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Exercise. Prove the converse

Proof. Consider the spanning tree. □

A graph is 2-connected if and only if it has no cut-vertices

Definition. A block is a maximal subgraph with no cut-vertices.

Proposition. *A cycle must lie completely inside a block*

Proof. A vertex of a cycle cannot be a cut-vertex □

Definition. Given a graph G, let C denote the cut vertices of G,
Let $\mathcal{B}$ denote the set of blocks
A block graph of G is a graph G' so that
$V(G') := \mathcal{B} \cup C$
$E(G') := \{v, B \mid v \in C, B \in \mathcal{B}, v \in B\}$

Definition. A graph G is n-partite if $V(G)$ can be partitioned into n classes so that each edge has ends in different classes. If $n = 2$, the graph is called bipartite.