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Example. Count the number of ways to form n pairs

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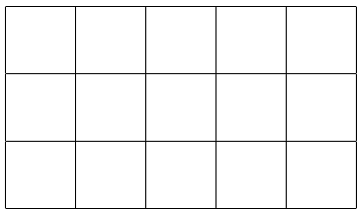

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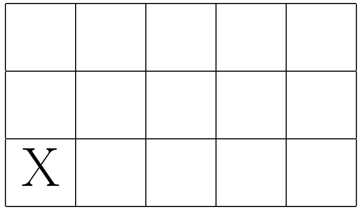

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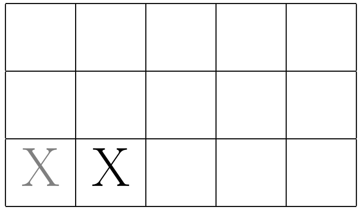

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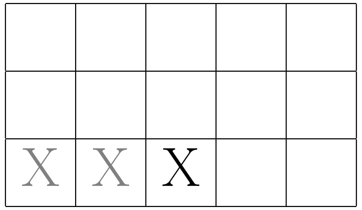

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X	X	X		

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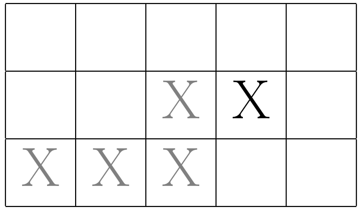

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			X	X
		X	X	
X	X	X		

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		X	X	
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			X	X
		X	X	
X	X	X		

Solution. Y:

Example. Count the number of n-tuples of $0, 1$ with exactly m 0s and $n - m$ 1s.

Solution. X^n_m: set of such n-tuples

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			X	X
		X	X	
X	X	X		

Solution. Y: set of such routes

Example. Count the number of n-tuples of $0, 1$ with exactly m 0s and $n - m$ 1s.

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			X	X
		X	X	
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Solution. Y: set of such routes

$$\to Y$$

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Solution. Y: set of such routes

$X_{m-1}^{n-1} \to Y$

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$$X^{n-1}_{m-1} \to Y$$

where X

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$$f(0,0,1,0,1,0) = \begin{array}{|c|c|c|c|c|}\hline & & & \mathrm{X} & \mathrm{X} \\ \hline & & \mathrm{X} & \mathrm{X} & \\ \hline \mathrm{X} & \mathrm{X} & \mathrm{X} & & \\ \hline \end{array}$$

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			X	X
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X	X	X		

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			X	X
		X	X	
X	X	X		

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where $\mathcal{C}_m(\{1, \ldots, n\}) := \{A \subset \{1, \ldots, n\} \mid |A| = m\}$
and $f(A) = (x_1, \ldots, x_i, \ldots, x_n)$,
such that $x_i = 0$ if and only if $i \in A$

$$|X^n_m| = |\mathcal{C}_m(\{1, \ldots, n\})| = \binom{n}{m}$$

Example. Count the number of ways to go from the bottom left corner of an $m \times n$-grid to the top right corner in the shortest way.

$$f(0,0,1,0,1,0) = \begin{array}{|c|c|c|c|c|} \hline & & & \mathrm{X} & \mathrm{X} \\ \hline & & \mathrm{X} & \mathrm{X} & \\ \hline \mathrm{X} & \mathrm{X} & \mathrm{X} & & \\ \hline \end{array}$$

Solution. Y: set of such routes
$f : X_{m-1}^{n-1} \to Y$
where X : is defined as before
and $f(x_1, \ldots, x_i, \ldots, x_{m+n-2}) =$ a route, where,
ith step is right if $x_i = 0$ and up if $x_i = 1$.

Example. Count the number of n-tuples of $0, 1$ with exactly m 0s and $n - m$ 1s.

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$$|Y|$$

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$$|Y| = |X^{n-1}_{m-1}| = \binom{m+n-2}{m-1}$$

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