Exercise sheet 2

Curves and Surfaces, MTH201

1. For $\mathbf{v} : (\alpha, \beta) \to \mathbf{R}^2$ and $\mathbf{w} : (\alpha, \beta) \to \mathbf{R}^2$, show that $(\mathbf{v}(t).\mathbf{w}(t))' = \mathbf{v}'(t).\mathbf{w}(t) + \mathbf{v}(t).\mathbf{w}'(t)$.

2. If $\mathbf{n} : (\alpha, \beta) \to \mathbf{R}^2$ is such that $||\mathbf{n}(t)||$ is constant, then prove that $\dot{\mathbf{n}}(t)$ is either 0 or perpendicular to $\mathbf{n}(t)$.

3. if we denote,
$$s_\alpha(t) := \int_{t_\alpha}^{t} ||\dot{\gamma}(u)|| \mathrm{d}u$$
$$s_\beta(t) := \int_{t_\beta}^{t} ||\dot{\gamma}(u)|| \mathrm{d}u$$
prove that $s_\beta(t) - s_\alpha(t)$ is a constant **(assume that $t_\alpha < t_\beta$).**

4. If $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ is a smooth **and regular** parametrization, then show that $||\dot{\gamma}(t)|| : (\alpha, \beta) \to \mathbb{R}$ is smooth.

5. For the parametrization $\gamma : (-\pi/2, \pi/2) \to \mathbb{R}^2$ given by $\gamma(t) = (5\cos(t), 5\sin(t))$,
   (a) Find the arc-length function $s(t)$ (starting at, say, 0)
   (b) Find a reparametrization map ϕ so that $\gamma(\phi(t))$ is a unit-speed parametrization.