Graph theory

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Definition.

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Definition (Graph)**.** A graph “on a set V”

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Definition. Given a graph G, a vertex $v \in V(G)$ is incident to an edge $e \in E(G)$ if $v \in e$

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The number of vertices with odd degree is even

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Deleting v_0 with $d_G(v_0) \leq \epsilon(G)$, to obtain G_1,
Observe $\epsilon(G_1) \geq \epsilon(G)$

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