

Exercise sheet 1

Curves and Surfaces, MTH201

1. In how many ways can 7 students and 5 teachers be seated around a round table of no two teachers are adjacent?
2. Derive a formula for the number of surjective maps from a set X of cardinality n to a set Y of cardinality m in terms of $S(n, m)$.
3. Let $s(n, m)$ denote the number of ways to arrange $\{1, 2, \dots, n\}$ around m distinct circles so that each circle has at least one number. Note the difference with $S(n, m)$ that was done during a lecture ($s(n, m)$ are called Stirling numbers of the first kind, and $S(n, m)$ are called Stirling numbers of the second kind).
 - (a) Prove that $s(n, m) = s(n-1, m-1) + (n-1)s(n-1, m)$.
 - (b) Compute, $s(n, 0)$ ($n \geq 1$), $s(n, n)$ ($n \geq 0$), $s(n, 1)$ ($n \geq 2$), and $s(n, n-1)$ ($n \geq 2$).
 - (c) Compute $s(3, 2)$, $s(4, 2)$, and $s(4, 3)$.