Exercise sheet 2

Set theory and Logic, MTH303

1. Given a sequence of propositional forms $p_0, p_1, \ldots$ that is consistent and a proposition q so that $p_0, p_1, \ldots \vdash q$, is it true that the sequence $q, p_0, p_1, \ldots$ is also consisent? Justify your answer.

2. Prove that for any propositional forms p, q so that $p \implies q$ (q can be derived from p by an inference rule), then $p \to q$ is a tautology. You will have to check this for each of the inference rules.

3. Given a sequence of propositional forms $p_0, p_1, \ldots$ and a propositional form q for which $p_0, p_1, \ldots \nvdash \neg q$, then prove that the sequence $q, p_0, p_1, \ldots$ is also consistent.

4. Given a sequence of propositional forms $p_0, p_1, \ldots$ that is consistent and a proposition q, prove that $q, p_0, p_1, \ldots$ is also consisent, where $r = q$ or $\neg q$

5. Given a maximally consistent sequence of propositional forms $p_0, p_1, \ldots$ if $q \to r$ for some i, and $q = p_j$ for some j, then $r = p_k$ for some k.

6. Consider a maxiamlly consistent sequence of propositional forms that extends the FL axioms. Does it contain all the tautologies? Does it contain only tautologies?