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$$f : A \times B \to X$$

$$f((x_1, x_2, x_3), (y_1, y_2)) = x_1x_2x_3y_1y_2$$

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Size of each equivalence class: $3! \times 2!$

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Size of each equivalence class: $3! \times 2!$

Final answer: $\frac{5!}{3! \times 2!}$

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Example. Count the number of ways to select m balls from an infinite supply of red, green, and blue ones.

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Solution. If $m = 5$, examples of selections:
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Final answer: $\frac{5!}{3!\times 2!}$

Example. Count the number of ways to select m balls from an infinite supply of red, green, and blue ones.

Solution. If $m = 5$, examples of selections:
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Alternatively,

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number of ways to permute $A_1A_2A_3B_1B_2$. *Answer:* $5!$

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Final answer: $\frac{5!}{3! \times 2!}$

Example. Count the number of ways to select m balls from an infinite supply of red, green, and blue ones.

Solution. If $m = 5$, examples of selections:
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Alternatively,
$XXX|X|X$,

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Size of each equivalence class: $3! \times 2!$

Final answer: $\frac{5!}{3! \times 2!}$

Example. Count the number of ways to select m balls from an infinite supply of red, green, and blue ones.

Solution. If $m = 5$, examples of selections:
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Alternatively,
$XXX|X|X$, $XX|XX|X$, $X|XXXX|$

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Size of each equivalence class: $3! \times 2!$

Final answer: $\frac{5!}{3!\times 2!}$

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Size of each equivalence class: $3! \times 2!$

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Example. Count the number of ways to select m balls from an infinite supply of red, green, and blue ones.

Solution. If $m = 5$, examples of selections: $RRRGB$, $RRGGB$, $RGGGG$

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Final answer: $\binom{m+2}{m}$

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Example. Count the number of ways to select m integers out of $\{1, \ldots, n\}$ so that they contain no consecutive integers.

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Example. Count the number of ways to select m integers out of $\{1, \ldots, n\}$ so that they contain no consecutive integers.

Solution. Consider such a set, $A = \{x_1, \ldots, x_m\}$. Main idea:
Order the elements

$$A := \{x_1, x_2, x_3, \ldots, x_m\}$$

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Solution. If $m = 5$, examples of selections:
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Alternatively,
$XXX|X|X$, $XX|XX|X$, $X|XXXX|$

Final answer: $\binom{m+2}{m}$

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