

## Exercise sheet 2

Curves and Surfaces, MTH201

1. Prove that in any group with at least 2 people, there must exist at least two individuals who know the same number of people.
2. Prove that if from a set of  $n$  integers, none of which are a multiple of  $n$ , one can choose two whose difference is a multiple of  $n$ .
3. Prove that a subset of  $\{1, 2, \dots, 2n + 1\}$  with cardinality  $n + 1$  has a pair of coprime elements.
4. Consider 5 points in  $\mathbb{R}^2$  with integer coordinates. Show that there are at least two whose mid-point of the line segment joining them also have integer coordinates.
5. Compute the Ramsey numbers  $R(1, n)$  and  $R(2, n)$  for any  $n$ . Prove that  $R(m, n) = R(n, m)$ .
6. Consider the set  $\{2^1 - 1, 2^2 - 1, \dots, 2^{n-1} - 1\}$ , where  $n \geq 3$  is odd. Prove that there is a number from this set divisible by  $n$ .