

Chain rule for mult-variable functions

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Chain rule for mult-variable functions

$$f : \mathbb{R}^2$$

Chain rule for mult-variable functions

$$f : \mathbb{R}^2 \rightarrow \mathbb{R}$$

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$$f : \mathbb{R}^2 \rightarrow \mathbb{R}$$

$$\gamma$$

Chain rule for mult-variable functions

$$f : \mathbb{R}^2 \rightarrow \mathbb{R}$$

$$\gamma : (\alpha, \beta)$$

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$$f : \mathbb{R}^2 \rightarrow \mathbb{R}$$

$$\gamma : (\alpha, \beta) \rightarrow \mathbb{R}^2$$

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$$\gamma(t)$$

Chain rule for mult-variable functions

$$f : \mathbb{R}^2 \rightarrow \mathbb{R}$$

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$$\gamma(t) = (x(t), y(t))$$

Chain rule for mult-variable functions

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$$f \circ \gamma$$

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$$(f \circ \gamma)'(t_0)$$

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$$f \circ \gamma : (\alpha, \beta) \rightarrow \mathbb{R}$$

$$(f \circ \gamma)'(t_0) = f_x(x(t_0), y(t_0))x'(t_0) + f_y(x(t_0), y(t_0))y'(t_0)$$

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$$f_y(x(t_0), y(t_0))y'(t_0) = \nabla(f)(x(t_0), y(t_0)) \cdot \dot{\gamma}(t_0),$$

$$\text{where } \nabla(f)(x, y) = (f_x(x, y), f_y(x, y)),$$

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$$[f_{\mathbf{v}}(x(t_0), y(t_0)) := (f \circ \gamma)'(t_0) = \nabla(f)(x(t_0), y(t_0)) \cdot \dot{\gamma}(t_0)],$$

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$$\gamma(u, v)$$

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$$f \circ \gamma : \mathbb{R}^2 \rightarrow \mathbb{R}$$

$$(f \circ \gamma)_u(u_0, v_0)$$

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$$\begin{aligned} f &: \mathbb{R}^2 \rightarrow \mathbb{R} \\ \gamma &: (\alpha, \beta) \rightarrow \mathbb{R}^2 \\ \gamma(t) &= (x(t), y(t)) \\ f \circ \gamma &: (\alpha, \beta) \rightarrow \mathbb{R} \end{aligned}$$

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$$\begin{aligned} &(f \circ \gamma)_u(u_0, v_0) \\ &= f_x(x(u_0, v_0), y(u_0, v_0))x_u(u_0, v_0) \\ &+ f_y(x(u_0, v_0), y(u_0, v_0))y_u(u_0, v_0) \end{aligned}$$

Curves on surfaces

Definition. $\mathbf{v} \in \mathbb{R}^3$ is a tangent vector of the surface S at a point p , if there is a γ

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Definition. $\mathbf{v} \in \mathbb{R}^3$ is a tangent vector of the surface S at a point p , if there is a $\gamma : (\alpha, \beta)$

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Definition. $\mathbf{v} \in \mathbb{R}^3$ is a tangent vector of the surface S at a point p , if there is a $\gamma : (\alpha, \beta) \rightarrow S$

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Definition. $\mathbf{v} \in \mathbb{R}^3$ is a tangent vector of the surface S at a point p , if there is a $\gamma : (\alpha, \beta) \rightarrow S \subset \mathbb{R}^3$ so that $p = \gamma(t)$ and

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Curves on surfaces

Definition. $\mathbf{v} \in \mathbb{R}^3$ is a tangent vector of the surface S at a point p , if there is a $\gamma : (\alpha, \beta) \rightarrow S \subset \mathbb{R}^3$ so that $p = \gamma(t)$ and $\mathbf{v} = \dot{\gamma}(t)$

$\gamma : (\alpha, \beta) \rightarrow S \subset \mathbb{R}^3$ is a curve.
 $\sigma : U \rightarrow S$ a surface patch.

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$$\begin{aligned}(f \circ \gamma)_u(u_0, v_0) \\ &= f_x(x(u_0, v_0), y(u_0, v_0))x_u(u_0, v_0) \\ &+ f_y(x(u_0, v_0), y(u_0, v_0))y_u(u_0, v_0)\end{aligned}$$

Curves on surfaces

Definition. $\mathbf{v} \in \mathbb{R}^3$ is a tangent vector of the surface S at a point p , if there is a $\gamma : (\alpha, \beta) \rightarrow S \subset \mathbb{R}^3$ so that $p = \gamma(t)$ and $\mathbf{v} = \dot{\gamma}(t)$

$\gamma : (\alpha, \beta) \rightarrow S \subset \mathbb{R}^3$ is a curve.

$\sigma : U \rightarrow S$ a surface patch.

So, $\gamma(t) = \sigma(x(t), y(t))$

Chain rule for mult-variable functions

$$f : \mathbb{R}^2 \rightarrow \mathbb{R}$$

$$\gamma : (\alpha, \beta) \rightarrow \mathbb{R}^2$$

$$\gamma(t) = (x(t), y(t))$$

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$$\boxed{f_{\mathbf{v}}(x(t_0), y(t_0)) := (f \circ \gamma)'(t_0) = \nabla(f)(p) \cdot \mathbf{v}},$$

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$$\dot{\gamma}(t) = x'(t)\sigma_x(p) + y'(t)\sigma_y(p)$$

The tangent vectors at $p \in S \subset \mathbb{R}^3$ always belong to the span of $\sigma_x(p)$ and $\sigma_y(p)$.

Exercise. Show that any vector that belongs to the span of $\sigma_x(p)$ and $\sigma_y(p)$, is a tangent vector.

Chain rule for mult-variable functions

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Curves on surfaces

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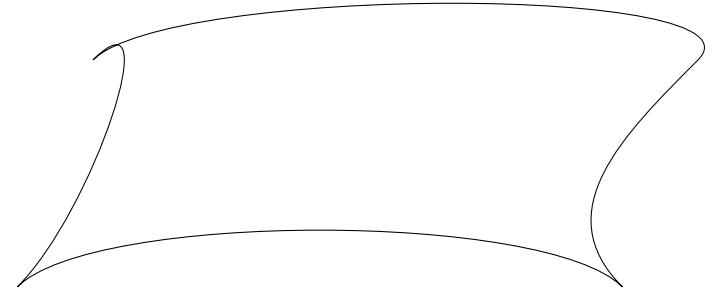
Exercise. Show that any vector that belongs to the span of $\sigma_x(p)$ and $\sigma_y(p)$, is a tangent vector.

Exercise. Show that σ is regular at p if and only if the tangent vectors at p form a two dimensional subspace of \mathbb{R}^3 .

A curve on a surface

Note:: This was not part of the lecture, however, since there seemed to be some confusion, this adds a diagram to support along with “subtitles”.

$$S \subset \mathbb{R}^3$$

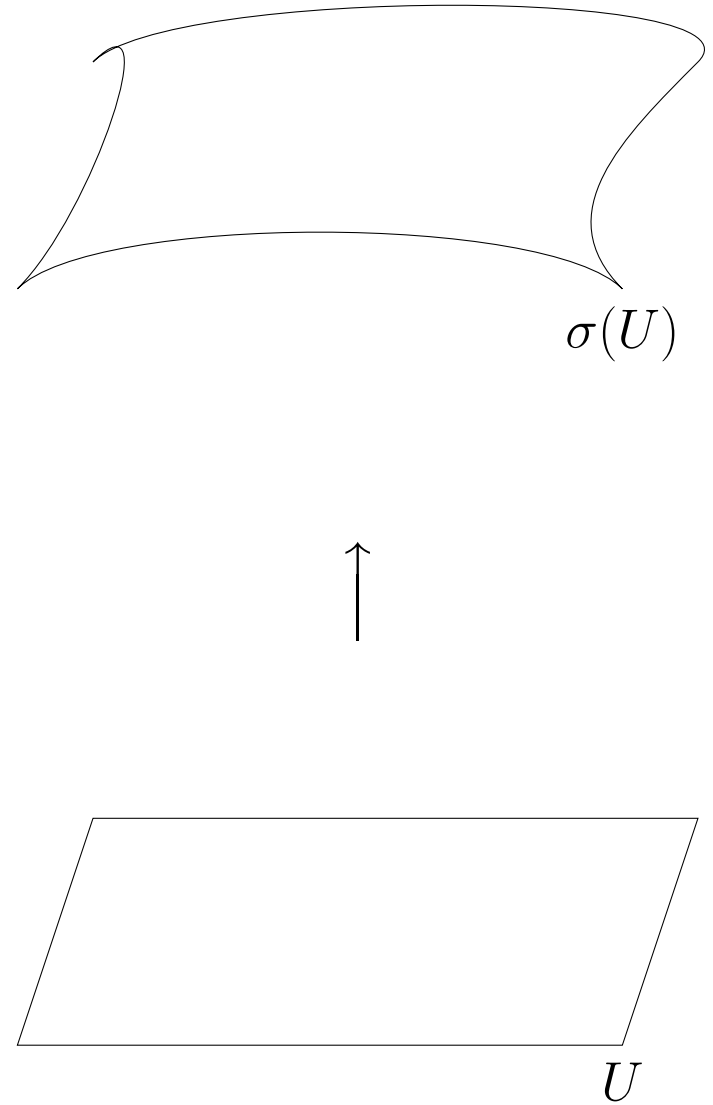


Consider a surface in space

A curve on a surface

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$$U \rightarrow S \subset \mathbb{R}^3$$

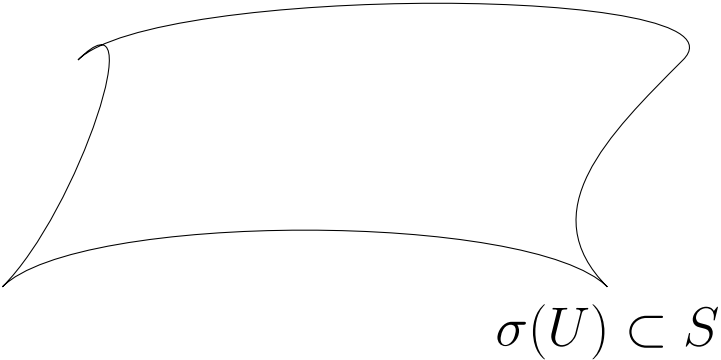


and a surface patch which is a map

A curve on a surface

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$$U \rightarrow S \subset \mathbb{R}^3$$

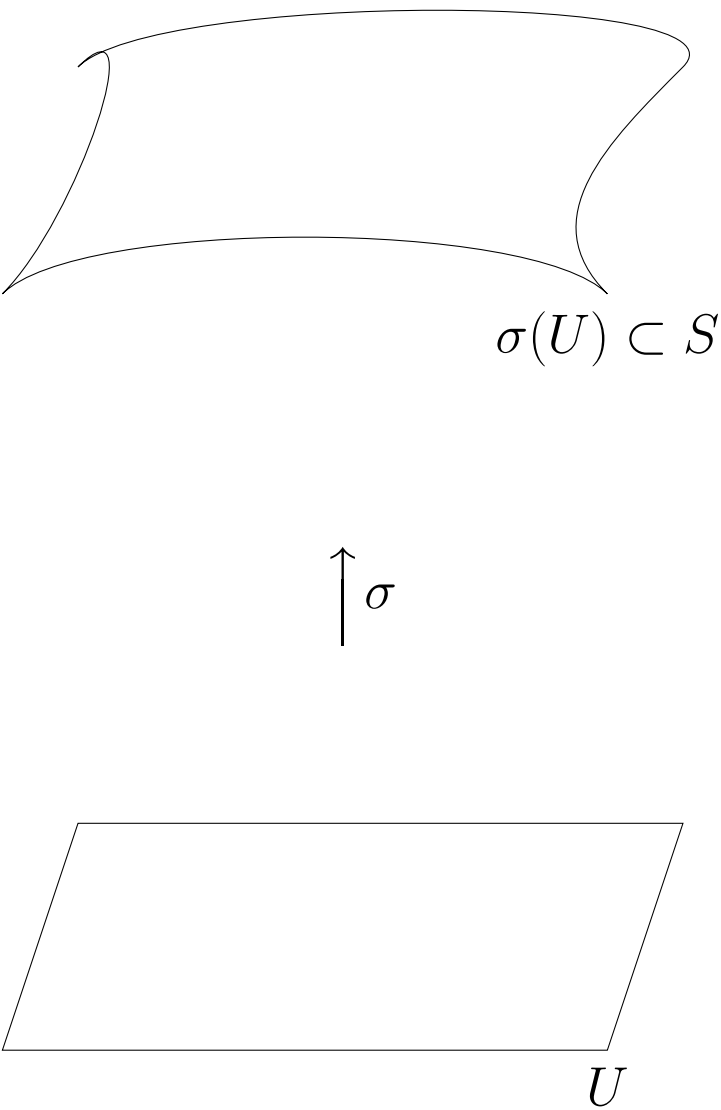


onto a part of the surface

A curve on a surface

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$$\sigma : U \rightarrow S \subset \mathbb{R}^3$$



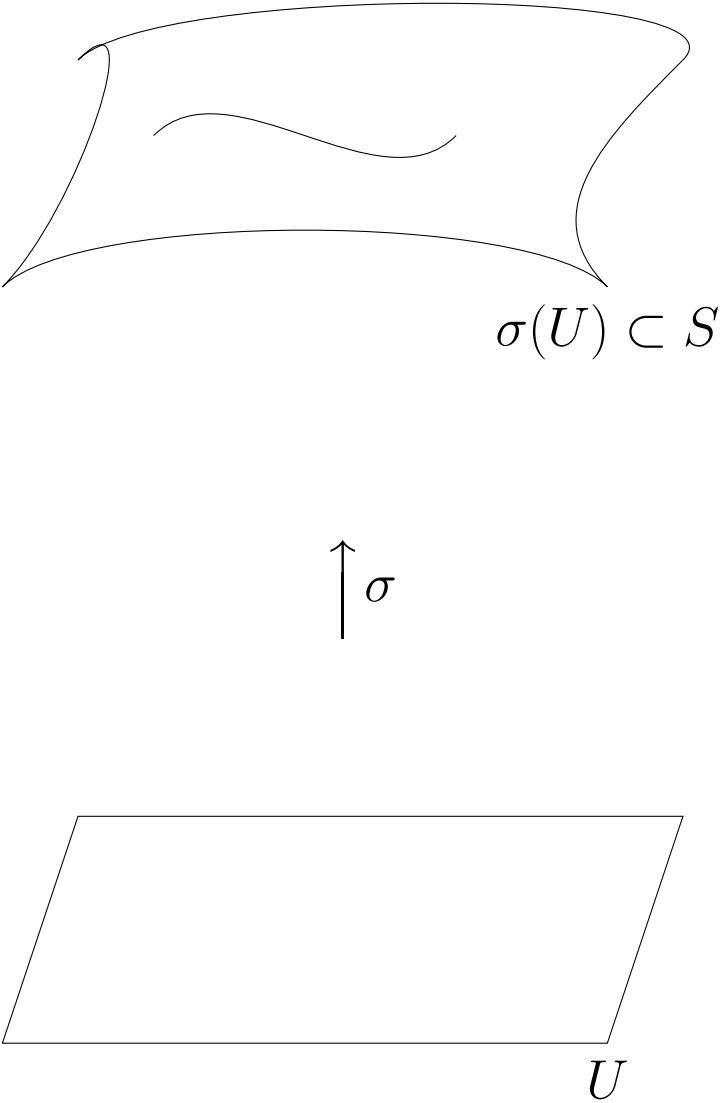
As usual we denote it by σ .

A curve on a surface

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$$\sigma : U \rightarrow S \subset \mathbb{R}^3$$

$$\gamma : (\alpha, \beta) \rightarrow S$$



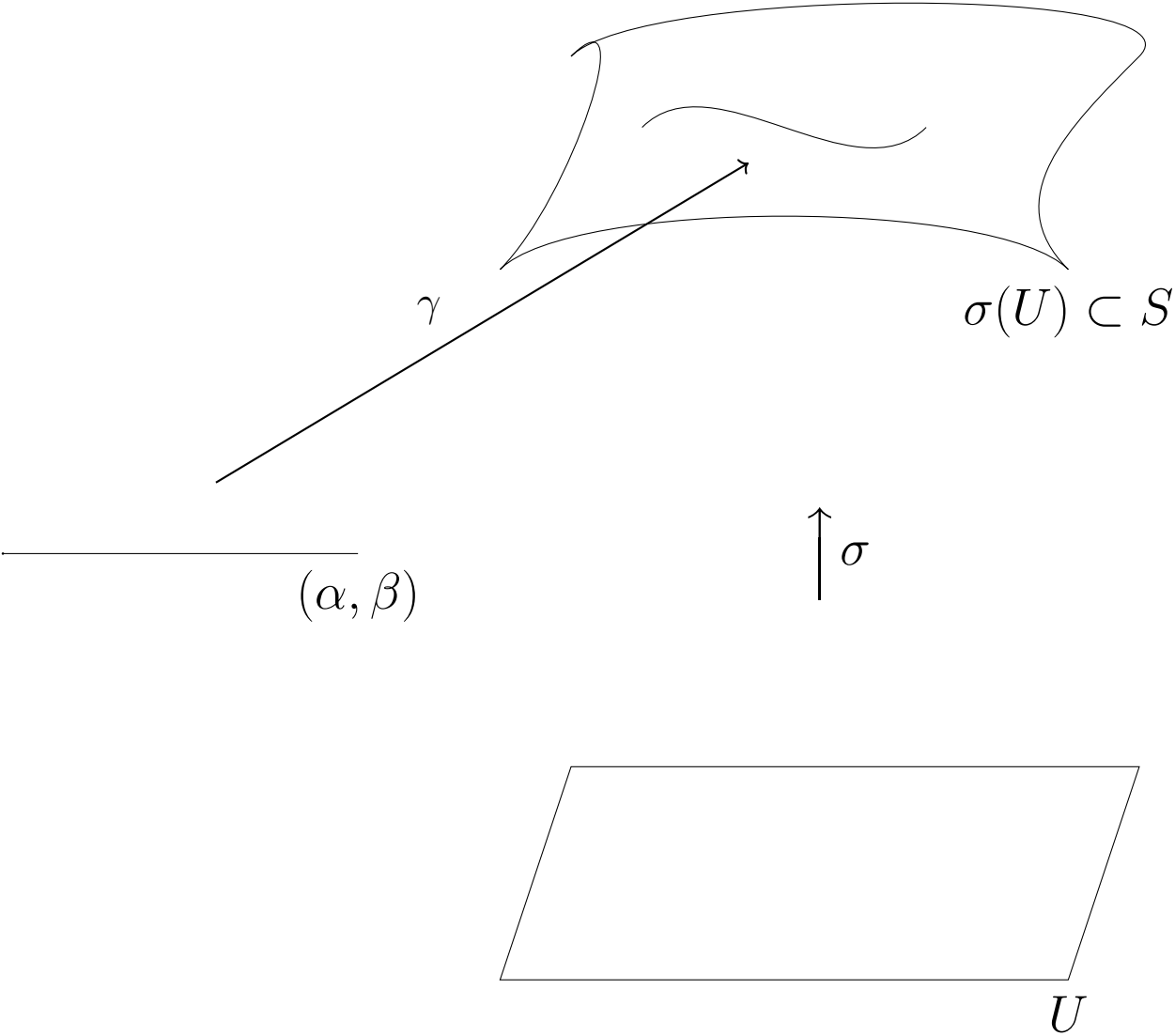
Now consider a curve on the surface

A curve on a surface

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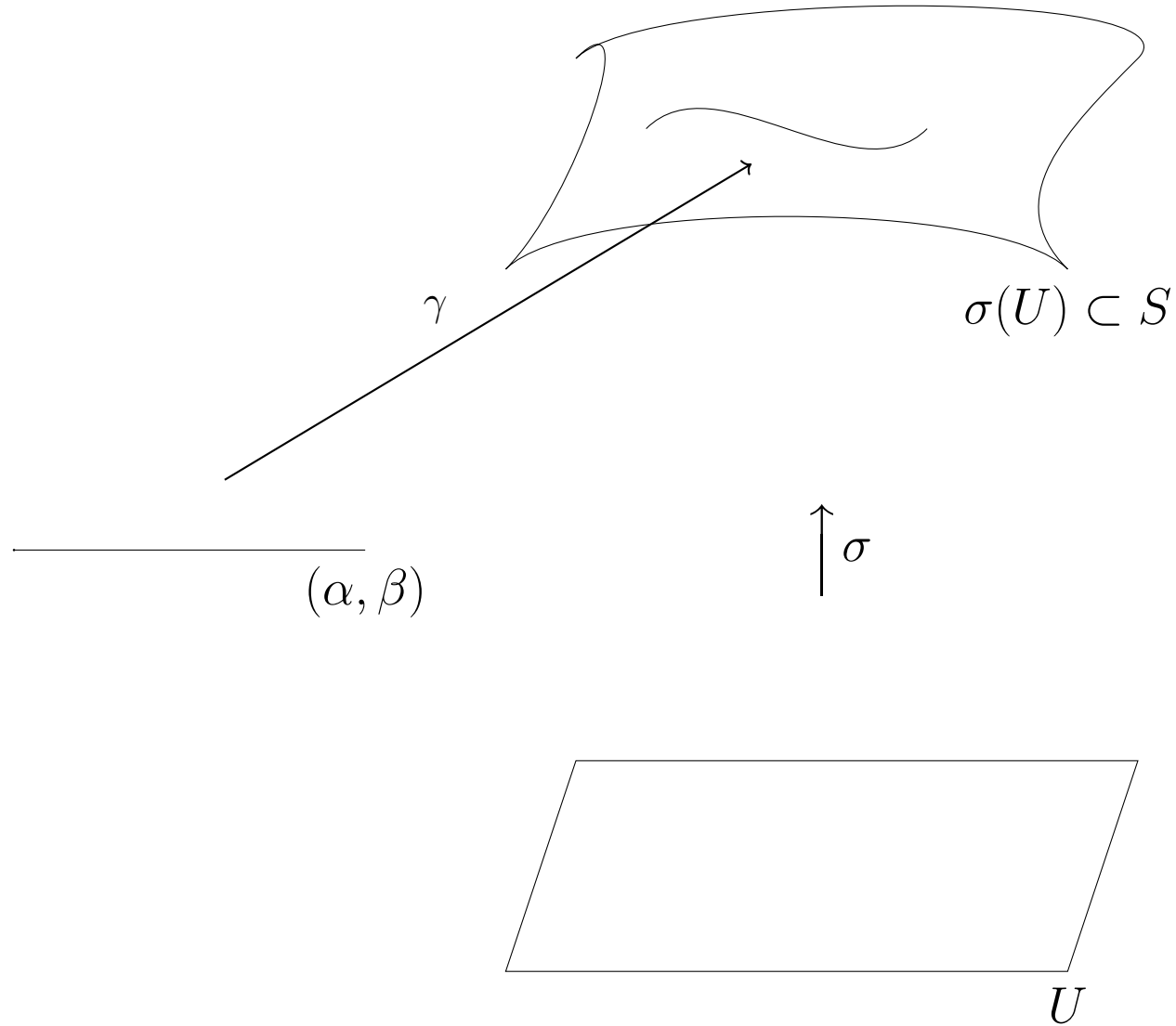
parametrized by γ

A curve on a surface

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$$\sigma : U \rightarrow S \subset \mathbb{R}^3$$

$$\gamma : (\alpha, \beta) \rightarrow \sigma(U) \subset S$$



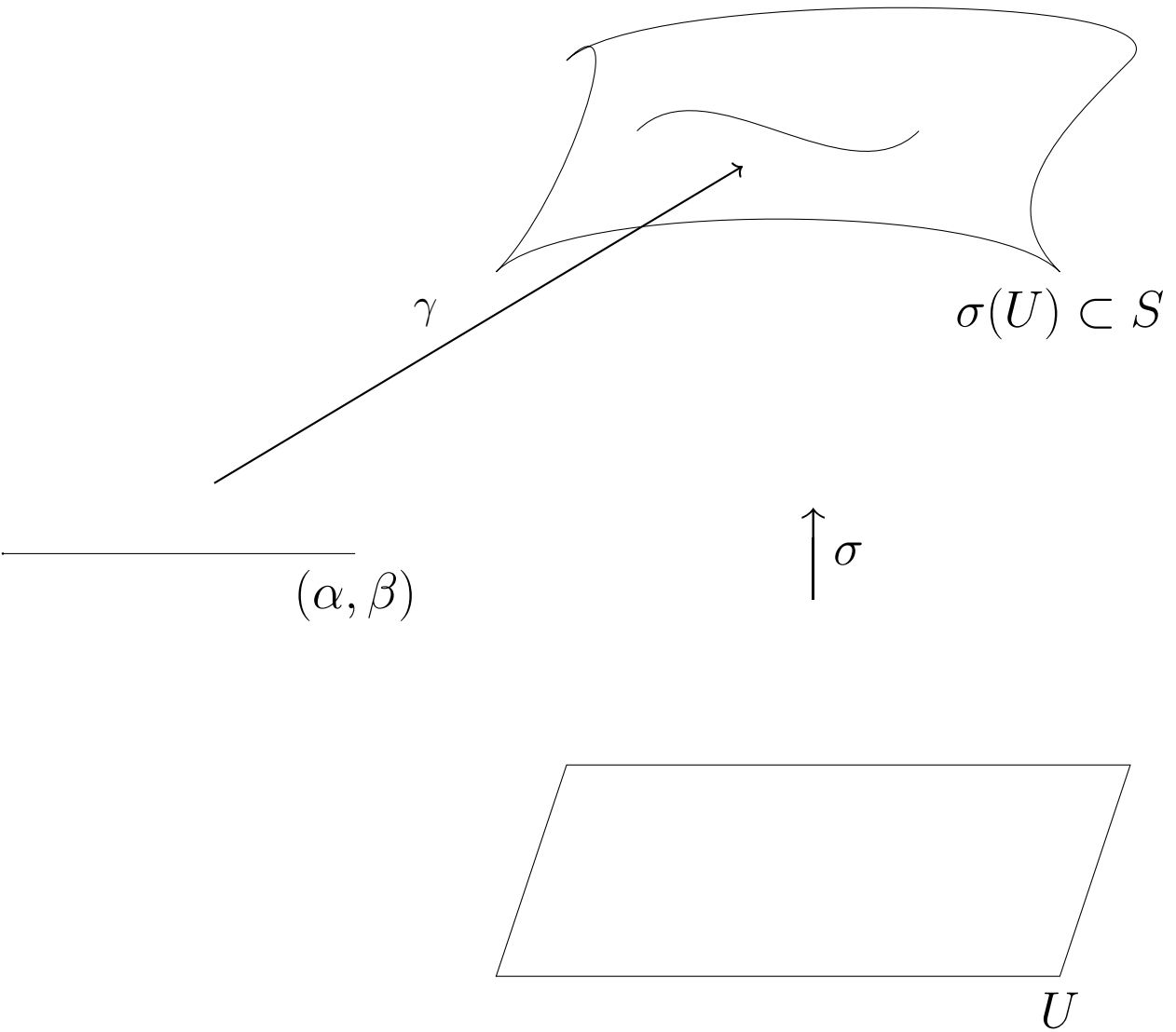
and let us assume it lies in the image of the surface patch

A curve on a surface

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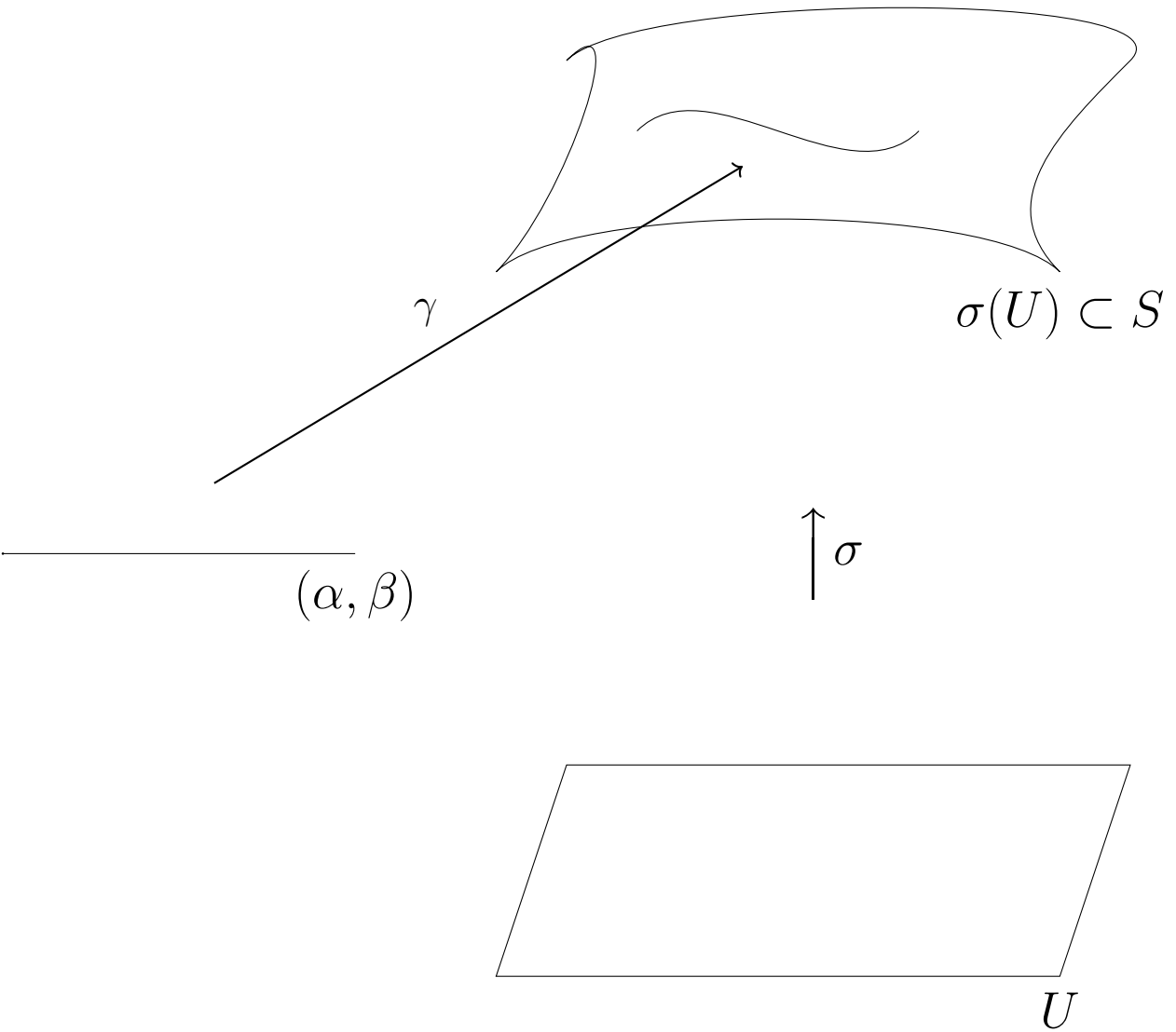
But it is also a curve in space

A curve on a surface

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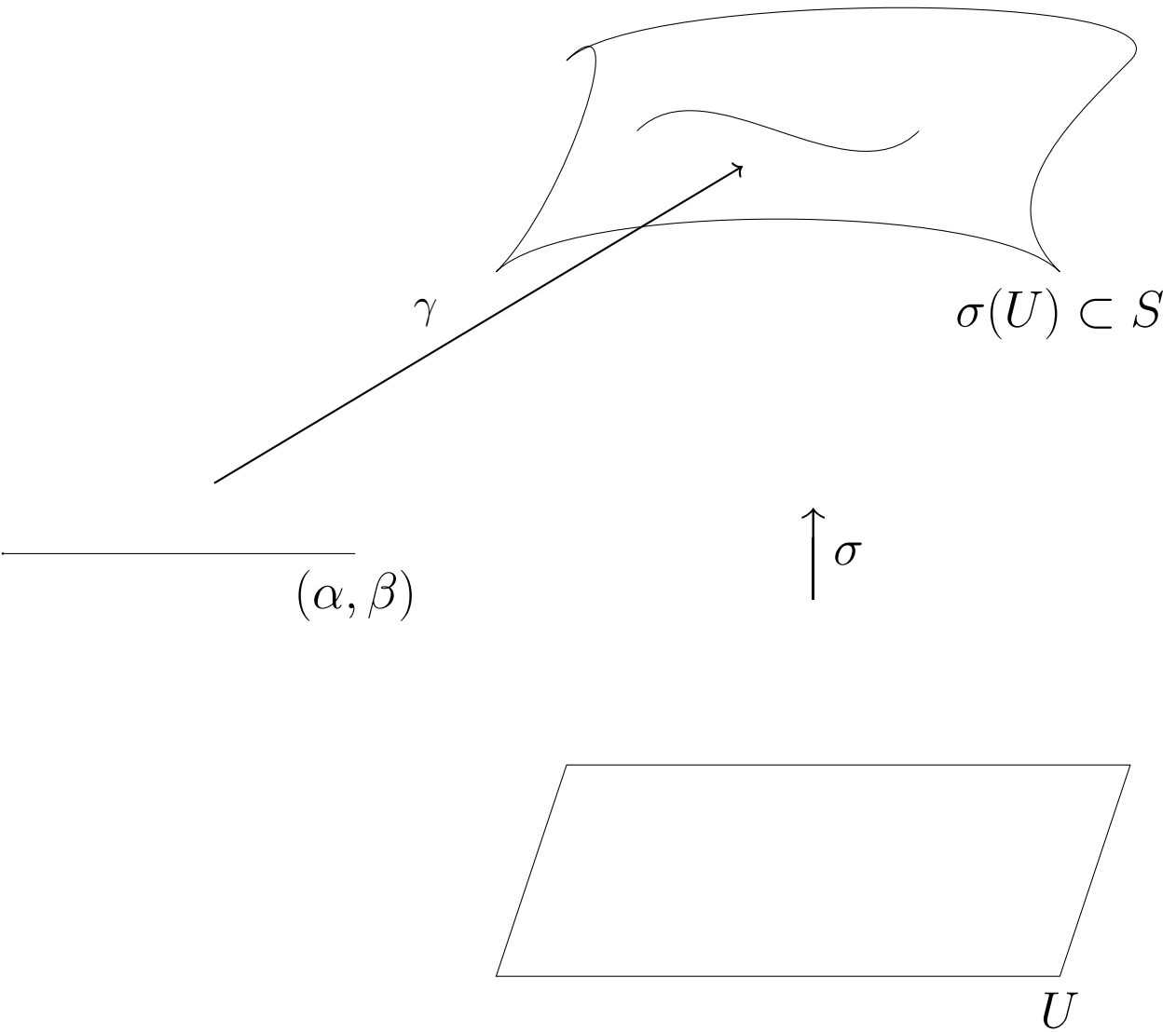
that happens to lie on a surface.

A curve on a surface

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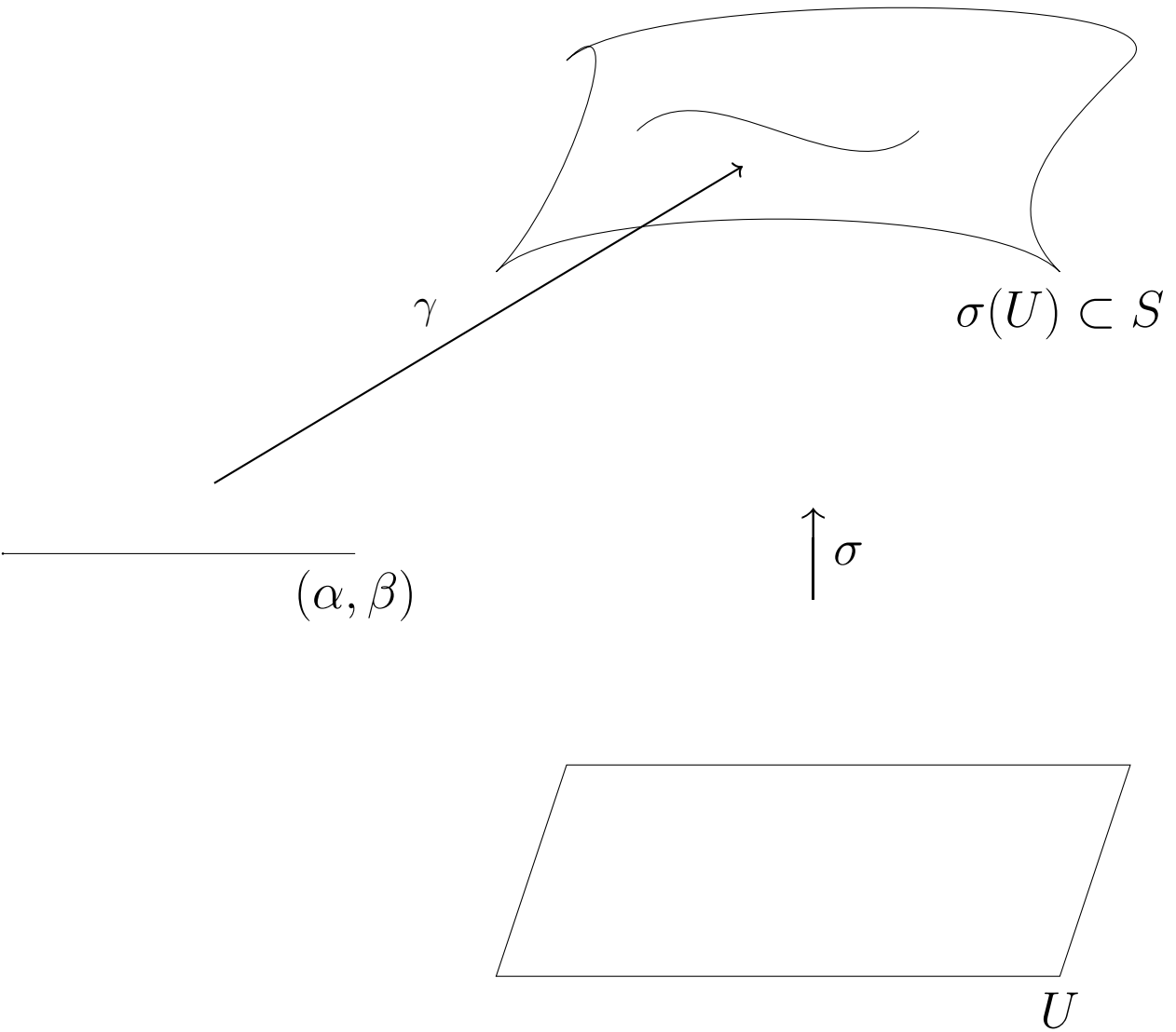
Let us see what lying on a surface

A curve on a surface

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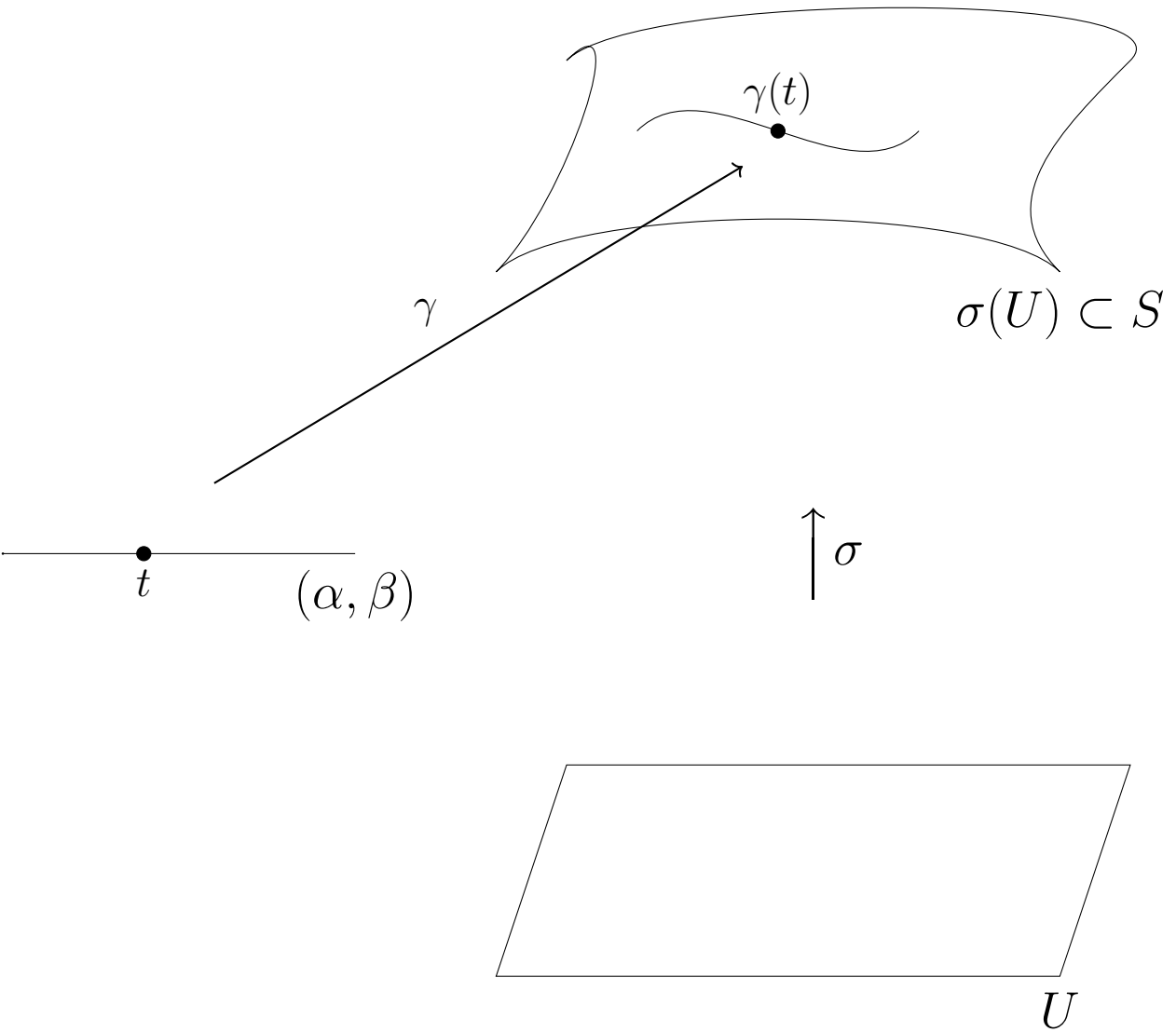
tells us about this space curve.

A curve on a surface

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A parameter t goes to $\gamma(t)$

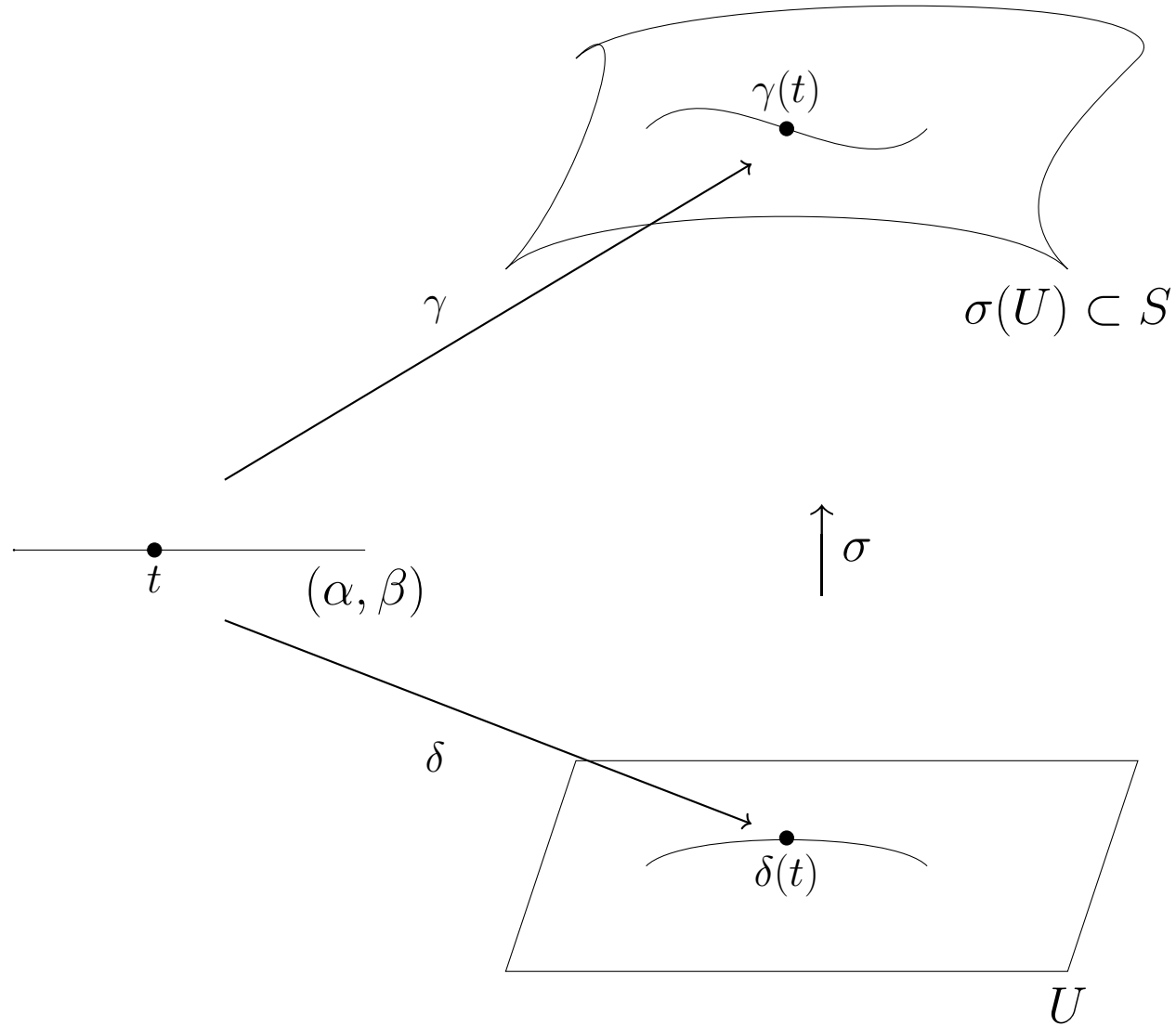
A curve on a surface

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$$\gamma : (\alpha, \beta) \rightarrow \sigma(U) \subset S \subset \mathbb{R}^3$$

$$\delta : (\alpha, \beta) \rightarrow U$$



But to each $\gamma(t) \in \sigma(U)$, σ corresponds a $\delta(t) \in U$

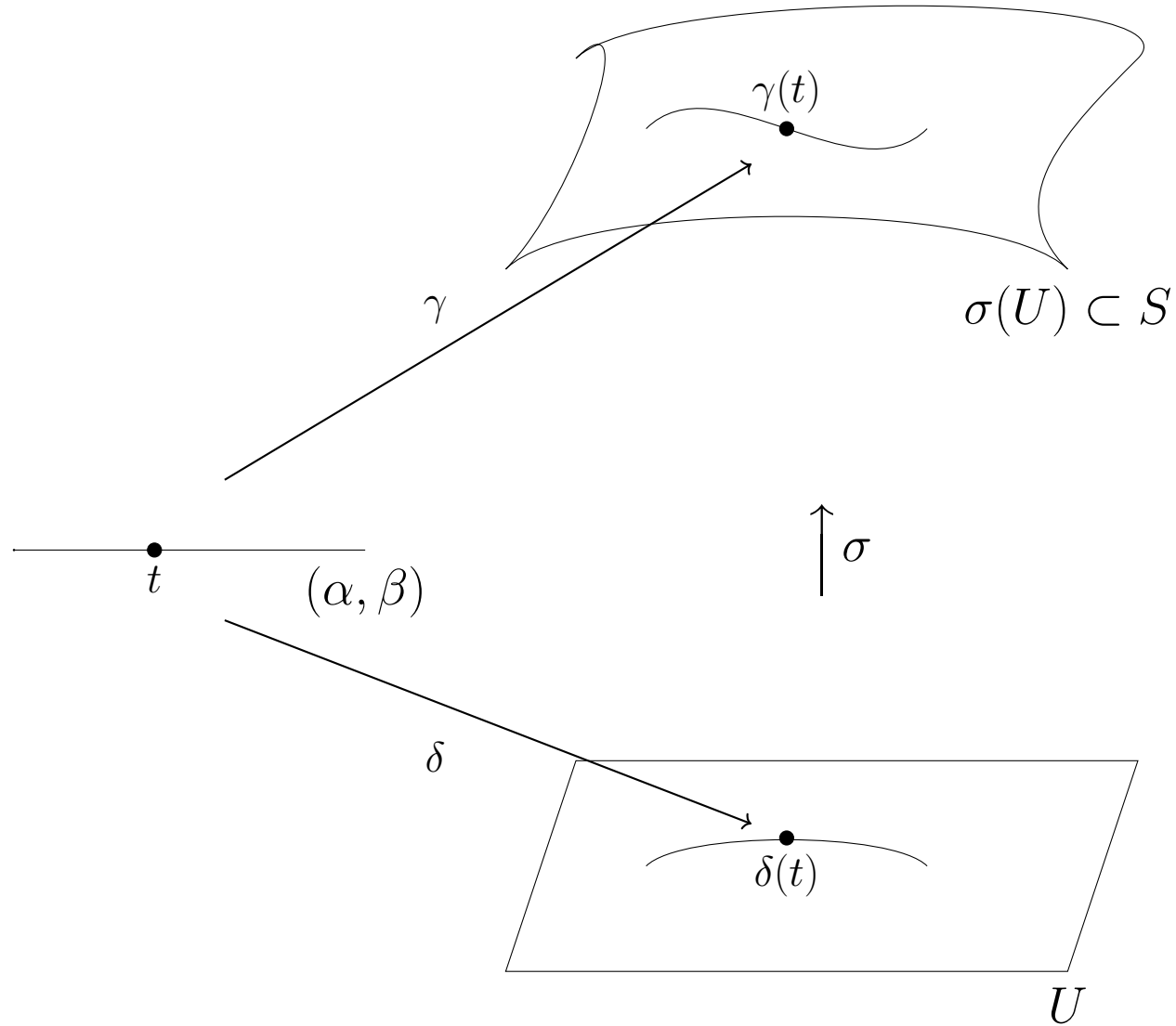
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so that $\gamma(t) = \sigma(\delta(t))$

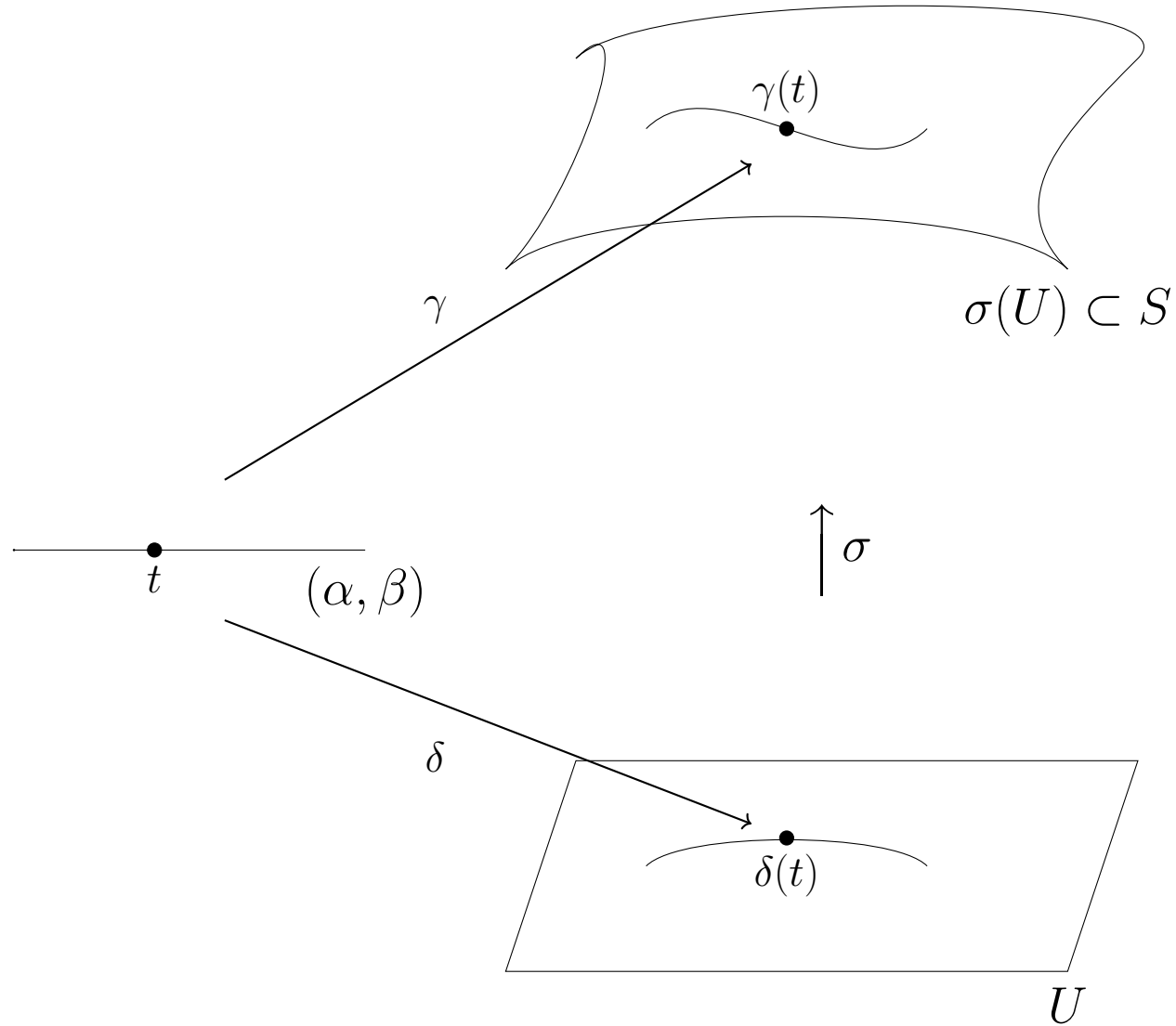
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Note that this gives a $\delta(t)$ for each t

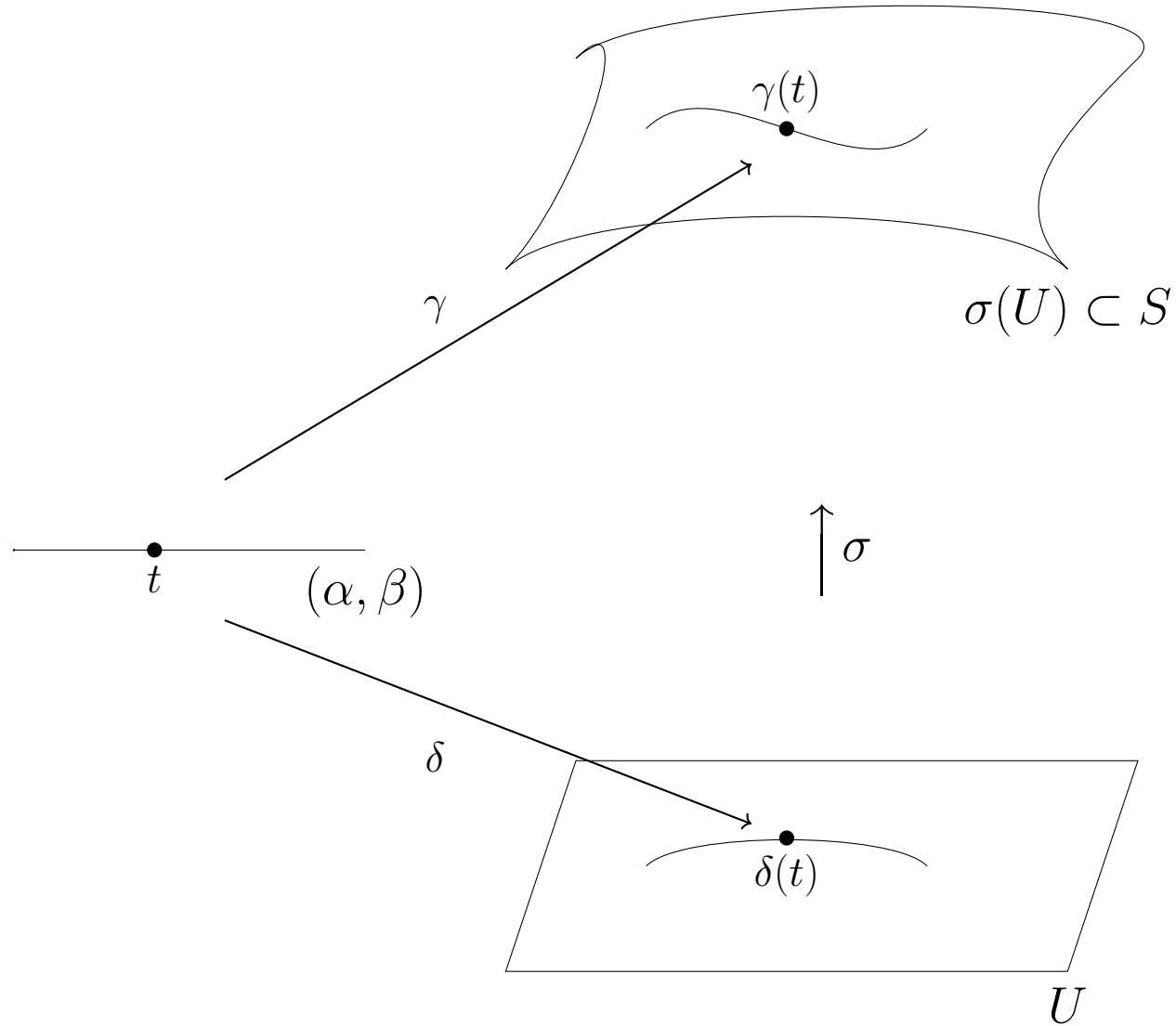
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so it defines a map.

A curve on a surface

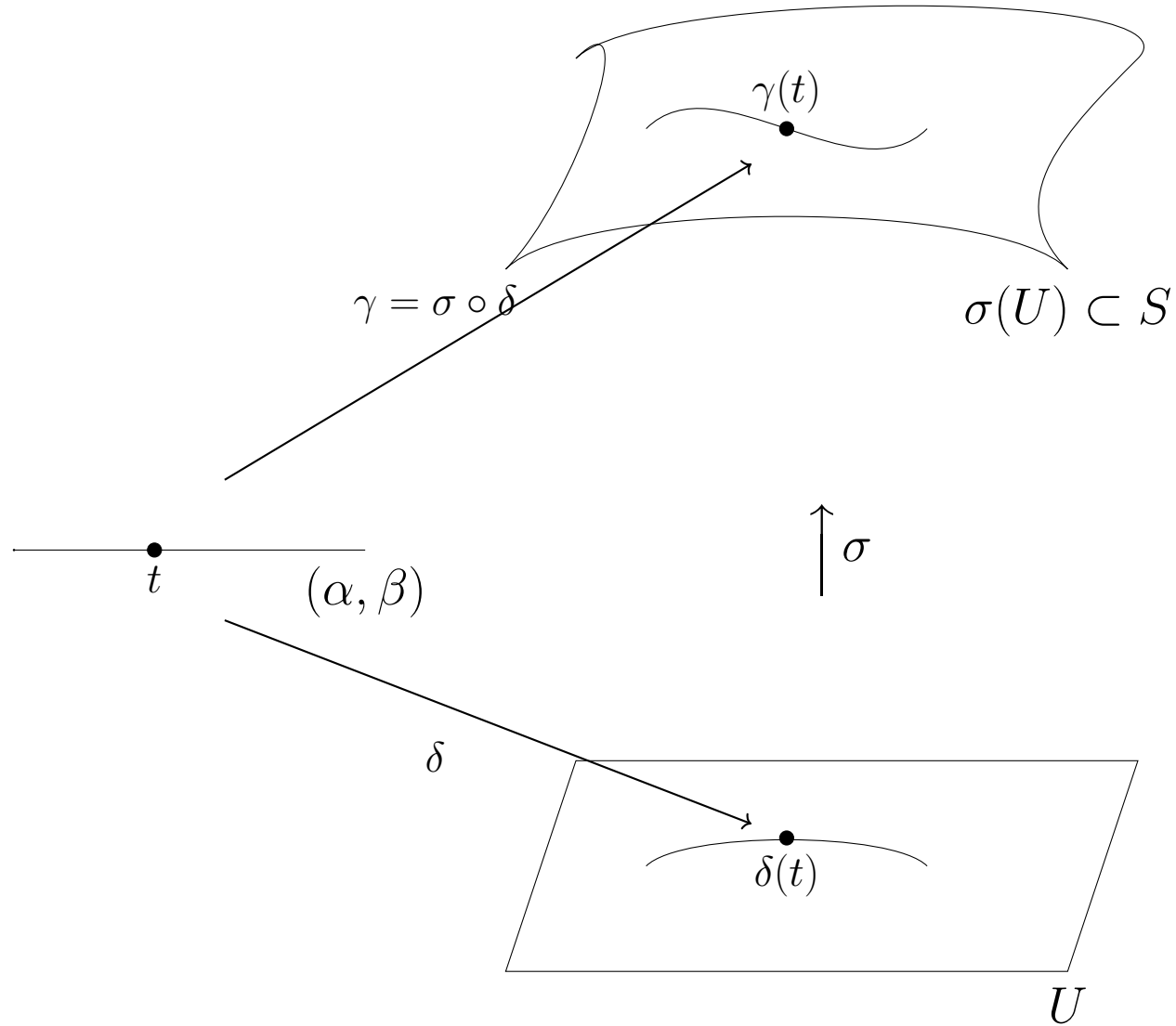
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$$\delta : (\alpha, \beta) \rightarrow U$$

$$\gamma(t) = \sigma(\delta(t))$$



Its smoothness takes some work, but assume it for now.

A curve on a surface

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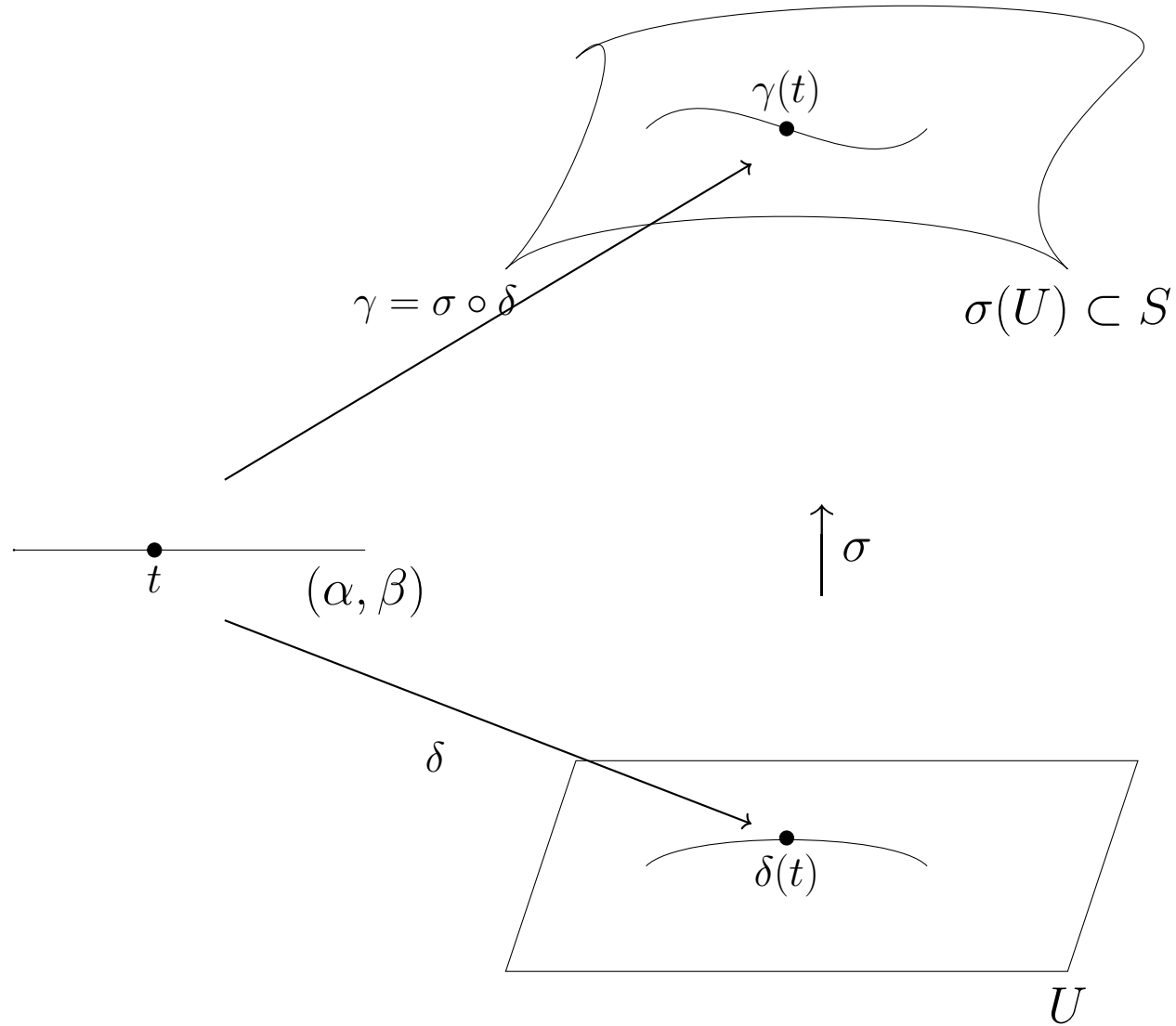
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$$\gamma(t) = \sigma(\delta(t))$$

$$\delta(t) = (x(t), y(t))$$



If we let $x(t)$ and $y(t)$ denote the coordinates of $\delta(t)$,

A curve on a surface

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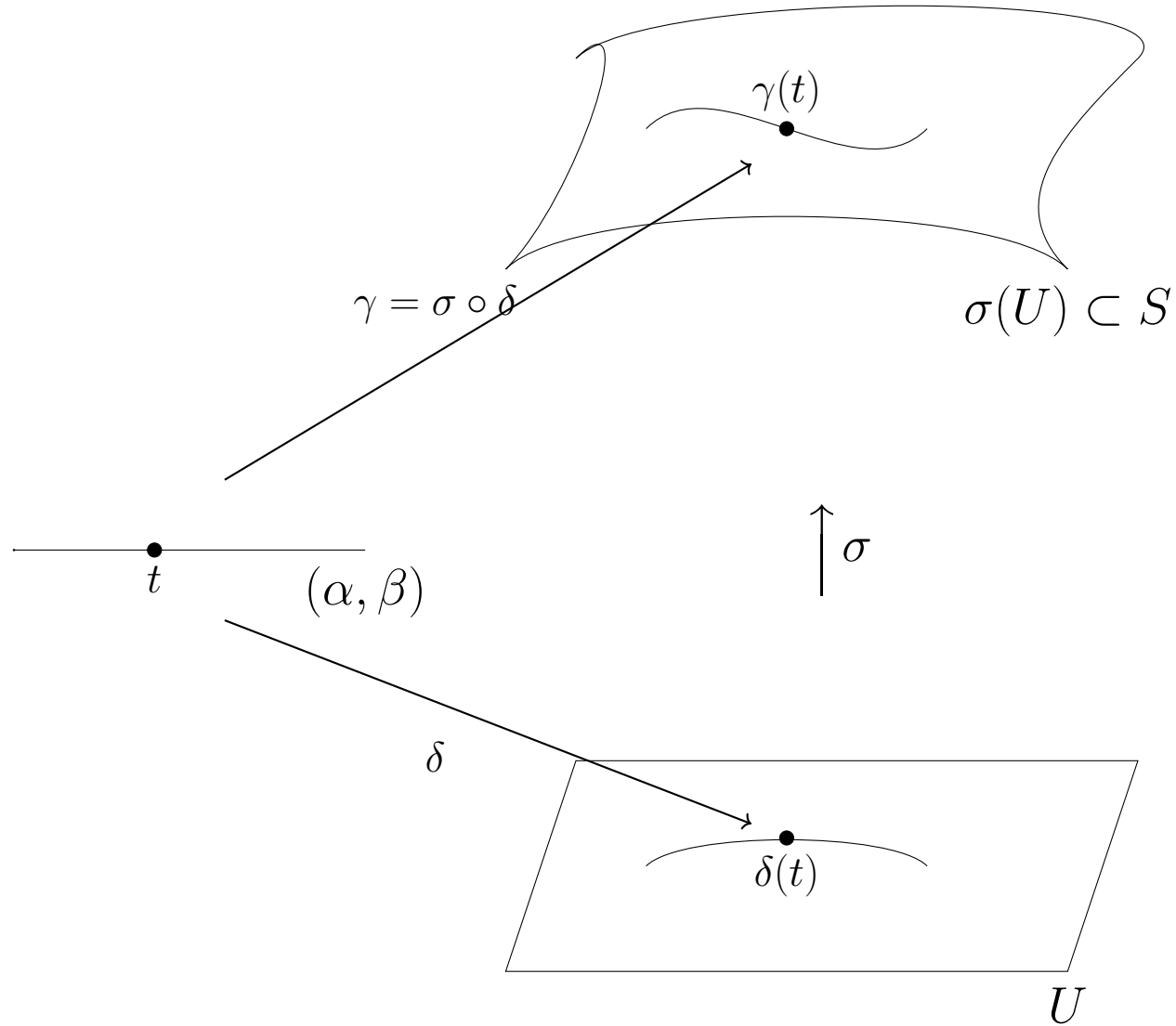
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chain rule allows us to express the derivatives

A curve on a surface

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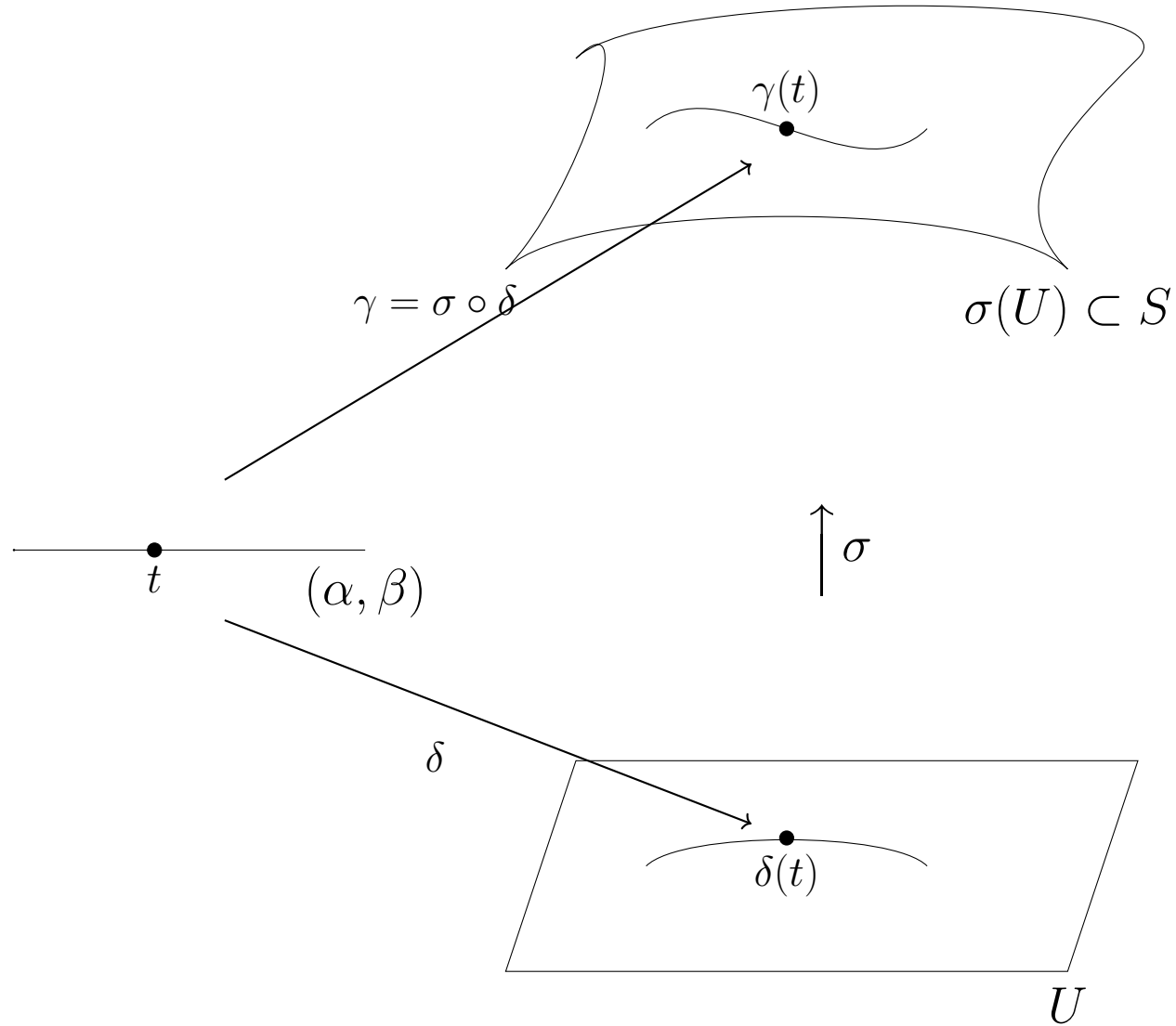
$$\gamma : (\alpha, \beta) \rightarrow \sigma(U) \subset S \subset \mathbb{R}^3$$

$$\delta : (\alpha, \beta) \rightarrow U$$

$$\gamma(t) = \sigma(\delta(t))$$

$$\delta(t) = (x(t), y(t))$$

$$\dot{\gamma}(t) = x'(t)\sigma_x(\delta(t)) +$$



entirely in terms of the derivatives of δ and σ

A curve on a surface

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$$\sigma : U \rightarrow S \subset \mathbb{R}^3$$

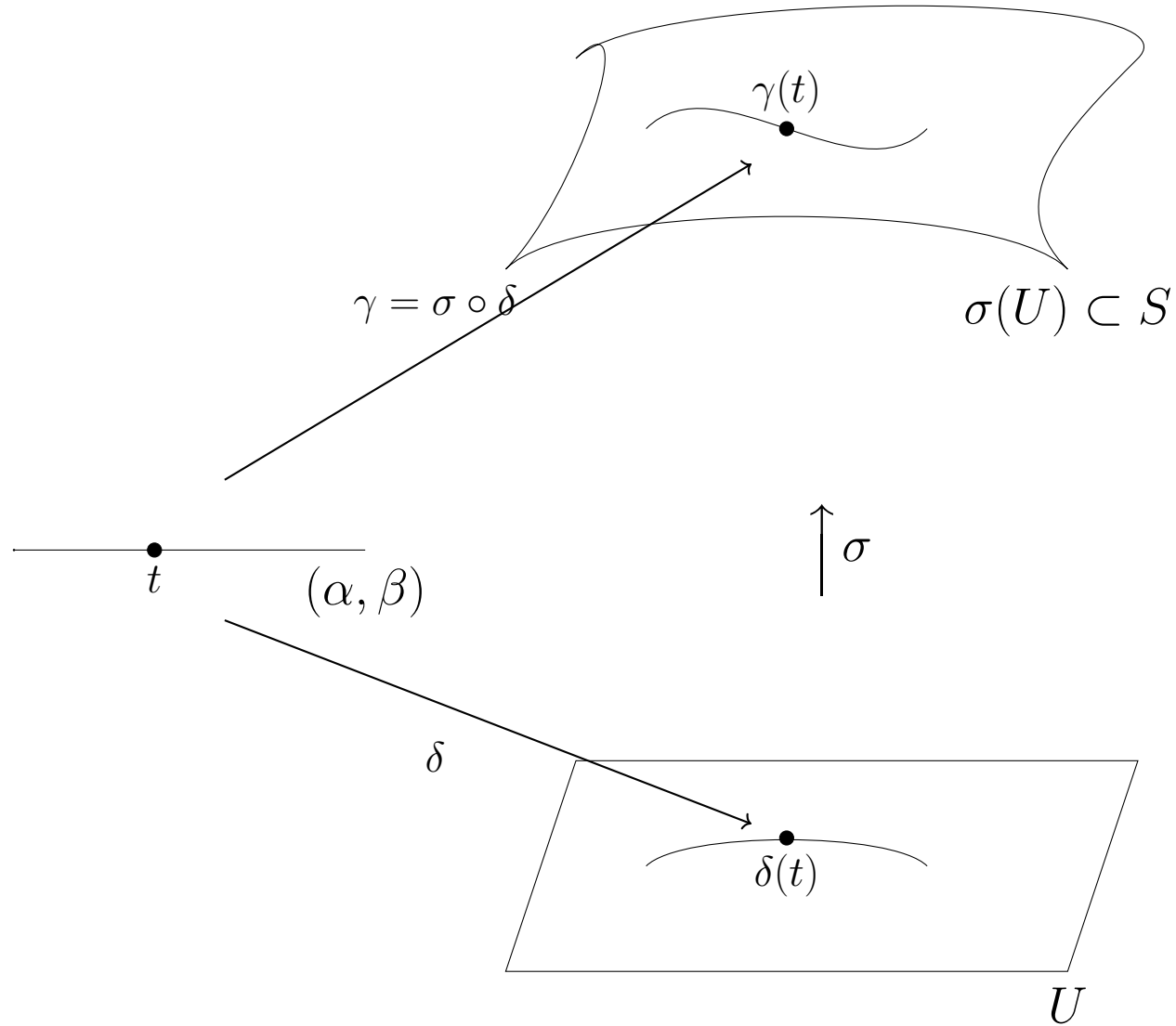
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A curve on a surface

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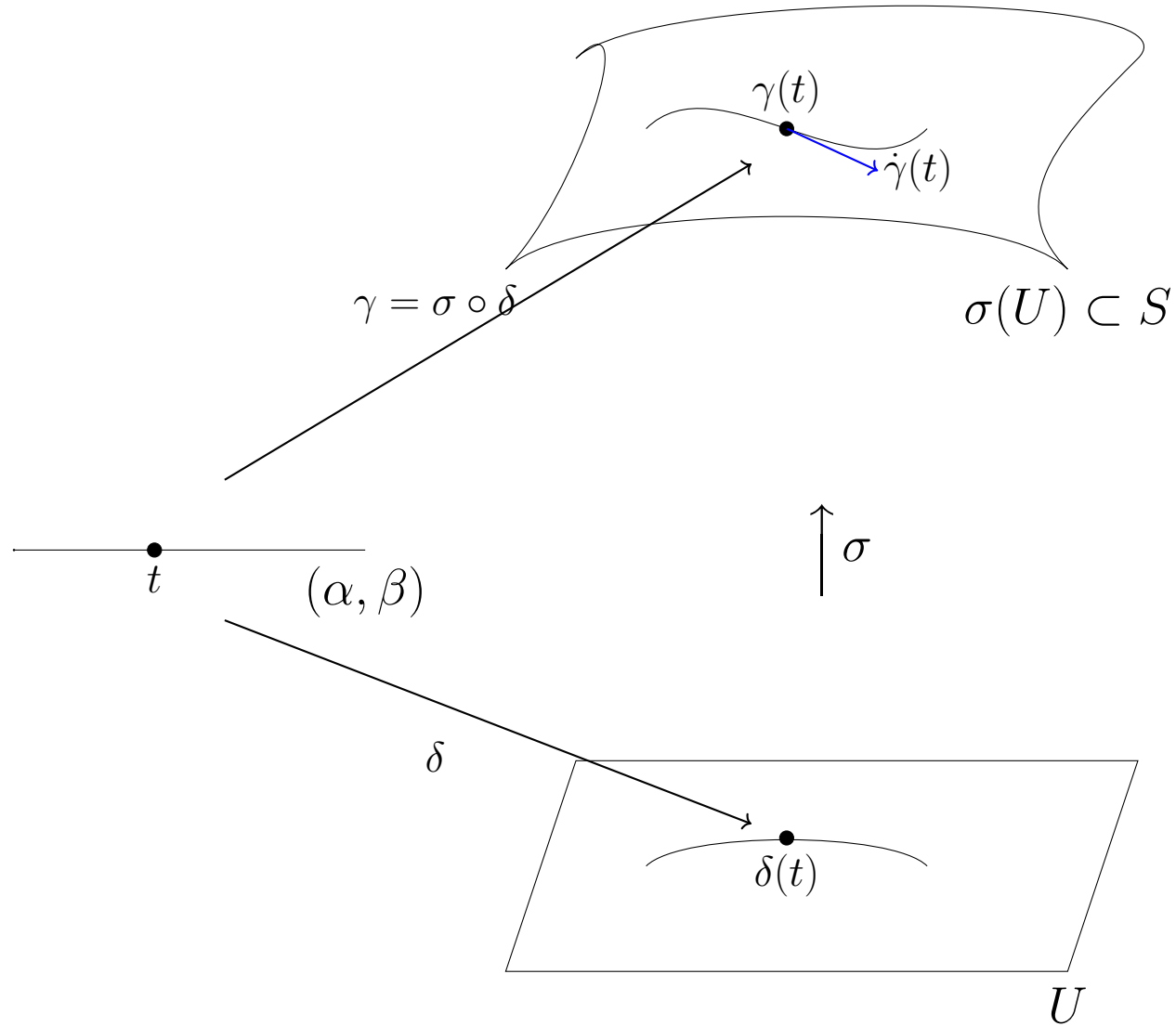
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The left hand side is the velocity vector of γ in space

A curve on a surface

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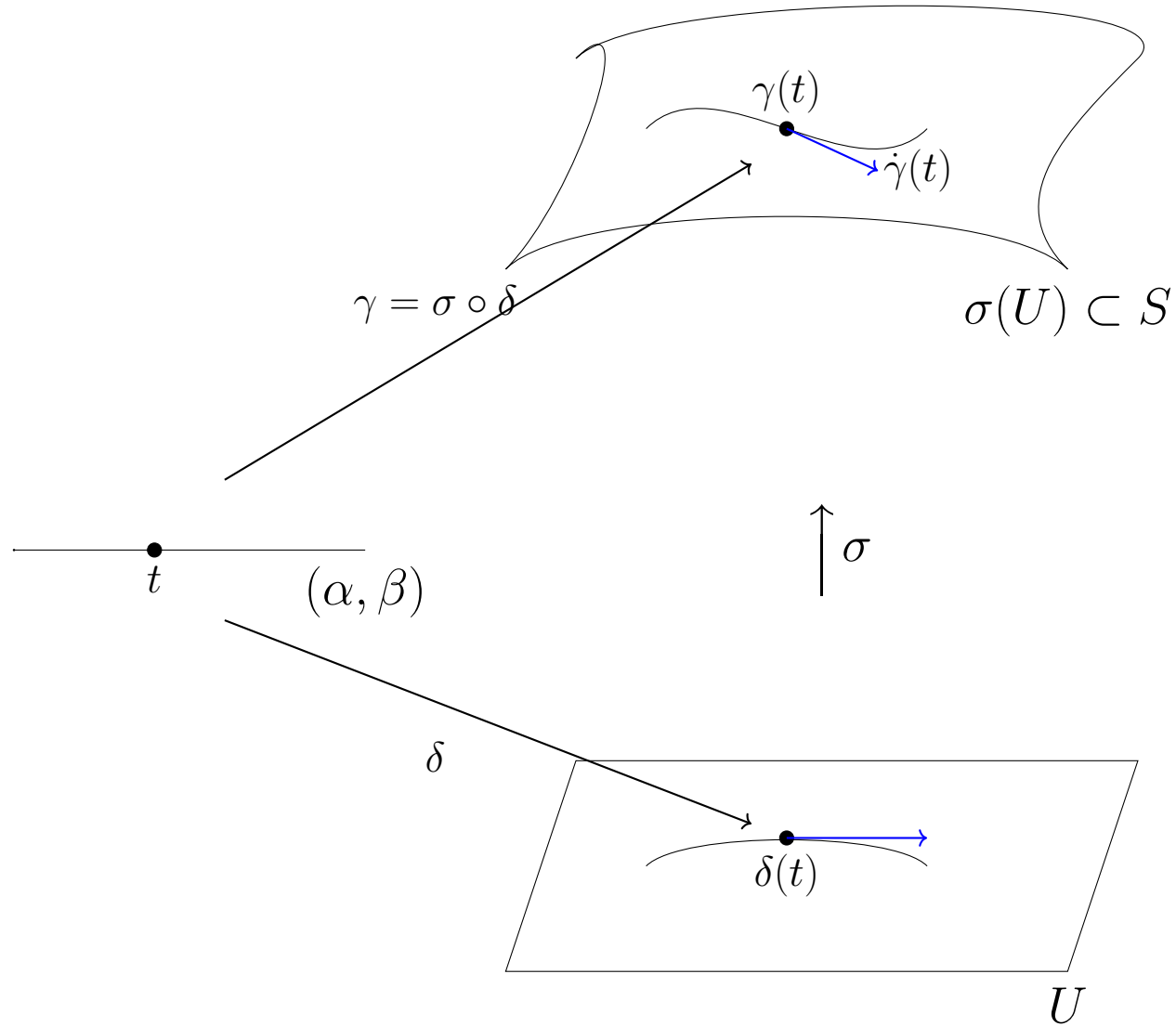
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$$\delta(t) = (x(t), y(t))$$

$$\dot{\gamma}(t) = x'(t)\sigma_x(\delta(t)) + y'(t)\sigma_y(\delta(t))$$



The right hand side expresses it in terms of the patch

A curve on a surface

Note:: This was not part of the lecture, however, since there seemed to be some confusion, this adds a diagram to support along with “subtitles”.

$$\sigma : U \rightarrow S \subset \mathbb{R}^3$$

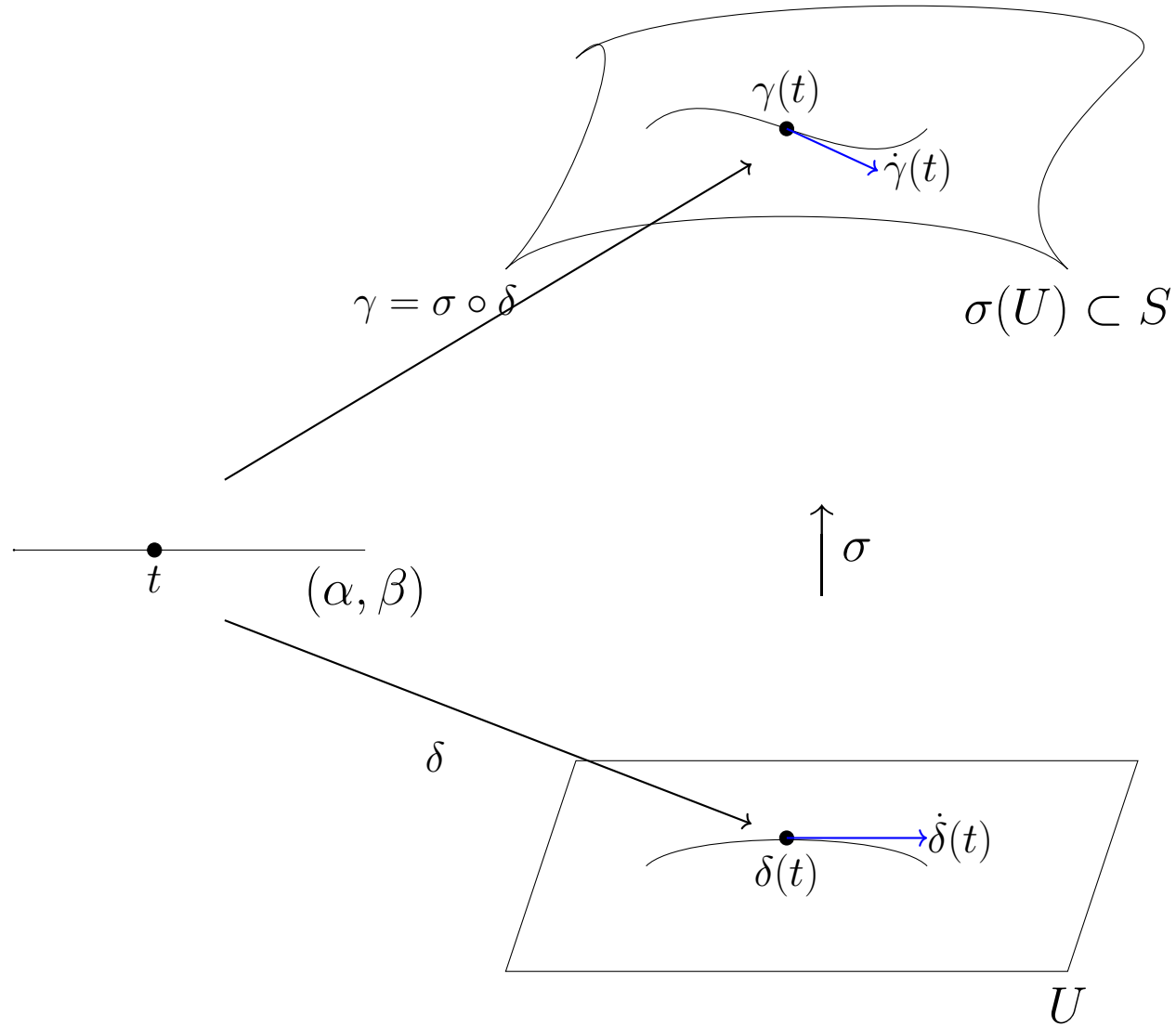
$$\gamma : (\alpha, \beta) \rightarrow \sigma(U) \subset S \subset \mathbb{R}^3$$

$$\delta : (\alpha, \beta) \rightarrow U$$

$$\gamma(t) = \sigma(\delta(t))$$

$$\delta(t) = (x(t), y(t))$$

$$\dot{\gamma}(t) = x'(t)\sigma_x(\delta(t)) + y'(t)\sigma_y(\delta(t))$$



i.e., in terms of the velocity of δ .

A curve on a surface

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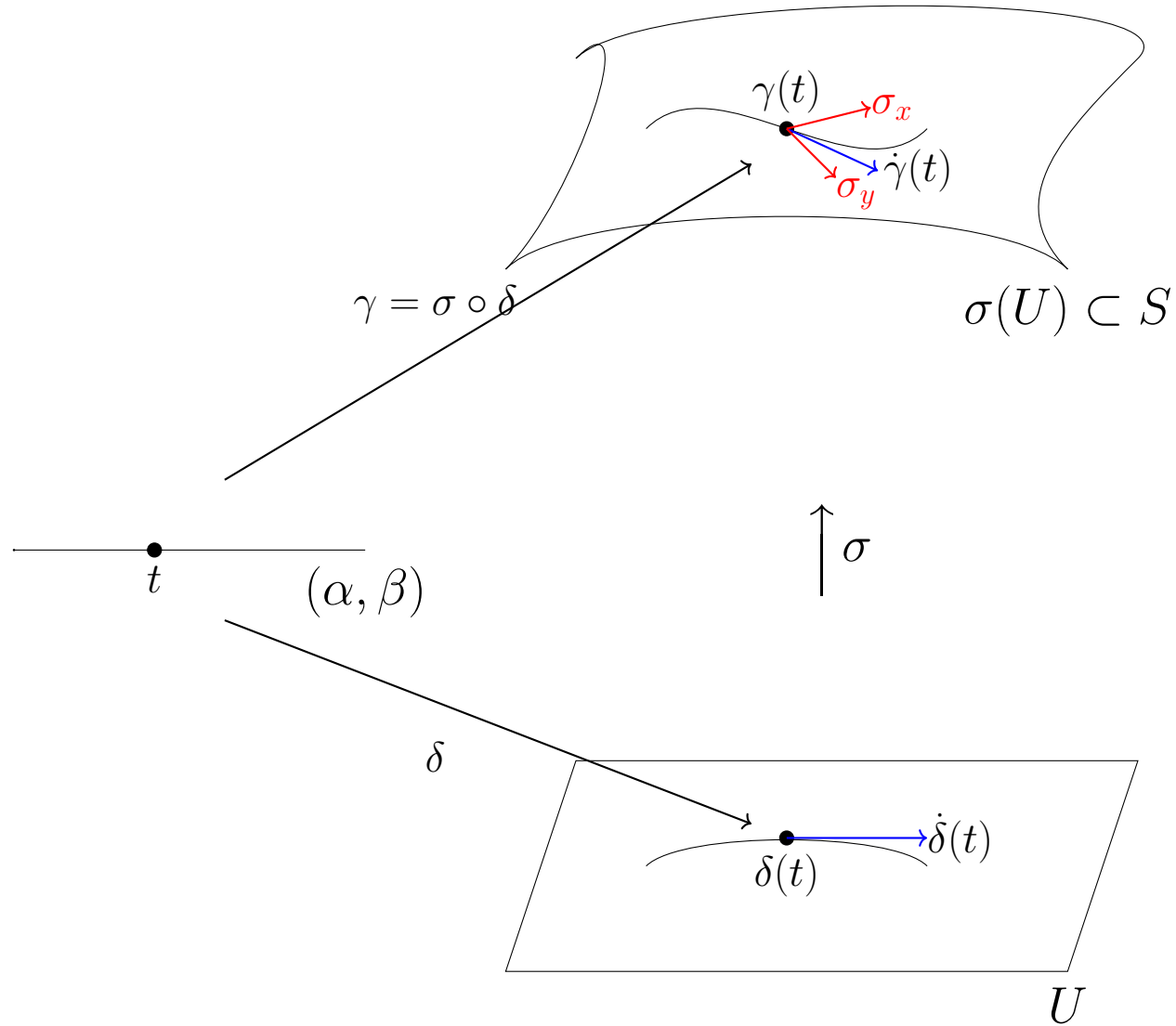
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Essentially, $\dot{\gamma}(t)$ can be written in terms of the surface patch, specifically, σ_x and σ_y .

A curve on a surface

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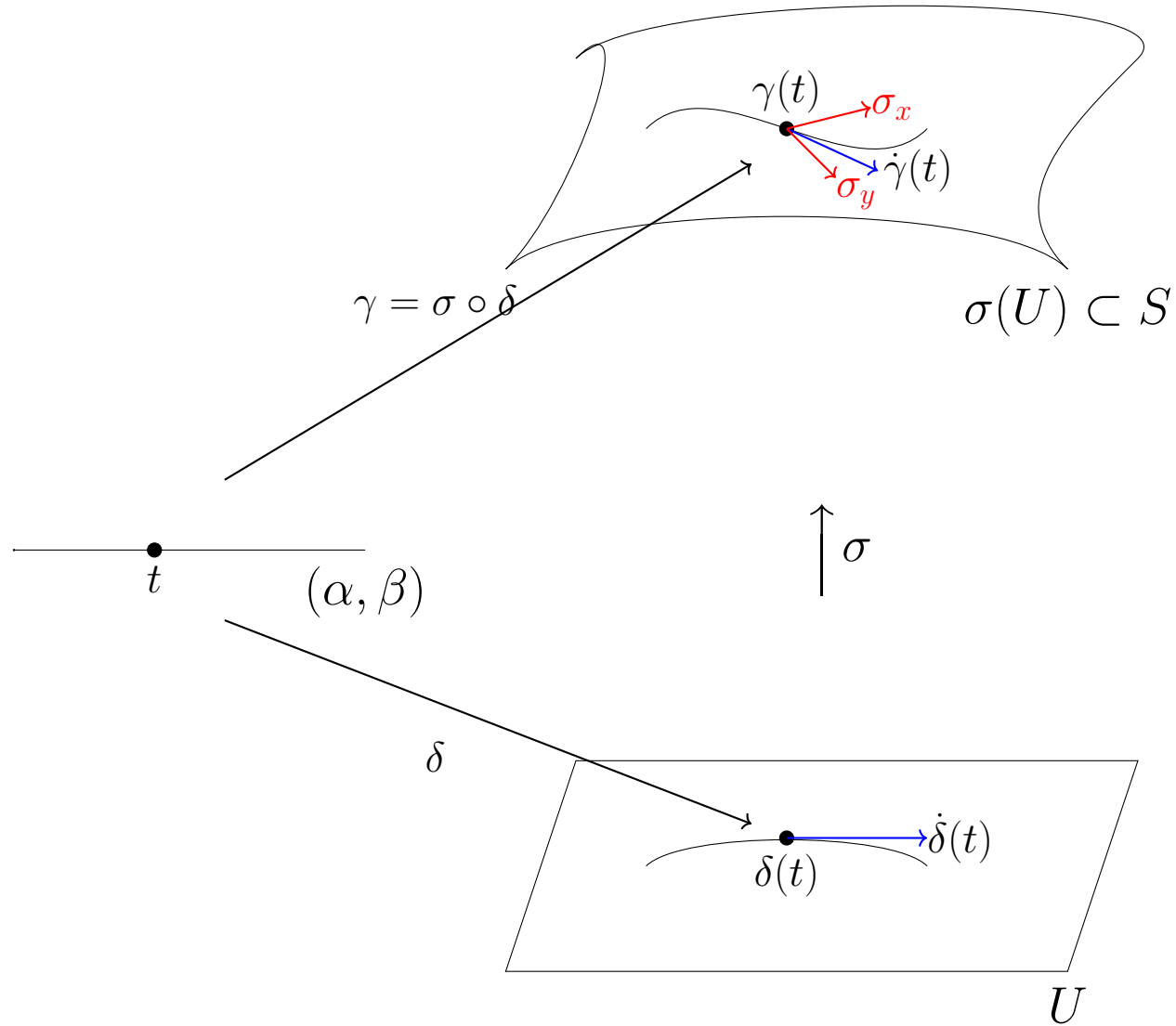
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The coefficients come from $\dot{\delta}(t)$

A curve on a surface

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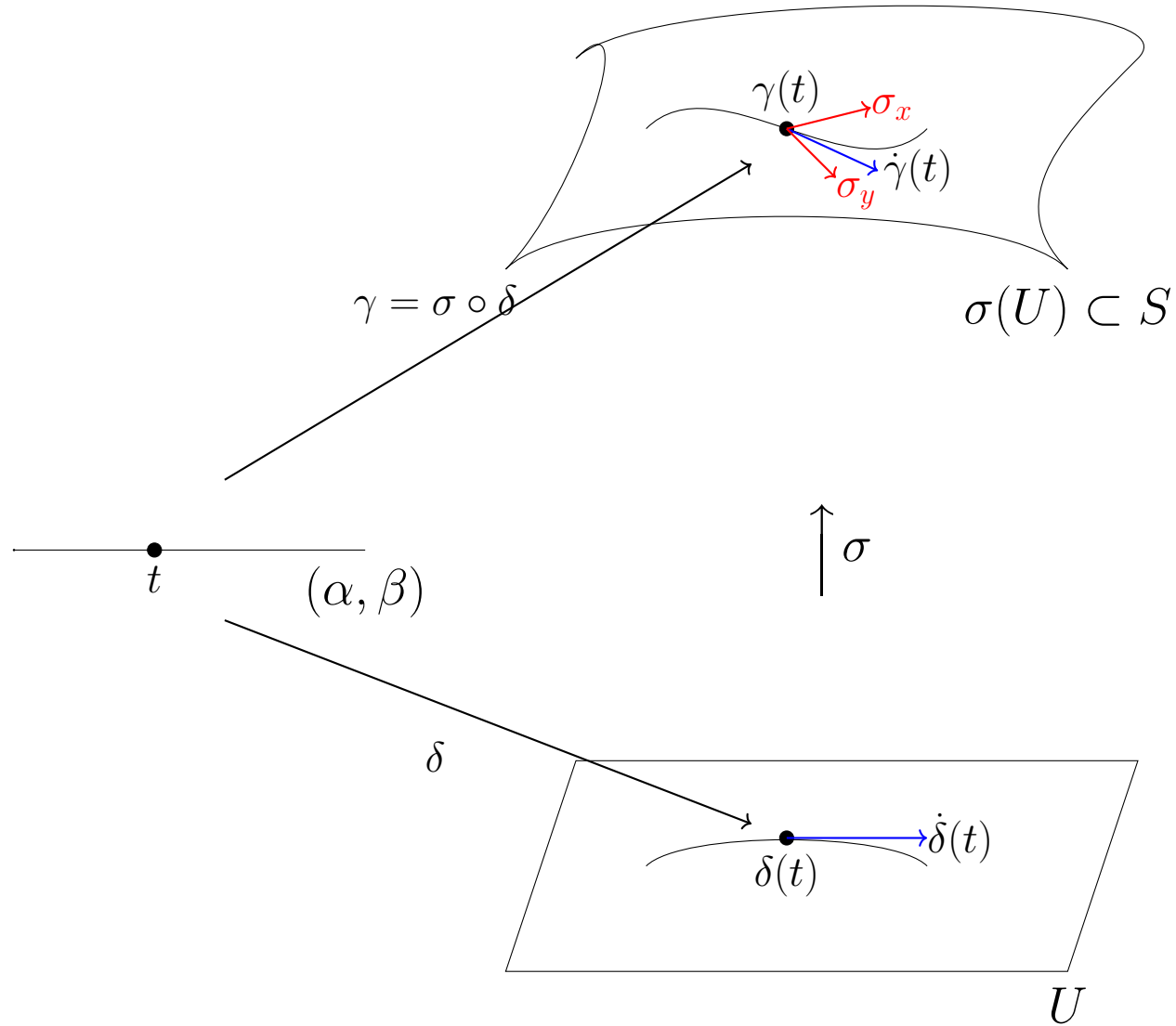
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(which is also the coordinates provides by the surface patch).

A curve on a surface

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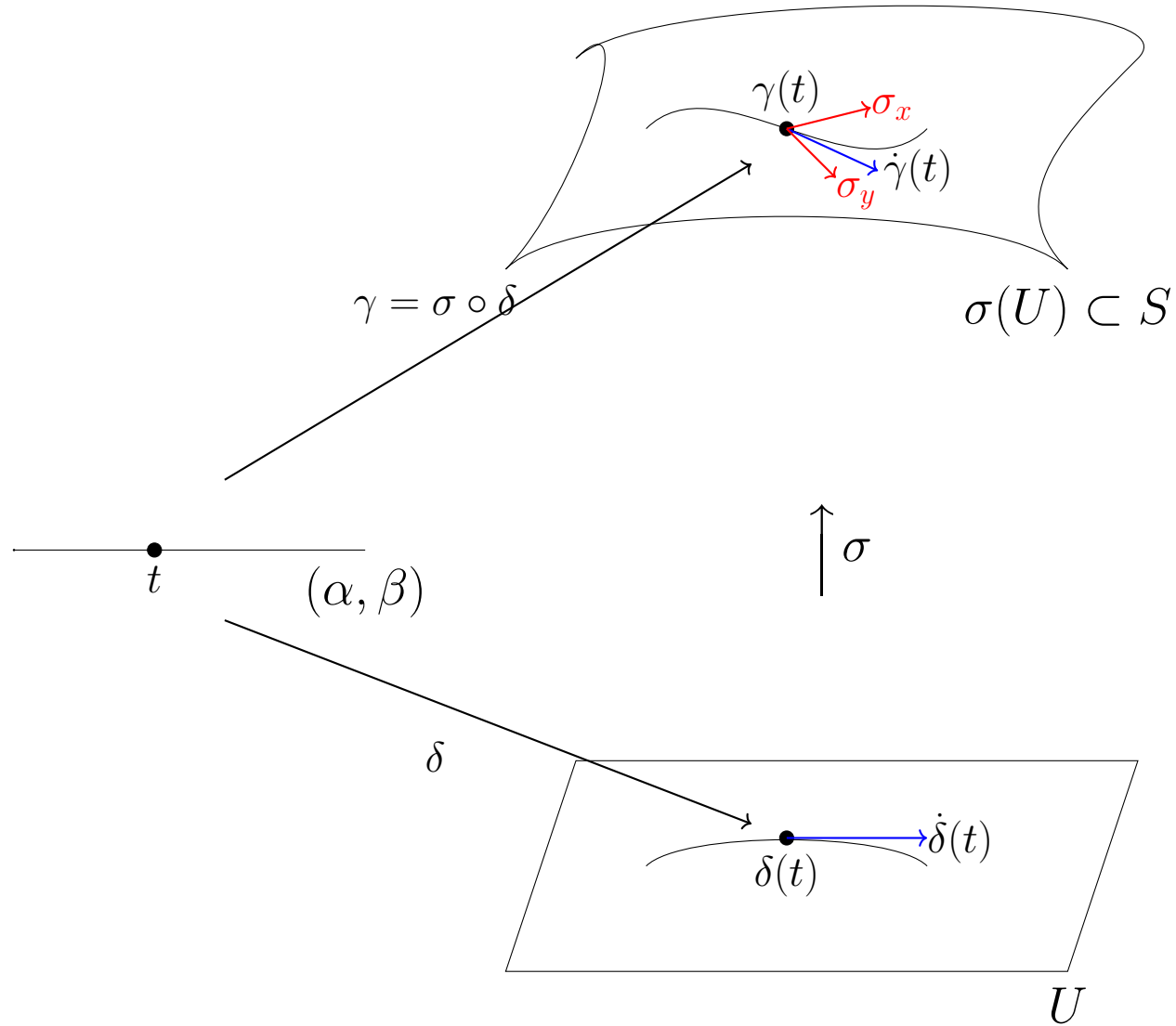
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This shows why partial derivatives feature at all

A curve on a surface

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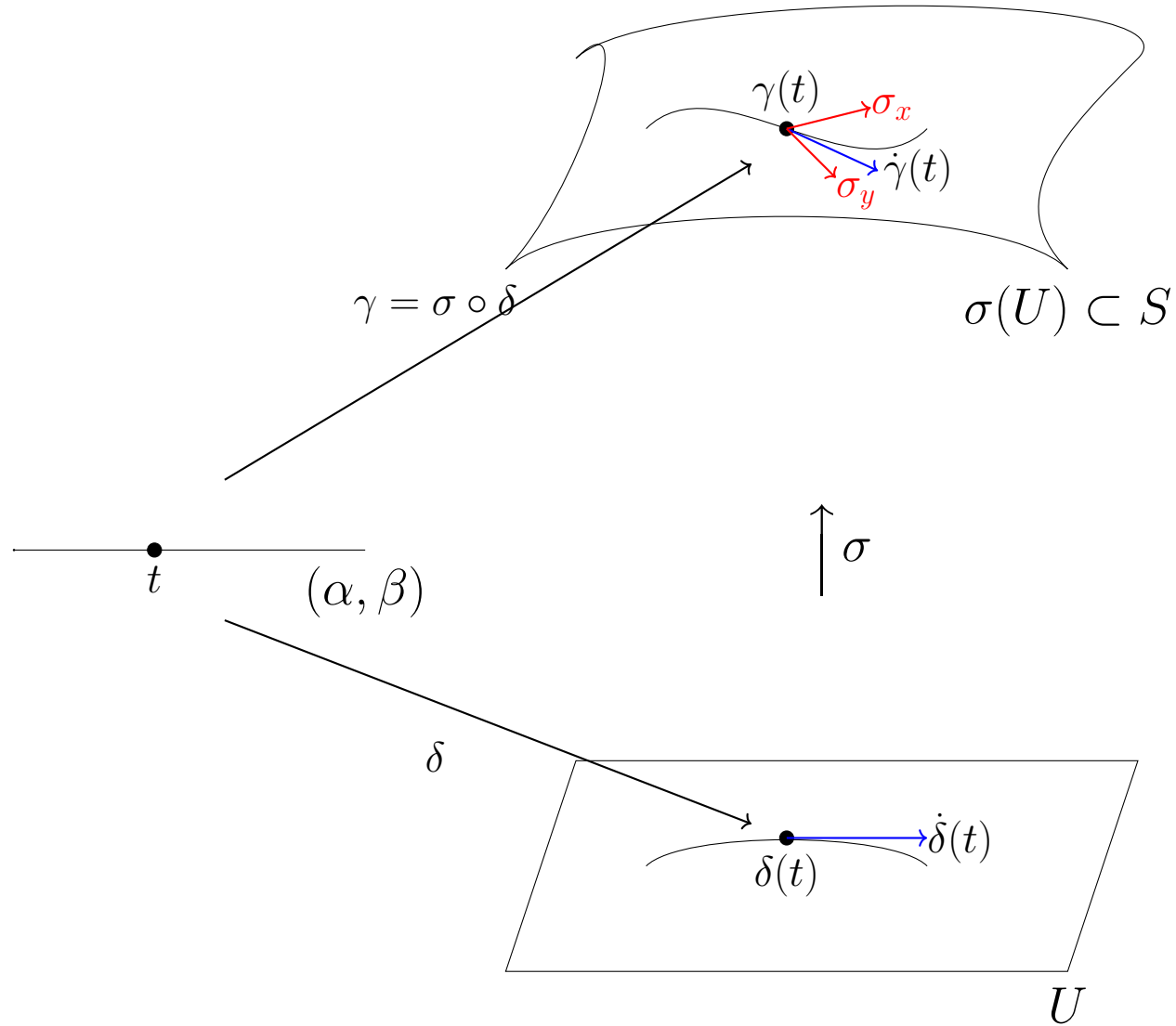
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and the importance of regularity...

A curve on a surface

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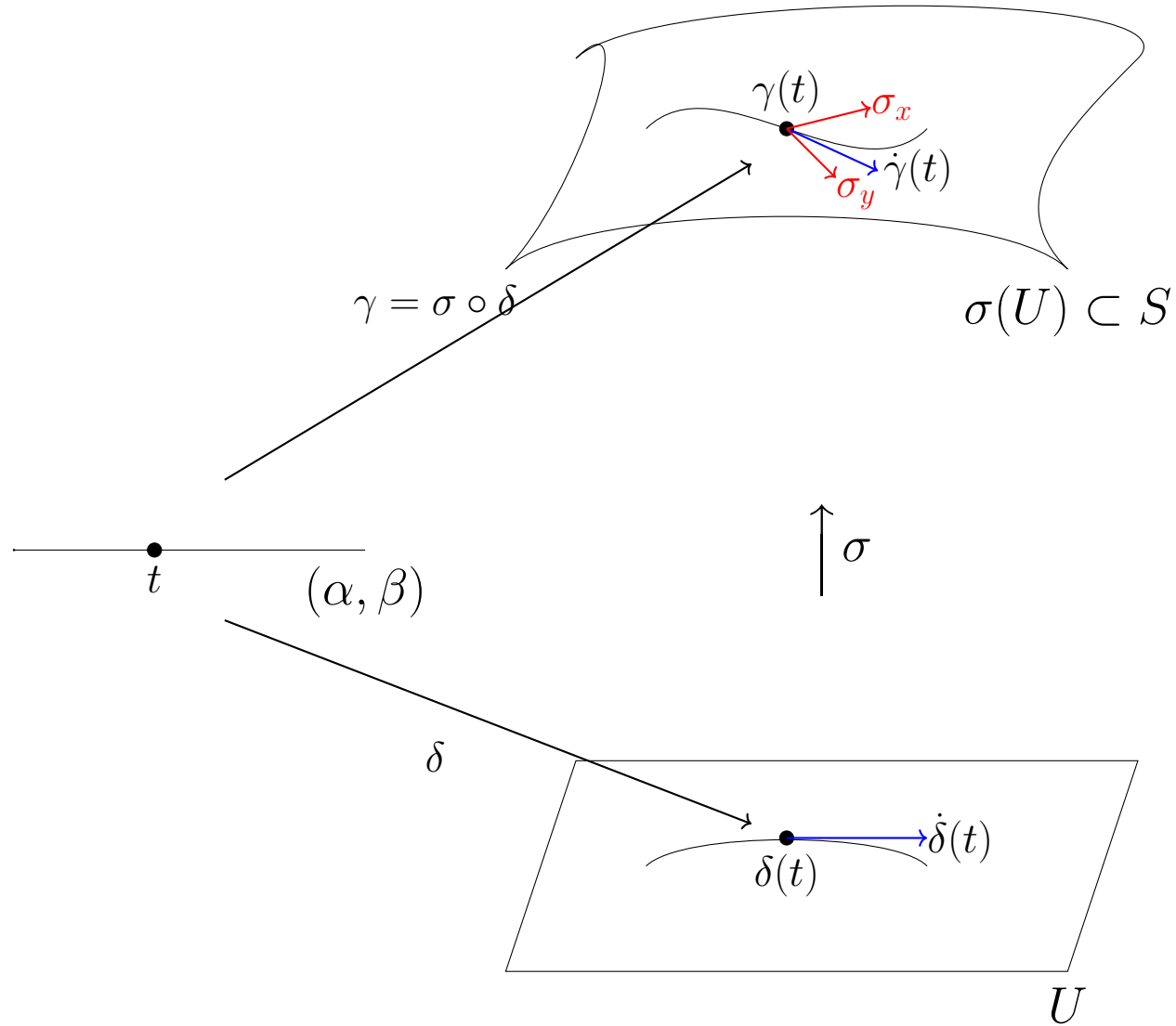
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...to ensure σ_x and σ_y are linearly independent