Exercise sheet 3

Curves and Surfaces, MTH201

1. In an earlier exercise you found the parametrization of a line segment joining two points. Use that parametrization to find the arc length of the line segment in terms of its end points. Try with some other parametrization too.

2. These steps will show that the line segment joining two points is the shortest possible curve joining the two points:

   (a) Show that $\mathbf{v}.\mathbf{w} \leq \|\mathbf{v}\|\|\mathbf{w}\|$ for any two vectors $\mathbf{v}$ and $\mathbf{w}$

   (b) Show that $\|\mathbf{v}\| = \mathbf{v}.\frac{\mathbf{v}}{\|\mathbf{v}\|}$. This provides another way to obtain the norm of a vector: take its dot product with a unit vector in the same direction.

   (c) The previous part shows that $\|\gamma(t_1)-\gamma(t_0)\| = (\gamma(t_1)-\gamma(t_0)).\frac{\gamma(t_1)-\gamma(t_0)}{\|\gamma(t_1)-\gamma(t_0)\|}$. Now use the fundamental theorem of calculus to (carefully!) prove that $\|\gamma(t_1) - \gamma(t_0)\| = \int_{t_0}^{t_1} \dot{\gamma}(t).\frac{\gamma(t_1)-\gamma(t_0)}{\|\gamma(t_1)-\gamma(t_0)\|}\mathrm{d}t$

   (d) Use the previous and first part to prove that $\|\gamma(t_1) - \gamma(t_0)\| \leq \int_{t_0}^{t_1} \|\dot{\gamma}(t)\|\mathrm{d}t$. Note that this shows that the distance between the end points is always less than or equal to the arc length of a curve joining the two end points.

3. If a parametrization $\gamma : (\alpha, \beta) \to \mathbb{R}^3$ satisfies the condition that $\|\ddot{\gamma}(t)\| = 0$ for all t, what kind of curve will it trace out?

4. If a parametrization $\gamma : (\alpha, \beta) \to \mathbb{R}^3$ satisfies the condition that $\ddot{\gamma}(t)$ is constant, what kind of curve will it trace out?

to be updated...