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Example. Find the number of surjective mappings from $\mathbb{N}_m := \{1, 2, \ldots, m\}$ to $\mathbb{N}_n := \{1, 2, \ldots, n\}$

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Example. Find the number of surjective mappings from $\mathbb{N}_m := \{1, 2, \dots, m\}$ to $\mathbb{N}_n := \{1, 2, \dots, n\}$ **Recall:**

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Set of *indexed* partititions

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Set of *indexed* partititions is in bijective correspondence with surjective mappings from $\mathbb{N}_m$ to $\mathbb{N}_n$.

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$$\boxed{S(m, n) = \frac{1}{n!}\Sigma_{k=0}^{n}(-1)^k\binom{n}{k}(n-k)^m}$$

Example. Count the number of ways

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Example. Count the number of ways to go from the bottom left most square of an $m \times n$-grid

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Example. Count the number of ways to go from the bottom left most square of an $m \times n$-grid to the top right most square

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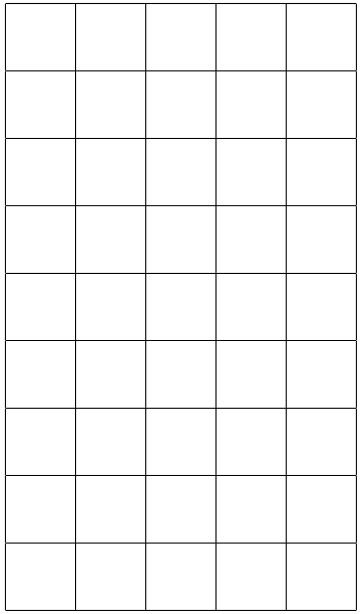
$n!S(m, n) = \Sigma_{k=0}^{n}(-1)^k \binom{n}{k}(n-k)^m$

$$\boxed{S(m, n) = \frac{1}{n!}\Sigma_{k=0}^{n}(-1)^k \binom{n}{k}(n-k)^m}$$

Example. Count the number of ways to go from the bottom left most square of an $m \times n$-grid to the top right most square in the shortest way excluding the X's.



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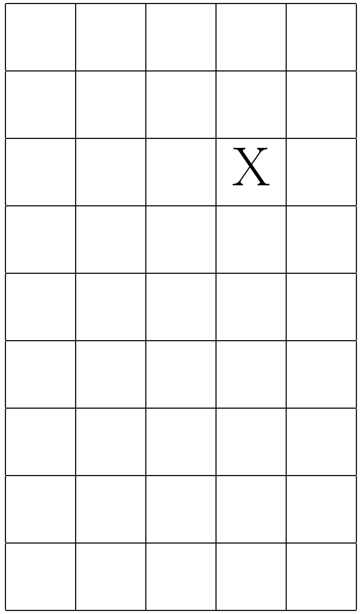

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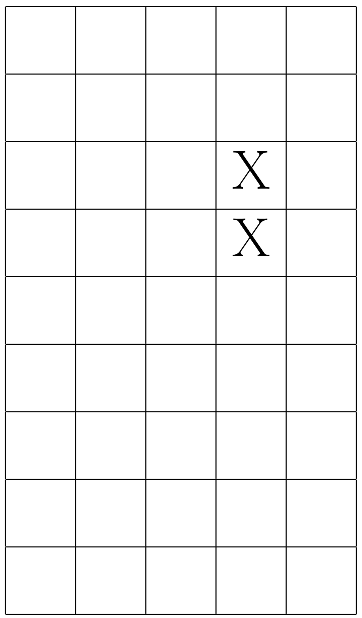

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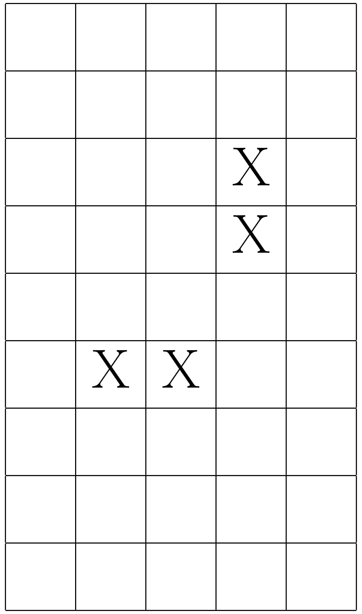

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