

Exercise sheet 2

Curves and Surfaces, MTH201

1. For $\mathbf{v} : (\alpha, \beta) \rightarrow \mathbf{R}^2$ and $\mathbf{w} : (\alpha, \beta) \rightarrow \mathbf{R}^2$, show that $(\mathbf{v}(t) \cdot \mathbf{w}(t))' = \mathbf{v}'(t) \cdot \mathbf{w}(t) + \mathbf{v}(t) \cdot \mathbf{w}'(t)$.
2. If $\mathbf{n} : (\alpha, \beta) \rightarrow \mathbf{R}^2$ is such that $\|\mathbf{n}(t)\|$ is constant, then prove that $\dot{\mathbf{n}}(t)$ is either 0 or perpendicular to $\mathbf{n}(t)$.
3. For $\mathbf{v} : (\alpha, \beta) \rightarrow \mathbf{R}^2$ and $\mathbf{w} : (\alpha, \beta) \rightarrow \mathbf{R}^2$, show that $(\mathbf{v}(t) \cdot \mathbf{w}(t))' = \mathbf{v}'(t) \cdot \mathbf{w}(t) + \mathbf{v}(t) \cdot \mathbf{w}'(t)$ (Assume that all the functions are smooth).
4. If we denote,

$$s_\alpha(t) := \int_{t_\alpha}^t \|\dot{\gamma}(u)\| du$$
$$s_\beta(t) := \int_{t_\beta}^t \|\dot{\gamma}(u)\| du$$

Prove that $s_\beta(t) - s_\alpha(t)$ is a constant.

5. If $\gamma : (\alpha, \beta) \rightarrow \mathbb{R}^2$ is a smooth parametrization, then show that $\|\dot{\gamma}(t)\| : (\alpha, \beta) \rightarrow \mathbb{R}$ is smooth.
6. For the parametrization $\gamma : (-\pi/2, \pi/2) \rightarrow \mathbb{R}^2$ given by $\gamma(t) = (5 \cos(t), 5 \sin(t))$,
 - (a) Find the arc-length function $s(t)$ (starting at, say, 0)
 - (b) Find a reparametrization map ϕ so that $\gamma(\phi(t))$ is a unit-speed parametrization.