Exercise sheet 4

Set theory and Logic, MTH303

1. Prove (from the axioms) the symmetry, reflexivity, and transitivity of $=$ in first order logic.
2. Let α be a well formed formula, and assume that $\alpha \vdash \alpha'$, then prove that $\forall x \alpha \vdash \forall x \alpha'$.
3. Let α and α' be well formed formulae such that $\alpha \vdash \alpha'$, t a term, and x a variable. Choose a variable z that does not occur in α', t, or x. Prove that $\forall y \alpha \vdash \forall z (\alpha')^y_z$
4. Let Γ denote a consistent set of well formed formulae. Prove that $\Gamma \cup \{\neg \forall x \alpha \to \neg \alpha^x_c\}$ is consistent where c is a constant symbol that has not been used before. Extend the proof to the union of Γ with countably many well formed formulae of the form $\neg \forall x \alpha \to \neg \alpha^x_c$.
5. Prove that to show that $\Gamma \vDash \alpha$ implies $\Gamma \vdash \alpha$ is equivalent to showing that any consistent set of well formed formulae is satisfiable.
6. Let $x_1, x_2, \ldots$ denote variables in a first order language, then will the set $\{\forall x_1 P x_1, P x_2, P x_3, \ldots\}$ be satisfiable and / or consistent?