

Exercise sheet 4

Curves and Surfaces, MTH201

1. Prove that $\ddot{\gamma}(t) \cdot \mathbf{n}(\gamma(t)) = -\dot{\gamma}(t) \cdot \frac{d}{dt} \mathbf{n}(\gamma(t))$. (*Hint*: You have done similar things many times before. Which rule helps you?)
2. Use the chain rule to show that $\frac{d}{dt}|_{t=t_0} \mathbf{n}(\gamma(t)) = \frac{d}{dt}|_{t=t_0} \mathbf{n}(\delta(t))$ if $\gamma(t_0) = \delta(t_0)$ and $\dot{\gamma}(t_0) = \dot{\delta}(t_0)$. In other words, the derivative is the same for curves which pass through the same point $\gamma(t_0)$ and have the same velocity vectors. (*Hint*: Look at everything from the point of view of the coordinate patch and apply chain rule).
3. Can you see how the previous two exercises show that for a unit speed parametrization γ , the quantity $\ddot{\gamma}(t) \cdot \mathbf{n}(\gamma(t))$ depends only on the point and direction.