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Example. Find the number of ways to distribute m **Example.** Find the number of ways identical objects in n distinct boxes.

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General term: $\binom{n+k-1}{k}x^{n+k}$

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Solution.

$$(x + x^{2} + \cdots)(x + x^{2} + \cdots) \dots (x + x^{2} + \cdots)$$
(because $a_{m}x^{m} = x^{r_{1}}x^{r_{2}} \dots x^{r_{n}} + \cdots$, such that $r_{1} + r_{2} + \cdots + r_{n} = m$)
$$= (x + x^{2} + x^{3} \cdots)^{n}$$

$$= x^{n}(1 + x + x^{2} + x^{3} \cdots)^{n}$$

$$= \frac{x^{n}}{(1-x)^{n}}$$

$$= x^{n}(1 + nx + \binom{n+1}{2}x^{2} + \binom{n+2}{3}x^{3} + \cdots)$$

$$= x^{n} + nx^{n+1} + \binom{n+1}{2}x^{n+2} + \binom{n+2}{3}x^{n+3} + \cdots)$$
General term: $\binom{n+k-1}{k}x^{n+k}$
Answer: $\binom{m-1}{m-n}$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution. $\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{}$ (because $a_m x^m = x^{r_1} x^{r_2} \dots x^{r_n} + \cdots$, such that $r_1 + r_2 + \cdots + r_n = m$ $= (x + x^2 + x^3 \cdots)^n$ $= x^{n} (1 + x + x^{2} + x^{3} \cdots)^{n}$ $= \frac{x^{n}}{(1-x)^{n}}$ $= x^{n}(1 + nx + {\binom{n+1}{2}}x^{2} + {\binom{n+2}{3}}x^{3} + \cdots)$ $= x^{n} + nx^{n+1} + {n+1 \choose 2}x^{n+2} + {n+2 \choose 3}x^{n+3} + \cdots$ General term: $\binom{n+k-1}{k}x^{n+k}$ Answer: $\binom{m-1}{m-n}$

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Solution.

$$(x + x^{2} + \cdots)(x + x^{2} + \cdots) \dots (x + x^{2} + \cdots)$$
(because $a_{m}x^{m} = x^{r_{1}}x^{r_{2}} \dots x^{r_{n}} + \cdots$, such that $r_{1} + r_{2} + \cdots + r_{n} = m$)
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Solution. Generating function:

$$(1+x)\dots(1+x)$$

Example. Number of ways to choose m elements out **Example.** Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

> Solution. $(x + x^2 + \cdots)(x + x^2 + \cdots) \dots (x + x^2 + \cdots)$ (because $a_m x^m = x^{r_1} x^{r_2} \dots x^{r_n} + \cdots$, such that $r_1 + r_2 + \cdots + r_n = m$ $= (x + x^2 + x^3 \cdots)^n$ $=x^{n}(1+x+x^{2}+x^{3}\cdots)^{n}$ $=x^{n}(1+nx+\binom{n+1}{2}x^{2}+\binom{n+2}{3}x^{3}+\cdots)$ $= x^{n} + nx^{n+1} + {n+1 \choose 2}x^{n+2} + {n+2 \choose 3}x^{n+3} + \cdots$ General term: $\binom{n+k-1}{k}x^{n+k}$ Answer: $\binom{m-1}{m}$

Solution. Generating function:

$$(1+x)\dots(1+x) = (1+x)^n$$

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Example. Number of ways to choose m elements out **Example.** Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

> Solution. $(x + x^2 + \cdots)(x + x^2 + \cdots) \dots (x + x^2 + \cdots)$ (because $a_m x^m = x^{r_1} x^{r_2} \dots x^{r_n} + \cdots$, such that $r_1 + r_2 + \cdots + r_n = m$ $= (x + x^2 + x^3 \cdots)^n$ $=x^{n}(1+x+x^{2}+x^{3}\cdots)^{n}$ $=x^{n}(1+nx+\binom{n+1}{2}x^{2}+\binom{n+2}{3}x^{3}+\cdots)$ $= x^{n} + nx^{n+1} + {n+1 \choose 2}x^{n+2} + {n+2 \choose 3}x^{n+3} + \cdots$ General term: $\binom{n+k-1}{k}x^{n+k}$

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Partitions of integers

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> Solution. $(x + x^2 + \cdots)(x + x^2 + \cdots) \dots (x + x^2 + \cdots)$

(because
$$a_m x^m = x^{r_1} x^{r_2} \dots x^{r_n} + \cdots$$
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General term: $\binom{n+k-1}{k} x^{n+k}$
Answer: $\binom{m-1}{m}$

Example. Number of ways to choose m elements out **Example.** Find the number of ways to distribute m of n elements

Solution. Generating function:

$$(1+x)\dots(1+x) = (1+x)^n$$

Answer: $\binom{n}{m}$

Partitions of integers

Examples

identical objects in n distinct boxes so that each box contains at least one object.

Solution.

$$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{\substack{n \text{ times} \\ persuse } a} \underbrace{x^m-x^{r_1}x^{r_2}}_{\substack{n \text{ times} \\ persuse } a} \underbrace{x^{r_n}+\cdots}_{\substack{n \text{ times} \\ persuse } a}$$

(because
$$a_m x^m = x^{r_1} x^{r_2} \dots x^{r_n} + \cdots$$
,
such that $r_1 + r_2 + \cdots + r_n = m$)
$$= (x + x^2 + x^3 + \cdots)^n$$

$$= x^n (1 + x + x^2 + x^3 + \cdots)^n$$

$$= \frac{x^n}{(1-x)^n}$$

$$= x^n (1 + nx + {n+1 \choose 2} x^2 + {n+2 \choose 3} x^3 + \cdots)$$

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General term: ${n+k-1 \choose 2} x^{n+k}$

General term: $\binom{n+k-1}{k}x^{n+k}$

Answer: $\binom{m-1}{m}$

Solution. Generating function:

$$(1+x)\dots(1+x) = (1+x)^n$$

Answer: $\binom{n}{m}$

Partitions of integers

Examples

Example. Number of ways to choose m elements out **Example.** Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

$$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{...}$$

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Partitions of integers

$$Examples \\ 3 = 1 + 1 + 1$$

Example. Number of ways to choose m elements out **Example.** Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.
$$(x + x^{2} + \cdots)(x + x^{2} + \cdots) \dots (x + x^{2} + \cdots)$$
(because $a_{m}x^{m} = x^{r_{1}}x^{r_{2}} \dots x^{r_{n}} + \cdots$,
such that $r_{1} + r_{2} + \cdots + r_{n} = m$)
$$= (x + x^{2} + x^{3} \cdots)^{n}$$

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Partitions of integers

Examples
$$3 = 1 + 1 + 1 = 2 + 1$$

Example. Number of ways to choose m elements out **Example.** Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

> Solution. $(x + x^2 + \cdots)(x + x^2 + \cdots) \dots (x + x^2 + \cdots)$ (because $a_m x^m = x^{r_1} x^{r_2} \dots x^{r_n} + \cdots$, such that $r_1 + r_2 + \cdots + r_n = m$ $= (x + x^2 + x^3 \cdots)^n$ $= x^n (1 + x + x^2 + x^3 \cdots)^n$ $= x^{n}(1 + nx + {n+1 \choose 2}x^{2} + {n+2 \choose 3}x^{3} + \cdots)$ $= x^{n} + nx^{n+1} + {n+1 \choose 2}x^{n+2} + {n+2 \choose 3}x^{n+3} + \cdots$ General term: $\binom{n+k-1}{k}x^{n+k}$ Answer: $\binom{m-1}{m}$

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Partitions of integers

Examples
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 4

Example. Number of ways to choose m elements out **Example.** Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

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Partitions of integers

Examples

$$3 = 1 + 1 + 1 = 2 + 1$$

$$4 = 1 + 1 + 1 + 1$$

Example. Number of ways to choose m elements out **Example.** Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

$$(x + x^2 + \cdots)(x + x^2 + \cdots) \dots (x + x^2 + \cdots)$$

(because
$$a_m x^m = x^{r_1} x^{r_2} \dots x^{r_n} + \cdots$$
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$$= (x + x^2 + x^3 + \cdots)^n$$

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General term: $\binom{n+k-1}{k} x^{n+k}$
Answer: $\binom{m-1}{m}$

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Partitions of integers

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Example. Number of ways to choose m elements out **Example.** Find the number of ways to distribute m of n elements

Solution. Generating function:

$$(1+x)\dots(1+x) = (1+x)^n$$

Answer: $\binom{n}{m}$

Partitions of integers

$$3 = 1 + 1 + 1 = 2 + 1$$

 $4 = 1 + 1 + 1 + 1 = 2 + 1 + 1 = 2 + 2$

identical objects in n distinct boxes so that each box contains at least one object.

Solution.

$$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{\substack{n \text{ times}}}$$
(because $a_m x^m = x^{r_1} x^{r_2} \dots x^{r_n} + \cdots$,
such that $r_1 + r_2 + \cdots + r_n = m$)
$$= (x+x^2+x^3\cdots)^n$$

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Partitions of integers

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$$3 = 1 + 1 + 1 = 2 + 1$$

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Partitions of integers

Examples

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Example. Find the number of partitions of m into in- Answer: $\binom{m-1}{m}$ tegers

Example. Number of ways to choose m elements out **Example.** Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

$$(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)$$

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General term: $\binom{n+k-1}{k} x^{n+k}$

Solution. Generating function:

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Partitions of integers

Examples

$$3 = 1 + 1 + 1 = 2 + 1$$

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Example. Find the number of partitions of m into in- Answer: $\binom{m-1}{m-n}$ tegers with allowed sizes 1, 2, 3.

Example. Number of ways to choose m elements out **Example.** Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

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Partitions of integers

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Partitions of integers

Examples

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 $4 = 1 + 1 + 1 + 1 = 2 + 1 + 1 = 2 + 2$

Example. Find the number of partitions of m into in- Answer: $\binom{m-1}{m}$ tegers with allowed sizes 1, 2, 3.

Solution.
$$(1+x+\cdots)(1+x^2+x^4+\cdots)(1+x^3+x^6+\cdots)$$

Example. Number of ways to choose m elements out **Example.** Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.

$$(x + x^2 + \cdots)(x + x^2 + \cdots) \dots (x + x^2 + \cdots)$$

(because $a_m x^m = x^{r_1} x^{r_2} \dots x^{r_n} + \cdots$, such that $r_1 + r_2 + \cdots + r_n = m$ $= (x + x^2 + x^3 \cdots)^n$ $= x^n (1 + x + x^2 + x^3 \cdots)^n$ $= x^{n}(1 + nx + {n+1 \choose 2}x^{2} + {n+2 \choose 3}x^{3} + \cdots)$ $= x^{n} + nx^{n+1} + {n+1 \choose 2}x^{n+2} + {n+2 \choose 3}x^{n+3} + \cdots$ General term: $\binom{n+k-1}{k}x^{n+k}$