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$\|\mathbf{e}_1(t)\| = 1$, $\|\mathbf{e}_2(t)\| = 1$, and $\mathbf{e}_1(t).\mathbf{e}_2(t) = 0$

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For any, $\mathbf{v}(t) \in \mathbb{R}^2$,

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$\mathbf{e}_1(t), \mathbf{e}_2(t) \in \mathbb{R}^2$

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$\{\mathbf{T}(t), \mathbf{N}_s(t)\}$ form an orthonormal basis,

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$\mathbf{e}_1(t), \mathbf{e}_2(t) \in \mathbb{R}^2$

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$\{\mathbf{T}(t), \mathbf{N}_s(t)\}$ form an orthonormal basis, for each t.

For any, $\mathbf{v}(t) \in \mathbb{R}^2$,

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for some $\alpha(t), \beta(t) \in \mathbb{R}$ (uniquely represented like this!)

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So, $\mathbf{v}(t), \mathbf{e}_1(t), \mathbf{e}_2(t)$ smooth $\implies \alpha(t)$, $\beta(t)$ smooth.

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$\dot{\mathbf{T}}(t) = 0\mathbf{T}(t) + \kappa_s(t)\mathbf{N}_s(t)$

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Exercise. If γ

Exercise. If $\gamma : (\alpha, \beta)$

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Solution. Let the curvature be κ.

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□

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Exercise. If $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a unit speed parametrization with constant (non-zero) curvature, then show that $\gamma(t)$ lies on a circle for every t.

Solution. Let the curvature be κ.
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