

Let us first consider the basic building block of logic

**Definition:** A **proposition**

A proposition!

**Definition:** A **proposition** is a statement that is true or false.

We do not care if the statement is really true or not

**Definition:** A **proposition** is a statement that is true or false.

only whether it makes sense to ask if it is true or false

**Definition:** A **proposition** is a statement that is true or false.

“If you do not attend the lectures regularly

Here is an example

**Definition:** A **proposition** is a statement that is true or false.

“If you do not attend the lectures regularly or you do not pay attention

of a complex proposition

**Definition:** A **proposition** is a statement that is true or false.

“If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time

involving many simpler propositions

**Definition:** A **proposition** is a statement that is true or false.

“If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult.”

combined by what we will call “logical operators”



**Definition:** A **proposition** is a statement that is true or false.

“If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time **then** you will find this course very difficult.”

Observe the “if...then”

**Definition:** A **proposition** is a statement that is true or false.

“If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time **then** you will find this course very difficult.”

“implies” operator:

We call this “implies”

**Definition:** A **proposition** is a statement that is true or false.

“If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time **then** you will find this course very difficult.”

“implies” operator:  $\rightarrow$

and denote it by this arrow

**Definition:** A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time **then** you will find this course very difficult."

"implies" operator:  $\rightarrow$

"You do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time"  $\rightarrow$  "you will find this course very difficult"

Now we rewrite using our new operator

**Definition:** A **proposition** is a statement that is true or false.

“If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult.”

“implies” operator:  $\rightarrow$

“You do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time”  $\rightarrow$  “you will find this course very difficult”

But this is not the only operator

**Definition:** A **proposition** is a statement that is true or false.

“If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult.”

“implies” operator:  $\rightarrow$

“You do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time”  $\rightarrow$  “you will find this course very difficult”

Notice the “or” in this sentence

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"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator:  $\rightarrow$

"You do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time"  $\rightarrow$  "you will find this course very difficult"

Here is another one

**Definition:** A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator:  $\rightarrow$

"or" operator:

"You do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time"  $\rightarrow$  "you will find this course very difficult"

This is called the "or operator"



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"If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult."

"implies" operator:  $\rightarrow$

"or" operator:  $\vee$

"You do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time"  $\rightarrow$  "you will find this course very difficult"

Which we denote by this symbol:  $\vee$

**Definition:** A **proposition** is a statement that is true or false.

"If you do not attend the lectures regularly **or** you do not pay attention **or** you do not clear your doubts in time then you will find this course very difficult."

"implies" operator:  $\rightarrow$

"or" operator:  $\vee$

"You do not attend the lectures regularly"  $\vee$  "you do not pay attention"  $\vee$  "you do not clear your doubts in time"  $\rightarrow$  "you will find this course very difficult"

Again we rewrite using our new operator

**Definition:** A **proposition** is a statement that is true or false.

"If you do **not** attend the lectures regularly or you do **not** pay attention or you do **not** clear your doubts in time then you will find this course very difficult."

"implies" operator:  $\rightarrow$

"or" operator:  $\vee$

"You do not attend the lectures regularly"  $\vee$  "you do not pay attention"  $\vee$  "you do not clear your doubts in time"  $\rightarrow$  "you will find this course very difficult"

Now observe that we have used another "logical" word (highlighted)

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"implies" operator:  $\rightarrow$

"or" operator:  $\vee$

"not" function:

"You do not attend the lectures regularly"  $\vee$  "you do not pay attention"  $\vee$  "you do not clear your doubts in time"  $\rightarrow$  "you will find this course very difficult"

The "not", also called "negation"

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"implies" operator:  $\rightarrow$

"or" operator:  $\vee$

"not" function:  $\neg$

"You do not attend the lectures regularly"  $\vee$  "you do not pay attention"  $\vee$  "you do not clear your doubts in time"  $\rightarrow$  "you will find this course very difficult"

which we denote with this symbol

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"implies" operator:  $\rightarrow$

"or" operator:  $\vee$

"not" function:  $\neg$

$\neg$  "You attend the lectures regularly"  $\vee$   $\neg$  "you pay attention"  $\vee$   $\neg$  "you clear your doubts in time"  $\rightarrow$   $\neg$  "you will find this course very easy"

again, we rewrite everything using this new symbol

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“implies” operator:  $\rightarrow$

“or” operator:  $\vee$

“not” function:  $\neg$

$\neg$  “You attend the lectures regularly”  $\vee \neg$  “you pay attention”  $\vee \neg$  “you clear your doubts in time”  $\rightarrow \neg$  “you will find this course very easy”

Now we will introduce an operator we do not strictly need

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"implies" operator:  $\rightarrow$

"or" operator:  $\vee$

"not" function:  $\neg$

"and" operator:

$\neg$  "You attend the lectures regularly"  $\vee$   $\neg$  "you pay attention"  $\vee$   $\neg$  "you clear your doubts in time"  $\rightarrow$   $\neg$  "you will find this course very easy"

In fact, it can be written in terms of the other ones



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“or” operator:  $\vee$

“not” function:  $\neg$

“and” operator:

$\neg$  “You attend the lectures regularly”  $\vee \neg$  “you pay attention”  $\vee \neg$  “you clear your doubts in time”  $\rightarrow \neg$  “you will find this course very easy”

It is called “and”

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"implies" operator:  $\rightarrow$

"or" operator:  $\vee$

"not" function:  $\neg$

"and" operator:  $\wedge$

$\neg$  "You attend the lectures regularly"  $\vee \neg$  "you pay attention"  $\vee \neg$  "you clear your doubts in time"  $\rightarrow \neg$  "you will find this course very easy"

and is denoted by  $\wedge$

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"implies" operator:  $\rightarrow$

"or" operator:  $\vee$

"not" function:  $\neg$

"and" operator:  $\wedge$

$\neg$  ("You attend the lectures regularly"  $\wedge$  "you pay attention"  $\wedge$  "you clear your doubts in time")  $\rightarrow \neg$  "you will find this course very easy"

Here we rewrite everything to use  $\wedge$  instead of  $\vee$

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"implies" operator:  $\rightarrow$

"or" operator:  $\vee$

"not" function:  $\neg$

"and" operator:  $\wedge$

"if and only if" operator:

$\neg$  ( "You attend the lectures regularly"  $\wedge$  "you pay attention"  $\wedge$  "you clear your doubts in time" )  $\rightarrow$   $\neg$  "you will find this course very easy"

We have one more operator which we will use later

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"or" operator:  $\vee$

"not" function:  $\neg$

"and" operator:  $\wedge$

"if and only if" operator:  $\leftrightarrow$

$\neg$  ("You attend the lectures regularly"  $\wedge$  "you pay attention"  $\wedge$  "you clear your doubts in time")  $\rightarrow \neg$  "you will find this course very easy"

denoted by  $\leftrightarrow$

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“implies” operator:  $\rightarrow$

“or” operator:  $\vee$

“not” function:  $\neg$

“and” operator:  $\wedge$

“if and only if” operator:  $\leftrightarrow$

*Variables:*  $A, B, C, \dots$ ,

$\neg$  (“You attend the lectures regularly”  $\wedge$  “you pay attention”  $\wedge$  “you clear your doubts in time”)  $\rightarrow \neg$  “you will find this course very easy”

Apart from operators, let us introduce “variables” in our “language”

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“if and only if” operator:  $\leftrightarrow$

*Variables:*  $A, B, C, \dots, P, Q, R,$

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We can use any of the upper case letters

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and even index them if we need too many



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$\neg (\text{"You attend the lectures regularly"} \wedge \text{"you pay attention"} \wedge \text{"you clear your doubts in time"}) \rightarrow \neg \text{"you will find this course very easy"}$

"implies" operator:  $\rightarrow$

"or" operator:  $\vee$

"not" function:  $\neg$

"and" operator:  $\wedge$

"if and only if" operator:  $\leftrightarrow$

*Variables:*  $A, B, C, \dots, P, Q, R, \dots, A_1, A_2, \dots$

*Example:*

Let us abstract out the specific propositions as variables

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*Variables:*  $A, B, C, \dots, P, Q, R, \dots, A_1, A_2, \dots$

*Example:*

$P :=$  “You attend the courses regularly”

$\neg (\text{“You attend the lectures regularly”} \wedge \text{“you pay attention”} \wedge \text{“you clear your doubts in time”}) \rightarrow \neg$   
“you will find this course very easy”

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$\neg ( \text{"You attend the lectures regularly"} \wedge \text{"you pay attention"} \wedge \text{"you clear your doubts in time"} ) \rightarrow \neg \text{"you will find this course very easy"}$

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$P :=$  "You attend the courses regularly"

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$\neg(P \wedge Q \wedge R) \rightarrow \neg S$

$\neg ( \text{"You attend the lectures regularly"} \wedge \text{"you pay attention"} \wedge \text{"you clear your doubts in time"} ) \rightarrow \neg \text{"you will find this course very easy"}$

Using these interpretations we get this expression

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$\neg P$

But we know that the following expression is also equivalent

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$\neg P \vee \neg Q$



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$\neg(P \wedge Q \wedge R) \rightarrow \neg S$

$\neg ( \text{“You attend the lectures regularly”} \wedge \text{“you pay attention”} \wedge \text{“you clear your doubts in time”} ) \rightarrow \neg \text{“you will find this course very easy”}$

$\neg P \vee \neg Q \vee \neg R$

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$\neg(P \wedge Q \wedge R) \rightarrow \neg S$

$\neg ("You\ attend\ the\ lectures\ regularly" \wedge "you\ pay\ attention" \wedge "you\ clear\ your\ doubts\ in\ time") \rightarrow \neg$   
"you will find this course very easy"

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$

$\neg$  "You attend the lectures regularly"

In natural language, it translates to this sentence

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$\neg (\text{"You attend the lectures regularly"} \wedge \text{"you pay attention"} \wedge \text{"you clear your doubts in time"}) \rightarrow \neg \text{"you will find this course very easy"}$

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$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$

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$S$

We will later see that we can always rewrite a the previous logical expression as this

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“If you do not attend the lectures regularly or you do not pay attention or you do not clear your doubts in time then you will find this course very difficult.”

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"If you find this course easy,

In natural language, it is this

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"If you find this course easy, then it implies you attend the lectures regularly, pay attention, and clear your doubts in time"

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$S \rightarrow P \wedge Q \wedge R$

"If you find this course easy, then it implies you attend the lectures regularly, pay attention, and clear your doubts in time"

$P$

Do you think this expression is equivalent to the above?

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We will see later that it is not

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$P \wedge Q \wedge R \rightarrow S?$  Not necessarily!!

You may attend the lectures regularly, pay attention, clear your doubts in time, but it may not be enough for this course!

And here is why it is necessarily true

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Let us now change interpretations of the variables

$P :=$  "You will keep up with the lecture"

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and observe that whether the expressions were equivalent or not

$P :=$  "You will keep up with the lecture"  
 $Q :=$  "You will find the lecture interesting"

$\neg ( \text{"You attend the lectures regularly"} \wedge \text{"you pay attention"} \wedge \text{"you clear your doubts in time"} ) \rightarrow \neg \text{"you will find this course very easy"}$

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$

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did not depend on the precise interpretation

$P :=$  "You will keep up with the lecture"  
 $Q :=$  "You will find the lecture interesting"  
 $R :=$  "You will understand everything"

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$$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$$

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Although each variable has an interpretation in natural language...

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in logic we are only concerned with deductions we can make

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about whether the compound propositions are true or false



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given the truth or falseness of the propositions that they are composed of.

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So we bypass the natural language interpretations of the variables

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$\neg ( \text{“You attend the lectures regularly”} \wedge \text{“you pay attention”} \wedge \text{“you clear your doubts in time”} ) \rightarrow \neg \text{“you will find this course very easy”}$

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and only care if the propositional variables are assigned  $T$  or  $F$  in a given context

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$\neg (\text{"You attend the lectures regularly"} \wedge \text{"you pay attention"} \wedge \text{"you clear your doubts in time"}) \rightarrow \neg \text{"you will find this course very easy"}$

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A valuation is, therefore, determined by its value on variables.

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Observe that if there are  $n$  variables, then there are  $2^n$  different valuations

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This is the “semantic” side, as opposed to the “syntactic” side that we will see later.



## Semantic

P		$\neg P$
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Let us consider the "not" function which has just one argument

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**Semantic**

P	$\neg P$
T	F

It will have just two possible functions, so we will have 2 rows

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**Semantic**

P	$\neg P$
T	F

If the input is  $T$ , the output is  $F$

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**Semantic**

P	$\neg P$
T	F
F	T

If the input is  $F$ , the output is  $T$

$P :=$  "You will keep up with the lecture"  
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2.  $\nu(p \wedge q) = T$  iff  $\nu(p) = T$ , and  $\nu(q) = T$
3.  $\nu(p \vee q) = F$  iff  $\nu(p) = F$ , and  $\nu(q) = F$
4.  $\nu(p \rightarrow q) = F$  iff  $\nu(p) = T$ , and  $\nu(q) = F$
5.  $\nu(p \leftrightarrow q) = T$  iff  $\nu(p) = \nu(q)$ ,  $F$

**Semantic**

P	$\neg P$
T	F
F	T

P	Q	$P \wedge Q$
---	---	--------------

Operators like  $\wedge$  will have two arguments

$P :=$  "You will keep up with the lecture"

$Q :=$  "You will find the lecture interesting"

$R :=$  "You will understand everything"

$S :=$  "You pay attention"

$S \rightarrow P \wedge Q \wedge R$

"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$

"If you are not keeping up with the lecture, or you are not finding the lecture interesting, or you do not understand something, then you are not paying attention."

Let  $S$  denote the set of propositional variables. Then a **valuation** is a function  $\nu : S \rightarrow \{T, F\}$ . if,

1.  $\nu(\neg p) = F$  iff  $\nu(p) = T$
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5.  $\nu(p \leftrightarrow q) = T$  iff  $\nu(p) = \nu(q)$

so 4 different valuations, represented as 4 rows

**Semantic**

P	$\neg P$
T	F
F	T

P	Q	$P \wedge Q$
T	T	T

$P :=$  "You will keep up with the lecture"

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$S \rightarrow P \wedge Q \wedge R$

"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$

"If you are not keeping up with the lecture, or you are not finding the lecture interesting, or you do not understand something, then you are not paying attention."

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**Semantic**

P	$\neg P$
T	F
F	T

P	Q	$P \wedge Q$
T	T	T

The first row is easy because keeping the usual interpretation of *land*

$P :=$  "You will keep up with the lecture"  
 $Q :=$  "You will find the lecture interesting"  
 $R :=$  "You will understand everything"  
 $S :=$  "You pay attention"  
 $S \rightarrow P \wedge Q \wedge R$

"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$

"If you are not keeping up with the lecture, or you are not finding the lecture interesting, or you do not understand something, then you are not paying attention."

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**Semantic**

P	$\neg P$
T	F
F	T

P	Q	$P \wedge Q$
T	T	T

The output should be true if both  $P$  and  $Q$  are true



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$S :=$  "You pay attention"

$S \rightarrow P \wedge Q \wedge R$

"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$

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5.  $\nu(p \leftrightarrow q) = T$  iff  $\nu(p) = \nu(q)$ ,  $F$

But false in all other circumstances

**Semantic**

P	$\neg P$
T	F
F	T

P	Q	$P \wedge Q$
T	T	T
T	F	F

$P :=$  "You will keep up with the lecture"

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$S \rightarrow P \wedge Q \wedge R$

"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$

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### Semantic

P	$\neg P$
T	F
F	T

P	Q	$P \wedge Q$
T	T	T
T	F	F
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$P :=$  "You will keep up with the lecture"

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$S \rightarrow P \wedge Q \wedge R$

"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$

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### Semantic

P	$\neg P$
T	F
F	T

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

$P :=$  "You will keep up with the lecture"

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"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

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**Semantic**

P	$\neg P$
T	F
F	T

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
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The "or" operator has the same number of arguments

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$S \rightarrow P \wedge Q \wedge R$

"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$

"If you are not keeping up with the lecture, or you are not finding the lecture interesting, or you do not understand something, then you are not paying attention."

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5.  $\nu(p \leftrightarrow q) = T$  iff  $\nu(p) = \nu(q)$

but now it is easier to define it by it can be false

**Semantic**

P	$\neg P$
T	F
F	T

P	Q	$P \vee Q$
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P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

F	F	F
---	---	---

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$S \rightarrow P \wedge Q \wedge R$

"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$

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### Semantic

P	$\neg P$
T	F
F	T

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
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P	Q	$P \vee Q$
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## Semantic

P	$\neg P$
T	F
F	T

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \rightarrow Q$
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The trickiest one is "implies"

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$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$

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## Semantic

P	$\neg P$
T	F
F	T

P	Q	$P \vee Q$
T	T	T
T	F	T
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F	F	F

P	Q	$P \wedge Q$
T	T	T
T	F	F
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P	Q	$P \rightarrow Q$
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Again, it is better to understand it by asking when it can be "wrong"



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"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

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## Semantic

P	$\neg P$
T	F
F	T

P	Q	$P \vee Q$
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F	F	F

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T	F	F
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"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$

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## Semantic

P	$\neg P$
T	F
F	T

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

An implication is false only if the assumption is true but what follows is still false

$P :=$  "You will keep up with the lecture"

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"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

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4.  $\nu(p \rightarrow q) = F$  iff  $\nu(p) = T$ , and  $\nu(q) = F$
5.  $\nu(p \leftrightarrow q) = T$  iff  $\nu(p) = \nu(q)$

However, if  $P$  is false, the implication does not "apply", so it is still "vaccuously" true

## Semantic

P	$\neg P$
T	F
F	T

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
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"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

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5.  $\nu(p \leftrightarrow q) = T$  iff  $\nu(p) = \nu(q)$

**Semantic**

P	$\neg P$
T	F
F	T

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	$P \wedge Q$	$P \wedge Q \rightarrow R$
---	---	---	--------------	----------------------------

Let us consider a final and more complex example

$P :=$  "You will keep up with the lecture"

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$S :=$  "You pay attention"

$S \rightarrow P \wedge Q \wedge R$

"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$

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Let  $S$  denote the set of propositional variables. Then a **valuation** is a function  $\nu : S \rightarrow \{T, F\}$ . if,

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## Semantic

P	$\neg P$
T	F
F	T

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	$P \wedge Q$	$P \wedge Q \rightarrow R$
T	T	T	T	T

We use the "and" truth table to figure out the  $T$

$P :=$  "You will keep up with the lecture"

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$S \rightarrow P \wedge Q \wedge R$

"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$

"If you are not keeping up with the lecture, or you are not finding the lecture interesting, or you do not understand something, then you are not paying attention."

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1.  $\nu(\neg p) = F$  iff  $\nu(p) = T$
2.  $\nu(p \wedge q) = T$  iff  $\nu(p) = T$ , and  $\nu(q) = T$
3.  $\nu(p \vee q) = F$  iff  $\nu(p) = F$ , and  $\nu(q) = F$
4.  $\nu(p \rightarrow q) = F$  iff  $\nu(p) = T$ , and  $\nu(q) = F$
5.  $\nu(p \leftrightarrow q) = T$  iff  $\nu(p) = \nu(q)$

## Semantic

P	$\neg P$
T	F
F	T

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	$P \wedge Q$	$P \wedge Q \rightarrow R$
T	T	T	T	T

and then we use the "implication" truth table

$P :=$  "You will keep up with the lecture"

$Q :=$  "You will find the lecture interesting"

$R :=$  "You will understand everything"

$S :=$  "You pay attention"

$S \rightarrow P \wedge Q \wedge R$

"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$

"If you are not keeping up with the lecture, or you are not finding the lecture interesting, or you do not understand something, then you are not paying attention."

Let  $S$  denote the set of propositional variables. Then a **valuation** is a function  $\nu : S \rightarrow \{T, F\}$ . if,

1.  $\nu(\neg p) = F$  iff  $\nu(p) = T$
2.  $\nu(p \wedge q) = T$  iff  $\nu(p) = T$ , and  $\nu(q) = T$
3.  $\nu(p \vee q) = F$  iff  $\nu(p) = F$ , and  $\nu(q) = F$
4.  $\nu(p \rightarrow q) = F$  iff  $\nu(p) = T$ , and  $\nu(q) = F$
5.  $\nu(p \leftrightarrow q) = T$  iff  $\nu(p) = \nu(q)$

## Semantic

P	$\neg P$
T	F
F	T

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	$P \wedge Q$	$P \wedge Q \rightarrow R$
T	T	T	T	T
T	T	F	T	F

Now see if you can see why each of the rest of the rows are correct

$P :=$  "You will keep up with the lecture"

$Q :=$  "You will find the lecture interesting"

$R :=$  "You will understand everything"

$S :=$  "You pay attention"

$S \rightarrow P \wedge Q \wedge R$

"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$

"If you are not keeping up with the lecture, or you are not finding the lecture interesting, or you do not understand something, then you are not paying attention."

Let  $S$  denote the set of propositional variables. Then a **valuation** is a function  $\nu : S \rightarrow \{T, F\}$ . if,

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2.  $\nu(p \wedge q) = T$  iff  $\nu(p) = T$ , and  $\nu(q) = T$
3.  $\nu(p \vee q) = F$  iff  $\nu(p) = F$ , and  $\nu(q) = F$
4.  $\nu(p \rightarrow q) = F$  iff  $\nu(p) = T$ , and  $\nu(q) = F$
5.  $\nu(p \leftrightarrow q) = T$  iff  $\nu(p) = \nu(q)$ ,  $F$

## Semantic

		P   Q		P $\wedge$ Q
P	$\neg P$	T	T	T
T	F	T	F	F
F	T	F	T	F
		F	F	F

  

P	Q	P $\vee$ Q	P	Q	P $\rightarrow$ Q
T	T	T	T	T	T
T	F	T	T	F	F
F	T	T	F	T	T
F	F	F	F	F	T

  

P	Q	R	P $\wedge$ Q	P $\wedge$ Q $\rightarrow$ R
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T



$P :=$  "You will keep up with the lecture"

$Q :=$  "You will find the lecture interesting"

$R :=$  "You will understand everything"

$S :=$  "You pay attention"

$S \rightarrow P \wedge Q \wedge R$

"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$

"If you are not keeping up with the lecture, or you are not finding the lecture interesting, or you do not understand something, then you are not paying attention."

Let  $S$  denote the set of propositional variables. Then a **valuation** is a function  $\nu : S \rightarrow \{T, F\}$ . if,

1.  $\nu(\neg p) = F$  iff  $\nu(p) = T$
2.  $\nu(p \wedge q) = T$  iff  $\nu(p) = T$ , and  $\nu(q) = T$
3.  $\nu(p \vee q) = F$  iff  $\nu(p) = F$ , and  $\nu(q) = F$
4.  $\nu(p \rightarrow q) = F$  iff  $\nu(p) = T$ , and  $\nu(q) = F$
5.  $\nu(p \leftrightarrow q) = T$  iff  $\nu(p) = \nu(q)$ ,  $F$

## Semantic

P	$\neg P$
T	F
F	T

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	$P \wedge Q$	$P \wedge Q \rightarrow R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T

$P :=$  "You will keep up with the lecture"

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$S :=$  "You pay attention"

$S \rightarrow P \wedge Q \wedge R$

"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$

"If you are not keeping up with the lecture, or you are not finding the lecture interesting, or you do not understand something, then you are not paying attention."

Let  $S$  denote the set of propositional variables. Then a **valuation** is a function  $\nu : S \rightarrow \{T, F\}$ . if,

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2.  $\nu(p \wedge q) = T$  iff  $\nu(p) = T$ , and  $\nu(q) = T$
3.  $\nu(p \vee q) = F$  iff  $\nu(p) = F$ , and  $\nu(q) = F$
4.  $\nu(p \rightarrow q) = F$  iff  $\nu(p) = T$ , and  $\nu(q) = F$
5.  $\nu(p \leftrightarrow q) = T$  iff  $\nu(p) = \nu(q)$ ,  $F$

## Semantic

P	$\neg P$
T	F
F	T

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	$P \wedge Q$	$P \wedge Q \rightarrow R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T

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$S :=$  "You pay attention"

$S \rightarrow P \wedge Q \wedge R$

"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$

"If you are not keeping up with the lecture, or you are not finding the lecture interesting, or you do not understand something, then you are not paying attention."

Let  $S$  denote the set of propositional variables. Then a **valuation** is a function  $\nu : S \rightarrow \{T, F\}$ . if,

1.  $\nu(\neg p) = F$  iff  $\nu(p) = T$
2.  $\nu(p \wedge q) = T$  iff  $\nu(p) = T$ , and  $\nu(q) = T$
3.  $\nu(p \vee q) = F$  iff  $\nu(p) = F$ , and  $\nu(q) = F$
4.  $\nu(p \rightarrow q) = F$  iff  $\nu(p) = T$ , and  $\nu(q) = F$
5.  $\nu(p \leftrightarrow q) = T$  iff  $\nu(p) = \nu(q)$ ,  $F$

### Semantic

P	$\neg P$
T	F
F	T

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	$P \wedge Q$	$P \wedge Q \rightarrow R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T

$P :=$  "You will keep up with the lecture"

$Q :=$  "You will find the lecture interesting"

$R :=$  "You will understand everything"

$S :=$  "You pay attention"

$S \rightarrow P \wedge Q \wedge R$

"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$

"If you are not keeping up with the lecture, or you are not finding the lecture interesting, or you do not understand something, then you are not paying attention."

Let  $S$  denote the set of propositional variables. Then a **valuation** is a function  $\nu : S \rightarrow \{T, F\}$ . if,

1.  $\nu(\neg p) = F$  iff  $\nu(p) = T$
2.  $\nu(p \wedge q) = T$  iff  $\nu(p) = T$ , and  $\nu(q) = T$
3.  $\nu(p \vee q) = F$  iff  $\nu(p) = F$ , and  $\nu(q) = F$
4.  $\nu(p \rightarrow q) = F$  iff  $\nu(p) = T$ , and  $\nu(q) = F$
5.  $\nu(p \leftrightarrow q) = T$  iff  $\nu(p) = \nu(q)$ ,  $F$

## Semantic

		P    Q         P $\wedge$ Q		
P	$\neg P$	T	T	T
T	F	T	F	F
F	T	F	T	F
		F	F	F

		P    Q         P $\vee$ Q		
P	Q	T	T	T
T	F	T	F	T
F	T	F	T	T
F	F	F	F	F

		P    Q         P $\rightarrow$ Q		
P	Q	T	T	T
T	F	T	F	F
F	T	F	T	T
F	F	F	F	T

P	Q	R	P $\wedge$ Q	P $\wedge$ Q $\rightarrow$ R
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T

$P :=$  "You will keep up with the lecture"

$Q :=$  "You will find the lecture interesting"

$R :=$  "You will understand everything"

$S :=$  "You pay attention"

$S \rightarrow P \wedge Q \wedge R$

"If you pay attention, then you will keep up with the lecture, find the lecture interesting, and understand everything"

$\neg P \vee \neg Q \vee \neg R \rightarrow \neg S$

"If you are not keeping up with the lecture, or you are not finding the lecture interesting, or you do not understand something, then you are not paying attention."

Let  $S$  denote the set of propositional variables. Then a **valuation** is a function  $\nu : S \rightarrow \{T, F\}$ . if,

1.  $\nu(\neg p) = F$  iff  $\nu(p) = T$
2.  $\nu(p \wedge q) = T$  iff  $\nu(p) = T$ , and  $\nu(q) = T$
3.  $\nu(p \vee q) = F$  iff  $\nu(p) = F$ , and  $\nu(q) = F$
4.  $\nu(p \rightarrow q) = F$  iff  $\nu(p) = T$ , and  $\nu(q) = F$
5.  $\nu(p \leftrightarrow q) = T$  iff  $\nu(p) = \nu(q)$ ,  $F$

## Semantic

		P    Q         P $\wedge$ Q		
P	$\neg P$	T	T	T
T	F	T	F	F
F	T	F	T	F
		F	F	F

		P    Q         P $\vee$ Q		
P	Q	T	T	T
T	T	T	F	T
T	F	F	T	T
F	T	F	F	F
F	F			

		P    Q         P $\rightarrow$ Q		
P	Q	T	T	T
T	T	T	F	F
T	F	F	T	T
F	T	F	F	T
F	F	F	F	T

P	Q	R	P $\wedge$ Q	P $\wedge$ Q $\rightarrow$ R
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

## Syntactic

## Semantic

P	$\neg P$
T	F
F	T

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	$P \wedge Q$	$P \wedge Q \rightarrow R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

We now consider the syntactic side

# Syntactic

## Semantic

P	$\neg P$
T	F
F	T

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	$P \wedge Q$	$P \wedge Q \rightarrow R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

where we are not concerned with the syntactic means

# Syntactic

## Semantic

P		P		P		P		P		P	
P		$\neg P$		P		Q		$P \wedge Q$		P	
T	T	F		T	T	T	T	T		T	T
T	F	T		T	F	F	F	F		F	F
F	T			F	T	F	T	F		F	F
F	F			F	F	F	F	F		F	F

  

P		P		P		P		P		P	
P		Q		$P \vee Q$		P		Q		$P \rightarrow Q$	
T	T	T	T	T		T	T	T	T	T	T
T	F	T	T	T		T	F	F	F	F	F
F	T	T	T	T		F	T	T	T	T	T
F	F	F	F	F		F	F	F	F	T	T

  

P			P		P		P		P		P	
P			Q		R		$P \wedge Q$		$P \wedge Q \rightarrow R$		$P \wedge Q \rightarrow R$	
T	T	T	T	T	T	T	T	T	T		T	T
T	T	F	T	T	F	F	F	F	F		F	F
T	F	T	T	F	T	T	T	T	T		T	T
T	F	F	T	F	F	F	F	F	F		T	T
F	T	T	F	T	T	T	T	T	T		T	T
F	T	F	F	T	F	F	F	F	F		T	T
F	F	T	F	F	T	T	T	T	T		T	T
F	F	F	F	F	F	F	F	F	F		T	T

but merely they relate with each other.



# Syntactic

## Semantic

P	$\neg P$
T	F
F	T

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	$P \wedge Q$	$P \wedge Q \rightarrow R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

Yet, as we will see, we will be able to prove everything that we could have derived from truth tables

# Syntactic

*Axioms*

# Semantic

P	$\neg P$
T	F
F	T

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	$P \wedge Q$	$P \wedge Q \rightarrow R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

In the next lecture we will be introduced to these axioms

## Syntactic

## Axioms

1.  $p \rightarrow (q \rightarrow p)$

## Semantic

**Semantic**

P	$\neg P$
T	F
F	T

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	$P \wedge Q$	$P \wedge Q \rightarrow R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

## Syntactic

## Axioms

1.  $p \rightarrow (q \rightarrow p)$
2.  $p \rightarrow (q \rightarrow r) \rightarrow (p \rightarrow q \rightarrow [p \rightarrow r])$

## Semantic

Semantics					
P	$\neg P$	P	Q	$P \wedge Q$	
T	F	T	T	T	
F	T	T	F	F	
		F	T	F	
		F	F	F	
P	Q	$P \vee Q$	P	Q	$P \rightarrow Q$
T	T	T	T	T	T
T	F	T	T	F	F
F	T	T	F	T	T
F	F	F	F	F	T
P	Q	R	$P \wedge Q$	$P \wedge Q \rightarrow R$	
T	T	T	T	T	
T	T	F	T	F	
T	F	T	F	T	
T	F	F	F	T	
F	T	T	F	T	
F	T	F	F	T	
F	F	T	F	T	
F	F	F	F	T	

## Syntactic

### Axioms

1.  $p \rightarrow (q \rightarrow p)$
2.  $p \rightarrow (q \rightarrow r) \rightarrow (p \rightarrow q \rightarrow [p \rightarrow r])$
3.  $\neg p \rightarrow \neg q \rightarrow (q \rightarrow p)$

## Semantic

P		Q		P $\wedge$ Q	
T	T	T	T	T	T
T	F	F	F	F	F
F	T	F	F	F	F
F	F	F	F	F	F

  

P		Q		P $\vee$ Q	
T	T	T	T	T	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	F	F	F	F

  

P		Q		P $\rightarrow$ Q	
T	T	T	T	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	F	F	F	T	T

  

P	Q	R	P $\wedge$ Q	P $\wedge$ Q $\rightarrow$ R
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

## Syntactic

### Axioms

1.  $p \rightarrow (q \rightarrow p)$
2.  $p \rightarrow (q \rightarrow r) \rightarrow (p \rightarrow q \rightarrow [p \rightarrow r])$
3.  $\neg p \rightarrow \neg q \rightarrow (q \rightarrow p)$

$$P \rightarrow Q, P \implies Q$$

## Semantic

P	$\neg P$	P	Q	$P \wedge Q$
T	F	T	T	T
T	F	T	F	F
F	T	F	T	F
F	T	F	F	F

P	Q	$P \vee Q$	P	Q	$P \rightarrow Q$
T	T	T	T	T	T
T	F	T	T	F	F
F	T	T	F	T	T
F	F	F	F	F	T

P	Q	R	$P \wedge Q$	$P \wedge Q \rightarrow R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

and this very important rule (among others)

## Syntactic

### Axioms

1.  $p \rightarrow (q \rightarrow p)$
2.  $p \rightarrow (q \rightarrow r) \rightarrow (p \rightarrow q \rightarrow [p \rightarrow r])$
3.  $\neg p \rightarrow \neg q \rightarrow (q \rightarrow p)$

$P \rightarrow Q, P \implies Q$  (Modus Ponens)

## Semantic

P	$\neg P$
T	F
F	T

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	R	$P \wedge Q$	$P \wedge Q \rightarrow R$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

called "Modus Ponens"