Exercise sheet 1

Curves and Surfaces, MTH201

1. In how many ways can 7 students and 5 teachers be seated around a round table of no two teachers are adjacent?

2. In how many ways can 3 distinct balls be placed in 5 boxes so that empty boxes are *not* adjacent?

3. Derive a formula for the number of surjective maps from a set X of cardinality n to a set Y of cardinality m in terms of $S(n, m)$.

4. Let $s(n, m)$ denote the number of ways to arrange $\{1, 2, \ldots, n\}$ around m distinct circles so that each circle has at least one number. Note the difference with $S(n, m)$ that was done during a lecture ($s(n, m)$ are called Stirling numbers of the first kind, and $S(n, m)$ are called Stirling numbers of the second kind).

   (a) Prove that $s(n, m) = s(n - 1, m - 1) + (n - 1)s(n - 1, m)$.

   (b) Compute, $s(n, 0)$ $(n \geq 1)$, $s(n, n)$ $(n \geq 0)$, $s(n, 1)$ $(n \geq 2)$, and $s(n, n - 1)$ $(n \geq 2)$.

   (c) Compute $s(3, 2)$, $s(4, 2)$, and $s(4, 3)$.