Hints / Solutions to Exercise sheet 3

Curves and Surfaces, MTH201

Question 1: In an earlier exercise you found the parametrization of a line segment joining two points. Use that parametrization to find the arc length of the line segment in terms of its end points. Try with some other parametrization too.

Solution 1: The line segment between p and q can be obtained by considering the displacement vector $q - p$. The parametrization should be so that at $t = 0$, p is not being displaced at all, but when $t = 1$, p is displaced to q:

$$\gamma(t) = p + t(q - p)$$
$$\dot{\gamma}(t) = (q - p)$$
$$\|\dot{\gamma}(t)\| = \|q - p\|$$
$$\int_0^1 \|\dot{\gamma}(t)\mathrm{d}t\| = \int_0^1 \|q - p\|\mathrm{d}t = \|q - p\|$$

Question 2: These steps will show that the line segment joining two points is the shortest possible curve joining the two points:

Solution 2: Remember that $\mathbf{v}.\mathbf{w} = \|\mathbf{v}\|\|\mathbf{w}\|\cos(\theta)$ where θ is the angle between $\mathbf{v}$ and $\mathbf{w}$. Part 1.. easily follows from $\cos(\theta) \leq 1$ for any angle θ.

For part 2,

$$\mathbf{v}.\frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{1}{\|\mathbf{v}\|}(\mathbf{v}.\mathbf{v}) = \frac{\|\mathbf{v}\|^2}{\|\mathbf{v}\|} = \|\mathbf{v}\|$$

This part allows you to express the norm in a way that will allow you to use the fundamental theorem of calculus in the next part.