Example. Find the number of ways to distribute m identical objects in n distinct boxes.

5 objects in 3 boxes
2 + 3 + 0 Monomial: $x^2x^3x^0$
0 + 2 + 3 Monomial: $x^0x^2x^3$
1 + 1 + 3 Monomial: $x^1x^1x^3$

$$(1+x+x^2+\cdots)(1+x+x^2+\cdots)(1+x+x^2+\cdots)$$

Example. Find the number of ways to distribute m identical objects in n distinct boxes.

5 objects in 3 boxes
$2 + 3 + 0$ Monomial: $x^2x^3x^0$
$0 + 2 + 3$ Monomial: $x^0x^2x^3$
$1 + 1 + 3$ Monomial: $x^1x^1x^3$

$$(1+x+x^2+\cdots)(1+x+x^2+\cdots)(1+x+x^2+\cdots)$$

Solution.

Example. Find the number of ways to distribute m identical objects in n distinct boxes.

5 objects in 3 boxes
$2 + 3 + 0$ Monomial: $x^2x^3x^0$
$0 + 2 + 3$ Monomial: $x^0x^2x^3$
$1 + 1 + 3$ Monomial: $x^1x^1x^3$

$$(1+x+x^2+\cdots)(1+x+x^2+\cdots)(1+x+x^2+\cdots)$$

Solution.
$$\underbrace{(1+x+x^2+\cdots)(1+x+x^2+\cdots)\ldots(1+x+x^2+\cdots)}_{n \text{ times}}$$

Example. Find the number of ways to distribute m identical objects in n distinct boxes.

5 objects in 3 boxes
$2 + 3 + 0$ Monomial: $x^2x^3x^0$
$0 + 2 + 3$ Monomial: $x^0x^2x^3$
$1 + 1 + 3$ Monomial: $x^1x^1x^3$

$$(1+x+x^2+\cdots)(1+x+x^2+\cdots)(1+x+x^2+\cdots)$$

Solution.
$$\underbrace{(1+x+x^2+\cdots)(1+x+x^2+\cdots)\dots(1+x+x^2+\cdots)}_{n \text{ times}}$$
(because $a_m x^m = x^{r_1}x^{r_2}\dots x^{r_n} + \cdots,$

Example. Find the number of ways to distribute m identical objects in n distinct boxes.

5 objects in 3 boxes
$2+3+0$ Monomial: $x^2x^3x^0$
$0+2+3$ Monomial: $x^0x^2x^3$
$1+1+3$ Monomial: $x^1x^1x^3$

$$(1+x+x^2+\cdots)(1+x+x^2+\cdots)(1+x+x^2+\cdots)$$

Solution.
$\underbrace{(1+x+x^2+\cdots)(1+x+x^2+\cdots)\dots(1+x+x^2+\cdots)}_{n \text{ times}}$
(because $a_m x^m = x^{r_1}x^{r_2}\dots x^{r_n} + \cdots,$
such that $r_1 + r_2 + \cdots + r_n = m$)

Example. Find the number of ways to distribute m identical objects in n distinct boxes.

5 objects in 3 boxes
$2+3+0$ Monomial: $x^2x^3x^0$
$0+2+3$ Monomial: $x^0x^2x^3$
$1+1+3$ Monomial: $x^1x^1x^3$

$$(1+x+x^2+\cdots)(1+x+x^2+\cdots)(1+x+x^2+\cdots)$$

Solution.
$\underbrace{(1+x+x^2+\cdots)(1+x+x^2+\cdots)\ldots(1+x+x^2+\cdots)}_{n \text{ times}}$
(because $a_m x^m = x^{r_1}x^{r_2}\ldots x^{r_n} + \cdots$,
such that $r_1 + r_2 + \cdots + r_n = m$)
$= (1+x+x^2+\cdots)^n$

Example. Find the number of ways to distribute m identical objects in n distinct boxes.

5 objects in 3 boxes
$2+3+0$ Monomial: $x^2x^3x^0$
$0+2+3$ Monomial: $x^0x^2x^3$
$1+1+3$ Monomial: $x^1x^1x^3$

$$(1+x+x^2+\cdots)(1+x+x^2+\cdots)(1+x+x^2+\cdots)$$

Solution.
$\underbrace{(1+x+x^2+\cdots)(1+x+x^2+\cdots)\ldots(1+x+x^2+\cdots)}_{n \text{ times}}$
(because $a_m x^m = x^{r_1}x^{r_2}\ldots x^{r_n}+\cdots$,
such that $r_1+r_2+\cdots+r_n=m$)
$=(1+x+x^2+\cdots)^n$
$=\frac{1}{(1-x)^n}$

Example. Find the number of ways to distribute m identical objects in n distinct boxes.

5 objects in 3 boxes
$2 + 3 + 0$ Monomial: $x^2x^3x^0$
$0 + 2 + 3$ Monomial: $x^0x^2x^3$
$1 + 1 + 3$ Monomial: $x^1x^1x^3$

$$(1+x+x^2+\cdots)(1+x+x^2+\cdots)(1+x+x^2+\cdots)$$

Solution.
$\underbrace{(1+x+x^2+\cdots)(1+x+x^2+\cdots)\ldots(1+x+x^2+\cdots)}_{n \text{ times}}$
(because $a_m x^m = x^{r_1}x^{r_2}\ldots x^{r_n} + \cdots$,
such that $r_1 + r_2 + \cdots + r_n = m$)
$= (1 + x + x^2 + \cdots)^n$
$= \frac{1}{(1-x)^n}$
$= 1 + nx + \binom{n+1}{2}x^2 + \binom{n+2}{3}x^3 + \cdots$

Example. Find the number of ways to distribute m identical objects in n distinct boxes.

5 objects in 3 boxes
$2+3+0$ Monomial: $x^2x^3x^0$
$0+2+3$ Monomial: $x^0x^2x^3$
$1+1+3$ Monomial: $x^1x^1x^3$

$$(1+x+x^2+\cdots)(1+x+x^2+\cdots)(1+x+x^2+\cdots)$$

Solution.
$\underbrace{(1+x+x^2+\cdots)(1+x+x^2+\cdots)\ldots(1+x+x^2+\cdots)}_{n \text{ times}}$
(because $a_m x^m = x^{r_1}x^{r_2}\ldots x^{r_n}+\cdots,$
such that $r_1+r_2+\cdots+r_n=m$)
$=(1+x+x^2+\cdots)^n$
$=\frac{1}{(1-x)^n}$
$=1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots$
Answer: Coefficient of x^m,

Example. Find the number of ways to distribute m identical objects in n distinct boxes.

5 objects in 3 boxes
$2+3+0$ Monomial: $x^2x^3x^0$
$0+2+3$ Monomial: $x^0x^2x^3$
$1+1+3$ Monomial: $x^1x^1x^3$

$$(1+x+x^2+\cdots)(1+x+x^2+\cdots)(1+x+x^2+\cdots)$$

Solution.
$\underbrace{(1+x+x^2+\cdots)(1+x+x^2+\cdots)\ldots(1+x+x^2+\cdots)}_{n\text{ times}}$
(because $a_m x^m = x^{r_1}x^{r_2}\ldots x^{r_n}+\cdots,$
such that $r_1+r_2+\cdots+r_n=m$)
$=(1+x+x^2+\cdots)^n$
$=\frac{1}{(1-x)^n}$
$=1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots$
Answer: Coefficient of x^m, i.e. $\binom{n+m-1}{m}$

Example. Find the number of ways to distribute m identical objects in n distinct boxes.

5 objects in 3 boxes
$2+3+0$ Monomial: $x^2x^3x^0$
$0+2+3$ Monomial: $x^0x^2x^3$
$1+1+3$ Monomial: $x^1x^1x^3$

$$(1+x+\cdots)(1+x+\cdots)(1+x+\cdots)$$

Solution.
$\underbrace{(1+x+\cdots)(1+x+\cdots)\ldots(1+x+\cdots)}_{n \text{ times}}$
(because $a_m x^m = x^{r_1}x^{r_2}\ldots x^{r_n}+\cdots$,
such that $r_1+r_2+\cdots+r_n=m$)
$=(1+x+x^2+\cdots)^n$
$=\frac{1}{(1-x)^n}$
$=1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots$
Answer: Coefficient of x^m, i.e. $\binom{n+m-1}{m}$

Example. Find the number of ways to distribute m **Example.** Find the number of ways
identical objects in n distinct boxes.

5 objects in 3 boxes
$2+3+0$ Monomial: $x^2x^3x^0$
$0+2+3$ Monomial: $x^0x^2x^3$
$1+1+3$ Monomial: $x^1x^1x^3$

$$(1+x+\cdots)(1+x+\cdots)(1+x+\cdots)$$

Solution.
$\underbrace{(1+x+\cdots)(1+x+\cdots)\dots(1+x+\cdots)}_{n \text{ times}}$
(because $a_m x^m = x^{r_1}x^{r_2}\dots x^{r_n}+\cdots,$
such that $r_1+r_2+\cdots+r_n=m$)
$=(1+x+x^2+\cdots)^n$
$=\frac{1}{(1-x)^n}$
$=1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots$
Answer: Coefficient of x^m, i.e. $\binom{n+m-1}{m}$

Example. Find the number of ways to distribute m identical objects in n distinct boxes.

5 objects in 3 boxes
$2+3+0$ Monomial: $x^2x^3x^0$
$0+2+3$ Monomial: $x^0x^2x^3$
$1+1+3$ Monomial: $x^1x^1x^3$

$$(1+x+\cdots)(1+x+\cdots)(1+x+\cdots)$$

Solution.
$\underbrace{(1+x+\cdots)(1+x+\cdots)\dots(1+x+\cdots)}_{n \text{ times}}$
(because $a_mx^m = x^{r_1}x^{r_2}\dots x^{r_n}+\cdots,$
such that $r_1+r_2+\cdots+r_n=m$)
$=(1+x+x^2+\cdots)^n$
$=\frac{1}{(1-x)^n}$
$=1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots$
Answer: Coefficient of x^m, i.e. $\binom{n+m-1}{m}$

Example. Find the number of ways to distribute m identical objects

Example. Find the number of ways to distribute m identical objects in n distinct boxes.

5 objects in 3 boxes
$2+3+0$ Monomial: $x^2x^3x^0$
$0+2+3$ Monomial: $x^0x^2x^3$
$1+1+3$ Monomial: $x^1x^1x^3$

$$(1+x+\cdots)(1+x+\cdots)(1+x+\cdots)$$

Solution.
$\underbrace{(1+x+\cdots)(1+x+\cdots)\ldots(1+x+\cdots)}_{n \text{ times}}$
(because $a_m x^m = x^{r_1}x^{r_2}\ldots x^{r_n}+\cdots$,
such that $r_1+r_2+\cdots+r_n=m$)
$=(1+x+x^2+\cdots)^n$
$=\frac{1}{(1-x)^n}$
$=1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots$
Answer: Coefficient of x^m, i.e. $\binom{n+m-1}{m}$

Example. Find the number of ways to distribute m identical objects in n distinct boxes

Example. Find the number of ways to distribute m identical objects in n distinct boxes.

5 objects in 3 boxes
$2+3+0$ Monomial: $x^2x^3x^0$
$0+2+3$ Monomial: $x^0x^2x^3$
$1+1+3$ Monomial: $x^1x^1x^3$

$$(1+x+\cdots)(1+x+\cdots)(1+x+\cdots)$$

Solution.
$\underbrace{(1+x+\cdots)(1+x+\cdots)\ldots(1+x+\cdots)}_{n \text{ times}}$
(because $a_m x^m = x^{r_1}x^{r_2}\ldots x^{r_n}+\cdots,$
such that $r_1+r_2+\cdots+r_n=m$)
$=(1+x+x^2+\cdots)^n$
$=\frac{1}{(1-x)^n}$
$=1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots$
Answer: Coefficient of x^m, i.e. $\binom{n+m-1}{m}$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Example. Find the number of ways to distribute m identical objects in n distinct boxes.

5 objects in 3 boxes
$2+3+0$ Monomial: $x^2x^3x^0$
$0+2+3$ Monomial: $x^0x^2x^3$
$1+1+3$ Monomial: $x^1x^1x^3$

$$(1+x+\cdots)(1+x+\cdots)(1+x+\cdots)$$

Solution.
$\underbrace{(1+x+\cdots)(1+x+\cdots)\dots(1+x+\cdots)}_{n \text{ times}}$
(because $a_m x^m = x^{r_1}x^{r_2}\dots x^{r_n} + \cdots$,
such that $r_1+r_2+\cdots+r_n = m$)
$= (1+x+x^2+\cdots)^n$
$= \frac{1}{(1-x)^n}$
$= 1 + nx + \binom{n+1}{2}x^2 + \binom{n+2}{3}x^3 + \cdots$
Answer: Coefficient of x^m, i.e. $\binom{n+m-1}{m}$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.

Example. Find the number of ways to distribute m identical objects in n distinct boxes.

5 objects in 3 boxes
$2+3+0$ Monomial: $x^2x^3x^0$
$0+2+3$ Monomial: $x^0x^2x^3$
$1+1+3$ Monomial: $x^1x^1x^3$

$$(1+x+\cdots)(1+x+\cdots)(1+x+\cdots)$$

Solution.
$\underbrace{(1+x+\cdots)(1+x+\cdots)\dots(1+x+\cdots)}_{n \text{ times}}$
(because $a_mx^m = x^{r_1}x^{r_2}\dots x^{r_n}+\cdots$,
such that $r_1+r_2+\cdots+r_n=m$)
$=(1+x+x^2+\cdots)^n$
$=\frac{1}{(1-x)^n}$
$=1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots$
Answer: Coefficient of x^m, i.e. $\binom{n+m-1}{m}$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.
$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{n \text{ times}}$

Example. Find the number of ways to distribute m identical objects in n distinct boxes.

5 objects in 3 boxes
$2+3+0$ Monomial: $x^2x^3x^0$
$0+2+3$ Monomial: $x^0x^2x^3$
$1+1+3$ Monomial: $x^1x^1x^3$

$$(1+x+\cdots)(1+x+\cdots)(1+x+\cdots)$$

Solution.
$\underbrace{(1+x+\cdots)(1+x+\cdots)\dots(1+x+\cdots)}_{n \text{ times}}$
(because $a_m x^m = x^{r_1}x^{r_2}\dots x^{r_n}+\cdots,$
such that $r_1+r_2+\cdots+r_n=m$)
$=(1+x+x^2+\cdots)^n$
$=\frac{1}{(1-x)^n}$
$=1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots$
Answer: Coefficient of x^m, i.e. $\binom{n+m-1}{m}$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.
$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{n \text{ times}}$
(because $a_m x^m = x^{r_1}x^{r_2}\dots x^{r_n}+\cdots,$

Example. Find the number of ways to distribute m identical objects in n distinct boxes.

5 objects in 3 boxes
$2+3+0$ Monomial: $x^2x^3x^0$
$0+2+3$ Monomial: $x^0x^2x^3$
$1+1+3$ Monomial: $x^1x^1x^3$

$$(1+x+\cdots)(1+x+\cdots)(1+x+\cdots)$$

Solution.
$\underbrace{(1+x+\cdots)(1+x+\cdots)\dots(1+x+\cdots)}_{n\text{ times}}$
(because $a_m x^m = x^{r_1}x^{r_2}\dots x^{r_n}+\cdots$,
such that $r_1+r_2+\cdots+r_n=m$)
$=(1+x+x^2+\cdots)^n$
$=\frac{1}{(1-x)^n}$
$=1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots$
Answer: Coefficient of x^m, i.e. $\binom{n+m-1}{m}$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.
$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{n\text{ times}}$
(because $a_m x^m = x^{r_1}x^{r_2}\dots x^{r_n}+\cdots$,
such that $r_1+r_2+\cdots+r_n=m$)

Example. Find the number of ways to distribute m identical objects in n distinct boxes.

5 objects in 3 boxes
$2+3+0$ Monomial: $x^2x^3x^0$
$0+2+3$ Monomial: $x^0x^2x^3$
$1+1+3$ Monomial: $x^1x^1x^3$

$$(1+x+\cdots)(1+x+\cdots)(1+x+\cdots)$$

Solution.
$\underbrace{(1+x+\cdots)(1+x+\cdots)\ldots(1+x+\cdots)}_{n \text{ times}}$
(because $a_mx^m = x^{r_1}x^{r_2}\ldots x^{r_n}+\cdots,$
such that $r_1+r_2+\cdots+r_n=m$)
$=(1+x+x^2+\cdots)^n$
$=\frac{1}{(1-x)^n}$
$=1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots$
Answer: Coefficient of x^m, i.e. $\binom{n+m-1}{m}$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.
$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\ldots(x+x^2+\cdots)}_{n \text{ times}}$
(because $a_mx^m = x^{r_1}x^{r_2}\ldots x^{r_n}+\cdots,$
such that $r_1+r_2+\cdots+r_n=m$)
$=(x+x^2+x^3\cdots)^n$

Example. Find the number of ways to distribute m identical objects in n distinct boxes.

5 objects in 3 boxes
$2+3+0$ Monomial: $x^2x^3x^0$
$0+2+3$ Monomial: $x^0x^2x^3$
$1+1+3$ Monomial: $x^1x^1x^3$

$$(1+x+\cdots)(1+x+\cdots)(1+x+\cdots)$$

Solution.
$\underbrace{(1+x+\cdots)(1+x+\cdots)\ldots(1+x+\cdots)}_{n \text{ times}}$
(because $a_m x^m = x^{r_1}x^{r_2}\ldots x^{r_n}+\cdots,$
such that $r_1+r_2+\cdots+r_n=m$)
$=(1+x+x^2+\cdots)^n$
$=\frac{1}{(1-x)^n}$
$=1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots$
Answer: Coefficient of x^m, i.e. $\binom{n+m-1}{m}$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.
$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\ldots(x+x^2+\cdots)}_{n \text{ times}}$
(because $a_m x^m = x^{r_1}x^{r_2}\ldots x^{r_n}+\cdots,$
such that $r_1+r_2+\cdots+r_n=m$)
$=(x+x^2+x^3\cdots)^n$
$=x^n(1+x+x^2+x^3\cdots)^n$

Example. Find the number of ways to distribute m identical objects in n distinct boxes.

5 objects in 3 boxes
$2+3+0$ Monomial: $x^2x^3x^0$
$0+2+3$ Monomial: $x^0x^2x^3$
$1+1+3$ Monomial: $x^1x^1x^3$

$$(1+x+\cdots)(1+x+\cdots)(1+x+\cdots)$$

Solution.
$\underbrace{(1+x+\cdots)(1+x+\cdots)\dots(1+x+\cdots)}_{n \text{ times}}$
(because $a_mx^m = x^{r_1}x^{r_2}\dots x^{r_n}+\cdots$,
such that $r_1+r_2+\cdots+r_n = m$)
$= (1+x+x^2+\cdots)^n$
$= \frac{1}{(1-x)^n}$
$= 1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots$
Answer: Coefficient of x^m, i.e. $\binom{n+m-1}{m}$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.
$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{n \text{ times}}$
(because $a_mx^m = x^{r_1}x^{r_2}\dots x^{r_n}+\cdots$,
such that $r_1+r_2+\cdots+r_n = m$)
$= (x+x^2+x^3\cdots)^n$
$= x^n(1+x+x^2+x^3\cdots)^n$
$= \frac{x^n}{(1-x)^n}$

Example. Find the number of ways to distribute m identical objects in n distinct boxes.

5 objects in 3 boxes
$2+3+0$ Monomial: $x^2x^3x^0$
$0+2+3$ Monomial: $x^0x^2x^3$
$1+1+3$ Monomial: $x^1x^1x^3$

$$(1+x+\cdots)(1+x+\cdots)(1+x+\cdots)$$

Solution.
$\underbrace{(1+x+\cdots)(1+x+\cdots)\dots(1+x+\cdots)}_{n \text{ times}}$
(because $a_m x^m = x^{r_1}x^{r_2}\dots x^{r_n}+\cdots$,
such that $r_1+r_2+\cdots+r_n=m$)
$=(1+x+x^2+\cdots)^n$
$=\frac{1}{(1-x)^n}$
$=1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots$
Answer: Coefficient of x^m, i.e. $\binom{n+m-1}{m}$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.
$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{n \text{ times}}$
(because $a_m x^m = x^{r_1}x^{r_2}\dots x^{r_n}+\cdots$,
such that $r_1+r_2+\cdots+r_n=m$)
$=(x+x^2+x^3\cdots)^n$
$=x^n(1+x+x^2+x^3\cdots)^n$
$=\frac{x^n}{(1-x)^n}$
$=x^n(1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots)$

Example. Find the number of ways to distribute m identical objects in n distinct boxes.

5 objects in 3 boxes
$2+3+0$ Monomial: $x^2x^3x^0$
$0+2+3$ Monomial: $x^0x^2x^3$
$1+1+3$ Monomial: $x^1x^1x^3$

$$(1+x+\cdots)(1+x+\cdots)(1+x+\cdots)$$

Solution.
$\underbrace{(1+x+\cdots)(1+x+\cdots)\dots(1+x+\cdots)}_{n \text{ times}}$
(because $a_mx^m = x^{r_1}x^{r_2}\dots x^{r_n}+\cdots$,
such that $r_1+r_2+\cdots+r_n=m$)
$=(1+x+x^2+\cdots)^n$
$=\frac{1}{(1-x)^n}$
$=1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots$
Answer: Coefficient of x^m, i.e. $\binom{n+m-1}{m}$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.
$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{n \text{ times}}$
(because $a_mx^m = x^{r_1}x^{r_2}\dots x^{r_n}+\cdots$,
such that $r_1+r_2+\cdots+r_n=m$)
$=(x+x^2+x^3\cdots)^n$
$=x^n(1+x+x^2+x^3\cdots)^n$
$=\frac{x^n}{(1-x)^n}$
$=x^n(1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots)$
$=x^n+nx^{n+1}+\binom{n+1}{2}x^{n+2}+\binom{n+2}{3}x^{n+3}+\cdots)$

Example. Find the number of ways to distribute m identical objects in n distinct boxes.

5 objects in 3 boxes
$2+3+0$ Monomial: $x^2x^3x^0$
$0+2+3$ Monomial: $x^0x^2x^3$
$1+1+3$ Monomial: $x^1x^1x^3$

$$(1+x+\cdots)(1+x+\cdots)(1+x+\cdots)$$

Solution.
$\underbrace{(1+x+\cdots)(1+x+\cdots)\dots(1+x+\cdots)}_{n\text{ times}}$
(because $a_mx^m = x^{r_1}x^{r_2}\dots x^{r_n}+\cdots,$
such that $r_1+r_2+\cdots+r_n=m$)
$=(1+x+x^2+\cdots)^n$
$=\frac{1}{(1-x)^n}$
$=1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots$
Answer: Coefficient of x^m, i.e. $\binom{n+m-1}{m}$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.
$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{n\text{ times}}$
(because $a_mx^m = x^{r_1}x^{r_2}\dots x^{r_n}+\cdots,$
such that $r_1+r_2+\cdots+r_n=m$)
$=(x+x^2+x^3\cdots)^n$
$=x^n(1+x+x^2+x^3\cdots)^n$
$=\frac{x^n}{(1-x)^n}$
$=x^n(1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots)$
$=x^n+nx^{n+1}+\binom{n+1}{2}x^{n+2}+\binom{n+2}{3}x^{n+3}+\cdots)$
General term: $\binom{n+k-1}{k}x^{n+k}$

Example. Find the number of ways to distribute m identical objects in n distinct boxes.

5 objects in 3 boxes
$2+3+0$ Monomial: $x^2x^3x^0$
$0+2+3$ Monomial: $x^0x^2x^3$
$1+1+3$ Monomial: $x^1x^1x^3$

$$(1+x+\cdots)(1+x+\cdots)(1+x+\cdots)$$

Solution.
$\underbrace{(1+x+\cdots)(1+x+\cdots)\dots(1+x+\cdots)}_{n \text{ times}}$
(because $a_mx^m = x^{r_1}x^{r_2}\dots x^{r_n}+\cdots$,
such that $r_1+r_2+\cdots+r_n=m$)
$=(1+x+x^2+\cdots)^n$
$=\frac{1}{(1-x)^n}$
$=1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots$
Answer: Coefficient of x^m, i.e. $\binom{n+m-1}{m}$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.
$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{n \text{ times}}$
(because $a_mx^m = x^{r_1}x^{r_2}\dots x^{r_n}+\cdots$,
such that $r_1+r_2+\cdots+r_n=m$)
$=(x+x^2+x^3\cdots)^n$
$=x^n(1+x+x^2+x^3\cdots)^n$
$=\frac{x^n}{(1-x)^n}$
$=x^n(1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots)$
$=x^n+nx^{n+1}+\binom{n+1}{2}x^{n+2}+\binom{n+2}{3}x^{n+3}+\cdots)$
General term: $\binom{n+k-1}{k}x^{n+k}$
Answer: $\binom{m-1}{m-n}$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.

$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\ldots(x+x^2+\cdots)}_{n\text{ times}}$

(because $a_m x^m = x^{r_1}x^{r_2}\ldots x^{r_n}+\cdots,$

such that $r_1+r_2+\cdots+r_n=m$)

$=(x+x^2+x^3\cdots)^n$

$=x^n(1+x+x^2+x^3\cdots)^n$

$=\frac{x^n}{(1-x)^n}$

$=x^n(1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots)$

$=x^n+nx^{n+1}+\binom{n+1}{2}x^{n+2}+\binom{n+2}{3}x^{n+3}+\cdots)$

General term: $\binom{n+k-1}{k}x^{n+k}$

Answer: $\binom{m-1}{m-n}$

Example.

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.

$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{n \text{ times}}$

(because $a_m x^m = x^{r_1}x^{r_2}\dots x^{r_n}+\cdots$,

such that $r_1+r_2+\cdots+r_n=m$)

$= (x+x^2+x^3\cdots)^n$

$= x^n(1+x+x^2+x^3\cdots)^n$

$= \frac{x^n}{(1-x)^n}$

$= x^n(1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots)$

$= x^n+nx^{n+1}+\binom{n+1}{2}x^{n+2}+\binom{n+2}{3}x^{n+3}+\cdots)$

General term: $\binom{n+k-1}{k}x^{n+k}$

Answer: $\binom{m-1}{m-n}$

Example. Number of ways to choose m elements out of n elements

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.

$$\underbrace{(x + x^2 + \cdots)(x + x^2 + \cdots)\ldots(x + x^2 + \cdots)}_{n \text{ times}}$$

(because $a_m x^m = x^{r_1} x^{r_2} \ldots x^{r_n} + \cdots,$

such that $r_1 + r_2 + \cdots + r_n = m$)

$= (x + x^2 + x^3 \cdots)^n$

$= x^n(1 + x + x^2 + x^3 \cdots)^n$

$= \frac{x^n}{(1-x)^n}$

$= x^n(1 + nx + \binom{n+1}{2}x^2 + \binom{n+2}{3}x^3 + \cdots)$

$= x^n + nx^{n+1} + \binom{n+1}{2}x^{n+2} + \binom{n+2}{3}x^{n+3} + \cdots)$

General term: $\binom{n+k-1}{k}x^{n+k}$

Answer: $\binom{m-1}{m-n}$

Example. Number of ways to choose m elements out of n elements

Solution.

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.

$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\ldots(x+x^2+\cdots)}_{n\text{ times}}$

(because $a_m x^m = x^{r_1}x^{r_2}\ldots x^{r_n}+\cdots$,

such that $r_1+r_2+\cdots+r_n=m$)

$=(x+x^2+x^3\cdots)^n$

$=x^n(1+x+x^2+x^3\cdots)^n$

$=\frac{x^n}{(1-x)^n}$

$=x^n(1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots)$

$=x^n+nx^{n+1}+\binom{n+1}{2}x^{n+2}+\binom{n+2}{3}x^{n+3}+\cdots)$

General term: $\binom{n+k-1}{k}x^{n+k}$

Answer: $\binom{m-1}{m-n}$

Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.

$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\ldots(x+x^2+\cdots)}_{n \text{ times}}$

(because $a_m x^m = x^{r_1}x^{r_2}\ldots x^{r_n}+\cdots$,

such that $r_1+r_2+\cdots+r_n=m$)

$= (x+x^2+x^3\cdots)^n$

$= x^n(1+x+x^2+x^3\cdots)^n$

$= \frac{x^n}{(1-x)^n}$

$= x^n(1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots)$

$= x^n+nx^{n+1}+\binom{n+1}{2}x^{n+2}+\binom{n+2}{3}x^{n+3}+\cdots)$

General term: $\binom{n+k-1}{k}x^{n+k}$

Answer: $\binom{m-1}{m-n}$

Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

$$(1+x)\dots(1+x)$$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.
$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{n \text{ times}}$
(because $a_m x^m = x^{r_1}x^{r_2}\dots x^{r_n}+\cdots$,
such that $r_1+r_2+\cdots+r_n=m$)
$=(x+x^2+x^3\cdots)^n$
$=x^n(1+x+x^2+x^3\cdots)^n$
$=\frac{x^n}{(1-x)^n}$
$=x^n(1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots)$
$=x^n+nx^{n+1}+\binom{n+1}{2}x^{n+2}+\binom{n+2}{3}x^{n+3}+\cdots)$
General term: $\binom{n+k-1}{k}x^{n+k}$
Answer: $\binom{m-1}{m-n}$

Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

$$(1+x)\dots(1+x) = (1+x)^n$$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.
$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{n \text{ times}}$
(because $a_m x^m = x^{r_1}x^{r_2}\dots x^{r_n} + \cdots$,
such that $r_1 + r_2 + \cdots + r_n = m$)
$= (x + x^2 + x^3 \cdots)^n$
$= x^n(1 + x + x^2 + x^3 \cdots)^n$
$= \frac{x^n}{(1-x)^n}$
$= x^n(1 + nx + \binom{n+1}{2}x^2 + \binom{n+2}{3}x^3 + \cdots)$
$= x^n + nx^{n+1} + \binom{n+1}{2}x^{n+2} + \binom{n+2}{3}x^{n+3} + \cdots)$
General term: $\binom{n+k-1}{k}x^{n+k}$
Answer: $\binom{m-1}{m-n}$

Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

$$(1+x)\dots(1+x) = (1+x)^n$$

Answer: $\binom{n}{m}$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.
$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{n \text{ times}}$
(because $a_m x^m = x^{r_1}x^{r_2}\dots x^{r_n}+\cdots$,
such that $r_1+r_2+\cdots+r_n = m$)
$=(x+x^2+x^3\cdots)^n$
$=x^n(1+x+x^2+x^3\cdots)^n$
$=\frac{x^n}{(1-x)^n}$
$=x^n(1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots)$
$=x^n+nx^{n+1}+\binom{n+1}{2}x^{n+2}+\binom{n+2}{3}x^{n+3}+\cdots)$
General term: $\binom{n+k-1}{k}x^{n+k}$
Answer: $\binom{m-1}{m-n}$

Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

$$(1+x)\dots(1+x) = (1+x)^n$$

Answer: $\binom{n}{m}$

Partitions of integers

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.
$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{n \text{ times}}$
(because $a_m x^m = x^{r_1}x^{r_2}\dots x^{r_n}+\cdots$,
such that $r_1+r_2+\cdots+r_n = m$)
$= (x+x^2+x^3\cdots)^n$
$= x^n(1+x+x^2+x^3\cdots)^n$
$= \frac{x^n}{(1-x)^n}$
$= x^n(1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots)$
$= x^n+nx^{n+1}+\binom{n+1}{2}x^{n+2}+\binom{n+2}{3}x^{n+3}+\cdots)$
General term: $\binom{n+k-1}{k}x^{n+k}$
Answer: $\binom{m-1}{m-n}$

Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

$$(1+x)\dots(1+x) = (1+x)^n$$

Answer: $\binom{n}{m}$

Partitions of integers

Examples

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.
$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{n \text{ times}}$
(because $a_m x^m = x^{r_1}x^{r_2}\dots x^{r_n}+\cdots$,
such that $r_1+r_2+\cdots+r_n = m$)
$= (x+x^2+x^3\cdots)^n$
$= x^n(1+x+x^2+x^3\cdots)^n$
$= \frac{x^n}{(1-x)^n}$
$= x^n(1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots)$
$= x^n+nx^{n+1}+\binom{n+1}{2}x^{n+2}+\binom{n+2}{3}x^{n+3}+\cdots)$
General term: $\binom{n+k-1}{k}x^{n+k}$
Answer: $\binom{m-1}{m-n}$

Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

$$(1+x)\dots(1+x) = (1+x)^n$$

Answer: $\binom{n}{m}$

Partitions of integers

Examples
3

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.
$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{n \text{ times}}$
(because $a_m x^m = x^{r_1}x^{r_2}\dots x^{r_n}+\cdots,$
such that $r_1+r_2+\cdots+r_n = m$)
$= (x+x^2+x^3\cdots)^n$
$= x^n(1+x+x^2+x^3\cdots)^n$
$= \frac{x^n}{(1-x)^n}$
$= x^n(1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots)$
$= x^n+nx^{n+1}+\binom{n+1}{2}x^{n+2}+\binom{n+2}{3}x^{n+3}+\cdots)$
General term: $\binom{n+k-1}{k}x^{n+k}$
Answer: $\binom{m-1}{m-n}$

Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

$$(1+x)\dots(1+x) = (1+x)^n$$

Answer: $\binom{n}{m}$

Partitions of integers

Examples

$3 = 1+1+1$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.

$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{n \text{ times}}$

(because $a_m x^m = x^{r_1}x^{r_2}\dots x^{r_n} + \cdots,$

such that $r_1 + r_2 + \cdots + r_n = m$)

$= (x+x^2+x^3\cdots)^n$

$= x^n(1+x+x^2+x^3\cdots)^n$

$= \frac{x^n}{(1-x)^n}$

$= x^n(1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots)$

$= x^n+nx^{n+1}+\binom{n+1}{2}x^{n+2}+\binom{n+2}{3}x^{n+3}+\cdots)$

General term: $\binom{n+k-1}{k}x^{n+k}$

Answer: $\binom{m-1}{m-n}$

Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

$$(1+x)\dots(1+x) = (1+x)^n$$

Answer: $\binom{n}{m}$

Partitions of integers

Examples
$3 = 1+1+1 = 2+1$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.
$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{n \text{ times}}$
(because $a_m x^m = x^{r_1}x^{r_2}\dots x^{r_n} + \cdots$,
such that $r_1+r_2+\cdots+r_n = m$)
$= (x+x^2+x^3\cdots)^n$
$= x^n(1+x+x^2+x^3\cdots)^n$
$= \frac{x^n}{(1-x)^n}$
$= x^n(1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots)$
$= x^n+nx^{n+1}+\binom{n+1}{2}x^{n+2}+\binom{n+2}{3}x^{n+3}+\cdots)$
General term: $\binom{n+k-1}{k}x^{n+k}$
Answer: $\binom{m-1}{m-n}$

Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

$$(1+x)\dots(1+x) = (1+x)^n$$

Answer: $\binom{n}{m}$

Partitions of integers

Examples

$3 = 1+1+1 = 2+1$

4

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.

$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{n \text{ times}}$

(because $a_m x^m = x^{r_1}x^{r_2}\dots x^{r_n}+\cdots$,

such that $r_1+r_2+\cdots+r_n = m$)

$= (x+x^2+x^3\cdots)^n$

$= x^n(1+x+x^2+x^3\cdots)^n$

$= \frac{x^n}{(1-x)^n}$

$= x^n(1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots)$

$= x^n+nx^{n+1}+\binom{n+1}{2}x^{n+2}+\binom{n+2}{3}x^{n+3}+\cdots)$

General term: $\binom{n+k-1}{k}x^{n+k}$

Answer: $\binom{m-1}{m-n}$

Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

$$(1+x)\dots(1+x) = (1+x)^n$$

Answer: $\binom{n}{m}$

Partitions of integers

Examples

$3 = 1+1+1 = 2+1$

$4 = 1+1+1+1$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.

$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{n \text{ times}}$

(because $a_m x^m = x^{r_1}x^{r_2}\dots x^{r_n} + \cdots$,

such that $r_1 + r_2 + \cdots + r_n = m$)

$= (x + x^2 + x^3 \cdots)^n$

$= x^n(1 + x + x^2 + x^3 \cdots)^n$

$= \frac{x^n}{(1-x)^n}$

$= x^n(1 + nx + \binom{n+1}{2}x^2 + \binom{n+2}{3}x^3 + \cdots)$

$= x^n + nx^{n+1} + \binom{n+1}{2}x^{n+2} + \binom{n+2}{3}x^{n+3} + \cdots)$

General term: $\binom{n+k-1}{k}x^{n+k}$

Answer: $\binom{m-1}{m-n}$

Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

$$(1+x)\dots(1+x) = (1+x)^n$$

Answer: $\binom{n}{m}$

Partitions of integers

Examples
$3 = 1+1+1 = 2+1$
$4 = 1+1+1+1 = 2+1+1$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.
$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{n \text{ times}}$
(because $a_m x^m = x^{r_1}x^{r_2}\dots x^{r_n} + \cdots$,
such that $r_1 + r_2 + \cdots + r_n = m$)
$= (x + x^2 + x^3 \cdots)^n$
$= x^n(1 + x + x^2 + x^3 \cdots)^n$
$= \frac{x^n}{(1-x)^n}$
$= x^n(1 + nx + \binom{n+1}{2}x^2 + \binom{n+2}{3}x^3 + \cdots)$
$= x^n + nx^{n+1} + \binom{n+1}{2}x^{n+2} + \binom{n+2}{3}x^{n+3} + \cdots)$
General term: $\binom{n+k-1}{k}x^{n+k}$
Answer: $\binom{m-1}{m-n}$

Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

$$(1+x)\dots(1+x)=(1+x)^n$$

Answer: $\binom{n}{m}$

Partitions of integers

Examples

$3=1+1+1=2+1$

$4=1+1+1+1=2+1+1=2{+}2$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.

$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{n \text{ times}}$

(because $a_m x^m = x^{r_1}x^{r_2}\dots x^{r_n}+\cdots,$

such that $r_1+r_2+\cdots+r_n=m$)

$=(x+x^2+x^3\cdots)^n$

$=x^n(1+x+x^2+x^3\cdots)^n$

$=\frac{x^n}{(1-x)^n}$

$=x^n(1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots)$

$=x^n+nx^{n+1}+\binom{n+1}{2}x^{n+2}+\binom{n+2}{3}x^{n+3}+\cdots)$

General term: $\binom{n+k-1}{k}x^{n+k}$

Answer: $\binom{m-1}{m-n}$

Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

$$(1+x)\dots(1+x) = (1+x)^n$$

Answer: $\binom{n}{m}$

Partitions of integers

Examples

$3 = 1+1+1 = 2+1$

$4 = 1+1+1+1 = 2+1+1 = 2+2$

Example.

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.

$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{n\text{ times}}$

(because $a_m x^m = x^{r_1}x^{r_2}\dots x^{r_n}+\cdots$,

such that $r_1+r_2+\cdots+r_n = m$)

$= (x+x^2+x^3\cdots)^n$

$= x^n(1+x+x^2+x^3\cdots)^n$

$= \frac{x^n}{(1-x)^n}$

$= x^n(1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots)$

$= x^n+nx^{n+1}+\binom{n+1}{2}x^{n+2}+\binom{n+2}{3}x^{n+3}+\cdots)$

General term: $\binom{n+k-1}{k}x^{n+k}$

Answer: $\binom{m-1}{m-n}$

Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

$$(1+x)\dots(1+x) = (1+x)^n$$

Answer: $\binom{n}{m}$

Partitions of integers

Examples

$3 = 1+1+1 = 2+1$

$4 = 1+1+1+1 = 2+1+1 = 2+2$

Example. Find the number of partitions of m into integers

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.

$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{n \text{ times}}$

(because $a_m x^m = x^{r_1}x^{r_2}\dots x^{r_n} + \cdots$,

such that $r_1 + r_2 + \cdots + r_n = m$)

$= (x + x^2 + x^3 \cdots)^n$

$= x^n(1 + x + x^2 + x^3 \cdots)^n$

$= \frac{x^n}{(1-x)^n}$

$= x^n(1 + nx + \binom{n+1}{2}x^2 + \binom{n+2}{3}x^3 + \cdots)$

$= x^n + nx^{n+1} + \binom{n+1}{2}x^{n+2} + \binom{n+2}{3}x^{n+3} + \cdots)$

General term: $\binom{n+k-1}{k}x^{n+k}$

Answer: $\binom{m-1}{m-n}$

Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

$$(1+x)\dots(1+x) = (1+x)^n$$

Answer: $\binom{n}{m}$

Partitions of integers

Examples

$3 = 1+1+1 = 2+1$

$4 = 1+1+1+1 = 2+1+1 = 2+2$

Example. Find the number of partitions of m into integers with allowed sizes 1, 2, 3.

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.

$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{n \text{ times}}$

(because $a_m x^m = x^{r_1}x^{r_2}\dots x^{r_n} + \cdots,$

such that $r_1+r_2+\cdots+r_n = m$)

$= (x+x^2+x^3\cdots)^n$

$= x^n(1+x+x^2+x^3\cdots)^n$

$= \frac{x^n}{(1-x)^n}$

$= x^n(1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots)$

$= x^n+nx^{n+1}+\binom{n+1}{2}x^{n+2}+\binom{n+2}{3}x^{n+3}+\cdots)$

General term: $\binom{n+k-1}{k}x^{n+k}$

Answer: $\binom{m-1}{m-n}$

Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

$$(1+x)\dots(1+x) = (1+x)^n$$

Answer: $\binom{n}{m}$

Partitions of integers

Examples

$3 = 1+1+1 = 2+1$

$4 = 1+1+1+1 = 2+1+1 = 2+2$

Example. Find the number of partitions of m into integers with allowed sizes 1, 2, 3.

Solution.

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.

$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{n \text{ times}}$

(because $a_m x^m = x^{r_1}x^{r_2}\dots x^{r_n}+\cdots,$

such that $r_1+r_2+\cdots+r_n = m$)

$= (x+x^2+x^3\cdots)^n$

$= x^n(1+x+x^2+x^3\cdots)^n$

$= \frac{x^n}{(1-x)^n}$

$= x^n(1+nx+\binom{n+1}{2}x^2+\binom{n+2}{3}x^3+\cdots)$

$= x^n+nx^{n+1}+\binom{n+1}{2}x^{n+2}+\binom{n+2}{3}x^{n+3}+\cdots)$

General term: $\binom{n+k-1}{k}x^{n+k}$

Answer: $\binom{m-1}{m-n}$

Example. Number of ways to choose m elements out of n elements

Solution. Generating function:

$$(1+x)\dots(1+x) = (1+x)^n$$

Answer: $\binom{n}{m}$

Partitions of integers

Examples

$3 = 1+1+1 = 2+1$

$4 = 1+1+1+1 = 2+1+1 = 2+2$

Example. Find the number of partitions of m into integers with allowed sizes 1, 2, 3.

Solution. $(1+x+\cdots)(1+x^2+x^4+\cdots)(1+x^3+x^6+\cdots)$

Example. Find the number of ways to distribute m identical objects in n distinct boxes so that each box contains at least one object.

Solution.

$\underbrace{(x+x^2+\cdots)(x+x^2+\cdots)\dots(x+x^2+\cdots)}_{n \text{ times}}$

(because $a_m x^m = x^{r_1} x^{r_2} \dots x^{r_n} + \cdots$,

such that $r_1 + r_2 + \cdots + r_n = m$)

$= (x + x^2 + x^3 \cdots)^n$

$= x^n(1 + x + x^2 + x^3 \cdots)^n$

$= \frac{x^n}{(1-x)^n}$

$= x^n(1 + nx + \binom{n+1}{2}x^2 + \binom{n+2}{3}x^3 + \cdots)$

$= x^n + nx^{n+1} + \binom{n+1}{2}x^{n+2} + \binom{n+2}{3}x^{n+3} + \cdots)$

General term: $\binom{n+k-1}{k}x^{n+k}$

Answer: $\binom{m-1}{m-n}$