

Theorem (Second Fundamental theorem of calculus).

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$\int \cos(t) + t^3dt$

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Products need care!

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Example.

$$\int \cos(t) + t^3 dt = \sin(t) + \frac{t^4}{4}$$

Similarly,

$$\int f_1(t) - f_2(t) = F_1(t) - F_2(t) = \int f_1(t) - \int f_2(t)$$

Example.

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Products need care!

$$(f(t)g(t))' = f'(t)g(t) + f(t)g'(t)$$

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Example.

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where $g(t) = \sin(t)$

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$$f(t)g(t) = \int f'(t)g(t) + \int f(t)g'(t)$$

$$\int f(t)g'(t) = f(t)g(t) - \int f'(t)g(t)$$

Example.

$$\int \underbrace{t}_{f(t)} \underbrace{\cos(t)}_{g'(t)} = t \sin(t) - \int \sin(t) = t \sin(t) + \cos(t)$$

where $g(t) = \sin(t)$

Substitution rule

Substitution rule

s

Substitution rule

$$s : [\alpha, \beta]$$

Substitution rule

$$s : [\alpha, \beta] \rightarrow \mathbb{R}$$

Substitution rule

$$s : [\alpha, \beta] \rightarrow \mathbb{R}$$

$$\phi$$

Substitution rule

$$s : [\alpha, \beta] \rightarrow \mathbb{R}$$

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Substitution rule

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$$(s(\phi(\tilde{t})))$$

Substitution rule

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$$(s(\phi(\tilde{t})))'$$

Substitution rule

$$s : [\alpha, \beta] \rightarrow \mathbb{R}$$

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$$(s(\phi(\tilde{t})))' = s'(\phi(\tilde{t}))$$

Substitution rule

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$$(s(\phi(\tilde{t})))' = s'(\phi(\tilde{t}))\phi'(\tilde{t})$$

Substitution rule

$$s : [\alpha, \beta] \rightarrow \mathbb{R}$$

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$$s'(\phi(\tilde{t}))\phi'(\tilde{t}) = (s(\phi(\tilde{t})))'$$

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$$s'(\phi(\tilde{t}))\phi'(\tilde{t}) = (s(\phi(\tilde{t})))'$$

$$\int_{\tilde{\alpha}}^{\tilde{\beta}} s'(\phi(\tilde{t}))\phi'(\tilde{t})$$

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Informally:

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Informally:

Substituting, $t = \phi(\tilde{t})$

$$\frac{dt}{d\tilde{t}} = \phi'(\tilde{t})$$

$$dt = \phi'(\tilde{t}) d\tilde{t}$$

Substitution rule

$$s : [\alpha, \beta] \rightarrow \mathbb{R}$$

$$\phi : [\tilde{\alpha}, \tilde{\beta}] \rightarrow [\alpha, \beta]$$

$$s'(\phi(\tilde{t}))\phi'(\tilde{t}) = (s(\phi(\tilde{t})))'$$

$$\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} \underbrace{s'(\phi(\tilde{t}))}_t \underbrace{\phi'(\tilde{t})}_{dt} d\tilde{t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} (s(\phi(\tilde{t})))' d\tilde{t} = s(\phi(\tilde{\beta})) - s(\phi(\tilde{\alpha})) = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} s'(t) dt$$

Informally:

Substituting, $t = \phi(\tilde{t})$

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Informally:

Substituting, $t = \phi(\tilde{t})$

$$\frac{\mathrm{d}t}{\mathrm{d}\tilde{t}} = \phi'(\tilde{t})$$

$$\mathrm{d}t = \phi'(\tilde{t})\mathrm{d}\tilde{t}$$

Substitution rule

$$s : [\alpha, \beta] \rightarrow \mathbb{R}$$

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$$\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} \underbrace{F(\phi(\tilde{t}))}_t \underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t)\mathrm{d}t$$

Informally:

Substituting, $t = \phi(\tilde{t})$

$$\frac{\mathrm{d}t}{\mathrm{d}\tilde{t}} = \phi'(\tilde{t})$$

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$$s : [\alpha, \beta] \rightarrow \mathbb{R}$$

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Substituting, $t = \phi(\tilde{t})$

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$$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$$

$$\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} \underbrace{F(\phi(\tilde{t}))}_t \underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t)\mathrm{d}t$$

$$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$$

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$$\text{Assume, } \phi'(t) > 0$$

$$\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} \underbrace{F(\phi(\tilde{t}))}_t \underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t)\mathrm{d}t$$

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$$\text{Assume, } \phi'(t) > 0$$

$$\|\dot{\tilde{\gamma}}(\tilde{t})\|$$

$$\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_t) \underbrace{\phi'(\tilde{t})}_{\mathrm{d}t} \mathrm{d}\tilde{t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t) \mathrm{d}t$$

$$\begin{aligned}\tilde{\gamma}(\tilde{t}) &= \gamma(\phi(\tilde{t})) \\ \dot{\tilde{\gamma}}(\tilde{t}) &= \dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t}) \\ \|\dot{\tilde{\gamma}}(\tilde{t})\| &= \|\dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\|\|\phi'(\tilde{t})\|\end{aligned}$$

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$$\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_t) \underbrace{\phi'(\tilde{t})}_{\mathrm{d}t} \mathrm{d}\tilde{t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t) \mathrm{d}t$$

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Assume, $\phi'(t) > 0$

$$\int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\tilde{\gamma}}(\tilde{t})\| \mathrm{d}\tilde{t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\gamma}(\phi(\tilde{t}))\|\phi'(\tilde{t}) \mathrm{d}\tilde{t}$$

$$\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_t) \underbrace{\phi'(\tilde{t})}_{\mathrm{d}t} \mathrm{d}\tilde{t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t) \mathrm{d}t$$

$$\begin{aligned}\tilde{\gamma}(\tilde{t}) &= \gamma(\phi(\tilde{t})) \\ \dot{\tilde{\gamma}}(\tilde{t}) &= \dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t}) \\ \|\dot{\tilde{\gamma}}(\tilde{t})\| &= \|\dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\|\|\phi'(\tilde{t})\|\end{aligned}$$

Assume, $\phi'(t) > 0$

$$\int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\tilde{\gamma}}(\tilde{t})\| \mathrm{d}\tilde{t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\gamma}(\underbrace{\phi(\tilde{t})}_t)\|\phi'(\tilde{t}) \mathrm{d}\tilde{t}$$

$$\boxed{\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} \underbrace{F(\phi(\tilde{t}))}_t \underbrace{\phi'(\tilde{t})}_{\mathrm{d}\tilde{t}} \mathrm{d}\tilde{t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t) \mathrm{d}t}$$

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Assume, $\phi'(t) > 0$

$$\int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\tilde{\gamma}}(\tilde{t})\| \mathrm{d}\tilde{t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} \underbrace{\|\dot{\gamma}(\phi(\tilde{t}))\|}_t \underbrace{\phi'(\tilde{t})}_{\mathrm{d}t} \mathrm{d}\tilde{t}$$

$$\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} \underbrace{F(\phi(\tilde{t}))}_t \underbrace{\phi'(\tilde{t})}_{\mathrm{d}t} \mathrm{d}\tilde{t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t) \mathrm{d}t$$

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$$\int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\tilde{\gamma}}(\tilde{t})\| \mathrm{d}\tilde{t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} \underbrace{\|\dot{\gamma}(\phi(\tilde{t}))\|}_t \underbrace{\phi'(\tilde{t})}_{\mathrm{d}t} \mathrm{d}\tilde{t} = \int_{\phi(\tilde{\alpha})}^{\phi(\tilde{\beta})}$$

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$$\int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\tilde{\gamma}}(\tilde{t})\| \mathrm{d}\tilde{t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\gamma}(\underbrace{\phi(\tilde{t}))}_t\| \underbrace{\phi'(\tilde{t})}_{\mathrm{d}\tilde{t}} \mathrm{d}\tilde{t} = \int_{\phi(\tilde{\alpha})}^{\phi(\tilde{\beta})} \|\dot{\gamma}(t)\|$$

$$\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} \underbrace{F(\phi(\tilde{t}))}_t \underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t)\mathrm{d}t$$

$$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$$

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$$\int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\tilde{\gamma}}(\tilde{t})\|\mathrm{d}\tilde{t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\gamma}(\underbrace{\phi(\tilde{t}))}_t)\|\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{\phi(\tilde{\alpha})}^{\phi(\tilde{\beta})} \|\dot{\gamma}(t)\|\mathrm{d}t$$

$$\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_t) \underbrace{\phi'(\tilde{t})}_{dt} d\tilde{t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t) dt$$

$$\begin{aligned} \tilde{\gamma}(\tilde{t}) &= \gamma(\phi(\tilde{t})) \\ \dot{\tilde{\gamma}}(\tilde{t}) &= \dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t}) \\ \|\dot{\tilde{\gamma}}(\tilde{t})\| &= \|\dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\|\|\phi'(\tilde{t})\| \end{aligned}$$

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We have proved,

$$\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_t) \underbrace{\phi'(\tilde{t})}_{dt} d\tilde{t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t) dt$$

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Assume, $\phi'(t) > 0$

$$\int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\tilde{\gamma}}(\tilde{t})\| d\tilde{t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\gamma}(\underbrace{\phi(\tilde{t})}_t)\| \underbrace{\phi'(\tilde{t})}_{dt} d\tilde{t} = \int_{\phi(\tilde{\alpha})}^{\phi(\tilde{\beta})} \|\dot{\gamma}(t)\| dt$$

We have proved,

Theorem. *The arc length*

$$\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_t) \underbrace{\phi'(\tilde{t})}_{dt} d\tilde{t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t) dt$$

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Assume, $\phi'(t) > 0$

$$\int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\tilde{\gamma}}(\tilde{t})\| d\tilde{t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\gamma}(\underbrace{\phi(\tilde{t})}_t)\| \underbrace{\phi'(\tilde{t})}_{dt} d\tilde{t} = \int_{\phi(\tilde{\alpha})}^{\phi(\tilde{\beta})} \|\dot{\gamma}(t)\| dt$$

We have proved,

Theorem. *The arc length is invariant*

$$\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_t) \underbrace{\phi'(\tilde{t})}_{dt} d\tilde{t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t) dt$$

$$\begin{aligned}\tilde{\gamma}(\tilde{t}) &= \gamma(\phi(\tilde{t})) \\ \dot{\tilde{\gamma}}(\tilde{t}) &= \dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t}) \\ \|\dot{\tilde{\gamma}}(\tilde{t})\| &= \|\dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\|\|\phi'(\tilde{t})\|\end{aligned}$$

Assume, $\phi'(t) > 0$

$$\int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\tilde{\gamma}}(\tilde{t})\| d\tilde{t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\gamma}(\underbrace{\phi(\tilde{t})}_t)\| \underbrace{\phi'(\tilde{t})}_{dt} d\tilde{t} = \int_{\phi(\tilde{\alpha})}^{\phi(\tilde{\beta})} \|\dot{\gamma}(t)\| dt$$

We have proved,

Theorem. *The arc length is invariant under reparametrization.*