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$= x^n(1 + x + x^2 + x^3 \cdots)^n$
$= \frac{x^n}{(1-x)^n}$
$= x^n(1 + nx + \binom{n+1}{2}x^2 + \binom{n+2}{3}x^3 + \cdots)$

Example. Find the number of ways to distribute m identical objects in n distinct boxes.

5 objects in 3 boxes
$2 + 3 + 0$ Monomial: $x^2 x^3 x^0$
$0 + 2 + 3$ Monomial: $x^0 x^2 x^3$
$1 + 1 + 3$ Monomial: $x^1 x^1 x^3$

$$(1 + x + \cdots)(1 + x + \cdots)(1 + x + \cdots)$$

Solution.
$$\underbrace{(1 + x + \cdots)(1 + x + \cdots)\ldots(1 + x + \cdots)}_{n \text{ times}}$$
(because $a_m x^m = x^{r_1} x^{r_2} \ldots x^{r_n} + \cdots$,
such that $r_1 + r_2 + \cdots + r_n = m$)
$$= (1 + x + x^2 + \cdots)^n$$
$$= \frac{1}{(1-x)^n}$$
$$= 1 + nx + \binom{n+1}{2}x^2 + \binom{n+2}{3}x^3 + \cdots$$
Answer: Coefficient of x^m, i.e. $\binom{n+m-1}{m}$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.
$$\underbrace{(x + x^2 + \cdots)(x + x^2 + \cdots)\ldots(x + x^2 + \cdots)}_{n \text{ times}}$$
(because $a_m x^m = x^{r_1} x^{r_2} \ldots x^{r_n} + \cdots$,
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Answer: $\binom{m-1}{m-n}$

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Example. Number of ways to choose

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

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$$\underbrace{(x + x^2 + \cdots)(x + x^2 + \cdots)\ldots(x + x^2 + \cdots)}_{n \text{ times}}$$

(because $a_m x^m = x^{r_1} x^{r_2} \ldots x^{r_n} + \cdots$,
such that $r_1 + r_2 + \cdots + r_n = m$)

$= (x + x^2 + x^3 \cdots)^n$

$= x^n (1 + x + x^2 + x^3 \cdots)^n$

$= \frac{x^n}{(1-x)^n}$

$= x^n (1 + nx + \binom{n+1}{2} x^2 + \binom{n+2}{3} x^3 + \cdots)$

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General term: $\binom{n+k-1}{k} x^{n+k}$

Answer: $\binom{m-1}{m-n}$

Example. Number of ways to choose m elements out of n elements

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.

$$\underbrace{(x + x^2 + \cdots)(x + x^2 + \cdots)\ldots(x + x^2 + \cdots)}_{n \text{ times}}$$

(because $a_m x^m = x^{r_1} x^{r_2} \ldots x^{r_n} + \cdots$,
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General term: $\binom{n+k-1}{k} x^{n+k}$

Answer: $\binom{m-1}{m-n}$

Example. Number of ways to choose m elements out of n elements

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(because $a_m x^m = x^{r_1} x^{r_2} \ldots x^{r_n} + \cdots$, such that $r_1 + r_2 + \cdots + r_n = m$)

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Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

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Answer: $\binom{m-1}{m-n}$

Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

$$(1+x)\dots(1+x)$$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.

$$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{n \text{ times}}$$

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$$= (x+x^2+x^3\cdots)^n$$
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General term: $\binom{n+k-1}{k}x^{n+k}$

Answer: $\binom{m-1}{m-n}$

Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

$$(1 + x) \ldots (1 + x) = (1 + x)^n$$

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$$\underbrace{(x + x^2 + \cdots)(x + x^2 + \cdots) \ldots (x + x^2 + \cdots)}_{n \text{ times}}$$

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Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

$$(1+x)\ldots(1+x) = (1+x)^n$$

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Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

$$(1 + x) \ldots (1 + x) = (1 + x)^n$$

Answer: $\binom{n}{m}$

Partitions of integers

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

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Partitions of integers

Examples

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

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Example. Number of ways to choose m elements out of n elements

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$$(1 + x) \ldots (1 + x) = (1 + x)^n$$

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Partitions of integers

Examples

3

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.

$$\underbrace{(x + x^2 + \cdots)(x + x^2 + \cdots) \ldots (x + x^2 + \cdots)}_{n \text{ times}}$$

(because $a_m x^m = x^{r_1} x^{r_2} \ldots x^{r_n} + \cdots$,
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Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

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Partitions of integers

Examples

$3 = 1 + 1 + 1$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.

$$\underbrace{(x + x^2 + \cdots)(x + x^2 + \cdots)\ldots(x + x^2 + \cdots)}_{n \text{ times}}$$

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Solution. Generating function:

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Answer: $\binom{n}{m}$

Partitions of integers

Examples

$3 = 1 + 1 + 1 = 2 + 1$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

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$$\underbrace{(x + x^2 + \cdots)(x + x^2 + \cdots) \ldots (x + x^2 + \cdots)}_{n \text{ times}}$$

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Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

$$(1 + x) \ldots (1 + x) = (1 + x)^n$$

Answer: $\binom{n}{m}$

Partitions of integers

Examples
$3 = 1 + 1 + 1 = 2 + 1$
4

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.
$$\underbrace{(x + x^2 + \cdots)(x + x^2 + \cdots) \ldots (x + x^2 + \cdots)}_{n \text{ times}}$$
(because $a_m x^m = x^{r_1} x^{r_2} \ldots x^{r_n} + \cdots$,
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Answer: $\binom{m-1}{m-n}$

Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

$$(1 + x) \ldots (1 + x) = (1 + x)^n$$

Answer: $\binom{n}{m}$

Partitions of integers

Examples
$3 = 1 + 1 + 1 = 2 + 1$
$4 = 1 + 1 + 1 + 1$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.
$$\underbrace{(x + x^2 + \cdots)(x + x^2 + \cdots) \ldots (x + x^2 + \cdots)}_{n \text{ times}}$$
(because $a_m x^m = x^{r_1} x^{r_2} \ldots x^{r_n} + \cdots$,
such that $r_1 + r_2 + \cdots + r_n = m$)
$= (x + x^2 + x^3 \cdots)^n$
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General term: $\binom{n+k-1}{k}x^{n+k}$
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Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

$$(1 + x) \ldots (1 + x) = (1 + x)^n$$

Answer: $\binom{n}{m}$

Partitions of integers

Examples
$3 = 1 + 1 + 1 = 2 + 1$
$4 = 1 + 1 + 1 + 1 = 2 + 1 + 1$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.
$$\underbrace{(x + x^2 + \cdots)(x + x^2 + \cdots) \ldots (x + x^2 + \cdots)}_{n \text{ times}}$$
(because $a_m x^m = x^{r_1} x^{r_2} \ldots x^{r_n} + \cdots$,
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Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

$$(1 + x) \ldots (1 + x) = (1 + x)^n$$

Answer: $\binom{n}{m}$

Partitions of integers

Examples
$3 = 1 + 1 + 1 = 2 + 1$
$4 = 1 + 1 + 1 + 1 = 2 + 1 + 1 = 2{+}2$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.
$$\underbrace{(x + x^2 + \cdots)(x + x^2 + \cdots) \ldots (x + x^2 + \cdots)}_{n \text{ times}}$$
(because $a_m x^m = x^{r_1} x^{r_2} \ldots x^{r_n} + \cdots$,
such that $r_1 + r_2 + \cdots + r_n = m$)
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$= x^n (1 + x + x^2 + x^3 \cdots)^n$
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$= x^n (1 + nx + \binom{n+1}{2}x^2 + \binom{n+2}{3}x^3 + \cdots)$
$= x^n + nx^{n+1} + \binom{n+1}{2}x^{n+2} + \binom{n+2}{3}x^{n+3} + \cdots)$
General term: $\binom{n+k-1}{k}x^{n+k}$
Answer: $\binom{m-1}{m-n}$

Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

$$(1 + x) \ldots (1 + x) = (1 + x)^n$$

Answer: $\binom{n}{m}$

Partitions of integers

Examples

$3 = 1 + 1 + 1 = 2 + 1$

$4 = 1 + 1 + 1 + 1 = 2 + 1 + 1 = 2{+}2$

Example.

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.

$$\underbrace{(x + x^2 + \cdots)(x + x^2 + \cdots) \ldots (x + x^2 + \cdots)}_{n \text{ times}}$$

(because $a_m x^m = x^{r_1} x^{r_2} \ldots x^{r_n} + \cdots$,
such that $r_1 + r_2 + \cdots + r_n = m$)

$= (x + x^2 + x^3 \cdots)^n$

$= x^n (1 + x + x^2 + x^3 \cdots)^n$

$= \frac{x^n}{(1-x)^n}$

$= x^n (1 + nx + \binom{n+1}{2} x^2 + \binom{n+2}{3} x^3 + \cdots)$

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Example. Find the number of partitions of m into integers

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Example. Find the number of partitions of m into integers with allowed sizes 1, 2, 3.

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.

$$\underbrace{(x + x^2 + \cdots)(x + x^2 + \cdots)\ldots(x + x^2 + \cdots)}_{n \text{ times}}$$

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such that $r_1 + r_2 + \cdots + r_n = m$)

$= (x + x^2 + x^3 \cdots)^n$

$= x^n (1 + x + x^2 + x^3 \cdots)^n$

$= \dfrac{x^n}{(1-x)^n}$

$= x^n \left(1 + nx + \binom{n+1}{2}x^2 + \binom{n+2}{3}x^3 + \cdots\right)$

$= x^n + nx^{n+1} + \binom{n+1}{2}x^{n+2} + \binom{n+2}{3}x^{n+3} + \cdots)$

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Example. Number of ways to choose m elements out of n elements

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Partitions of integers

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$3 = 1 + 1 + 1 = 2 + 1$
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Partitions of integers

Examples
$3 = 1 + 1 + 1 = 2 + 1$
$4 = 1 + 1 + 1 + 1 = 2 + 1 + 1 = 2+2$

Example. Find the number of partitions of m into integers with allowed sizes 1, 2, 3.

Solution. $(1+x+\cdots)(1+x^2+x^4+\cdots)(1+x^3+x^6+\cdots)$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.
$$\underbrace{(x + x^2 + \cdots)(x + x^2 + \cdots)\ldots(x + x^2 + \cdots)}_{n \text{ times}}$$
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