Curvature

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Exercise. Show that the curvature at any point of any line

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Exercise. Show that the curvature at any point of any line is 0.