Formal Language

Consider the English language, specifically the way it is represented in wirtten form.

Formal Language

Σ: alphabet (set of symbols)

It has an alphabet consisting of all the symbols that we need to write sentences

$$\Sigma = \{a, b, \ldots, z,$$

Formal Language

Σ: alphabet (set of symbols)

So we have the letters

$$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z,$$

Formal Language

Σ: alphabet (set of symbols)

but we also need the upper case letter

$$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ ,$$

Formal Language

Σ: alphabet (set of symbols)

and the space symbol so that we can tell apart words

$$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ , ., \ldots\}$$

Formal Language

Σ: alphabet (set of symbols)

and finally the punctuation marks

$$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ , ., \ldots\}$$

Formal Language

Σ: alphabet (set of symbols)

Σ^*: set of finite strings

Formal Language

Σ: alphabet (set of symbols)

Σ^*: set of finite strings

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ , ., \ldots\}$

"I broke the chalk",

Now sentences are strings over the alphabet

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ , ., \ldots\}$

"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Formal Language

Σ: alphabet (set of symbols)

Σ^*: set of finite strings

but it will also contain gibberish like this

$$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ , ., \ldots\}$$

"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Formal Language

Σ: alphabet (set of symbols)

Σ^*: set of finite strings

Now a language, at its most basic, specifies which strings are valid

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ \ ,., \ldots\}$

Formal Language

"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Σ: alphabet (set of symbols)

Σ^*: set of finite strings

and which are invalid

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ , ., \ldots\}$

"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Formal Language

Σ: alphabet (set of symbols)

Σ^*: set of finite strings

So we are interested in which elements of Σ^*, the set of strings, should be included and which should not be

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ , ., \ldots\}$

"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Formal Language

Σ: alphabet (set of symbols)

Σ^*: set of finite strings

A language over the alphabet Σ is $L \subset \Sigma^*$

So we define it as a subset of the set of string, Σ^*

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ , ., \ldots\}$

"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Formal Language

Σ: alphabet (set of symbols)

Σ^*: set of finite strings

A language over the alphabet Σ is $L \subset \Sigma^*$

Note that here we do not describe the rules that define the subset

$$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ , ., \ldots\}$$

"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Formal Language

Σ: alphabet (set of symbols)

Σ^*: set of finite strings

A language over the alphabet Σ is $L \subset \Sigma^*$

That may differ between languages

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ ,.,\ldots\}$

"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Formal Language

Σ: alphabet (set of symbols)

Σ^*: set of finite strings

A language over the alphabet Σ is $L \subset \Sigma^*$

In natural languages, this is usually done by specifying a grammar

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ , ., \ldots\}$

"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Formal Language

Σ: alphabet (set of symbols)

Σ^*: set of finite strings

A language over the alphabet Σ is $L \subset \Sigma^*$

Compilers or interpreters of computer languages look for syntax errors by testing it with a specified grammar

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ , ., \ldots\}$

"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Formal Language

Σ: alphabet (set of symbols)

Σ^*: set of finite strings

A language over the alphabet Σ is $L \subset \Sigma^*$

Let us now take stock of the symbols that are used to form expressions in logic

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ , ., \ldots\}$

"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Formal Language

Σ: alphabet (set of symbols)

Σ^*: set of finite strings

A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:

We will call the resulting alphabet a proposition alphabet

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ , ., \ldots\}$

"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Formal Language

Σ: alphabet (set of symbols)

Σ^*: set of finite strings

A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:

$\{A, B, \ldots,$

it must include variables

Formal Language

Σ: alphabet (set of symbols)

Σ^*: set of finite strings

A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:

$\{A, B, \ldots, A_1, A_2, \ldots$

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ \ , ., \ldots\}$

"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

including indexed variables if we need to many of them

$\Sigma = \{a, b, \dots, z, A, B, \dots, Z, \ , ., \dots\}$

"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Formal Language

Σ: alphabet (set of symbols)

Σ^*: set of finite strings

A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:

$\{A, B, \dots, A_1, A_2, \dots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

and the usual functions and operators

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ , ., \ldots\}$

"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Formal Language

Σ: alphabet (set of symbols)

Σ^*: set of finite strings

A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:

$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

and now we need to specify the language, to declare which strings are valid.

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ , ., \ldots\}$
"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)*

Definition A string is a propositional form if

We will simultaneously present an example

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ ,., \ldots\}$
"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $P \quad Q$

Definition A string is a propositional form if

1. it is a proposition variable

We of course need the variables

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ , ., \ldots\}$
"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $\neg P \quad Q$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p

and the negation of a propositional form

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ \ , ., \ldots\}$
"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $\neg P \quad Q$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p

note the self referencing nature of this rule.

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ \ , ., \ldots\}$
"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $\neg P \quad Q$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p

We have not yet completely defined what a valid propositional form is

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ , ., \ldots\}$

"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Formal Language

Σ: alphabet (set of symbols)

Σ^*: set of finite strings

A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:

$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $\neg P \quad Q$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p

However, if something is valid, its negation will also be valid

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ \ , ., \ldots\}$

"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Formal Language

Σ: alphabet (set of symbols)

Σ^*: set of finite strings

A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:

$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $\neg P \quad Q$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p

So, as will shortly see, since $(P \vee (P \wedge Q))$ is valid, this rule says so is $\neg(P \vee (P \wedge Q))$

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ ,., \ldots\}$
"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $\neg P \wedge Q$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

We finally include those that are composed by operators

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ \ , ., \ldots\}$
"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \to, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \to Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \to q), (p \leftrightarrow q)$ for some propositional forms p and q.

Note that we use parentheses to avoid confusion about which operator is first

Formal Language

Σ: alphabet (set of symbols)

Σ^*: set of finite strings

A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:

$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ ,., \ldots\}$

"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

We will therefore have extra parentheses but we can always add a few more rules

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ , ., \ldots\}$
"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

to allow us to drop the outermost ones

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \; ,., \ldots\}$
"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology**

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ , ., \ldots\}$
"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν.

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ , ., \ldots\}$
"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\vDash p$

Formal Language

Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ , ., \ldots\}$
"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\vDash p$

Examples:

$\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, \ , ., \ldots\}$
"I broke the chalk", "jklajsdlf ak vldsjaf ldaspfoa.3"

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\vDash p$

Examples: $P \vee \neg P$

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\models p$

Examples: $P \vee \neg P$, $\neg(P \wedge \neg Q)$

$p \leftrightarrow \neg\neg p$

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\models p$

Examples: $P \vee \neg P$, $\neg(P \wedge \neg Q)$

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\models p$

Examples: $P \vee \neg P$, $\neg(P \wedge \neg Q)$

$$p \leftrightarrow \neg\neg p$$

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\models p$

Examples: $P \vee \neg P$, $\neg(P \wedge \neg Q)$

$$p \leftrightarrow \neg\neg p$$

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$
$$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$$

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\vDash p$

Examples: $P \vee \neg P$, $\neg(P \wedge \neg Q)$

$p \leftrightarrow \neg\neg p$

$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$
$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$

$p \wedge p$

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\vDash p$

Examples: $P \vee \neg P$, $\neg(P \wedge \neg Q)$

$$p \leftrightarrow \neg\neg p$$

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$
$$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$$

$$p \wedge p \leftrightarrow p$$

Formal Language

Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\models p$

Examples: $P \vee \neg P$, $\neg(P \wedge \neg Q)$

$p \leftrightarrow \neg\neg p$

$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$
$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$

$p \wedge p \leftrightarrow p$
$p \vee p \leftrightarrow p$

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\models p$

Examples: $P \vee \neg P$, $\neg(P \wedge \neg Q)$

$p \leftrightarrow \neg\neg p$

$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$
$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$

$p \wedge p \leftrightarrow p$
$p \vee p \leftrightarrow p$

$p \wedge q$

Formal Language

Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\models p$

Examples: $P \vee \neg P$, $\neg(P \wedge \neg Q)$

$p \leftrightarrow \neg\neg p$

$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$
$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$

$p \wedge p \leftrightarrow p$
$p \vee p \leftrightarrow p$

$p \wedge q \leftrightarrow q \wedge p$

Formal Language

Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\vDash p$

Examples: $P \vee \neg P$, $\neg(P \wedge \neg Q)$

$p \leftrightarrow \neg\neg p$

$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$
$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$

$p \wedge p \leftrightarrow p$
$p \vee p \leftrightarrow p$

$p \wedge q \leftrightarrow q \wedge p$
$p \vee q \leftrightarrow q \vee p$

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\models p$

Examples: $P \vee \neg P$, $\neg(P \wedge \neg Q)$

$p \leftrightarrow \neg\neg p$

$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$
$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$

$p \wedge p \leftrightarrow p$
$p \vee p \leftrightarrow p$

$p \wedge q \leftrightarrow q \wedge p$
$p \vee q \leftrightarrow q \vee p$

$(p \rightarrow q)$

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\models p$

Examples: $P \vee \neg P$, $\neg(P \wedge \neg Q)$

$p \leftrightarrow \neg\neg p$

$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$
$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$

$p \wedge p \leftrightarrow p$
$p \vee p \leftrightarrow p$

$p \wedge q \leftrightarrow q \wedge p$
$p \vee q \leftrightarrow q \vee p$

$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\models p$

Examples: $P \vee \neg P$, $\neg(P \wedge \neg Q)$

$p \leftrightarrow \neg\neg p$

$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$
$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$

$p \wedge p \leftrightarrow p$
$p \vee p \leftrightarrow p$

$p \wedge q \leftrightarrow q \wedge p$
$p \vee q \leftrightarrow q \vee p$

$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$
$(p \rightarrow q)$

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\models p$

Examples: $P \vee \neg P$, $\neg(P \wedge \neg Q)$

$p \leftrightarrow \neg\neg p$

$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$
$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$

$p \wedge p \leftrightarrow p$
$p \vee p \leftrightarrow p$

$p \wedge q \leftrightarrow q \wedge p$
$p \vee q \leftrightarrow q \vee p$

$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$
$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\vDash p$

Examples: $P \vee \neg P$, $\neg(P \wedge \neg Q)$

$p \leftrightarrow \neg\neg p$

$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$
$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$

$p \wedge p \leftrightarrow p$
$p \vee p \leftrightarrow p$

$p \wedge q \leftrightarrow q \wedge p$
$p \vee q \leftrightarrow q \vee p$

$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$
$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

$(p \leftrightarrow q)$

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\vDash p$

Examples: $P \vee \neg P$, $\neg(P \wedge \neg Q)$

$p \leftrightarrow \neg\neg p$

$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$
$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$

$p \wedge p \leftrightarrow p$
$p \vee p \leftrightarrow p$

$p \wedge q \leftrightarrow q \wedge p$
$p \vee q \leftrightarrow q \vee p$

$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$
$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

$(p \leftrightarrow q) \leftrightarrow ((p \rightarrow q) \wedge (q \leftarrow p))$

Formal Language

Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\vDash p$

Examples: $P \vee \neg P$, $\neg(P \wedge \neg Q)$

$p \leftrightarrow \neg\neg p$

$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$
$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$

$p \wedge p \leftrightarrow p$
$p \vee p \leftrightarrow p$

$p \wedge q \leftrightarrow q \wedge p$
$p \vee q \leftrightarrow q \vee p$

$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$
$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

$(p \leftrightarrow q) \leftrightarrow ((p \rightarrow q) \wedge (q \leftarrow p))$

$(p \wedge q \rightarrow r)$

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\models p$

Examples: $P \vee \neg P$, $\neg(P \wedge \neg Q)$

$p \leftrightarrow \neg\neg p$

$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$
$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$

$p \wedge p \leftrightarrow p$
$p \vee p \leftrightarrow p$

$p \wedge q \leftrightarrow q \wedge p$
$p \vee q \leftrightarrow q \vee p$

$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$
$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

$(p \leftrightarrow q) \leftrightarrow ((p \rightarrow q) \wedge (q \leftarrow p))$

$(p \wedge q \rightarrow r) \leftrightarrow (p \rightarrow (q \rightarrow r))$

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\models p$

Examples: $P \vee \neg P$, $\neg(P \wedge \neg Q)$

$p \leftrightarrow \neg\neg p$

$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$
$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$

$p \wedge p \leftrightarrow p$
$p \vee p \leftrightarrow p$

$p \wedge q \leftrightarrow q \wedge p$
$p \vee q \leftrightarrow q \vee p$

$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$
$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

$(p \leftrightarrow q) \leftrightarrow ((p \rightarrow q) \wedge (q \leftarrow p))$

$(p \wedge q \rightarrow r) \leftrightarrow (p \rightarrow (q \rightarrow r))$

$(p \wedge (q \wedge r))$

Formal Language

Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\vDash p$

Examples: $P \vee \neg P$, $\neg(P \wedge \neg Q)$

$$p \leftrightarrow \neg\neg p$$

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$
$$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$$

$$p \wedge p \leftrightarrow p$$
$$p \vee p \leftrightarrow p$$

$$p \wedge q \leftrightarrow q \wedge p$$
$$p \vee q \leftrightarrow q \vee p$$

$$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$$
$$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$$

$$(p \leftrightarrow q) \leftrightarrow ((p \rightarrow q) \wedge (q \leftarrow p))$$

$$(p \wedge q \rightarrow r) \leftrightarrow (p \rightarrow (q \rightarrow r))$$

$$(p \wedge (q \wedge r)) \leftrightarrow ((p \wedge q) \wedge r)$$

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \to, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \to Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \to q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\models p$

Examples: $P \vee \neg P$, $\neg(P \wedge \neg Q)$

$p \leftrightarrow \neg\neg p$

$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$
$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$

$p \wedge p \leftrightarrow p$
$p \vee p \leftrightarrow p$

$p \wedge q \leftrightarrow q \wedge p$
$p \vee q \leftrightarrow q \vee p$

$(p \to q) \leftrightarrow (\neg p \vee q)$
$(p \to q) \leftrightarrow (\neg q \to \neg p)$

$(p \leftrightarrow q) \leftrightarrow ((p \to q) \wedge (q \leftarrow p))$

$(p \wedge q \to r) \leftrightarrow (p \to (q \to r))$

$(p \wedge (q \wedge r)) \leftrightarrow ((p \wedge q) \wedge r)$
$(p \vee (q \vee r))$

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\models p$

Examples: $P \vee \neg P$, $\neg(P \wedge \neg Q)$

$p \leftrightarrow \neg\neg p$

$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$
$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$

$p \wedge p \leftrightarrow p$
$p \vee p \leftrightarrow p$

$p \wedge q \leftrightarrow q \wedge p$
$p \vee q \leftrightarrow q \vee p$

$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$
$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

$(p \leftrightarrow q) \leftrightarrow ((p \rightarrow q) \wedge (q \leftarrow p))$

$(p \wedge q \rightarrow r) \leftrightarrow (p \rightarrow (q \rightarrow r))$

$(p \wedge (q \wedge r)) \leftrightarrow ((p \wedge q) \wedge r)$
$(p \vee (q \vee r)) \leftrightarrow ((p \vee q) \vee r)$

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\models p$

Examples: $P \vee \neg P$, $\neg(P \wedge \neg Q)$

$p \leftrightarrow \neg\neg p$

$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$
$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$

$p \wedge p \leftrightarrow p$
$p \vee p \leftrightarrow p$

$p \wedge q \leftrightarrow q \wedge p$
$p \vee q \leftrightarrow q \vee p$

$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$
$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

$(p \leftrightarrow q) \leftrightarrow ((p \rightarrow q) \wedge (q \leftarrow p))$

$(p \wedge q \rightarrow r) \leftrightarrow (p \rightarrow (q \rightarrow r))$

$(p \wedge (q \wedge r)) \leftrightarrow ((p \wedge q) \wedge r)$
$(p \vee (q \vee r)) \leftrightarrow ((p \vee q) \vee r)$

$(p \wedge q)$

Formal Language

Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\vDash p$

Examples: $P \vee \neg P$, $\neg(P \wedge \neg Q)$

$p \leftrightarrow \neg\neg p$

$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$
$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$

$p \wedge p \leftrightarrow p$
$p \vee p \leftrightarrow p$

$p \wedge q \leftrightarrow q \wedge p$
$p \vee q \leftrightarrow q \vee p$

$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$
$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

$(p \leftrightarrow q) \leftrightarrow ((p \rightarrow q) \wedge (q \leftarrow p))$

$(p \wedge q \rightarrow r) \leftrightarrow (p \rightarrow (q \rightarrow r))$

$(p \wedge (q \wedge r)) \leftrightarrow ((p \wedge q) \wedge r)$
$(p \vee (q \vee r)) \leftrightarrow ((p \vee q) \vee r)$

$(p \wedge q) \leftrightarrow (q \wedge p)$

Formal Language
Σ: alphabet (set of symbols)
Σ^*: set of finite strings
A language over the alphabet Σ is $L \subset \Sigma^*$

Proposition alphabet:
$\{A, B, \ldots, A_1, A_2, \ldots, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, (,), [,]\}$

Example *(string)* $(\neg P \wedge Q) \vee (P \rightarrow Q \wedge P)$

Definition A string is a propositional form if

1. it is a proposition variable
2. of the form $\neg p$ for some propositional form p
3. of the form $(p \vee q), (p \wedge q), (p \rightarrow q), (p \leftrightarrow q)$ for some propositional forms p and q.

A propositional for p is a **tautology** if $\nu(p) = T$ for any ν. Denoted $\models p$

Examples: $P \vee \neg P$, $\neg(P \wedge \neg Q)$

$p \leftrightarrow \neg\neg p$

$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$
$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$

$p \wedge p \leftrightarrow p$
$p \vee p \leftrightarrow p$

$p \wedge q \leftrightarrow q \wedge p$
$p \vee q \leftrightarrow q \vee p$

$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$
$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$

$(p \leftrightarrow q) \leftrightarrow ((p \rightarrow q) \wedge (q \leftarrow p))$

$(p \wedge q \rightarrow r) \leftrightarrow (p \rightarrow (q \rightarrow r))$

$(p \wedge (q \wedge r)) \leftrightarrow ((p \wedge q) \wedge r)$
$(p \vee (q \vee r)) \leftrightarrow ((p \vee q) \vee r)$

$(p \wedge q) \leftrightarrow (q \wedge p)$
$(p \vee q)$

$$p \leftrightarrow \neg\neg p$$

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$
$$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$$

$$p \wedge p \leftrightarrow p$$
$$p \vee p \leftrightarrow p$$

$$p \wedge q \leftrightarrow q \wedge p$$
$$p \vee q \leftrightarrow q \vee p$$

$$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$$
$$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$$

$$(p \leftrightarrow q) \leftrightarrow ((p \rightarrow q) \wedge (q \leftarrow p))$$

$$(p \wedge q \rightarrow r) \leftrightarrow (p \rightarrow (q \rightarrow r))$$

$$(p \wedge (q \wedge r)) \leftrightarrow ((p \wedge q) \wedge r)$$
$$(p \vee (q \vee r)) \leftrightarrow ((p \vee q) \vee r)$$

$$(p \wedge q) \leftrightarrow (q \wedge p)$$
$$(p \vee q) \leftrightarrow (q \vee p)$$

$(p \wedge (q \vee r))$

$$p \leftrightarrow \neg\neg p$$

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$
$$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$$

$$p \wedge p \leftrightarrow p$$
$$p \vee p \leftrightarrow p$$

$$p \wedge q \leftrightarrow q \wedge p$$
$$p \vee q \leftrightarrow q \vee p$$

$$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$$
$$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$$

$$(p \leftrightarrow q) \leftrightarrow ((p \rightarrow q) \wedge (q \leftarrow p))$$

$$(p \wedge q \rightarrow r) \leftrightarrow (p \rightarrow (q \rightarrow r))$$

$$(p \wedge (q \wedge r)) \leftrightarrow ((p \wedge q) \wedge r)$$
$$(p \vee (q \vee r)) \leftrightarrow ((p \vee q) \vee r)$$

$$(p \wedge q) \leftrightarrow (q \wedge p)$$
$$(p \vee q) \leftrightarrow (q \vee p)$$

$$(p \wedge (q \vee r)) \leftrightarrow ((p \wedge q) \vee (p \wedge r))$$

$$p \leftrightarrow \neg\neg p$$

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$
$$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$$

$$p \wedge p \leftrightarrow p$$
$$p \vee p \leftrightarrow p$$

$$p \wedge q \leftrightarrow q \wedge p$$
$$p \vee q \leftrightarrow q \vee p$$

$$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$$
$$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$$

$$(p \leftrightarrow q) \leftrightarrow ((p \rightarrow q) \wedge (q \leftarrow p))$$

$$(p \wedge q \rightarrow r) \leftrightarrow (p \rightarrow (q \rightarrow r))$$

$$(p \wedge (q \wedge r)) \leftrightarrow ((p \wedge q) \wedge r)$$
$$(p \vee (q \vee r)) \leftrightarrow ((p \vee q) \vee r)$$

$$(p \wedge q) \leftrightarrow (q \wedge p)$$
$$(p \vee q) \leftrightarrow (q \vee p)$$

$$(p \wedge (q \vee r)) \leftrightarrow ((p \wedge q) \vee (p \wedge r))$$
$$(p \vee (q \wedge r))$$

$$p \leftrightarrow \neg\neg p$$

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$
$$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$$

$$p \wedge p \leftrightarrow p$$
$$p \vee p \leftrightarrow p$$

$$p \wedge q \leftrightarrow q \wedge p$$
$$p \vee q \leftrightarrow q \vee p$$

$$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$$
$$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$$

$$(p \leftrightarrow q) \leftrightarrow ((p \rightarrow q) \wedge (q \leftarrow p))$$

$$(p \wedge q \rightarrow r) \leftrightarrow (p \rightarrow (q \rightarrow r))$$

$$(p \wedge (q \wedge r)) \leftrightarrow ((p \wedge q) \wedge r)$$
$$(p \vee (q \vee r)) \leftrightarrow ((p \vee q) \vee r)$$

$$(p \wedge q) \leftrightarrow (q \wedge p)$$
$$(p \vee q) \leftrightarrow (q \vee p)$$

$$(p \wedge (q \vee r)) \leftrightarrow ((p \wedge q) \vee (p \wedge r))$$
$$(p \vee (q \wedge r)) \leftrightarrow ((p \vee q) \wedge (p \vee r))$$

$$p \leftrightarrow \neg\neg p$$

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$
$$\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$$

$$p \wedge p \leftrightarrow p$$
$$p \vee p \leftrightarrow p$$

$$p \wedge q \leftrightarrow q \wedge p$$
$$p \vee q \leftrightarrow q \vee p$$

$$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$$
$$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$$

$$(p \leftrightarrow q) \leftrightarrow ((p \rightarrow q) \wedge (q \leftarrow p))$$

$$(p \wedge q \rightarrow r) \leftrightarrow (p \rightarrow (q \rightarrow r))$$

$$(p \wedge (q \wedge r)) \leftrightarrow ((p \wedge q) \wedge r)$$
$$(p \vee (q \vee r)) \leftrightarrow ((p \vee q) \vee r)$$

$$(p \wedge q) \leftrightarrow (q \wedge p)$$
$$(p \vee q) \leftrightarrow (q \vee p)$$