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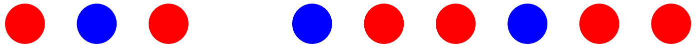
Solution. 

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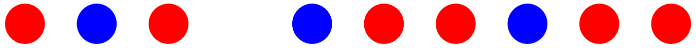
Solution. 

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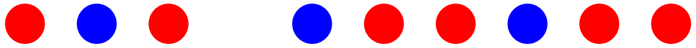
Solution. 

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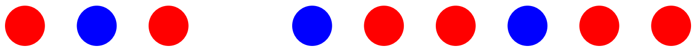
Solution. 

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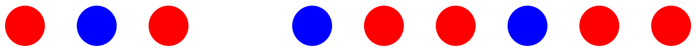
Solution. 

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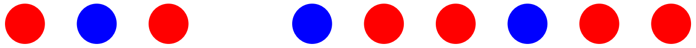
Solution. 

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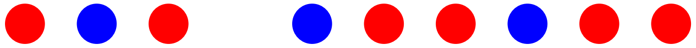
Solution. 

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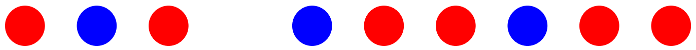
Solution. 

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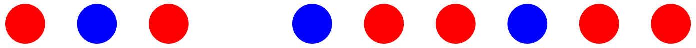
Solution. 

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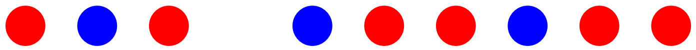
Solution. 

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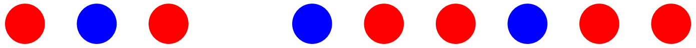
Solution. 

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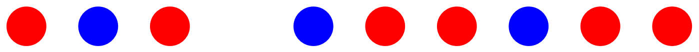
Solution. 

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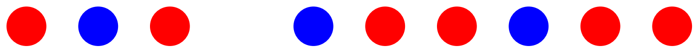
Solution. 

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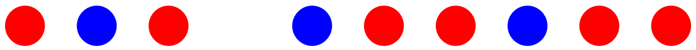
Solution. 

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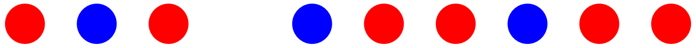
Solution. 

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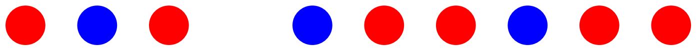
Solution. 

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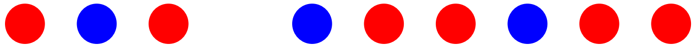
Solution. 

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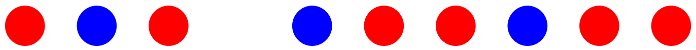
Solution. 

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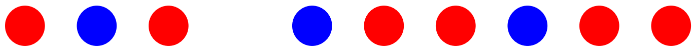
Solution. 

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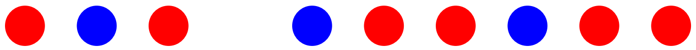
Solution. 

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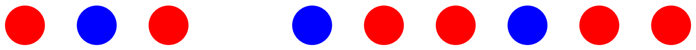
Solution. 

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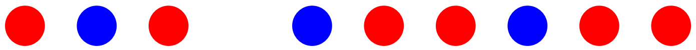
2. 1 is always accompanied

Example. Count the number of ways to arrange m blue balls, and n red balls so that none of the blue balls are adjacent and the ends are red balls?

Solution. 

After placing the blue balls, there are $m+1$ "boxes"

$m-1$ red balls must be placed to separate the blues

2 red balls must be placed at the edges

Remaining $n-m-1$ red calls must be placed in $m+1$ boxes

Answer: $\binom{n-1}{n-m-1}$ □

Example. Count the number of non-negative integer solutions to

$$x_1 + x_2 + \cdots + x_n = m$$

Solution. n boxes. m 1s. x_is are the number of 1s in each box. *Answer:* $\binom{n+m-1}{m}$ □

Example. Number of ways to partition $\{1, \ldots, n\}$ into a m subsets.

Denoted $S(n, m)$ (Sterling number of the second kind).

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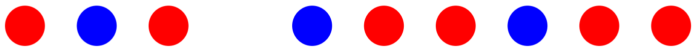
Solution. 

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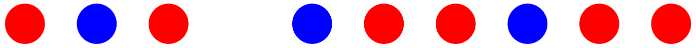
Solution. 

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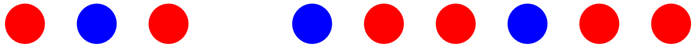
Solution. 

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First place $\{2, \ldots, n\}$ into the m subsets!

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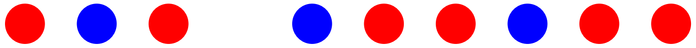
Solution. 

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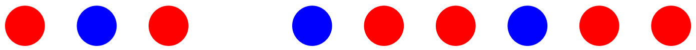
Solution. 

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then place 1 into one of the m subsets!

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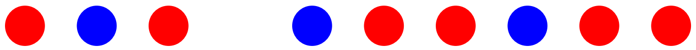
Solution. 

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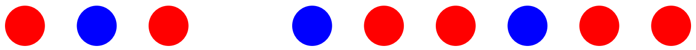
Solution. 

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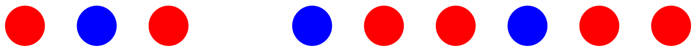
Solution. 

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Answer: $S(n, m) = S(n-1, m-1) + S(n-1, m) \times m$

□

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Examples.

1. $S(3, 2)$

Example. Number of ways to partition $\{1, \ldots, n\}$ into a m subsets.
Denoted $S(n, m)$ (Sterling number of the second kind).

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$S(n,m) = 0$ if $n < m$
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Examples.

1. $S(3,2) = S(2,1) +$

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Examples.

1. $S(3, 2) = S(2, 1) + S(2, 2)$

Example. Number of ways to partition $\{1, \ldots, n\}$ into a m subsets.
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Answer: $S(n, m) = S(n-1, m-1) + S(n-1, m) \times m$

$\square$

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Examples.

1. $S(3,2) = S(2,1) + S(2,2) \times 2$

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$S(n, m) = 0$ if $n < m$
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Examples.

1. $S(3, 2) = S(2, 1) + S(2, 2) \times 2 = 1 + 1 \times 2$

$S(n, m) = 0$ if $n < m$
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$S(n, n) = 1$

Examples.

1. $S(3, 2) = S(2, 1) + S(2, 2) \times 2 = 1 + 1 \times 2 = 3$

Example. Number of ways to partition $\{1, \ldots, n\}$ into a m subsets.
Denoted $S(n, m)$ (Sterling number of the second kind).

Solution. Idea: Split the partitions into two disjoint types.

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   # of such partitions: $S(n-1, m-1)$.

2. 1 is always accompanied
   # of such partitions: $S(n-1, m) \times m$.

Answer: $S(n, m) = S(n-1, m-1) + S(n-1, m) \times m$

□

$S(n, m) = 0$ if $n < m$
$S(n, 1) = 1$
$S(n, n) = 1$

Examples.

1. $S(3, 2) = S(2, 1) + S(2, 2) \times 2 = 1 + 1 \times 2 = 3$
2. $S(4, 2)$

Example. Number of ways to partition $\{1, \ldots, n\}$ into a m subsets.
Denoted $S(n, m)$ (Sterling number of the second kind).

Solution. *Idea:* Split the partitions into two disjoint types.

1. 1 forms a singletons subset
   # of such partitions: $S(n-1, m-1)$.
2. 1 is always accompanied
   # of such partitions: $S(n-1, m) \times m$.

Answer: $S(n, m) = S(n-1, m-1) + S(n-1, m) \times m$

□

$S(n, m) = 0$ if $n < m$
$S(n, 1) = 1$
$S(n, n) = 1$

Examples.

1. $S(3, 2) = S(2, 1) + S(2, 2) \times 2 = 1 + 1 \times 2 = 3$
2. $S(4, 2) = S(3, 1) +$

Example. Number of ways to partition $\{1, \ldots, n\}$ into a m subsets.
Denoted $S(n, m)$ (Sterling number of the second kind).

Solution. *Idea:* Split the partitions into two disjoint types.

1. 1 forms a singletons subset
   # of such partitions: $S(n-1, m-1)$.
2. 1 is always accompanied
   # of such partitions: $S(n-1, m) \times m$.

Answer: $S(n, m) = S(n-1, m-1) + S(n-1, m) \times m$

$\square$

$S(n, m) = 0$ if $n < m$
$S(n, 1) = 1$
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Examples.

1. $S(3, 2) = S(2, 1) + S(2, 2) \times 2 = 1 + 1 \times 2 = 3$
2. $S(4, 2) = S(3, 1) + S(3, 2)$

Example. Number of ways to partition $\{1, \ldots, n\}$ into a m subsets.
Denoted $S(n, m)$ (Sterling number of the second kind).

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   # of such partitions: $S(n-1, m-1)$.
2. 1 is always accompanied
   # of such partitions: $S(n-1, m) \times m$.

Answer: $S(n, m) = S(n-1, m-1) + S(n-1, m) \times m$

□

$S(n,m) = 0$ if $n < m$
$S(n,1) = 1$
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Examples.

1. $S(3,2) = S(2,1) + S(2,2) \times 2 = 1 + 1 \times 2 = 3$
2. $S(4,2) = S(3,1) + S(3,2) \times 2$

Example. Number of ways to partition $\{1, \ldots, n\}$ into a m subsets.
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Answer: $S(n,m) = S(n-1,m-1) + S(n-1,m) \times m$

$\square$

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Examples.

1. $S(3,2) = S(2,1) + S(2,2) \times 2 = 1 + 1 \times 2 = 3$
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2. $S(4, 2) = S(3, 1) + S(3, 2) \times 2 = 1 + 3 \times 2 = 7$

Example. Number of ways to partition $\{1, \dots, n\}$ into a m subsets.
Denoted $S(n, m)$ (Sterling number of the second kind).

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Answer: $S(n, m) = S(n-1, m-1) + S(n-1, m) \times m$

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$S(n, m) = 0$ if $n < m$
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Examples.

1. $S(3, 2) = S(2, 1) + S(2, 2) \times 2 = 1 + 1 \times 2 = 3$
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3. $S(4, 3)$

Example. Number of ways to partition $\{1, \ldots, n\}$ into a m subsets.
Denoted $S(n, m)$ (Sterling number of the second kind).

Solution. *Idea:* Split the partitions into two disjoint types.

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