Exercise sheet 4

Set theory and Logic, MTH303

1. Show that $\{\forall x(\alpha \to \beta), \forall x\alpha\} \vDash \forall x\beta$.
2. Show that $\alpha \vDash \forall x\alpha$ as long as x is *not* free in α.
3. Define a new quantifier $\exists!$, so that $\exists!\alpha$ which is satisfied by s in a structure $\mathcal{U}$ iff there exists exactly one $k \in \mathcal{U}$ such tht $\vDash_{\mathcal{U}} \alpha[s(x|k)]$. Show that there $\exists! x\alpha$ is always logically equivalent to a formula that involves the letters provided by a usual first order alphabet.
4. Consider a first order language with just a one binary predicate symbol and no function symbol. Up to isomorphism, how many structures does it have with just two elements?
5. Let P denote a predicate symbol. Use the axioms to show that $\vdash Px \to \exists Py$. What is this theorem telling you?
6. Show that if $\vdash \alpha \to \beta$ then $\vdash \forall x\alpha \to \forall x\beta$.