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$A_1 \cap A_2 = \emptyset$ (M_1 cannot sit both left and right)

$A_2 \cap A_3 = \emptyset$ (W_2 's left is taken up by M_1)

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Case 1: $k > n$ then $\omega(k) = 0$. By pigeonhole principle

Case 2: $k \leq n$

$\omega(k) = \sum |A_{i_1} \cap \dots \cap A_{i_k}|$

Equivalent to:

There are two slots for each W_i , left or right

Each arrangement corresponds to

A choice of k indices from $1, 2, \dots, 2n$, around a cicle
 so that non-adjacent

(even to be interpreted as left odd as right)

Count the number of subsets with 1
and consider 1 as first
 1, $2n$ cannot belong to subsets
 The rest of $k - 1$ are chosen from $2n - 3$
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 In bijective correspondence with $k - 1$ distinct numbers

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