Hints / Solutions to Exercise sheet 3

Curves and Surfaces, MTH201

Question 1: In an earlier exercise you found the parametrization of a line segment joining two points. Use that parametrization to find the arc length of the line segment in terms of its end points. Try with some other parametrization too.

Solution 1: The line segment between p and q can be obtained by considering the displacement vector q - p. The parametrization should be so that at t = 0, p is not being displaced at all, but when t = 1, p is displaced to q:

$$\begin{split} \gamma(t) &= p + t(q-p) \\ \dot{\gamma}(t) &= (q-p) \\ \|\dot{\gamma}(t)\| &= \|q-p\| \\ \int_0^1 \|\dot{\gamma}(t)\mathrm{d}t\| &= \int_0^1 \|q-p\|\mathrm{d}t = \|q-p\| \end{split}$$

Question 2: These steps will show that the line segment joining two points is the shortest possible curve joining the two points:

Solution 2: Remember that $\mathbf{v}.\mathbf{w} = \|\mathbf{v}\|\|\mathbf{w}\|\cos(\theta)$ where θ is the angle between \mathbf{v} and \mathbf{w} . Part 1.. easily follows from $\cos(\theta) \leq 1$ for any angle θ .

For part 2,

$$\mathbf{v} \cdot \frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{1}{\|\mathbf{v}\|} (\mathbf{v} \cdot \mathbf{v}) = \frac{\|\mathbf{v}\|^2}{\|\mathbf{v}\|} = \|\mathbf{v}\|$$

This part allows you to express the norm in a way that will allow you to use the fundamental theorem of calculus in the next part.