Dicussion before the lecture

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Regular parametrization

Definition.
The point $\gamma(t)$

Regular parametrization

Definition.
The point $\gamma(t)$ of γ :

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \rightarrow$

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Definition.
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Proposition. *A reparametrization of a regular parametrization*

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$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\tilde{\gamma} : (\tilde{\alpha}, \tilde{\beta}) \to$ $\square$

Regular parametrization

Definition.
The point $\gamma(t)$ of $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is regular point if $\dot{\gamma}(t) \neq 0$ and singular if $\dot{\gamma}(t) = 0$. The parametrization γ is called regular if $\dot{\gamma}(t) \neq 0$ for *every* $t \in (\alpha, \beta)$.

Example. $\gamma(t) = (t^3, t^3)$ is singular at 0 because $\dot{\gamma}(t) = (3t^2, 3t^2)$.

Lemma.
If $\phi : (\tilde{\alpha}, \tilde{\beta}) \to (\alpha, \beta)$ *is invertible with inverse* $\psi : (\alpha, \beta) \to (\tilde{\alpha}, \tilde{\beta})$, *then* $\phi'(t) \neq 0$ *for all* $t \in (\tilde{\alpha}, \tilde{\beta})$ *and* $\psi'(t) \neq 0$ *for all* $t \in (\alpha, \beta)$.

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$\psi(\phi(t)) = t$ for *each* t
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So, $\psi'(\phi(t)) \neq 0$ and $\phi'(t) \neq 0$ for *each* t
So, $\psi'(t) \neq 0$ and $\phi'(t) \neq 0$ for *each* t because ϕ is bijective. $\qquad\square$

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□

Inner product:

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$v = (2, 3)$

Inner product:

$v = (2, 3)$

$w = (2, 1)$

Inner product:

$v = (2, 3)$

$w = (2, 1)$

$v.w$

Inner product:

$v = (2, 3)$

$w = (2, 1)$

$v.w := (2, 3).(2, 1)$

Inner product:

$v = (2, 3)$

$w = (2, 1)$

$v.w := (2, 3).(2, 1) = 2 \times 2 + 3 \times 1$

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$(x_1, y_1).(x_2, y_2) := x_1 x_2 + y_1 y_2$

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θ is the angle between v and w

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