Definition. An *n-clique*

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Generalized Pigeonhole principle

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A

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A and B

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A and B finite sets

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f

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Example. Consider a graph with 6 vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue.

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Example. Consider a graph with 6 vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue. Show that there are three vertices so that the edges joining each pair of them are of the same colour.

Example.

Example. In a group of 6 people, there is either a subset of 3 who know each other, or a subset of 3 who do not know each other.

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Example. Consider a graph with 17 vertices, such that **Definition** (Ramsay Number). every pair vertices is joined by exactly one edge which can be coloured either red, blue, or green. Show that there are three vertices so that the edges joining each pair of them are of the same colour.

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Example. Consider a graph with 17 vertices, such that **Definition** (Ramsay Number). R(p,q) the smallest every pair vertices is joined by exactly one edge which number ncan be coloured either red, blue, or green. Show that there are three vertices so that the edges joining each pair of them are of the same colour.

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Example. Consider a graph with 17 vertices, such that **Definition** (Ramsay Number). R(p,q) the smallest every pair vertices is joined by exactly one edge which number n such that a graph with n vertices, and each can be coloured either red, blue, or green. Show that pair of vertices is joined by exactly one edge, and each there are three vertices so that the edges joining each coloured by either red or blue, pair of them are of the same colour.

Solution.

Let the set of vertices be, $V := \{v_1, v_2, \dots, v_{17}\}$ v_1 is joined with 16 vertices.

 $16 = 3 \times 5 + 1$, so by pigeonhole principle,

At least 5 + 1 = 6 edges containing v_1 are of the same colour C_1

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