

## Exercise sheet 2

Curves and Surfaces, MTH201

1. For  $\mathbf{v} : (\alpha, \beta) \rightarrow \mathbf{R}^2$  and  $\mathbf{w} : (\alpha, \beta) \rightarrow \mathbf{R}^2$ , show that  $(\mathbf{v}(t) \cdot \mathbf{w}(t))' = \mathbf{v}'(t) \cdot \mathbf{w}(t) + \mathbf{v}(t) \cdot \mathbf{w}'(t)$ .
2. If  $\mathbf{n} : (\alpha, \beta) \rightarrow \mathbf{R}^2$  is such that  $\|\mathbf{n}(t)\|$  is constant, then prove that  $\dot{\mathbf{n}}(t)$  is either 0 or perpendicular to  $\mathbf{n}(t)$ .
3. For  $\mathbf{v} : (\alpha, \beta) \rightarrow \mathbf{R}^2$  and  $\mathbf{w} : (\alpha, \beta) \rightarrow \mathbf{R}^2$ , show that  $(\mathbf{v}(t) \cdot \mathbf{w}(t))' = \mathbf{v}'(t) \cdot \mathbf{w}(t) + \mathbf{v}(t) \cdot \mathbf{w}'(t)$  (Assume that all the functions are smooth).
4. If we denote,

$$s_\alpha(t) := \int_{t_\alpha}^t \|\dot{\gamma}(u)\| du$$
$$s_\beta(t) := \int_{t_\beta}^t \|\dot{\gamma}(u)\| du$$

Prove that  $s_\beta(t) - s_\alpha(t)$  is a constant.

5. If  $\gamma : (\alpha, \beta) \rightarrow \mathbf{R}^2$  is a smooth parametrization, then show that  $\|\dot{\gamma}(t)\| : (\alpha, \beta) \rightarrow \mathbf{R}$  is smooth.
6. For the parametrization  $\gamma : (-\pi/2, \pi/2) \rightarrow \mathbf{R}^2$  given by  $\gamma(t) = (5 \cos(t), 5 \sin(t))$ ,
  - (a) Find the arc-length function  $s(t)$  (starting at, say, 0)
  - (b) Find a reparametrization map  $\phi$  so that  $\gamma(\phi(t))$  is a unit-speed parametrization.

To be updated...