Dicussion before the lecture

Reparametrization example by "shifting the phase"

$\dot{\tilde{\gamma}}(t) = \dot{\gamma}(\phi(t))\phi'(t)$

Dicussion before the lecture

Reparametrization example by "shifting the phase"

$\dot{\tilde{\gamma}}(t) = \dot{\gamma}(\phi(t))\phi'(t)$
$\gamma(t) := (\cos(t), \sin(t))$

Dicussion before the lecture

Reparametrization example by “shifting the phase”

$$\dot{\tilde{\gamma}}(t) = \dot{\gamma}(\phi(t))\phi'(t)$$
$$\gamma(t) := (\cos(t), \sin(t))$$
$$\tilde{\gamma}(t) := (\cos(t + \pi/2), \sin(t + \pi/2))$$

Dicussion before the lecture

Reparametrization example by “shifting the phase”

$$\dot{\tilde{\gamma}}(t) = \dot{\gamma}(\phi(t))\phi'(t)$$
$$\gamma(t) := (\cos(t), \sin(t))$$
$$\tilde{\gamma}(t) := (\cos(t + \pi/2), \sin(t + \pi/2))$$
$$\phi(t) = t + \pi/2$$

Dicussion before the lecture

Reparametrization example by “shifting the phase”

$$\dot{\tilde{\gamma}}(t) = \dot{\gamma}(\phi(t))\phi'(t)$$
$$\gamma(t) := (\cos(t), \sin(t))$$
$$\tilde{\gamma}(t) := (\cos(t + \pi/2), \sin(t + \pi/2))$$
$$\phi(t) = t + \pi/2$$
$$\tilde{\gamma}(t) := \gamma(\phi(t))$$

Regular parametrization

Definition.
The point $\gamma(t)$

Regular parametrization

Definition.
The point $\gamma(t)$ of γ :

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$.

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If ϕ

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ *is invertible*

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ *is invertible with inverse* ψ

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ *is invertible with inverse* $\psi : (\alpha, \beta) \to$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ *is invertible with inverse* $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ *is invertible with inverse* $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$*, then* $\phi'(t) \neq 0$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ *is invertible with inverse* $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$*, then* $\phi'(t) \neq 0$ *for all* $t \in (\tilde{\alpha}, \tilde{\beta})$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ *is invertible with inverse* $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$*, then* $\phi'(t) \neq 0$ *for all* $t \in (\tilde{\alpha}, \tilde{\beta})$ *and* $\psi'(t) \neq 0$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ *is invertible with inverse* $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$*, then* $\phi'(t) \neq 0$ *for all* $t \in (\tilde{\alpha}, \tilde{\beta})$ *and* $\psi'(t) \neq 0$ *for all* $t \in (\alpha, \beta)$*.*

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ is invertible with inverse $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$, then $\phi'(t) \neq 0$ for all $t \in (\tilde{\alpha}, \tilde{\beta})$ and $\psi'(t) \neq 0$ for all $t \in (\alpha, \beta)$.

Proof.
$\psi(\phi(t)) = t$ $\square$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot\gamma(t) \neq 0$ and singular if $\dot\gamma(t) = 0$. The parametrization γ is called regular if $\dot\gamma(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot\gamma(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde\alpha, \tilde\beta) \to (\alpha, \beta)$ is invertible with inverse $\psi : (\alpha, \beta) \to (\tilde\alpha, \tilde\beta)$, then $\phi'(t) \neq 0$ for all $t \in (\tilde\alpha, \tilde\beta)$ and $\psi'(t) \neq 0$ for all $t \in (\alpha, \beta)$.

Proof.
$\psi(\phi(t)) = t$ for *each* t $\square$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ is invertible with inverse $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$, then $\phi'(t) \neq 0$ for all $t \in (\tilde{\alpha}, \tilde{\beta})$ and $\psi'(t) \neq 0$ for all $t \in (\alpha, \beta)$.

Proof.
$\psi(\phi(t)) = t$ for *each* t
$\psi'(\phi(t))\phi'(t) = 1$ $\square$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ is invertible with inverse $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$, then $\phi'(t) \neq 0$ for all $t \in (\tilde{\alpha}, \tilde{\beta})$ and $\psi'(t) \neq 0$ for all $t \in (\alpha, \beta)$.

Proof.
$\psi(\phi(t)) = t$ for *each* t
$\psi'(\phi(t))\phi'(t) = 1$ for *each* t $\square$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ is invertible with inverse $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$, then $\phi'(t) \neq 0$ for all $t \in (\tilde{\alpha}, \tilde{\beta})$ and $\psi'(t) \neq 0$ for all $t \in (\alpha, \beta)$.

Proof.
$\psi(\phi(t)) = t$ for *each* t
$\psi'(\phi(t))\phi'(t) = 1$ for *each* t
So, $\psi'(\phi(t)) \neq 0$ and $\phi'(t) \neq 0$ $\square$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ is invertible with inverse $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$, then $\phi'(t) \neq 0$ for all $t \in (\tilde{\alpha}, \tilde{\beta})$ and $\psi'(t) \neq 0$ for all $t \in (\alpha, \beta)$.

Proof.
$\psi(\phi(t)) = t$ for *each* t
$\psi'(\phi(t))\phi'(t) = 1$ for *each* t
So, $\psi'(\phi(t)) \neq 0$ and $\phi'(t) \neq 0$ for *each* t $\square$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ is invertible with inverse $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$, then $\phi'(t) \neq 0$ for all $t \in (\tilde{\alpha}, \tilde{\beta})$ and $\psi'(t) \neq 0$ for all $t \in (\alpha, \beta)$.

Proof.
$\psi(\phi(t)) = t$ for *each* t
$\psi'(\phi(t))\phi'(t) = 1$ for *each* t
So, $\psi'(\phi(t)) \neq 0$ and $\phi'(t) \neq 0$ for *each* t
So, $\psi'(t) \neq 0$ and $\phi'(t) \neq 0$ $\square$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ is invertible with inverse $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$, then $\phi'(t) \neq 0$ for all $t \in (\tilde{\alpha}, \tilde{\beta})$ and $\psi'(t) \neq 0$ for all $t \in (\alpha, \beta)$.

Proof.
$\psi(\phi(t)) = t$ for *each* t
$\psi'(\phi(t))\phi'(t) = 1$ for *each* t
So, $\psi'(\phi(t)) \neq 0$ and $\phi'(t) \neq 0$ for *each* t
So, $\psi'(t) \neq 0$ and $\phi'(t) \neq 0$ for *each* t $\square$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ is invertible with inverse $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$, then $\phi'(t) \neq 0$ for all $t \in (\tilde{\alpha}, \tilde{\beta})$ and $\psi'(t) \neq 0$ for all $t \in (\alpha, \beta)$.

Proof.
$\psi(\phi(t)) = t$ for *each* t
$\psi'(\phi(t))\phi'(t) = 1$ for *each* t
So, $\psi'(\phi(t)) \neq 0$ and $\phi'(t) \neq 0$ for *each* t
So, $\psi'(t) \neq 0$ and $\phi'(t) \neq 0$ for *each* t because ϕ is bijective. $\square$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ is invertible with inverse $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$, then $\phi'(t) \neq 0$ for all $t \in (\tilde{\alpha}, \tilde{\beta})$ and $\psi'(t) \neq 0$ for all $t \in (\alpha, \beta)$.

Proof.
$\psi(\phi(t)) = t$ for *each* t
$\psi'(\phi(t))\phi'(t) = 1$ for *each* t
So, $\psi'(\phi(t)) \neq 0$ and $\phi'(t) \neq 0$ for *each* t
So, $\psi'(t) \neq 0$ and $\phi'(t) \neq 0$ for *each* t because ϕ is bijective. $\square$

Proposition. *A reparametrization of a regular parametrization*

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ *is invertible with inverse* $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$*, then* $\phi'(t) \neq 0$ *for all* $t \in (\tilde{\alpha}, \tilde{\beta})$ *and* $\psi'(t) \neq 0$ *for all* $t \in (\alpha, \beta)$*.*

Proof.
$\psi(\phi(t)) = t$ for *each* t
$\psi'(\phi(t))\phi'(t) = 1$ for *each* t
So, $\psi'(\phi(t)) \neq 0$ and $\phi'(t) \neq 0$ for *each* t
So, $\psi'(t) \neq 0$ and $\phi'(t) \neq 0$ for *each* t because ϕ is bijective. □

Proposition. *A reparametrization of a regular parametrization is regular.*

Proof.
$\gamma :$ □

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ is invertible with inverse $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$, then $\phi'(t) \neq 0$ for all $t \in (\tilde{\alpha}, \tilde{\beta})$ and $\psi'(t) \neq 0$ for all $t \in (\alpha, \beta)$.

Proof.
$\psi(\phi(t)) = t$ for *each* t
$\psi'(\phi(t))\phi'(t) = 1$ for *each* t
So, $\psi'(\phi(t)) \neq 0$ and $\phi'(t) \neq 0$ for *each* t
So, $\psi'(t) \neq 0$ and $\phi'(t) \neq 0$ for *each* t because ϕ is bijective. $\square$

Proposition. *A reparametrization of a regular parametrization is regular.*

Proof.
$\gamma : (\alpha, \beta) \to$ $\square$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ is invertible with inverse $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$, then $\phi'(t) \neq 0$ for all $t \in (\tilde{\alpha}, \tilde{\beta})$ and $\psi'(t) \neq 0$ for all $t \in (\alpha, \beta)$.

Proof.
$\psi(\phi(t)) = t$ for *each* t
$\psi'(\phi(t))\phi'(t) = 1$ for *each* t
So, $\psi'(\phi(t)) \neq 0$ and $\phi'(t) \neq 0$ for *each* t
So, $\psi'(t) \neq 0$ and $\phi'(t) \neq 0$ for *each* t because ϕ is bijective. $\square$

Proposition. *A reparametrization of a regular parametrization is regular.*

Proof.
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$ $\square$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ is invertible with inverse $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$, then $\phi'(t) \neq 0$ for all $t \in (\tilde{\alpha}, \tilde{\beta})$ and $\psi'(t) \neq 0$ for all $t \in (\alpha, \beta)$.

Proof.
$\psi(\phi(t)) = t$ for *each* t
$\psi'(\phi(t))\phi'(t) = 1$ for *each* t
So, $\psi'(\phi(t)) \neq 0$ and $\phi'(t) \neq 0$ for *each* t
So, $\psi'(t) \neq 0$ and $\phi'(t) \neq 0$ for *each* t because ϕ is bijective. □

Proposition. *A reparametrization of a regular parametrization is regular.*

Proof.
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\tilde{\gamma} :$ □

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ is invertible with inverse $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$, then $\phi'(t) \neq 0$ for all $t \in (\tilde{\alpha}, \tilde{\beta})$ and $\psi'(t) \neq 0$ for all $t \in (\alpha, \beta)$.

Proof.
$\psi(\phi(t)) = t$ for *each* t
$\psi'(\phi(t))\phi'(t) = 1$ for *each* t
So, $\psi'(\phi(t)) \neq 0$ and $\phi'(t) \neq 0$ for *each* t
So, $\psi'(t) \neq 0$ and $\phi'(t) \neq 0$ for *each* t because ϕ is bijective. $\square$

Proposition. *A reparametrization of a regular parametrization is regular.*

Proof.
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to$ $\square$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ is invertible with inverse $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$, then $\phi'(t) \neq 0$ for all $t \in (\tilde{\alpha}, \tilde{\beta})$ and $\psi'(t) \neq 0$ for all $t \in (\alpha, \beta)$.

Proof.
$\psi(\phi(t)) = t$ for *each* t
$\psi'(\phi(t))\phi'(t) = 1$ for *each* t
So, $\psi'(\phi(t)) \neq 0$ and $\phi'(t) \neq 0$ for *each* t
So, $\psi'(t) \neq 0$ and $\phi'(t) \neq 0$ for *each* t because ϕ is bijective. □

Proposition. *A reparametrization of a regular parametrization is regular.*

Proof.
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to \mathbb{R}^2$ □

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ *is invertible with inverse* $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$*, then* $\phi'(t) \neq 0$ *for all* $t \in (\tilde{\alpha}, \tilde{\beta})$ *and* $\psi'(t) \neq 0$ *for all* $t \in (\alpha, \beta)$*.*

Proof.
$\psi(\phi(t)) = t$ for *each* t
$\psi'(\phi(t))\phi'(t) = 1$ for *each* t
So, $\psi'(\phi(t)) \neq 0$ and $\phi'(t) \neq 0$ for *each* t
So, $\psi'(t) \neq 0$ and $\phi'(t) \neq 0$ for *each* t because ϕ is bijective. □

Proposition. *A reparametrization of a regular parametrization is regular.*

Proof.
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to \mathbb{R}^2$
$\tilde{\gamma}(\tilde{t})$ □

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ is invertible with inverse $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$, then $\phi'(t) \neq 0$ for all $t \in (\tilde{\alpha}, \tilde{\beta})$ and $\psi'(t) \neq 0$ for all $t \in (\alpha, \beta)$.

Proof.
$\psi(\phi(t)) = t$ for *each* t
$\psi'(\phi(t))\phi'(t) = 1$ for *each* t
So, $\psi'(\phi(t)) \neq 0$ and $\phi'(t) \neq 0$ for *each* t
So, $\psi'(t) \neq 0$ and $\phi'(t) \neq 0$ for *each* t because ϕ is bijective. $\square$

Proposition. *A reparametrization of a regular parametrization is regular.*

Proof.
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to \mathbb{R}^2$
$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$ $\square$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ is invertible with inverse $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$, then $\phi'(t) \neq 0$ for all $t \in (\tilde{\alpha}, \tilde{\beta})$ and $\psi'(t) \neq 0$ for all $t \in (\alpha, \beta)$.

Proof.
$\psi(\phi(t)) = t$ for *each* t
$\psi'(\phi(t))\phi'(t) = 1$ for *each* t
So, $\psi'(\phi(t)) \neq 0$ and $\phi'(t) \neq 0$ for *each* t
So, $\psi'(t) \neq 0$ and $\phi'(t) \neq 0$ for *each* t because ϕ is bijective. $\square$

Proposition. *A reparametrization of a regular parametrization is regular.*

Proof.
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to \mathbb{R}^2$
$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$
$\tilde{\gamma}'(\tilde{t})$ $\square$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ is invertible with inverse $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$, then $\phi'(t) \neq 0$ for all $t \in (\tilde{\alpha}, \tilde{\beta})$ and $\psi'(t) \neq 0$ for all $t \in (\alpha, \beta)$.

Proof.
$\psi(\phi(t)) = t$ for *each* t
$\psi'(\phi(t))\phi'(t) = 1$ for *each* t
So, $\psi'(\phi(t)) \neq 0$ and $\phi'(t) \neq 0$ for *each* t
So, $\psi'(t) \neq 0$ and $\phi'(t) \neq 0$ for *each* t because ϕ is bijective. $\square$

Proposition. *A reparametrization of a regular parametrization is regular.*

Proof.
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to \mathbb{R}^2$
$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$
$\tilde{\gamma}'(\tilde{t}) = \gamma'(\phi(\tilde{t}))\phi'(\tilde{t})$ $\square$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ is invertible with inverse $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$, then $\phi'(t) \neq 0$ for all $t \in (\tilde{\alpha}, \tilde{\beta})$ and $\psi'(t) \neq 0$ for all $t \in (\alpha, \beta)$.

Proof.
$\psi(\phi(t)) = t$ for *each* t
$\psi'(\phi(t))\phi'(t) = 1$ for *each* t
So, $\psi'(\phi(t)) \neq 0$ and $\phi'(t) \neq 0$ for *each* t
So, $\psi'(t) \neq 0$ and $\phi'(t) \neq 0$ for *each* t because ϕ is bijective. □

Proposition. *A reparametrization of a regular parametrization is regular.*

Proof.
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to \mathbb{R}^2$
$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$
$\tilde{\gamma}'(\tilde{t}) = \gamma'(\phi(\tilde{t}))\phi'(\tilde{t})$
But, $\gamma'(t) \neq 0$ □

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ is invertible with inverse $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$, then $\phi'(t) \neq 0$ for all $t \in (\tilde{\alpha}, \tilde{\beta})$ and $\psi'(t) \neq 0$ for all $t \in (\alpha, \beta)$.

Proof.
$\psi(\phi(t)) = t$ for *each* t
$\psi'(\phi(t))\phi'(t) = 1$ for *each* t
So, $\psi'(\phi(t)) \neq 0$ and $\phi'(t) \neq 0$ for *each* t
So, $\psi'(t) \neq 0$ and $\phi'(t) \neq 0$ for *each* t because ϕ is bijective. □

Proposition. *A reparametrization of a regular parametrization is regular.*

Proof.
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to \mathbb{R}^2$
$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$
$\tilde{\gamma}'(\tilde{t}) = \gamma'(\phi(\tilde{t}))\phi'(\tilde{t})$
But, $\gamma'(t) \neq 0$ for all t, □

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ is invertible with inverse $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$, then $\phi'(t) \neq 0$ for all $t \in (\tilde{\alpha}, \tilde{\beta})$ and $\psi'(t) \neq 0$ for all $t \in (\alpha, \beta)$.

Proof.
$\psi(\phi(t)) = t$ for *each* t
$\psi'(\phi(t))\phi'(t) = 1$ for *each* t
So, $\psi'(\phi(t)) \neq 0$ and $\phi'(t) \neq 0$ for *each* t
So, $\psi'(t) \neq 0$ and $\phi'(t) \neq 0$ for *each* t because ϕ is bijective. $\square$

Proposition. *A reparametrization of a regular parametrization is regular.*

Proof.
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to \mathbb{R}^2$
$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$
$\tilde{\gamma}'(\tilde{t}) = \gamma'(\phi(\tilde{t}))\phi'(\tilde{t})$
But, $\gamma'(t) \neq 0$ for all t, therefore, even for $\phi(\tilde{t})$ $\square$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ is invertible with inverse $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$, then $\phi'(t) \neq 0$ for all $t \in (\tilde{\alpha}, \tilde{\beta})$ and $\psi'(t) \neq 0$ for all $t \in (\alpha, \beta)$.

Proof.
$\psi(\phi(t)) = t$ for *each* t
$\psi'(\phi(t))\phi'(t) = 1$ for *each* t
So, $\psi'(\phi(t)) \neq 0$ and $\phi'(t) \neq 0$ for *each* t
So, $\psi'(t) \neq 0$ and $\phi'(t) \neq 0$ for *each* t because ϕ is bijective. $\square$

Proposition. *A reparametrization of a regular parametrization is regular.*

Proof.
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to \mathbb{R}^2$
$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$
$\tilde{\gamma}'(\tilde{t}) = \gamma'(\phi(\tilde{t}))\phi'(\tilde{t})$
But, $\gamma'(t) \neq 0$ for all t, therefore, even for $\phi(\tilde{t})$
and, $\phi'(\tilde{t}) \neq 0$ $\square$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ is invertible with inverse $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$, then $\phi'(t) \neq 0$ for all $t \in (\tilde{\alpha}, \tilde{\beta})$ and $\psi'(t) \neq 0$ for all $t \in (\alpha, \beta)$.

Proof.
$\psi(\phi(t)) = t$ for *each* t
$\psi'(\phi(t))\phi'(t) = 1$ for *each* t
So, $\psi'(\phi(t)) \neq 0$ and $\phi'(t) \neq 0$ for *each* t
So, $\psi'(t) \neq 0$ and $\phi'(t) \neq 0$ for *each* t because ϕ is bijective. □

Proposition. *A reparametrization of a regular parametrization is regular.*

Proof.
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to \mathbb{R}^2$
$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$
$\tilde{\gamma}'(\tilde{t}) = \gamma'(\phi(\tilde{t}))\phi'(\tilde{t})$
But, $\gamma'(t) \neq 0$ for all t, therefore, even for $\phi(\tilde{t})$
and, $\phi'(\tilde{t}) \neq 0$ for all $\tilde{t}$ □

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ is invertible with inverse $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$, then $\phi'(t) \neq 0$ for all $t \in (\tilde{\alpha}, \tilde{\beta})$ and $\psi'(t) \neq 0$ for all $t \in (\alpha, \beta)$.

Proof.
$\psi(\phi(t)) = t$ for *each* t
$\psi'(\phi(t))\phi'(t) = 1$ for *each* t
So, $\psi'(\phi(t)) \neq 0$ and $\phi'(t) \neq 0$ for *each* t
So, $\psi'(t) \neq 0$ and $\phi'(t) \neq 0$ for *each* t because ϕ is bijective. $\square$

Proposition. *A reparametrization of a regular parametrization is regular.*

Proof.
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to \mathbb{R}^2$
$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$
$\tilde{\gamma}'(\tilde{t}) = \gamma'(\phi(\tilde{t}))\phi'(\tilde{t})$
But, $\gamma'(t) \neq 0$ for all t, therefore, even for $\phi(\tilde{t})$
and, $\phi'(\tilde{t}) \neq 0$ for all $\tilde{t}$

$\square$

Inner product:

Inner product:
$v = (2, 3)$

Inner product:
$v = (2, 3)$
$w = (2, 1)$

Inner product:
$v = (2, 3)$
$w = (2, 1)$
$v.w$

Inner product:
$v = (2, 3)$
$w = (2, 1)$
$v.w := (2, 3).(2, 1)$

Inner product:
$v = (2, 3)$
$w = (2, 1)$
$v.w := (2, 3).(2, 1) = 2 \times 2 + 3 \times 1$

Inner product:

$v = (2, 3)$

$w = (2, 1)$

$v.w := (2, 3).(2, 1) = 2 \times 2 + 3 \times 1 = 7$

Inner product:
$v = (2, 3)$
$w = (2, 1)$
$v.w := (2, 3).(2, 1) = 2 \times 2 + 3 \times 1 = 7$
In general:

Inner product:
$v = (2, 3)$
$w = (2, 1)$
$v.w := (2, 3).(2, 1) = 2 \times 2 + 3 \times 1 = 7$
In general:
$(x_1, y_1).(x_2, y_2) := x_1 x_2 + y_1 y_2$

Inner product:
$v = (2, 3)$
$w = (2, 1)$
$v.w := (2, 3).(2, 1) = 2 \times 2 + 3 \times 1 = 7$
In general:
$(x_1, y_1).(x_2, y_2) := x_1 x_2 + y_1 y_2$

Exercise. For $\mathbf{v} : (\alpha, \beta) \to \mathbf{R}^2$

Inner product:
$v = (2, 3)$
$w = (2, 1)$
$v.w := (2, 3).(2, 1) = 2 \times 2 + 3 \times 1 = 7$
In general:
$(x_1, y_1).(x_2, y_2) := x_1 x_2 + y_1 y_2$

Exercise. For $\mathbf{v} : (\alpha, \beta) \to \mathbf{R}^2$ and $\mathbf{w} : (\alpha, \beta) \to \mathbf{R}^2$,

Inner product:
$v = (2, 3)$
$w = (2, 1)$
$v.w := (2, 3).(2, 1) = 2 \times 2 + 3 \times 1 = 7$
In general:
$(x_1, y_1).(x_2, y_2) := x_1 x_2 + y_1 y_2$

Exercise. For $\mathbf{v} : (\alpha, \beta) \to \mathbf{R}^2$ and $\mathbf{w} : (\alpha, \beta) \to \mathbf{R}^2$,
show that $(\mathbf{v}(t).\mathbf{w}(t))' = \mathbf{v}'(t).\mathbf{w}(t) + \mathbf{v}(t).\mathbf{w}'(t)$

Inner product:
$v = (2, 3)$
$w = (2, 1)$
$v.w := (2, 3).(2, 1) = 2 \times 2 + 3 \times 1 = 7$
In general:
$(x_1, y_1).(x_2, y_2) := x_1 x_2 + y_1 y_2$

Exercise. For $\mathbf{v} : (\alpha, \beta) \to \mathbf{R}^2$ and $\mathbf{w} : (\alpha, \beta) \to \mathbf{R}^2$,
show that $(\mathbf{v}(t).\mathbf{w}(t))' = \mathbf{v}'(t).\mathbf{w}(t) + \mathbf{v}(t).\mathbf{w}'(t)$

$v = (x, y)$

Inner product:
$v = (2,3)$
$w = (2,1)$
$v.w := (2,3).(2,1) = 2 \times 2 + 3 \times 1 = 7$
In general:
$(x_1, y_1).(x_2, y_2) := x_1 x_2 + y_1 y_2$

Exercise. For $\mathbf{v} : (\alpha, \beta) \to \mathbf{R}^2$ and $\mathbf{w} : (\alpha, \beta) \to \mathbf{R}^2$,
show that $(\mathbf{v}(t).\mathbf{w}(t))' = \mathbf{v}'(t).\mathbf{w}(t) + \mathbf{v}(t).\mathbf{w}'(t)$

$v = (x, y)$
$v.v$

Inner product:
$v = (2,3)$
$w = (2,1)$
$v.w := (2,3).(2,1) = 2 \times 2 + 3 \times 1 = 7$
In general:
$(x_1, y_1).(x_2, y_2) := x_1 x_2 + y_1 y_2$

Exercise. For $\mathbf{v} : (\alpha, \beta) \to \mathbf{R}^2$ and $\mathbf{w} : (\alpha, \beta) \to \mathbf{R}^2$,
show that $(\mathbf{v}(t).\mathbf{w}(t))' = \mathbf{v}'(t).\mathbf{w}(t) + \mathbf{v}(t).\mathbf{w}'(t)$

$v = (x, y)$
$v.v = (x, y).(x, y) = x^2 + y^2$

Inner product:
$v = (2,3)$
$w = (2,1)$
$v.w := (2,3).(2,1) = 2 \times 2 + 3 \times 1 = 7$
In general:
$(x_1, y_1).(x_2, y_2) := x_1 x_2 + y_1 y_2$

Exercise. For $\mathbf{v} : (\alpha, \beta) \to \mathbf{R}^2$ and $\mathbf{w} : (\alpha, \beta) \to \mathbf{R}^2$, show that $(\mathbf{v}(t).\mathbf{w}(t))' = \mathbf{v}'(t).\mathbf{w}(t) + \mathbf{v}(t).\mathbf{w}'(t)$

$v = (x, y)$
$v.v = (x,y).(x,y) = x^2 + y^2$
$||(x,y)|| = \sqrt{x^2 + y^2}$

Inner product:
$v = (2, 3)$
$w = (2, 1)$
$v.w := (2, 3).(2, 1) = 2 \times 2 + 3 \times 1 = 7$
In general:
$(x_1, y_1).(x_2, y_2) := x_1x_2 + y_1y_2$

Exercise. For $\mathbf{v} : (\alpha, \beta) \to \mathbf{R}^2$ and $\mathbf{w} : (\alpha, \beta) \to \mathbf{R}^2$,
show that $(\mathbf{v}(t).\mathbf{w}(t))' = \mathbf{v}'(t).\mathbf{w}(t) + \mathbf{v}(t).\mathbf{w}'(t)$

$v = (x, y)$
$v.v = (x, y).(x, y) = x^2 + y^2$
$||(x, y)|| = \sqrt{x^2 + y^2}$

$v = (x, y)$

Inner product:
$v = (2, 3)$
$w = (2, 1)$
$v.w := (2, 3).(2, 1) = 2 \times 2 + 3 \times 1 = 7$
In general:
$(x_1, y_1).(x_2, y_2) := x_1 x_2 + y_1 y_2$

Exercise. For $\mathbf{v} : (\alpha, \beta) \to \mathbf{R}^2$ and $\mathbf{w} : (\alpha, \beta) \to \mathbf{R}^2$, show that $(\mathbf{v}(t).\mathbf{w}(t))' = \mathbf{v}'(t).\mathbf{w}(t) + \mathbf{v}(t).\mathbf{w}'(t)$

$v = (x, y)$
$v.v = (x, y).(x, y) = x^2 + y^2$
$||(x, y)|| = \sqrt{x^2 + y^2}$

$v = (x, y)$
$v.w =$

Inner product:
$v = (2, 3)$
$w = (2, 1)$
$v.w := (2, 3).(2, 1) = 2 \times 2 + 3 \times 1 = 7$
In general:
$(x_1, y_1).(x_2, y_2) := x_1 x_2 + y_1 y_2$

Exercise. For $\mathbf{v} : (\alpha, \beta) \to \mathbf{R}^2$ and $\mathbf{w} : (\alpha, \beta) \to \mathbf{R}^2$,
show that $(\mathbf{v}(t).\mathbf{w}(t))' = \mathbf{v}'(t).\mathbf{w}(t) + \mathbf{v}(t).\mathbf{w}'(t)$

$v = (x, y)$
$v.v = (x, y).(x, y) = x^2 + y^2$
$||(x, y)|| = \sqrt{x^2 + y^2}$

$v = (x, y)$
$v.w = ||v||||w|| \cos(\theta)$ where,
θ is the angle between v and w

Inner product:
$v = (2,3)$
$w = (2,1)$
$v.w := (2,3).(2,1) = 2 \times 2 + 3 \times 1 = 7$
In general:
$(x_1, y_1).(x_2, y_2) := x_1 x_2 + y_1 y_2$

Exercise. For $\mathbf{v} : (\alpha, \beta) \to \mathbf{R}^2$ and $\mathbf{w} : (\alpha, \beta) \to \mathbf{R}^2$, show that $(\mathbf{v}(t).\mathbf{w}(t))' = \mathbf{v}'(t).\mathbf{w}(t) + \mathbf{v}(t).\mathbf{w}'(t)$

$v = (x, y)$
$v.v = (x,y).(x,y) = x^2 + y^2$
$||(x,y)|| = \sqrt{x^2 + y^2}$

$v = (x, y)$
$v.w = ||v||||w|| \cos(\theta)$ where,
θ is the angle between v and w

Definition. $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a unit speed parametrization

Inner product:
$v = (2, 3)$
$w = (2, 1)$
$v.w := (2, 3).(2, 1) = 2 \times 2 + 3 \times 1 = 7$
In general:
$(x_1, y_1).(x_2, y_2) := x_1x_2 + y_1y_2$

Exercise. For $\mathbf{v} : (\alpha, \beta) \to \mathbf{R}^2$ and $\mathbf{w} : (\alpha, \beta) \to \mathbf{R}^2$, show that $(\mathbf{v}(t).\mathbf{w}(t))' = \mathbf{v}'(t).\mathbf{w}(t) + \mathbf{v}(t).\mathbf{w}'(t)$

$v = (x, y)$
$v.v = (x, y).(x, y) = x^2 + y^2$
$||(x, y)|| = \sqrt{x^2 + y^2}$

$v = (x, y)$
$v.w = ||v||||w|| \cos(\theta)$ where,
θ is the angle between v and w

Definition. $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a unit speed parametrization if $||\dot{\gamma}(t)|| = 1$

Inner product:
$v = (2, 3)$
$w = (2, 1)$
$v.w := (2, 3).(2, 1) = 2 \times 2 + 3 \times 1 = 7$
In general:
$(x_1, y_1).(x_2, y_2) := x_1x_2 + y_1y_2$

Exercise. For $\mathbf{v} : (\alpha, \beta) \to \mathbf{R}^2$ and $\mathbf{w} : (\alpha, \beta) \to \mathbf{R}^2$, show that $(\mathbf{v}(t).\mathbf{w}(t))' = \mathbf{v}'(t).\mathbf{w}(t) + \mathbf{v}(t).\mathbf{w}'(t)$

$v = (x, y)$
$v.v = (x, y).(x, y) = x^2 + y^2$
$||(x, y)|| = \sqrt{x^2 + y^2}$

$v = (x, y)$
$v.w = ||v||||w|| \cos(\theta)$ where,
θ is the angle between v and w

Definition. $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a unit speed parametrization if $||\dot{\gamma}(t)|| = 1$ for each $t \in (\alpha, \beta)$

Inner product:
$v = (2,3)$
$w = (2,1)$
$v.w := (2,3).(2,1) = 2 \times 2 + 3 \times 1 = 7$
In general:
$(x_1, y_1).(x_2, y_2) := x_1x_2 + y_1y_2$

Exercise. For $\mathbf{v} : (\alpha, \beta) \to \mathbf{R}^2$ and $\mathbf{w} : (\alpha, \beta) \to \mathbf{R}^2$, show that $(\mathbf{v}(t).\mathbf{w}(t))' = \mathbf{v}'(t).\mathbf{w}(t) + \mathbf{v}(t).\mathbf{w}'(t)$

$v = (x, y)$
$v.v = (x,y).(x,y) = x^2 + y^2$
$||(x,y)|| = \sqrt{x^2 + y^2}$

$v = (x, y)$
$v.w = ||v||||w|| \cos(\theta)$ where,
θ is the angle between v and w

Definition. $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a unit speed parametrization if $||\dot{\gamma}(t)|| = 1$ for each $t \in (\alpha, \beta)$

Theorem. *If $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a unit speed parametrization*

Inner product:
$v = (2, 3)$
$w = (2, 1)$
$v.w := (2, 3).(2, 1) = 2 \times 2 + 3 \times 1 = 7$
In general:
$(x_1, y_1).(x_2, y_2) := x_1x_2 + y_1y_2$

Exercise. For $\mathbf{v} : (\alpha, \beta) \to \mathbf{R}^2$ and $\mathbf{w} : (\alpha, \beta) \to \mathbf{R}^2$, show that $(\mathbf{v}(t).\mathbf{w}(t))' = \mathbf{v}'(t).\mathbf{w}(t) + \mathbf{v}(t).\mathbf{w}'(t)$

$v = (x, y)$
$v.v = (x, y).(x, y) = x^2 + y^2$
$||(x, y)|| = \sqrt{x^2 + y^2}$

$v = (x, y)$
$v.w = ||v||||w|| \cos(\theta)$ where,
θ is the angle between v and w

Definition. $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a unit speed parametrization if $||\dot{\gamma}(t)|| = 1$ for each $t \in (\alpha, \beta)$

Theorem. *If $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a unit speed parametrization, then $\dot{\gamma}(t).\ddot{\gamma}(t) = 0$*

Inner product:
$v = (2, 3)$
$w = (2, 1)$
$v.w := (2, 3).(2, 1) = 2 \times 2 + 3 \times 1 = 7$
In general:
$(x_1, y_1).(x_2, y_2) := x_1x_2 + y_1y_2$

Exercise. For $\mathbf{v} : (\alpha, \beta) \to \mathbf{R}^2$ and $\mathbf{w} : (\alpha, \beta) \to \mathbf{R}^2$, show that $(\mathbf{v}(t).\mathbf{w}(t))' = \mathbf{v}'(t).\mathbf{w}(t) + \mathbf{v}(t).\mathbf{w}'(t)$

$v = (x, y)$
$v.v = (x, y).(x, y) = x^2 + y^2$
$||(x, y)|| = \sqrt{x^2 + y^2}$

$v = (x, y)$
$v.w = ||v||||w|| \cos(\theta)$ where,
θ is the angle between v and w

Definition. $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a unit speed parametrization if $||\dot{\gamma}(t)|| = 1$ for each $t \in (\alpha, \beta)$

Theorem. *If $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a unit speed parametrization, then $\dot{\gamma}(t).\ddot{\gamma}(t) = 0$ for all $t \in (\alpha, \beta)$.*

Inner product:
$v = (2,3)$
$w = (2,1)$
$v.w := (2,3).(2,1) = 2 \times 2 + 3 \times 1 = 7$
In general:
$(x_1, y_1).(x_2, y_2) := x_1 x_2 + y_1 y_2$

Exercise. For $\mathbf{v} : (\alpha, \beta) \to \mathbf{R}^2$ and $\mathbf{w} : (\alpha, \beta) \to \mathbf{R}^2$, show that $(\mathbf{v}(t).\mathbf{w}(t))' = \mathbf{v}'(t).\mathbf{w}(t) + \mathbf{v}(t).\mathbf{w}'(t)$

$v = (x, y)$
$v.v = (x, y).(x, y) = x^2 + y^2$
$||(x, y)|| = \sqrt{x^2 + y^2}$

$v = (x, y)$
$v.w = ||v||||w|| \cos(\theta)$ where,
θ is the angle between v and w

Definition. $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a unit speed parametrization if $||\dot{\gamma}(t)|| = 1$ for each $t \in (\alpha, \beta)$

Theorem. *If $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a unit speed parametrization, then $\dot{\gamma}(t).\ddot{\gamma}(t) = 0$ for all $t \in (\alpha, \beta)$.*

Proof. $||\dot{\gamma}(t)|| = 1$

□

Inner product:
$v = (2, 3)$
$w = (2, 1)$
$v.w := (2, 3).(2, 1) = 2 \times 2 + 3 \times 1 = 7$
In general:
$(x_1, y_1).(x_2, y_2) := x_1x_2 + y_1y_2$

Exercise. For $\mathbf{v} : (\alpha, \beta) \to \mathbf{R}^2$ and $\mathbf{w} : (\alpha, \beta) \to \mathbf{R}^2$, show that $(\mathbf{v}(t).\mathbf{w}(t))' = \mathbf{v}'(t).\mathbf{w}(t) + \mathbf{v}(t).\mathbf{w}'(t)$

$v = (x, y)$
$v.v = (x, y).(x, y) = x^2 + y^2$
$||(x, y)|| = \sqrt{x^2 + y^2}$

$v = (x, y)$
$v.w = ||v||||w|| \cos(\theta)$ where,
θ is the angle between v and w

Definition. $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a unit speed parametrization if $||\dot{\gamma}(t)|| = 1$ for each $t \in (\alpha, \beta)$

Theorem. *If $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a unit speed parametrization, then $\dot{\gamma}(t).\ddot{\gamma}(t) = 0$ for all $t \in (\alpha, \beta)$.*

Proof. $||\dot{\gamma}(t)|| = 1$
$\dot{\gamma}(t).\dot{\gamma}(t) = 1$

□

Inner product:
$v = (2,3)$
$w = (2,1)$
$v.w := (2,3).(2,1) = 2 \times 2 + 3 \times 1 = 7$
In general:
$(x_1, y_1).(x_2, y_2) := x_1 x_2 + y_1 y_2$

Exercise. For $\mathbf{v} : (\alpha, \beta) \to \mathbf{R}^2$ and $\mathbf{w} : (\alpha, \beta) \to \mathbf{R}^2$, show that $(\mathbf{v}(t).\mathbf{w}(t))' = \mathbf{v}'(t).\mathbf{w}(t) + \mathbf{v}(t).\mathbf{w}'(t)$

$v = (x, y)$
$v.v = (x, y).(x, y) = x^2 + y^2$
$||(x, y)|| = \sqrt{x^2 + y^2}$

$v = (x, y)$
$v.w = ||v||||w|| \cos(\theta)$ where,
θ is the angle between v and w

Definition. $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a unit speed parametrization if $||\dot\gamma(t)|| = 1$ for each $t \in (\alpha, \beta)$

Theorem. *If $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a unit speed parametrization, then $\dot\gamma(t).\ddot\gamma(t) = 0$ for all $t \in (\alpha, \beta)$.*

Proof. $||\dot\gamma(t)|| = 1$
$\dot\gamma(t).\dot\gamma(t) = 1$
$(\dot\gamma(t).\dot\gamma(t))' = 0$

□

Inner product:
$v = (2, 3)$
$w = (2, 1)$
$v.w := (2, 3).(2, 1) = 2 \times 2 + 3 \times 1 = 7$
In general:
$(x_1, y_1).(x_2, y_2) := x_1 x_2 + y_1 y_2$

Exercise. For $\mathbf{v} : (\alpha, \beta) \to \mathbf{R}^2$ and $\mathbf{w} : (\alpha, \beta) \to \mathbf{R}^2$, show that $(\mathbf{v}(t).\mathbf{w}(t))' = \mathbf{v}'(t).\mathbf{w}(t) + \mathbf{v}(t).\mathbf{w}'(t)$

$v = (x, y)$
$v.v = (x, y).(x, y) = x^2 + y^2$
$||(x, y)|| = \sqrt{x^2 + y^2}$

$v = (x, y)$
$v.w = ||v||||w|| \cos(\theta)$ where,
θ is the angle between v and w

Definition. $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a unit speed parametrization if $||\dot{\gamma}(t)|| = 1$ for each $t \in (\alpha, \beta)$

Theorem. *If $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a unit speed parametrization, then $\dot{\gamma}(t).\ddot{\gamma}(t) = 0$ for all $t \in (\alpha, \beta)$.*

Proof. $||\dot{\gamma}(t)|| = 1$
$\dot{\gamma}(t).\dot{\gamma}(t) = 1$
$(\dot{\gamma}(t).\dot{\gamma}(t))' = 0$
$\ddot{\gamma}(t).\dot{\gamma}(t) + \dot{\gamma}(t).\ddot{\gamma}(t) = 0$

$\square$

Inner product:
$v = (2, 3)$
$w = (2, 1)$
$v.w := (2, 3).(2, 1) = 2 \times 2 + 3 \times 1 = 7$
In general:
$(x_1, y_1).(x_2, y_2) := x_1x_2 + y_1y_2$

Exercise. For $\mathbf{v} : (\alpha, \beta) \to \mathbf{R}^2$ and $\mathbf{w} : (\alpha, \beta) \to \mathbf{R}^2$, show that $(\mathbf{v}(t).\mathbf{w}(t))' = \mathbf{v}'(t).\mathbf{w}(t) + \mathbf{v}(t).\mathbf{w}'(t)$

$v = (x, y)$
$v.v = (x, y).(x, y) = x^2 + y^2$
$||(x, y)|| = \sqrt{x^2 + y^2}$

$v = (x, y)$
$v.w = ||v||||w|| \cos(\theta)$ where,
θ is the angle between v and w

Definition. $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a unit speed parametrization if $||\dot{\gamma}(t)|| = 1$ for each $t \in (\alpha, \beta)$

Theorem. *If $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a unit speed parametrization, then $\dot{\gamma}(t).\ddot{\gamma}(t) = 0$ for all $t \in (\alpha, \beta)$.*

Proof. $||\dot{\gamma}(t)|| = 1$
$\dot{\gamma}(t).\dot{\gamma}(t) = 1$
$(\dot{\gamma}(t).\dot{\gamma}(t))' = 0$
$\ddot{\gamma}(t).\dot{\gamma}(t) + \dot{\gamma}(t).\ddot{\gamma}(t) = 0$
$2\ddot{\gamma}(t).\dot{\gamma}(t) = 0$

$\square$

Inner product:
$v = (2, 3)$
$w = (2, 1)$
$v.w := (2, 3).(2, 1) = 2 \times 2 + 3 \times 1 = 7$
In general:
$(x_1, y_1).(x_2, y_2) := x_1x_2 + y_1y_2$

Exercise. For $\mathbf{v} : (\alpha, \beta) \to \mathbf{R}^2$ and $\mathbf{w} : (\alpha, \beta) \to \mathbf{R}^2$, show that $(\mathbf{v}(t).\mathbf{w}(t))' = \mathbf{v}'(t).\mathbf{w}(t) + \mathbf{v}(t).\mathbf{w}'(t)$

$v = (x, y)$
$v.v = (x, y).(x, y) = x^2 + y^2$
$||(x, y)|| = \sqrt{x^2 + y^2}$

$v = (x, y)$
$v.w = ||v||||w|| \cos(\theta)$ where,
θ is the angle between v and w

Definition. $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a unit speed parametrization if $||\dot{\gamma}(t)|| = 1$ for each $t \in (\alpha, \beta)$

Theorem. *If $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a unit speed parametrization, then $\dot{\gamma}(t).\ddot{\gamma}(t) = 0$ for all $t \in (\alpha, \beta)$.*

Proof. $||\dot{\gamma}(t)|| = 1$
$\dot{\gamma}(t).\dot{\gamma}(t) = 1$
$(\dot{\gamma}(t).\dot{\gamma}(t))' = 0$
$\ddot{\gamma}(t).\dot{\gamma}(t) + \dot{\gamma}(t).\ddot{\gamma}(t) = 0$
$2\ddot{\gamma}(t).\dot{\gamma}(t) = 0$
$\ddot{\gamma}(t).\dot{\gamma}(t) = 0$

□

Inner product:
$v = (2,3)$
$w = (2,1)$
$v.w := (2,3).(2,1) = 2 \times 2 + 3 \times 1 = 7$
In general:
$(x_1, y_1).(x_2, y_2) := x_1 x_2 + y_1 y_2$

Exercise. For $\mathbf{v} : (\alpha, \beta) \to \mathbf{R}^2$ and $\mathbf{w} : (\alpha, \beta) \to \mathbf{R}^2$, show that $(\mathbf{v}(t).\mathbf{w}(t))' = \mathbf{v}'(t).\mathbf{w}(t) + \mathbf{v}(t).\mathbf{w}'(t)$

$v = (x, y)$
$v.v = (x, y).(x, y) = x^2 + y^2$
$||(x, y)|| = \sqrt{x^2 + y^2}$

$v = (x, y)$
$v.w = ||v|| ||w|| \cos(\theta)$ where,
θ is the angle between v and w

Definition. $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a unit speed parametrization if $||\dot{\gamma}(t)|| = 1$ for each $t \in (\alpha, \beta)$

Theorem. *If $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a unit speed parametrization, then $\dot{\gamma}(t).\ddot{\gamma}(t) = 0$ for all $t \in (\alpha, \beta)$.*

Proof. $||\dot{\gamma}(t)|| = 1$
$\dot{\gamma}(t).\dot{\gamma}(t) = 1$
$(\dot{\gamma}(t).\dot{\gamma}(t))' = 0$
$\ddot{\gamma}(t).\dot{\gamma}(t) + \dot{\gamma}(t).\ddot{\gamma}(t) = 0$
$2\ddot{\gamma}(t).\dot{\gamma}(t) = 0$
$\ddot{\gamma}(t).\dot{\gamma}(t) = 0$

$\square$

Dicussion after the lecture

$$\ddot{\tilde{\gamma}}(t) = \ddot{\gamma}(\phi(t))\phi'(t)\phi'(t)$$

Dicussion after the lecture

$$\ddot{\tilde{\gamma}}(t) = \ddot{\gamma}(\phi(t))\phi'(t)\phi'(t) + \dot{\gamma}(\phi(t))\phi''(t)$$

Dicussion after the lecture

$\ddot{\tilde{\gamma}}(t) = \ddot{\gamma}(\phi(t))\phi'(t)\phi'(t) + \dot{\gamma}(\phi(t))\phi''(t)$
(may change the direction of acceleration)

Dicussion after the lecture

$\ddot{\tilde{\gamma}}(t) = \ddot{\gamma}(\phi(t))\phi'(t)\phi'(t) + \dot{\gamma}(\phi(t))\phi''(t)$
(may change the direction of acceleration)

If the direction of velocity does not change with t

Dicussion after the lecture

$\ddot{\tilde{\gamma}}(t) = \ddot{\gamma}(\phi(t))\phi'(t)\phi'(t) + \dot{\gamma}(\phi(t))\phi''(t)$
(may change the direction of acceleration)

If the direction of velocity does not change with t

$\dot{\gamma}(t) =$

Dicussion after the lecture

$\ddot{\tilde{\gamma}}(t) = \ddot{\gamma}(\phi(t))\phi'(t)\phi'(t) + \dot{\gamma}(\phi(t))\phi''(t)$
(may change the direction of acceleration)

If the direction of velocity does not change with t

$\dot{\gamma}(t) = f(t)\mathbf{v} =$

Dicussion after the lecture

$\ddot{\tilde{\gamma}}(t) = \ddot{\gamma}(\phi(t))\phi'(t)\phi'(t) + \dot{\gamma}(\phi(t))\phi''(t)$
(may change the direction of acceleration)

If the direction of velocity does not change with t

$\dot{\gamma}(t) = f(t)\mathbf{v} = (f(t)v_1, f(t)v_2)$

Dicussion after the lecture

$\ddot{\tilde{\gamma}}(t) = \ddot{\gamma}(\phi(t))\phi'(t)\phi'(t) + \dot{\gamma}(\phi(t))\phi''(t)$
(may change the direction of acceleration)

If the direction of velocity does not change with t

$\dot{\gamma}(t) = f(t)\mathbf{v} = (f(t)v_1, f(t)v_2)$
$\ddot{\gamma}(t) =$

Dicussion after the lecture

$\ddot{\tilde{\gamma}}(t) = \ddot{\gamma}(\phi(t))\phi'(t)\phi'(t) + \dot{\gamma}(\phi(t))\phi''(t)$
(may change the direction of acceleration)

If the direction of velocity does not change with t

$\dot{\gamma}(t) = f(t)\mathbf{v} = (f(t)v_1, f(t)v_2)$
$\ddot{\gamma}(t) = (f'(t)v_1, f'(t)v_2) =$

Dicussion after the lecture

$\ddot{\tilde{\gamma}}(t) = \ddot{\gamma}(\phi(t))\phi'(t)\phi'(t) + \dot{\gamma}(\phi(t))\phi''(t)$
(may change the direction of acceleration)

If the direction of velocity does not change with t

$\dot{\gamma}(t) = f(t)\mathbf{v} = (f(t)v_1, f(t)v_2)$
$\ddot{\gamma}(t) = (f'(t)v_1, f'(t)v_2) = f'(t)\mathbf{v}$

Dicussion after the lecture

$\ddot{\tilde{\gamma}}(t) = \ddot{\gamma}(\phi(t))\phi'(t)\phi'(t) + \dot{\gamma}(\phi(t))\phi''(t)$
(may change the direction of acceleration)

If the direction of velocity does not change with t

$\dot{\gamma}(t) = f(t)\mathbf{v} = (f(t)v_1, f(t)v_2)$
$\ddot{\gamma}(t) = (f'(t)v_1, f'(t)v_2) = f'(t)\mathbf{v}$
Direction of acceleration also unchanged