

Space curves

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$$\mathbf{v} = \alpha_1 \mathbf{e}_1 + \alpha_2 \mathbf{e}_2 + \alpha_3 \mathbf{e}_3$$

$$\mathbf{w} = \beta_1 \mathbf{e}_1 + \beta_2 \mathbf{e}_2 + \beta_3 \mathbf{e}_3$$

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$$\mathbf{v} \times \mathbf{w} = (\alpha_1 \mathbf{e}_1 + \alpha_2 \mathbf{e}_2 + \alpha_3 \mathbf{e}_3) \times (\beta_1 \mathbf{e}_1 + \beta_2 \mathbf{e}_2 + \beta_3 \mathbf{e}_3)$$

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$$\begin{aligned} \mathbf{v} \times \mathbf{w} &= (\alpha_1 \mathbf{e}_1 + \alpha_2 \mathbf{e}_2 + \alpha_3 \mathbf{e}_3) \times (\beta_1 \mathbf{e}_1 + \beta_2 \mathbf{e}_2 + \beta_3 \mathbf{e}_3) \\ &= (\beta_2 \alpha_3 - \beta_3 \alpha_2) \mathbf{e}_1 + \cdots \end{aligned}$$

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$$\mathbf{v}(t) = \alpha_1(t)\mathbf{e}_1 + \alpha_2(t)\mathbf{e}_2 + \alpha_3(t)\mathbf{e}_3$$

$$\mathbf{w}(t) = \beta_1(t)\mathbf{e}_1 + \beta_2(t)\mathbf{e}_2 + \beta_3(t)\mathbf{e}_3$$

$$\begin{aligned}\mathbf{v}(t) \times \mathbf{w}(t) &= (\alpha_1(t)\mathbf{e}_1 + \alpha_2(t)\mathbf{e}_2 + \alpha_3(t)\mathbf{e}_3) \times (\beta_1(t)\mathbf{e}_1 + \beta_2(t)\mathbf{e}_2 + \beta_3(t)\mathbf{e}_3) \\ &= (\beta_2(t)\alpha_3(t) - \beta_3(t)\alpha_2(t))\mathbf{e}_1 + \cdots\end{aligned}$$

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So, if $\mathbf{v}(t)$ and $\mathbf{w}(t)$ are smooth,

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So, if $\mathbf{v}(t)$ and $\mathbf{w}(t)$ are smooth, then $\mathbf{v}(t) \times \mathbf{w}(t)$ is smooth.

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for some (unique!) $x(t)$,

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$\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$, (unit) vector perpendicular to $\mathbf{T}(t)$

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for each $t \in (\alpha, \beta)$.

So, any vector field,

$$\mathbf{v}(t) = x(t)\mathbf{T}(t) + y(t)\mathbf{N}(t) + z(t)\mathbf{B}(t)$$

for some (unique!) $x(t), y(t), z(t) \in \mathbb{R}$

So, $x : (\alpha, \beta) \rightarrow \mathbb{R}$, $y : (\alpha, \beta) \rightarrow \mathbb{R}$,

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What are $\dot{\mathbf{T}}(t)$, $\dot{\mathbf{N}}(t)$, and $\dot{\mathbf{B}}(t)$ in terms of the basis?

Space curves

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Space curves

Given $\gamma : (\alpha, \beta) \rightarrow \mathbb{R}^3$ **unit speed** parametrization,
 $\kappa(t) \neq 0$

$\mathbf{T}(t) := \dot{\gamma}(t)$, (unit) vector in direction of velocity

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So, any vector field,

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Frenet-Serret equations

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Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t .

Planes

$$\begin{aligned}P &= \{(x,y,z) \in \mathbb{R}^3 \mid a(x-x_0)+b(y-y_0)+c(z-z_0) = 0\} \\ P &= \{\mathbf{v} \in \mathbb{R}^3 \mid \mathbf{n}.\mathbf{v} = 0\}\end{aligned}$$

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$$\begin{aligned}P &= \{(x,y,z) \in \mathbb{R}^3 \mid a(x-x_0)+b(y-y_0)+c(z-z_0) = 0\} \\ P &= \{\mathbf{v} \in \mathbb{R}^3 \mid \mathbf{n}.\mathbf{v} = 0\}\end{aligned}$$

$$\gamma : (\alpha, \beta)$$

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$$\gamma : (\alpha, \beta) \rightarrow \mathbb{R}^3 \text{ parametrizes a curve}$$

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$\gamma : (\alpha, \beta) \rightarrow \mathbb{R}^3$ parametrizes a curve that lies on the plane, P ,

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$\gamma : (\alpha,\beta) \rightarrow \mathbb{R}^3$ parametrizes a curve that lies on the plane, P , if and only if

$$\mathbf{n}.(\gamma(t) - \gamma(t_0)) = 0$$

Frenet-Serret equations

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Planes

$$\begin{aligned}P &= \{(x,y,z) \in \mathbb{R}^3 \mid a(x-x_0)+b(y-y_0)+c(z-z_0) = 0\} \\ P &= \{\mathbf{v} \in \mathbb{R}^3 \mid \mathbf{n}.\mathbf{v} = 0\}\end{aligned}$$

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$$\mathbf{n}.(\gamma(t) - \gamma(t_0)) = 0$$

for all t

Frenet-Serret equations

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$\gamma : (\alpha,\beta) \rightarrow \mathbb{R}^3$ parametrizes a curve that lies on the plane, P , if and only if

$$\mathbf{n}.(\gamma(t) - \gamma(t_0)) = 0$$

for all $t \in (\alpha,\beta)$.

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$

$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$

$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

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If $\tau(t) = 0$

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Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t .

If $\tau(t) = 0$ for all t

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$

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If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,

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If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,
then $\dot{\mathbf{B}}(t)$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$

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If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,
then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t)$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$

$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$

$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

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If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,
then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$

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If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,

then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$

So, $\mathbf{B}(t)$ is constant,

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$

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So, $\mathbf{B}(t)$ is constant, say, \mathbf{B}

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$$((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))'$$

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So, $\mathbf{B}(t)$ is constant, say, \mathbf{B}

$$((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' = ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t)+$$

$$\begin{aligned}\dot{\mathbf{T}}(t) &= \kappa(t)\mathbf{N}(t) \\ \dot{\mathbf{N}}(t) &= -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t) \\ \dot{\mathbf{B}}(t) &= -\tau(t)\mathbf{N}(t)\end{aligned}$$

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$$((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' = ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t)$$

$$\begin{aligned}\dot{\mathbf{T}}(t) &= \kappa(t)\mathbf{N}(t) \\ \dot{\mathbf{N}}(t) &= -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t) \\ \dot{\mathbf{B}}(t) &= -\tau(t)\mathbf{N}(t)\end{aligned}$$

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So, $\mathbf{B}(t)$ is constant, say, \mathbf{B}

$$\begin{aligned}((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) -\end{aligned}$$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$

$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$

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Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t .

If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,

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So, $\mathbf{B}(t)$ is constant, say, \mathbf{B}

$$\begin{aligned} ((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \end{aligned}$$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$

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So, $\mathbf{B}(t)$ is constant, say, \mathbf{B}

$$\begin{aligned} ((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \end{aligned}$$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$

$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$

$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

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If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,

then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$

So, $\mathbf{B}(t)$ is constant, say, \mathbf{B}

$$\begin{aligned} ((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \\ &= -\mathbf{T}(t).\mathbf{B}(t) \end{aligned}$$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$

$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$

$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

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If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,

then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$

So, $\mathbf{B}(t)$ is constant, say, \mathbf{B}

$$\begin{aligned} ((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \\ &= -\mathbf{T}(t).\mathbf{B}(t) \\ &= 0 \end{aligned}$$

$$(\gamma(t_0) - \gamma(t)).\mathbf{B}(t)$$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$

$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$

$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t .

If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,

then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$

So, $\mathbf{B}(t)$ is constant, say, \mathbf{B}

$$\begin{aligned} ((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \\ &= -\mathbf{T}(t).\mathbf{B}(t) \\ &= 0 \end{aligned}$$

$$(\gamma(t_0) - \gamma(t)).\mathbf{B}(t) = (\gamma(t_0) - \gamma(t)).\mathbf{B}$$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$

$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$

$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t .

If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,

then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$

So, $\mathbf{B}(t)$ is constant, say, \mathbf{B}

$$\begin{aligned} ((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \\ &= -\mathbf{T}(t).\mathbf{B}(t) \\ &= 0 \end{aligned}$$

$$(\gamma(t_0) - \gamma(t)).\mathbf{B}(t) = (\gamma(t_0) - \gamma(t)).\mathbf{B} = c$$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$

At $t = t_0$,

$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$

$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t .

If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,

then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$

So, $\mathbf{B}(t)$ is constant, say, \mathbf{B}

$$\begin{aligned} ((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \\ &= -\mathbf{T}(t).\mathbf{B}(t) \\ &= 0 \end{aligned}$$

$$(\gamma(t_0) - \gamma(t)).\mathbf{B}(t) = (\gamma(t_0) - \gamma(t)).\mathbf{B} = c$$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$

$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$

$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

At $t = t_0$,

$$c = (\gamma(t_0) - \gamma(t_0)).\mathbf{B} = 0$$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t .

If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,

then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$

So, $\mathbf{B}(t)$ is constant, say, \mathbf{B}

$$\begin{aligned} ((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \\ &= -\mathbf{T}(t).\mathbf{B}(t) \\ &= 0 \end{aligned}$$

$$(\gamma(t_0) - \gamma(t)).\mathbf{B}(t) = (\gamma(t_0) - \gamma(t)).\mathbf{B} = c$$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$

$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$

$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

At $t = t_0$,

$$c = (\gamma(t_0) - \gamma(t_0)).B = 0$$

$$\text{So, } (\gamma(t_0) - \gamma(t)).B = 0$$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t .

If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,

then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$

So, $\mathbf{B}(t)$ is constant, say, \mathbf{B}

$$\begin{aligned} ((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \\ &= -\mathbf{T}(t).\mathbf{B}(t) \\ &= 0 \end{aligned}$$

$$(\gamma(t_0) - \gamma(t)).\mathbf{B}(t) = (\gamma(t_0) - \gamma(t)).\mathbf{B} = c$$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$

$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$

$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

At $t = t_0$,

$$c = (\gamma(t_0) - \gamma(t_0)).\mathbf{B} = 0$$

$$\text{So, } (\gamma(t_0) - \gamma(t)).\mathbf{B} = 0$$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t . If the torsion is a constant 0, the curve lies on a plane.

Notation:

If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,

then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$

So, $\mathbf{B}(t)$ is constant, say, \mathbf{B}

$$\begin{aligned} ((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \\ &= -\mathbf{T}(t).\mathbf{B}(t) \\ &= 0 \end{aligned}$$

$$(\gamma(t_0) - \gamma(t)).\mathbf{B}(t) = (\gamma(t_0) - \gamma(t)).\mathbf{B} = c$$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$

$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$

$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

At $t = t_0$,

$$c = (\gamma(t_0) - \gamma(t_0)).B = 0$$

$$\text{So, } (\gamma(t_0) - \gamma(t)).B = 0$$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t . If the torsion is a constant 0, the curve lies on a plane.

If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,

then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$

So, $\mathbf{B}(t)$ is constant, say, \mathbf{B}

Notation:

$\mathbf{T}(t)$:

$$\begin{aligned} ((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \\ &= -\mathbf{T}(t).\mathbf{B}(t) \\ &= 0 \end{aligned}$$

$$(\gamma(t_0) - \gamma(t)).\mathbf{B}(t) = (\gamma(t_0) - \gamma(t)).\mathbf{B} = c$$

$$\begin{aligned}\dot{\mathbf{T}}(t) &= \kappa(t)\mathbf{N}(t) \\ \dot{\mathbf{N}}(t) &= -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t) \\ \dot{\mathbf{B}}(t) &= -\tau(t)\mathbf{N}(t)\end{aligned}$$

At $t = t_0$,
 $c = (\gamma(t_0) - \gamma(t)).B = 0$
 So, $(\gamma(t_0) - \gamma(t)).B = 0$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t . If the torsion is a constant 0, the curve lies on a plane.

Notation:
 $\mathbf{T}(t)$: unit **tangent** vector at t

If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,
 then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$
 So, $\mathbf{B}(t)$ is constant, say, \mathbf{B}

$$\begin{aligned}((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \\ &= -\mathbf{T}(t).\mathbf{B}(t) \\ &= 0\end{aligned}$$

$$(\gamma(t_0) - \gamma(t)).\mathbf{B}(t) = (\gamma(t_0) - \gamma(t)).\mathbf{B} = c$$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$

$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$

$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

At $t = t_0$,

$$c = (\gamma(t_0) - \gamma(t)).\mathbf{B} = 0$$

$$\text{So, } (\gamma(t_0) - \gamma(t)).\mathbf{B} = 0$$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t . If the torsion is a constant 0, the curve lies on a plane.

If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,
then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$
So, $\mathbf{B}(t)$ is constant, say, \mathbf{B}

Notation:

$\mathbf{T}(t)$: unit **tangent** vector at t

$\mathbf{N}(t)$:

$$\begin{aligned} ((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \\ &= -\mathbf{T}(t).\mathbf{B}(t) \\ &= 0 \end{aligned}$$

$$(\gamma(t_0) - \gamma(t)).\mathbf{B}(t) = (\gamma(t_0) - \gamma(t)).\mathbf{B} = c$$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$

$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$

$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

At $t = t_0$,

$$c = (\gamma(t_0) - \gamma(t)).\mathbf{B} = 0$$

$$\text{So, } (\gamma(t_0) - \gamma(t)).\mathbf{B} = 0$$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t . If the torsion is a constant 0, the curve lies on a plane.

If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,

$$\text{then } \dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$$

So, $\mathbf{B}(t)$ is constant, say, \mathbf{B}

Notation:

$\mathbf{T}(t)$: unit **tangent** vector at t

$\mathbf{N}(t)$: unit **normal** vector at t

$$\begin{aligned} ((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \\ &= -\mathbf{T}(t).\mathbf{B}(t) \\ &= 0 \end{aligned}$$

$$(\gamma(t_0) - \gamma(t)).\mathbf{B}(t) = (\gamma(t_0) - \gamma(t)).\mathbf{B} = c$$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$

$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$

$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

At $t = t_0$,

$$c = (\gamma(t_0) - \gamma(t)).\mathbf{B} = 0$$

$$\text{So, } (\gamma(t_0) - \gamma(t)).\mathbf{B} = 0$$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t . If the torsion is a constant 0, the curve lies on a plane.

If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,

$$\text{then } \dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$$

So, $\mathbf{B}(t)$ is constant, say, \mathbf{B}

Notation:

$\mathbf{T}(t)$: unit **tangent** vector at t

$\mathbf{N}(t)$: unit **normal** vector at t

$\mathbf{B}(t)$:

$$\begin{aligned} ((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0 \\ &= -\mathbf{T}(t).\mathbf{B}(t) \\ &= 0 \end{aligned}$$

$$(\gamma(t_0) - \gamma(t)).\mathbf{B}(t) = (\gamma(t_0) - \gamma(t)).\mathbf{B} = c$$

$$\dot{\mathbf{T}}(t) = \kappa(t)\mathbf{N}(t)$$

$$\dot{\mathbf{N}}(t) = -\kappa(t)\mathbf{T}(t) + \tau(t)\mathbf{B}(t)$$

$$\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$$

At $t = t_0$,

$$c = (\gamma(t_0) - \gamma(t)).\mathbf{B} = 0$$

So, $(\gamma(t_0) - \gamma(t)).\mathbf{B} = 0$

Definition. $\tau(t)$, defined so that $\dot{\mathbf{B}}(t) = -\tau(t)\mathbf{N}(t)$ is called the torsion of γ at t . If the torsion is a constant 0, the curve lies on a plane.

If $\tau(t) = 0$ for all $t \in (\alpha, \beta)$,
then $\dot{\mathbf{B}}(t) = 0\mathbf{N}(t) = 0$
So, $\mathbf{B}(t)$ is constant, say, \mathbf{B}

Notation:

$\mathbf{T}(t)$: unit **tangent** vector at t

$\mathbf{N}(t)$: unit **normal** vector at t

$\mathbf{B}(t)$: unit **binormal** vector at t

$$\begin{aligned} ((\gamma(t_0) - \gamma(t)).\mathbf{B}(t))' &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) + (\gamma(t_0) - \gamma(t)).\dot{\mathbf{B}}(t)) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - \tau(t)(\gamma(t_0) - \gamma(t)).\mathbf{N}(t)) \\ &= ((\gamma(t_0) - \gamma(t))'.\mathbf{B}(t) - 0) \\ &= -\mathbf{T}(t).\mathbf{B}(t) \\ &= 0 \end{aligned}$$

$$(\gamma(t_0) - \gamma(t)).\mathbf{B}(t) = (\gamma(t_0) - \gamma(t)).\mathbf{B} = c$$