

# Exercise sheet 1

Set theory and Logic, MTH303

1. Use truth tables to prove that the following are tautologies:

(a)  $P \rightarrow (Q \rightarrow P)$

(b)  $(P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow [P \rightarrow R])$

(c)  $(\neg P \rightarrow \neg Q) \rightarrow (Q \rightarrow P)$

Yes, they have to do with the FL axioms!

2. Show that the first FL axiom, along with modus ponens, allows you to show:  $p, q \vdash p$ .
3. Show that the second FL axiom, along with modus ponens, allows you to show:  $p \rightarrow q, q \rightarrow r, p \vdash r$
4. Show that the third FL axiom, along with modus ponens, allows you to show:  $\neg q \rightarrow \neg p, p \vdash q$  (Does this remind you of proving by considering the contrapositive?)
5. Show that the addition inference rule follows from the axioms and replacement rules, i.e. show that  $p \vdash_* p \vee q$ .