

Exercise sheet 4

Set theory and Logic, MTH303

1. Let α be a well formed formula, and assume that $\alpha \vdash \alpha'$, then prove that $\forall x \alpha \vdash \forall x \alpha'$.
2. Let α and α' be well formed formulae, t a term, and x a variable. Choose a variable z that does not occur in α' , t , or x . Prove that $\forall y \alpha \vdash \forall z (\alpha')_z^y$.
3. Let Γ denote a consistent set of well formed formulae. Prove that $\Gamma \cup \{\neg \forall x \alpha \rightarrow \neg \alpha_c^x\}$ is consistent. Extend the proof to the union of Γ with countably many well formed formulae of the form $\neg \forall x \alpha \rightarrow \neg \alpha_c^x$.