

Formal Language

Consider the English language, specifically the way it is represented in written form.

Formal Language

Σ : alphabet (set of symbols)

It has an alphabet consisting of all the symbols that we need to write sentences

$$\Sigma = \{a, b, \dots, z,$$

Formal Language

Σ : alphabet (set of symbols)

So we have the letters

$$\Sigma = \{a, b, \dots, z, A, B, \dots, Z,$$

Formal Language

Σ : alphabet (set of symbols)

but we also need the upper case letter

$$\Sigma = \{a, b, \dots, z, A, B, \dots, Z, \text{ ,}$$

Formal Language

Σ : alphabet (set of symbols)

and the space symbol so that we can tell apart words

$$\Sigma = \{a, b, \dots, z, A, B, \dots, Z, \text{ }, \dots\}$$

Formal Language

Σ : alphabet (set of symbols)

and finally the punctuation marks

$$\Sigma = \{a, b, \dots, z, A, B, \dots, Z, \text{ }, \dots\}$$

Formal Language

Σ : alphabet (set of symbols)

Σ^* : set of finite strings

Formal Language

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$\Sigma = \{a, b, \dots, z, A, B, \dots, Z, \text{ }, \dots\}$

"I broke the chalk",

Now sentences are strings over the alphabet

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"I broke the chalk", "jklajsdlf ak vl dsjaf l daspfoa.3"

but it will also contain gibberish like this

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"I broke the chalk", "jklajsdlf ak vlidsjaf ldispfoa.3"

Now a language, at its most basic, specifies which strings are valid

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and which are invalid

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So we are interested in which elements of Σ^* , the set of strings, should be included and which should not be

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A language over the alphabet Σ is $L \subset \Sigma^*$

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So we define it as a subset of the set of string, Σ^*

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Note that here we do not describe the rules that define the subset

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That may differ between languages

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In natural languages, this is usually done by specifying a grammar

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Let us now take stock of the symbols that are used to form expressions in logic

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Proposition alphabet:

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We will call the resulting alphabet a proposition alphabet

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Proposition alphabet:

$\{A, B, \dots,$

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it must include variables

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Proposition alphabet:

$\{A, B, \dots, A_1, A_2, \dots$

$\Sigma = \{a, b, \dots, z, A, B, \dots, Z, ., \dots\}$

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including indexed variables if we need many of them

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"I broke the chalk", "jklajsdlf ak vldsjafl daspfoa.3"

and the usual functions and operators

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"I broke the chalk", "jklajsdlf ak vldsjafl daspfoa.3"

and now we need to specify the language, to declare which strings are valid.

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Example (*string*)

Definition A string is a propositional form if

We will simultaneously present an example

$\Sigma = \{a, b, \dots, z, A, B, \dots, Z, \dots\}$
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Example (*string*) P Q

Definition A string is a propositional form if

1. it is a proposition variable

We of course need the variables

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Example (*string*) $\neg P \quad Q$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p

and the negation of a propositional form

$\Sigma = \{a, b, \dots, z, A, B, \dots, Z, \dots\}$
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Example (*string*) $\neg P \quad Q$

Definition A string is a propositional form if

1. it is a proposition variable
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note the self referencing nature of this rule.

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Example (*string*) $\neg P \quad Q$

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We have not yet completely defined what a valid propositional form is

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Example (*string*) $\neg P \quad Q$

Definition A string is a propositional form if

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However, if something is valid, its negation will also be valid

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Example (*string*) $\neg P \quad Q$

Definition A string is a propositional form if

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So, as will shortly see, since $(P \vee (P \wedge Q))$ is valid, this rule says so is $\neg(P \vee (P \wedge Q))$

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Example (*string*) $\neg P \wedge Q$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q)$, $(p \wedge q)$, $(p \rightarrow q)$, $(p \leftrightarrow q)$ for some propositional forms p and q .

We finally include those that are composed by operators

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Example (*string*) $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

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Note that we use parentheses to avoid confusion about which operator is first

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Example (string) $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

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We will therefore have extra parentheses but we can always add a few more rules

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to allow us to drop the outermost ones

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A propositional form p is a **tautology**

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A propositional form p is a **tautology** if $\nu(p) = T$ for any ν .

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A propositional form p is a **tautology** if $\nu(p) = T$ for any ν . Denoted $\models p$

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Examples:

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Examples: $P \vee \neg P$

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Examples: $P \vee \neg P, \neg(P \wedge \neg Q)$

$$p \leftrightarrow \neg\neg p$$

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Examples: $P \vee \neg P, \neg(P \wedge \neg Q)$

$$p \leftrightarrow \neg\neg p$$

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$

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$$p \leftrightarrow \neg\neg p$$

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$

$$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$$

Proposition alphabet:

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Examples: $P \vee \neg P$, $\neg(P \wedge \neg Q)$

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Examples: $P \vee \neg P, \neg(P \wedge \neg Q)$

$$p \leftrightarrow \neg\neg p$$

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$

$$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$$

$$p \wedge p$$

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Examples: $P \vee \neg P, \neg(P \wedge \neg Q)$

$$p \leftrightarrow \neg\neg p$$

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$

$$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$$

$$p \wedge p \leftrightarrow p$$

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Examples: $P \vee \neg P, \neg(P \wedge \neg Q)$

$$p \leftrightarrow \neg\neg p$$

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$

$$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$$

$$p \wedge p \leftrightarrow p$$

$$p \vee p \leftrightarrow p$$

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$$(p \leftrightarrow q) \leftrightarrow ((p \rightarrow q) \wedge (q \leftarrow p))$$

$$(p \wedge q \rightarrow r) \leftrightarrow (p \rightarrow (q \rightarrow r))$$

$$(p \wedge (q \wedge r)) \leftrightarrow ((p \wedge q) \wedge r)$$

$$(p \vee (q \vee r)) \leftrightarrow ((p \vee q) \vee r)$$

$$(p \wedge q) \leftrightarrow (q \wedge p)$$

$$(p \vee q) \leftrightarrow (q \vee p)$$

$$(p \wedge (q \vee r)) \leftrightarrow ((p \wedge q) \vee (p \wedge r))$$

$$(p \vee (q \wedge r))$$

$$p \leftrightarrow \neg \neg p$$

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$

$$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$$

$$p \wedge p \leftrightarrow p$$

$$p \vee p \leftrightarrow p$$

$$p \wedge q \leftrightarrow q \wedge p$$

$$p \vee q \leftrightarrow q \vee p$$

$$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$$

$$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$$

$$(p \leftrightarrow q) \leftrightarrow ((p \rightarrow q) \wedge (q \leftarrow p))$$

$$(p \wedge q \rightarrow r) \leftrightarrow (p \rightarrow (q \rightarrow r))$$

$$(p \wedge (q \wedge r)) \leftrightarrow ((p \wedge q) \wedge r)$$

$$(p \vee (q \vee r)) \leftrightarrow ((p \vee q) \vee r)$$

$$(p \wedge q) \leftrightarrow (q \wedge p)$$

$$(p \vee q) \leftrightarrow (q \vee p)$$

$$(p \wedge (q \vee r)) \leftrightarrow ((p \wedge q) \vee (p \wedge r))$$

$$(p \vee (q \wedge r)) \leftrightarrow ((p \vee q) \wedge (p \vee r))$$

$$p \leftrightarrow \neg\neg p$$

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$

$$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$$

$$p \wedge p \leftrightarrow p$$

$$p \vee p \leftrightarrow p$$

$$p \wedge q \leftrightarrow q \wedge p$$

$$p \vee q \leftrightarrow q \vee p$$

$$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$$

$$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$$

$$(p \leftrightarrow q) \leftrightarrow ((p \rightarrow q) \wedge (q \leftarrow p))$$

$$(p \wedge q \rightarrow r) \leftrightarrow (p \rightarrow (q \rightarrow r))$$

$$(p \wedge (q \wedge r)) \leftrightarrow ((p \wedge q) \wedge r)$$

$$(p \vee (q \vee r)) \leftrightarrow ((p \vee q) \vee r)$$

$$(p \wedge q) \leftrightarrow (q \wedge p)$$

$$(p \vee q) \leftrightarrow (q \vee p)$$