

# Exercise sheet 1

Curves and Surfaces, MTH201

1. Find a parametrization  $\gamma(t)$  for a line segment joining two given points  $(x_1, y_1)$  and  $(x_2, y_2)$ . Find  $\dot{\gamma}(t)$ .
2. What does the parametrization trace out  $\gamma(t) = (\frac{2t}{1+t^2}, \frac{1-t^2}{1+t^2})$ ?
3. Show that the parametrization  $\gamma(t) := (t^2 - 1, t(t^2 - 1))$  is not injective, i.e. there are two *distinct* real numbers  $t_1$  and  $t_2$  so that  $\gamma(t_1) = \gamma(t_2)$ . Can you deduce the shape<sup>1</sup> of this curve? Can you express the set of points defined by this curve as the zero set<sup>2</sup> of some function  $f(x, y)$ ?
4. Remember that  $\mathbb{R}^2$  can be given the structure of a vector space by defining,  $(x_1, y_1) + (x_2, y_2) := (x_1 + x_2, y_1 + y_2)$  (vector addition) and  $c(x, y) := (cx, cy)$  (scalar multiplication) for some real number  $c$ . Let  $\mathbf{v}_1 : (\alpha, \beta) \rightarrow \mathbb{R}^2$  and  $\mathbf{v}_2 : (\alpha, \beta) \rightarrow \mathbb{R}^2$  be smooth “vector valued” functions. Show that,
  - (a)  $(\mathbf{v}_1(t) + \mathbf{v}_2(t))' = \mathbf{v}_1'(t) + \mathbf{v}_2'(t)$
  - (b)  $(\mathbf{v}_1(t) - \mathbf{v}_2(t))' = \mathbf{v}_1'(t) - \mathbf{v}_2'(t)$
  - (c)  $\mathbf{v}(\phi(t))' = \mathbf{v}'(\phi(t))\phi'(t)$ , where  $\phi : (\alpha', \beta') \rightarrow (\alpha, \beta)$  is a smooth function.
  - (d) During the lecture we defined  $\mathbf{v}'(t)$ , where  $\mathbf{v}(t) = (f(t), g(t))$  to be  $(f'(t), g'(t))$ . Show that,

$$\mathbf{v}'(t) = \lim_{h \rightarrow 0} 1/h(\mathbf{v}(t+h) - \mathbf{v}(t))$$

Remember that the subtraction above is vector subtraction and the multiplication by  $1/h$  is scalar multiplication.

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<sup>1</sup>Just a rough drawing showing where the curve intersects the axes and where it self-intersects etc.

<sup>2</sup>The zero set of a function  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  is  $\{(x, y) \mid f(x, y) = 0\}$ , i.e. the set of points  $(x, y)$  in the plane for which  $f(x, y) = 0$