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Example. Count the number of ways to form n pairs

Example. Count the number of ways to form n pairs out of a set X of $2n$ elements.

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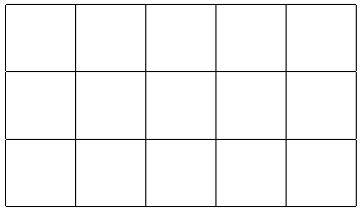

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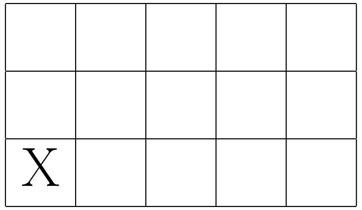

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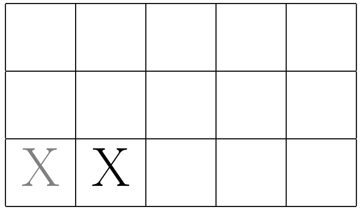

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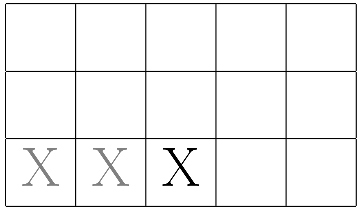

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$X^{n-1}_{m-1} \to Y$

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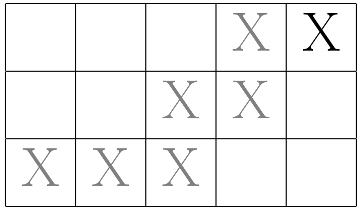

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where $\mathcal{C}_m(\{1, \ldots, n\}) := \{A \subset \{1, \ldots, n\} \mid |A| = m\}$
and $f(A) = (x_1, \ldots, x_i, \ldots, x_n)$,
such that $x_i = 0$ if and only if $i \in A$

$$|X^n_m| = |\mathcal{C}_m(\{1, \ldots, n\})| = \binom{n}{m}$$

Example. Count the number of ways to go from the bottom left corner of an $m \times n$-grid to the top right corner in the shortest way.

$f(0, 0, 1, 0, 1, 0) =$

			X	X
		X	X	
X	X	X		

Solution. Y: set of such routes
$f : X_{m-1}^{n-1} \to Y$
where X : is defined as before
and $f(x_1, \ldots, x_i, \ldots, x_{m+n-2}) =$ a route, where,
ith step is right if $x_i = 0$ and up if $x_i = 1$.

Example. Count the number of n-tuples of $0, 1$ with exactly m 0s and $n - m$ 1s.

Solution. X_m^n: set of such n-tuples
$f : \mathcal{C}_m(\{1, \ldots, n\}) \to X_m^n$
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Example. Count the number of ways to go from the bottom left corner of an $m \times n$-grid to the top right corner in the shortest way.

$$f(0,0,1,0,\textcolor{red}{1,}0) = \begin{array}{|c|c|c|c|c|}\hline & & & \mathrm{X} & \mathrm{X} \\ \hline & & \mathrm{X} & \mathrm{X} & \\ \hline \mathrm{X} & \mathrm{X} & \mathrm{X} & & \\ \hline \end{array}$$

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