Chain rule for mult-variable functions

f

Chain rule for mult-variable functions

$f : \mathbb{R}^2$

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
γ

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : (\alpha, \beta)$

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\gamma(t)$

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\gamma(t) = (x(t), y(t))$

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
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$\gamma(t) = (x(t), y(t))$
$f \circ \gamma$

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
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$f \circ \gamma : (\alpha, \beta)$

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$f \circ \gamma : (\alpha, \beta) \to \mathbb{R}$

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$(f \circ \gamma)'(t_0)$

Chain rule for mult-variable functions

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$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\gamma(t) = (x(t), y(t))$
$f \circ \gamma : (\alpha, \beta) \to \mathbb{R}$

$$(f \circ \gamma)'(t_0) = f_x(x(t_0), y(t_0))x'(t_0) + f_y(x(t_0), y(t_0))y'(t_0)$$

Chain rule for mult-variable functions

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$(f \circ \gamma)'(t_0) = f_x(x(t_0), y(t_0))x'(t_0) + f_y(x(t_0), y(t_0))y'(t_0) = (f_x(x, y), f_y(x, y)).\dot{\gamma}(t_0)$

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\gamma(t) = (x(t), y(t))$
$f \circ \gamma : (\alpha, \beta) \to \mathbb{R}$

$(f \circ \gamma)'(t_0) = f_x(x(t_0), y(t_0))x'(t_0) + f_y(x(t_0), y(t_0))y'(t_0) = \nabla(f)(x(t_0), y(t_0)).\dot{\gamma}(t_0)$,
where $\nabla(f)(x, y) = (f_x(x, y).f_y(x, y))$,

Chain rule for mult-variable functions

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$\boxed{f_{\mathbf{v}}(x(t_0), y(t_0)) := (f \circ \gamma)'(t_0) = \nabla(f)(x(t_0), y(t_0)).\mathbf{v}}$,
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$\mathbf{v} = \dot{\gamma}(t_0)$,

Chain rule for mult-variable functions

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$\boxed{f_{\mathbf{v}}(x(t_0), y(t_0)) := (f \circ \gamma)'(t_0) = \nabla(f)(p).\mathbf{v}}$,
where $\nabla(f)(x, y) = (f_x(x, y).f_y(x, y))$,
$\mathbf{v} = \dot{\gamma}(t_0)$,
and $p = (x(t_0), y(t_0))$

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$f : \mathbb{R}^2$

Chain rule for mult-variable functions

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$f : \mathbb{R}^2 \to \mathbb{R}$

Chain rule for mult-variable functions

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$f : \mathbb{R}^2 \to \mathbb{R}$
γ

Chain rule for mult-variable functions

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$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : \mathbb{R}^2$

Chain rule for mult-variable functions

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$f : \mathbb{R}^2 \to \mathbb{R}$
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Chain rule for mult-variable functions

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$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : \mathbb{R}^2 \to \mathbb{R}^2$
$\gamma(u, v)$

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$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
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$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : \mathbb{R}^2 \to \mathbb{R}^2$
$\gamma(u, v) = (x(u, v), y(u, v))$
$f \circ \gamma : \mathbb{R}^2$

Chain rule for mult-variable functions

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$f : \mathbb{R}^2 \to \mathbb{R}$
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$\gamma(u, v) = (x(u, v), y(u, v))$
$f \circ \gamma : \mathbb{R}^2 \to \mathbb{R}$

Chain rule for mult-variable functions

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$\gamma : \mathbb{R}^2 \to \mathbb{R}^2$
$\gamma(u, v) = (x(u, v), y(u, v))$
$f \circ \gamma : \mathbb{R}^2 \to \mathbb{R}$

$(f \circ \gamma)_u(u_0, v_0)$

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
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$\gamma : \mathbb{R}^2 \to \mathbb{R}^2$
$\gamma(u, v) = (x(u, v), y(u, v))$
$f \circ \gamma : \mathbb{R}^2 \to \mathbb{R}$

$(f \circ \gamma)_u(u_0, v_0)$
$= f_x(x(u_0, v_0), y(u_0, v_0))x_u(u_0, v_0)$
$+ f_y(x(u_0, v_0), y(u_0, v_0))y_u(u_0, v_0)$

Curves on surfaces

Definition. $\mathbf{v} \in \mathbb{R}^3$ is a tangent vector of the surface S at a point p, if there is a γ

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
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$f \circ \gamma : (\alpha, \beta) \to \mathbb{R}$

$\boxed{f_{\mathbf{v}}(x(t_0), y(t_0)) := (f \circ \gamma)'(t_0) = \nabla(f)(p).\mathbf{v}}$,
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and $p = (x(t_0), y(t_0))$

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : \mathbb{R}^2 \to \mathbb{R}^2$
$\gamma(u, v) = (x(u, v), y(u, v))$
$f \circ \gamma : \mathbb{R}^2 \to \mathbb{R}$

$(f \circ \gamma)_u(u_0, v_0)$
$= f_x(x(u_0, v_0), y(u_0, v_0))x_u(u_0, v_0)$
$+ f_y(x(u_0, v_0), y(u_0, v_0))y_u(u_0, v_0)$

Curves on surfaces

Definition. $\mathbf{v} \in \mathbb{R}^3$ is a tangent vector of the surface S at a point p, if there is a $\gamma : (\alpha, \beta)$

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
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$\boxed{f_{\mathbf{v}}(x(t_0), y(t_0)) := (f \circ \gamma)'(t_0) = \nabla(f)(p).\mathbf{v}}$,
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$\gamma : \mathbb{R}^2 \to \mathbb{R}^2$
$\gamma(u, v) = (x(u, v), y(u, v))$
$f \circ \gamma : \mathbb{R}^2 \to \mathbb{R}$

$(f \circ \gamma)_u(u_0, v_0)$
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Curves on surfaces

Definition. $\mathbf{v} \in \mathbb{R}^3$ is a tangent vector of the surface S at a point p, if there is a $\gamma : (\alpha, \beta) \to S$

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\gamma(t) = (x(t), y(t))$
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$\boxed{f_{\mathbf{v}}(x(t_0), y(t_0)) := (f \circ \gamma)'(t_0) = \nabla(f)(p).\mathbf{v}}$,
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and $p = (x(t_0), y(t_0))$

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : \mathbb{R}^2 \to \mathbb{R}^2$
$\gamma(u, v) = (x(u, v), y(u, v))$
$f \circ \gamma : \mathbb{R}^2 \to \mathbb{R}$

$(f \circ \gamma)_u(u_0, v_0)$
$= f_x(x(u_0, v_0), y(u_0, v_0))x_u(u_0, v_0)$
$+f_y(x(u_0, v_0), y(u_0, v_0))y_u(u_0, v_0)$

Curves on surfaces

Definition. $\mathbf{v} \in \mathbb{R}^3$ is a tangent vector of the surface S at a point p, if there is a $\gamma : (\alpha, \beta) \to S \subset \mathbb{R}^3$ so that $p = \gamma(t)$ and

Chain rule for mult-variable functions

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$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\gamma(t) = (x(t), y(t))$
$f \circ \gamma : (\alpha, \beta) \to \mathbb{R}$

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and $p = (x(t_0), y(t_0))$

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : \mathbb{R}^2 \to \mathbb{R}^2$
$\gamma(u, v) = (x(u, v), y(u, v))$
$f \circ \gamma : \mathbb{R}^2 \to \mathbb{R}$

$(f \circ \gamma)_u(u_0, v_0)$
$= f_x(x(u_0, v_0), y(u_0, v_0))x_u(u_0, v_0)$
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Curves on surfaces

Definition. $\mathbf{v} \in \mathbb{R}^3$ is a tangent vector of the surface S at a point p, if there is a $\gamma : (\alpha, \beta) \to S \subset \mathbb{R}^3$ so that $p = \gamma(t)$ and $\mathbf{v} = \dot{\gamma}(t)$

$\gamma : (\alpha, \beta) \to S \subset \mathbb{R}^3$ is a curve.

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\gamma(t) = (x(t), y(t))$
$f \circ \gamma : (\alpha, \beta) \to \mathbb{R}$

$\boxed{f_{\mathbf{v}}(x(t_0), y(t_0)) := (f \circ \gamma)'(t_0) = \nabla(f)(p).\mathbf{v}}$,
where $\nabla(f)(x, y) = (f_x(x, y).f_y(x, y))$,
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and $p = (x(t_0), y(t_0))$

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : \mathbb{R}^2 \to \mathbb{R}^2$
$\gamma(u, v) = (x(u, v), y(u, v))$
$f \circ \gamma : \mathbb{R}^2 \to \mathbb{R}$

$(f \circ \gamma)_u(u_0, v_0)$
$= f_x(x(u_0, v_0), y(u_0, v_0))x_u(u_0, v_0)$
$+f_y(x(u_0, v_0), y(u_0, v_0))y_u(u_0, v_0)$

Curves on surfaces

Definition. $\mathbf{v} \in \mathbb{R}^3$ is a tangent vector of the surface S at a point p, if there is a $\gamma : (\alpha, \beta) \to S \subset \mathbb{R}^3$ so that $p = \gamma(t)$ and $\mathbf{v} = \dot{\gamma}(t)$

$\gamma : (\alpha, \beta) \to S \subset \mathbb{R}^3$ is a curve.
$\sigma : U \to S$ a surface patch.

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\gamma(t) = (x(t), y(t))$
$f \circ \gamma : (\alpha, \beta) \to \mathbb{R}$

$\boxed{f_{\mathbf{v}}(x(t_0), y(t_0)) := (f \circ \gamma)'(t_0) = \nabla(f)(p).\mathbf{v}}$,
where $\nabla(f)(x, y) = (f_x(x, y).f_y(x, y))$,
$\mathbf{v} = \dot{\gamma}(t_0)$,
and $p = (x(t_0), y(t_0))$

$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : \mathbb{R}^2 \to \mathbb{R}^2$
$\gamma(u, v) = (x(u, v), y(u, v))$
$f \circ \gamma : \mathbb{R}^2 \to \mathbb{R}$

$(f \circ \gamma)_u(u_0, v_0)$
$= f_x(x(u_0, v_0), y(u_0, v_0))x_u(u_0, v_0)$
$+f_y(x(u_0, v_0), y(u_0, v_0))y_u(u_0, v_0)$

Curves on surfaces

Definition. $\mathbf{v} \in \mathbb{R}^3$ is a tangent vector of the surface S at a point p, if there is a $\gamma : (\alpha, \beta) \to S \subset \mathbb{R}^3$ so that $p = \gamma(t)$ and $\mathbf{v} = \dot{\gamma}(t)$

$\gamma : (\alpha, \beta) \to S \subset \mathbb{R}^3$ is a curve.
$\sigma : U \to S$ a surface patch.
So, $\gamma(t) = \sigma(x(t), y(t))$

Chain rule for mult-variable functions

$f : \mathbb{R}^2 \to \mathbb{R}$
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Curves on surfaces

Definition. $\mathbf{v} \in \mathbb{R}^3$ is a tangent vector of the surface S at a point p, if there is a $\gamma : (\alpha, \beta) \to S \subset \mathbb{R}^3$ so that $p = \gamma(t)$ and $\mathbf{v} = \dot{\gamma}(t)$

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Chain rule for mult-variable functions

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Exercise. Show that any vector that belongs to the span of $\sigma_x(p)$ and $\sigma_y(p)$, is a tangent vector.

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Exercise. Show that σ is regular at p if and only if the tangent vectors at p form a two dimensional subspace of $\mathbb{R}^3$.