

**Definition.** A “parametrized plane curve”

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Image  $\gamma$

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**Examples.**

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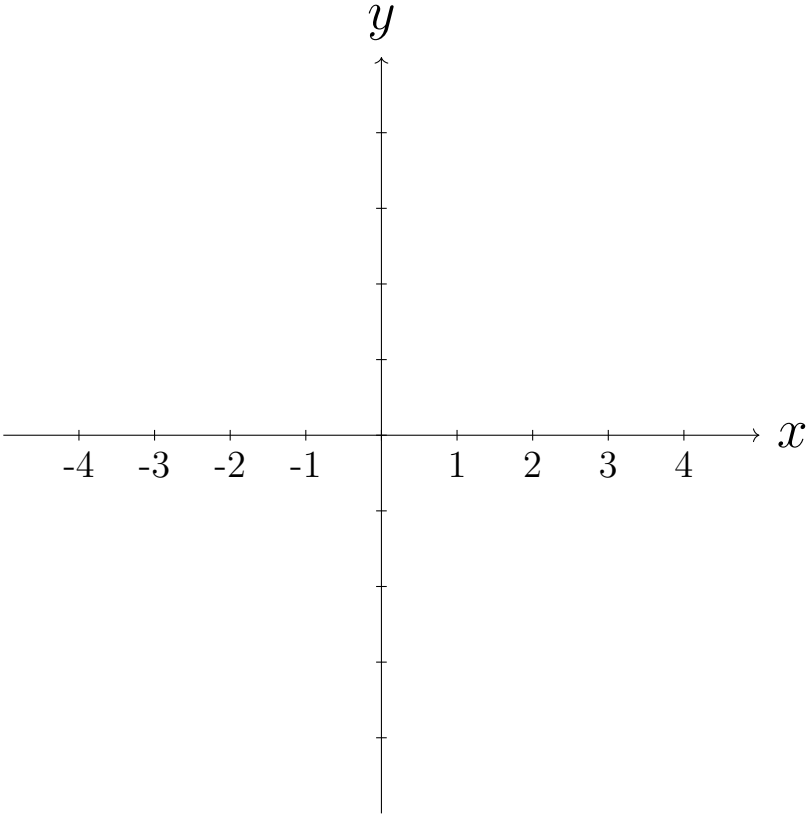
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# Parametrizing a circle

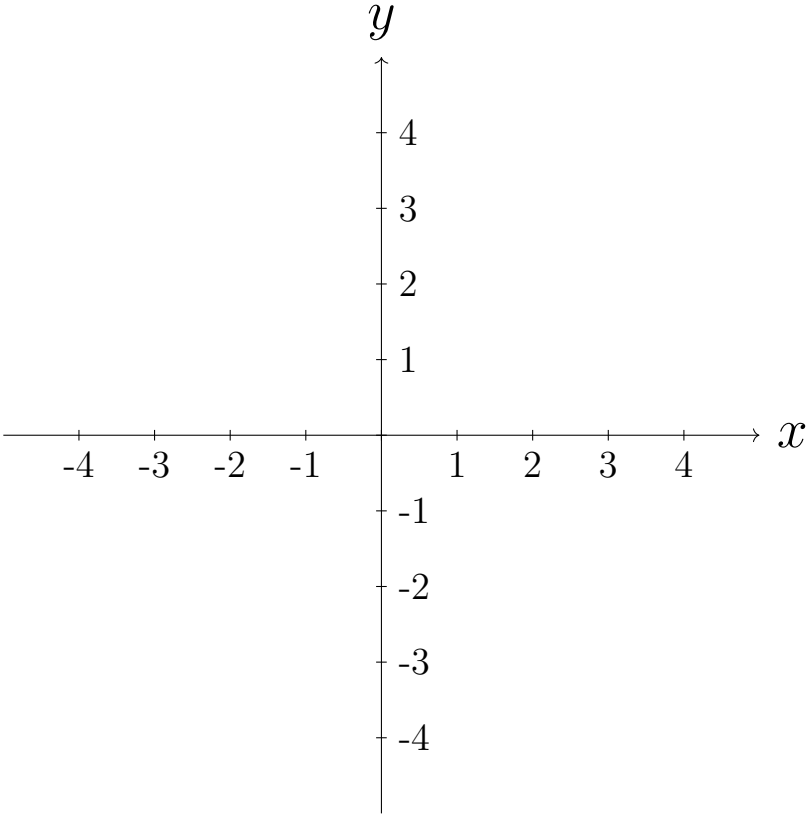




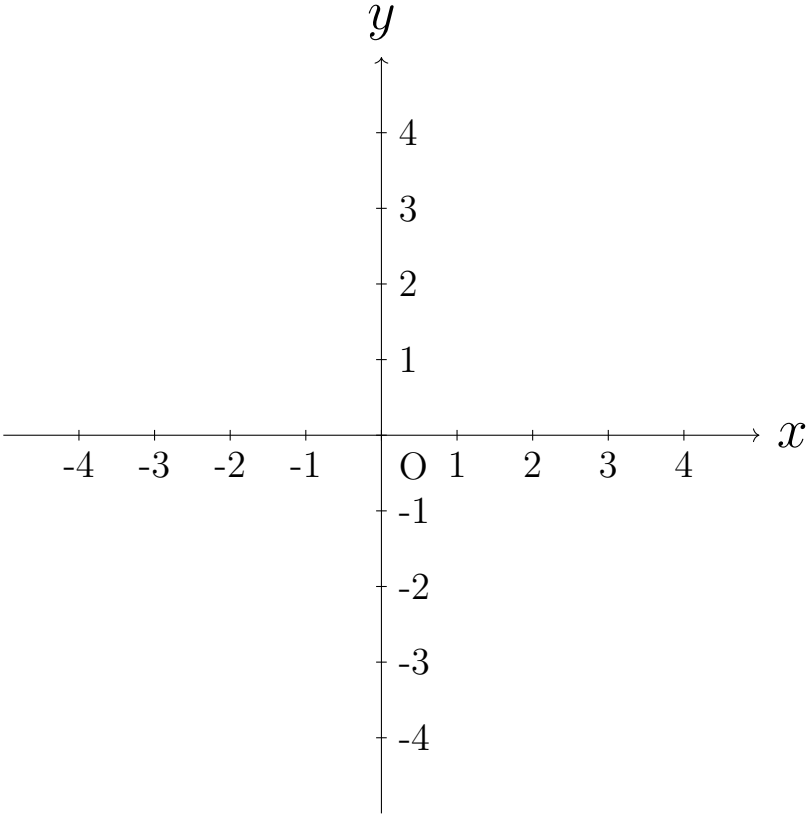
# Parametrizing a circle



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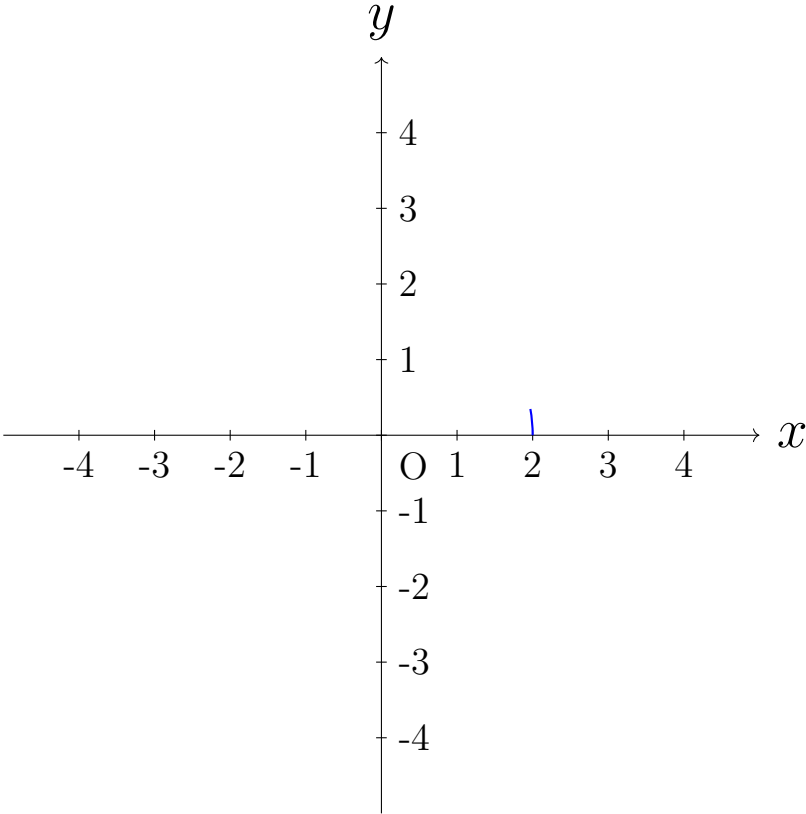


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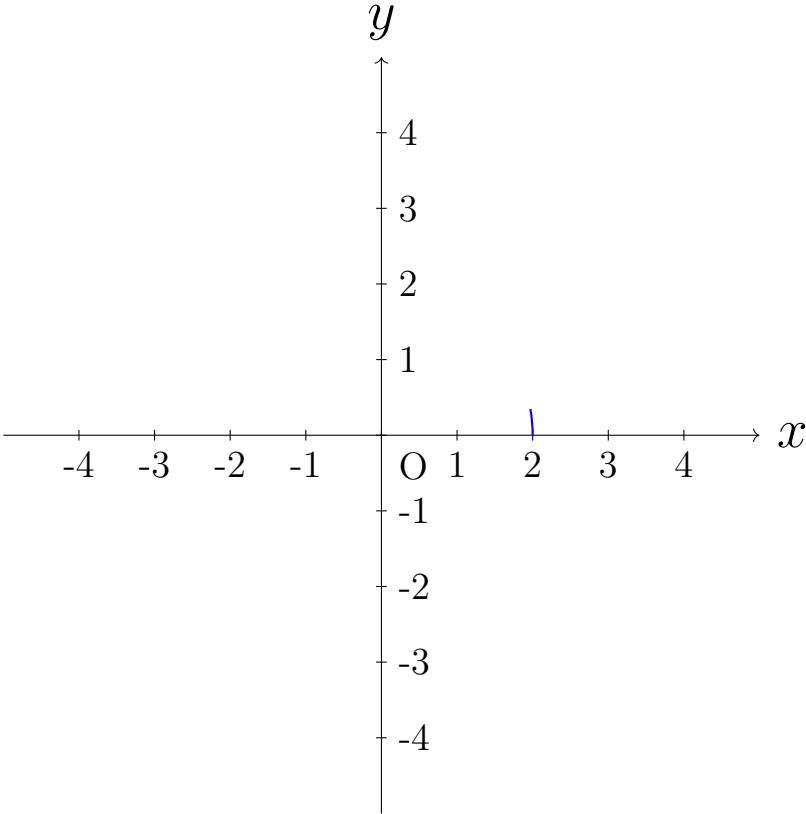


# Parametrizing a circle

$$\gamma : (0, \pi/18) \rightarrow \mathbb{R}^2$$



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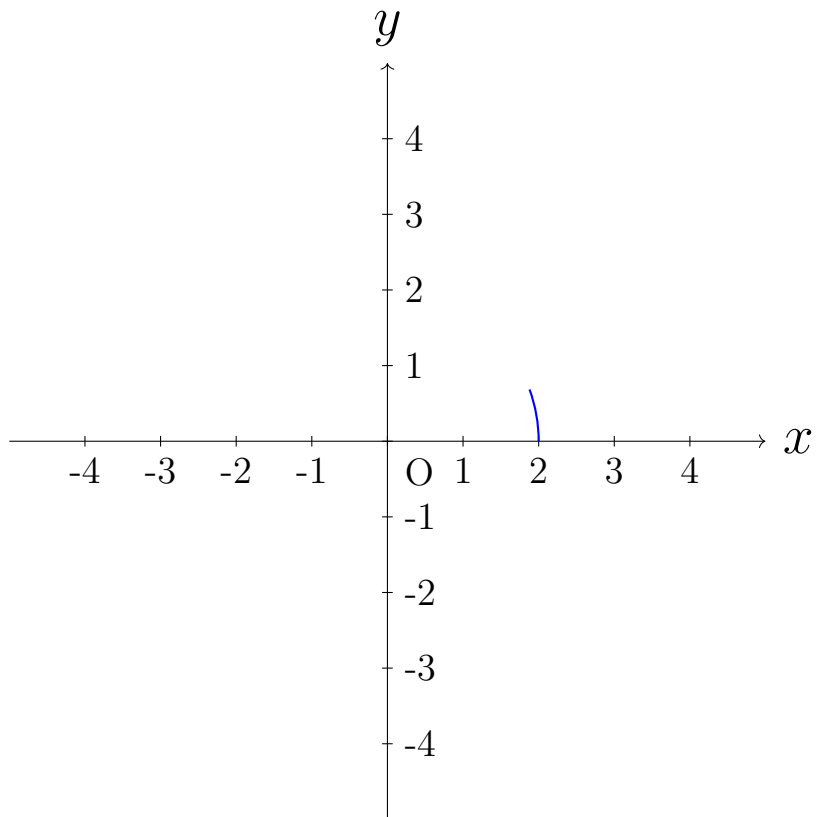


$$\gamma : (0, \pi/18) \rightarrow \mathbb{R}^2$$
$$\gamma(t) := (2 \cos(t), 2 \sin(t))$$

# Parametrizing a circle

$$\gamma : (0, 2\pi/18) \rightarrow \mathbb{R}^2$$

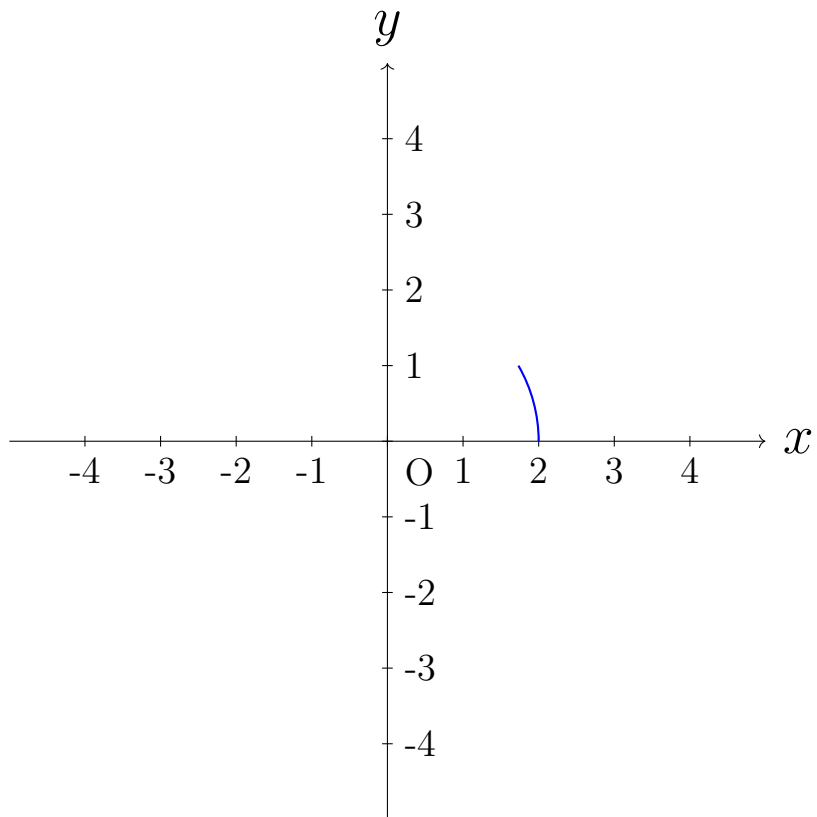
$$\gamma(t) := (2 \cos(t), 2 \sin(t))$$



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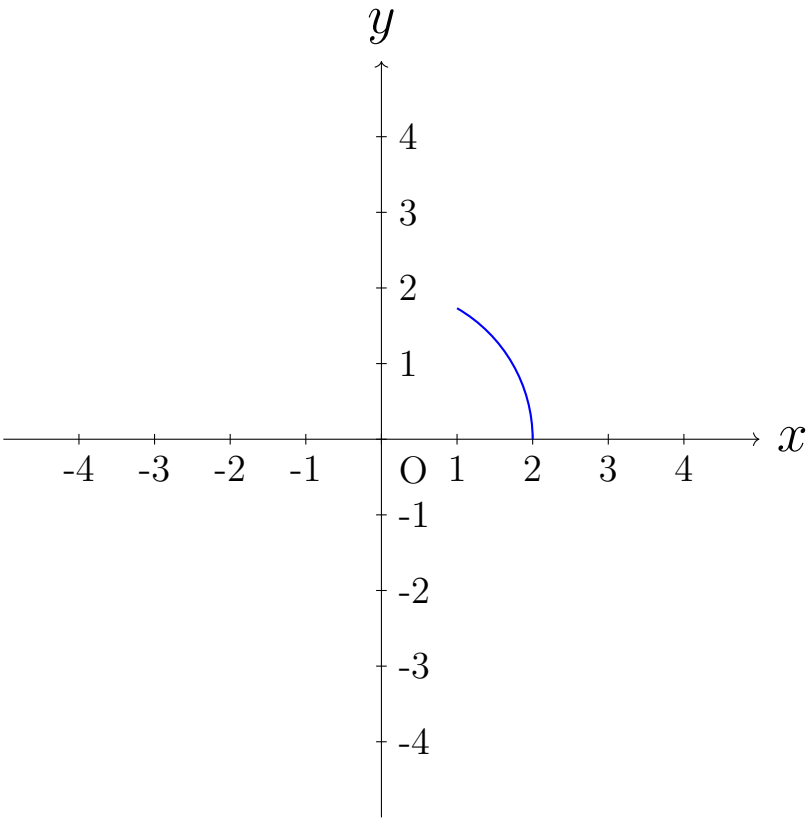
$$\gamma : (0, 3\pi/18) \rightarrow \mathbb{R}^2$$

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# Parametrizing a circle

$$\gamma : (0, 6\pi/18) \rightarrow \mathbb{R}^2$$
$$\gamma(t) := (2 \cos(t), 2 \sin(t))$$

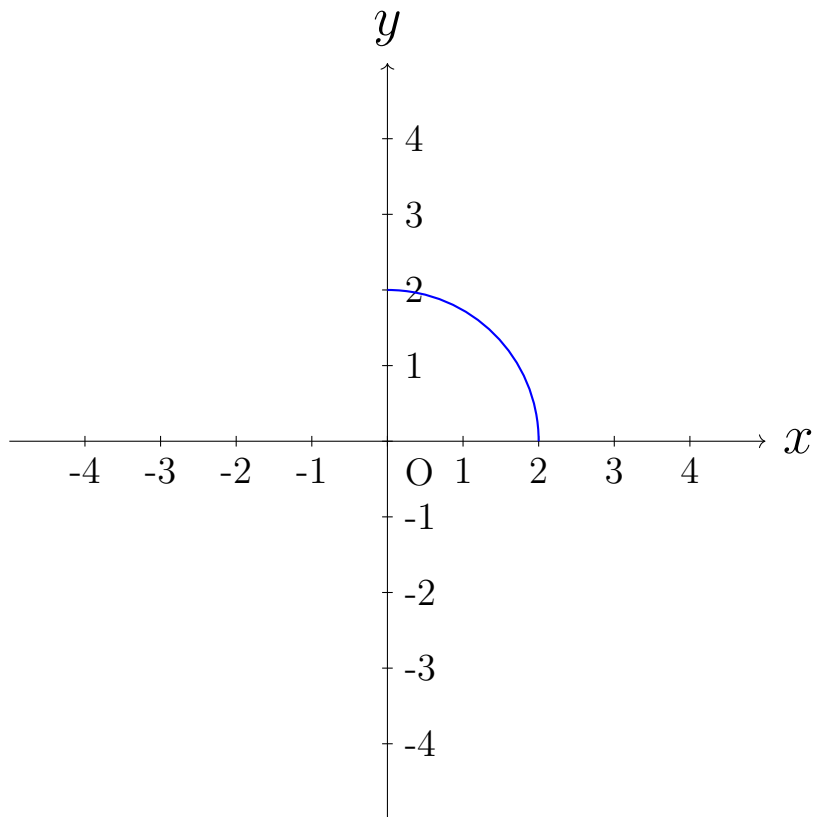




# Parametrizing a circle

$$\gamma : (0, 9\pi/18) \rightarrow \mathbb{R}^2$$

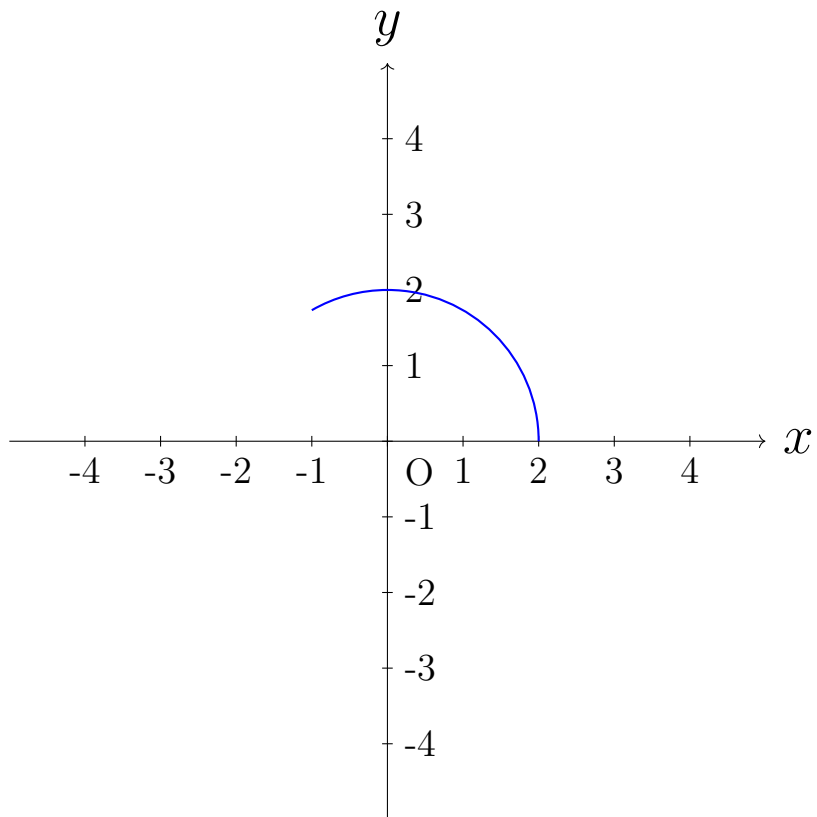
$$\gamma(t) := (2 \cos(t), 2 \sin(t))$$



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$$\gamma : (0, 12\pi/18) \rightarrow \mathbb{R}^2$$

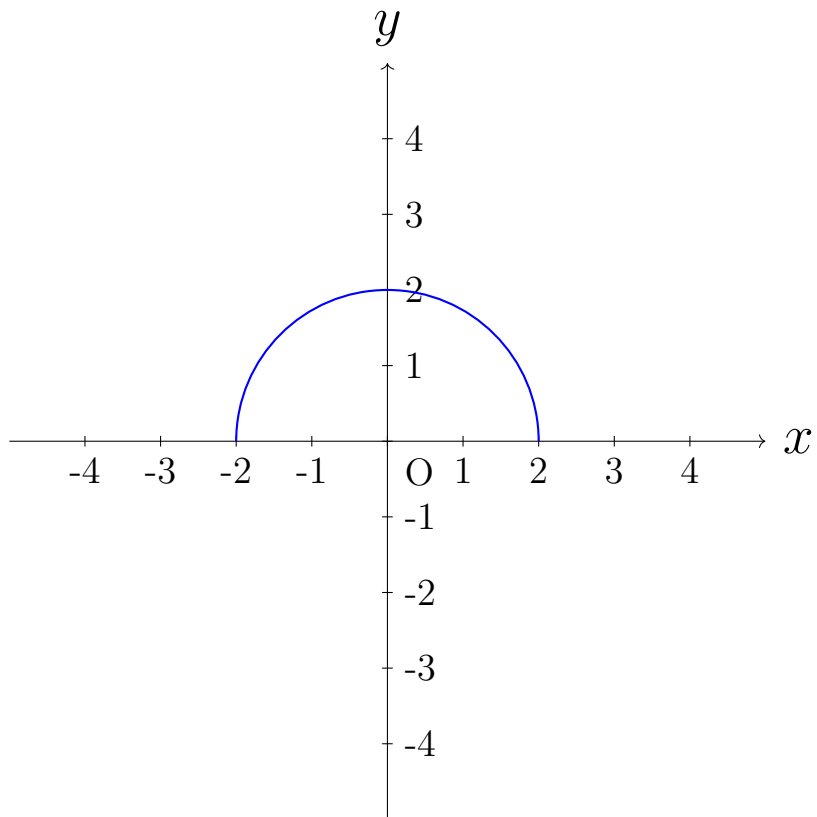
$$\gamma(t) := (2 \cos(t), 2 \sin(t))$$



# Parametrizing a circle

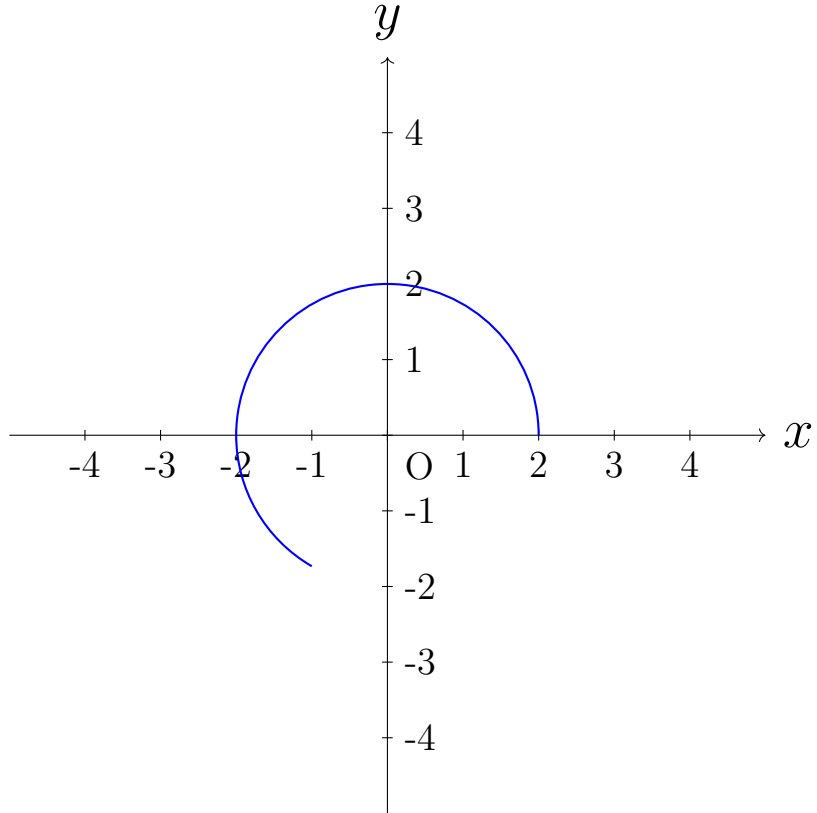
$$\gamma : (0, 18\pi/18) \rightarrow \mathbb{R}^2$$

$$\gamma(t) := (2 \cos(t), 2 \sin(t))$$



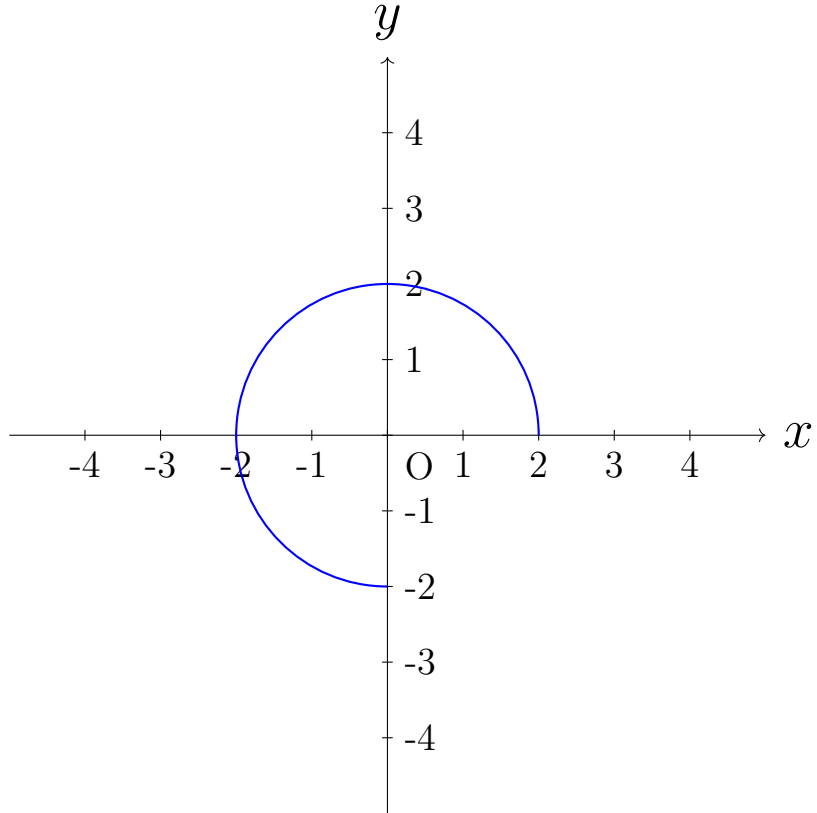
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$$\gamma : (0, 24\pi/18) \rightarrow \mathbb{R}^2$$
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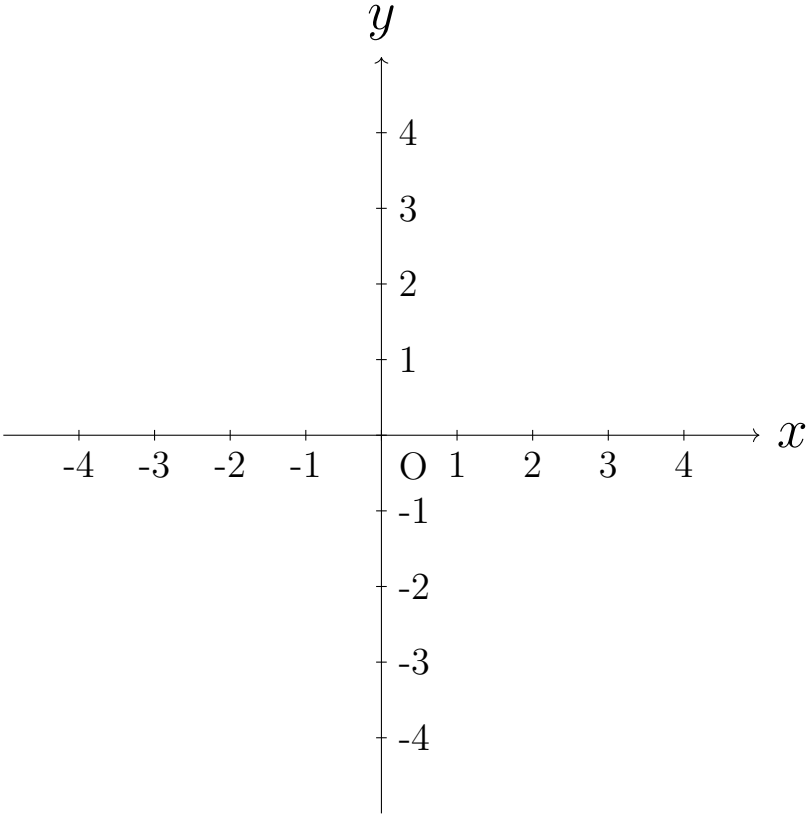


# Parametrizing a circle

$$\gamma : (0, 27\pi/18) \rightarrow \mathbb{R}^2$$
$$\gamma(t) := (2 \cos(t), 2 \sin(t))$$



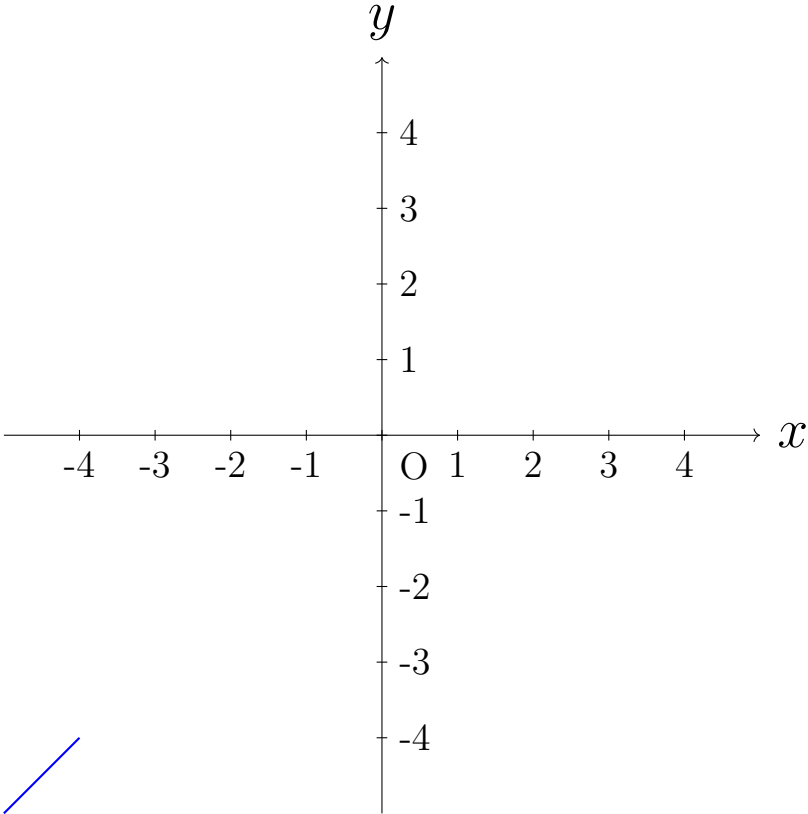
# Parametrizing a line



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# Parametrizing a line

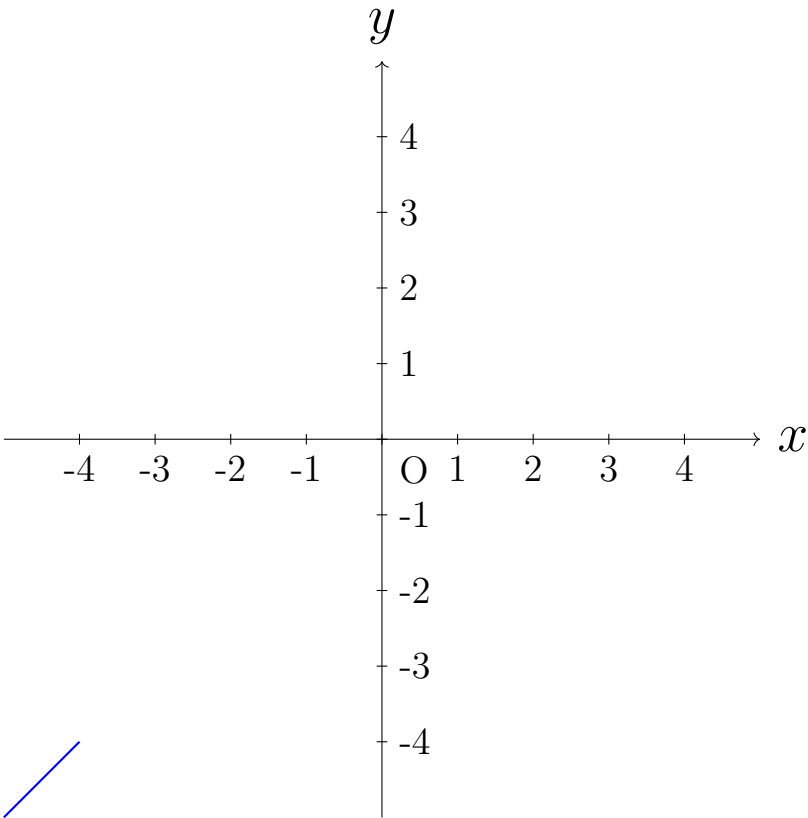
$$\gamma : (-5, -4) \rightarrow \mathbb{R}^2$$



# Parametrizing a line

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$$\gamma(t) := (t, t)$$

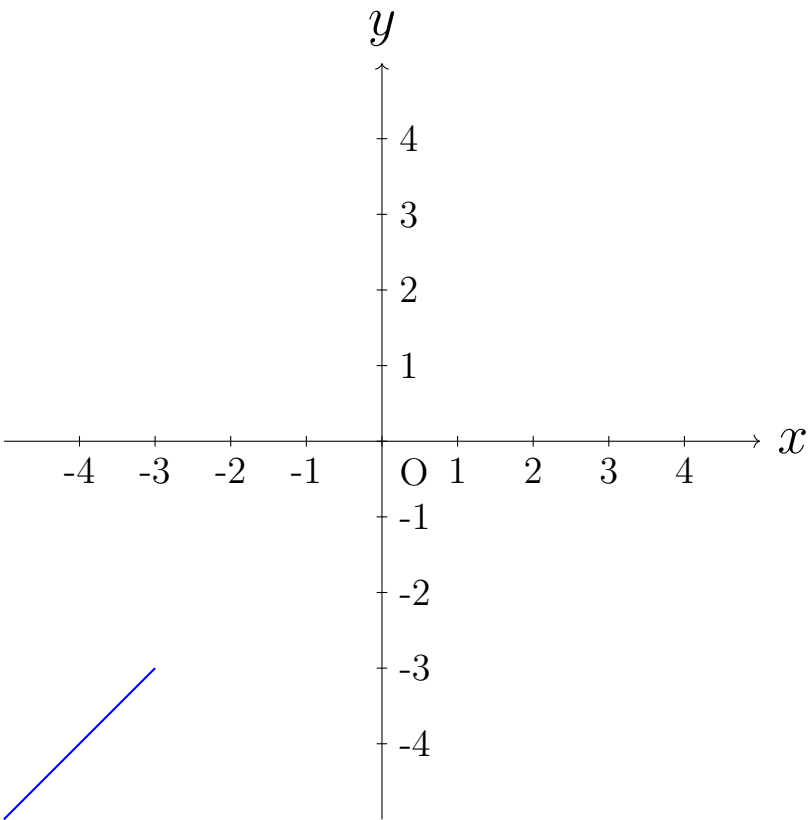




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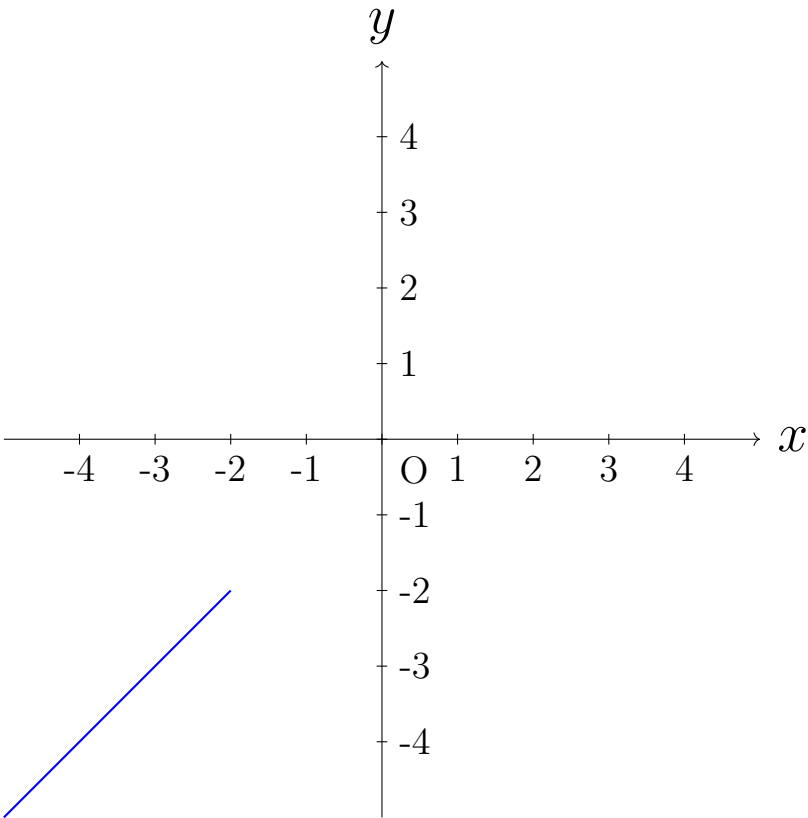
$$\gamma(t) := (t, t)$$



# Parametrizing a line

$$\gamma : (-5, -2) \rightarrow \mathbb{R}^2$$

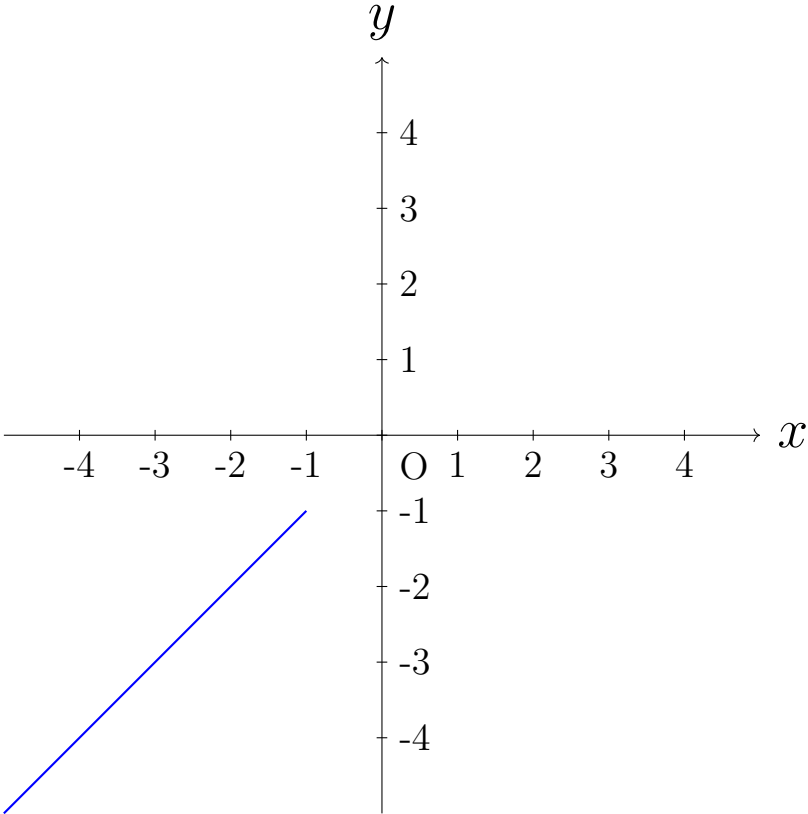
$$\gamma(t) := (t, t)$$



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$$\gamma : (-5, -1) \rightarrow \mathbb{R}^2$$

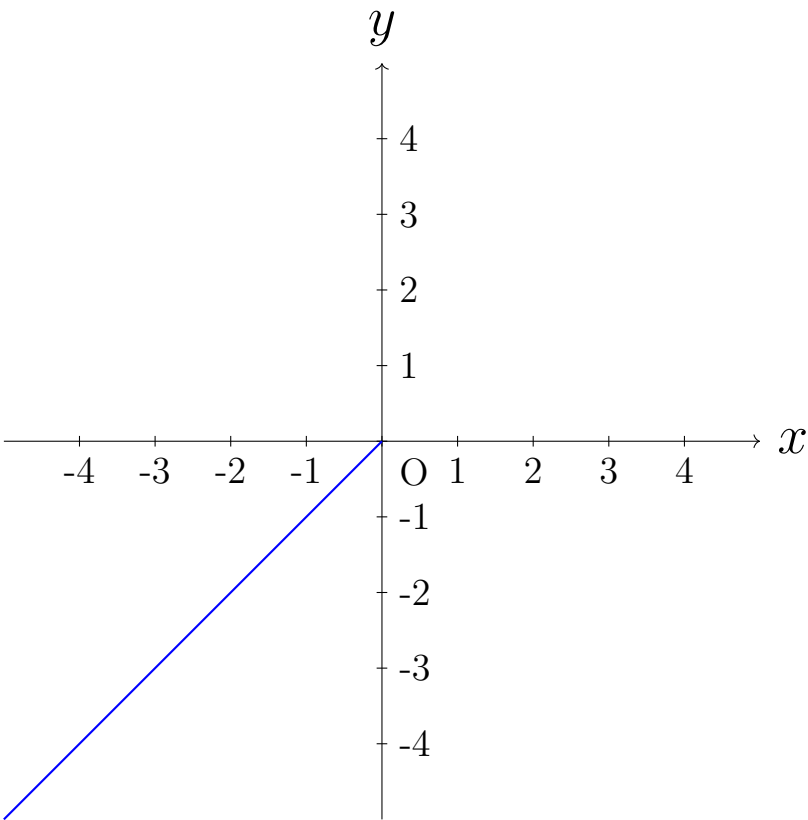
$$\gamma(t) := (t, t)$$



# Parametrizing a line

$$\gamma : (-5, 0) \rightarrow \mathbb{R}^2$$

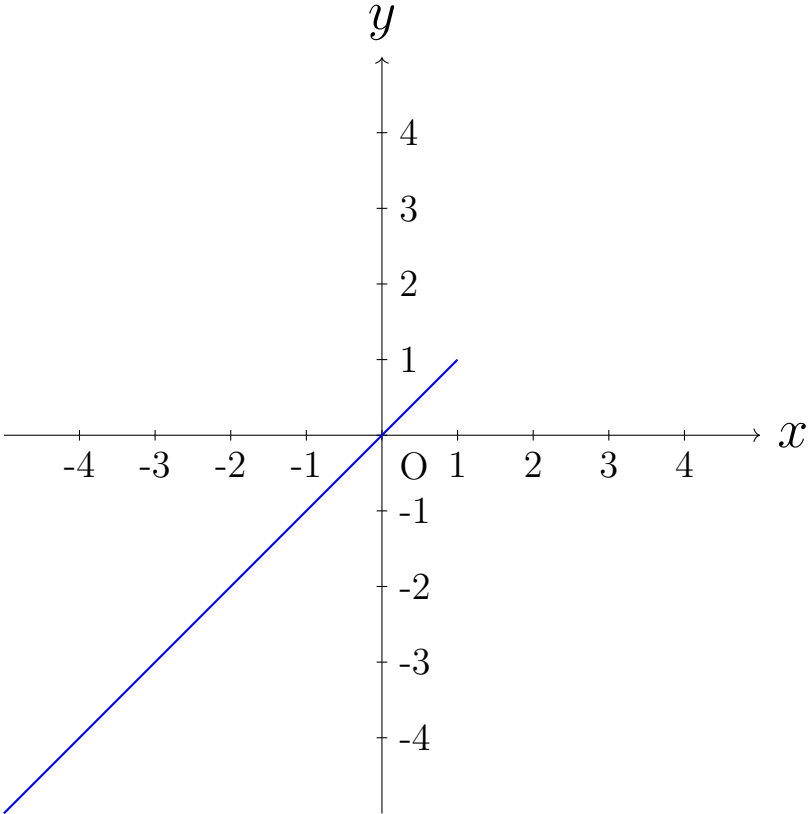
$$\gamma(t) := (t, t)$$



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$$\gamma : (-5, 1) \rightarrow \mathbb{R}^2$$

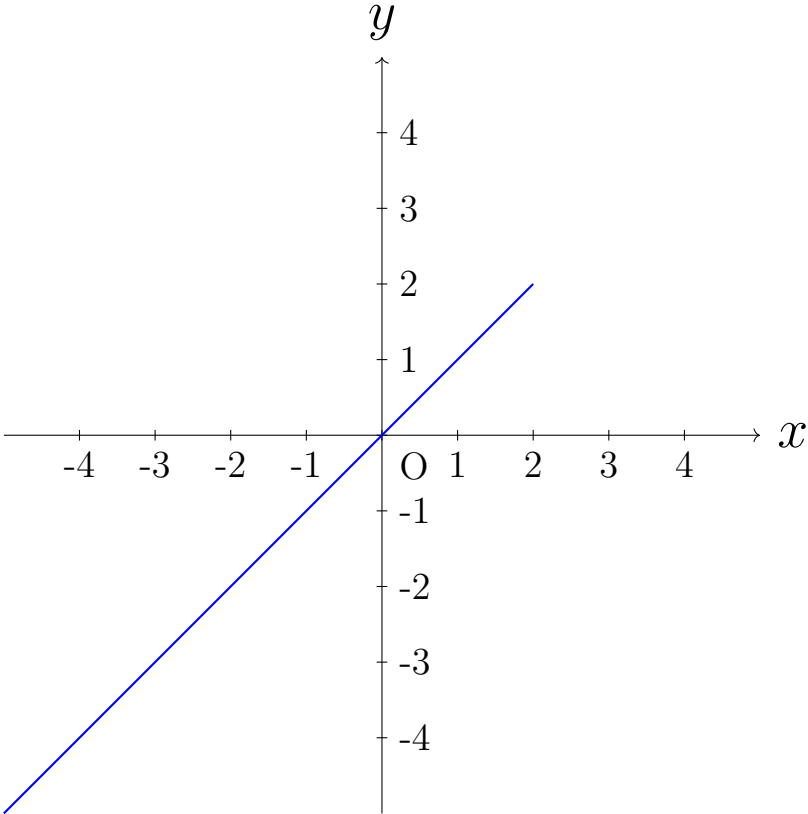
$$\gamma(t) := (t, t)$$



# Parametrizing a line

$$\gamma : (-5, 2) \rightarrow \mathbb{R}^2$$

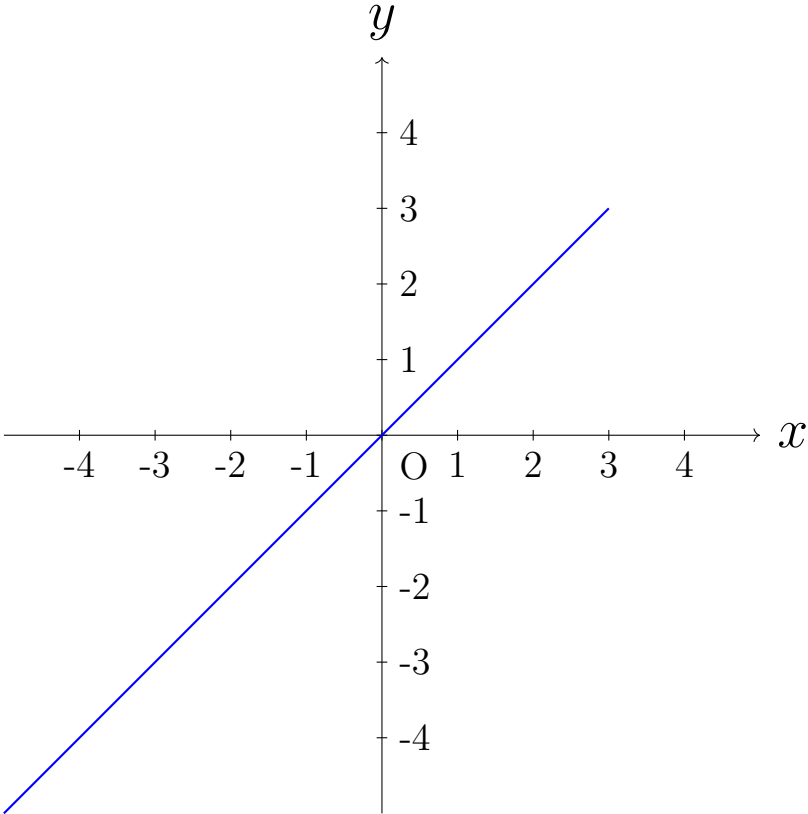
$$\gamma(t) := (t, t)$$



# Parametrizing a line

$$\gamma : (-5, 3) \rightarrow \mathbb{R}^2$$

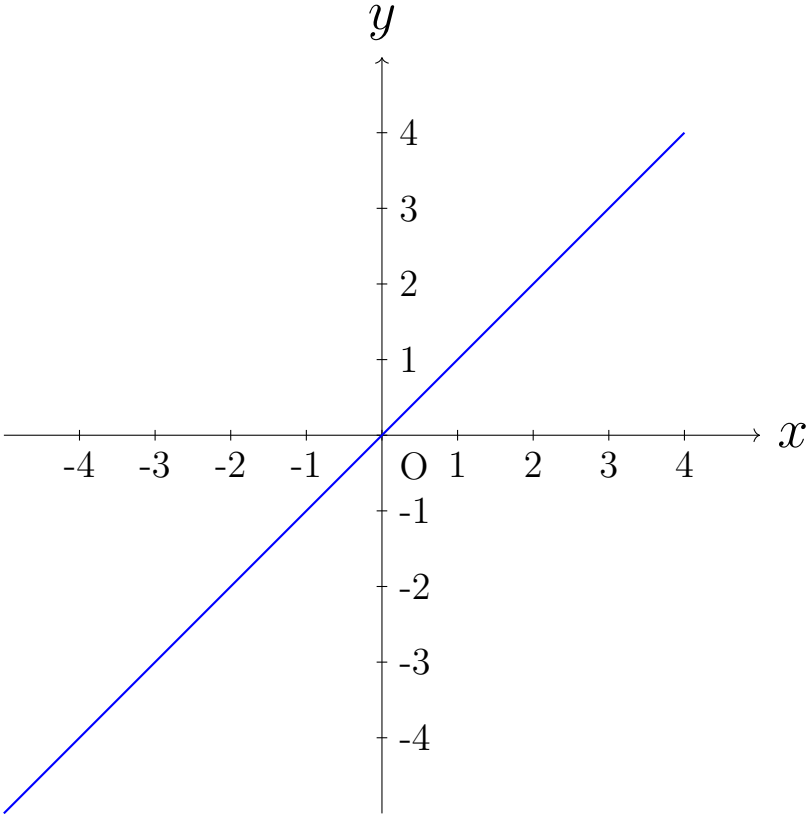
$$\gamma(t) := (t, t)$$



# Parametrizing a line

$$\gamma : (-5, 4) \rightarrow \mathbb{R}^2$$

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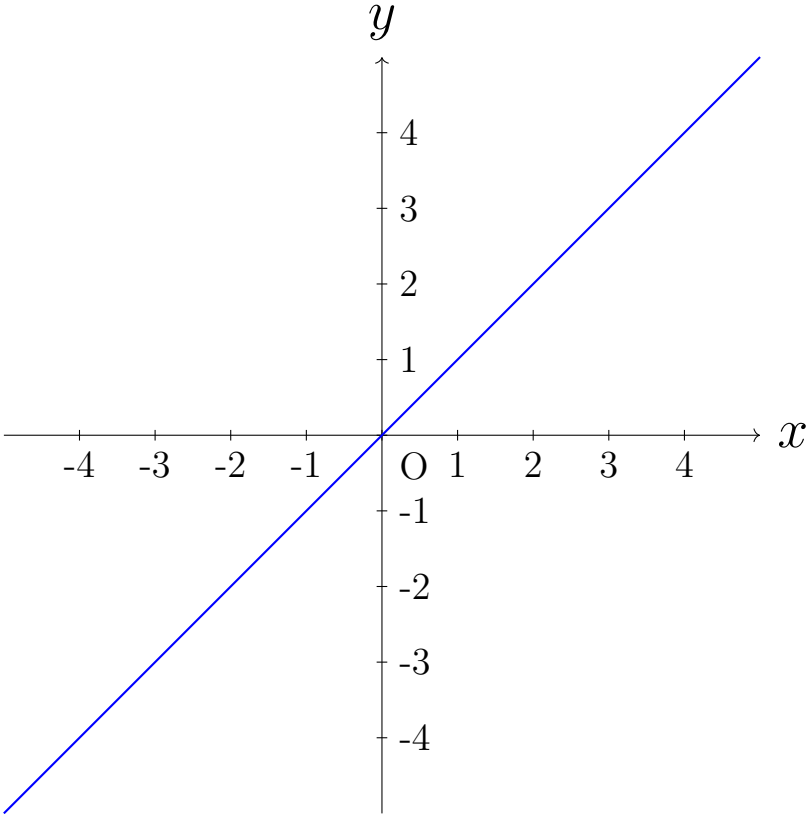




# Parametrizing a line

$$\gamma : (-5, 5) \rightarrow \mathbb{R}^2$$

$$\gamma(t) := (t, t)$$



# Quick review: Derivative

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