Let us first consider the basic building block of logic

Definition: A **proposition**

A proposition!

Definition: A **proposition** is a statement that is true or false.

We do not care if the statement is really true or not

Definition: A **proposition** is a statement that is true or false.

only whether it makes sense to ask if it is true or false

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly

Here is an example

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention

of a complex proposition

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time

involving many simpler propositions

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

combined by what we will call "logical operators"

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

Observe the "if...then"

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator:

We call this "implies"

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$

and denote it by this arrow

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$

"You do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time" $\rightarrow$ "you will find this course very difficult"

Now we rewrite using our new operator

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$

"You do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time" $\rightarrow$ "you will find this course very difficult"

But this is not the only operator

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$

"You do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time" $\rightarrow$ "you will find this course very difficult"

Notice the "or" in this sentence

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$

"You do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time" $\rightarrow$ "you will find this course very difficult"

Here is another one

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$
"or" operator:

"You do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time" $\rightarrow$ "you will find this course very difficult"

This is called the "or operator"

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$
"or" operator: $\vee$

"You do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time" $\rightarrow$ "you will find this course very difficult"

Which we denote by this symbol: $\vee$

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$
"or" operator: $\vee$

"You do not attend the lectures regularly" $\vee$ "you do not pay attention" $\vee$ "you do not clear your doubts in time" $\rightarrow$ "you will find this course very difficult"

Again we rewrite using our new operator

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$
"or" operator: $\vee$

"You do not attend the lectures regularly" $\vee$ "you do not pay attention" $\vee$ "you do not clear your doubts in time" $\rightarrow$ "you will find this course very difficult"

Now observe that we have used another "logical" word (highlighted)

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$
"or" operator: $\vee$
"not" function:

"You do not attend the lectures regularly" $\vee$ "you do not pay attention" $\vee$ "you do not clear your doubts in time" $\rightarrow$ "you will find this course very difficult"

The "not", also called "negation"

Definition: A **proposition** is a statement that is true or false.

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"implies" operator: $\rightarrow$
"or" operator: $\vee$
"not" function: $\neg$

"You do not attend the lectures regularly" $\vee$ "you do not pay attention" $\vee$ "you do not clear your doubts in time" $\rightarrow$ "you will find this course very difficult"

which we denote with this symbol

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$
"or" operator: $\vee$
"not" function: $\neg$

$\neg$ "You attend the lectures regularly" $\vee$ $\neg$ "you pay attention" $\vee$ $\neg$ "you clear your doubts in time" $\rightarrow$ $\neg$ "you will find this course very easy"

again, we rewrite everything using this new symbol

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$
"or" operator: $\vee$
"not" function: $\neg$

$\neg$ "You attend the lectures regularly" $\vee$ $\neg$ "you pay attention" $\vee$ $\neg$ "you clear your doubts in time" $\rightarrow$ $\neg$ "you will find this course very easy"

Now we will introduce an operator we do not strictly need

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$
"or" operator: $\vee$
"not" function: $\neg$
"and" operator:

$\neg$ "You attend the lectures regularly" $\vee$ $\neg$ "you pay attention" $\vee$ $\neg$ "you clear your doubts in time" $\rightarrow$ $\neg$ "you will find this course very easy"

In fact, it can be written in terms of the other ones

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$
"or" operator: $\vee$
"not" function: $\neg$
"and" operator:

$\neg$ "You attend the lectures regularly" $\vee$ $\neg$ "you pay attention" $\vee$ $\neg$ "you clear your doubts in time" $\rightarrow$ $\neg$ "you will find this course very easy"

It is called "and"

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$
"or" operator: $\vee$
"not" function: $\neg$
"and" operator: $\wedge$

$\neg$ "You attend the lectures regularly" $\vee$ $\neg$ "you pay attention" $\vee$ $\neg$ "you clear your doubts in time" $\rightarrow$ $\neg$ "you will find this course very easy"

and is denoted by $\wedge$

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$
"or" operator: $\vee$
"not" function: $\neg$
"and" operator: $\wedge$

$\neg$ ("You attend the lectures regularly" $\wedge$ "you pay attention" $\wedge$ "you clear your doubts in time") $\rightarrow$ $\neg$ "you will find this course very easy"

Here we rewrite everything to use $\wedge$ instead of $\vee$

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$
"or" operator: $\vee$
"not" function: $\neg$
"and" operator: $\wedge$
"if and only if" operator:

$\neg$ ("You attend the lectures regularly" $\wedge$ "you pay attention" $\wedge$ "you clear your doubts in time") $\rightarrow$ $\neg$ "you will find this course very easy"

We have one more operator which we will use later

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"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$
"or" operator: $\vee$
"not" function: $\neg$
"and" operator: $\wedge$
"if and only if" operator: $\leftrightarrow$

$\neg$ ("You attend the lectures regularly" $\wedge$ "you pay attention" $\wedge$ "you clear your doubts in time") $\rightarrow$ $\neg$ "you will find this course very easy"

denoted by $\leftrightarrow$

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$
"or" operator: $\vee$
"not" function: $\neg$
"and" operator: $\wedge$
"if and only if" operator: $\leftrightarrow$

Variables: A, B, C, $\ldots$,

$\neg$ ("You attend the lectures regularly" $\wedge$ "you pay attention" $\wedge$ "you clear your doubts in time") $\rightarrow$ $\neg$ "you will find this course very easy"

Apart from operators, let us introduce "variables" in our "language"

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"implies" operator: $\rightarrow$
"or" operator: $\vee$
"not" function: $\neg$
"and" operator: $\wedge$
"if and only if" operator: $\leftrightarrow$

Variables: A, B, C, $\ldots$, P, Q, R,

$\neg$ ("You attend the lectures regularly" $\wedge$ "you pay attention" $\wedge$ "you clear your doubts in time") $\rightarrow$ $\neg$ "you will find this course very easy"

We can use any of the upper case letters

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$
"or" operator: $\vee$
"not" function: $\neg$
"and" operator: $\wedge$
"if and only if" operator: $\leftrightarrow$

Variables: A, B, C, $\ldots$, P, Q, R, $\ldots$, A_1, A_2, $\ldots$

$\neg$ ("You attend the lectures regularly" $\wedge$ "you pay attention" $\wedge$ "you clear your doubts in time") $\rightarrow$ $\neg$ "you will find this course very easy"

and even index them if we need too many

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$
"or" operator: $\vee$
"not" function: $\neg$
"and" operator: $\wedge$
"if and only if" operator: $\leftrightarrow$

Variables: A, B, C, ..., P, Q, R, ..., A_1, A_2, ...

Example:

$\neg$ ("You attend the lectures regularly" $\wedge$ "you pay attention" $\wedge$ "you clear your doubts in time") $\rightarrow$ $\neg$ "you will find this course very easy"

Let us abstract out the specific propositions as variables

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$
"or" operator: $\vee$
"not" function: $\neg$
"and" operator: $\wedge$
"if and only if" operator: $\leftrightarrow$

Variables: A, B, C, ..., P, Q, R, ..., A_1, A_2, ...

Example:
$P :=$ "You attend the courses regularly"

$\neg$ ("You attend the lectures regularly" $\wedge$ "you pay attention" $\wedge$ "you clear your doubts in time") $\rightarrow \neg$ "you will find this course very easy"

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$
"or" operator: $\vee$
"not" function: $\neg$
"and" operator: $\wedge$
"if and only if" operator: $\leftrightarrow$

Variables: A, B, C, ..., P, Q, R, ..., A_1, A_2, ...

Example:
$P :=$ "You attend the courses regularly"
$Q :=$ "You pay attention"

$\neg$ ("You attend the lectures regularly" $\wedge$ "you pay attention" $\wedge$ "you clear your doubts in time") $\rightarrow \neg$ "you will find this course very easy"

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$
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Variables: A, B, C, ..., P, Q, R, ..., A_1, A_2, ...

Example:
$P :=$ "You attend the courses regularly"
$Q :=$ "You pay attention"
$R :=$ "You clear your doubts in time"

$\neg$ ("You attend the lectures regularly" $\wedge$ "you pay attention" $\wedge$ "you clear your doubts in time") $\rightarrow$ $\neg$ "you will find this course very easy"

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Variables: A, B, C, ..., P, Q, R, ..., A_1, A_2, ...

Example:
$P :=$ "You attend the courses regularly"
$Q :=$ "You pay attention"
$R :=$ "You clear your doubts in time"
$S :=$ "You will find this course easy"

$\neg$ ("You attend the lectures regularly" $\wedge$ "you pay attention" $\wedge$ "you clear your doubts in time") $\rightarrow$ $\neg$ "you will find this course very easy"

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$
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Variables: A, B, C, ..., P, Q, R, ..., A_1, A_2, ...

Example:
$P :=$ "You attend the courses regularly"
$Q :=$ "You pay attention"
$R :=$ "You clear your doubts in time"
$S :=$ "You will find this course easy"
$\neg(P \wedge Q \wedge R) \rightarrow \neg S$

Using these interpretations we get this expression

$\neg$ ("You attend the lectures regularly" $\wedge$ "you pay attention" $\wedge$ "you clear your doubts in time") $\rightarrow \neg$ "you will find this course very easy"

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$
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Variables: A, B, C, ..., P, Q, R, ..., A_1, A_2, ...

Example:
$P :=$ "You attend the courses regularly"
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$\neg(P \wedge Q \wedge R) \rightarrow \neg S$

$\neg$ ("You attend the lectures regularly" $\wedge$ "you pay attention" $\wedge$ "you clear your doubts in time") $\rightarrow \neg$ "you will find this course very easy"

$\neg P$

But we know that the following expression is also equivalent

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"implies" operator: $\rightarrow$
"or" operator: $\vee$
"not" function: $\neg$
"and" operator: $\wedge$
"if and only if" operator: $\leftrightarrow$

Variables: A, B, C, ..., P, Q, R, ..., A_1, A_2, ...

Example:
$P :=$ "You attend the courses regularly"
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$S :=$ "You will find this course easy"
$\neg(P \wedge Q \wedge R) \rightarrow \neg S$

$\neg$ ("You attend the lectures regularly" $\wedge$ "you pay attention" $\wedge$ "you clear your doubts in time") $\rightarrow \neg$ "you will find this course very easy"

$\neg P \vee \neg Q$

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Variables: A, B, C, ..., P, Q, R, ..., A_1, A_2, ...

Example:
$P :=$ "You attend the courses regularly"
$Q :=$ "You pay attention"
$R :=$ "You clear your doubts in time"
$S :=$ "You will find this course easy"
$\neg(P \wedge Q \wedge R) \rightarrow \neg S$

$\neg$ ("You attend the lectures regularly" $\wedge$ "you pay attention" $\wedge$ "you clear your doubts in time") $\rightarrow \neg$ "you will find this course very easy"

$\neg P \vee \neg Q \vee \neg R$

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$
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"not" function: $\neg$
"and" operator: $\wedge$
"if and only if" operator: $\leftrightarrow$

Variables: A, B, C, ..., P, Q, R, ..., A_1, A_2, ...

Example:
$P :=$ "You attend the courses regularly"
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$\neg(P \wedge Q \wedge R) \rightarrow \neg S$

$\neg$ ("You attend the lectures regularly" $\wedge$ "you pay attention" $\wedge$ "you clear your doubts in time") $\rightarrow \neg$ "you will find this course very easy"

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$
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Variables: A, B, C, ..., P, Q, R, ..., A_1, A_2, ...

Example:
$P :=$ "You attend the courses regularly"
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$S :=$ "You will find this course easy"
$\neg(P \wedge Q \wedge R) \rightarrow \neg S$

$\neg$ ("You attend the lectures regularly" $\wedge$ "you pay attention" $\wedge$ "you clear your doubts in time") $\rightarrow \neg$ "you will find this course very easy"

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$
$\neg$ "You attend the lectures regularly"

In natural language, it translates to this sentence

Definition: A **proposition** is a statement that is true or false.

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"implies" operator: $\rightarrow$
"or" operator: $\vee$
"not" function: $\neg$
"and" operator: $\wedge$
"if and only if" operator: $\leftrightarrow$

Variables: $A, B, C, \ldots, P, Q, R, \ldots, A_1, A_2, \ldots$

Example:
$P :=$ "You attend the courses regularly"
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$\neg$ ("You attend the lectures regularly" $\wedge$ "you pay attention" $\wedge$ "you clear your doubts in time") $\rightarrow \neg$ "you will find this course very easy"

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$
$\neg$ "You attend the lectures regularly" $\vee \neg$ "you pay attention"

Definition: A **proposition** is a statement that is true or false.

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"implies" operator: $\rightarrow$
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Variables: A, B, C, ..., P, Q, R, ..., A_1, A_2, ...

Example:
$P :=$ "You attend the courses regularly"
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$\neg(P \wedge Q \wedge R) \rightarrow \neg S$

$\neg$ ("You attend the lectures regularly" $\wedge$ "you pay attention" $\wedge$ "you clear your doubts in time") $\rightarrow \neg$ "you will find this course very easy"

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$
$\neg$ "You attend the lectures regularly" $\vee \neg$ "you pay attention" $\vee \neg$ "you clear your doubts in time"

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

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Variables: A, B, C, ..., P, Q, R, ..., A_1, A_2, ...

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$\neg$ ("You attend the lectures regularly" $\wedge$ "you pay attention" $\wedge$ "you clear your doubts in time") $\rightarrow \neg$ "you will find this course very easy"

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$
$\neg$ "You attend the lectures regularly" $\vee \neg$ "you pay attention" $\vee \neg$ "you clear your doubts in time" $\rightarrow \neg$ "you will find this course very easy"

S

We will later see that we can always rewrite a the previous logical expression as this

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Example:
$P :=$ "You attend the courses regularly"
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$\neg$ ("You attend the lectures regularly" $\wedge$ "you pay attention" $\wedge$ "you clear your doubts in time") $\rightarrow \neg$ "you will find this course very easy"

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$
$\neg$ "You attend the lectures regularly" $\vee \neg$ "you pay attention" $\vee \neg$ "you clear your doubts in time" $\rightarrow \neg$ "you will find this course very easy"

$S \rightarrow P$

Definition: A **proposition** is a statement that is true or false.

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Variables: A, B, C, ..., P, Q, R, ..., A_1, A_2, ...

Example:
$P :=$ "You attend the courses regularly"
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$\neg(P \wedge Q \wedge R) \rightarrow \neg S$

$\neg$ ("You attend the lectures regularly" $\wedge$ "you pay attention" $\wedge$ "you clear your doubts in time") $\rightarrow \neg$ "you will find this course very easy"

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$
$\neg$ "You attend the lectures regularly" $\vee \neg$ "you pay attention" $\vee \neg$ "you clear your doubts in time" $\rightarrow \neg$ "you will find this course very easy"

$S \rightarrow P \wedge Q$

Definition: A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator: $\rightarrow$
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Variables: A, B, C, ..., P, Q, R, ..., A_1, A_2, ...

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In natural language, it is this

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Do you think this expression is equivalent to the above?

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And here is why it is necessarily true

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Let us now change interpretations of the variables

$P :=$ "You will keep up with the lecture"

$\neg$ ("You attend the lectures regularly" $\wedge$ "you pay attention" $\wedge$ "you clear your doubts in time") $\rightarrow \neg$ "you will find this course very easy"

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and observe that whether the expressions were equivalent or not

$P :=$ "You will keep up with the lecture"
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did not depend on the precise interpretation

$P :=$ "You will keep up with the lecture"
$Q :=$ "You will find the lecture interesting"
$R :=$ "You will understand everything"

$\neg$ ("You attend the lectures regularly" $\wedge$ "you pay attention" $\wedge$ "you clear your doubts in time") $\rightarrow \neg$ "you will find this course very easy"

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Using this interpretation this expression

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$S \to P$

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$\neg$ ("You attend the lectures regularly" $\wedge$ "you pay attention" $\wedge$ "you clear your doubts in time") $\to \neg$ "you will find this course very easy"

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You may attend the lectures regularly, pay attention, clear your doubts in time, but it may not be enough for this course! I might make the course very difficult!

Translates to this sentence in English

$P :=$ "You will keep up with the lecture"
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Although each variable has an interpretation in natural language...

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in logic we are only concerned with deductions we can make

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about whether the compound propositions are true or false

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given the truth or falseness of the propositions that they are composed of.

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So we bypass the natural language interpretations of the variables

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and only care if the propositional variables are assigned True or False in a given context

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Observe that if there are n variables, then then there are 2^n different functions

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This is the "semantic" side, as opposed to the "syntactic" side that we will see later.

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Let us consider the "not" function which has just one argument

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Semantic

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T	F

It will have just two possible functions, so we will have 2 rows

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Semantic

P	¬P
T	F

If the input is T, the output is F

$P :=$ "You will keep up with the lecture"
$Q :=$ "You will find the lecture interesting"
$R :=$ "You will understand everything"
$S :=$ "You pay attention"
$S \to P \wedge Q \wedge R$
"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

$\neg P \vee \neg Q \vee \neg R \to \neg S$
"If you are not keeping up with the lecture, or you are not finding the lecture interesting, or you do not understand something, then you are not paying attention."

Let S denote the set of propositional variables. Then a **valuation** is a function $\nu : S \to \{True, False, \}$.

Semantic

P	¬P
T	F
F	T

If the input is F, the output is T

$P :=$ "You will keep up with the lecture"
$Q :=$ "You will find the lecture interesting"
$R :=$ "You will understand everything"
$S :=$ "You pay attention"
$S \to P \wedge Q \wedge R$
"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

$\neg P \vee \neg Q \vee \neg R \to \neg S$
"If you are not keeping up with the lecture, or you are not finding the lecture interesting, or you do not understand something, then you are not paying attention."

Let S denote the set of propositional variables. Then a **valuation** is a function $\nu : S \to \{True, False, \}$.

Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q

Operators like $\wedge$ will have two arguments

$P :=$ "You will keep up with the lecture"
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$S \to P \wedge Q \wedge R$
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Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T

so 4 different valuations, represented as 4 rows

$P :=$ "You will keep up with the lecture"
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Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T

The first row is easy because keeping the usual interpretation of $land$

$P :=$ "You will keep up with the lecture"
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Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T

The output should be true if both P *and* Q are true

$P :=$ "You will keep up with the lecture"
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$R :=$ "You will understand everything"
$S :=$ "You pay attention"
$S \rightarrow P \wedge Q \wedge R$
"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$
"If you are not keeping up with the lecture, or you are not finding the lecture interesting, or you do not understand something, then you are not paying attention."

Let S denote the set of propositional variables. Then a **valuation** is a function $\nu : S \rightarrow \{True, False, \}$.

Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F

But false in all other circumstances

$P :=$ "You will keep up with the lecture"
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$S \to P \wedge Q \wedge R$
"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

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Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F

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T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

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Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q

The "or" operator has the same number of arguments

$P :=$ "You will keep up with the lecture"
$Q :=$ "You will find the lecture interesting"
$R :=$ "You will understand everything"
$S :=$ "You pay attention"
$S \to P \wedge Q \wedge R$
"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

$\neg P \vee \neg Q \vee \neg R \to \neg S$
"If you are not keeping up with the lecture, or you are not finding the lecture interesting, or you do not understand something, then you are not paying attention."

Let S denote the set of propositional variables. Then a **valuation** is a function $\nu : S \to \{True, False, \}$.

Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
F	F	F

but now it is easier to define it by it can be false

$P :=$ "You will keep up with the lecture"
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$R :=$ "You will understand everything"
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$S \to P \wedge Q \wedge R$
"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

$\neg P \vee \neg Q \vee \neg R \to \neg S$
"If you are not keeping up with the lecture, or you are not finding the lecture interesting, or you do not understand something, then you are not paying attention."

Let S denote the set of propositional variables. Then a **valuation** is a function $\nu : S \to \{True, False, \}$.

Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

$P :=$ "You will keep up with the lecture"
$Q :=$ "You will find the lecture interesting"
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$S \to P \wedge Q \wedge R$
"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

$\neg P \vee \neg Q \vee \neg R \to \neg S$
"If you are not keeping up with the lecture, or you are not finding the lecture interesting, or you do not understand something, then you are not paying attention."

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Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q

The trickiest one is "implies"

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$S \to P \wedge Q \wedge R$
"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

$\neg P \vee \neg Q \vee \neg R \to \neg S$
"If you are not keeping up with the lecture, or you are not finding the lecture interesting, or you do not understand something, then you are not paying attention."

Let S denote the set of propositional variables. Then a **valuation** is a function $\nu : S \to \{True, False, \}$.

Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q

Again, it is better to understand it by asking when it can be "wrong"

$P :=$ "You will keep up with the lecture"
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$R :=$ "You will understand everything"
$S :=$ "You pay attention"
$S \to P \wedge Q \wedge R$
"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

$\neg P \vee \neg Q \vee \neg R \to \neg S$
"If you are not keeping up with the lecture, or you are not finding the lecture interesting, or you do not understand something, then you are not paying attention."

Let S denote the set of propositional variables. Then a **valuation** is a function $\nu : S \to \{True, False, \}$.

Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q

$P :=$ "You will keep up with the lecture"
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$\neg P \vee \neg Q \vee \neg R \to \neg S$
"If you are not keeping up with the lecture, or you are not finding the lecture interesting, or you do not understand something, then you are not paying attention."

Let S denote the set of propositional variables. Then a **valuation** is a function $\nu : S \to \{True, False, \}$.

Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	F	F

An implication is false only if the assumption is true but what follows is still false

$P :=$ "You will keep up with the lecture"
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$S \to P \wedge Q \wedge R$
"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

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P	¬P
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P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

However, if P is false, the implication does not "apply", so it is still "vaccuously" true

$P :=$ "You will keep up with the lecture"
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"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

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T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \to R$

Let us consider a final and more complex example

$P :=$ "You will keep up with the lecture"
$Q :=$ "You will find the lecture interesting"
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$S \to P \wedge Q \wedge R$
"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

$\neg P \vee \neg Q \vee \neg R \to \neg S$
"If you are not keeping up with the lecture, or you are not finding the lecture interesting, or you do not understand something, then you are not paying attention."

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Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
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F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \to R$
T	T	T	T	

We use the "and" truth table to figure out the T

$P :=$ "You will keep up with the lecture"
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$\neg P \vee \neg Q \vee \neg R \to \neg S$
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P	¬P
T	F
F	T

P	Q	P ∧ Q
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T	F	F
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F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \to R$
T	T	T	T	T

and then we use the "implication" truth table

$P :=$ "You will keep up with the lecture"
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$S \to P \wedge Q \wedge R$
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P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \to R$
T	T	T	T	T
T	T	F	T	F

Now see if you can see why each of the rest of the rows are correct

$P :=$ "You will keep up with the lecture"
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$S \to P \wedge Q \wedge R$
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"If you are not keeping up with the lecture, or you are not finding the lecture interesting, or you do not understand something, then you are not paying attention."

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Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \to R$
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T	F	T
F	T	T
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T	F	F
F	T	T
F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \to R$
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T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
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T	F	T
F	T	T
F	F	F

P	Q	P → Q
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F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \to R$
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T	T	F	T	F
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T	F	F	F	T
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Semantic

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T	F
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T	T	T
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P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \to R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
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Semantic

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T	T	T
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P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \to R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T

Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \rightarrow R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

Syntactic

Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \rightarrow R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

We now consider the syntactic side

Syntactic

Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \rightarrow R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

where we are not concerned with the syntactic means

Syntactic

Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \rightarrow R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

but merely they relate with each other.

Syntactic

Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \rightarrow R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

Yet, as we will see, we will be able to prove everything that we could have derived from truth tables

Syntactic

Axioms

Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \rightarrow R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

In the next lecture we will be introduced to these axioms

Syntactic

Axioms

1. $p \to (q \to p)$

Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \to R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

Syntactic

Axioms

1. $p \to (q \to p)$
2. $p \to (q \to r) \to (p \to q \to [p \to r])$

Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \to R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

Syntactic

Axioms

1. $p \to (q \to p)$
2. $p \to (q \to r) \to (p \to q \to [p \to r])$
3. $\neg p \to \neg q \to (q \to p)$

Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \to R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

Syntactic

Axioms

1. $p \to (q \to p)$
2. $p \to (q \to r) \to (p \to q \to [p \to r])$
3. $\neg p \to \neg q \to (q \to p)$

$P \to Q, P \implies Q$

Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \to R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

and this very important rule

Syntactic

Axioms

1. $p \to (q \to p)$
2. $p \to (q \to r) \to (p \to q \to [p \to r])$
3. $\neg p \to \neg q \to (q \to p)$

$P \to Q, P \implies Q$ (Modus Ponens)

Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \to R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

called "Modus Ponens"

Syntactic

Axioms

1. $p \to (q \to p)$
2. $p \to (q \to r) \to (p \to q \to [p \to r])$
3. $\neg p \to \neg q \to (q \to p)$

$P \to Q, P \implies Q$ (Modus Ponens)

Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \to R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

Syntactic

Axioms

1. $p \to (q \to p)$
2. $p \to (q \to r) \to (p \to q \to [p \to r])$
3. $\neg p \to \neg q \to (q \to p)$

$P \to Q, P \implies Q$ (Modus Ponens)

First order logic

↑

Propositional Logic

Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \to R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

Syntactic

Axioms

1. $p \to (q \to p)$
2. $p \to (q \to r) \to (p \to q \to [p \to r])$
3. $\neg p \to \neg q \to (q \to p)$

$P \to Q, P \implies Q$ (Modus Ponens)

Set theory
↑
First order logic
↑
Propositional Logic

Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \to R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

Syntactic

Axioms

1. $p \to (q \to p)$
2. $p \to (q \to r) \to (p \to q \to [p \to r])$
3. $\neg p \to \neg q \to (q \to p)$

$P \to Q, P \implies Q$ (Modus Ponens)

Syntactic

Set theory

↑

First order logic

↑

Propositional Logic

Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \to R$
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T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

Syntactic

Axioms

1. $p \to (q \to p)$
2. $p \to (q \to r) \to (p \to q \to [p \to r])$
3. $\neg p \to \neg q \to (q \to p)$

$P \to Q, P \implies Q$ (Modus Ponens)

Syntactic Semantic

Set theory

↑

First order logic

↑

Propositional Logic

Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	T	T
T	F	F
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F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \to R$
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T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

Syntactic

Axioms

1. $p \to (q \to p)$
2. $p \to (q \to r) \to (p \to q \to [p \to r])$
3. $\neg p \to \neg q \to (q \to p)$

$P \to Q, P \implies Q$ (Modus Ponens)

Syntactic

Semantic

Set theory

↑

First order logic

↑

Propositional Logic

Truth tables

Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \to R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

Syntactic

Axioms

1. $p \to (q \to p)$
2. $p \to (q \to r) \to (p \to q \to [p \to r])$
3. $\neg p \to \neg q \to (q \to p)$

$P \to Q, P \implies Q$ (Modus Ponens)

Syntactic	Semantic
Set theory	
↑	
First order logic	Model
↑	
Propositional Logic	Truth tables

Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \to R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

Syntactic

Axioms

1. $p \to (q \to p)$
2. $p \to (q \to r) \to (p \to q \to [p \to r])$
3. $\neg p \to \neg q \to (q \to p)$

$P \to Q, P \implies Q$ (Modus Ponens)

Syntactic	Semantic
Set theory	Von-Neumann Universe
↑	
First order logic	Model
↑	
Propositional Logic	Truth tables

Semantic

P	¬P
T	F
F	T

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	P → Q
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T	F	F
F	T	T
F	F	T

P	Q	R	P ∧ Q	$P \wedge Q \to R$
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T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T