Exercise sheet 5

Set theory and Logic, MTH303

1. Prove that the power set of a the union of a family of sets is a set.
2. Show that a subset axiom implies that there can be no set that contains every other set?
3. Prove that for any function $f : X \to Y$, there is a maximal subset of X so that f restricted to that subset is 1-1.
4. Prove that $(X \setminus Y) \cup (Y \setminus X)$ is a set if X and Y are sets.
5. Prove that the cartesian product of sets is a set and that the cartesian product of non-empty sets is non-empty.
6. Consider a subset A of ω. Note that A is a family of sets. Prove that if $\cup A = A$, then $A = \omega$