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$$(x^2 + x^4 + x^6 + \cdots)^5$$
$$= x^{10}(1 + x^2 + x^4 + \cdots)^5$$

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$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

Example. Sequence: $(1, 2, 3, \ldots)$
Generating function:

$$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + \cdots$$

Example. Sequence: $(\binom{n}{0}, \binom{n}{1}, \binom{n}{2}, \ldots, \binom{n}{n}, 0, 0, \ldots)$
Generating function:

$$(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \cdots$$

Example. Sequence: $(1, \binom{n}{1}, \binom{n+1}{2}, \binom{n+2}{3}, \ldots, \binom{n+k-1}{k}, \ldots$
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Example. Find the coefficient of x^{30} in $(x^2 + x^4 + x^6 + \cdots)^5$.

Solution.

$$\begin{aligned}
&(x^2 + x^4 + x^6 + \cdots)^5 \\
&= x^{10}(1 + x^2 + x^4 + \cdots)^5 \\
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\end{aligned}$$

Answer:

Generating functions

Definition. Given a sequence $(a_0, a_1, a_2, \ldots)$, its generating function is the power series,

$$A(x) := a_0 + a_1 x + a_2 x^2 \cdots$$

Example. Sequence: $(1, 1, 1, \ldots)$
Generating function:

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\end{aligned}$$

Answer: $\binom{14}{10}$

$(a_0 + a_1 x + a_2 x^2 \cdots) + (b_0 + b_1 x + b_2 x^2 \cdots)$

Example. Sequence: $(1, \binom{n}{1}, \binom{n+1}{2}, \binom{n+2}{3}, \ldots, \binom{n+k-1}{k}, \ldots$
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$(a_0 + a_1x + a_2x^2 \cdots) + (b_0 + b_1x + b_2x^2 \cdots)$
$= (a_0 + b_0) + (a_1 + b_1)x + (a_2 + b_2)x^2 \cdots$

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Example. Sequence: $(1, \binom{n}{1}, \binom{n+1}{2}, \binom{n+2}{3}, \ldots, \binom{n+k-1}{k}, \ldots$
Alternatively, $(1, n, \frac{n(n+1)}{2!}, \frac{n(n+1)(n+2)}{3!}, \ldots)$
Generating function:

$$\frac{1}{(1-x)^n} = 1 + nx + \frac{n(n+1)}{2!}x^2 + \frac{n(n+1)(n+2)}{3!}x^3 + \cdots$$

Example. Find the coefficient of x^{30} in $(x^2 + x^4 + x^6 + \cdots)^5$.

Solution.

$$\begin{aligned}
&(x^2 + x^4 + x^6 + \cdots)^5 \\
&= x^{10}(1 + x^2 + x^4 + \cdots)^5 \\
&= x^{10}\frac{1}{(1-x^2)^5} \\
&= x^{10}\left(1 + \binom{5}{1}x^2 + \cdots + \binom{5+k-1}{k}x^{2k} + \cdots\right)
\end{aligned}$$

Answer: $\binom{14}{10}$

$(a_0 + a_1x + a_2x^2 \cdots) + (b_0 + b_1x + b_2x^2 \cdots)$
$= (a_0 + b_0) + (a_1 + b_1)x + (a_2 + b_2)x^2 \cdots$

$c(a_0 + a_1x + a_2x^2 \cdots)$
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What is the right choice of A_is?

$A_1 := \{1, 2, 4, 8, \ldots, 2^s\}$, whese s is such that $2^{s+1} > 60$
If it contained $2^r m$, where m is odd,
then 2^{r+1} would not divide it

$A_3 := \{3, 2 \times 3, 2^2 \times 3, \ldots, 2^? \times 3\}$

$A_m := \{m, 2 \times m, 2^2 \times m, \ldots, 2^? \times m\}$
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Given the set A_{m_1} and A_{m_2}, $m_1 \neq m_2$
choose $a_{m_1} \in A_{m_1}$ and $a_{m_2} \in A_{m_2}$
so that it has the highest power of 2

i.e. there exists m_1 and m_2 odd and k_1 and k_2
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Midsem question

What is the minimum number of numbers one should select from $\{1, 2, 3, \ldots, 60\}$ to guarantee that there is at least one number which divides another?

Solution. $\{1, 2, 3, \ldots, 60\} = \sqcup_{i=1}^{k} A_i$
Given any two $x, y \in A_i$, either $x|y$ or $y|x$.
So by pigeonhole principle, if we choose at least $k+1$ numbers
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If $2^{k_1} m_1 | 2^{k_2} m_2$ (m_1 and m_2 are odd)
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If $2^{k_1} m_1 | 2^{k_2} m_2$ (m_1 and m_2 are odd)
$\implies m_1 | m_2 \implies m_2 \geq 2m_1$
If $k_2 \geq k_1$, then $2^{k_2} m_2 \geq 2^{k_1+1} m_1 > 60$.

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which contradicts the choice of k_2 $\square$