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Prove that $s_{\beta}(t) - s_{\alpha}(t)$ is a constant.

Theorem (First Fundamental theorem of calculus).

f

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Corollary. *The arc length function*

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Proof. □

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$f : [\alpha, \beta] \rightarrow \mathbb{R}$ continuous

$$F(t) := \int_{t_0}^t f(u) du$$

then, $F'(t) = f(t)$

Corollary. $\gamma : (\alpha, \beta) \rightarrow \mathbb{R}^2$ is a smooth and regular parametrization.

and $s(t)$ its arc-length function beginning at t_0 then,

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$\gamma : (\alpha, \beta) \rightarrow \mathbb{R}^2$ *a regular smooth parametrization*

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Theorem.

$\gamma : (\alpha, \beta) \rightarrow \mathbb{R}^2$ a regular smooth parametrization $s(t)$,

$$\begin{aligned} \gamma &: (\alpha, \beta) \rightarrow \mathbb{R}^2 \\ \tilde{\gamma} &: (\tilde{\alpha}, \tilde{\beta}) \rightarrow \mathbb{R}^2 \\ \tilde{\gamma}(\tilde{t}) &= \gamma(\phi(\tilde{t})) \\ \tilde{\gamma}'(\tilde{t}) &= \gamma'(\phi(\tilde{t}))\phi'(\tilde{t}) \end{aligned}$$

If $\phi(t) = s^{-1}(t)$,

$$\tilde{\gamma}'(\tilde{t}) = \gamma'(s^{-1}(\tilde{t}))(s^{-1})'(\tilde{t}) = \gamma'(s^{-1}(\tilde{t}))\frac{1}{||\gamma'(s^{-1}(\tilde{t}))||}$$

$$||\tilde{\gamma}'(\tilde{t})|| = ||\gamma'(s^{-1}(\tilde{t}))||\frac{1}{||\gamma'(s^{-1}(\tilde{t}))||} = 1$$

$$(s^{-1})'(\tilde{t}) = \frac{1}{s'(s^{-1}(\tilde{t}))} = \frac{1}{||\gamma'(s^{-1}(\tilde{t}))||}$$

Proved that,

Theorem.

$\gamma : (\alpha, \beta) \rightarrow \mathbb{R}^2$ a regular smooth parametrization $s(t)$, the arc-length from t_0 to t

$$\begin{aligned} \gamma &: (\alpha, \beta) \rightarrow \mathbb{R}^2 \\ \tilde{\gamma} &: (\tilde{\alpha}, \tilde{\beta}) \rightarrow \mathbb{R}^2 \\ \tilde{\gamma}(\tilde{t}) &= \gamma(\phi(\tilde{t})) \\ \tilde{\gamma}'(\tilde{t}) &= \gamma'(\phi(\tilde{t}))\phi'(\tilde{t}) \end{aligned}$$

If $\phi(t) = s^{-1}(t)$,

$$\tilde{\gamma}'(\tilde{t}) = \gamma'(s^{-1}(\tilde{t}))(s^{-1})'(\tilde{t}) = \gamma'(s^{-1}(\tilde{t}))\frac{1}{||\gamma'(s^{-1}(\tilde{t}))||}$$

$$||\tilde{\gamma}'(\tilde{t})|| = ||\gamma'(s^{-1}(\tilde{t}))||\frac{1}{||\gamma'(s^{-1}(\tilde{t}))||} = 1$$

$$(s^{-1})'(\tilde{t}) = \frac{1}{s'(s^{-1}(\tilde{t}))} = \frac{1}{||\gamma'(s^{-1}(\tilde{t}))||}$$

Proved that,

Theorem.

$\gamma : (\alpha, \beta) \rightarrow \mathbb{R}^2$ a regular smooth parametrization $s(t)$, the arc-length from t_0 to t (where $t, t_0 \in (\alpha, \beta)$)

$$\begin{aligned} \gamma &: (\alpha, \beta) \rightarrow \mathbb{R}^2 \\ \tilde{\gamma} &: (\tilde{\alpha}, \tilde{\beta}) \rightarrow \mathbb{R}^2 \\ \tilde{\gamma}(\tilde{t}) &= \gamma(\phi(\tilde{t})) \\ \tilde{\gamma}'(\tilde{t}) &= \gamma'(\phi(\tilde{t}))\phi'(\tilde{t}) \end{aligned}$$

If $\phi(t) = s^{-1}(t)$,

$$\tilde{\gamma}'(\tilde{t}) = \gamma'(s^{-1}(\tilde{t}))(s^{-1})'(\tilde{t}) = \gamma'(s^{-1}(\tilde{t}))\frac{1}{||\gamma'(s^{-1}(\tilde{t}))||}$$

$$||\tilde{\gamma}'(\tilde{t})|| = ||\gamma'(s^{-1}(\tilde{t}))||\frac{1}{||\gamma'(s^{-1}(\tilde{t}))||} = 1$$

$$(s^{-1})'(\tilde{t}) = \frac{1}{s'(s^{-1}(\tilde{t}))} = \frac{1}{||\gamma'(s^{-1}(\tilde{t}))||}$$

Proved that,

Theorem.

$\gamma : (\alpha, \beta) \rightarrow \mathbb{R}^2$ a regular smooth parametrization $s(t)$, the arc-length from t_0 to t (where $t, t_0 \in (\alpha, \beta)$)
 $\tilde{\gamma}(t) = \gamma(s^{-1}(t))$, then

$$\begin{aligned} \gamma &: (\alpha, \beta) \rightarrow \mathbb{R}^2 \\ \tilde{\gamma} &: (\tilde{\alpha}, \tilde{\beta}) \rightarrow \mathbb{R}^2 \\ \tilde{\gamma}(\tilde{t}) &= \gamma(\phi(\tilde{t})) \\ \tilde{\gamma}'(\tilde{t}) &= \gamma'(\phi(\tilde{t}))\phi'(\tilde{t}) \end{aligned}$$

If $\phi(t) = s^{-1}(t)$,

$$\tilde{\gamma}'(\tilde{t}) = \gamma'(s^{-1}(\tilde{t}))(s^{-1})'(\tilde{t}) = \gamma'(s^{-1}(\tilde{t}))\frac{1}{||\gamma'(s^{-1}(\tilde{t}))||}$$

$$||\tilde{\gamma}'(\tilde{t})|| = ||\gamma'(s^{-1}(\tilde{t}))||\frac{1}{||\gamma'(s^{-1}(\tilde{t}))||} = 1$$

$$(s^{-1})'(\tilde{t}) = \frac{1}{s'(s^{-1}(\tilde{t}))} = \frac{1}{\|\gamma'(s^{-1}(\tilde{t}))\|}$$

Proved that,

Theorem.

$\gamma : (\alpha, \beta) \rightarrow \mathbb{R}^2$ a regular smooth parametrization
 $s(t)$, the arc-length from t_0 to t (where $t, t_0 \in (\alpha, \beta)$)
 $\tilde{\gamma}(t) = \gamma(s^{-1}(t))$, then
 $\tilde{\gamma}$ is a unit speed re-parametrization.

$$\begin{aligned}\gamma &: (\alpha, \beta) \rightarrow \mathbb{R}^2 \\ \tilde{\gamma} &: (\tilde{\alpha}, \tilde{\beta}) \rightarrow \mathbb{R}^2 \\ \tilde{\gamma}(\tilde{t}) &= \gamma(\phi(\tilde{t})) \\ \tilde{\gamma}'(\tilde{t}) &= \gamma'(\phi(\tilde{t}))\phi'(\tilde{t})\end{aligned}$$

$$\begin{aligned}\text{If } \phi(t) &= s^{-1}(t), \\ \tilde{\gamma}'(\tilde{t}) &= \gamma'(s^{-1}(\tilde{t}))(s^{-1})'(\tilde{t}) = \gamma'(s^{-1}(\tilde{t}))\frac{1}{\|\gamma'(s^{-1}(\tilde{t}))\|}\end{aligned}$$

$$\|\tilde{\gamma}'(\tilde{t})\| = \|\gamma'(s^{-1}(\tilde{t}))\|\frac{1}{\|\gamma'(s^{-1}(\tilde{t}))\|} = 1$$