Definition. A "parametrized plane curve"

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Explicitly,
$\gamma(t) = (f_1(t), f_2(t))$, for planes

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Examples.

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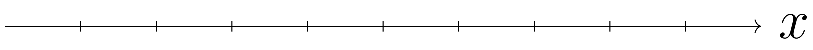
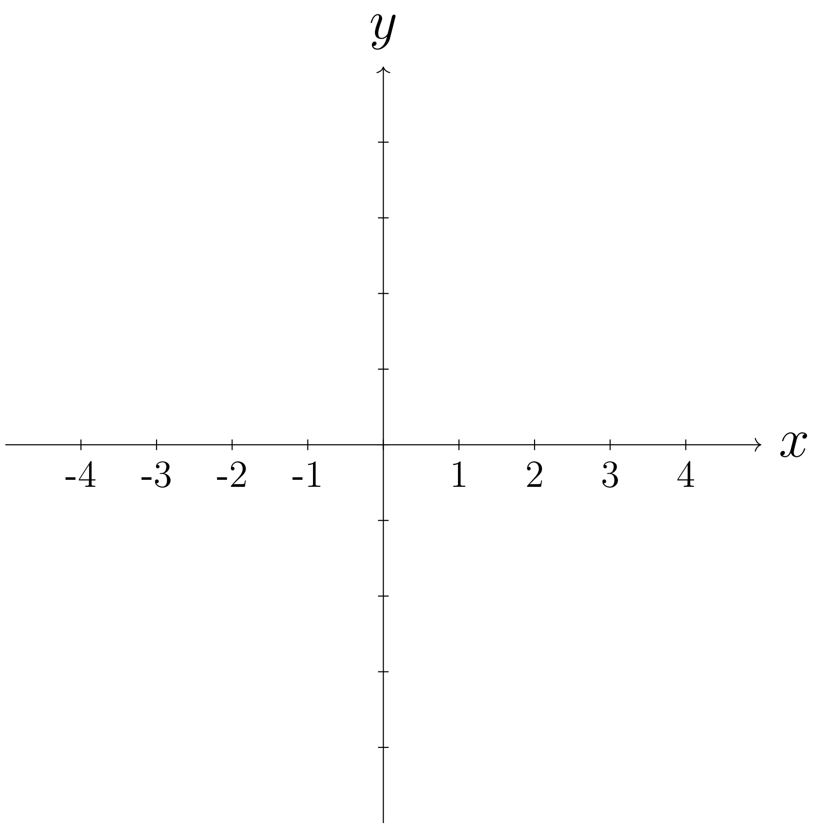
Parametrizing a circle



Parametrizing a circle



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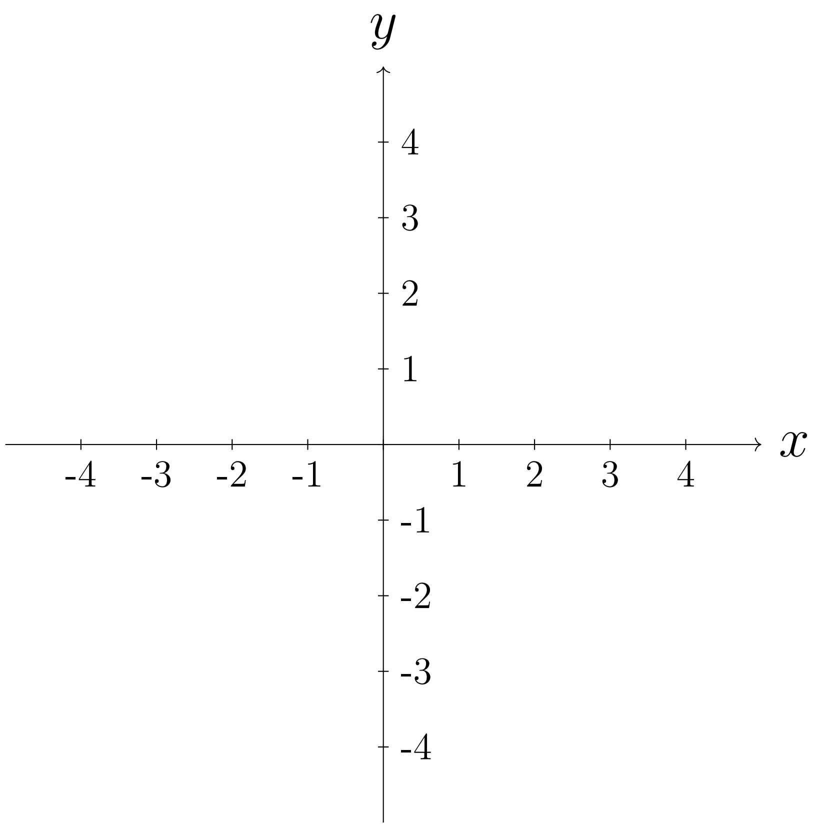

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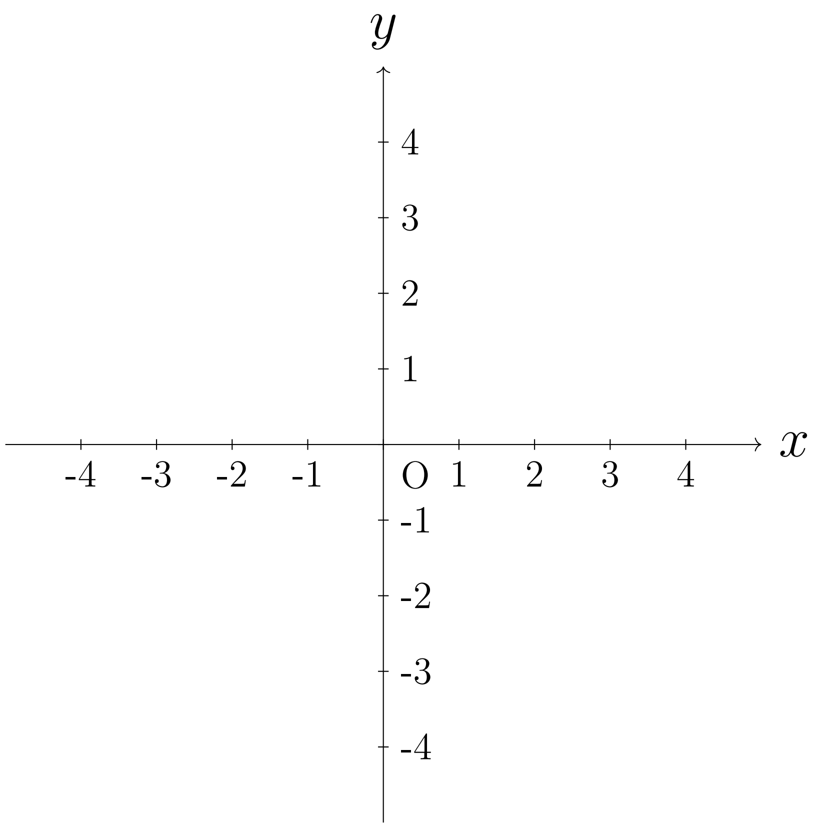
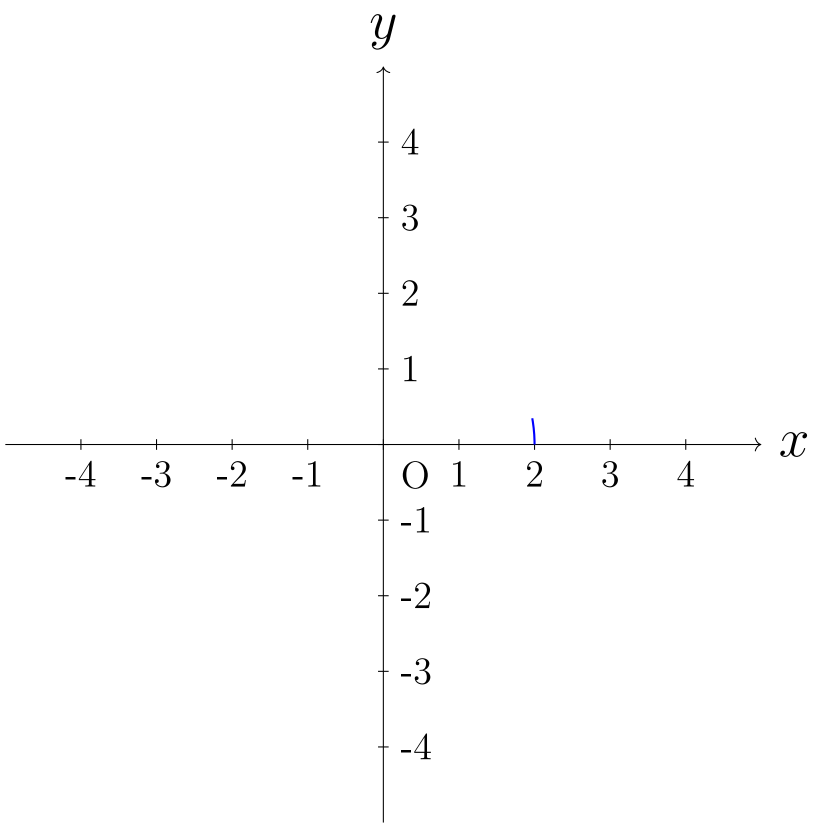


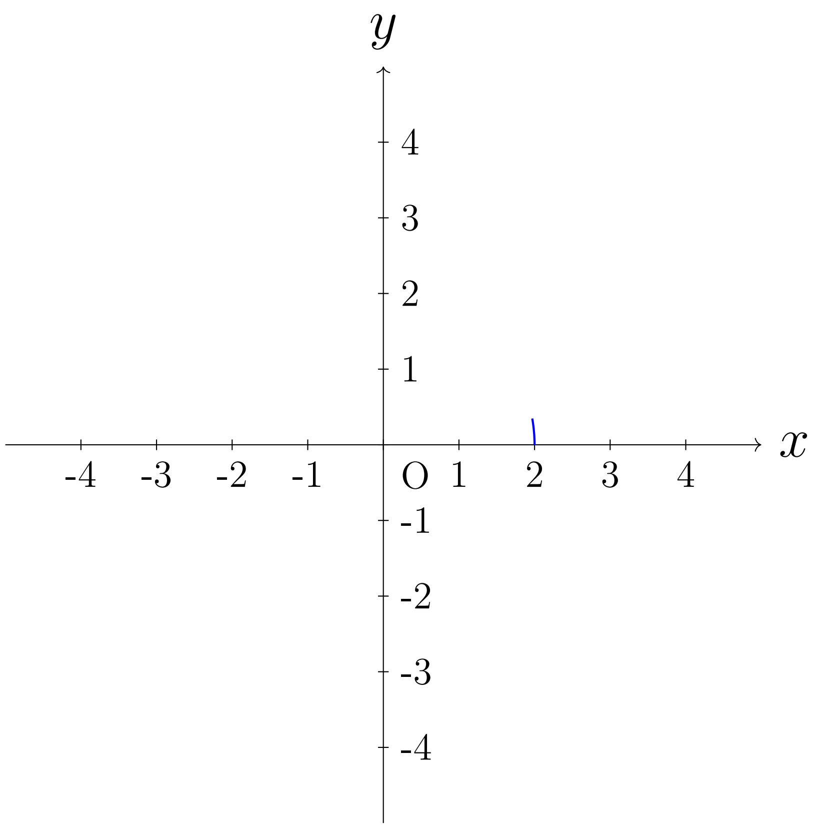
Parametrizing a circle

$$\gamma : (0, \pi/18) \to \mathbb{R}^2$$



Parametrizing a circle

$$\gamma : (0, \pi/18) \to \mathbb{R}^2$$
$$\gamma(t) := (2\cos(t), 2\sin(t))$$



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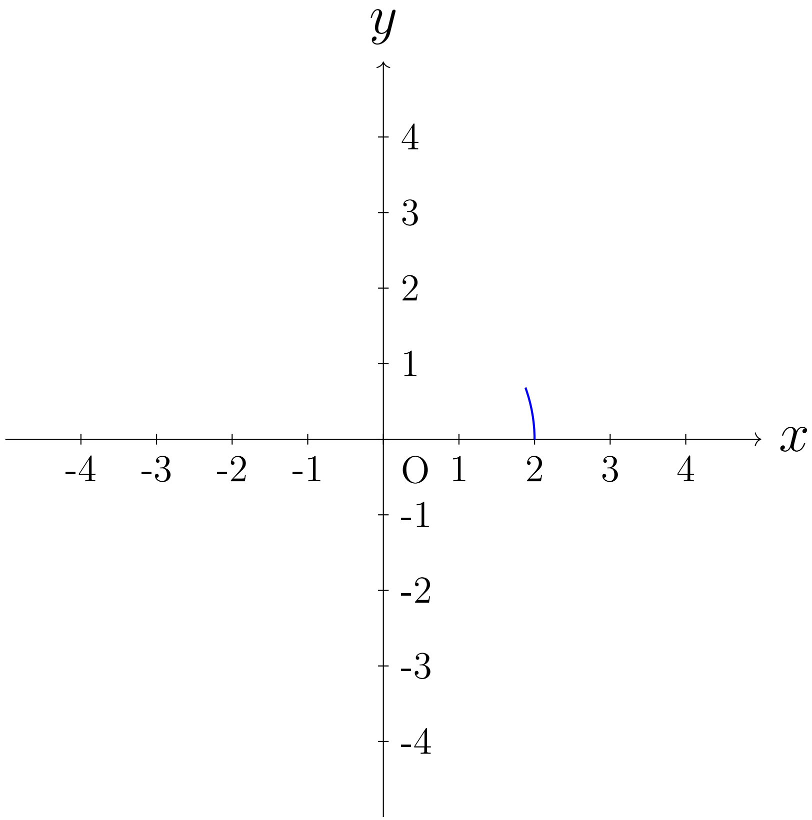
$$\gamma : (0, 2\pi/18) \to \mathbb{R}^2$$
$$\gamma(t) := (2\cos(t), 2\sin(t))$$



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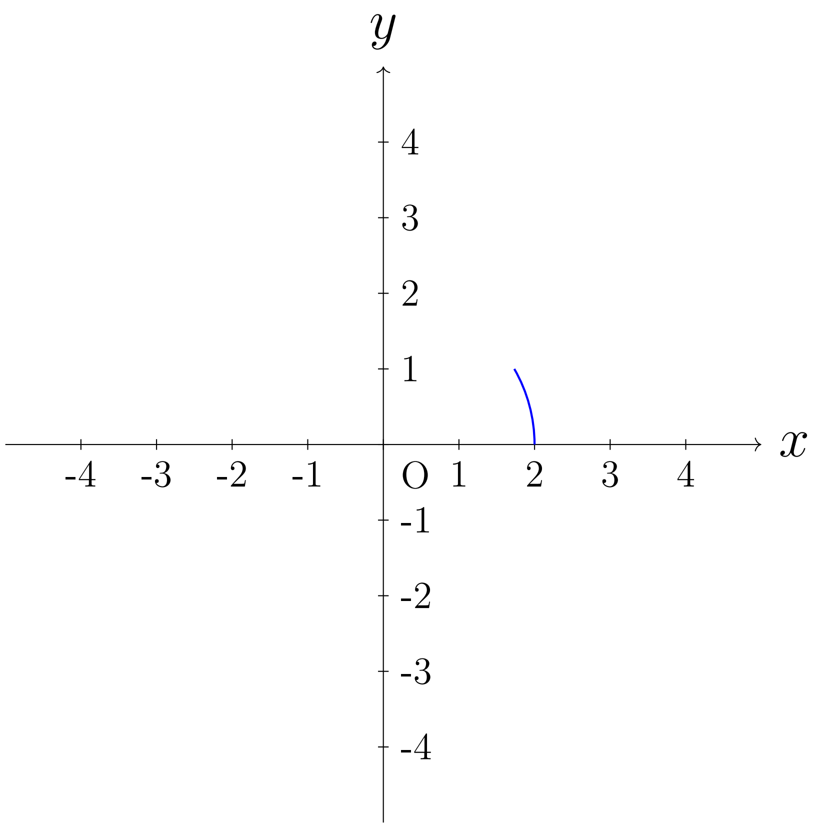

$$\gamma : (0, 3\pi/18) \to \mathbb{R}^2$$
$$\gamma(t) := (2\cos(t), 2\sin(t))$$

Parametrizing a circle

$$\gamma : (0, 6\pi/18) \to \mathbb{R}^2$$
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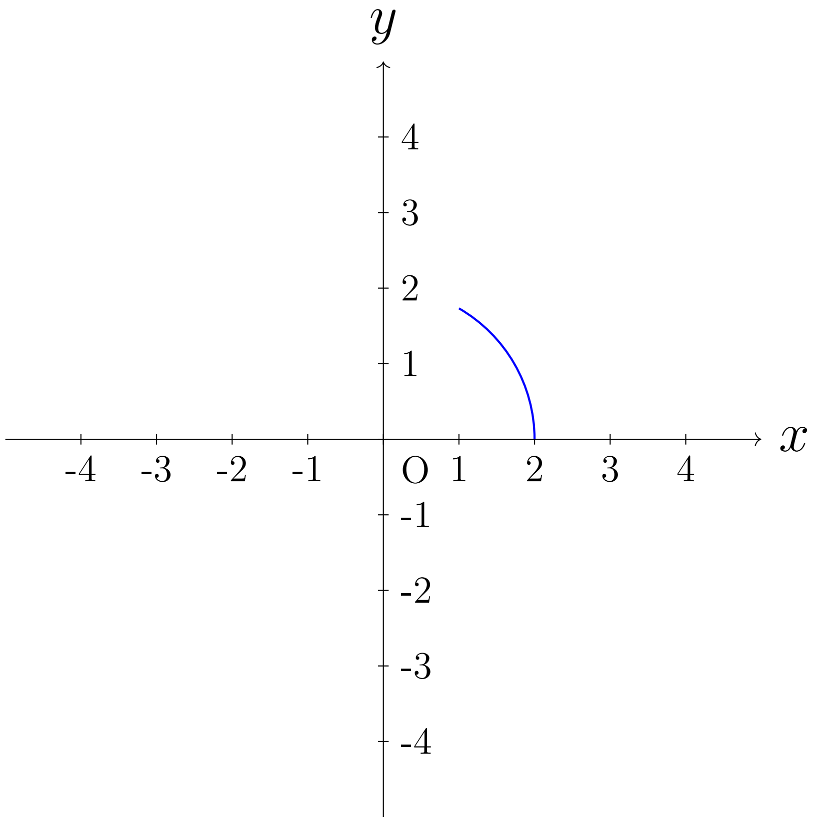
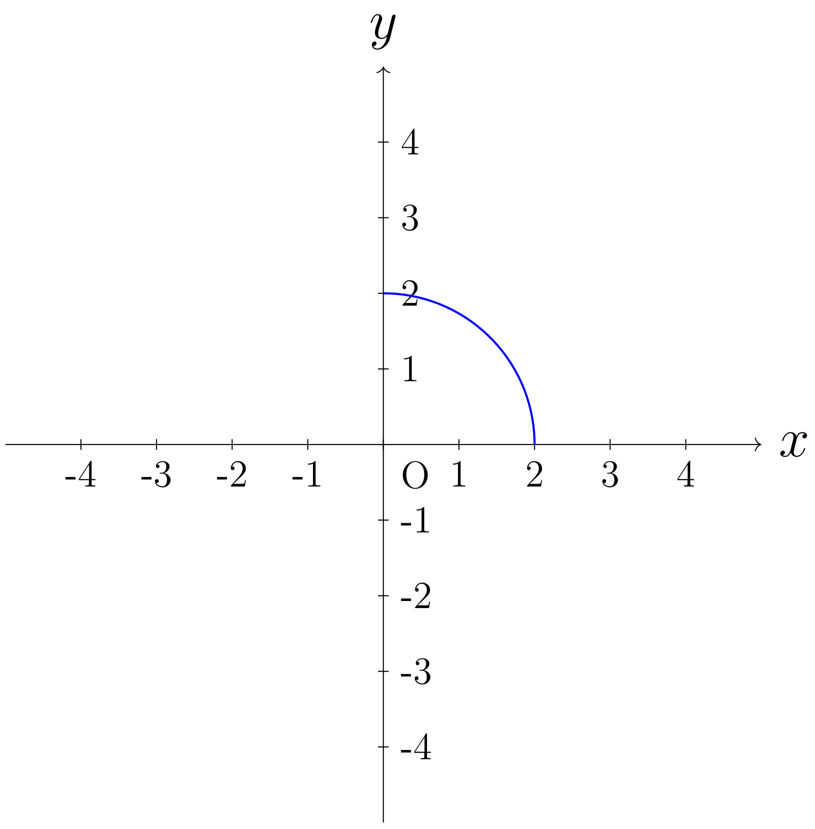


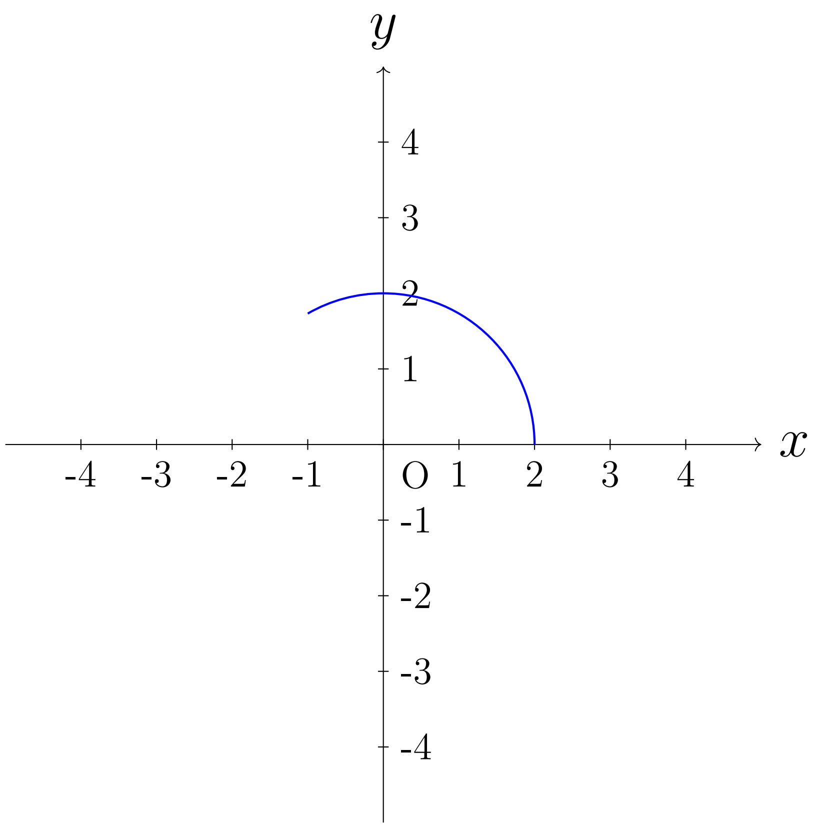
Parametrizing a circle

$$\gamma : (0, 9\pi/18) \to \mathbb{R}^2$$
$$\gamma(t) := (2\cos(t), 2\sin(t))$$



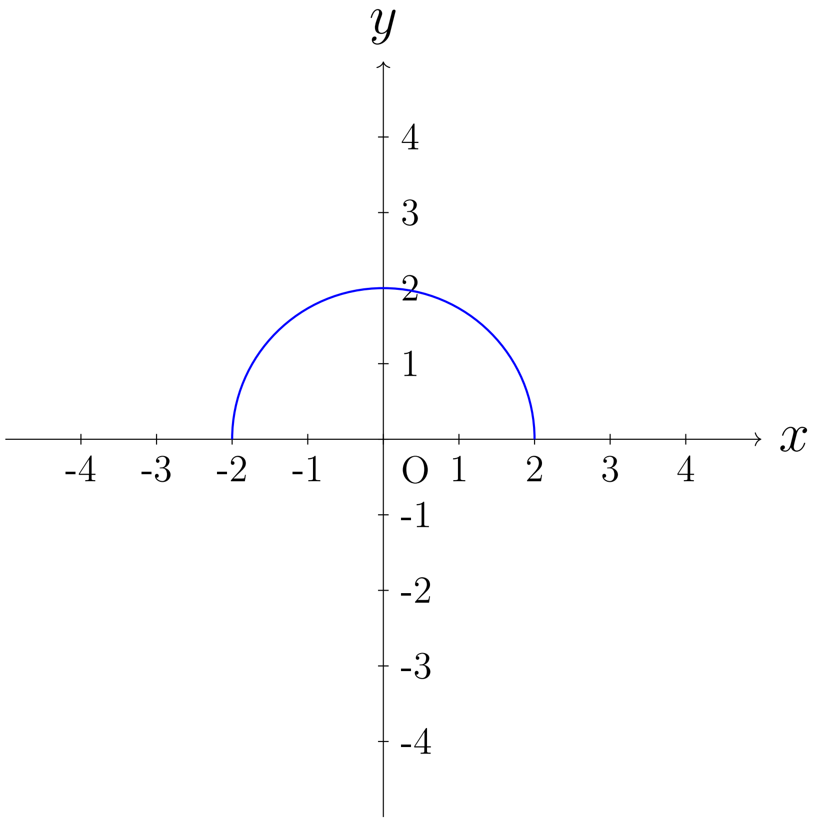
Parametrizing a circle

$$\gamma : (0, 12\pi/18) \to \mathbb{R}^2$$
$$\gamma(t) := (2\cos(t), 2\sin(t))$$



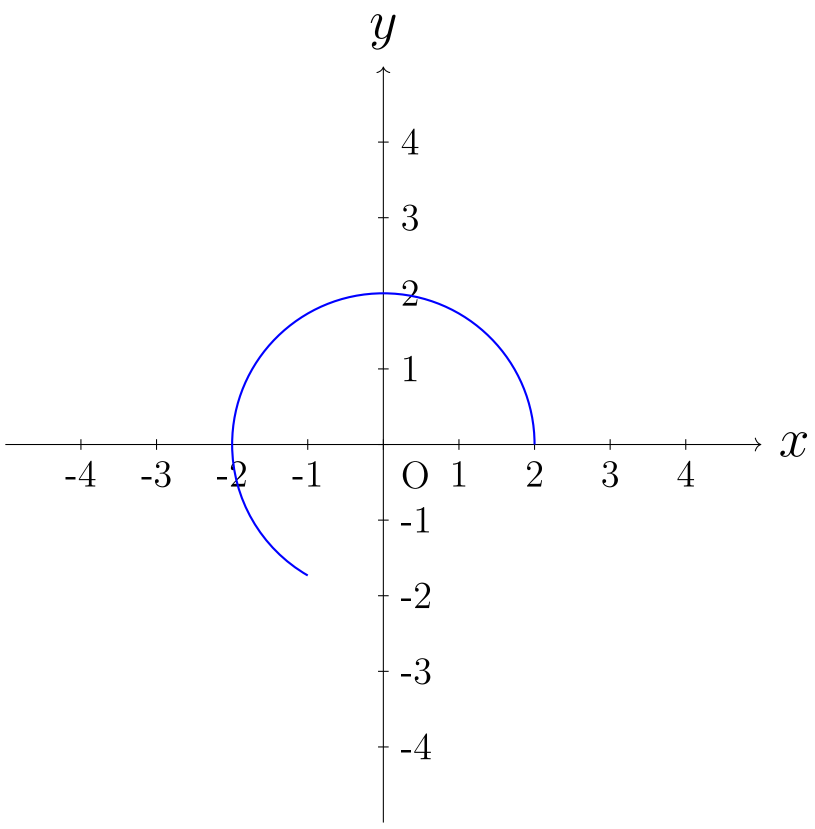
Parametrizing a circle

$$\gamma : (0, 18\pi/18) \to \mathbb{R}^2$$
$$\gamma(t) := (2\cos(t), 2\sin(t))$$



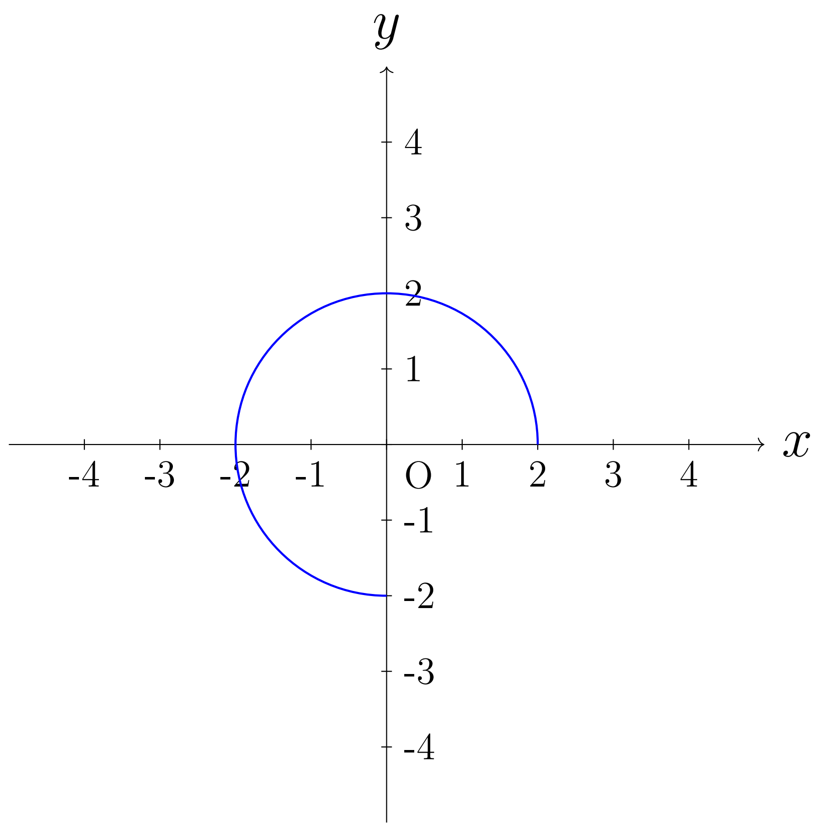
Parametrizing a circle

$$\gamma : (0, 24\pi/18) \to \mathbb{R}^2$$
$$\gamma(t) := (2\cos(t), 2\sin(t))$$



Parametrizing a circle

$$\gamma : (0, 27\pi/18) \to \mathbb{R}^2$$
$$\gamma(t) := (2\cos(t), 2\sin(t))$$



Parametrizing a line

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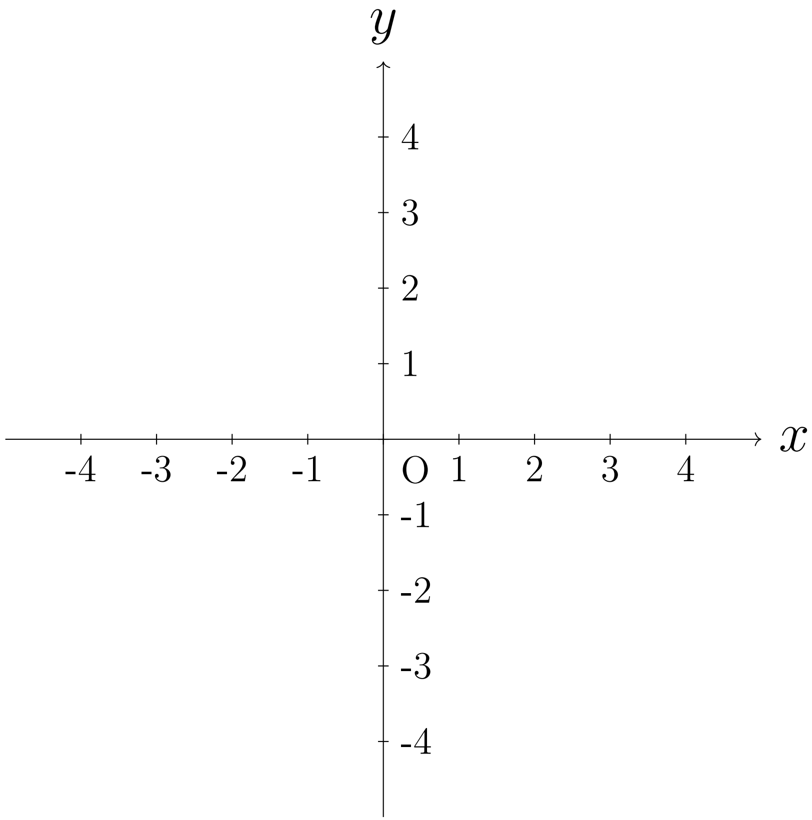
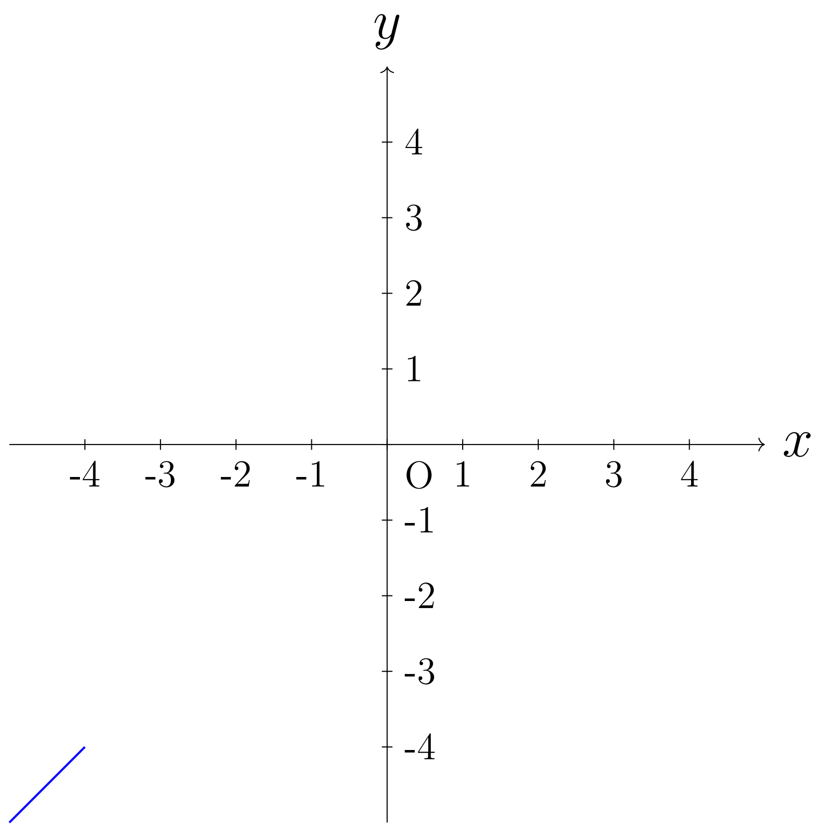
$$\gamma(t) := (2\cos(t), 2\sin(t))$$



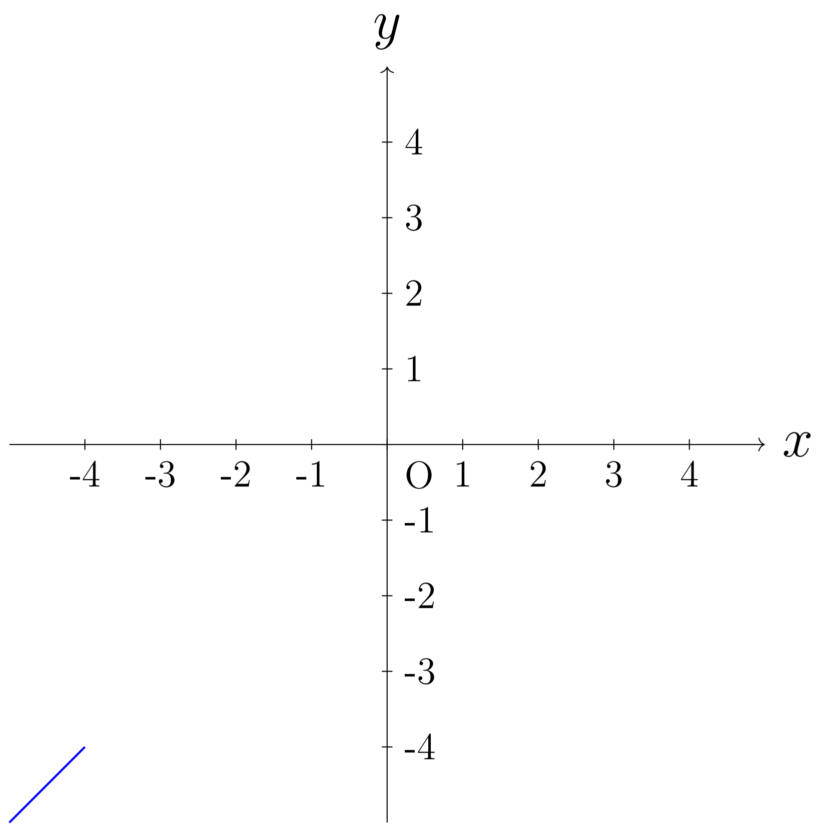
Parametrizing a line

$$\gamma : (-5, -4) \to \mathbb{R}^2$$



Parametrizing a line

$$\gamma : (-5, -4) \to \mathbb{R}^2$$
$$\gamma(t) := (t, t)$$



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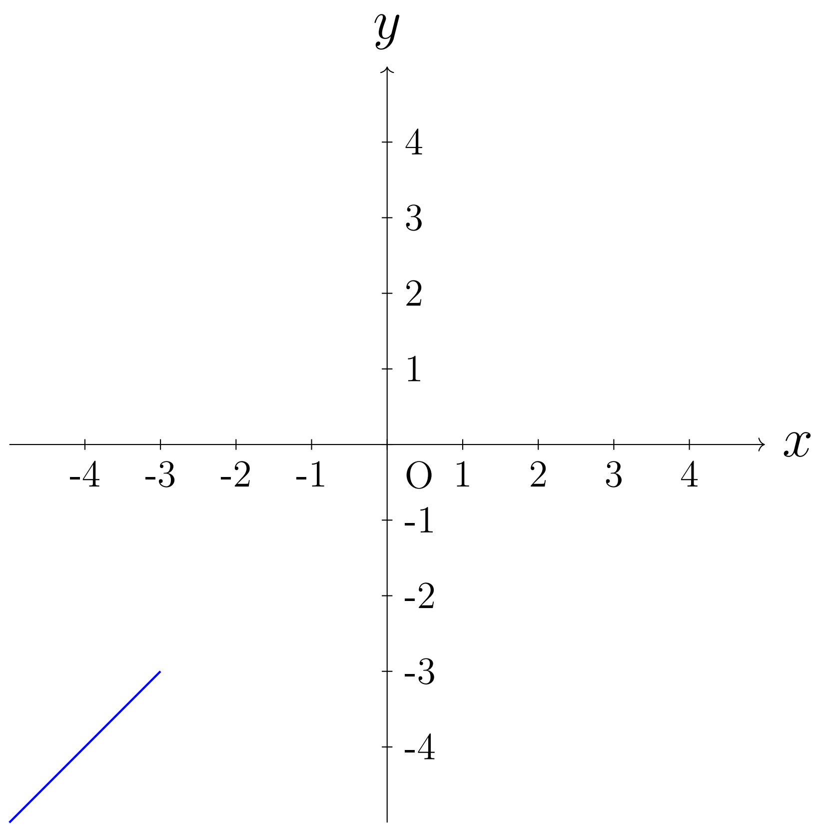

Parametrizing a line

$$\gamma : (-5, -2) \to \mathbb{R}^2$$
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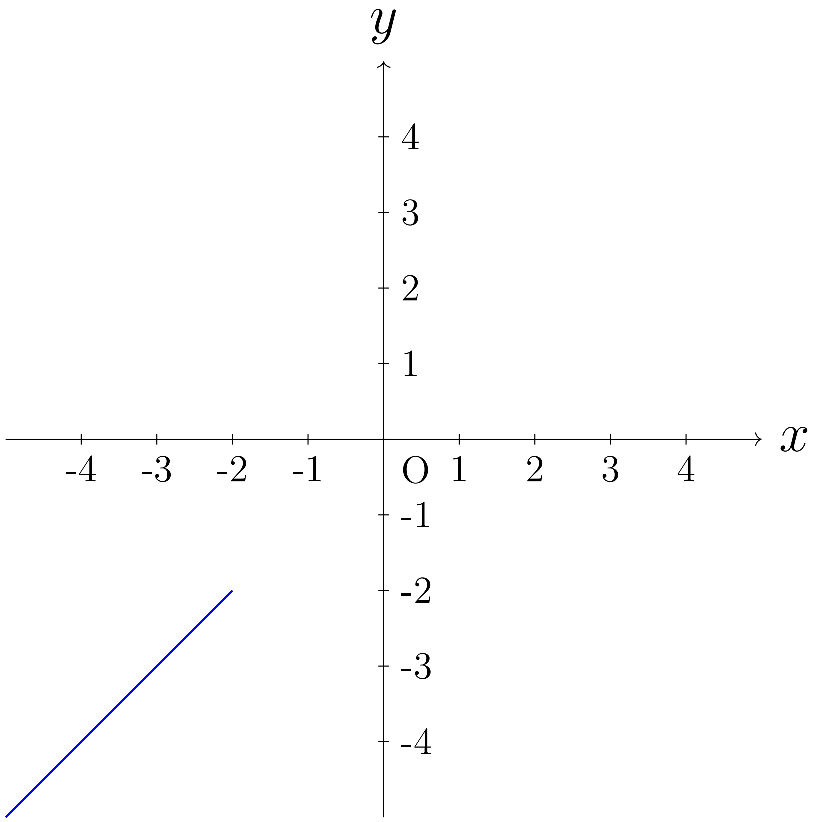

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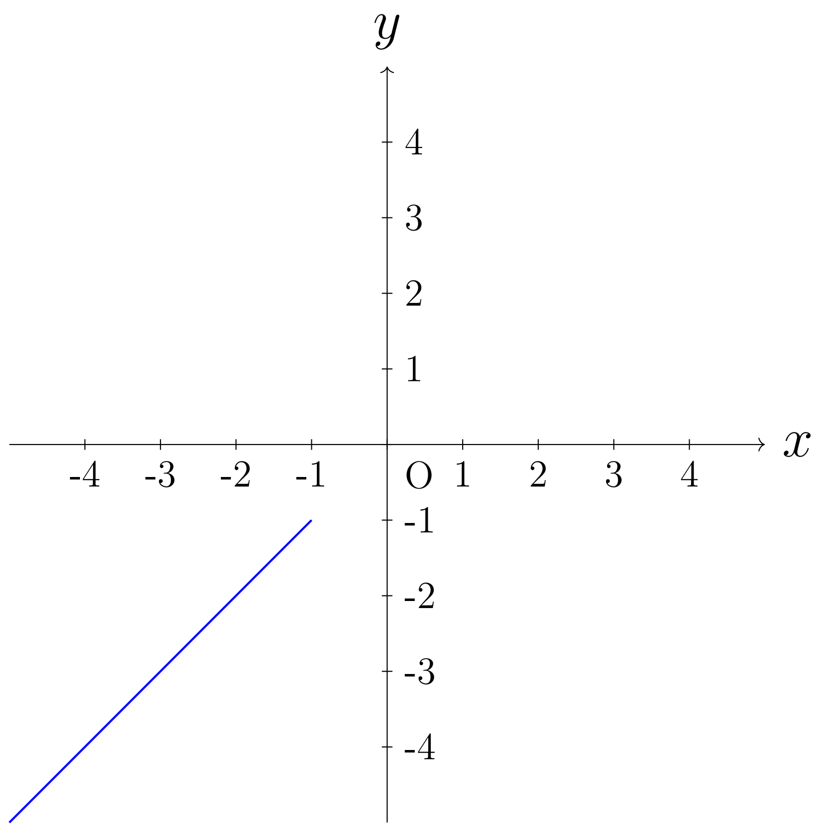
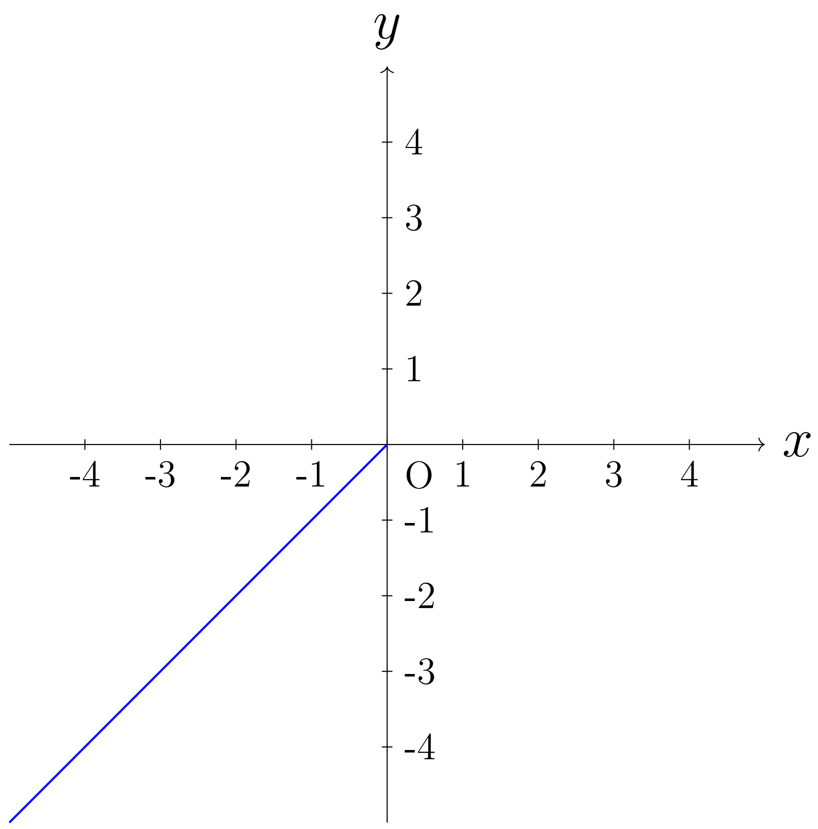


Parametrizing a line

$$\gamma : (-5, 0) \to \mathbb{R}^2$$
$$\gamma(t) := (t, t)$$



Parametrizing a line

$$\gamma : (-5, 1) \to \mathbb{R}^2$$
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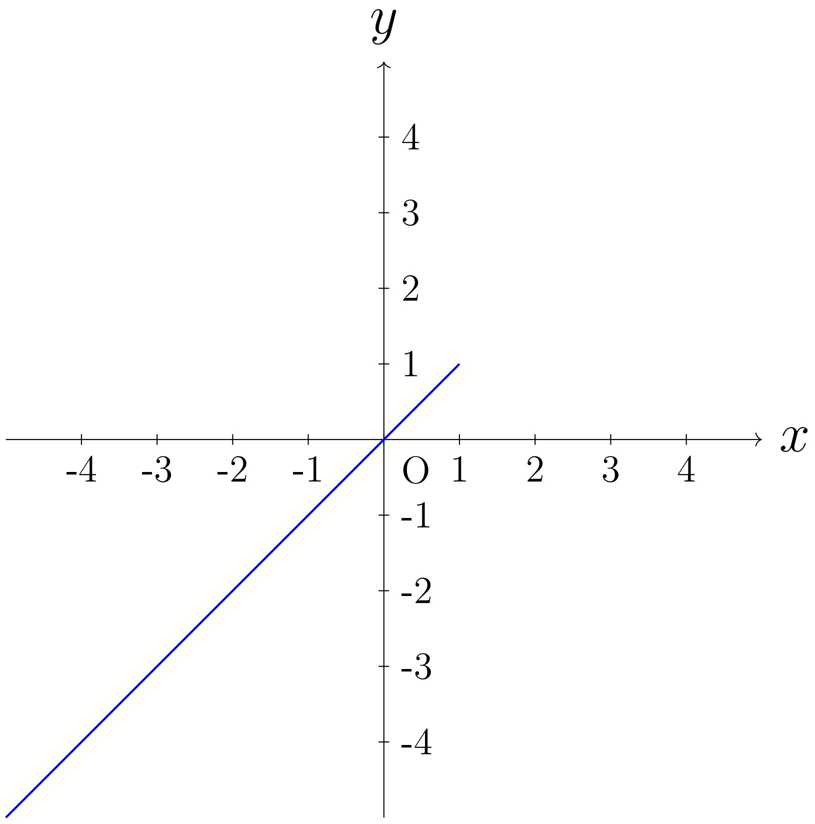
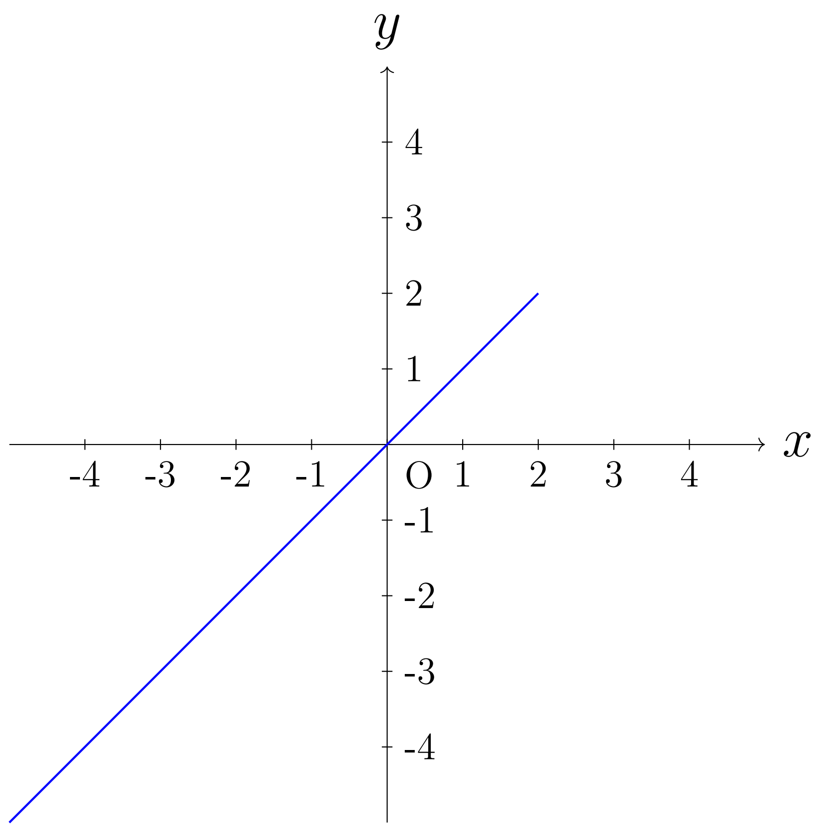


Parametrizing a line

$$\gamma : (-5, 2) \to \mathbb{R}^2$$
$$\gamma(t) := (t, t)$$



Parametrizing a line

$$\gamma : (-5, 3) \to \mathbb{R}^2$$
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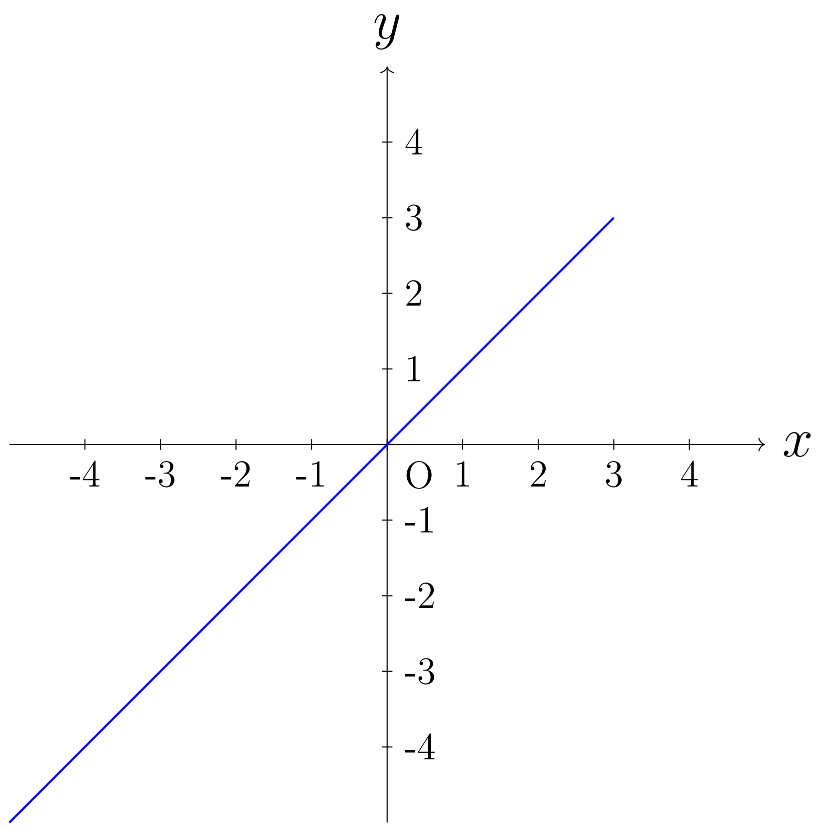
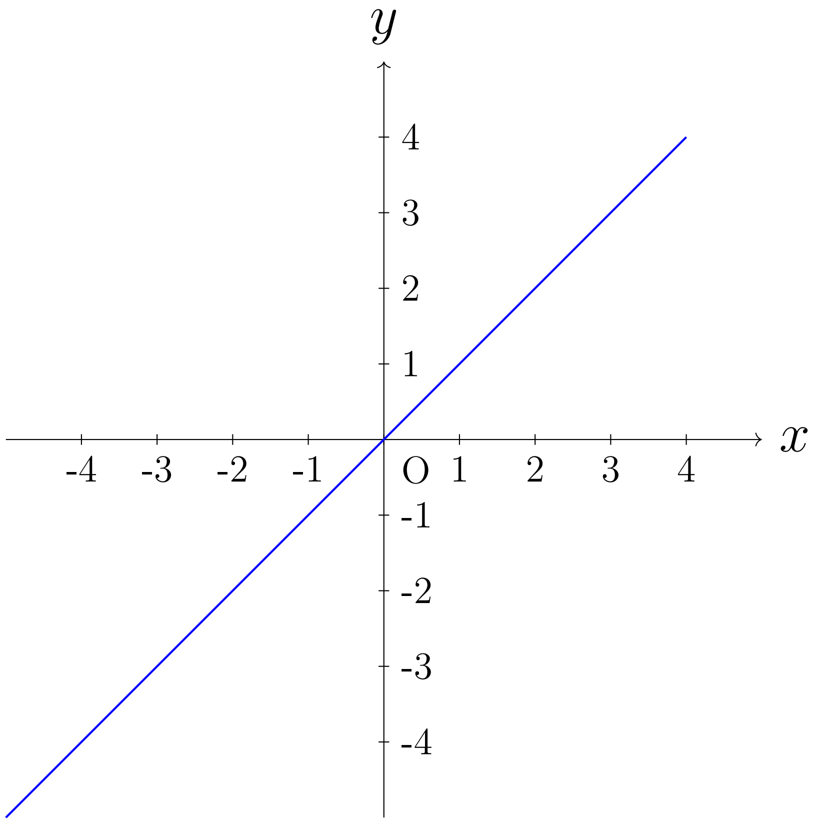


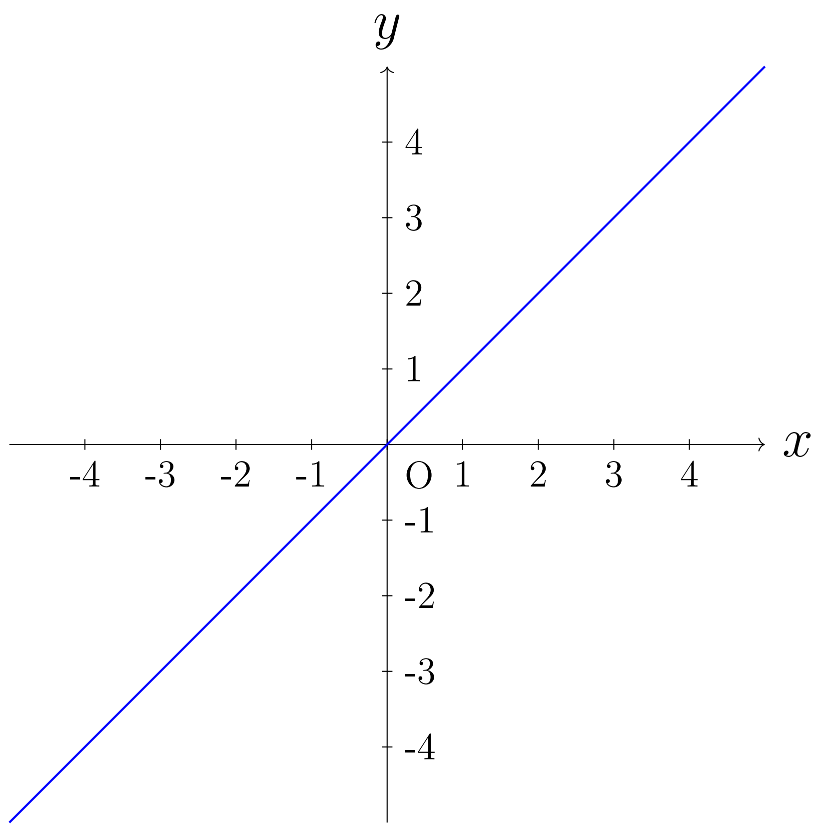
Parametrizing a line

$\gamma : (-5, 4) \to \mathbb{R}^2$
$\gamma(t) := (t, t)$



Parametrizing a line

$$\gamma : (-5, 5) \to \mathbb{R}^2$$
$$\gamma(t) := (t, t)$$



Quick review: Derivative

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Example. $f : \mathbb{R} \to \mathbb{R}$

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$$f(x) = \{$$

Quick review: Derivative

Example. $f : \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} x^2 & x < 5 \end{cases}$$

Quick review: Derivative

Example. $f : \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} x^2 & x < 5 \\ 0 & x = 5 \end{cases}$$

Quick review: Derivative

Example. $f : \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} x^2 & x < 5 \\ 0 & x = 5 \\ x^3 & x > 5 \end{cases}$$

Quick review: Derivative

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$f(5) = 0$

Quick review: Derivative

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$\lim_{x \to 5^-} f(x)$

Quick review: Derivative

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$\lim_{x \to 5^+} f(x)$

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$\lim_{x \to 5^-} f(x) = 5^2$
$\lim_{x \to 5^+} f(x) = 5^3$

Quick review: Derivative

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$\lim_{x \to 5^+} f(x) = 5^3$

Example.

$$f(x) = \begin{cases} x^2 & x < 5 \\ 0 & x = 5 \\ x^{\color{red}2} & x > 5 \end{cases}$$

Quick review: Derivative

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Quick review: Derivative

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