{B}$

$$((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))'$$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$
$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

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If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,
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So, $\mathbf{B}(t)$ is constant, say, $\mathbf{B}$

$$((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' \;=\; ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) +$$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$
$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

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If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,
then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$
So, $\mathbf{B}(t)$ is constant, say, $\mathbf{B}$

$$((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' \; = \; ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t)$$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$
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If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,
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So, $\mathbf{B}(t)$ is constant, say, $\mathbf{B}$

$$\begin{aligned}((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) -\end{aligned}$$

$$
\begin{aligned}
\dot{\mathbf{T}}(t) &= \kappa(t)\mathbf{N}(t)\\
\dot{\mathbf{N}}(t) &= -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)\\
\dot{\mathbf{B}}(t) &= -\tau(t)\mathbf{N}(t)
\end{aligned}
$$

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If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,
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So, $\mathbf{B}(t)$ is constant, say, $\mathbf{B}$

$$
\begin{aligned}
((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t)\\
&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t)
\end{aligned}
$$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
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So, $\mathbf{B}(t)$ is constant, say, $\mathbf{B}$

$$\begin{aligned}
((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\
&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\
&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0
\end{aligned}$$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
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If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,
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So, $\mathbf{B}(t)$ is constant, say, $\mathbf{B}$

$$\begin{aligned}
((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\
&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\
&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \\
&= -\mathbf{T}(t).\mathbf{B}(t)
\end{aligned}$$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$
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So, $\mathbf{B}(t)$ is constant, say, $\mathbf{B}$

$$\begin{aligned}
((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\
&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\
&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \\
&= -\mathbf{T}(t).\mathbf{B}(t) \\
&= 0
\end{aligned}$$

$$(\gamma(t_0) - \gamma(t)).\mathbf{B}(t)$$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$
$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

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If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,
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So, $\mathbf{B}(t)$ is constant, say, $\mathbf{B}$

$$\begin{aligned}
((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\
&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\
&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \\
&= -\mathbf{T}(t).\mathbf{B}(t) \\
&= 0
\end{aligned}$$

$$(\gamma(t_0) - \gamma(t)).\mathbf{B}(t) = (\gamma(t_0) - \gamma(t)).\mathbf{B}$$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$
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Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t.

If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,
then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$
So, $\mathbf{B}(t)$ is constant, say, $\mathbf{B}$

$$\begin{aligned}
((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\
&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\
&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \\
&= -\mathbf{T}(t).\mathbf{B}(t) \\
&= 0
\end{aligned}$$

$$(\gamma(t_0) - \gamma(t)).\mathbf{B}(t) = (\gamma(t_0) - \gamma(t)).\mathbf{B} = c$$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$
$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

At $t = t_0$,

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t.

If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,
then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$
So, $\mathbf{B}(t)$ is constant, say, $\mathbf{B}$

$$\begin{aligned}
((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\
&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\
&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \\
&= -\mathbf{T}(t).\mathbf{B}(t) \\
&= 0
\end{aligned}$$

$$(\gamma(t_0) - \gamma(t)).\mathbf{B}(t) = (\gamma(t_0) - \gamma(t)).\mathbf{B} = c$$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$
$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t.

If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,
then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$
So, $\mathbf{B}(t)$ is constant, say, $\mathbf{B}$

$$\begin{aligned}
((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\
&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\
&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \\
&= -\mathbf{T}(t).\mathbf{B}(t) \\
&= 0
\end{aligned}$$

$$(\gamma(t_0) - \gamma(t)).\mathbf{B}(t) = (\gamma(t_0) - \gamma(t)).\mathbf{B} = c$$

At $t = t_0$,
$c = (\gamma(t_0) - \gamma(t_0)).B = 0$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$
$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t.

If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,
then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$
So, $\mathbf{B}(t)$ is constant, say, $\mathbf{B}$

$$\begin{aligned}
((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\
&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\
&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \\
&= -\mathbf{T}(t).\mathbf{B}(t) \\
&= 0
\end{aligned}$$

$$(\gamma(t_0) - \gamma(t)).\mathbf{B}(t) = (\gamma(t_0) - \gamma(t)).\mathbf{B} = c$$

At $t = t_0$,
$c = (\gamma(t_0) - \gamma(t_0)).B = 0$

So, $(\gamma(t_0) - \gamma(t)).B = 0$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$
$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t.

If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,
then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$
So, $\mathbf{B}(t)$ is constant, say, $\mathbf{B}$

$$\begin{aligned}
((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\
&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\
&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \\
&= -\mathbf{T}(t).\mathbf{B}(t) \\
&= 0
\end{aligned}$$

$$(\gamma(t_0) - \gamma(t)).\mathbf{B}(t) = (\gamma(t_0) - \gamma(t)).\mathbf{B} = c$$

At $t = t_0$,
$c = (\gamma(t_0) - \gamma(t_0)).B = 0$

So, $(\gamma(t_0) - \gamma(t)).B = 0$

If the torsion is a constant 0, the curve lies on a plane.

Notation:

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$
$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t.

If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,
then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$
So, $\mathbf{B}(t)$ is constant, say, $\mathbf{B}$

$$\begin{aligned}((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \\ &= -\mathbf{T}(t).\mathbf{B}(t) \\ &= 0\end{aligned}$$

$$(\gamma(t_0) - \gamma(t)).\mathbf{B}(t) = (\gamma(t_0) - \gamma(t)).\mathbf{B} = c$$

At $t = t_0$,
$c = (\gamma(t_0) - \gamma(t_0)).B = 0$

So, $(\gamma(t_0) - \gamma(t)).B = 0$

If the torsion is a constant 0, the curve lies on a plane.

Notation:
$\mathbf{T}(t)$:

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$
$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t.

If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,
then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$
So, $\mathbf{B}(t)$ is constant, say, $\mathbf{B}$

$$\begin{aligned}
((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\
&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\
&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \\
&= -\mathbf{T}(t).\mathbf{B}(t) \\
&= 0
\end{aligned}$$

$$(\gamma(t_0) - \gamma(t)).\mathbf{B}(t) = (\gamma(t_0) - \gamma(t)).\mathbf{B} = c$$

At $t = t_0$,
$c = (\gamma(t_0) - \gamma(t_0)).B = 0$

So, $(\gamma(t_0) - \gamma(t)).B = 0$

If the torsion is a constant 0, the curve lies on a plane.

Notation:
$\mathbf{T}(t)$: unit **tangent** vector at t

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$
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&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\
&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \\
&= -\mathbf{T}(t).\mathbf{B}(t) \\
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\end{aligned}$$

$$(\gamma(t_0) - \gamma(t)).\mathbf{B}(t) = (\gamma(t_0) - \gamma(t)).\mathbf{B} = c$$

At $t = t_0$,
$c = (\gamma(t_0) - \gamma(t_0)).B = 0$

So, $(\gamma(t_0) - \gamma(t)).B = 0$

If the torsion is a constant 0, the curve lies on a plane.

Notation:
$\mathbf{T}(t)$: unit **tangent** vector at t
$\mathbf{N}(t)$:

$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$
$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$
$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$

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((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\
&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\
&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \\
&= -\mathbf{T}(t).\mathbf{B}(t) \\
&= 0
\end{aligned}
$$

$(\gamma(t_0) - \gamma(t)).\mathbf{B}(t) = (\gamma(t_0) - \gamma(t)).\mathbf{B} = c$

At $t = t_0$,
$c = (\gamma(t_0) - \gamma(t_0)).B = 0$

So, $(\gamma(t_0) - \gamma(t)).B = 0$

If the torsion is a constant 0, the curve lies on a plane.

Notation:
$\mathbf{T}(t)$: unit **tangent** vector at t
$\mathbf{N}(t)$: unit **normal** vector at t

$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$
$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$
$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t.

If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,
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So, $\mathbf{B}(t)$ is constant, say, $\mathbf{B}$

$$
\begin{aligned}
((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\
&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\
&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \\
&= -\mathbf{T}(t).\mathbf{B}(t) \\
&= 0
\end{aligned}
$$

$(\gamma(t_0) - \gamma(t)).\mathbf{B}(t) = (\gamma(t_0) - \gamma(t)).\mathbf{B} = c$

At $t = t_0$,
$c = (\gamma(t_0) - \gamma(t_0)).B = 0$

So, $(\gamma(t_0) - \gamma(t)).B = 0$

If the torsion is a constant 0, the curve lies on a plane.

Notation:
$\mathbf{T}(t)$: unit **tangent** vector at t
$\mathbf{N}(t)$: unit **normal** vector at t
$\mathbf{B}(t)$:

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$
$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$
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&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\
&= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \\
&= -\mathbf{T}(t).\mathbf{B}(t) \\
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\end{aligned}$$

$$(\gamma(t_0) - \gamma(t)).\mathbf{B}(t) = (\gamma(t_0) - \gamma(t)).\mathbf{B} = c$$

At $t = t_0$,
$c = (\gamma(t_0) - \gamma(t_0)).B = 0$

So, $(\gamma(t_0) - \gamma(t)).B = 0$

If the torsion is a constant 0, the curve lies on a plane.

Notation:
$\mathbf{T}(t)$: unit **tangent** vector at t
$\mathbf{N}(t)$: unit **normal** vector at t
$\mathbf{B}(t)$: unit **binormal** vector at t