

Exercise sheet 4

Curves and Surfaces, MTH201

1. Show that the curvature at any point of a line segment is always 0.
2. Find a parametrization of an ellipse, i.e. $\{(x, y) \in \mathbb{R}^2 \mid \frac{x^2}{a} + \frac{y^2}{b} = 1\}$ and use it to compute its curvature function $\kappa(t)$.
3. Given *any* smooth parametrization, $\gamma : (\alpha, \beta) \rightarrow \mathbb{R}^2$, is the curvature function $\kappa(t)$ always smooth? Do you need to add some condition? What is it?
4. Compute the signed curvature of the circle parametrized by $\gamma(t) = (5\cos(t), -5\sin(t))$.
5. If $\gamma : (\alpha, \beta) \rightarrow \mathbb{R}^2$ parametrizes a curve, compute the curvature of the curve parametrized by $\tilde{\gamma}(t) = \gamma(-t)$ in terms of the curvature of γ . What about the relation between the signed curvatures of γ and $\tilde{\gamma}$?
6. Compare the signed curvatures of a curve and its reflection, i.e. $\gamma(t)$ and $-\gamma(t)$.
7. By finding a unit speed parametrization of a circle of radius r , compute its curvature. Let $\gamma(t)$ be some other *constant* speed parametrization of a circle of radius r , where $v := \|\dot{\gamma}(t)\|$ is the (constant) speed, and prove that $\|\ddot{\gamma}(t)\| = v^2/r$ (Do you recognize the significance of this?).
8. Can you draw a curve whose signed curvature in terms of a unit speed parametrization is $\kappa_s(t) = t$?

to be updated...