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A component of a graph

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Definition.

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Proof. Let $x, y \in V(G)$, so that $d(x, y) = \text{diam}(X)$.

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Let u be a point in the other component.
Consider the shortest path P linking u and y

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$d(u, x) > d(x, y)$, so $d(x, y)$ cannot define the diameter

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