Example.

Example. Solve the recurrence relation

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2,$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ (*not* $a_0 = 1$)

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ $(not\ a_0 = 1)$ i.e.

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ $(not\ a_0 = 1)$ i.e.

$$f(n) - 2f(n-1) = 3n^2$$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ $(not\ a_0 = 1)$ i.e.

$$f(n) - 2f(n-1) = 3n^2$$

$$g(n) - 2g(n-1) = 3n^2$$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ $(not\ a_0 = 1)$ i.e.

$$f(n) - 2f(n-1) = 3n^2$$

$$g(n) - 2g(n-1) = 3n^2$$

$$(f-g)(n) - 2(f-g)(n-1) = 0$$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ (*not* $a_0 = 1$) i.e.

$$f(n) - 2f(n-1) = 3n^2$$

$$g(n) - 2g(n-1) = 3n^2$$

$$(f-g)(n) - 2(f-g)(n-1) = 0$$

then, $f(n) - g(n)$ is a solution to

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ $(not\ a_0 = 1)$ i.e.

$$f(n) - 2f(n-1) = 3n^2$$

$$g(n) - 2g(n-1) = 3n^2$$

$$(f-g)(n) - 2(f-g)(n-1) = 0$$

then, $f(n) - g(n)$ is a solution to $a_n - 2a_{n-1} = 0$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ $(not\ a_0 = 1)$ i.e.

$$f(n) - 2f(n-1) = 3n^2$$

$$g(n) - 2g(n-1) = 3n^2$$

$$(f-g)(n) - 2(f-g)(n-1) = 0$$

then, $f(n) - g(n)$ is a solution to $a_n - 2a_{n-1} = 0$

Solve, $f(n) - g(n)$ by the usual method,

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ (*not* $a_0 = 1$) i.e.

$$f(n) - 2f(n-1) = 3n^2$$

$$g(n) - 2g(n-1) = 3n^2$$

$$(f-g)(n) - 2(f-g)(n-1) = 0$$

then, $f(n) - g(n)$ is a solution to $a_n - 2a_{n-1} = 0$

Solve, $f(n)-g(n)$ by the usual method, satisfying $a_0 = 1$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ $(not\ a_0 = 1)$ i.e.

$$f(n) - 2f(n-1) = 3n^2$$

$$g(n) - 2g(n-1) = 3n^2$$

$$(f - g)(n) - 2(f - g)(n-1) = 0$$

then, $f(n) - g(n)$ is a solution to $a_n - 2a_{n-1} = 0$

Solve, $f(n) - g(n)$ by the usual method, satisfying $a_0 = 1$

$$f(n) - g(n) = 2^n$$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ $(not\ a_0 = 1)$ i.e.

$$f(n) - 2f(n-1) = 3n^2$$

$$g(n) - 2g(n-1) = 3n^2$$

$$(f-g)(n) - 2(f-g)(n-1) = 0$$

then, $f(n) - g(n)$ is a solution to $a_n - 2a_{n-1} = 0$

Solve, $f(n) - g(n)$ by the usual method, satisfying $a_0 = 1$

$$f(n) - g(n) = 2^n$$

Let $g(n)$ be a solution of only $a_n = 2a_{n-1} + 3n^2$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ (*not* $a_0 = 1$) i.e.

$$f(n) - 2f(n-1) = 3n^2$$

$$g(n) - 2g(n-1) = 3n^2$$

$$(f-g)(n) - 2(f-g)(n-1) = 0$$

then, $f(n) - g(n)$ is a solution to $a_n - 2a_{n-1} = 0$

Solve, $f(n) - g(n)$ by the usual method, satisfying $a_0 = 1$

$$f(n) - g(n) = 2^n$$

Let $g(n)$ be a solution of only $a_n = 2a_{n-1} + 3n^2$
E.g. $g(n) = A + Bn + Cn^2$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ $(not\ a_0 = 1)$ i.e.

$$f(n) - 2f(n-1) = 3n^2$$

$$g(n) - 2g(n-1) = 3n^2$$

$$(f-g)(n) - 2(f-g)(n-1) = 0$$

then, $f(n) - g(n)$ is a solution to $a_n - 2a_{n-1} = 0$

Solve, $f(n) - g(n)$ by the usual method, satisfying $a_0 = 1$

$$f(n) - g(n) = 2^n$$

Let $g(n)$ be a solution of only $a_n = 2a_{n-1} + 3n^2$
E.g. $g(n) = A + Bn + Cn^2$ find A, B, C

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ $(not\ a_0 = 1)$ i.e.

$$f(n) - 2f(n-1) = 3n^2$$

$$g(n) - 2g(n-1) = 3n^2$$

$$(f-g)(n) - 2(f-g)(n-1) = 0$$

then, $f(n) - g(n)$ is a solution to $a_n - 2a_{n-1} = 0$

Solve, $f(n) - g(n)$ by the usual method, satisfying $a_0 = 1$

$$f(n) - g(n) = 2^n$$

Let $g(n)$ be a solution of only $a_n = 2a_{n-1} + 3n^2$
E.g. $g(n) = A + Bn + Cn^2$ find A, B, C
$f(n) = (f-g)(n) + g(n)$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ $(not\ a_0 = 1)$ i.e.

$$f(n) - 2f(n-1) = 3n^2$$

$$g(n) - 2g(n-1) = 3n^2$$

$$(f-g)(n) - 2(f-g)(n-1) = 0$$

then, $f(n) - g(n)$ is a solution to $a_n - 2a_{n-1} = 0$

Solve, $f(n) - g(n)$ by the usual method, satisfying $a_0 = 1$

$$f(n) - g(n) = 2^n$$

Let $g(n)$ be a solution of only $a_n = 2a_{n-1} + 3n^2$
E.g. $g(n) = A + Bn + Cn^2$ find A, B, C
$f(n) = (f-g)(n) + g(n)$

Example.

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ $(not\ a_0 = 1)$ i.e.

$$f(n) - 2f(n-1) = 3n^2$$

$$g(n) - 2g(n-1) = 3n^2$$

$$(f-g)(n) - 2(f-g)(n-1) = 0$$

then, $f(n) - g(n)$ is a solution to $a_n - 2a_{n-1} = 0$

Solve, $f(n) - g(n)$ by the usual method, satisfying $a_0 = 1$

$$f(n) - g(n) = 2^n$$

Let $g(n)$ be a solution of only $a_n = 2a_{n-1} + 3n^2$
E.g. $g(n) = A + Bn + Cn^2$ find A, B, C
$f(n) = (f-g)(n) + g(n)$

Example. Solve the recurrence relation

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ ($not\ a_0 = 1$) i.e.

$$f(n) - 2f(n-1) = 3n^2$$

$$g(n) - 2g(n-1) = 3n^2$$

$$(f-g)(n) - 2(f-g)(n-1) = 0$$

then, $f(n) - g(n)$ is a solution to $a_n - 2a_{n-1} = 0$

Solve, $f(n) - g(n)$ by the usual method, satisfying $a_0 = 1$

$$f(n) - g(n) = 2^n$$

Let $g(n)$ be a solution of only $a_n = 2a_{n-1} + 3n^2$
E.g. $g(n) = A + Bn + Cn^2$ find A, B, C
$f(n) = (f-g)(n) + g(n)$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3^n$,

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ $(not\ a_0 = 1)$ i.e.

$$f(n) - 2f(n-1) = 3n^2$$

$$g(n) - 2g(n-1) = 3n^2$$

$$(f-g)(n) - 2(f-g)(n-1) = 0$$

then, $f(n) - g(n)$ is a solution to $a_n - 2a_{n-1} = 0$

Solve, $f(n)-g(n)$ by the usual method, satisfying $a_0 = 1$

$$f(n) - g(n) = 2^n$$

Let $g(n)$ be a solution of only $a_n = 2a_{n-1} + 3n^2$
E.g. $g(n) = A + Bn + Cn^2$ find A, B, C
$f(n) = (f-g)(n) + g(n)$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3^n$, $a_0 = 1$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ ($not\ a_0 = 1$) i.e.

$$f(n) - 2f(n-1) = 3n^2$$

$$g(n) - 2g(n-1) = 3n^2$$

$$(f-g)(n) - 2(f-g)(n-1) = 0$$

then, $f(n) - g(n)$ is a solution to $a_n - 2a_{n-1} = 0$

Solve, $f(n)-g(n)$ by the usual method, satisfying $a_0 = 1$

$$f(n) - g(n) = 2^n$$

Let $g(n)$ be a solution of only $a_n = 2a_{n-1} + 3n^2$
E.g. $g(n) = A + Bn + Cn^2$ find A, B, C
$f(n) = (f-g)(n) + g(n)$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3^n$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3^n$$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ $(not\ a_0 = 1)$ i.e.

$$f(n) - 2f(n-1) = 3n^2$$

$$g(n) - 2g(n-1) = 3n^2$$

$$(f-g)(n) - 2(f-g)(n-1) = 0$$

then, $f(n) - g(n)$ is a solution to $a_n - 2a_{n-1} = 0$

Solve, $f(n) - g(n)$ by the usual method, satisfying $a_0 = 1$

$$f(n) - g(n) = 2^n$$

Let $g(n)$ be a solution of only $a_n = 2a_{n-1} + 3n^2$
E.g. $g(n) = A + Bn + Cn^2$ find A, B, C
$f(n) = (f-g)(n) + g(n)$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3^n$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3^n$$

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ ($not\ a_0 = 1$) i.e.

$$f(n) - 2f(n-1) = 3n^2$$

$$g(n) - 2g(n-1) = 3n^2$$

$$(f-g)(n) - 2(f-g)(n-1) = 0$$

then, $f(n) - g(n)$ is a solution to $a_n - 2a_{n-1} = 0$

Solve, $f(n) - g(n)$ by the usual method, satisfying $a_0 = 1$

$$f(n) - g(n) = 2^n$$

Let $g(n)$ be a solution of only $a_n = 2a_{n-1} + 3n^2$
E.g. $g(n) = A + Bn + Cn^2$ find A, B, C
$f(n) = (f-g)(n) + g(n)$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3^n$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3^n$$

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$$

$$2xf(x) = 2a_0x + 2a_1x^2 + 2a_2x^2 + 2a_2x^4 + \cdots$$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ (*not* $a_0 = 1$) i.e.

$$f(n) - 2f(n-1) = 3n^2$$

$$g(n) - 2g(n-1) = 3n^2$$

$$(f-g)(n) - 2(f-g)(n-1) = 0$$

then, $f(n) - g(n)$ is a solution to $a_n - 2a_{n-1} = 0$

Solve, $f(n) - g(n)$ by the usual method, satisfying $a_0 = 1$

$$f(n) - g(n) = 2^n$$

Let $g(n)$ be a solution of only $a_n = 2a_{n-1} + 3n^2$
E.g. $g(n) = A + Bn + Cn^2$ find A, B, C
$f(n) = (f-g)(n) + g(n)$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3^n$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3^n$$

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$$

$$2xf(x) = 2a_0x + 2a_1x^2 + 2a_2x^2 + 2a_2x^4 + \cdots$$

$$f(x) - 2xf(x) = a_0 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots$$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ (*not* $a_0 = 1$) i.e.

$$f(n) - 2f(n-1) = 3n^2$$

$$g(n) - 2g(n-1) = 3n^2$$

$$(f-g)(n) - 2(f-g)(n-1) = 0$$

then, $f(n) - g(n)$ is a solution to $a_n - 2a_{n-1} = 0$

Solve, $f(n) - g(n)$ by the usual method, satisfying $a_0 = 1$

$$f(n) - g(n) = 2^n$$

Let $g(n)$ be a solution of only $a_n = 2a_{n-1} + 3n^2$
E.g. $g(n) = A + Bn + Cn^2$ find A, B, C
$f(n) = (f-g)(n) + g(n)$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3^n$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3^n$$

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$$

$$2xf(x) = 2a_0x + 2a_1x^2 + 2a_2x^2 + 2a_2x^4 + \cdots$$

$$f(x) - 2xf(x) = a_0 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots$$

$$f(x)(1-2x) = a_0 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots$$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ $(not\ a_0 = 1)$ i.e.

$$f(n) - 2f(n-1) = 3n^2$$

$$g(n) - 2g(n-1) = 3n^2$$

$$(f-g)(n) - 2(f-g)(n-1) = 0$$

then, $f(n) - g(n)$ is a solution to $a_n - 2a_{n-1} = 0$

Solve, $f(n) - g(n)$ by the usual method, satisfying $a_0 = 1$

$$f(n) - g(n) = 2^n$$

Let $g(n)$ be a solution of only $a_n = 2a_{n-1} + 3n^2$
E.g. $g(n) = A + Bn + Cn^2$ find A, B, C
$f(n) = (f-g)(n) + g(n)$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3^n$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3^n$$

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$$

$$2xf(x) = 2a_0x + 2a_1x^2 + 2a_2x^2 + 2a_2x^4 + \cdots$$

$$f(x) - 2xf(x) = a_0 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots$$

$$f(x)(1-2x) = 1 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots$$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ (*not* $a_0 = 1$) i.e.

$$f(n) - 2f(n-1) = 3n^2$$

$$g(n) - 2g(n-1) = 3n^2$$

$$(f-g)(n) - 2(f-g)(n-1) = 0$$

then, $f(n) - g(n)$ is a solution to $a_n - 2a_{n-1} = 0$

Solve, $f(n)-g(n)$ by the usual method, satisfying $a_0 = 1$

$$f(n) - g(n) = 2^n$$

Let $g(n)$ be a solution of only $a_n = 2a_{n-1} + 3n^2$
E.g. $g(n) = A + Bn + Cn^2$ find A, B, C
$f(n) = (f-g)(n) + g(n)$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3^n$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3^n$$

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$$

$$2xf(x) = 2a_0x + 2a_1x^2 + 2a_2x^2 + 2a_2x^4 + \cdots$$

$$f(x) - 2xf(x) = a_0 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots$$

$$f(x)(1-2x) = 1 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots$$
$$= \frac{1}{1-3x}$$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ (*not* $a_0 = 1$) i.e.

$$f(n) - 2f(n-1) = 3n^2$$

$$g(n) - 2g(n-1) = 3n^2$$

$$(f-g)(n) - 2(f-g)(n-1) = 0$$

then, $f(n) - g(n)$ is a solution to $a_n - 2a_{n-1} = 0$

Solve, $f(n) - g(n)$ by the usual method, satisfying $a_0 = 1$

$$f(n) - g(n) = 2^n$$

Let $g(n)$ be a solution of only $a_n = 2a_{n-1} + 3n^2$
E.g. $g(n) = A + Bn + Cn^2$ find A, B, C
$f(n) = (f-g)(n) + g(n)$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3^n$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3^n$$

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$$

$$2xf(x) = 2a_0x + 2a_1x^2 + 2a_2x^2 + 2a_2x^4 + \cdots$$

$$f(x) - 2xf(x) = a_0 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots$$

$$f(x)(1-2x) = 1 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots$$
$$= \frac{1}{1-3x}$$

$$f(x) = \frac{1}{(1-2x)(1-3x)}$$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ (*not* $a_0 = 1$) i.e.

$$f(n) - 2f(n-1) = 3n^2$$

$$g(n) - 2g(n-1) = 3n^2$$

$$(f-g)(n) - 2(f-g)(n-1) = 0$$

then, $f(n) - g(n)$ is a solution to $a_n - 2a_{n-1} = 0$

Solve, $f(n)-g(n)$ by the usual method, satisfying $a_0 = 1$

$$f(n) - g(n) = 2^n$$

Let $g(n)$ be a solution of only $a_n = 2a_{n-1} + 3n^2$
E.g. $g(n) = A + Bn + Cn^2$ find A, B, C
$f(n) = (f-g)(n) + g(n)$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3^n$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3^n$$

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$$

$$2xf(x) = 2a_0x + 2a_1x^2 + 2a_2x^2 + 2a_2x^4 + \cdots$$

$$f(x) - 2xf(x) = a_0 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots$$

$$f(x)(1-2x) = 1 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots$$
$$= \frac{1}{1-3x}$$

$$f(x) = \frac{1}{(1-2x)(1-3x)}$$
$$= \frac{A}{1-2x} + \frac{B}{1-3x}$$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3n^2$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3n^2$$

Observe:
If $f(n), g(n)$ are solutions of only
$a_n = 2a_{n-1} + 3n^2$ (*not* $a_0 = 1$) i.e.

$$f(n) - 2f(n-1) = 3n^2$$

$$g(n) - 2g(n-1) = 3n^2$$

$$(f-g)(n) - 2(f-g)(n-1) = 0$$

then, $f(n) - g(n)$ is a solution to $a_n - 2a_{n-1} = 0$

Solve, $f(n) - g(n)$ by the usual method, satisfying $a_0 = 1$

$$f(n) - g(n) = 2^n$$

Let $g(n)$ be a solution of only $a_n = 2a_{n-1} + 3n^2$
E.g. $g(n) = A + Bn + Cn^2$ find A, B, C
$f(n) = (f-g)(n) + g(n)$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3^n$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3^n$$

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$$

$$2xf(x) = 2a_0x + 2a_1x^2 + 2a_2x^2 + 2a_2x^4 + \cdots$$

$$f(x) - 2xf(x) = a_0 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots$$

$$\begin{aligned} f(x)(1-2x) &= 1 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots \\ &= \frac{1}{1-3x} \end{aligned}$$

$$\begin{aligned} f(x) &= \frac{1}{(1-2x)(1-3x)} \\ &= \frac{A}{1-2x} + \frac{B}{1-3x} \\ &= A\sum(2x)^n + B\sum(3x)^n \end{aligned}$$

General term: $A2^n + B3^n$

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3^n$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3^n$$

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$$

$$2xf(x) = 2a_0x + 2a_1x^2 + 2a_2x^2 + 2a_2x^4 + \cdots$$

$$f(x) - 2xf(x) = a_0 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots$$

$$\begin{aligned} f(x)(1-2x) &= 1 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots \\ &= \frac{1}{1-3x} \end{aligned}$$

$$\begin{aligned} f(x) &= \frac{1}{(1-2x)(1-3x)} \\ &= \frac{A}{1-2x} + \frac{B}{1-3x} \\ &= A\sum(2x)^n + B\sum(3x)^n \end{aligned}$$

General term: $A2^n + B3^n$

$$1 = a_0 = A + B$$

Example. Solve the recurrence relation $a_n = 2a_{n-1} + 3^n$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3^n$$

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$$
$$2xf(x) = 2a_0x + 2a_1x^2 + 2a_2x^2 + 2a_2x^4 + \cdots$$
$$f(x) - 2xf(x) = a_0 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots$$

$$\begin{aligned} f(x)(1-2x) &= 1 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots \\ &= \frac{1}{1-3x} \end{aligned}$$

$$\begin{aligned} f(x) &= \frac{1}{(1-2x)(1-3x)} \\ &= \frac{A}{1-2x} + \frac{B}{1-3x} \\ &= A\sum(2x)^n + B\sum(3x)^n \end{aligned}$$

General term: $A2^n + B3^n$

$$1 = a_0 = A + B$$

$$5 = a_1 = 2A + 3B$$

Example. Solve the recurrence relation $a_n = 2a_{n-1} + 3^n$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3^n$$

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$$
$$2xf(x) = 2a_0x + 2a_1x^2 + 2a_2x^2 + 2a_2x^4 + \cdots$$
$$f(x) - 2xf(x) = a_0 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots$$

$$\begin{aligned} f(x)(1-2x) &= 1 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots \\ &= \frac{1}{1-3x} \end{aligned}$$

$$\begin{aligned} f(x) &= \frac{1}{(1-2x)(1-3x)} \\ &= \frac{A}{1-2x} + \frac{B}{1-3x} \\ &= A\sum(2x)^n + B\sum(3x)^n \end{aligned}$$

General term: $A2^n + B3^n$

$$1 = a_0 = A + B$$

$$5 = a_1 = 2A + 3B$$

$$B = 3,$$

Example. Solve the recurrence relation $a_n = 2a_{n-1} + 3^n$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3^n$$

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$
$$2xf(x) = 2a_0 x + 2a_1 x^2 + 2a_2 x^2 + 2a_2 x^4 + \cdots$$
$$f(x) - 2xf(x) = a_0 + 3x + 3^2 x^2 + 3^3 x^3 + 3^4 x^4 + \cdots$$

$$\begin{aligned} f(x)(1-2x) &= 1 + 3x + 3^2 x^2 + 3^3 x^3 + 3^4 x^4 + \cdots \\ &= \frac{1}{1-3x} \end{aligned}$$

$$\begin{aligned} f(x) &= \frac{1}{(1-2x)(1-3x)} \\ &= \frac{A}{1-2x} + \frac{B}{1-3x} \\ &= A\sum(2x)^n + B\sum(3x)^n \end{aligned}$$

General term: $A2^n + B3^n$

$$1 = a_0 = A + B$$

$$5 = a_1 = 2A + 3B$$

$$B = 3, A = -2$$

Example. Solve the recurrence relation $a_n = 2a_{n-1} + 3^n$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3^n$$

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

$$2xf(x) = 2a_0 x + 2a_1 x^2 + 2a_2 x^2 + 2a_2 x^4 + \cdots$$

$$f(x) - 2xf(x) = a_0 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots$$

$$\begin{aligned} f(x)(1-2x) &= 1 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots \\ &= \frac{1}{1-3x} \end{aligned}$$

$$\begin{aligned} f(x) &= \frac{1}{(1-2x)(1-3x)} \\ &= \frac{A}{1-2x} + \frac{B}{1-3x} \\ &= A\sum(2x)^n + B\sum(3x)^n \end{aligned}$$

General term: $A2^n + B3^n$

$$1 = a_0 = A + B$$

$$5 = a_1 = 2A + 3B$$

$$B = 3, A = -2$$

Answer: $3^{n+1} - 2^{n+1}$

Example. Solve the recurrence relation $a_n = 2a_{n-1} + 3^n$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3^n$$

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

$$2xf(x) = 2a_0 x + 2a_1 x^2 + 2a_2 x^2 + 2a_2 x^4 + \cdots$$

$$f(x) - 2xf(x) = a_0 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots$$

$$\begin{aligned} f(x)(1-2x) &= 1 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots \\ &= \frac{1}{1-3x} \end{aligned}$$

$$\begin{aligned} f(x) &= \frac{1}{(1-2x)(1-3x)} \\ &= \frac{A}{1-2x} + \frac{B}{1-3x} \\ &= A\sum(2x)^n + B\sum(3x)^n \end{aligned}$$

General term: $A2^n + B3^n$

$$1 = a_0 = A + B$$

$$5 = a_1 = 2A + 3B$$

$$B = 3, A = -2$$

Answer: $3^{n+1} - 2^{n+1}$

Example.

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3^n$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3^n$$

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$$
$$2xf(x) = 2a_0x + 2a_1x^2 + 2a_2x^2 + 2a_2x^4 + \cdots$$
$$f(x) - 2xf(x) = a_0 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots$$

$$\begin{aligned} f(x)(1-2x) &= 1 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots \\ &= \frac{1}{1-3x} \end{aligned}$$

$$\begin{aligned} f(x) &= \frac{1}{(1-2x)(1-3x)} \\ &= \frac{A}{1-2x} + \frac{B}{1-3x} \\ &= A\sum(2x)^n + B\sum(3x)^n \end{aligned}$$

General term: $A2^n + B3^n$

$$1 = a_0 = A + B$$

$$5 = a_1 = 2A + 3B$$

$$B = 3, A = -2$$

Answer: $3^{n+1} - 2^{n+1}$

Example.

$$a_n + 2a_{n-1} + 3b_{n-1} = 0$$

Example. Solve the recurrence relation $a_n = 2a_{n-1} + 3^n$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3^n$$

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$$

$$2xf(x) = 2a_0x + 2a_1x^2 + 2a_2x^2 + 2a_2x^4 + \cdots$$

$$f(x) - 2xf(x) = a_0 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots$$

$$\begin{aligned} f(x)(1-2x) &= 1 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots \\ &= \frac{1}{1-3x} \end{aligned}$$

$$\begin{aligned} f(x) &= \frac{1}{(1-2x)(1-3x)} \\ &= \frac{A}{1-2x} + \frac{B}{1-3x} \\ &= A\sum(2x)^n + B\sum(3x)^n \end{aligned}$$

General term: $A2^n + B3^n$

$$1 = a_0 = A + B$$

$$5 = a_1 = 2A + 3B$$

$$B = 3, A = -2$$

Answer: $3^{n+1} - 2^{n+1}$

Example.

$$a_n + 2a_{n-1} + 3b_{n-1} = 0$$

$$b_n + 2a_{n-1} = 0$$

Example. Solve the recurrence relation $a_n = 2a_{n-1} + 3^n$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3^n$$

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$$

$$2xf(x) = 2a_0x + 2a_1x^2 + 2a_2x^2 + 2a_2x^4 + \cdots$$

$$f(x) - 2xf(x) = a_0 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots$$

$$\begin{aligned} f(x)(1-2x) &= 1 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots \\ &= \frac{1}{1-3x} \end{aligned}$$

$$\begin{aligned} f(x) &= \frac{1}{(1-2x)(1-3x)} \\ &= \frac{A}{1-2x} + \frac{B}{1-3x} \\ &= A\sum(2x)^n + B\sum(3x)^n \end{aligned}$$

General term: $A2^n + B3^n$

$$1 = a_0 = A + B$$

$$5 = a_1 = 2A + 3B$$

$$B = 3, A = -2$$

Answer: $3^{n+1} - 2^{n+1}$

Example.

$$a_n + 2a_{n-1} + 3b_{n-1} = 0$$

$$b_n + 2a_{n-1} = 0$$

Subsitute

Example. Solve the recurrence relation $a_n = 2a_{n-1} + 3^n$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3^n$$

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$$

$$2xf(x) = 2a_0x + 2a_1x^2 + 2a_2x^2 + 2a_2x^4 + \cdots$$

$$f(x) - 2xf(x) = a_0 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots$$

$$\begin{aligned} f(x)(1-2x) &= 1 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots \\ &= \frac{1}{1-3x} \end{aligned}$$

$$\begin{aligned} f(x) &= \frac{1}{(1-2x)(1-3x)} \\ &= \frac{A}{1-2x} + \frac{B}{1-3x} \\ &= A\sum(2x)^n + B\sum(3x)^n \end{aligned}$$

General term: $A2^n + B3^n$

$$1 = a_0 = A + B$$

$$5 = a_1 = 2A + 3B$$

$$B = 3, A = -2$$

Answer: $3^{n+1} - 2^{n+1}$

Example.

$$a_n + 2a_{n-1} + 3b_{n-1} = 0$$

$$b_n + 2a_{n-1} = 0$$

Subsitute $b_{n-1} = -2a_{n-2}$ in,

Example. Solve the recurrence relation
$a_n = 2a_{n-1} + 3^n$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3^n$$

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$$

$$2xf(x) = 2a_0x + 2a_1x^2 + 2a_2x^2 + 2a_2x^4 + \cdots$$

$$f(x) - 2xf(x) = a_0 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots$$

$$\begin{aligned} f(x)(1-2x) &= 1 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots \\ &= \frac{1}{1-3x} \end{aligned}$$

$$\begin{aligned} f(x) &= \frac{1}{(1-2x)(1-3x)} \\ &= \frac{A}{1-2x} + \frac{B}{1-3x} \\ &= A\sum(2x)^n + B\sum(3x)^n \end{aligned}$$

General term: $A2^n + B3^n$

$$1 = a_0 = A + B$$

$$5 = a_1 = 2A + 3B$$

$$B = 3, A = -2$$

Answer: $3^{n+1} - 2^{n+1}$

Example.

$$a_n + 2a_{n-1} + 3b_{n-1} = 0$$

$$b_n + 2a_{n-1} = 0$$

Subsitute $b_{n-1} = -2a_{n-2}$ in,

$$a_n + 2a_{n-1} + 3b_{n-1} = 0$$

to get,

Example. Solve the recurrence relation $a_n = 2a_{n-1} + 3^n$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3^n$$

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$$

$$2xf(x) = 2a_0x + 2a_1x^2 + 2a_2x^2 + 2a_2x^4 + \cdots$$

$$f(x) - 2xf(x) = a_0 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots$$

$$f(x)(1-2x) = 1 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots$$
$$= \frac{1}{1-3x}$$

$$f(x) = \frac{1}{(1-2x)(1-3x)}$$
$$= \frac{A}{1-2x} + \frac{B}{1-3x}$$
$$= A\sum(2x)^n + B\sum(3x)^n$$

General term: $A2^n + B3^n$

$$1 = a_0 = A + B$$

$$5 = a_1 = 2A + 3B$$

$$B = 3, A = -2$$

Answer: $3^{n+1} - 2^{n+1}$

Example.

$$a_n + 2a_{n-1} + 3b_{n-1} = 0$$

$$b_n + 2a_{n-1} = 0$$

Subsitute $b_{n-1} = -2a_{n-2}$ in,

$$a_n + 2a_{n-1} + 3b_{n-1} = 0$$

to get,

$$a_n + 2a_{n-1} - 6a_{n-2} = 0$$

Example. Solve the recurrence relation $a_n = 2a_{n-1} + 3^n$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3^n$$

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$$

$$2xf(x) = 2a_0x + 2a_1x^2 + 2a_2x^2 + 2a_2x^4 + \cdots$$

$$f(x) - 2xf(x) = a_0 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots$$

$$\begin{aligned} f(x)(1-2x) &= 1 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots \\ &= \frac{1}{1-3x} \end{aligned}$$

$$\begin{aligned} f(x) &= \frac{1}{(1-2x)(1-3x)} \\ &= \frac{A}{1-2x} + \frac{B}{1-3x} \\ &= A\sum(2x)^n + B\sum(3x)^n \end{aligned}$$

General term: $A2^n + B3^n$

$$1 = a_0 = A + B$$

$$5 = a_1 = 2A + 3B$$

$$B = 3, A = -2$$

Answer: $3^{n+1} - 2^{n+1}$

Example.

$$a_n + 2a_{n-1} + 3b_{n-1} = 0$$

$$b_n + 2a_{n-1} = 0$$

Subsitute $b_{n-1} = -2a_{n-2}$ in,

$$a_n + 2a_{n-1} + 3b_{n-1} = 0$$

to get,

$$a_n + 2a_{n-1} - 6a_{n-2} = 0$$

$\ldots$

Example. Solve the recurrence relation $a_n = 2a_{n-1} + 3^n$, $a_0 = 1$

$$a_n - 2a_{n-1} = 3^n$$

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$$

$$2xf(x) = 2a_0x + 2a_1x^2 + 2a_2x^2 + 2a_2x^4 + \cdots$$

$$f(x) - 2xf(x) = a_0 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots$$

$$\begin{aligned} f(x)(1-2x) &= 1 + 3x + 3^2x^2 + 3^3x^3 + 3^4x^4 + \cdots \\ &= \frac{1}{1-3x} \end{aligned}$$

$$\begin{aligned} f(x) &= \frac{1}{(1-2x)(1-3x)} \\ &= \frac{A}{1-2x} + \frac{B}{1-3x} \\ &= A\sum(2x)^n + B\sum(3x)^n \end{aligned}$$

General term: $A2^n + B3^n$

$$1 = a_0 = A + B$$

$$5 = a_1 = 2A + 3B$$

$$B = 3, A = -2$$

Answer: $3^{n+1} - 2^{n+1}$

Example.

$$a_n + 2a_{n-1} + 3b_{n-1} = 0$$

$$b_n + 2a_{n-1} = 0$$

Subsitute $b_{n-1} = -2a_{n-2}$ in,

$$a_n + 2a_{n-1} + 3b_{n-1} = 0$$

to get,

$$a_n + 2a_{n-1} - 6a_{n-2} = 0$$

$\ldots$

Exercise. Find a solution for the system of recurrence relations,

General term: $A2^n + B3^n$

$$1 = a_0 = A + B$$

$$5 = a_1 = 2A + 3B$$

$$B = 3, A = -2$$

Answer: $3^{n+1} - 2^{n+1}$

Example.

$$a_n + 2a_{n-1} + 3b_{n-1} = 0$$

$$b_n + 2a_{n-1} = 0$$

Subsitute $b_{n-1} = -2a_{n-2}$ in,

$$a_n + 2a_{n-1} + 3b_{n-1} = 0$$

to get,

$$a_n + 2a_{n-1} - 6a_{n-2} = 0$$

$\ldots$

Exercise. Find a solution for the system of recurrence relations,

$$a_n + 2a_{n-1} + 3b_{n-1} = 0$$

General term: $A2^n + B3^n$

$$1 = a_0 = A + B$$

$$5 = a_1 = 2A + 3B$$

$$B = 3, A = -2$$

Answer: $3^{n+1} - 2^{n+1}$

Example.

$$a_n + 2a_{n-1} + 3b_{n-1} = 0$$

$$b_n + 2a_{n-1} = 0$$

Subsitute $b_{n-1} = -2a_{n-2}$ in,

$$a_n + 2a_{n-1} + 3b_{n-1} = 0$$

to get,

$$a_n + 2a_{n-1} - 6a_{n-2} = 0$$

$\ldots$

Exercise. Find a solution for the system of recurrence relations,

$$a_n + 2a_{n-1} + 3b_{n-1} = 0$$

$$b_n + a_{n-1} + b_{n-1} = 0$$

Solution.

General term: $A2^n + B3^n$

$$1 = a_0 = A + B$$

$$5 = a_1 = 2A + 3B$$

$$B = 3, A = -2$$

Answer: $3^{n+1} - 2^{n+1}$

Example.

$$a_n + 2a_{n-1} + 3b_{n-1} = 0$$

$$b_n + 2a_{n-1} = 0$$

Subsitute $b_{n-1} = -2a_{n-2}$ in,

$$a_n + 2a_{n-1} + 3b_{n-1} = 0$$

to get,

$$a_n + 2a_{n-1} - 6a_{n-2} = 0$$

...

Exercise. Find a solution for the system of recurrence relations,

$$a_n + 2a_{n-1} + 3b_{n-1} = 0$$

$$b_n + a_{n-1} + b_{n-1} = 0$$

Solution. Use generating functions for $f(x)$ and $g(x)$ for a_n and b_n respectively ...