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$(f/g)' = \frac{f'g - fg'}{g^2}$	$(\frac{\sin(x)}{x^2})' = \frac{\cos(x)x^2 - \sin(x)2x}{x^4}$
$(f(g(x)))' = f'(g(x))g'(x)$	$(\sin(x^2))' = \cos(x^2)2x$

Derivative facts

- 1. $c' = 0$
- 2. $(x^n)' = nx^{n-1}$
- 3. $(\sin(x))' = \cos(x)$
- 4. $(\cos(x))' = -\sin(x)$
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- 6. $(\log(x))' = \frac{1}{x}$
- 7. $(e^x)' = e^x$

Example.

$$(x^2 \sin(x^3) + \cos(x))' =$$

Rule

$$(cf)' = cf',$$

where $c \in \mathbb{R}$

$$(f + g)' = f' + g'$$

$$(f - g)' = f' - g'$$

$$(fg)' = f'g + fg'$$

$$(f/g)' = \frac{f'g - fg'}{g^2}$$

$$(f(g(x)))' = f'(g(x))g'(x)$$

Example

$$(2 \sin(x))' = 2 \cos(x)$$

$$(\sin(x) + x^3)' = \cos(x) + 3x^2$$

$$(\sin(x) - x^3)' = \cos(x) - 3x^2$$

$$(x^2 \sin(x))' = 2x \sin(x) + x^2 \cos(x)$$

$$\left(\frac{\sin(x)}{x^2}\right)' = \frac{\cos(x)x^2 - \sin(x)2x}{x^4}$$

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Derivative facts

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Example.

$$(x^2 \sin(x^3) + \cos(x))' = 2x \sin(x^3) + x^2(\sin(x^3))'$$

Rule	Example
$(cf)' = cf'$, where $c \in \mathbb{R}$	$(2 \sin(x))' = 2 \cos(x)$
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$$\begin{aligned}(x^2 \sin(x^3) + \cos(x))' &= 2x \sin(x^3) + x^2(\sin(x^3))' \\ &\quad - \sin(x) \\ &= \end{aligned}$$

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Example

Example

γ :

Example

$$\gamma : (-\pi,$$

Example

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Example

$$\gamma : (-\pi, \pi) \rightarrow \mathbb{R}^2$$

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$$\gamma(t)$$

Example

$$\gamma : (-\pi, \pi) \rightarrow \mathbb{R}^2$$

$$\gamma(t) := (r \cos(t), r \sin(t))$$

Example

$$\begin{aligned}\gamma &: (-\pi, \pi) \rightarrow \mathbb{R}^2 \\ \gamma(t) &:= (r \cos(t), r \sin(t)) \\ \dot{\gamma}(t)\end{aligned}$$

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The “speed” at time t is defined as

$$\sqrt{v_1(t)^2 + v_2(t)^2}$$

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Can we find a parametrization of the circle

Example

Making the “speed” 1

$$\begin{aligned}\gamma &: (-\pi, \pi) \rightarrow \mathbb{R}^2 \\ \gamma(t) &:= (r \cos(t), r \sin(t)) \\ \dot{\gamma}(t) &:= (\underbrace{-r \sin(t)}_{v_1(t)}, \underbrace{r \cos(t)}_{v_2(t)})\end{aligned}$$

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Can we find a parametrization of the circle to ensure the speed is 1?

Example

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Making the “speed” 1

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Making the “speed” 1

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Making the “speed” 1

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Making the “speed” 1

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Making the “speed” 1

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$$\sqrt{v_1(t)^2 + v_2(t)^2}$$

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Can we find a parametrization of the circle to ensure the speed is 1?

Making the “speed” 1

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