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Then $|E(G')| = |V(G)| - 1$ so G' is a tree.
But $V(G') = V(G)$, i.e. G' spans G

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Assume that if $|E(G)| = |V(G)| + k$

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Definition. A cut vertex $v \in V(G)$ splits G into more
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Assume the result holds if $|E(G)| = k$

If $|E(G)| = k + 1$, delete an edge to get G' .

$V(G') = V(G)$ and $|E(G')| = |E(G)| - 1 = k$

G' , has at least $|V(G')| - |E(G')|$ components.

G , has at least $|V(G')| - |E(G')| - 1$ components. \square

Definition. A cut vertex $v \in V(G)$ splits G into more components

Theorem. *$v \in V(G)$ is a cut vertex if and only if there exist $x, y \in V(G)$, such that every path joining x and y contains v .*

Assume that if $|E(G)| = |V(G)| + k$

for some $k \geq 0$

then G has a spanning tree. If $|E(G)| = |V(G)| + k + 1$,

then it is not a tree. i.e. it has a cycle

Delete one edge from the cycle to get G'

$V(G') = V(G)$

By induction, there is a tree, T ,

so that $V(T) = V(G') = V(G)$, so T also spans G

Theorem. *A graph G , has at least $|V(G)| - |E(G)|$ components.*

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Definition. A cut vertex $v \in V(G)$ splits G into more components

Theorem. *$v \in V(G)$ is a cut vertex if and only if there exist $x, y \in V(G)$, such that every path joining x and y contains v .*

Theorem. *If $|V(G)| \geq 2$ then G has at least two vertices which are not cut vertices.*