

Exercise sheet 3

Set theory and Logic, MTH303

1. Prove that $\exists x p(x), \forall x (p(x) \rightarrow q(x)) \vdash \exists x q(x)$
2. Prove that $\exists p(x) \rightarrow q(y) \iff \forall x (p(x) \rightarrow q(y))$
3. Prove that the power set of a the union of a family of sets is a set.
4. Show that a subset axiom implies that there can be no set of all sets.
5. Prove that for any function $f : X \rightarrow Y$, there is a maximal subset of X so that f restricted to that subset is 1-1.
6. Prove that $(X \setminus Y) \cup (Y \setminus X)$ is a set if X and Y are sets.
7. Prove that the cartesian product of non-empty sets is non-empty.