Chain rule for mult-variable functions

f

Chain rule for mult-variable functions

$f : \mathbb{R}^2$

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$

γ

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : (\alpha, \beta)$

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\gamma(t)$

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\gamma(t) = (x(t), y(t))$

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\gamma(t) = (x(t), y(t))$
$f \circ \gamma$

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
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$\gamma(t) = (x(t), y(t))$
$f \circ \gamma : (\alpha, \beta)$

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$f \circ \gamma : (\alpha, \beta) \to \mathbb{R}$

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$(f \circ \gamma)'(t_0)$

Chain rule for mult-variable functions

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$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\gamma(t) = (x(t), y(t))$
$f \circ \gamma : (\alpha, \beta) \to \mathbb{R}$

$$(f \circ \gamma)'(t_0) = f_x(x(t_0), y(t_0))x'(t_0) + f_y(x(t_0), y(t_0))y'(t_0)$$

Chain rule for mult-variable functions

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$(f \circ \gamma)'(t_0) = f_x(x(t_0), y(t_0))x'(t_0) + f_y(x(t_0), y(t_0))y'(t_0) = (f_x(x, y), f_y(x, y)).\dot{\gamma}(t_0)$

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\gamma(t) = (x(t), y(t))$
$f \circ \gamma : (\alpha, \beta) \to \mathbb{R}$

$(f \circ \gamma)'(t_0) = f_x(x(t_0), y(t_0))x'(t_0) + f_y(x(t_0), y(t_0))y'(t_0) = \nabla(f)(x(t_0), y(t_0)).\dot{\gamma}(t_0)$,
where $\nabla(f)(x, y) = (f_x(x, y).f_y(x, y))$,

Chain rule for mult-variable functions

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$\boxed{(f \circ \gamma)'(t_0) = \nabla(f)(x(t_0), y(t_0)).\dot{\gamma}(t_0)}$,
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$\boxed{f_{\mathbf{v}}(x(t_0), y(t_0)) := (f \circ \gamma)'(t_0) = \nabla(f)(x(t_0), y(t_0)).\mathbf{v}}$,
where $\nabla(f)(x, y) = (f_x(x, y).f_y(x, y))$,
$\mathbf{v} = \dot{\gamma}(t_0)$,

Chain rule for mult-variable functions

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$\boxed{f_{\mathbf{v}}(x(t_0), y(t_0)) := (f \circ \gamma)'(t_0) = \nabla(f)(p).\mathbf{v}}$,
where $\nabla(f)(x, y) = (f_x(x, y).f_y(x, y))$,
$\mathbf{v} = \dot{\gamma}(t_0)$,
and $p = (x(t_0), y(t_0))$

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$f : \mathbb{R}^2$

Chain rule for mult-variable functions

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$f : \mathbb{R}^2 \to \mathbb{R}$

Chain rule for mult-variable functions

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$f : \mathbb{R}^2 \to \mathbb{R}$
γ

Chain rule for mult-variable functions

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$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : \mathbb{R}^2$

Chain rule for mult-variable functions

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$f : \mathbb{R}^2 \to \mathbb{R}$
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Chain rule for mult-variable functions

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$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : \mathbb{R}^2 \to \mathbb{R}^2$
$\gamma(u, v)$

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$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
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$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : \mathbb{R}^2 \to \mathbb{R}^2$
$\gamma(u, v) = (x(u, v), y(u, v))$
$f \circ \gamma : \mathbb{R}^2$

Chain rule for mult-variable functions

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$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : \mathbb{R}^2 \to \mathbb{R}^2$
$\gamma(u, v) = (x(u, v), y(u, v))$
$f \circ \gamma : \mathbb{R}^2 \to \mathbb{R}$

Chain rule for mult-variable functions

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$\gamma : \mathbb{R}^2 \to \mathbb{R}^2$
$\gamma(u, v) = (x(u, v), y(u, v))$
$f \circ \gamma : \mathbb{R}^2 \to \mathbb{R}$

$(f \circ \gamma)_u(u_0, v_0)$

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
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$\gamma : \mathbb{R}^2 \to \mathbb{R}^2$
$\gamma(u, v) = (x(u, v), y(u, v))$
$f \circ \gamma : \mathbb{R}^2 \to \mathbb{R}$

$(f \circ \gamma)_u(u_0, v_0)$
$= f_x(x(u_0, v_0), y(u_0, v_0))x_u(u_0, v_0)$
$+ f_y(x(u_0, v_0), y(u_0, v_0))y_u(u_0, v_0)$

Curves on surfaces

Definition. $\mathbf{v} \in \mathbb{R}^3$ is a tangent vector of the surface S at a point p, if there is a γ

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\gamma(t) = (x(t), y(t))$
$f \circ \gamma : (\alpha, \beta) \to \mathbb{R}$

$\boxed{f_{\mathbf{v}}(x(t_0), y(t_0)) := (f \circ \gamma)'(t_0) = \nabla(f)(p).\mathbf{v}}$,
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and $p = (x(t_0), y(t_0))$

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : \mathbb{R}^2 \to \mathbb{R}^2$
$\gamma(u, v) = (x(u, v), y(u, v))$
$f \circ \gamma : \mathbb{R}^2 \to \mathbb{R}$

$(f \circ \gamma)_u(u_0, v_0)$
$= f_x(x(u_0, v_0), y(u_0, v_0))x_u(u_0, v_0)$
$+ f_y(x(u_0, v_0), y(u_0, v_0))y_u(u_0, v_0)$

Curves on surfaces

Definition. $\mathbf{v} \in \mathbb{R}^3$ is a tangent vector of the surface S at a point p, if there is a $\gamma : (\alpha, \beta)$

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
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$\boxed{f_{\mathbf{v}}(x(t_0), y(t_0)) := (f \circ \gamma)'(t_0) = \nabla(f)(p).\mathbf{v}}$,
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$\gamma : \mathbb{R}^2 \to \mathbb{R}^2$
$\gamma(u, v) = (x(u, v), y(u, v))$
$f \circ \gamma : \mathbb{R}^2 \to \mathbb{R}$

$(f \circ \gamma)_u(u_0, v_0)$
$= f_x(x(u_0, v_0), y(u_0, v_0))x_u(u_0, v_0)$
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Curves on surfaces

Definition. $\mathbf{v} \in \mathbb{R}^3$ is a tangent vector of the surface S at a point p, if there is a $\gamma : (\alpha, \beta) \to S$

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\gamma(t) = (x(t), y(t))$
$f \circ \gamma : (\alpha, \beta) \to \mathbb{R}$

$\boxed{f_{\mathbf{v}}(x(t_0), y(t_0)) := (f \circ \gamma)'(t_0) = \nabla(f)(p).\mathbf{v}}$,
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and $p = (x(t_0), y(t_0))$

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : \mathbb{R}^2 \to \mathbb{R}^2$
$\gamma(u, v) = (x(u, v), y(u, v))$
$f \circ \gamma : \mathbb{R}^2 \to \mathbb{R}$

$(f \circ \gamma)_u(u_0, v_0)$
$= f_x(x(u_0, v_0), y(u_0, v_0))x_u(u_0, v_0)$
$+f_y(x(u_0, v_0), y(u_0, v_0))y_u(u_0, v_0)$

Curves on surfaces

Definition. $\mathbf{v} \in \mathbb{R}^3$ is a tangent vector of the surface S at a point p, if there is a $\gamma : (\alpha, \beta) \to S \subset \mathbb{R}^3$ so that $p = \gamma(t)$ and

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\gamma(t) = (x(t), y(t))$
$f \circ \gamma : (\alpha, \beta) \to \mathbb{R}$

$\boxed{f_{\mathbf{v}}(x(t_0), y(t_0)) := (f \circ \gamma)'(t_0) = \nabla(f)(p).\mathbf{v}}$,
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and $p = (x(t_0), y(t_0))$

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : \mathbb{R}^2 \to \mathbb{R}^2$
$\gamma(u, v) = (x(u, v), y(u, v))$
$f \circ \gamma : \mathbb{R}^2 \to \mathbb{R}$

$(f \circ \gamma)_u(u_0, v_0)$
$= f_x(x(u_0, v_0), y(u_0, v_0))x_u(u_0, v_0)$
$+f_y(x(u_0, v_0), y(u_0, v_0))y_u(u_0, v_0)$

Curves on surfaces

Definition. $\mathbf{v} \in \mathbb{R}^3$ is a tangent vector of the surface S at a point p, if there is a $\gamma : (\alpha, \beta) \to S \subset \mathbb{R}^3$ so that $p = \gamma(t)$ and $\mathbf{v} = \dot{\gamma}(t)$

$\gamma : (\alpha, \beta) \to S \subset \mathbb{R}^3$ is a curve.

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\gamma(t) = (x(t), y(t))$
$f \circ \gamma : (\alpha, \beta) \to \mathbb{R}$

$\boxed{f_{\mathbf{v}}(x(t_0), y(t_0)) := (f \circ \gamma)'(t_0) = \nabla(f)(p).\mathbf{v}}$,
where $\nabla(f)(x, y) = (f_x(x, y).f_y(x, y))$,
$\mathbf{v} = \dot{\gamma}(t_0)$,
and $p = (x(t_0), y(t_0))$

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : \mathbb{R}^2 \to \mathbb{R}^2$
$\gamma(u, v) = (x(u, v), y(u, v))$
$f \circ \gamma : \mathbb{R}^2 \to \mathbb{R}$

$(f \circ \gamma)_u(u_0, v_0)$
$= f_x(x(u_0, v_0), y(u_0, v_0))x_u(u_0, v_0)$
$+f_y(x(u_0, v_0), y(u_0, v_0))y_u(u_0, v_0)$

Curves on surfaces

Definition. $\mathbf{v} \in \mathbb{R}^3$ is a tangent vector of the surface S at a point p, if there is a $\gamma : (\alpha, \beta) \to S \subset \mathbb{R}^3$ so that $p = \gamma(t)$ and $\mathbf{v} = \dot{\gamma}(t)$

$\gamma : (\alpha, \beta) \to S \subset \mathbb{R}^3$ is a curve.
$\sigma : U \to S$ a surface patch.

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\gamma(t) = (x(t), y(t))$
$f \circ \gamma : (\alpha, \beta) \to \mathbb{R}$

$\boxed{f_{\mathbf{v}}(x(t_0), y(t_0)) := (f \circ \gamma)'(t_0) = \nabla(f)(p).\mathbf{v}}$,
where $\nabla(f)(x, y) = (f_x(x, y).f_y(x, y))$,
$\mathbf{v} = \dot{\gamma}(t_0)$,
and $p = (x(t_0), y(t_0))$

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : \mathbb{R}^2 \to \mathbb{R}^2$
$\gamma(u, v) = (x(u, v), y(u, v))$
$f \circ \gamma : \mathbb{R}^2 \to \mathbb{R}$

$(f \circ \gamma)_u(u_0, v_0)$
$= f_x(x(u_0, v_0), y(u_0, v_0))x_u(u_0, v_0)$
$+ f_y(x(u_0, v_0), y(u_0, v_0))y_u(u_0, v_0)$

Curves on surfaces

Definition. $\mathbf{v} \in \mathbb{R}^3$ is a tangent vector of the surface S at a point p, if there is a $\gamma : (\alpha, \beta) \to S \subset \mathbb{R}^3$ so that $p = \gamma(t)$ and $\mathbf{v} = \dot{\gamma}(t)$

$\gamma : (\alpha, \beta) \to S \subset \mathbb{R}^3$ is a curve.
$\sigma : U \to S$ a surface patch.
So, $\gamma(t) = \sigma(x(t), y(t))$

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\gamma(t) = (x(t), y(t))$
$f \circ \gamma : (\alpha, \beta) \to \mathbb{R}$

$\boxed{f_{\mathbf{v}}(x(t_0), y(t_0)) := (f \circ \gamma)'(t_0) = \nabla(f)(p).\mathbf{v}}$,
where $\nabla(f)(x, y) = (f_x(x, y).f_y(x, y))$,
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$(f \circ \gamma)_u(u_0, v_0)$
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Curves on surfaces

Definition. $\mathbf{v} \in \mathbb{R}^3$ is a tangent vector of the surface S at a point p, if there is a $\gamma : (\alpha, \beta) \to S \subset \mathbb{R}^3$ so that $p = \gamma(t)$ and $\mathbf{v} = \dot{\gamma}(t)$

$\gamma : (\alpha, \beta) \to S \subset \mathbb{R}^3$ is a curve.
$\sigma : U \to S$ a surface patch.
So, $\gamma(t) = \sigma(x(t), y(t)) = p \in S$

Chain rule for mult-variable functions

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$$\dot{\gamma}(t) = x'(t)\sigma_x(x(t), y(t)) + y'(t)\sigma_y(x(t), y(t))$$

Chain rule for mult-variable functions

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and $p=(x(t_0),y(t_0))$

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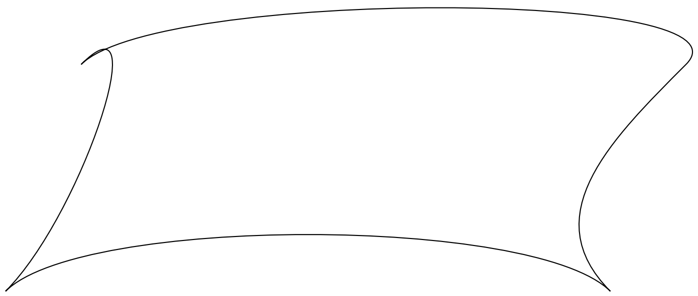
Exercise. Show that σ is regular at p if and only if the tangent vectors at p form a two dimensional subspace of $\mathbb{R}^3$.

A curve on a surface

Note:: This was not part of the lecture, however, since there seemed to be some confusion, this adds a diagram to support along with “subtitles”.

$S \subset \mathbb{R}^3$



Consider a surface in space

Chain rule for mult-variable functions

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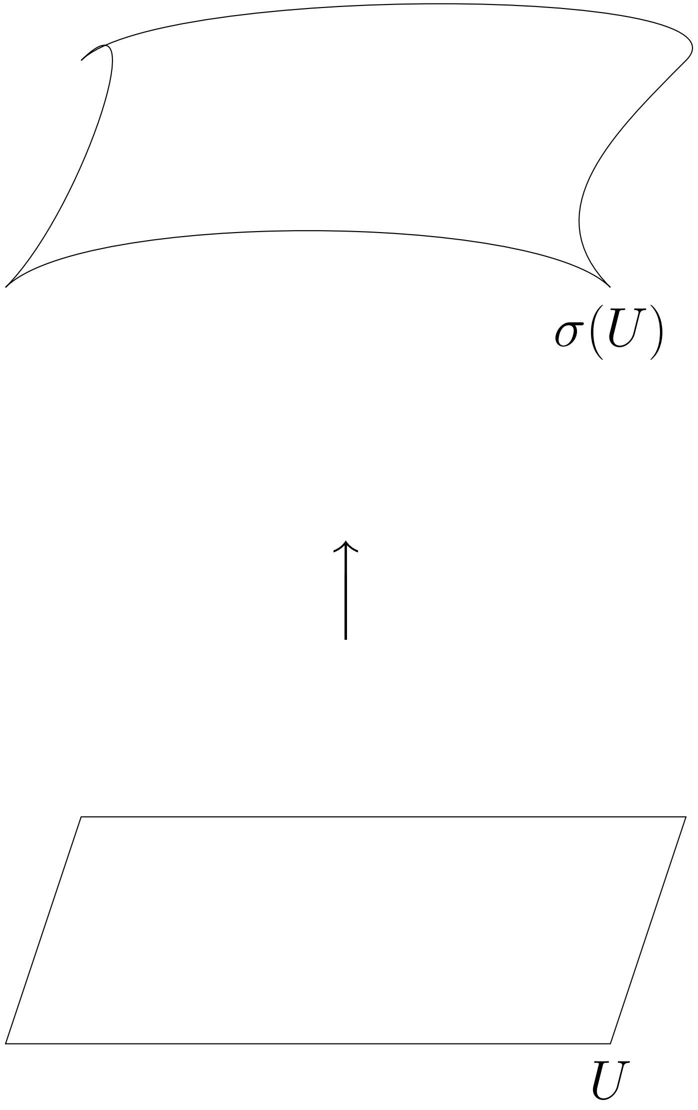
$U \to S \subset \mathbb{R}^3$



and a surface patch which is a map

Chain rule for mult-variable functions

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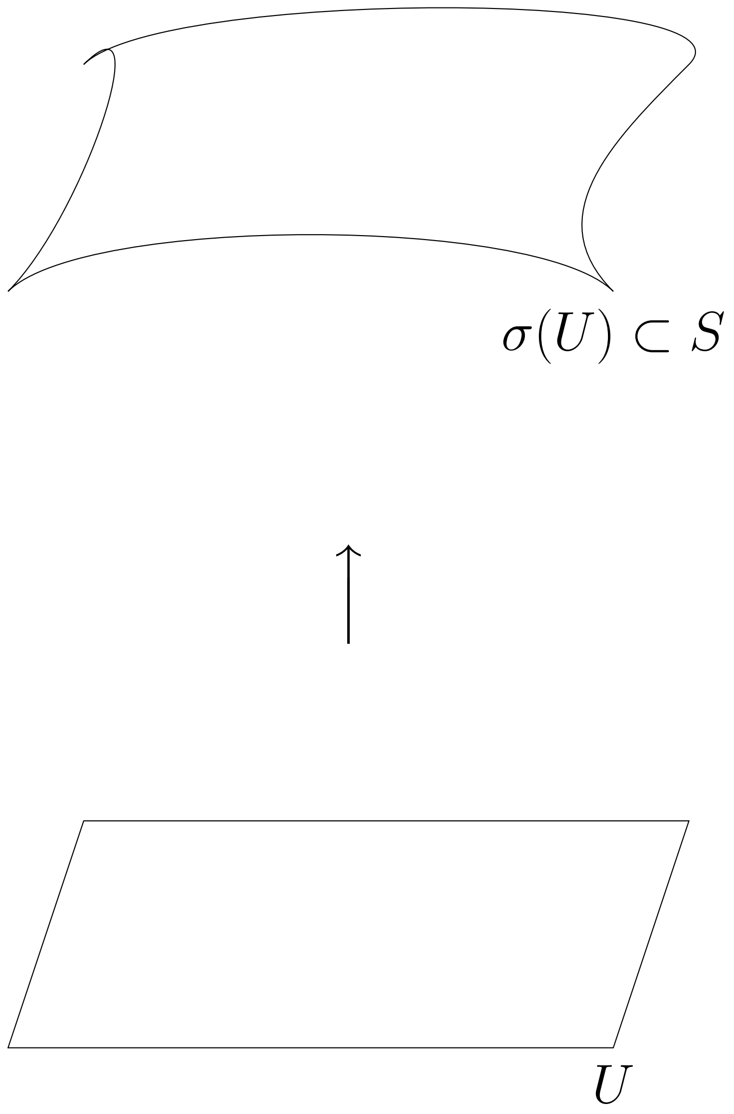

onto a part of the surface

Chain rule for mult-variable functions

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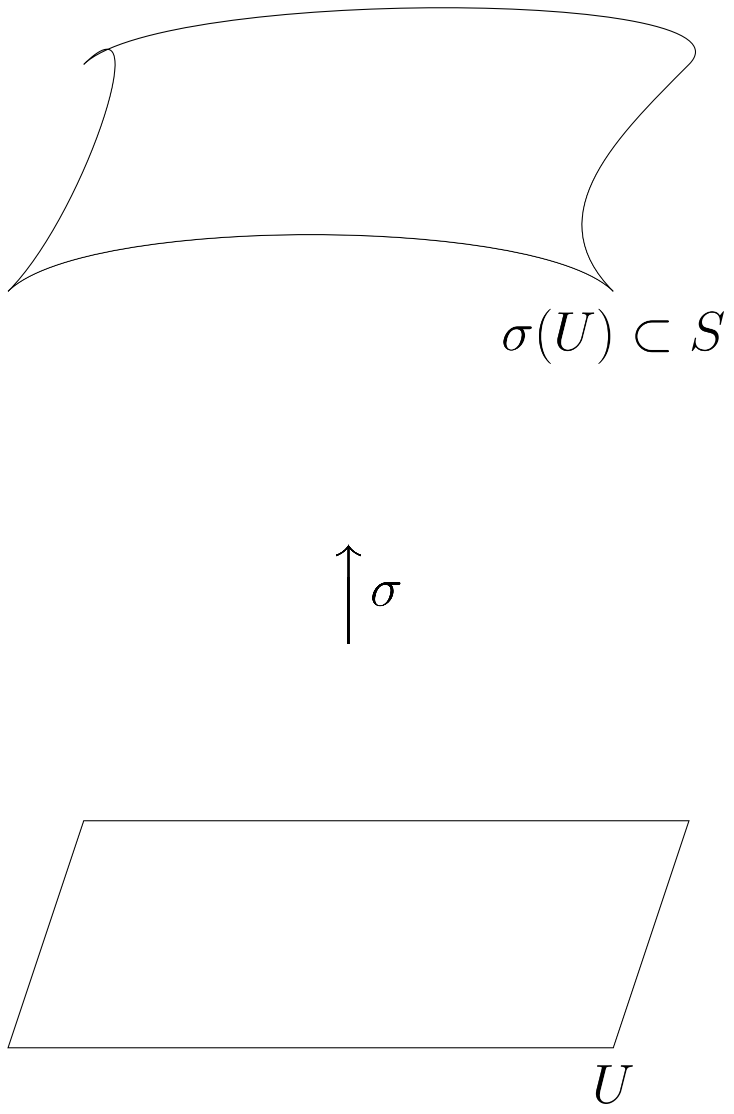

As usual we denote it by σ.

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A curve on a surface

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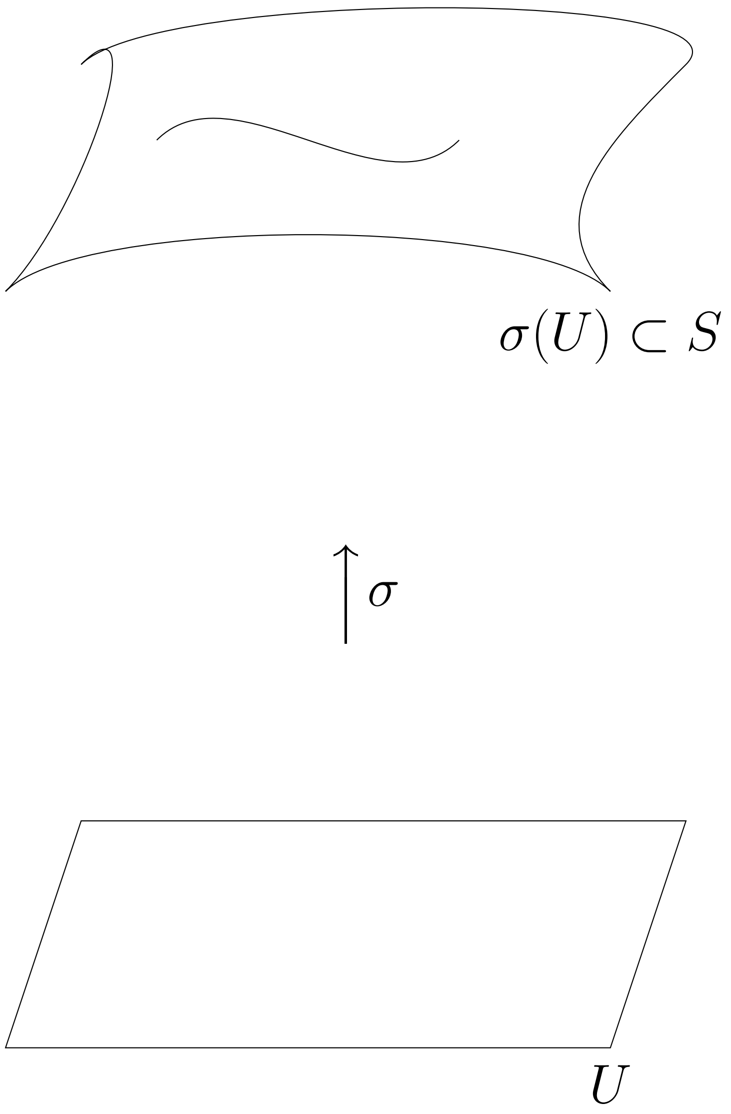

Now consider a curve on the surface

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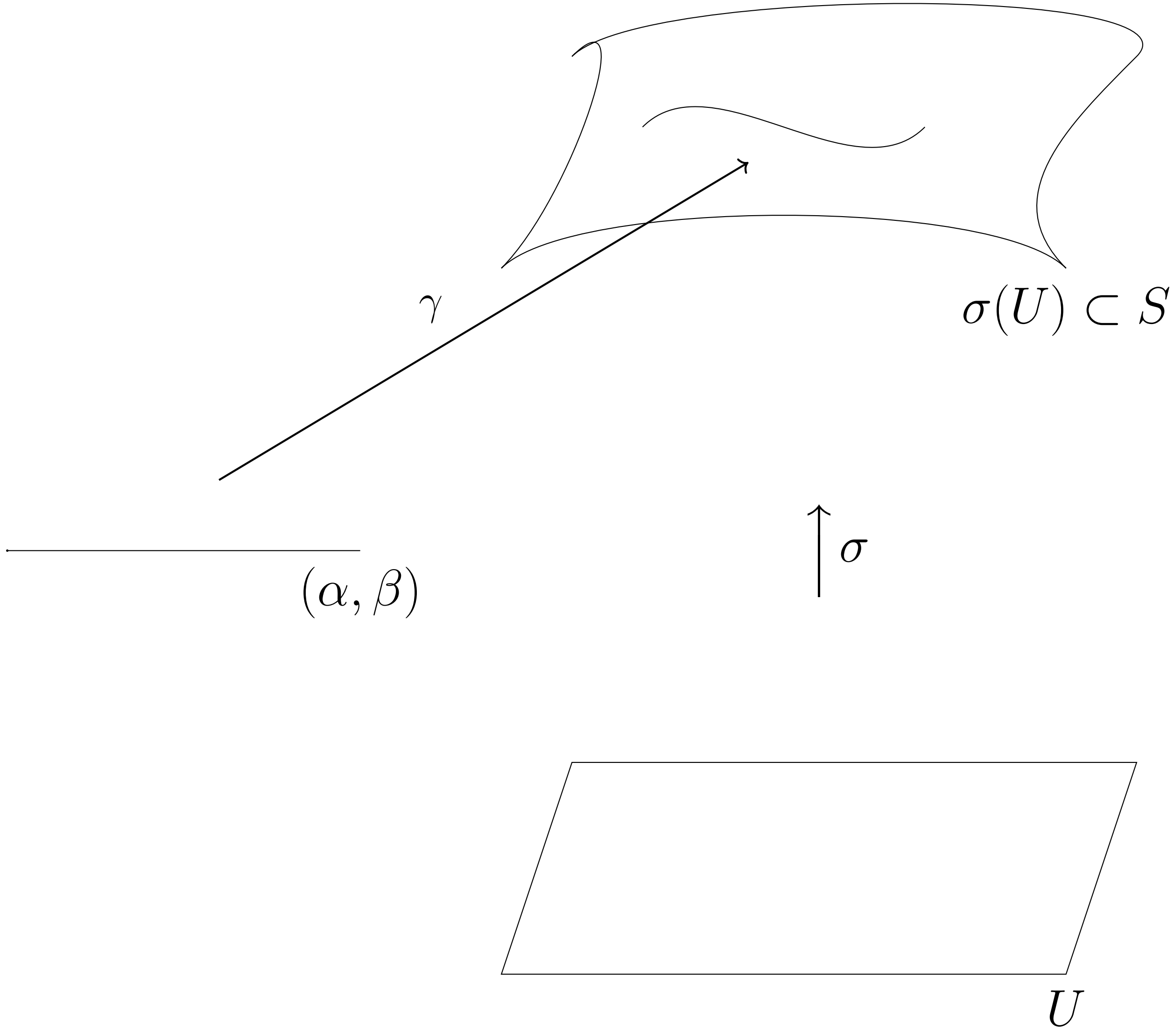

parametrized by γ

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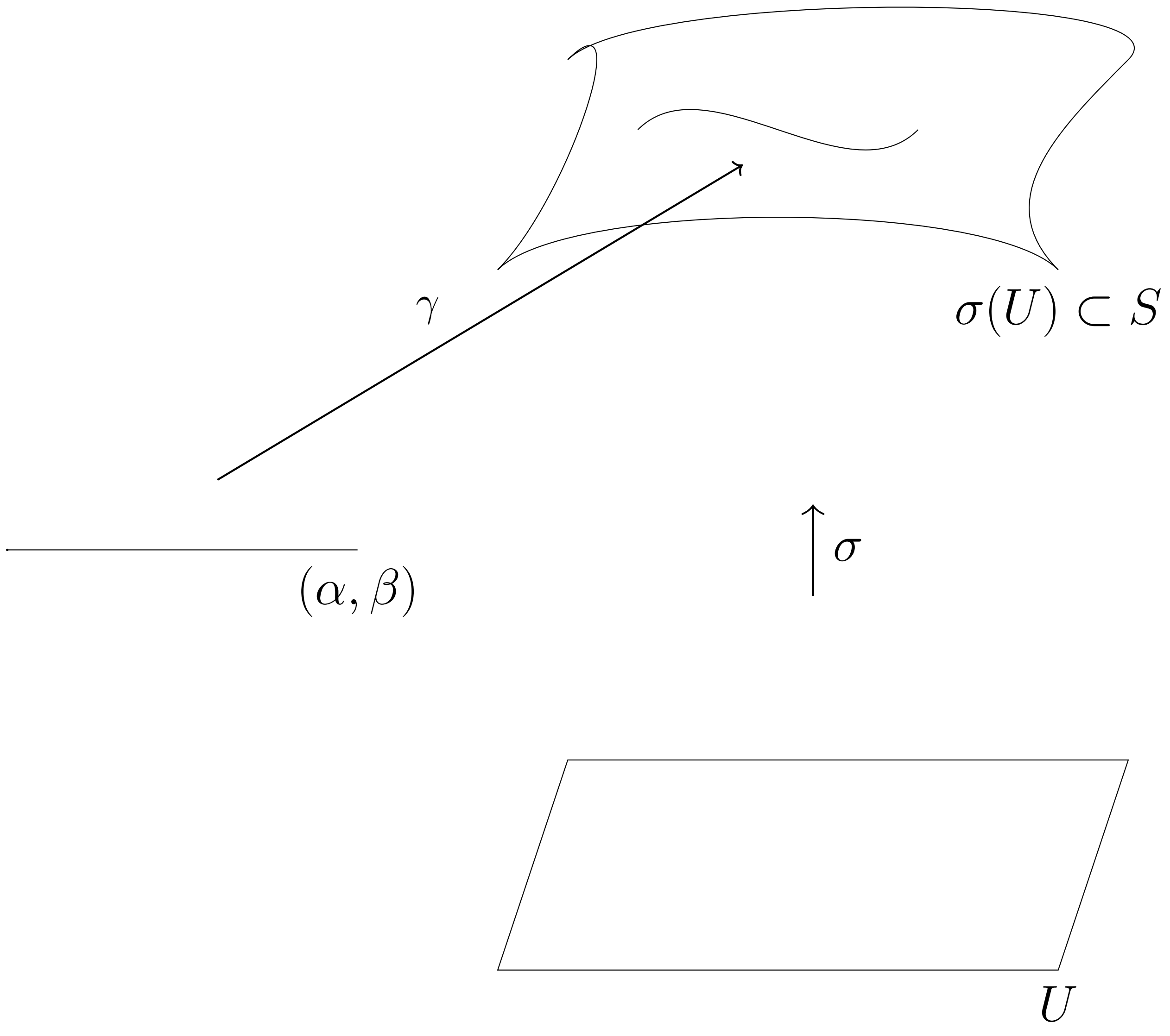

and let us assume it lies in the image of the surface patch

Chain rule for mult-variable functions

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Curves on surfaces

Definition. $\mathbf{v} \in \mathbb{R}^3$ is a tangent vector of the surface S at a point p, if there is a $\gamma : (\alpha, \beta) \to S \subset \mathbb{R}^3$ so that $p = \gamma(t)$ and $\mathbf{v} = \dot{\gamma}(t)$

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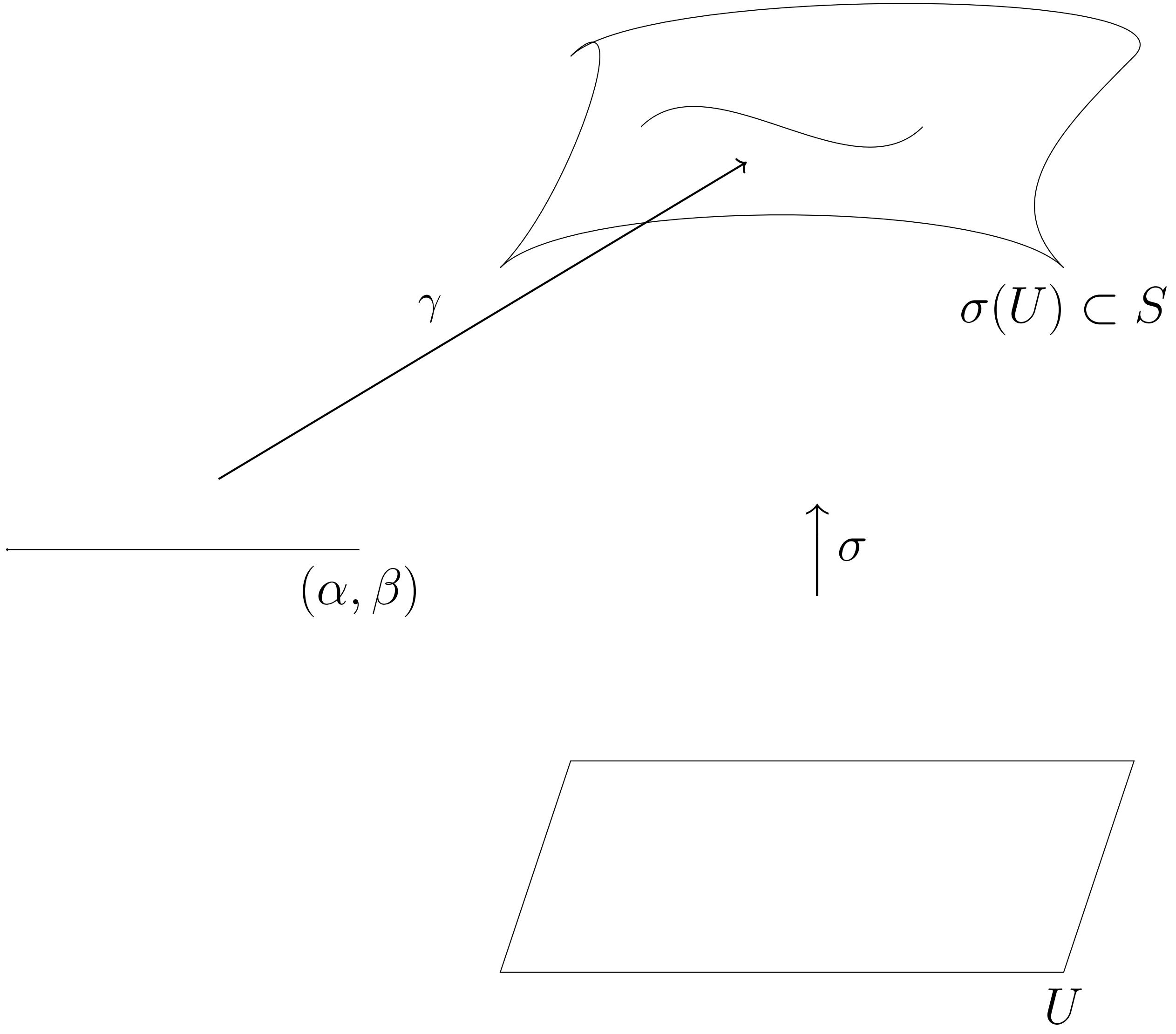

But it is also a curve in space

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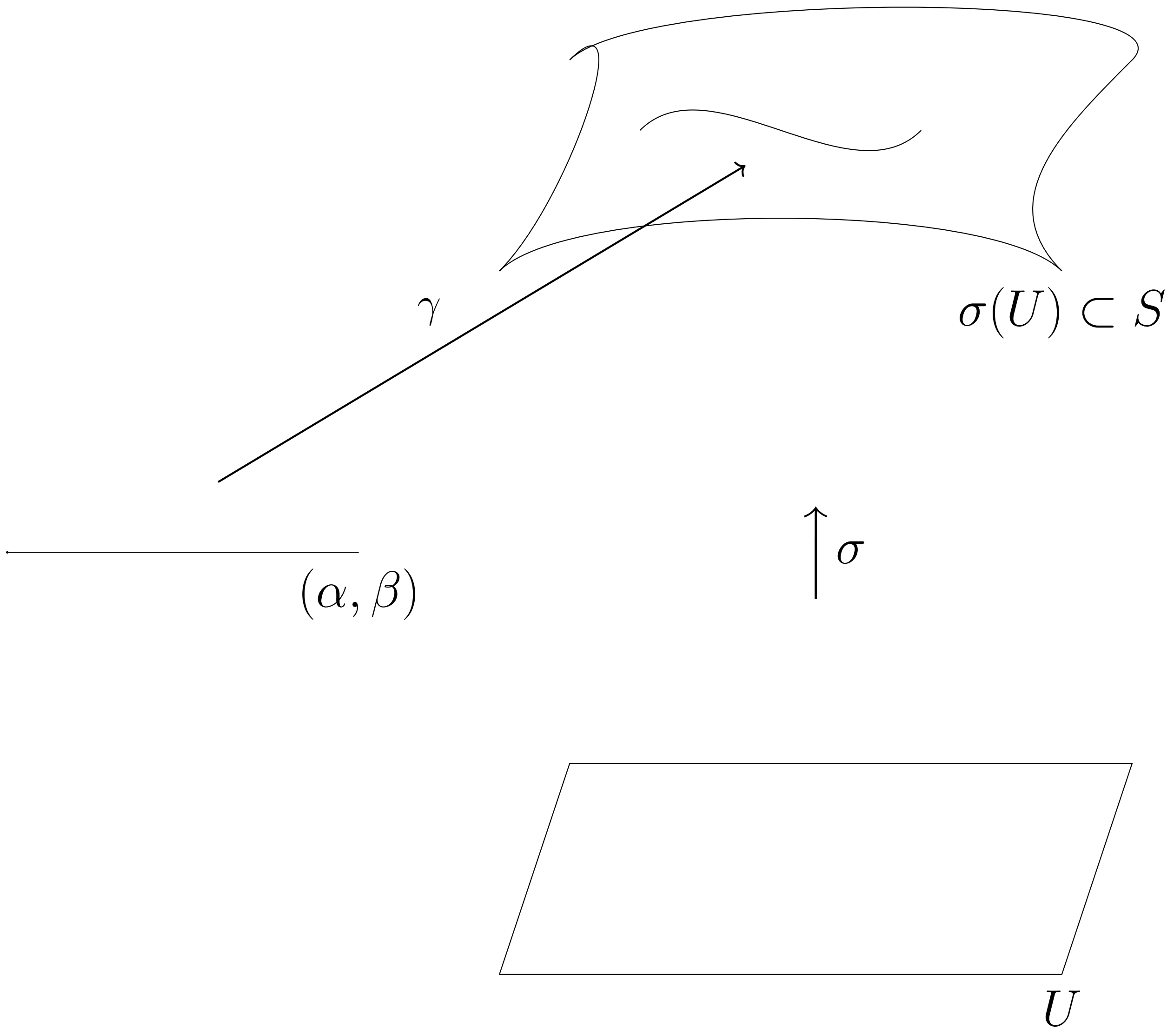

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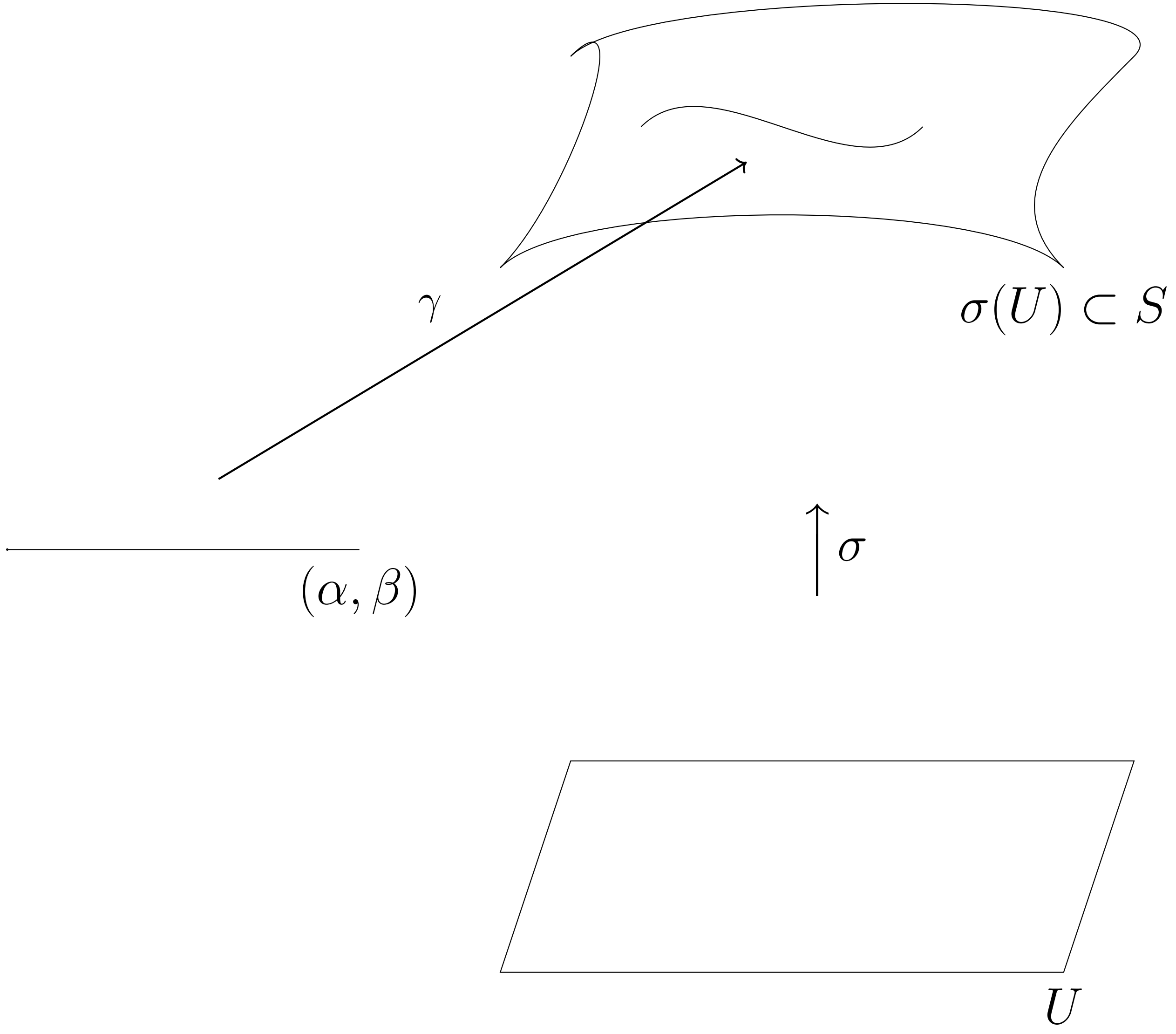

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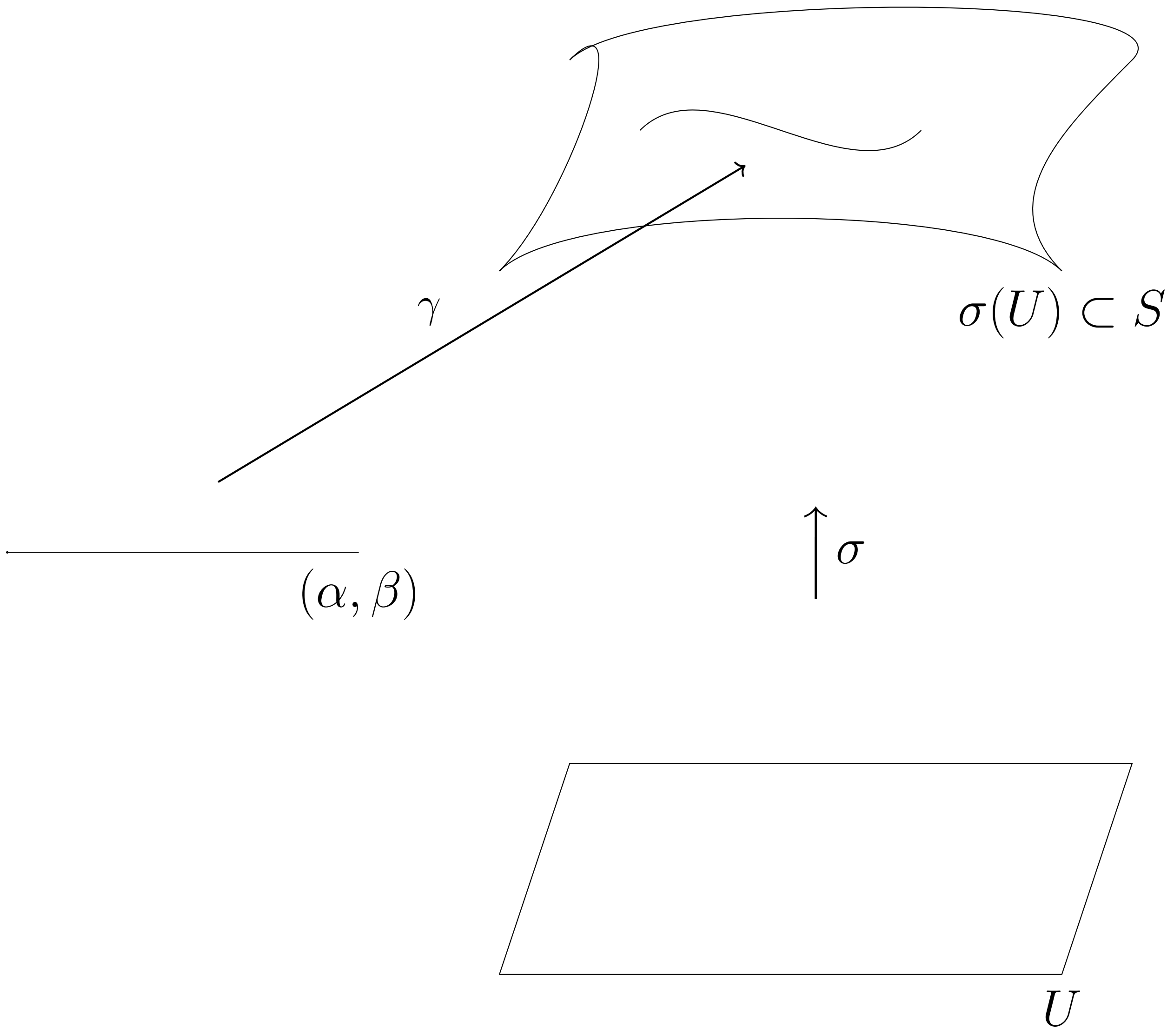

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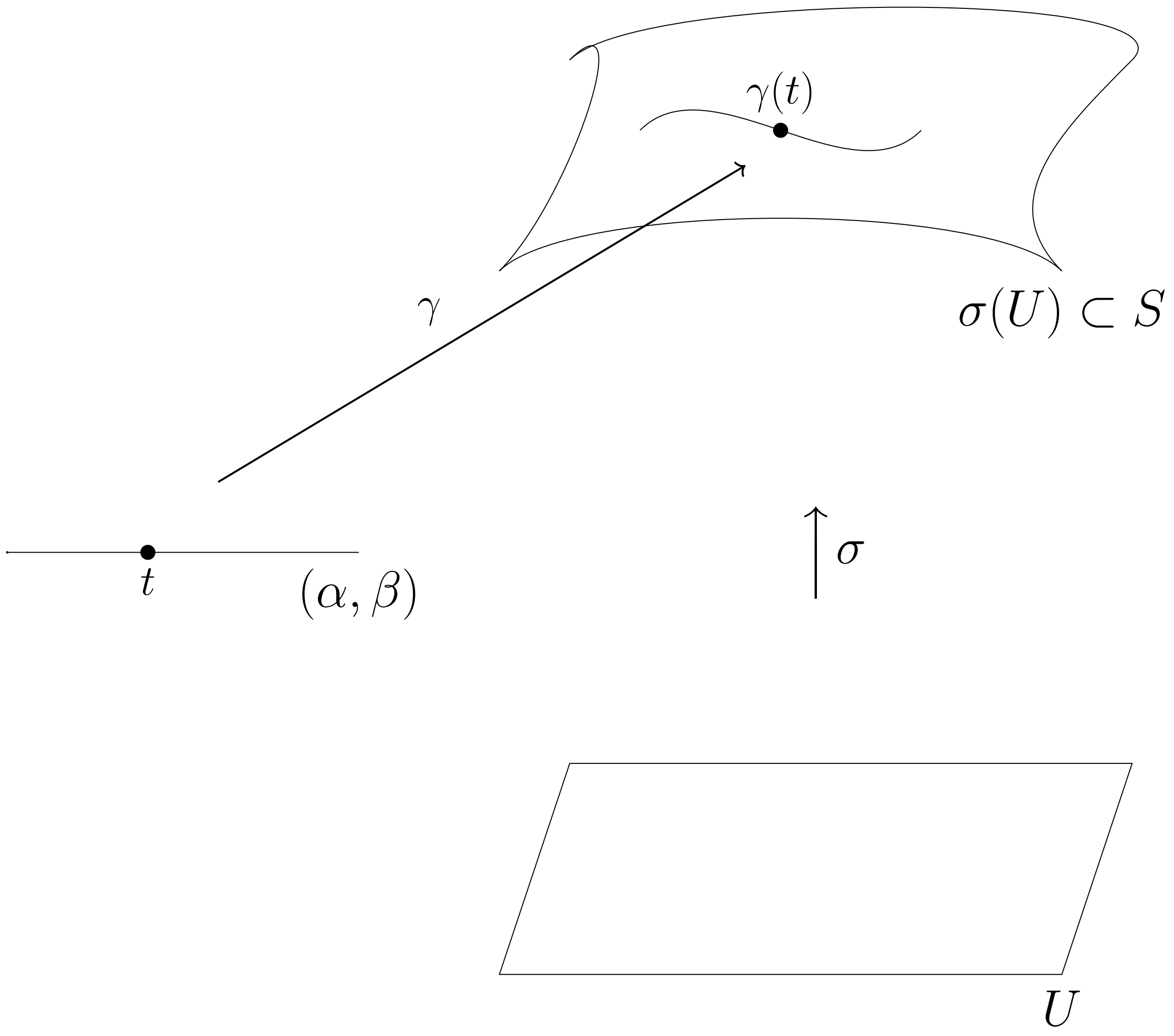

A parameter t goes to $\gamma(t)$

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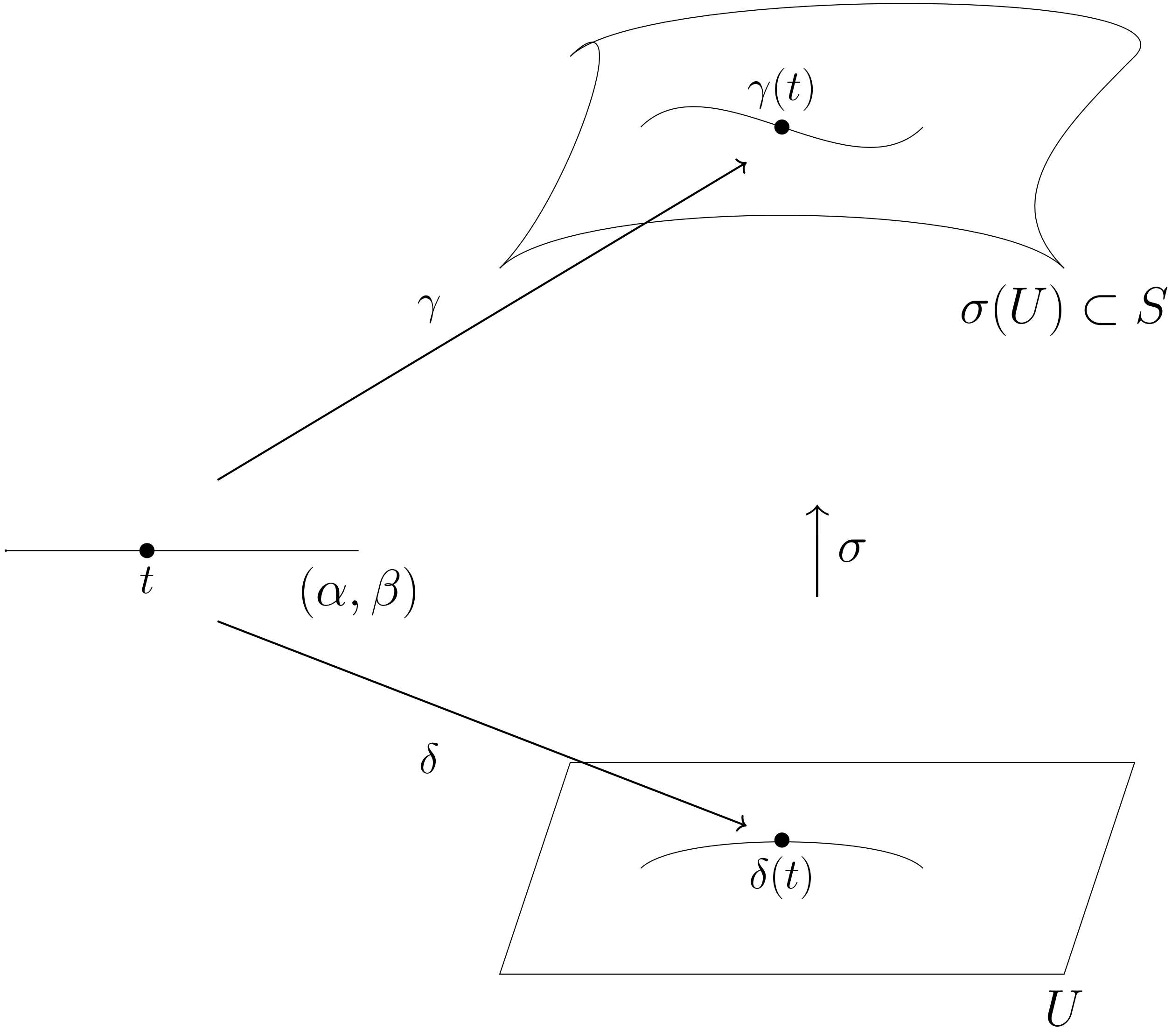

But to each $\gamma(t) \in \sigma(U)$, σ corresponds a $\delta(t) \in U$

.

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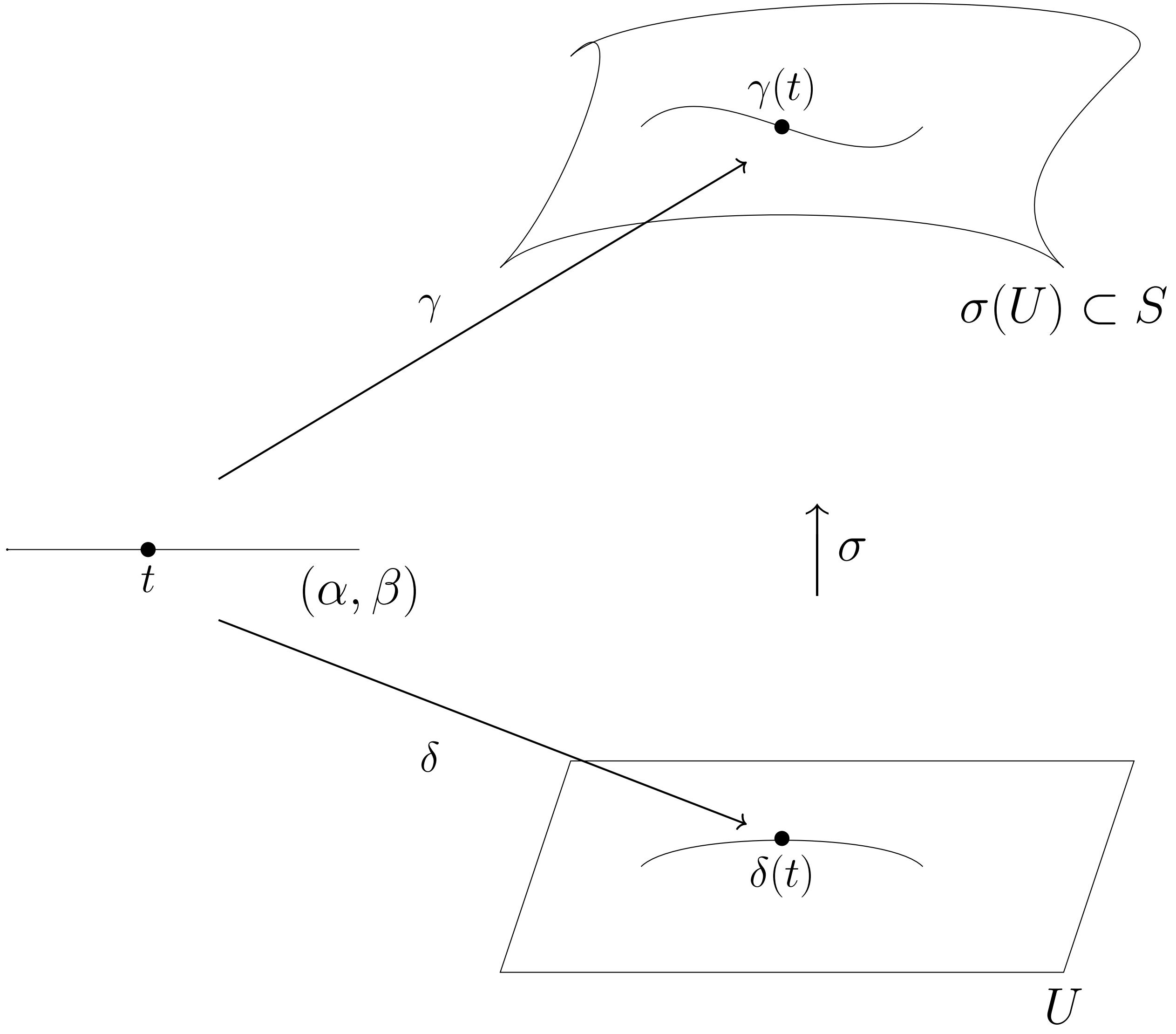

so that $\gamma(t) = \sigma(\delta(t))$

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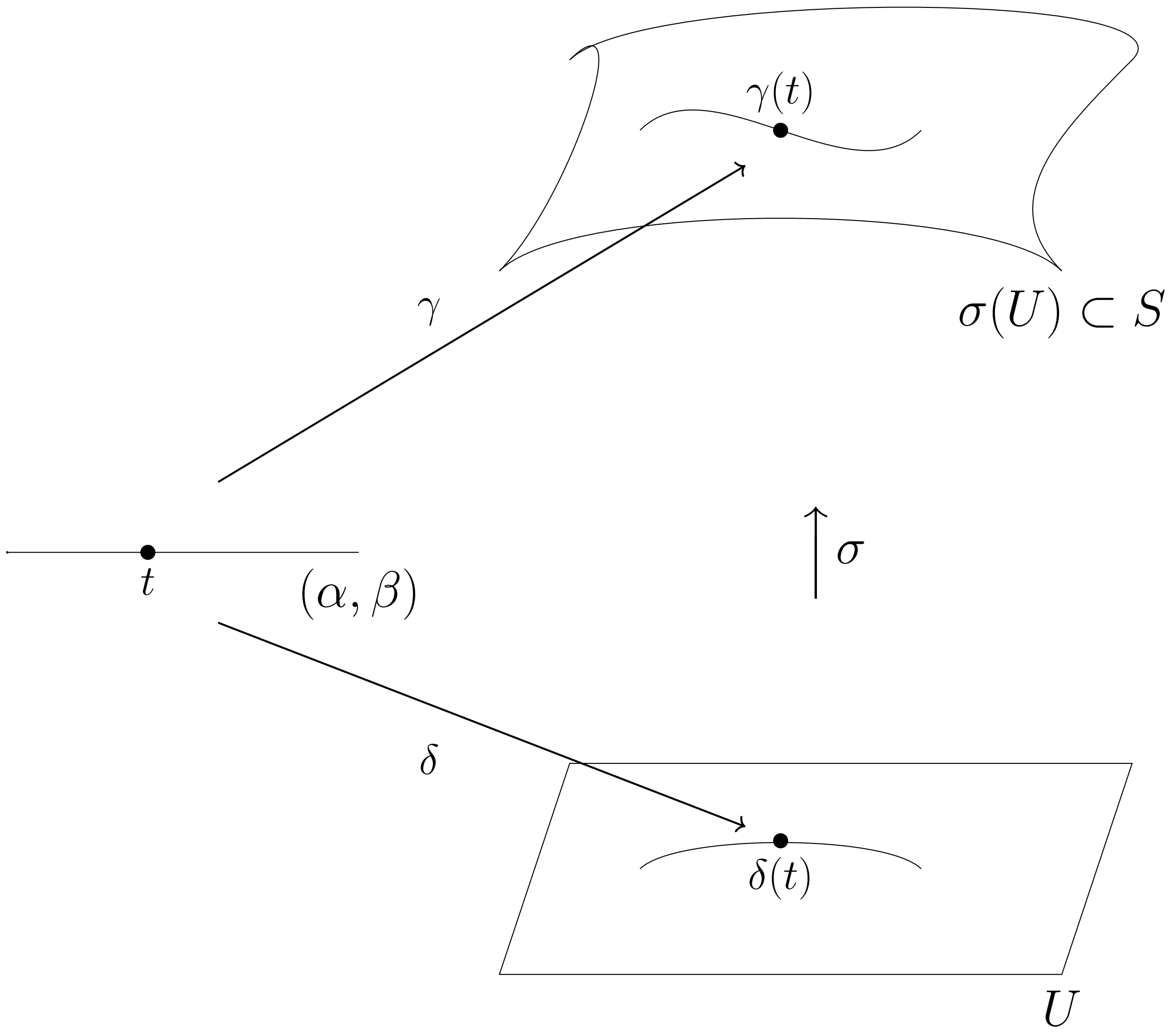

Note that this gives a $\delta(t)$ for each t

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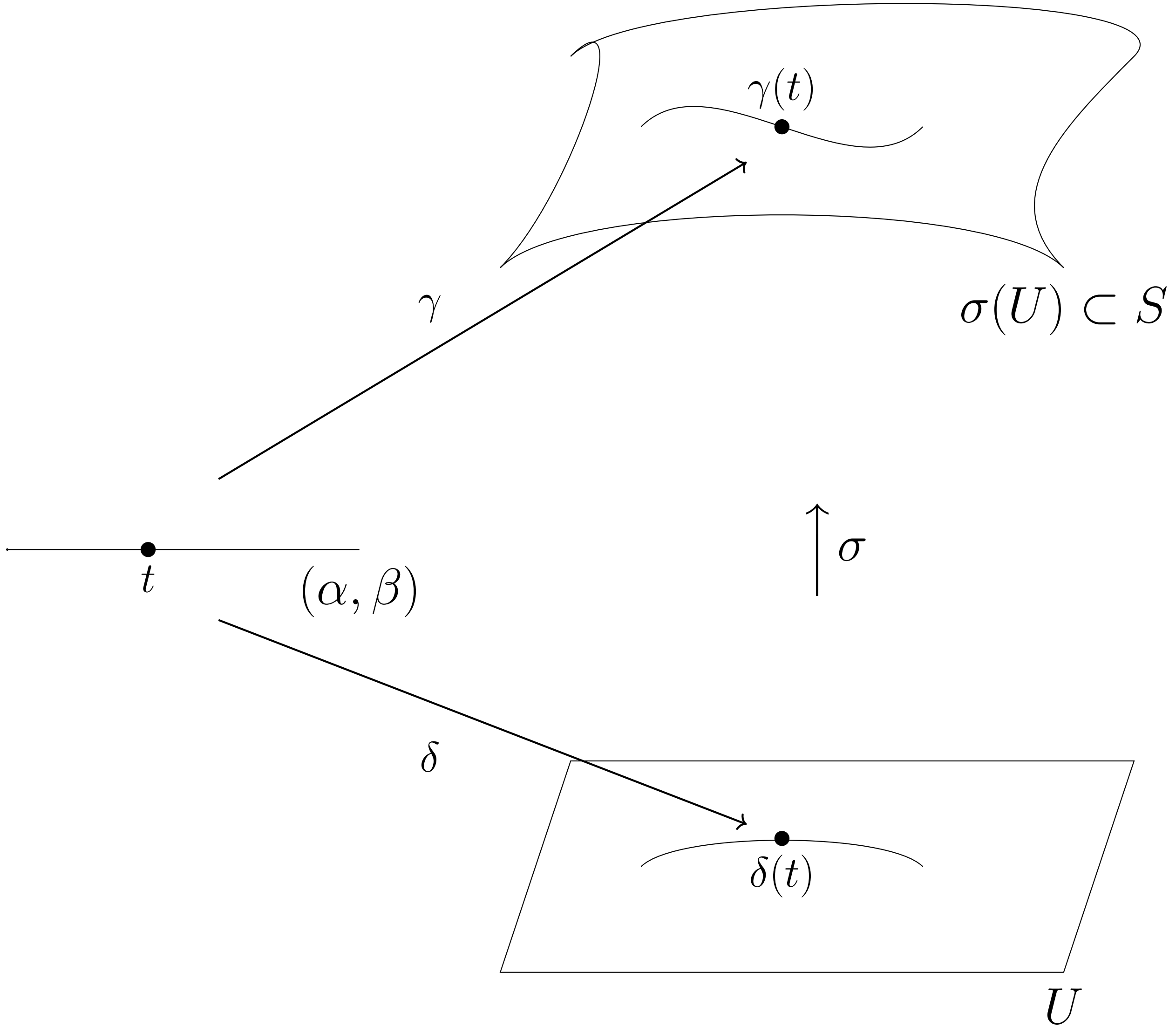

so it defines a map.

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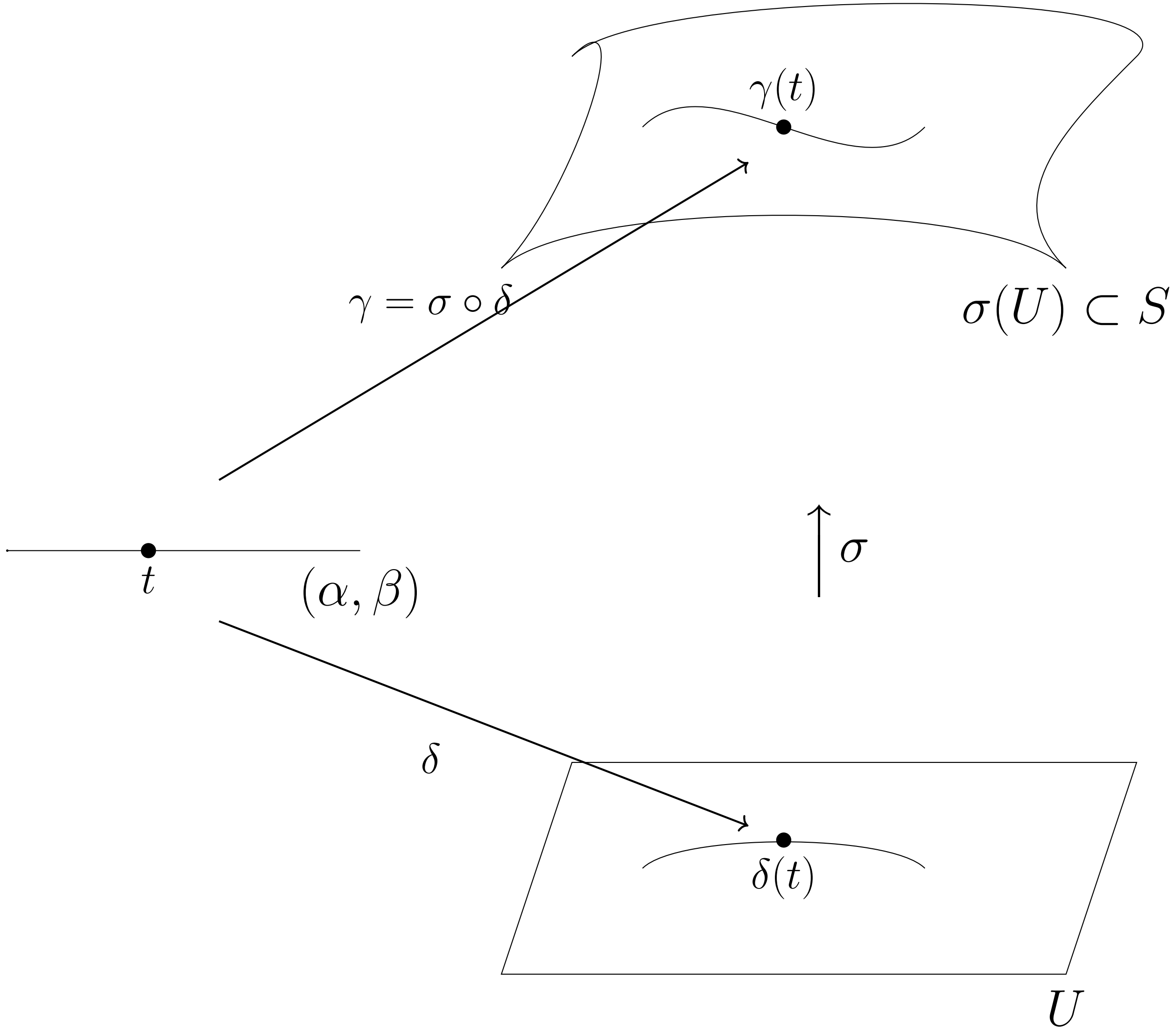

Its smoothness takes some work, but assume it for now.

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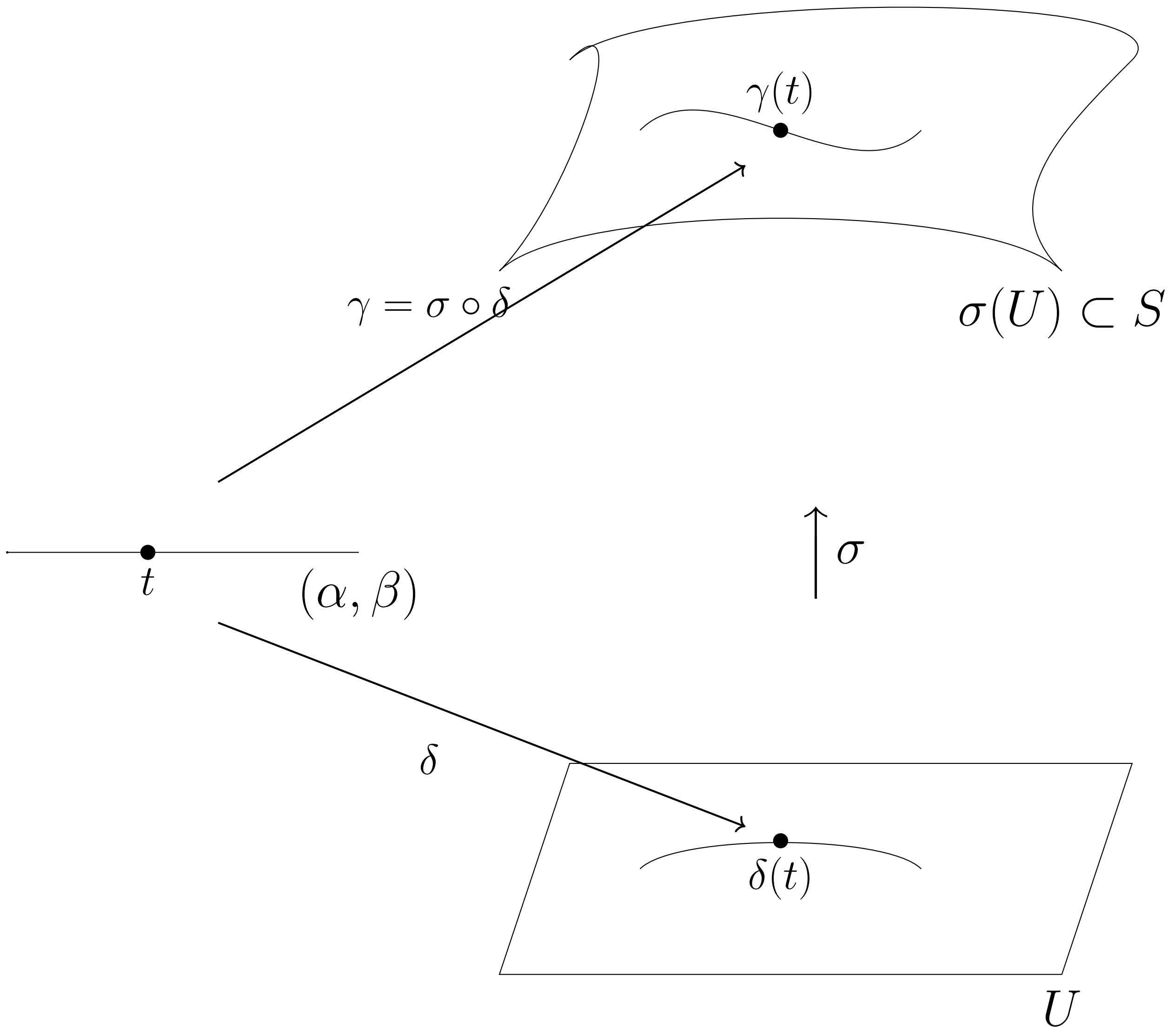

If we let $x(t)$ and $y(t)$ denote the coordinates of $\delta(t)$,

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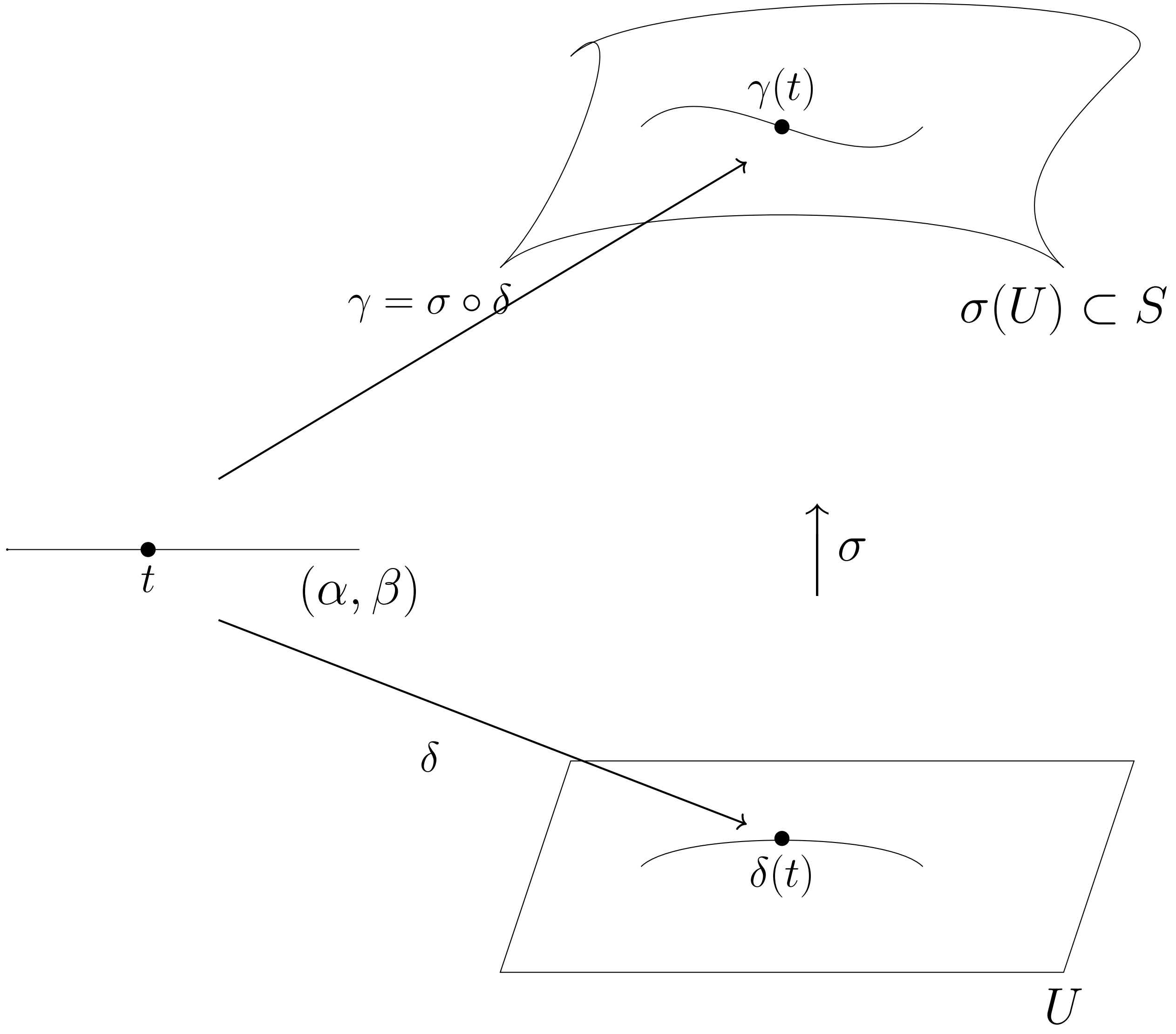

chain rule allows us to express the derivatives

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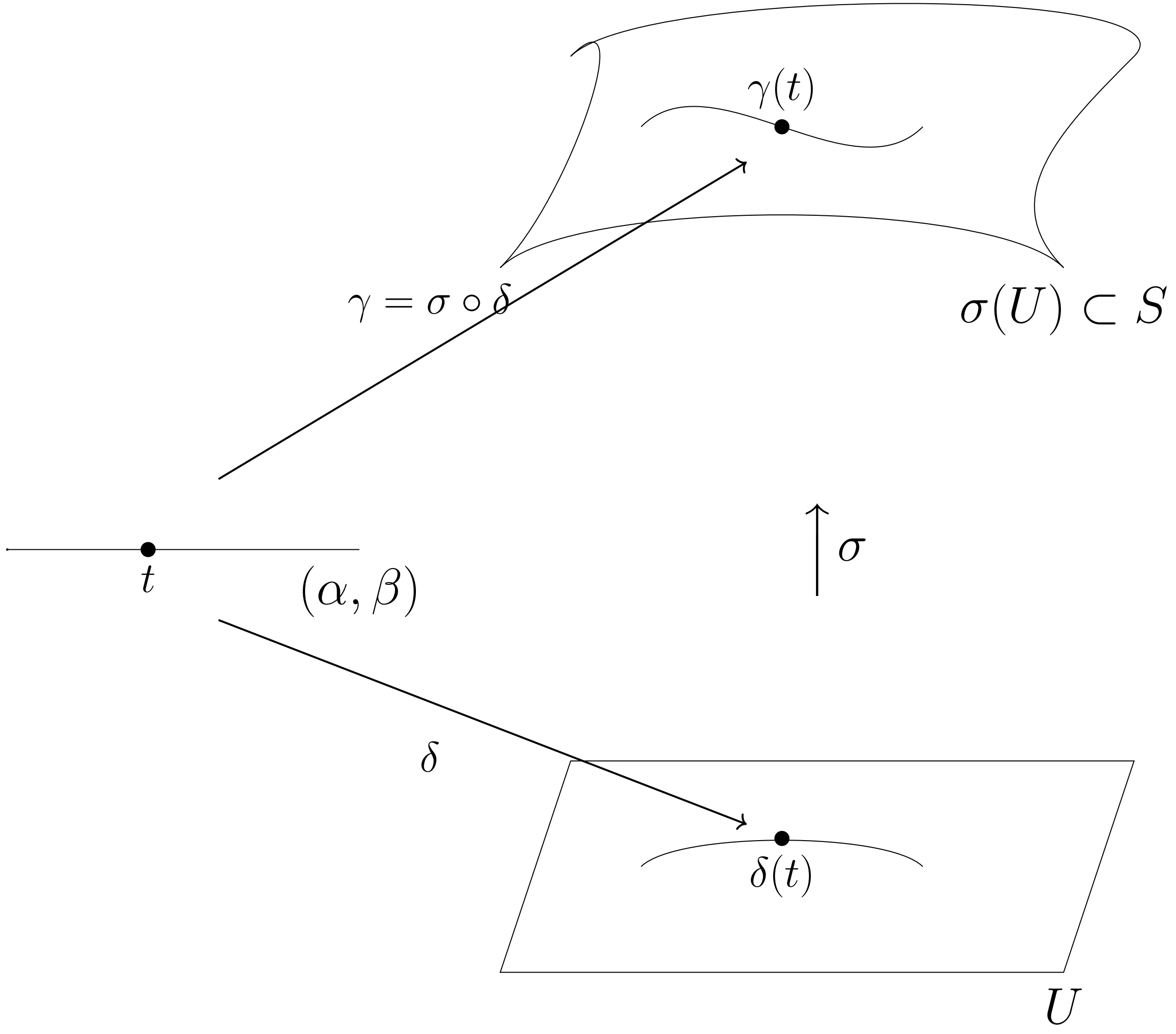

entirely in terms of the derivatives of δ and σ

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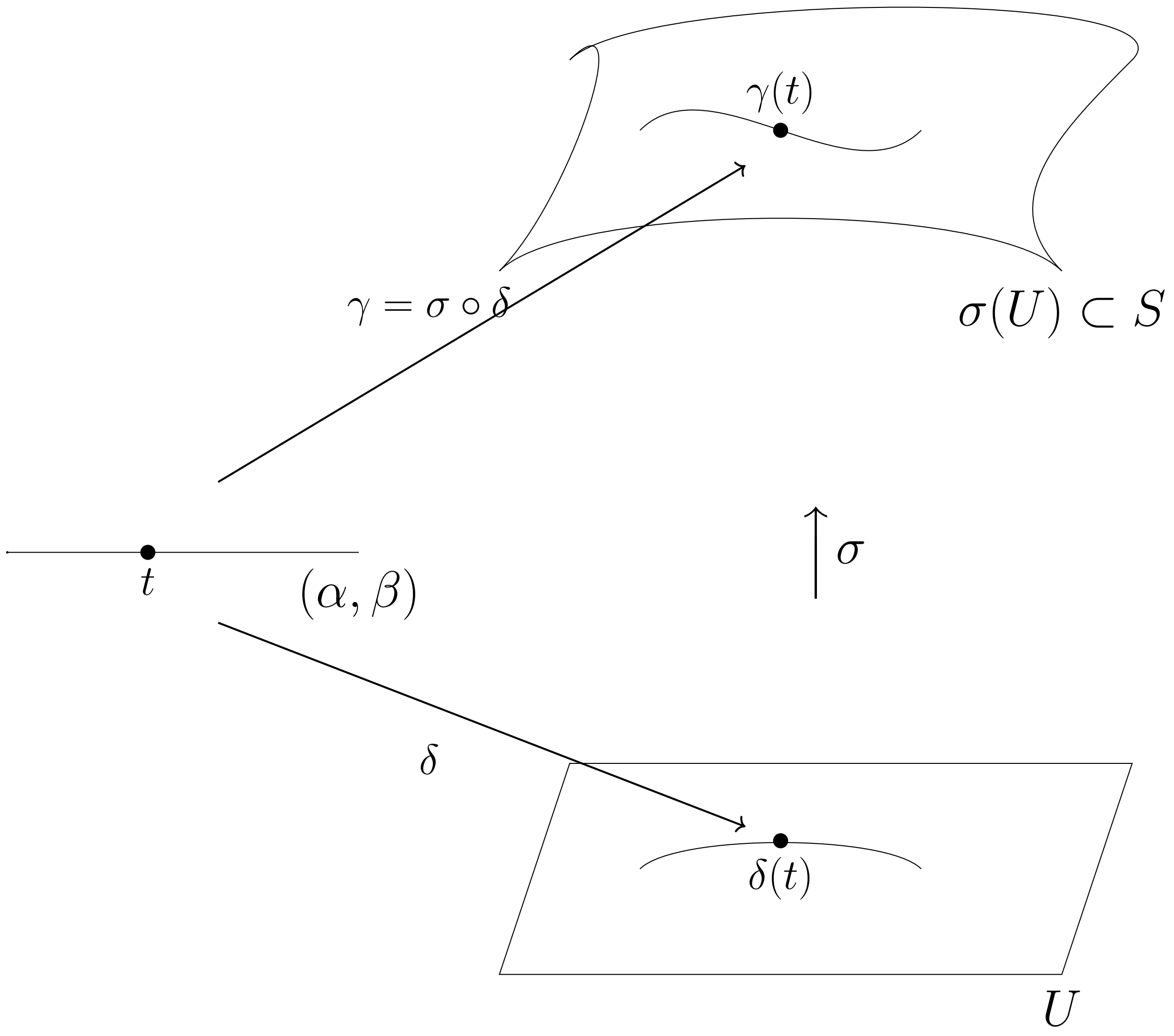

■

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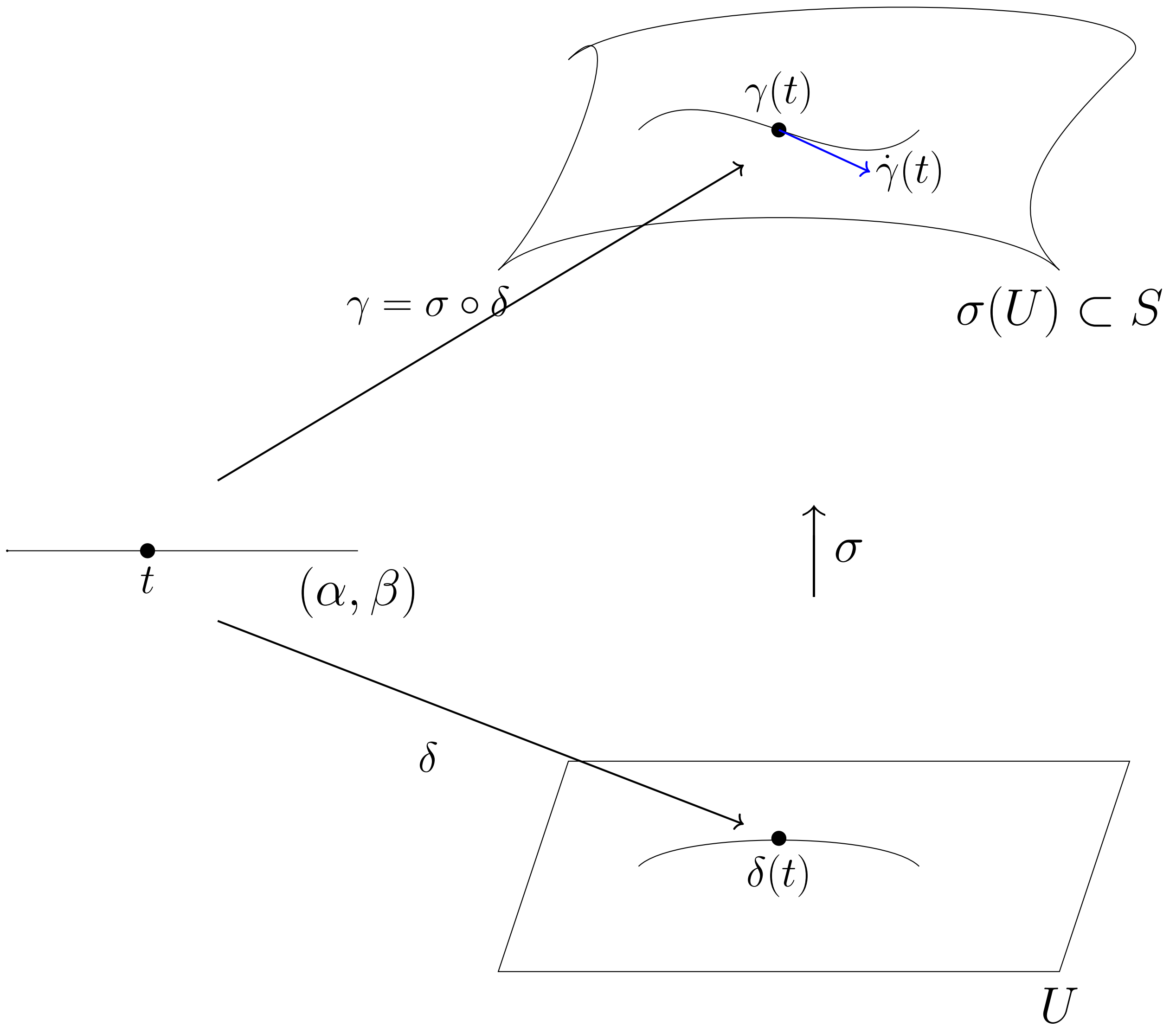

The left hand side is the velocity vector of γ in space

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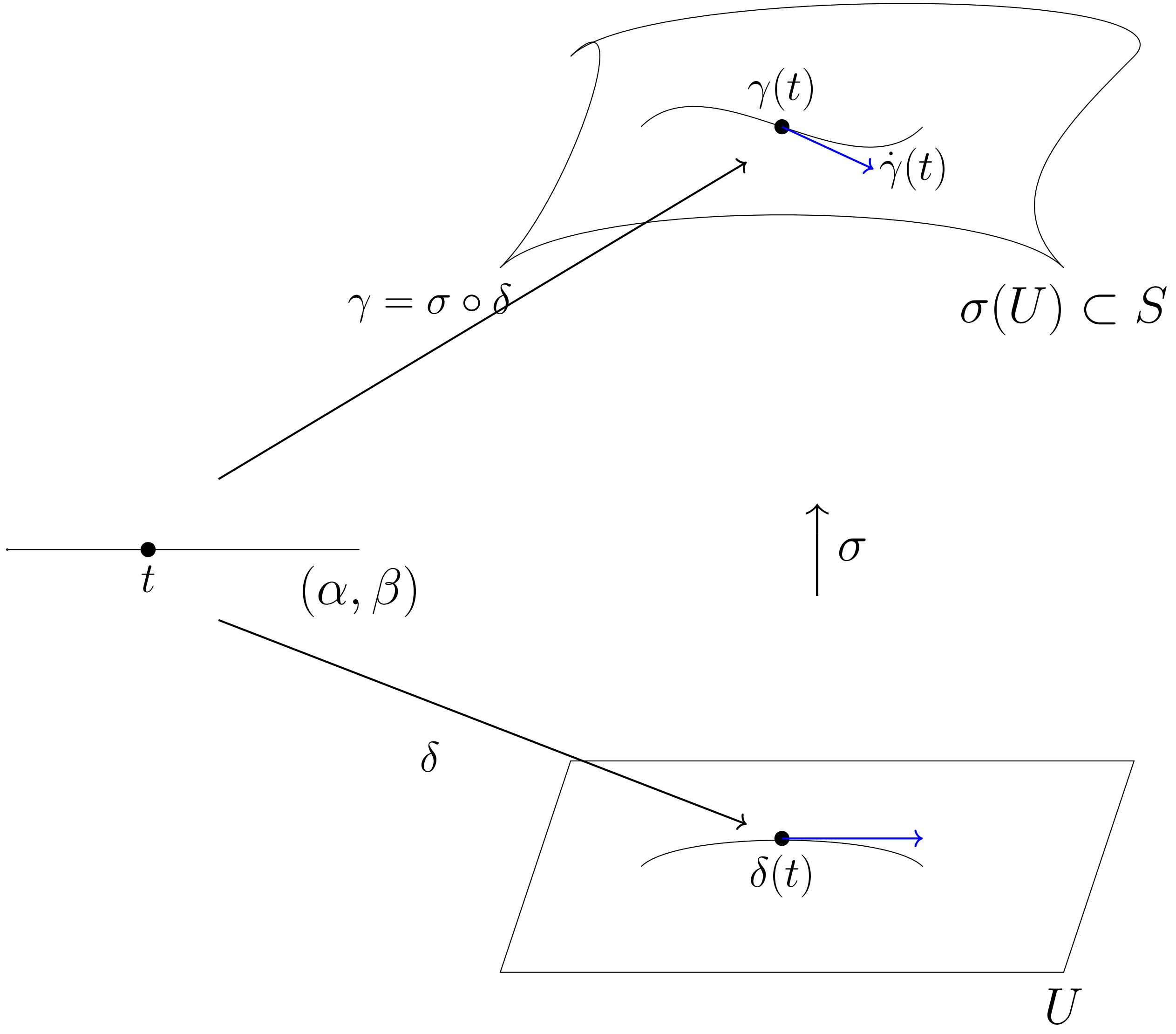

The right hand side expresses it in terms of the patch

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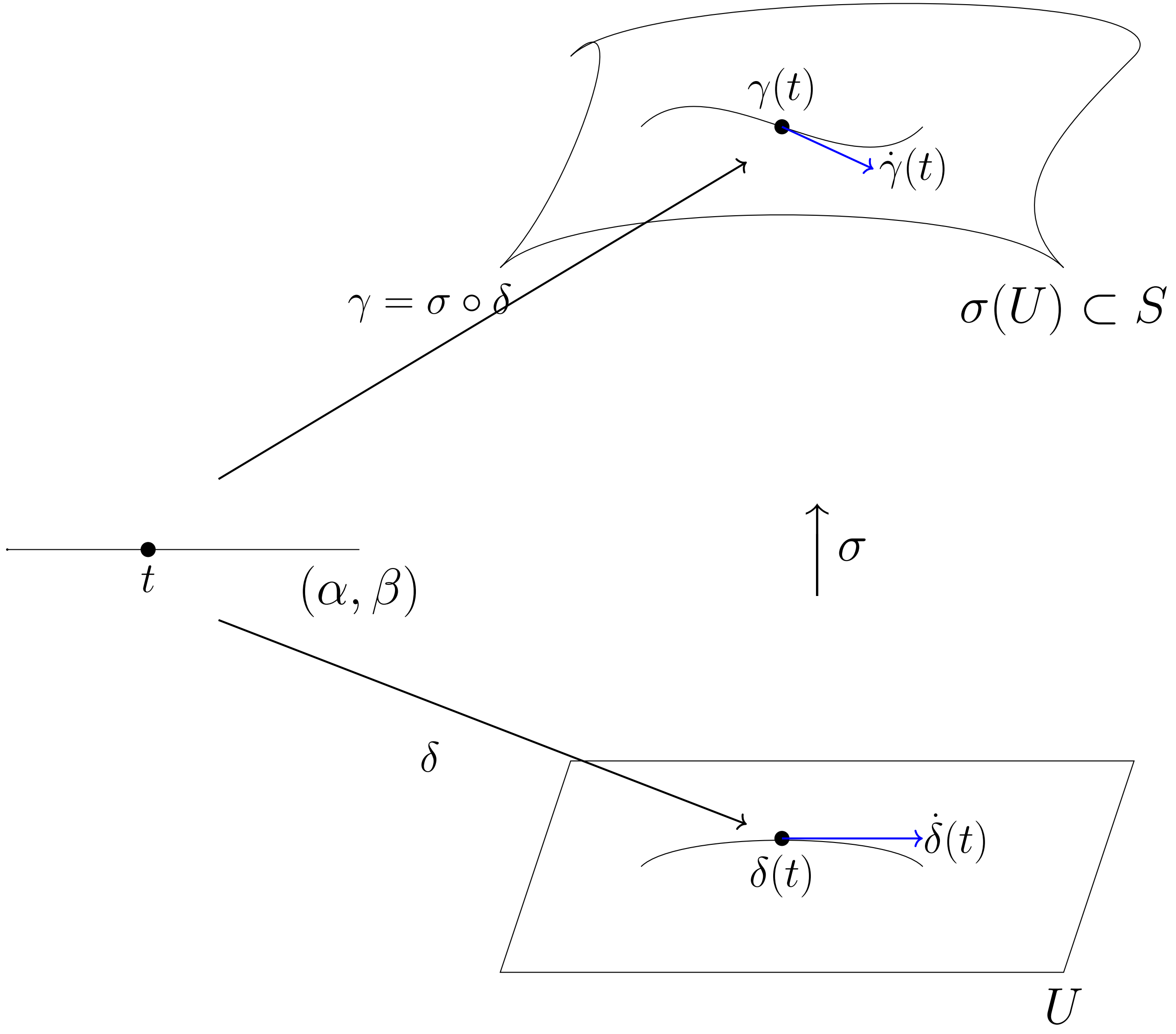

i.e., in terms of the velocity of δ.

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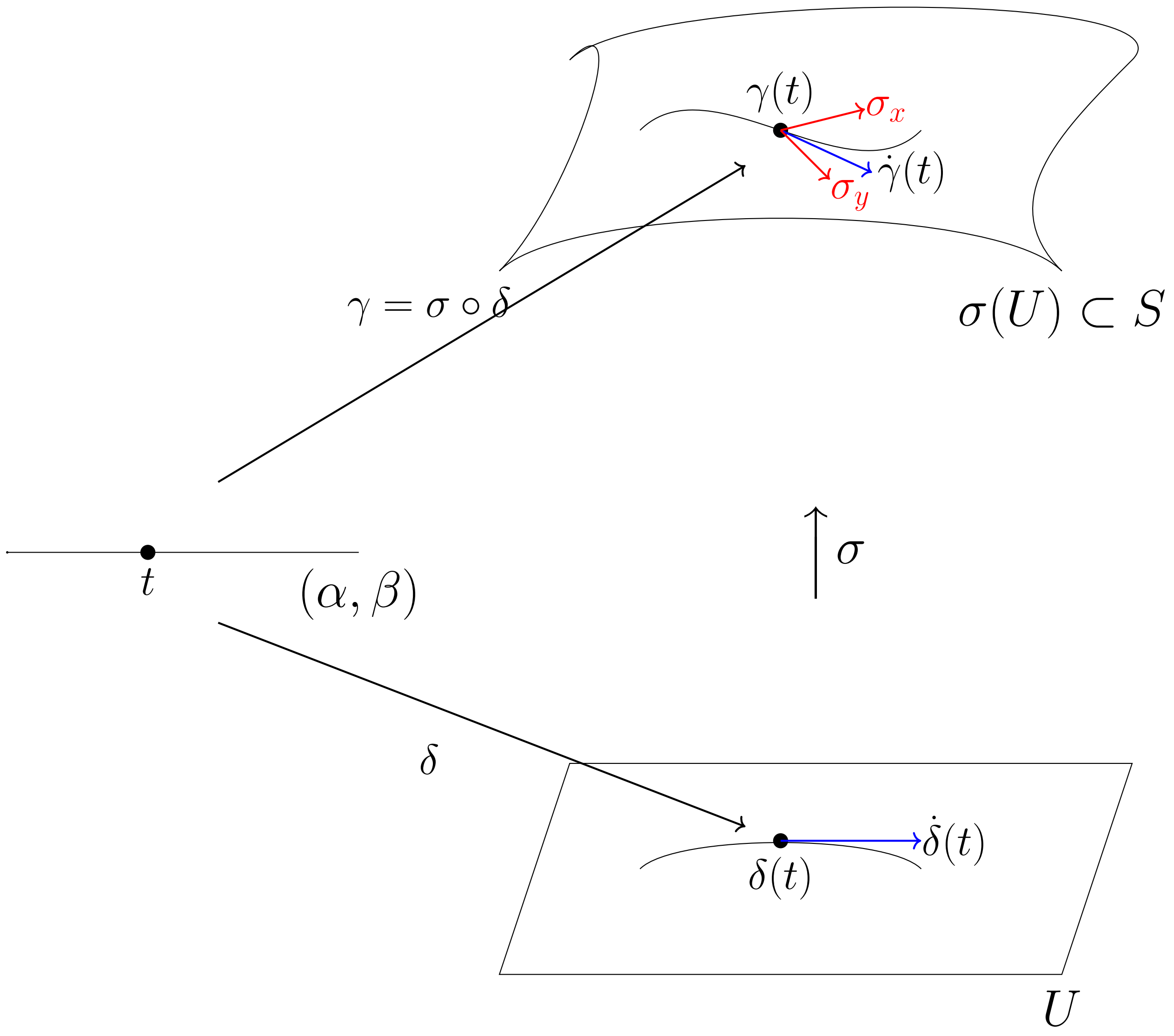

Essentially, $\dot{\gamma}(t)$ can be written in terms of the surface patch, specifically, σ_x and σ_y.

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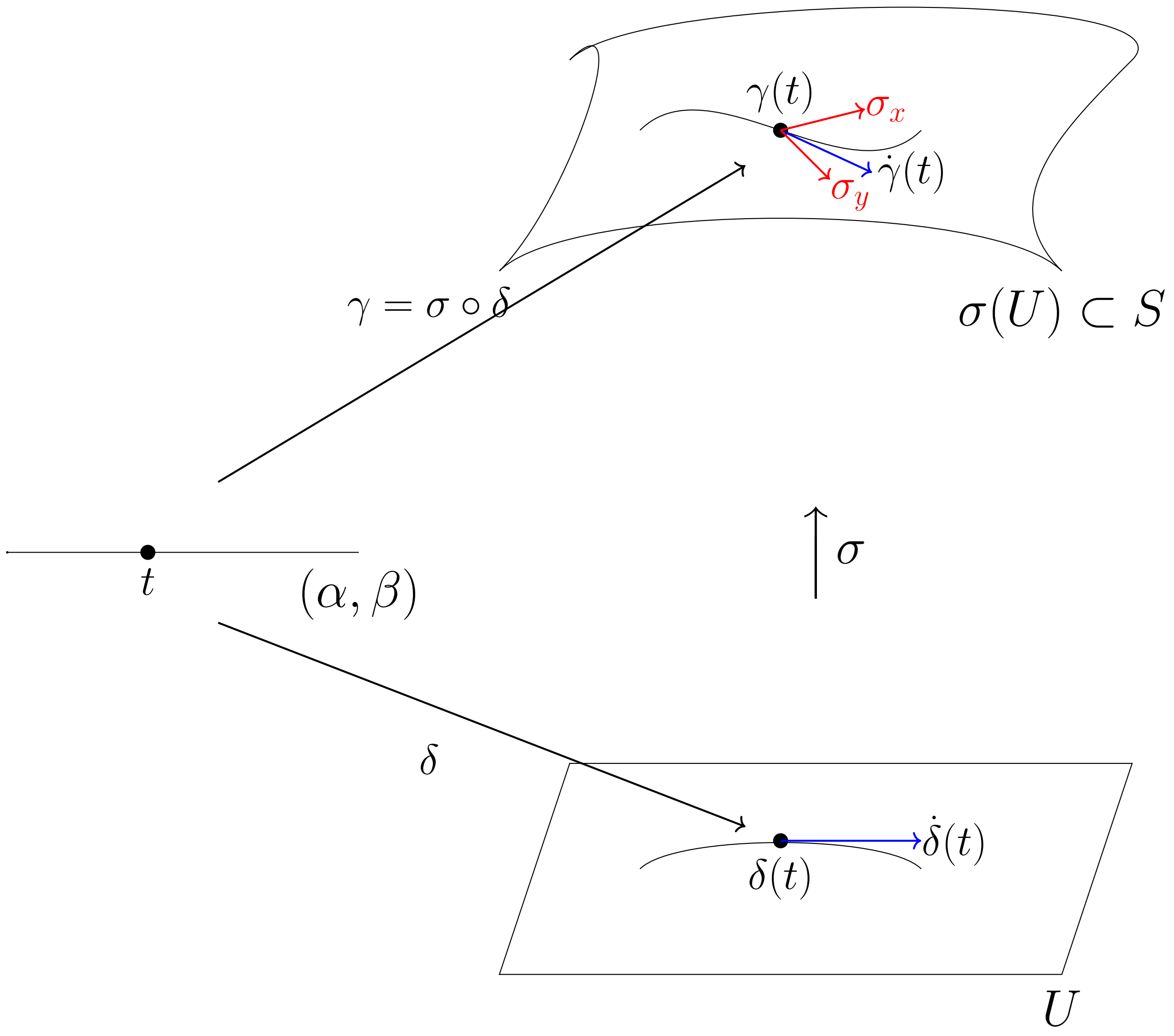

The coefficients come from $\dot{\delta}(t)$

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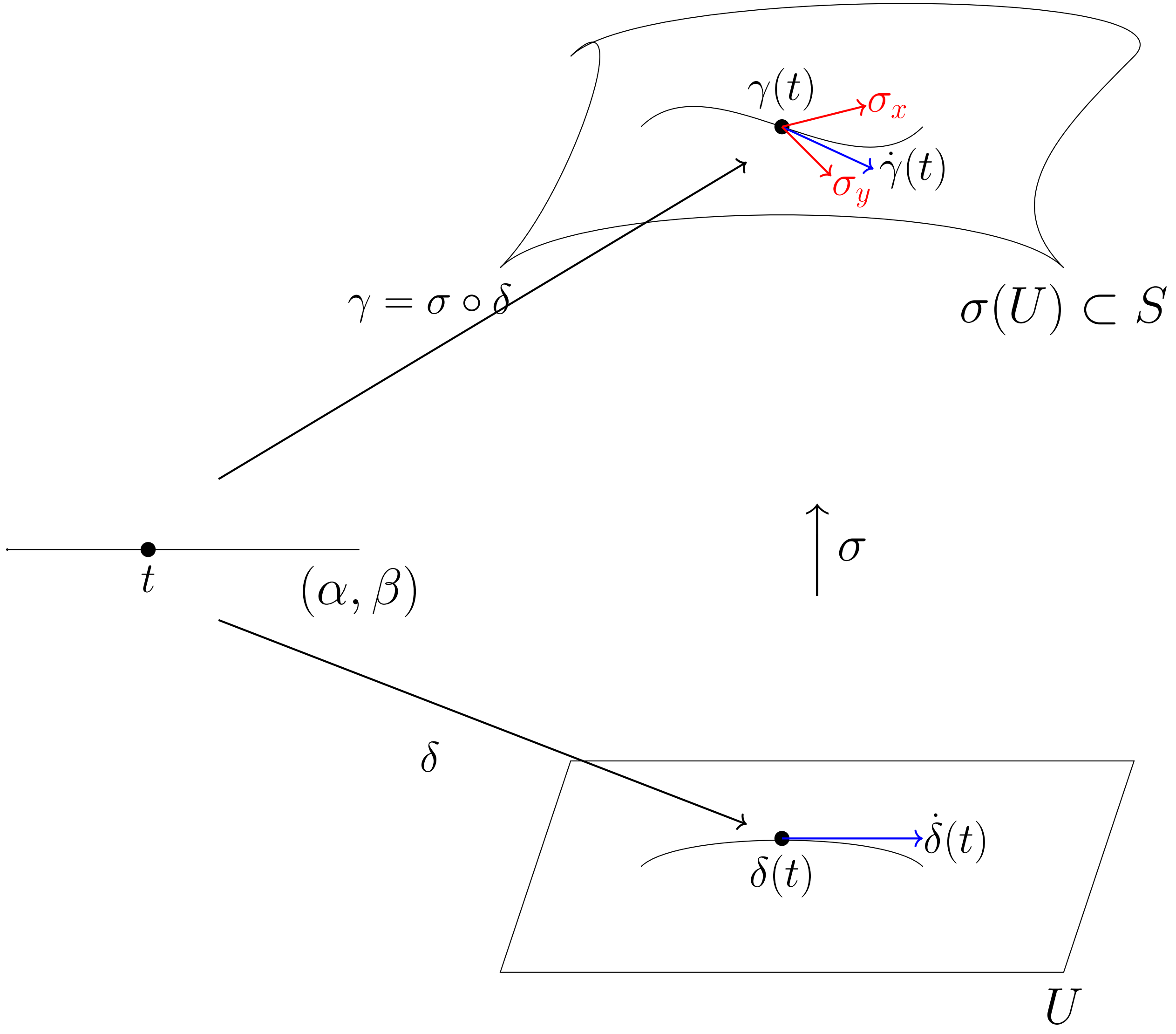

(which is also the coordinates provides by the surface patch).

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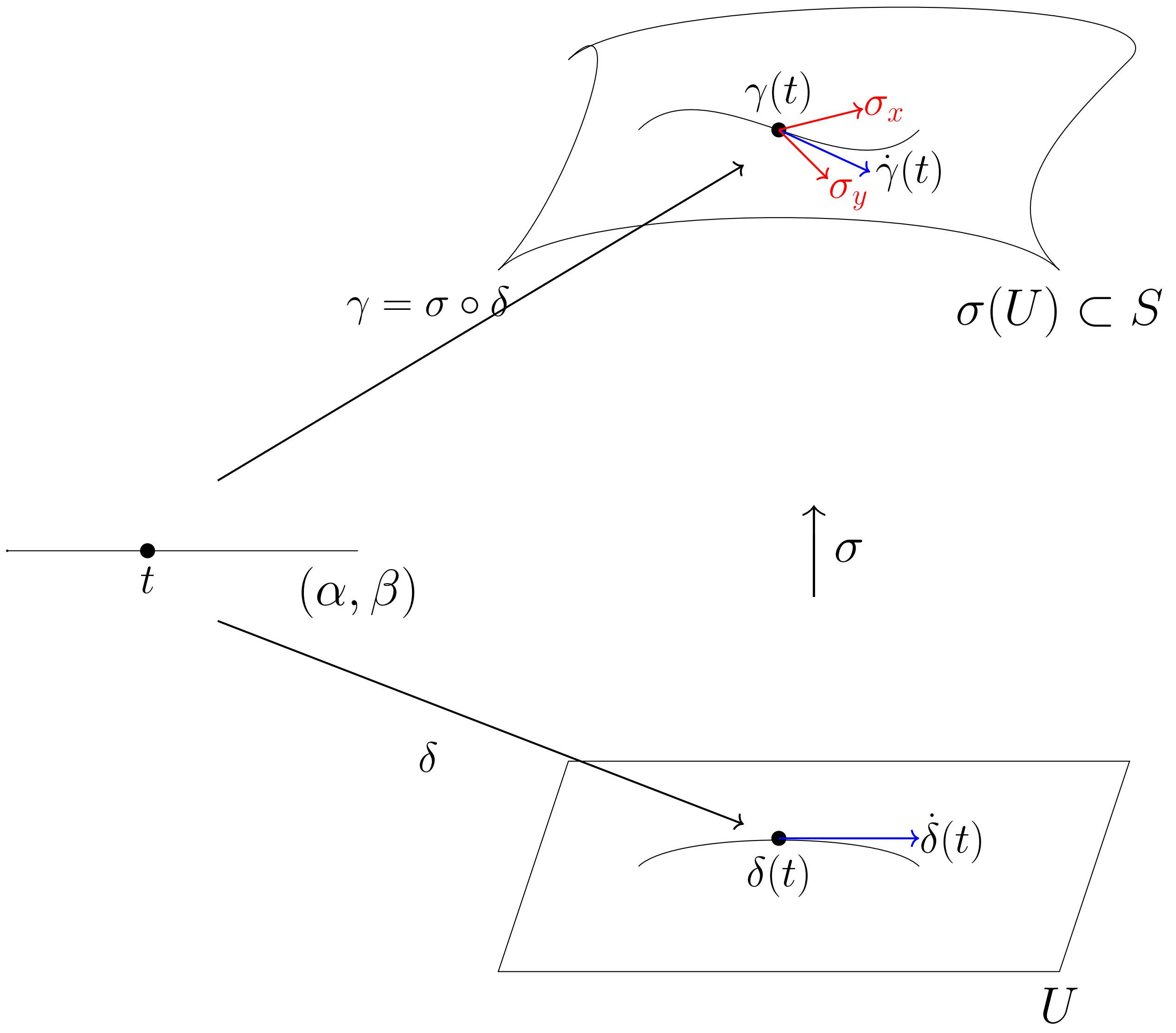

This shows why partial derivatives feature at all

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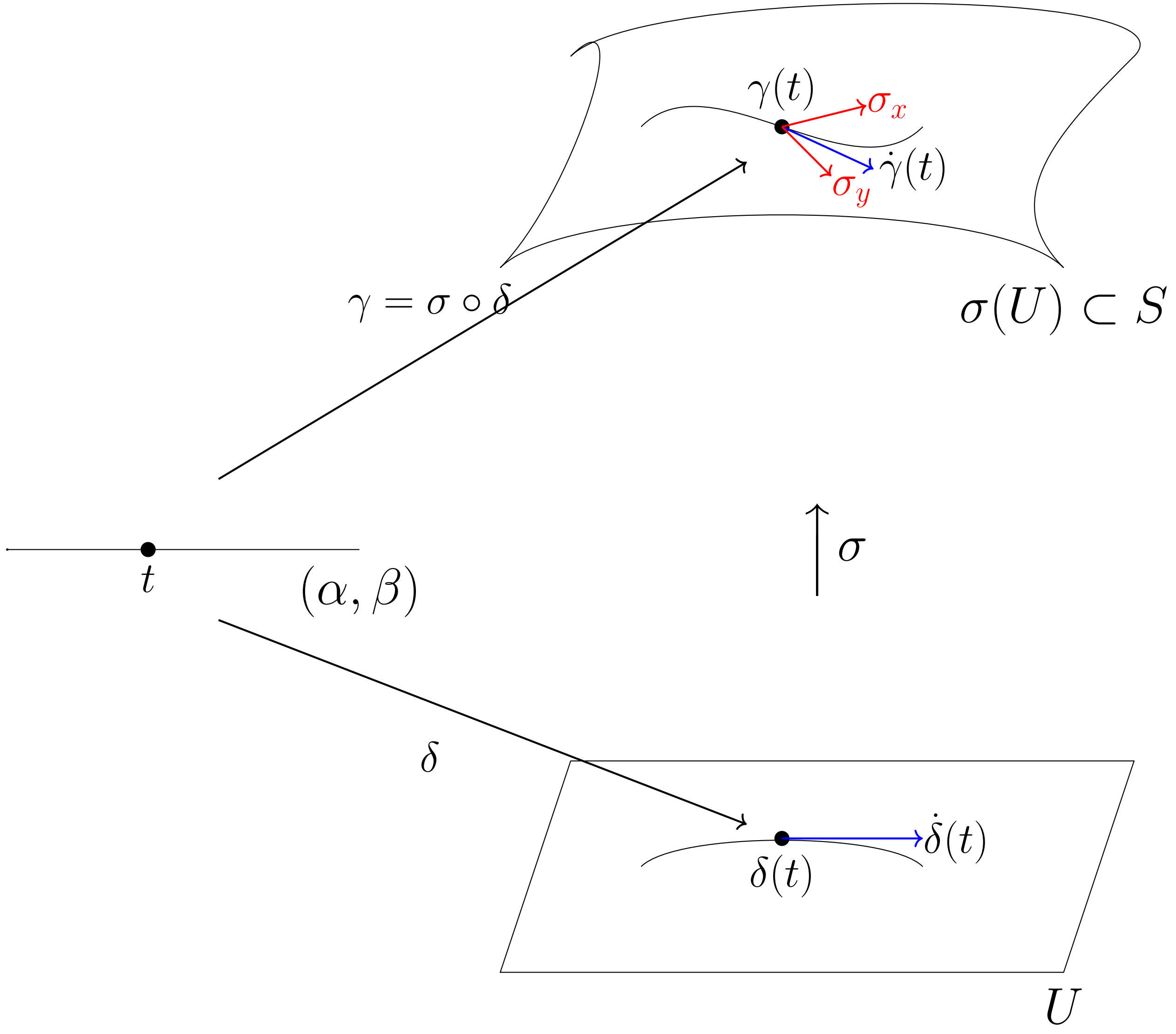

and the importance of regularity...

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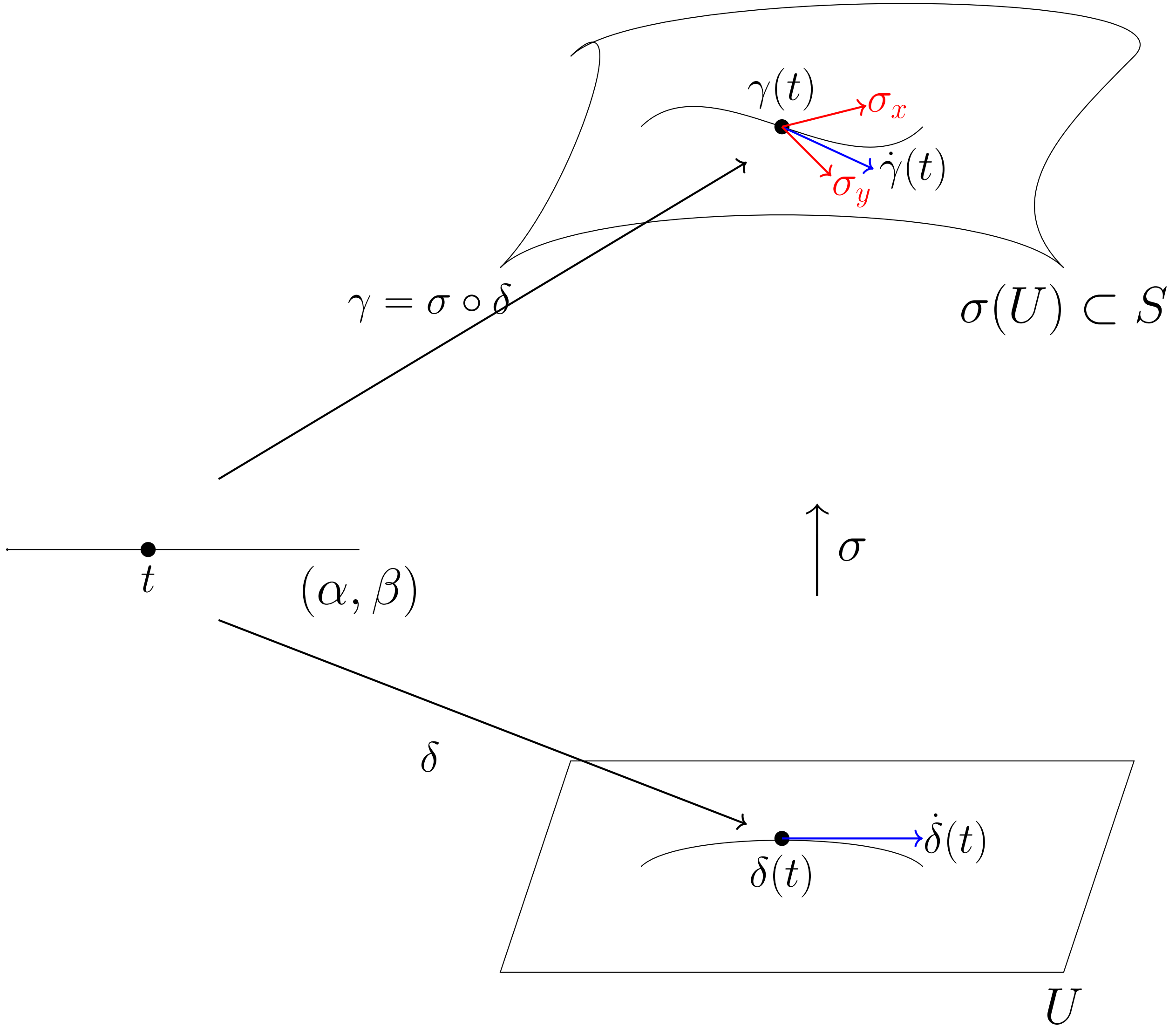

...to ensure σ_x and σ_y are linearly independent