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$$\begin{aligned} \mathbf{e}_1(t), \mathbf{e}_2(t) &\in \mathbb{R}^2 & \{\mathbf{T}(t), \} \\ \|\mathbf{e}_1(t)\| = 1, \|\mathbf{e}_2(t)\| = 1, \text{ and } \mathbf{e}_1(t) \cdot \mathbf{e}_2(t) &= 0 \\ \text{“}\mathbf{e}_1(t) \text{ and } \mathbf{e}_2(t) \text{ form an orthonormal basis”} \end{aligned}$$

$$\begin{aligned} \text{For any, } \mathbf{v}(t) &\in \mathbb{R}^2, \\ \mathbf{v}(t) &= \alpha(t)\mathbf{e}_1(t) + \beta(t)\mathbf{e}_2(t) \\ \text{for some } \alpha(t), \beta(t) &\in \mathbb{R} \text{ (uniquely represented like this!)} \end{aligned}$$

$$\begin{aligned} \text{Recovering the coefficients } \alpha(t), \beta(t): \\ \alpha(t) &= \mathbf{v}(t) \cdot \mathbf{e}_1(t) \\ \beta(t) &= \mathbf{v}(t) \cdot \mathbf{e}_2(t) \\ \text{So, } \mathbf{v}(t), \mathbf{e}_1(t), \mathbf{e}_2(t) \text{ smooth} &\implies \alpha(t), \beta(t) \text{ smooth.} \end{aligned}$$

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$$\{\mathbf{T}(t), \mathbf{N}_s(t)\}$$

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$$\|\mathbf{e}_1(t)\| = 1, \|\mathbf{e}_2(t)\| = 1, \text{ and } \mathbf{e}_1(t) \cdot \mathbf{e}_2(t) = 0$$

“ $\mathbf{e}_1(t)$ and $\mathbf{e}_2(t)$ form an orthonormal basis”

$\{\mathbf{T}(t), \mathbf{N}_s(t)\}$ form an orthonormal basis,

For any, $\mathbf{v}(t) \in \mathbb{R}^2$,

$$\mathbf{v}(t) = \alpha(t)\mathbf{e}_1(t) + \beta(t)\mathbf{e}_2(t)$$

for some $\alpha(t), \beta(t) \in \mathbb{R}$ (uniquely represented like this!)

Recovering the coefficients $\alpha(t), \beta(t)$:

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$$\dot{\mathbf{T}}(t) = \kappa_s(t)\mathbf{N}_s(t)$$

$\mathbf{e}_1(t), \mathbf{e}_2(t) \in \mathbb{R}^2$
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$$\dot{\mathbf{T}}(t) = 0\mathbf{T}(t) + \kappa_s(t)\mathbf{N}_s(t)$$

$\mathbf{e}_1(t), \mathbf{e}_2(t) \in \mathbb{R}^2$
 $\|\mathbf{e}_1(t)\| = 1, \|\mathbf{e}_2(t)\| = 1$, and $\mathbf{e}_1(t) \cdot \mathbf{e}_2(t) = 0$
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$$\begin{aligned}\dot{\mathbf{T}}(t) &= 0\mathbf{T}(t) + \kappa_s(t)\mathbf{N}_s(t) \\ \dot{\mathbf{N}}_s(t) &= ??\mathbf{T}(t) + ??\mathbf{N}_s(t)\end{aligned}$$

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$$\begin{aligned} \dot{\mathbf{T}}(t) &= 0\mathbf{T}(t) + \kappa_s(t)\mathbf{N}_s(t) \\ \dot{\mathbf{N}}_s(t) &= ??\mathbf{T}(t) + 0\mathbf{N}_s(t) \\ \dot{\mathbf{N}}_s(t) \cdot \mathbf{T}(t) & \end{aligned}$$

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$$\begin{aligned} \dot{\mathbf{T}}(t) &= 0\mathbf{T}(t) + \kappa_s(t)\mathbf{N}_s(t) \\ \dot{\mathbf{N}}_s(t) &= ??\mathbf{T}(t) + 0\mathbf{N}_s(t) \\ \dot{\mathbf{N}}_s(t) \cdot \mathbf{T}(t) + \mathbf{N}_s(t) \cdot \dot{\mathbf{T}}(t) &= (\mathbf{N}_s(t) \cdot \mathbf{T}(t))' \end{aligned}$$

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$$\begin{aligned} \dot{\mathbf{T}}(t) &= 0\mathbf{T}(t) + \kappa_s(t)\mathbf{N}_s(t) \\ \dot{\mathbf{N}}_s(t) &= ??\mathbf{T}(t) + 0\mathbf{N}_s(t) \\ \dot{\mathbf{N}}_s(t) \cdot \mathbf{T}(t) + \underbrace{\mathbf{N}_s(t) \cdot \dot{\mathbf{T}}(t)}_{\kappa_s(t)} &= \underbrace{(\mathbf{N}_s(t) \cdot \mathbf{T}(t))'}_0 \end{aligned}$$

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$$\begin{aligned} \dot{\mathbf{T}}(t) &= 0\mathbf{T}(t) + \kappa_s(t)\mathbf{N}_s(t) \\ \dot{\mathbf{N}}_s(t) &= -\kappa_s(t)\mathbf{T}(t) + 0\mathbf{N}_s(t) \\ \dot{\mathbf{N}}_s(t) \cdot \mathbf{T}(t) + \underbrace{\mathbf{N}_s(t) \cdot \dot{\mathbf{T}}(t)}_{\kappa_s(t)} &= \underbrace{(\mathbf{N}_s(t) \cdot \mathbf{T}(t))'}_0 \end{aligned}$$

Exercise. If γ

Exercise. If $\gamma : (\alpha, \beta)$

Exercise. If $\gamma : (\alpha, \beta) \rightarrow \mathbb{R}^2$

Exercise. If $\gamma : (\alpha, \beta) \rightarrow \mathbb{R}^2$ is a unit speed parametrization

Exercise. If $\gamma : (\alpha, \beta) \rightarrow \mathbb{R}^2$ is a unit speed parametrization with constant (non-zero) curvature,

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