Exercise sheet 5

Curves and Surfaces, MTH201

1. Prove that each vertex of a graph belongs to exactly one component.
2. Prove that every tree, T, has at least $\Delta(T)$ leaves.
3. Given a tree, T, let $f : V(T) \to V(T)$ be a bijection so that $\{x, y\} \in E(T) \iff \{f(x), f(y)\} \in E(T)$. Prove that f either fixes at least one vertex or fixes an edge.
4. Prove that every connected graph contains a path of length at least $\min\{2\delta(G), |G| - 1\}$.
5. Prove that every 2-connected graph contains a cycle.