$$f(x) :=$$

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \ldots$$

$$g(x) :=$$

$$f(x) := a_0 + a_1 \frac{x}{1!} + a_2 \frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1 \frac{x}{1!} + b_2 \frac{x^2}{2!} + \dots$$

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) :=$$

$$f(x) := a_0 + a_1 \frac{x}{1!} + a_2 \frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1 \frac{x}{1!} + b_2 \frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1 \frac{x}{1!} + c_2 \frac{x^2}{2!} + \dots$$

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \dots$$

$$c_n =$$

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \dots$$

$$c_n = a_n + b_n$$

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \dots$$

$$c_n = a_n + b_n$$

$$f(x)g(x) :=$$

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \dots$$

$$c_n = a_n + b_n$$

$$f(x)g(x) := d_0 + d_1\frac{x}{1!} + d_2\frac{x^2}{2!} + \dots$$

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \dots$$

$$c_n = a_n + b_n$$

$$f(x)g(x) := d_0 + d_1\frac{x}{1!} + d_2\frac{x^2}{2!} + \dots$$

$$d_n =$$

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \dots$$

$$c_n = a_n + b_n$$

$$f(x)g(x) := d_0 + d_1\frac{x}{1!} + d_2\frac{x^2}{2!} + \dots$$

$$d_n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} a_i b_{n-i}$$

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \dots$$

$$c_n = a_n + b_n$$

$$f(x)g(x) := d_0 + d_1\frac{x}{1!} + d_2\frac{x^2}{2!} + \dots$$

$$d_n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} a_i b_{n-i} = \sum_{i=0}^{n} \binom{n}{i} a_i b_{n-i}$$

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \dots$$

$$c_n = a_n + b_n$$

$$f(x)g(x) := d_0 + d_1\frac{x}{1!} + d_2\frac{x^2}{2!} + \dots$$

$$d_n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} a_i b_{n-i} = \sum_{i=0}^{n} \binom{n}{i} a_i b_{n-i}$$

Number of ways to partition

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \dots$$

$$c_n = a_n + b_n$$

$$f(x)g(x) := d_0 + d_1\frac{x}{1!} + d_2\frac{x^2}{2!} + \dots$$

$$d_n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} a_i b_{n-i} = \sum_{i=0}^{n} \binom{n}{i} a_i b_{n-i}$$

Number of structures on first subset

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \dots$$

$$c_n = a_n + b_n$$

$$f(x)g(x) := d_0 + d_1\frac{x}{1!} + d_2\frac{x^2}{2!} + \dots$$

$$d_n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} a_i b_{n-i} = \sum_{i=0}^{n} \binom{n}{i} a_i b_{n-i}$$

Number of structures on second subset

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \dots$$

$$c_n = a_n + b_n$$

$$f(x)g(x) := d_0 + d_1\frac{x}{1!} + d_2\frac{x^2}{2!} + \dots$$

$$d_n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} a_i b_{n-i} = \sum_{i=0}^{n} \binom{n}{i} a_i b_{n-i}$$

Example.

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \dots$$

$$c_n = a_n + b_n$$

$$f(x)g(x) := d_0 + d_1\frac{x}{1!} + d_2\frac{x^2}{2!} + \dots$$

$$d_n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} a_i b_{n-i} = \sum_{i=0}^{n} \binom{n}{i} a_i b_{n-i}$$

Example. Number of subsets of an n-element set.

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \dots$$

$$c_n = a_n + b_n$$

$$f(x)g(x) := d_0 + d_1\frac{x}{1!} + d_2\frac{x^2}{2!} + \dots$$

$$d_n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} a_i b_{n-i} = \sum_{i=0}^{n} \binom{n}{i} a_i b_{n-i}$$

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \dots$$

$$c_n = a_n + b_n$$

$$f(x)g(x) := d_0 + d_1\frac{x}{1!} + d_2\frac{x^2}{2!} + \dots$$

$$d_n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} a_i b_{n-i} = \sum_{i=0}^{n} \binom{n}{i} a_i b_{n-i}$$

Example. Number of subsets of an n-element set.

Solution.

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \dots$$

$$c_n = a_n + b_n$$

$$f(x)g(x) := d_0 + d_1\frac{x}{1!} + d_2\frac{x^2}{2!} + \dots$$

$$d_n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} a_i b_{n-i} = \sum_{i=0}^{n} \binom{n}{i} a_i b_{n-i}$$

Example. Number of subsets of an n-element set.

Solution. Number of ways to “form a set”:

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \dots$$

$$c_n = a_n + b_n$$

$$f(x)g(x) := d_0 + d_1\frac{x}{1!} + d_2\frac{x^2}{2!} + \dots$$

$$d_n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!}a_i b_{n-i} = \sum_{i=0}^{n} \binom{n}{i} a_i b_{n-i}$$

Example. Number of subsets of an n-element set.

Solution. Number of ways to "form a set":

$$1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \cdots$$

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \dots$$

$c_n = a_n + b_n$

$$f(x)g(x) := d_0 + d_1\frac{x}{1!} + d_2\frac{x^2}{2!} + \dots$$

$$d_n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} a_i b_{n-i} = \sum_{i=0}^{n} \binom{n}{i} a_i b_{n-i}$$

Example. Number of subsets of an n-element set.

Solution. Number of ways to “form a set”:

$$1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \dots = e^x$$

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \dots$$

$$c_n = a_n + b_n$$

$$f(x)g(x) := d_0 + d_1\frac{x}{1!} + d_2\frac{x^2}{2!} + \dots$$

$$d_n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} a_i b_{n-i} = \sum_{i=0}^{n} \binom{n}{i} a_i b_{n-i}$$

Example. Number of subsets of an n-element set.

Solution. Number of ways to "form a set":

$$1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \cdots = e^x$$

$$\left(1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \cdots\right)^2$$

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \dots$$

$$c_n = a_n + b_n$$

$$f(x)g(x) := d_0 + d_1\frac{x}{1!} + d_2\frac{x^2}{2!} + \dots$$

$$d_n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!}a_i b_{n-i} = \sum_{i=0}^{n} \binom{n}{i} a_i b_{n-i}$$

Example. Number of subsets of an n-element set.

Solution. Number of ways to "form a set":

$$1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \cdots = e^x$$

$$(1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \cdots)^2 = e^{2x}$$

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \dots$$

$c_n = a_n + b_n$

$$f(x)g(x) := d_0 + d_1\frac{x}{1!} + d_2\frac{x^2}{2!} + \dots$$

$$d_n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} a_i b_{n-i} = \sum_{i=0}^{n} \binom{n}{i} a_i b_{n-i}$$

Example. Number of subsets of an n-element set.

Solution. Number of ways to "form a set":

$$1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \cdots = e^x$$

$$\left(1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \cdots\right)^2 = e^{2x}$$

Answer: 2^n

Example.

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \dots$$

$c_n = a_n + b_n$

$$f(x)g(x) := d_0 + d_1\frac{x}{1!} + d_2\frac{x^2}{2!} + \dots$$

$$d_n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} a_i b_{n-i} = \sum_{i=0}^{n} \binom{n}{i} a_i b_{n-i}$$

Example. Number of subsets of an n-element set.

Solution. Number of ways to "form a set":

$$1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \dots = e^x$$

$$\left(1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \dots\right)^2 = e^{2x}$$

Answer: 2^n

Example. Number of functions from an n-element set to $\{1, \dots, m\}$ for a fixed m.

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \dots$$

$c_n = a_n + b_n$

$$f(x)g(x) := d_0 + d_1\frac{x}{1!} + d_2\frac{x^2}{2!} + \dots$$

$$d_n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} a_i b_{n-i} = \sum_{i=0}^{n} \binom{n}{i} a_i b_{n-i}$$

Example. Number of subsets of an n-element set.

Solution. Number of ways to "form a set":

$$1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \cdots = e^x$$

$$(1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \cdots)^2 = e^{2x}$$

Answer: 2^n

Example. Number of functions from an n-element set to $\{1, \dots, m\}$ for a fixed m.

Solution.

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \dots$$

$c_n = a_n + b_n$

$$f(x)g(x) := d_0 + d_1\frac{x}{1!} + d_2\frac{x^2}{2!} + \dots$$

$$d_n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} a_i b_{n-i} = \sum_{i=0}^{n} \binom{n}{i} a_i b_{n-i}$$

Example. Number of subsets of an n-element set.

Solution. Number of ways to "form a set":

$$1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \cdots = e^x$$

$$(1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \cdots)^2 = e^{2x}$$

Answer: 2^n

Example. Number of functions from an n-element set to $\{1, \dots, m\}$ for a fixed m.

Solution. Each function partitions the set into m subsets

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \dots$$

$c_n = a_n + b_n$

$$f(x)g(x) := d_0 + d_1\frac{x}{1!} + d_2\frac{x^2}{2!} + \dots$$

$$d_n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} a_i b_{n-i} = \sum_{i=0}^{n} \binom{n}{i} a_i b_{n-i}$$

Example. Number of subsets of an n-element set.

Solution. Number of ways to "form a set":

$$1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \dots = e^x$$

$$(1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \dots)^2 = e^{2x}$$

Answer: 2^n

Example. Number of functions from an n-element set to $\{1, \dots, m\}$ for a fixed m.

Solution. Each function partitions the set into m subsets (of preimages)

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \dots$$

$c_n = a_n + b_n$

$$f(x)g(x) := d_0 + d_1\frac{x}{1!} + d_2\frac{x^2}{2!} + \dots$$

$$d_n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} a_i b_{n-i} = \sum_{i=0}^{n} \binom{n}{i} a_i b_{n-i}$$

Example. Number of subsets of an n-element set.

Solution. Number of ways to "form a set":

$$1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \cdots = e^x$$

$$\left(1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \cdots\right)^2 = e^{2x}$$

Answer: 2^n

Example. Number of functions from an n-element set to $\{1, \dots, m\}$ for a fixed m.

Solution. Each function partitions the set into m subsets (of preimages)

$$1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \cdots$$

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \dots$$

$c_n = a_n + b_n$

$$f(x)g(x) := d_0 + d_1\frac{x}{1!} + d_2\frac{x^2}{2!} + \dots$$

$$d_n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} a_i b_{n-i} = \sum_{i=0}^{n} \binom{n}{i} a_i b_{n-i}$$

Example. Number of subsets of an n-element set.

Solution. Number of ways to "form a set":

$$1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \dots = e^x$$

$$\left(1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \dots\right)^2 = e^{2x}$$

Answer: 2^n

Example. Number of functions from an n-element set to $\{1, \dots, m\}$ for a fixed m.

Solution. Each function partitions the set into m subsets (of preimages)

$$1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \dots = e^x$$

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

$$g(x) := b_0 + b_1\frac{x}{1!} + b_2\frac{x^2}{2!} + \dots$$

$$f(x) + g(x) := c_0 + c_1\frac{x}{1!} + c_2\frac{x^2}{2!} + \dots$$

$c_n = a_n + b_n$

$$f(x)g(x) := d_0 + d_1\frac{x}{1!} + d_2\frac{x^2}{2!} + \dots$$

$$d_n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} a_i b_{n-i} = \sum_{i=0}^{n} \binom{n}{i} a_i b_{n-i}$$

Example. Number of subsets of an n-element set.

Solution. Number of ways to "form a set":

$$1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \dots = e^x$$

$$(1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \dots)^2 = e^{2x}$$

Answer: 2^n

Example. Number of functions from an n-element set to $\{1, \dots, m\}$ for a fixed m.

Solution. Each function partitions the set into m subsets (of preimages)

$$1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \dots = e^x$$

$$(1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \dots)^m$$

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

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$$f(x)g(x) := d_0 + d_1\frac{x}{1!} + d_2\frac{x^2}{2!} + \dots$$

$$d_n = \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} a_i b_{n-i} = \sum_{i=0}^{n} \binom{n}{i} a_i b_{n-i}$$

Example. Number of subsets of an n-element set.

Solution. Number of ways to "form a set":

$$1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \dots = e^x$$

$$(1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \dots)^2 = e^{2x}$$

Answer: 2^n

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Example. If c_n is the number ways to arrange n balls chosen out of 2 red and 3 yellow

$$f(x) := a_0 + a_1\frac{x}{1!} + a_2\frac{x^2}{2!} + \dots$$

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$$(1 + \frac{x}{1!} + \frac{x^2}{2!})$$

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Solution.

$$(1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots)^2$$

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Example. If c_n is the number of ways of distributing n different objects in 2 distinct boxes so that the first box has even number of objects and the second has odd number of objects

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Example. If c_n is the number of n-digit binary sequences with even number of 0 and odd number of 1s, find the exponential generating function of c_n.

Solution.

$$(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots)(\frac{x}{1!} + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots)$$
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