Example. Number of ways to choose r distinct objects from a set of X of n objects.

Solution.

$$\mathcal{C}_r^n = \{ \qquad \qquad \qquad \qquad \}$$

$\square$

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$$\mathcal{C}_r^n = \{\{x_1, x_2, \ldots, x_r\} \qquad\qquad\qquad\qquad \}$$

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$$\mathcal{C}_r^n = \{\{x_1, x_2, \ldots, x_r\} \mid x_i \in X \qquad\qquad\qquad\qquad \}$$

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Example. Number of ways to choose r distinct objects from a set of X of n objects.

Solution.

$$\begin{aligned} \mathcal{C}_r^n &= \{\{x_1, x_2, \ldots, x_r\} \mid x_i \in X, x_i \text{ distinct}\} \\ &= \{ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \} \end{aligned}$$

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$$\begin{aligned}\mathcal{C}_r^n &= \{\{x_1, x_2, \ldots, x_r\} \mid x_i \in X, x_i \text{ distinct}\} \\ &= \{[(x_1, x_2, \ldots, x_r)] \qquad\qquad\qquad\qquad \}\end{aligned}$$

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Size of each equivalence class: $\square$

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Size of each equivalence class: $\frac{r!}{(r-r)!}$ $\square$

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$\square$

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Number of equivalence classes: $\square$

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number of arrangements =

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□

Example. Number of ways to choose r distinct objects from a set of X of n objects.

number of arrangements $= |\mathcal{C}_r^n| \times r!$

Solution.

$$\begin{aligned}
\mathcal{C}_r^n &= \{\{x_1, x_2, \ldots, x_r\} \mid x_i \in X, x_i \text{ distinct}\} \\
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Example. Number of ways to choose r distinct objects from a set of X of n objects.

$$\frac{n!}{(n-r)!} = \text{number of arrangements} = |\mathcal{C}_r^n| \times r!$$

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$$\frac{n!}{(n-r)!} = \text{number of arrangements} = |\mathcal{C}_r^n| \times r!$$

$$\binom{n}{r} := |\mathcal{C}_r^n| = \frac{n!}{(n-r)!r!}$$

Example. Find the number of positive divisors of 50.

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Solution.

$$50 = 5 \times 10$$

$\square$

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Solution.

$$50 = 5 \times 10 = 5 \times 5 \times 2$$

$\square$

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Solution.

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Set of divisors: $\mathcal{D}_{50} := \{5^i \times 2^j\}$ $\square$

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$$f : [[2]] \times [[1]]$$

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$$f : [[2]] \times [[1]] \to \mathcal{D}_{50}$$

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$|\mathcal{D}_{50}| = |[[2]]| \times |[[1]]| = 3 \times 2$ $\square$

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$|\mathcal{D}_{50}| = |[[2]]| \times |[[1]]| = 3 \times 2 = 6$ $\square$

Example. Count the number of pairs (a, b) so that $a, b \in \{1, 2, \ldots, 10\}$ and a is smaller.

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Solution.

$$S := \{ \qquad\qquad\qquad\qquad \}$$

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$$S := \{(a, b) \mid a, b \in \{1, 2, \ldots, 10\}, \qquad \}$$

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$\square$

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$$50 = 5 \times 10 = 5 \times 5 \times 2 = 5^2 \times 2$$

Set of divisors: $\mathcal{D}_{50} := \{5^i \times 2^j \mid 0 \leq i \leq 2, 0 \leq j \leq 1\}$
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