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Products need care!

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$f(t)g(t)$

$f_1(t) = F_1'(t)$ and $f_2(t) = F_2'(t)$

$$\boxed{\int f_1(t) + f_2(t)\mathrm{dt} = F_1(t) + F_2(t) = \int f_1(t) + \int f_2(t)}$$

because,

$f_1(t) + f_2(t) = F_1'(t) + F_2'(t) = (F_1(t) + F_2(t))'$

Example.

$$\int \cos(t) + t^3 \mathrm{d}t = \sin(t) + \frac{t^4}{4}$$

Similarly,

$$\boxed{\int f_1(t) - f_2(t) = F_1(t) - F_2(t) = \int f_1(t) - \int f_2(t)}$$

Example.

$$\int \cos(t) - t^3 \mathrm{d}t = \sin(t) - \frac{t^4}{4}$$

Products need care!

$(f(t)g(t))' = f'(t)g(t) + f(t)g'(t)$

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Example.

$$\int t\cos(t)$$

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Example.

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Example.

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Example.

$$\int \underbrace{t}_{f(t)} \underbrace{\cos(t)}_{g'(t)} =$$

where $g(t) = \sin(t)$

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Example.

$$\int \underbrace{t}_{f(t)} \underbrace{\cos(t)}_{g'(t)} = t\sin(t)$$

where $g(t) = \sin(t)$

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$$\boxed{\int f(t)g'(t) = f(t)g(t) - \int f'(t)g(t)}$$

Example.

$$\int \underbrace{t}_{f(t)}\underbrace{\cos(t)}_{g'(t)} = t\sin(t) - \int \sin(t)$$

where $g(t) = \sin(t)$

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$$\boxed{\int f(t)g'(t) = f(t)g(t) - \int f'(t)g(t)}$$

Example.

$$\int \underbrace{t}_{f(t)} \underbrace{\cos(t)}_{g'(t)} = t\sin(t) - \int \sin(t) = t\sin(t) + \cos(t)$$

where $g(t) = \sin(t)$

Substitution rule

Substitution rule

s

Substitution rule

$s : [\alpha, \beta]$

Substitution rule

$s : [\alpha, \beta] \to \mathbb{R}$

Substitution rule

$s : [\alpha, \beta] \to \mathbb{R}$
ϕ

Substitution rule

$s : [\alpha, \beta] \to \mathbb{R}$
$\phi : [\tilde{\alpha}, \tilde{\beta}]$

Substitution rule

$s : [\alpha, \beta] \to \mathbb{R}$
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Substitution rule

$s : [\alpha, \beta] \to \mathbb{R}$
$\phi : [\tilde{\alpha}, \tilde{\beta}] \to [\alpha, \beta]$

$(s(\phi(\tilde{t})))$

Substitution rule

$s : [\alpha, \beta] \to \mathbb{R}$
$\phi : [\tilde{\alpha}, \tilde{\beta}] \to [\alpha, \beta]$

$(s(\phi(\tilde{t})))'$

Substitution rule

$s : [\alpha, \beta] \to \mathbb{R}$
$\phi : [\tilde{\alpha}, \tilde{\beta}] \to [\alpha, \beta]$

$(s(\phi(\tilde{t})))' = s'(\phi(\tilde{t}))$

Substitution rule

$s : [\alpha, \beta] \to \mathbb{R}$
$\phi : [\tilde{\alpha}, \tilde{\beta}] \to [\alpha, \beta]$

$$(s(\phi(\tilde{t})))' = s'(\phi(\tilde{t}))\phi'(\tilde{t})$$

Substitution rule

$s : [\alpha, \beta] \to \mathbb{R}$
$\phi : [\tilde{\alpha}, \tilde{\beta}] \to [\alpha, \beta]$

$s'(\phi(\tilde{t}))\phi'(\tilde{t}) = (s(\phi(\tilde{t})))'$

Substitution rule

$s : [\alpha, \beta] \to \mathbb{R}$
$\phi : [\tilde{\alpha}, \tilde{\beta}] \to [\alpha, \beta]$

$s'(\phi(\tilde{t}))\phi'(\tilde{t}) = (s(\phi(\tilde{t})))'$

$\int_{\tilde{\alpha}}^{\tilde{\beta}} s'(\phi(\tilde{t}))\phi'(\tilde{t})$

Substitution rule

$s : [\alpha, \beta] \to \mathbb{R}$
$\phi : [\tilde{\alpha}, \tilde{\beta}] \to [\alpha, \beta]$

$s'(\phi(\tilde{t}))\phi'(\tilde{t}) = (s(\phi(\tilde{t})))'$

$\int_{\tilde{\alpha}}^{\tilde{\beta}} s'(\phi(\tilde{t}))\phi'(\tilde{t})\mathrm{d}\tilde{t}$

Substitution rule

$s : [\alpha, \beta] \to \mathbb{R}$
$\phi : [\tilde{\alpha}, \tilde{\beta}] \to [\alpha, \beta]$

$s'(\phi(\tilde{t}))\phi'(\tilde{t}) = (s(\phi(\tilde{t})))'$

$\int_{\tilde{\alpha}}^{\tilde{\beta}} s'(\phi(\tilde{t}))\phi'(\tilde{t})\mathrm{d}\tilde{t} = \int_{\tilde{\alpha}}^{\tilde{\beta}}(s(\phi(\tilde{t})))'$

Substitution rule

$s : [\alpha, \beta] \to \mathbb{R}$
$\phi : [\tilde{\alpha}, \tilde{\beta}] \to [\alpha, \beta]$

$s'(\phi(\tilde{t}))\phi'(\tilde{t}) = (s(\phi(\tilde{t})))'$

$\int_{\tilde{\alpha}}^{\tilde{\beta}} s'(\phi(\tilde{t}))\phi'(\tilde{t})\mathrm{d}\tilde{t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} (s(\phi(\tilde{t})))'\mathrm{d}\tilde{t}$

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$\phi : [\tilde{\alpha}, \tilde{\beta}] \to [\alpha, \beta]$

$s'(\phi(\tilde{t}))\phi'(\tilde{t}) = (s(\phi(\tilde{t})))'$

$\int_{\tilde{\alpha}}^{\tilde{\beta}} s'(\phi(\tilde{t}))\phi'(\tilde{t})\mathrm{d}\tilde{t} = \int_{\tilde{\alpha}}^{\tilde{\beta}}(s(\phi(\tilde{t})))'\mathrm{d}\tilde{t} = s(\phi(\tilde{\beta})) - s(\phi(\tilde{\alpha}))$

Substitution rule

$s : [\alpha, \beta] \to \mathbb{R}$
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$s'(\phi(\tilde{t}))\phi'(\tilde{t}) = (s(\phi(\tilde{t})))'$

$\int_{\tilde{\alpha}}^{\tilde{\beta}} s'(\phi(\tilde{t}))\phi'(\tilde{t})\mathrm{d}\tilde{t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} (s(\phi(\tilde{t})))'\mathrm{d}\tilde{t} = s(\phi(\tilde{\beta})) - s(\phi(\tilde{\alpha})) = \int_{\phi(\tilde{\alpha})}^{\phi(\tilde{\beta})}$

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$s'(\phi(\tilde{t}))\phi'(\tilde{t}) = (s(\phi(\tilde{t})))'$

$\int_{\tilde{\alpha}}^{\tilde{\beta}} s'(\phi(\tilde{t}))\phi'(\tilde{t})\mathrm{d}\tilde{t} = \int_{\tilde{\alpha}}^{\tilde{\beta}}(s(\phi(\tilde{t})))'\mathrm{d}\tilde{t} = s(\phi(\tilde{\beta})) - s(\phi(\tilde{\alpha})) = \int_{\phi(\tilde{\alpha})}^{\phi(\tilde{\beta})} s'(t)$

Substitution rule

$s : [\alpha, \beta] \to \mathbb{R}$
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$s'(\phi(\tilde{t}))\phi'(\tilde{t}) = (s(\phi(\tilde{t})))'$

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$$s'(\phi(\tilde{t}))\phi'(\tilde{t}) = (s(\phi(\tilde{t})))'$$

$$\int_{\tilde{\alpha}}^{\tilde{\beta}} s'(\underbrace{\phi(\tilde{t})}_{t})\, \underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} (s(\phi(\tilde{t})))'\mathrm{d}\tilde{t} = s(\phi(\tilde{\beta})) - s(\phi(\tilde{\alpha})) = \int_{\phi(\tilde{\alpha})}^{\phi(\tilde{\beta})} s'(t)\mathrm{d}t$$

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$s : [\alpha, \beta] \to \mathbb{R}$
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$$s'(\phi(\tilde{t}))\phi'(\tilde{t}) = (s(\phi(\tilde{t})))'$$

$$\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} s'(\underbrace{\phi(\tilde{t})}_{t})\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} (s(\phi(\tilde{t})))'\mathrm{d}\tilde{t} = s(\phi(\tilde{\beta})) - s(\phi(\tilde{\alpha})) = \int_{\phi(\tilde{\alpha})}^{\phi(\tilde{\beta})} s'(t)\mathrm{d}t$$

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$s : [\alpha, \beta] \to \mathbb{R}$
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$\phi : [\tilde{\alpha}, \tilde{\beta}] \to [\alpha, \beta]$

$$s'(\phi(\tilde{t}))\phi'(\tilde{t}) = (s(\phi(\tilde{t})))'$$

$$\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} s'(\underbrace{\phi(\tilde{t})}_{t})\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} (s(\phi(\tilde{t})))'\mathrm{d}\tilde{t} = s(\phi(\tilde{\beta})) - s(\phi(\tilde{\alpha})) = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} s'(t)\mathrm{d}t$$

Informally:
Substituting, $t = \phi(\tilde{t})$

Substitution rule

$s : [\alpha, \beta] \to \mathbb{R}$
$\phi : [\tilde{\alpha}, \tilde{\beta}] \to [\alpha, \beta]$

$$s'(\phi(\tilde{t}))\phi'(\tilde{t}) = (s(\phi(\tilde{t})))'$$

$$\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} s'(\underbrace{\phi(\tilde{t})}_{t})\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} (s(\phi(\tilde{t})))'\mathrm{d}\tilde{t} = s(\phi(\tilde{\beta})) - s(\phi(\tilde{\alpha})) = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} s'(t)\mathrm{d}t$$

Informally:
Substituting, $t = \phi(\tilde{t})$
$\frac{\mathrm{d}t}{\mathrm{d}\tilde{t}} = \phi'(\tilde{t})$

Substitution rule

$s : [\alpha, \beta] \to \mathbb{R}$
$\phi : [\tilde{\alpha}, \tilde{\beta}] \to [\alpha, \beta]$

$$s'(\phi(\tilde{t}))\phi'(\tilde{t}) = (s(\phi(\tilde{t})))'$$

$$\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} s'(\underbrace{\phi(\tilde{t})}_{t})\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} (s(\phi(\tilde{t})))'\mathrm{d}\tilde{t} = s(\phi(\tilde{\beta})) - s(\phi(\tilde{\alpha})) = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} s'(t)\mathrm{d}t$$

Informally:
Substituting, $t = \phi(\tilde{t})$
$\frac{\mathrm{d}t}{\mathrm{d}\tilde{t}} = \phi'(\tilde{t})$
$\mathrm{d}t = \phi'(\tilde{t})\mathrm{d}\tilde{t}$

Substitution rule

$s : [\alpha, \beta] \to \mathbb{R}$
$\phi : [\tilde{\alpha}, \tilde{\beta}] \to [\alpha, \beta]$

$s'(\phi(\tilde{t}))\phi'(\tilde{t}) = (s(\phi(\tilde{t})))'$

$$\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} s'(\underbrace{\phi(\tilde{t})}_{t})\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} (s(\phi(\tilde{t})))'\mathrm{d}\tilde{t} = s(\phi(\tilde{\beta})) - s(\phi(\tilde{\alpha})) = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} s'(t)\mathrm{d}t$$

Informally:
Substituting, $t = \phi(\tilde{t})$
$\frac{\mathrm{d}t}{\mathrm{d}\tilde{t}} = \phi'(\tilde{t})$
$\mathrm{d}t = \phi'(\tilde{t})\mathrm{d}\tilde{t}$

Substitution rule

$s : [\alpha, \beta] \to \mathbb{R}$
$\phi : [\tilde{\alpha}, \tilde{\beta}] \to [\alpha, \beta]$

$$s'(\phi(\tilde{t}))\phi'(\tilde{t}) = (s(\phi(\tilde{t})))'$$

$$\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} s'(\underbrace{\phi(\tilde{t})}_{t})\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} s'(t)\mathrm{d}t$$

Informally:
Substituting, $t = \phi(\tilde{t})$
$\frac{\mathrm{d}t}{\mathrm{d}\tilde{t}} = \phi'(\tilde{t})$
$\mathrm{d}t = \phi'(\tilde{t})\mathrm{d}\tilde{t}$

Substitution rule

$s : [\alpha, \beta] \to \mathbb{R}$
$\phi : [\tilde{\alpha}, \tilde{\beta}] \to [\alpha, \beta]$

$s'(\phi(\tilde{t}))\phi'(\tilde{t}) = (s(\phi(\tilde{t})))'$

$\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_{t})\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t)\mathrm{d}t$

Informally:
Substituting, $t = \phi(\tilde{t})$
$\frac{\mathrm{d}t}{\mathrm{d}\tilde{t}} = \phi'(\tilde{t})$
$\mathrm{d}t = \phi'(\tilde{t})\mathrm{d}\tilde{t}$

Substitution rule

$s : [\alpha, \beta] \to \mathbb{R}$
$\phi : [\tilde{\alpha}, \tilde{\beta}] \to [\alpha, \beta]$

$$s'(\phi(\tilde{t}))\phi'(\tilde{t}) = (s(\phi(\tilde{t})))'$$

$$\boxed{\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_{t})\, \underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t)\mathrm{d}t}$$

Informally:
Substituting, $t = \phi(\tilde{t})$
$\frac{\mathrm{d}t}{\mathrm{d}\tilde{t}} = \phi'(\tilde{t})$
$\mathrm{d}t = \phi'(\tilde{t})\mathrm{d}\tilde{t}$

$$\boxed{\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_{t})\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t)\mathrm{d}t}$$

$$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$$

$$\boxed{\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_{t})\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t)\mathrm{d}t}$$

$$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$$
$$\dot{\tilde{\gamma}}(\tilde{t})$$

$$\boxed{\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_{t})\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t)\mathrm{d}t}$$

$$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$$
$$\dot{\tilde{\gamma}}(\tilde{t}) = \dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})$$

$$\boxed{\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_{t})\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t)\mathrm{d}t}$$

$$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$$
$$\dot{\tilde{\gamma}}(\tilde{t}) = \dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})$$
$$\|\dot{\tilde{\gamma}}(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})\|$$

$$\boxed{\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_{t})\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t)\mathrm{d}t}$$

$$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$$
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$$\|\dot{\tilde{\gamma}}(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\||\phi'(\tilde{t})|$$

$$\boxed{\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_{t})\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t)\mathrm{d}t}$$

$$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$$
$$\dot{\tilde{\gamma}}(\tilde{t}) = \dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})$$
$$\|\dot{\tilde{\gamma}}(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\||\phi'(\tilde{t})|$$

Assume, $\phi'(t) > 0$

$$\boxed{\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_{t})\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t)\mathrm{d}t}$$

$$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$$
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$$\|\dot{\tilde{\gamma}}(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\||\phi'(\tilde{t})|$$

Assume, $\phi'(t) > 0$

$$\|\dot{\tilde{\gamma}}(\tilde{t})\|$$

$$\boxed{\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_{t})\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t)\mathrm{d}t}$$

$$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$$
$$\dot{\tilde{\gamma}}(\tilde{t}) = \dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})$$
$$\|\dot{\tilde{\gamma}}(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\||\phi'(\tilde{t})|$$

Assume, $\phi'(t) > 0$

$$\|\dot{\tilde{\gamma}}(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\|\phi'(\tilde{t})$$

$$\boxed{\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_{t})\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t)\mathrm{d}t}$$

$$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$$
$$\dot{\tilde{\gamma}}(\tilde{t}) = \dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})$$
$$\|\dot{\tilde{\gamma}}(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\||\phi'(\tilde{t})|$$

Assume, $\phi'(t) > 0$

$$\|\dot{\tilde{\gamma}}(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\|\phi'(\tilde{t})$$

$$\boxed{\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_{t})\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t)\mathrm{d}t}$$

$$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$$
$$\dot{\tilde{\gamma}}(\tilde{t}) = \dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})$$
$$\|\dot{\tilde{\gamma}}(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\||\phi'(\tilde{t})|$$

Assume, $\phi'(t) > 0$

$$\int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\tilde{\gamma}}(\tilde{t})\|\mathrm{d}\tilde{t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\gamma}(\phi(\tilde{t}))\|\phi'(\tilde{t})\mathrm{d}\tilde{t}$$

$$\boxed{\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_{t})\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t)\mathrm{d}t}$$

$$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$$
$$\dot{\tilde{\gamma}}(\tilde{t}) = \dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})$$
$$\|\dot{\tilde{\gamma}}(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\||\phi'(\tilde{t})|$$

Assume, $\phi'(t) > 0$

$$\int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\tilde{\gamma}}(\tilde{t})\|\mathrm{d}\tilde{t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\gamma}(\underbrace{\phi(\tilde{t})}_{t})\|\phi'(\tilde{t})\mathrm{d}\tilde{t}$$

$$\boxed{\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_{t})\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t)\mathrm{d}t}$$

$$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$$
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$$\|\dot{\tilde{\gamma}}(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\||\phi'(\tilde{t})|$$

Assume, $\phi'(t) > 0$

$$\int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\tilde{\gamma}}(\tilde{t})\|\mathrm{d}\tilde{t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\gamma}(\underbrace{\phi(\tilde{t})}_{t})\|\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t}$$

$$\boxed{\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_{t})\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t)\mathrm{d}t}$$

$$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$$
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$$\|\dot{\tilde{\gamma}}(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\||\phi'(\tilde{t})|$$

Assume, $\phi'(t) > 0$

$$\int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\tilde{\gamma}}(\tilde{t})\|\mathrm{d}\tilde{t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\gamma}(\underbrace{\phi(\tilde{t})}_{t})\|\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{\phi(\tilde{\alpha})}^{\phi(\tilde{\beta})}$$

$$\boxed{\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_{t})\, \underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t)\mathrm{d}t}$$

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$$\|\dot{\tilde{\gamma}}(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\||\phi'(\tilde{t})|$$

Assume, $\phi'(t) > 0$

$$\int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\tilde{\gamma}}(\tilde{t})\|\mathrm{d}\tilde{t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\gamma}(\underbrace{\phi(\tilde{t})}_{t})\|\, \underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{\phi(\tilde{\alpha})}^{\phi(\tilde{\beta})} \|\dot{\gamma}(t)\|$$

$$\boxed{\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_{t})\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t)\mathrm{d}t}$$

$$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$$
$$\dot{\tilde{\gamma}}(\tilde{t}) = \dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})$$
$$\|\dot{\tilde{\gamma}}(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\||\phi'(\tilde{t})|$$

Assume, $\phi'(t) > 0$

$$\int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\tilde{\gamma}}(\tilde{t})\|\mathrm{d}\tilde{t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\gamma}(\underbrace{\phi(\tilde{t})}_{t})\|\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{\phi(\tilde{\alpha})}^{\phi(\tilde{\beta})} \|\dot{\gamma}(t)\|\mathrm{d}t$$

$$\boxed{\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_{t})\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t)\mathrm{d}t}$$

$$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$$
$$\dot{\tilde{\gamma}}(\tilde{t}) = \dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})$$
$$\|\dot{\tilde{\gamma}}(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\||\phi'(\tilde{t})|$$

Assume, $\phi'(t) > 0$

$$\int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\tilde{\gamma}}(\tilde{t})\|\mathrm{d}\tilde{t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\gamma}(\underbrace{\phi(\tilde{t})}_{t})\|\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{\phi(\tilde{\alpha})}^{\phi(\tilde{\beta})} \|\dot{\gamma}(t)\|\mathrm{d}t$$

We have proved,

$$\boxed{\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_{t})\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t)\mathrm{d}t}$$

$$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$$
$$\dot{\tilde{\gamma}}(\tilde{t}) = \dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})$$
$$\|\dot{\tilde{\gamma}}(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\||\phi'(\tilde{t})|$$

Assume, $\phi'(t) > 0$

$$\int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\tilde{\gamma}}(\tilde{t})\|\mathrm{d}\tilde{t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\gamma}(\underbrace{\phi(\tilde{t})}_{t})\|\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{\phi(\tilde{\alpha})}^{\phi(\tilde{\beta})} \|\dot{\gamma}(t)\|\mathrm{d}t$$

We have proved,

Theorem. *The arc length*

$$\boxed{\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_{t})\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t)\mathrm{d}t}$$

$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$
$\dot{\tilde{\gamma}}(\tilde{t}) = \dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})$
$\|\dot{\tilde{\gamma}}(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\||\phi'(\tilde{t})|$

Assume, $\phi'(t) > 0$

$$\int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\tilde{\gamma}}(\tilde{t})\|\mathrm{d}\tilde{t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\gamma}(\underbrace{\phi(\tilde{t})}_{t})\|\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{\phi(\tilde{\alpha})}^{\phi(\tilde{\beta})} \|\dot{\gamma}(t)\|\mathrm{d}t$$

We have proved,

Theorem. *The arc length is invariant*

$$\boxed{\int_{\tilde{t}=\tilde{\alpha}}^{\tilde{t}=\tilde{\beta}} F(\underbrace{\phi(\tilde{t})}_{t})\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{t=\phi(\tilde{\alpha})}^{t=\phi(\tilde{\beta})} F(t)\mathrm{d}t}$$

$$\tilde{\gamma}(\tilde{t}) = \gamma(\phi(\tilde{t}))$$
$$\dot{\tilde{\gamma}}(\tilde{t}) = \dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})$$
$$\|\dot{\tilde{\gamma}}(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\phi'(\tilde{t})\| = \|\dot{\gamma}(\phi(\tilde{t}))\||\phi'(\tilde{t})|$$

Assume, $\phi'(t) > 0$

$$\int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\tilde{\gamma}}(\tilde{t})\|\mathrm{d}\tilde{t} = \int_{\tilde{\alpha}}^{\tilde{\beta}} \|\dot{\gamma}(\underbrace{\phi(\tilde{t})}_{t})\|\,\underbrace{\phi'(\tilde{t})\mathrm{d}\tilde{t}}_{\mathrm{d}t} = \int_{\phi(\tilde{\alpha})}^{\phi(\tilde{\beta})} \|\dot{\gamma}(t)\|\mathrm{d}t$$

We have proved,

Theorem. *The arc length is invariant under reparametrization.*