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Theorem. $\Delta(G) = d$

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Theorem. $\Delta(G) = d, \text{rad } G = k \implies |G| \leq k^2$

$$m_c := \max \{d_G(v, c) \mid c \in V(G)\}$$
$$\min \{m_c \mid c \in V(G)\}$$

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Theorem. $\Delta(G) = d, \text{rad } G = k \implies |G| \leq \binom{d}{k} + \binom{d}{k-1}$

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Theorem. $\Delta(G) = d, \text{rad } G = k \implies |G| \leq dk + 1$

Proof. □

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Proof. $c \in V(G)$ central



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Theorem. $\Delta(G) = d, \text{rad } G = k \implies |G| \leq d \cdot k$

Proof. $c \in V(G)$ central
 $D_i := \{v \in V(G) \mid d_G(v, c) = i\}$

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$$D_i := \{v \in V(G) \mid d_G(v, c) = i\}$$

$$|D_0| = 1 \qquad \square$$

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$$|D_1| \leq \sum_{v \in D_0} |N(v)| \leq \sum_{v \in D_0} d \leq d \cdot 1 = d$$

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$$|D_i| \leq (d-1)^i |D_0| \leq \sum_{i=0}^d \binom{d}{i} (d-1)^i = \sum_{i=0}^d \binom{d}{i} (k-1)^i$$

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$$\sqcup D_i$$



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$$V(G) = \bigsqcup_i D_i$$

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$$|D_i| \leq (d-1)^i |D_1| \leq d(d-1)^{i-1}$$

$$V(G) = \sqcup D_i$$

$$|G| \leq \sum_{i=0}^d |D_i| \leq d \sum_{i=0}^{d-1} (d-1)^i = d \frac{(d-1)^d - 1}{(d-1) - 1} = \frac{d((d-1)^d - 1)}{d-2}$$

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$$|D_i| \leq (d-1)^i |D_1| \leq d(d-1)^{i-1}$$

$$V(G) = \bigsqcup D_i$$

$$|G| \leq 1 + d + d(d-1) + \dots + d(d-1)^{k-1} = 1 + d + d((d-1)^k - 1) = d(d-1)^k + 1 \leq (d+1)^k$$

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$$V(G) = \cup D_i$$

$$|G| \leq 1 + d + d(d - 1) + \dots + d(d - 1)^{d-1} \leq \frac{d+1}{2} \cdot \frac{d+1}{2} \quad \square$$

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Theorem. $\Delta(G) = d, \text{rad } G = k \implies |G| < ??$

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$$V(G) = \sqcup D_i$$

$$|G| \leq 1 + d + d(d - 1) + d(d - 1)^2 + \cdots + d(d - 1)^{k-1} \quad \square$$

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$$V(G) = \bigsqcup D_i$$

$$\begin{aligned} |G| &\leq 1 + d + d(d-1) + d(d-1)^2 + \dots + d(d-1)^{k-1} \\ &= 1 + \frac{d}{d-1}((d-1)^k - 1) \end{aligned}$$

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Theorem. $\Delta(G) = d, \text{rad } G = k \implies |G| \leq ?$

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$$V(G) = \sqcup D_i$$

$$|G| \leq 1 + d + d(d-1) + d(d-1)^2 + \dots + d(d-1)^{k-1}$$

$$= 1 + \frac{d}{d-1}((d-1)^k - 1)$$

$$= \frac{d}{d-1}(d-1)^k - \frac{d}{d-1}$$

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$$|G| \leq 1 + d + d(d-1) + d(d-1)^2 + \dots + d(d-1)^{k-1}$$

$$= 1 + \frac{d}{d-2}((d-1)^k - 1)$$

$$= \frac{d}{d-2}(d-1)^k - \frac{2}{d-2}$$

$$< \frac{d}{d-2}(d-1)^k$$

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Theorem. $\Delta(G) = d \geq 3, \text{rad } G = k \implies |G| < \frac{d}{d-2}(d-1)^k$

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Definition. A (non-empty) graph, G , is connected if any two of its vertices are linked by a path.

$U \subset V(G)$ is connected if $G[U]$ is connected.

Proposition. G connected, then we can choose an ordering v_1, v_2, \dots, v_n

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Proof. $G[v_1]$ is trivially connected.

Assume that $G_k := G[v_1, \dots, v_k]$ is connected for $k \leq m$.

Pick $v \in G \setminus G_k$

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Let v_{k+1} be the first vertex on this path outside G_k □

Definition (component).

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Definition. $A, B \subset V(G)$. $X \subset V(G) \cup E(G)$ separates A and B in G if every $A - B$ path contains an edge / vertex in X

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$$\{x, y\} \in E(G_1) \iff \{\phi(x), \phi(y)\} \in E(G_2)$$

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