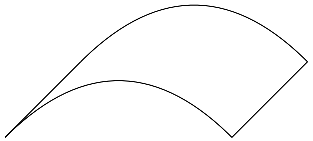
Surfaces



Definition (Surface patch).

$$\sigma :$$

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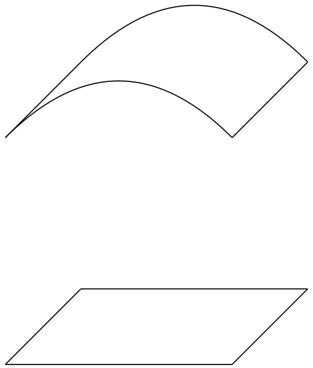

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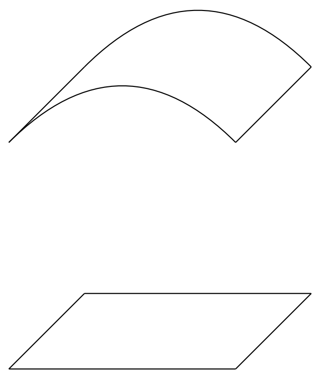

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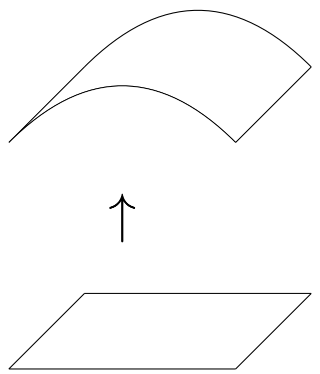

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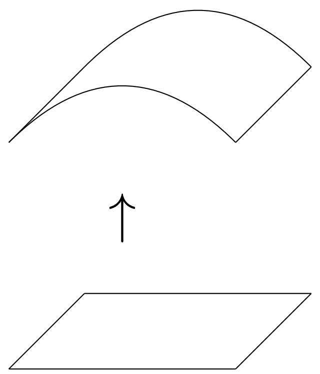

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$f : \mathbb{R}^2 \to \mathbb{R}$
$\gamma : (\alpha, \beta) \to \mathbb{R}^2$
$\gamma(t) = (x(t), y(t))$
$f \circ \gamma : (\alpha, \beta) \to \mathbb{R}$

$(f \circ \gamma)'(t_0) = f_x(x(t_0), y(t_0))x'(t_0) + f_y(x(t_0), y(t_0))y'(t_0) = (f_x(x, y), f_y(x, y)).\dot{\gamma}(t_0)$

Coordinate transformation

$\sigma : U \to \mathbb{R}^3$
$\tilde{\sigma} : \tilde{U} \to \mathbb{R}^3$
$\Phi : \tilde{U} \to U$ smooth, invertible, and inverse smooth
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where $\nabla(f)(x, y) = (f_x(x, y), f_y(x, y))$,
$\mathbf{v} = \dot{\gamma}(t_0)$,
and $p = (x(t_0), y(t_0))$