

# Hints / Solutions to Exercise sheet 4

Curves and Surfaces, MTH201

**Question 1:** Show that the curvature at any point of a line segment is always 0.

**Solution 1:** A line segment is parametrized by  $\gamma(t) = p + \mathbf{v}t$ . Note that  $\dot{\gamma}(t) = \mathbf{v}$  and so  $\|\dot{\gamma}(t)\| = \|\mathbf{v}\|$ . So a unit speed reparametrization is  $\tilde{\gamma}(\tilde{t}) = p + \frac{\mathbf{v}}{\|\mathbf{v}\|}\tilde{t}$ . Now  $\ddot{\gamma}(\tilde{t}) = 0$ .

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**Question 3:** Given *any* smooth parametrization,  $\gamma : (\alpha, \beta) \rightarrow \mathbb{R}^2$ , is the curvature function  $\kappa(t)$  always smooth? Do you need to add some condition? What is it?

**Hint 3:** The definition of curvature involves a norm, which involves taking a square root.  $\sqrt{f(t)}$  is smooth if and only if  $f(t) > 0$ . So it is smooth only if the acceleration is not 0, which is itself equivalent to the curvature being non-zero.

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**Question 5:** If  $\gamma : (\alpha, \beta) \rightarrow \mathbb{R}^2$  parametrizes a curve, compute the curvature of the curve parametrized by  $\tilde{\gamma}(t) = \gamma(-t)$  in terms of the curvature of  $\gamma$ . What about the relation between the signed curvatures of  $\gamma$  and  $\tilde{\gamma}$ ?

**Hint 5:** After computing, you will realize that they differ only by a sign. Intuitively, if you move along a curve in the opposite direction, then you “turn” in the other direction.