Definition.

Definition. A *tree*

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Observations about trees:

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2. Deleting any edge

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1. Any two vertices of a tree are connected by exactly one path
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Theorem. *G connected, $e \in E(G)$, $G - e$ can have at most 2 components*

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Proof. $e = \{x, y\}$.

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Case 1: There is another path P linking x and y

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Proof. $e = \{x, y\}$.

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Claim: $G - e$ is connected.

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Replace e by P

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Definition. $e \in E(G)$ is a bridge if G has fewer components than $G - e$.

□ **Theorem.**

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Exercise.

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Theorem. *$e \in E(G)$ is a bridge if and only if e is not part of a cycle.*

Exercise. G connected. G is a tree if and only if every edge is a bridge.

Proof. □

Definition.

Define $A := \{v \in V(G) \mid \text{ there is a path from v to x not containing } e\}$

A is in one connected component containing x

Any $v \in A^c$, can only be connected to x via path containing e.
e is the last edge of the path.
v links with y.

A^c is in one connected component containing y

Definition. $e \in E(G)$ is a bridge if G has fewer components than $G - e$.

Theorem. *$e \in E(G)$ is a bridge if and only if e is not part of a cycle.*

Exercise. G connected. G is a tree if and only if every edge is a bridge.

Proof. □

Definition. $H \subset G$ is a spanning subgraph if $V(H) = V(G)$.

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Definition. $e \in E(G)$ is a bridge if G has fewer components than $G - e$.

Theorem. *$e \in E(G)$ is a bridge if and only if e is not part of a cycle.*

Exercise. G connected. G is a tree if and only if every edge is a bridge.

Proof. □

Definition. $H \subset G$ is a spanning subgraph if $V(H) = V(G)$. If H is a tree, it is called a spanning tree.

Theorem. *A graph is connected if and only if it has a spanning tree.*

Define $A := \{v \in V(G) \mid \text{ there is a path from v to x not containing } e\}$

A is in one connected component containing x

Any $v \in A^c$, can only be connected to x via path containing e.
e is the last edge of the path.
v links with y.

A^c is in one connected component containing y

Definition. $e \in E(G)$ is a bridge if G has fewer components than $G - e$.

Theorem. $e \in E(G)$ *is a bridge if and only if* e *is not part of a cycle.*

Exercise. G connected. G is a tree if and only if every edge is a bridge.