Exercise sheet 1

Curves and Surfaces, MTH201

1. In how many ways can 7 students and 5 teachers be seated around a round table of no two teachers are adjacent?

2. In how many ways can 3 distinct balls be placed in 5 boxes so that empty boxes are *not* adjacent?

3. Derive a formula for the number of surjective maps from a set X of cardinality n to a set Y of cardinality m in terms of $S(n, m)$.

4. Let $s(n, m)$ denote the number of ways to arrange $\{1, 2, \ldots, n\}$ around m distinct circles so that each circle has at least one number. Note the difference with $S(n, m)$ that was done during a lecture ($s(n, m)$ are called Stirling numbers of the first kind, and $S(n, m)$ are called Stirling numbers of the second kind).

   (a) Prove that $s(n, m) = s(n-1, m-1) + (n-1)s(n-1, m)$.

   (b) Compute, $s(n, 0)$ $(n \geq 1)$, $s(n, n)$ $(n \geq 0)$, $s(n, 1)$ $(n \geq 2)$, and $s(n, n-1)$ $(n \geq 2)$.

   (c) Compute $s(3, 2)$, $s(4, 2)$, and $s(4, 3)$.

5. Find the number of ways to permute them 3 blue balls, 5 red balls, and 4 green balls (balls of the same colour are indistinguishable). Now generalize this: If there are m objects of types m_1, m_2, $\ldots$, m_k, and the within each type the balls are indistinguishable, then count the number of permutations of them. Try to do this in two ways, one by relating it with the simpler modification of when the balls are distinct, and the other, by selecting the places which meant for each colour. Show that the answers are the same.