Exercise sheet 1

Set theory and Logic, MTH303

1. Use truth tables to prove that the following are tautologies:

   (a) $P \to (Q \to P)$

   (b) $(P \to (Q \to R)) \to ((P \to Q) \to [P \to R])$

   (c) $(\neg P \to \neg Q) \to (Q \to P)$

   Yes, they have to do with the FL axioms!

2. Show that the first FL axiom, along with modus ponens, allows you to show: $p, q \vdash p$.

3. Show that the second FL axiom, along with modus ponens, allows you to show: $p \to q, q \to r, p \vdash r$

4. Show that the third FL axiom, along with modus ponens, allows you to show: $\neg q \to \neg p, p \vdash q$ (Does this remind you of proving by considering the contrapositive?)

5. Show that the addition inference rule follows from the axioms and replacement rules, i.e. show that $p \vdash_* p \vee q$.