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Size of each equivalence class: $3! \times 2!$

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Example. Count the number of ways to select m balls from an infinite supply of red, green, and blue ones.

Solution. If $m = 5$, examples of selections:
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Alternatively,

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Solution. If $m = 5$, examples of selections:
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Alternatively,
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Alternatively,
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Size of each equivalence class: $3! \times 2!$

Final answer: $\frac{5!}{3! \times 2!}$

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Solution. If $m = 5$, examples of selections:
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Final answer: $\binom{m+2}{m}$

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Example. Count the number of ways to select m integers out of $\{1, \ldots, n\}$ so that they contain no consecutive integers.

Solution. Consider such a set, $A = \{x_1, \ldots, x_m\}$. Main idea:
Order the elements

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Solution. If $m = 5$, examples of selections:
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Alternatively,
$XXX|X|X$, $XX|XX|X$, $X|XXXX|$

Final answer: $\binom{m+2}{m}$

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