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Place 1 next to any of the $n-1$ numbers that are already placed.
To the right the number.

Example: 245 376 89

$s(n,n) = 1$
$s(n,m) = 0$ if $n < m$
$s(n,1) = (n-1)!$
$s(n,n-1) = \binom{n}{2}$

$s(n, m) = s(n-1, m-1) + (n-1)s(n-1, m).$

$$\begin{aligned} s(n, n-1) &= s(n-1, n-2) + (n-1)s(n-1, n-1) \\ &= s(n-1, n-2) + (n-1) \\ &= s(n-2, n-3) + (n-2) + (n-1) \\ &= \cdots \\ &= 1 + 2 + \cdots n - 1 \\ &= \frac{n(n-1)}{2} = \binom{n}{2} \end{aligned}$$

$\{A \subset B \subset X \mid |B| = k\} \leftrightarrow \{C \subset X \setminus A \mid | |C| = k - |A|\}$