Exercise sheet 4

Set theory and Logic, MTH303

1. Prove that if X is a set, then so is its successor X^+.
2. Consider a subset A of ω. Note that A is a family of sets. Prove that if $\cup A = A$, then $A = \omega$
3. Recall the (recursive) definition of $m + n$, for $m, n \in \omega$. Prove that $l + (m + n) = (l + m) + n$ for any $l, m, n \in \omega$
4. Recall the (recursive) definition of $m.n$, for $m, n \in \omega$. Prove that $l.(m.n) = (l.m).n$ for any $l, m, n \in \omega$. Also prove $m.n = n.m$
5. Prove that if $m + a = n + a$ for some $m, n, a \in \omega$, then $m = n$.
6. Prove that if $m.a = n.a$ for some $m, n, a \in \omega$, where $a \neq 0$, then $m = n$.