# Exercise sheet 4

### Set theory and Logic, MTH303

1. Prove (from the axioms) the symmetry, reflexivity, and transitivity of $=$ in first order logic.

2. Let $\alpha$ be a well formed formula, and assume that $\alpha \vdash \alpha'$, then prove that $\forall x \alpha \vdash \forall x \alpha'$.

3. Let $\alpha$ and $\alpha'$ be well formed formulae, $t$ a term, and $x$ a variable. Choose a variable $z$ that does not occur in $\alpha'$, $t$, or $x$. Prove that $\forall y \alpha \vdash \forall z (\alpha')^y_z$

4. Let $\Gamma$ denote a consistent set of well formed formulae. Prove that $\Gamma \cup \{\neg \forall x \alpha \to \neg \alpha^x_c\}$ is consistent. Extent the proof to the union of $\Gamma$ with countably many well formed formulae of the form $\neg \forall x \alpha \to \neg \alpha^x_c$.

5. Prove that to show that $\Gamma \vDash \alpha$ implies $\Gamma \vdash \alpha$ is equivalent to showing that any consistent set of well formed formulae is satisfiable.

6. Let $x_1, x_2, \ldots$ denote variables in a first order language, then will the set $\{\forall x_1 P x_1, P x_2, P x_3, \ldots\}$ be satisfiable and / or consistent?