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Theorem. $\Delta(G) = d, rad\ G = k \implies |G| <??$

$m_c := \max \{d_G(v, c) \mid c \in V(G)\}$
$\min \{m_c \mid c \in V(G)\}$

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$D_i := \{v \in V(G) \mid d_G(v, c) = i\}$

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$U \subset V(G)$ is connected if $G[U]$ is connected.

Proposition. *G connected, then we can choose an ordering $v_1, v_2, \dots, v_n$*

of $V(G)$ so that $G[v_1, v_2, \dots, v_i]$ is connected for all i.

Proof. $G[v_1]$ is trivially connected.

Assume that $G_k := G[v_1, \dots, v_k]$ is connected for $k \leq m$.
Pick $v \in G \setminus G_k$
Consider $v_k - v$ (the path linking v with v_k).
Let v_{k+1} be the first vertex on this path outside G_i □

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