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Definition (A path)**.** A graph $P = (V(P), E(P))$ on a set of vertices $V(P) = \{x_0, x_1, \ldots, x_n\}$ is called a path of length n linking the "ends", x_0 and x_n, if its set of edges is of the form, $E(P) = \{x_0x_1, x_1x_2, \ldots, x_{n-1}x_n\}$.

Notation:
$P := x_0x_1x_2 \ldots x_n$ denotes a path linking x_0 with x_n.
$Q := Px_{n+1} = x_0x_1x_2 \ldots x_nx_{n+1}$

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Proof. Main observation: If a path P ends at x_n, but does not contain a "neighbour", x_{n+1} of x_n, it can be made longer by including x_{n+1} to give Px_{n+1}

A path P with the largest length ending at x_n must contain all of x_ns neighbours. P's length $\geq d(x_n) \geq \delta(G)$ □

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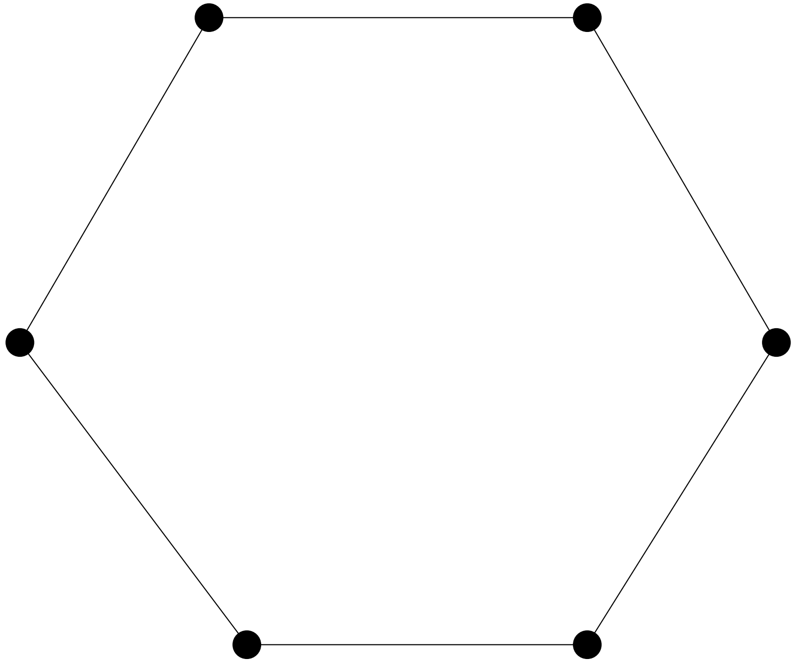

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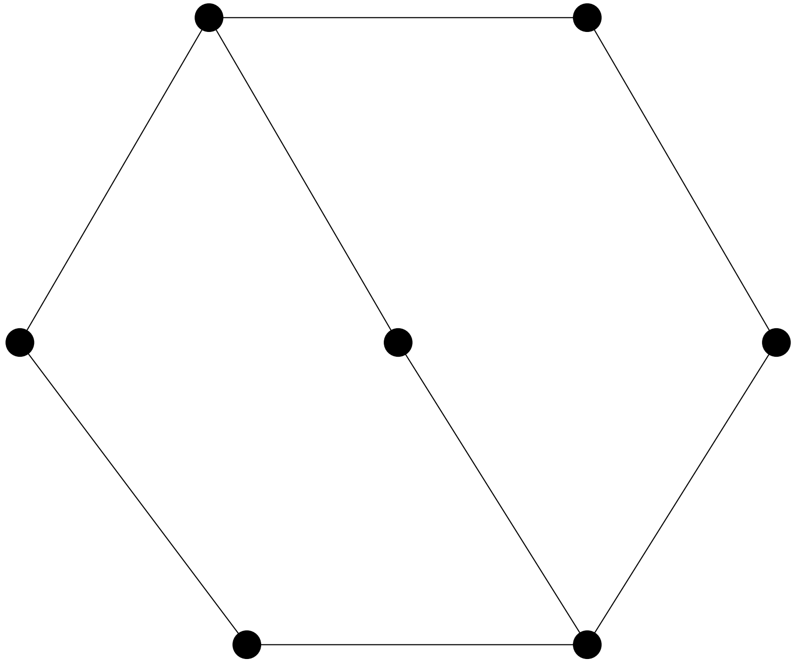

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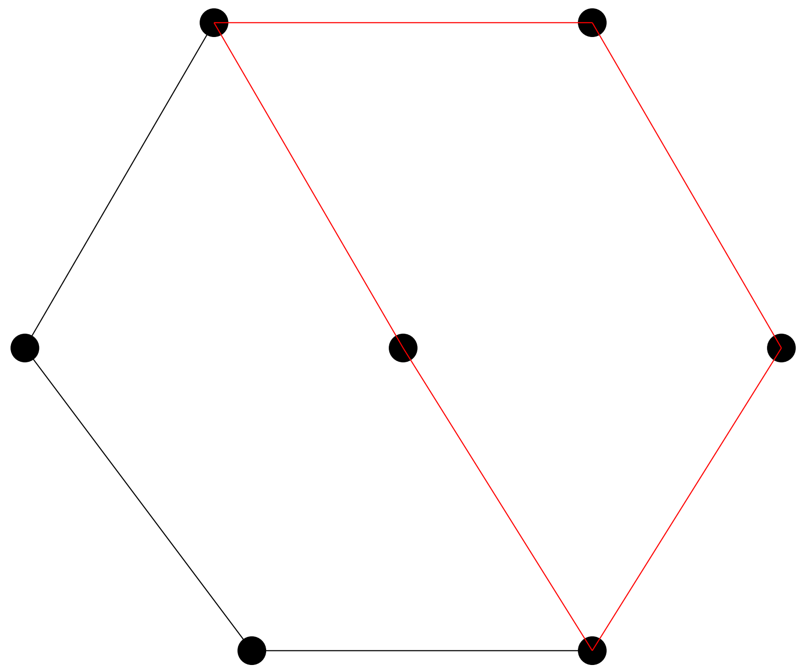

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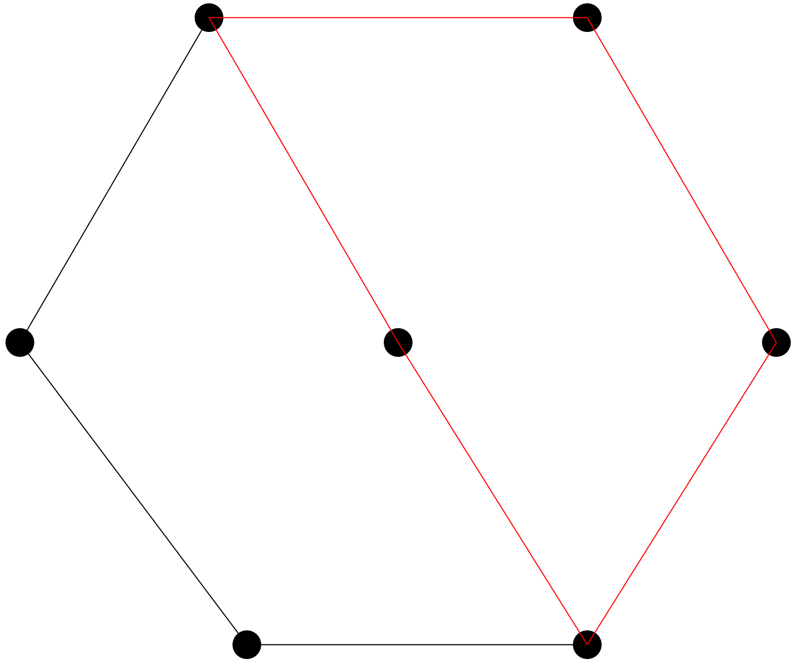

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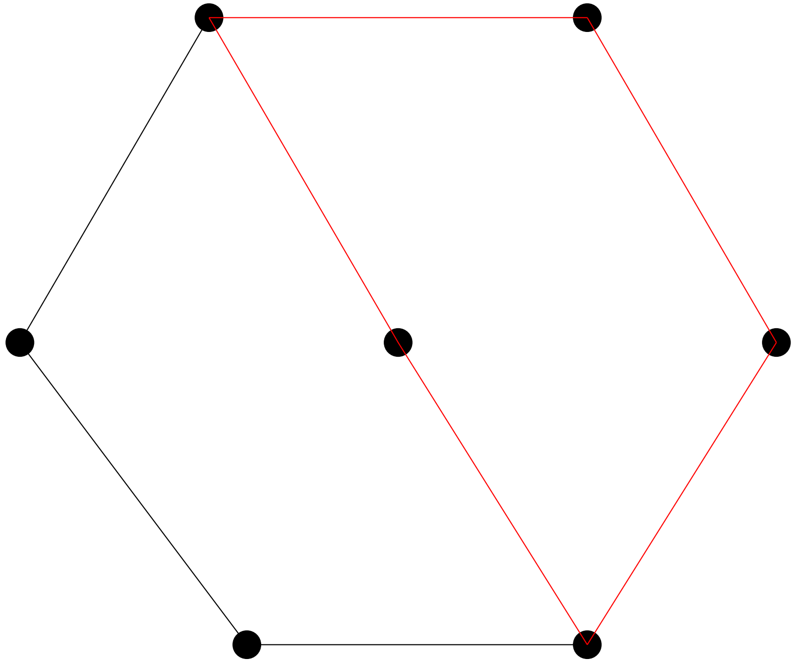

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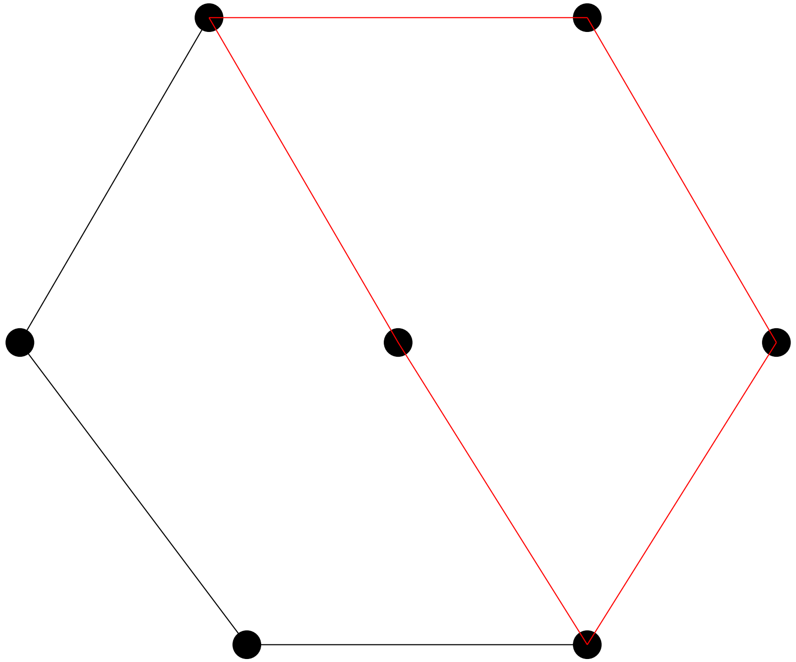

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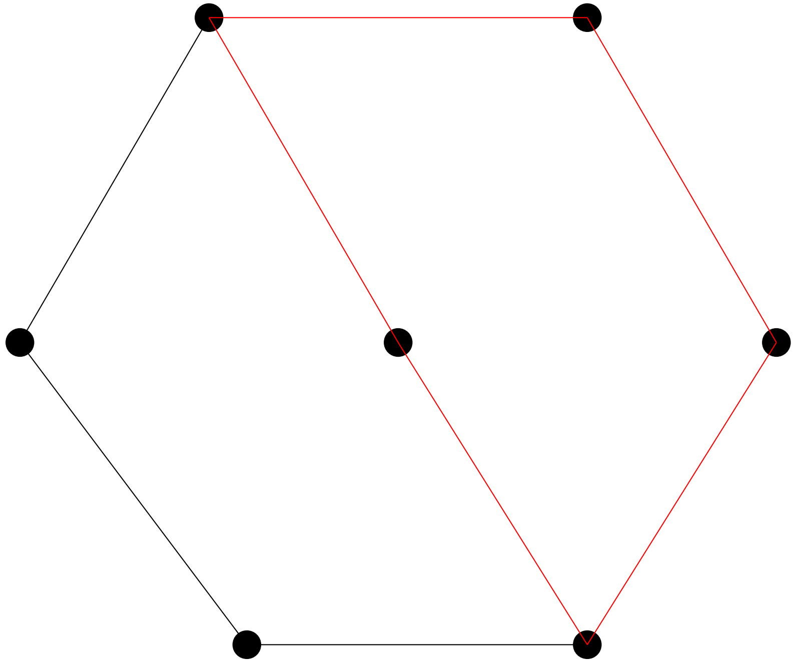

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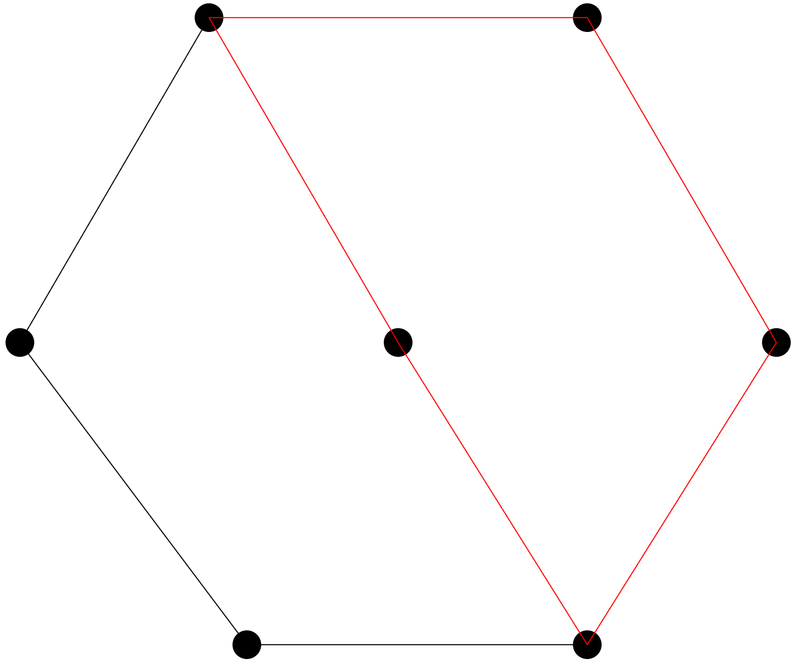

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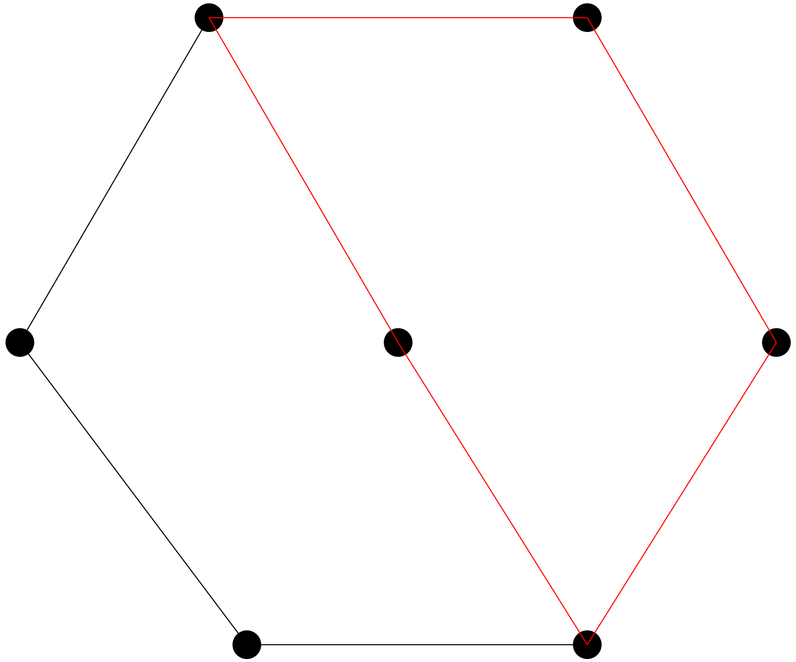
Definition. A (non-empty) graph, G, is connected



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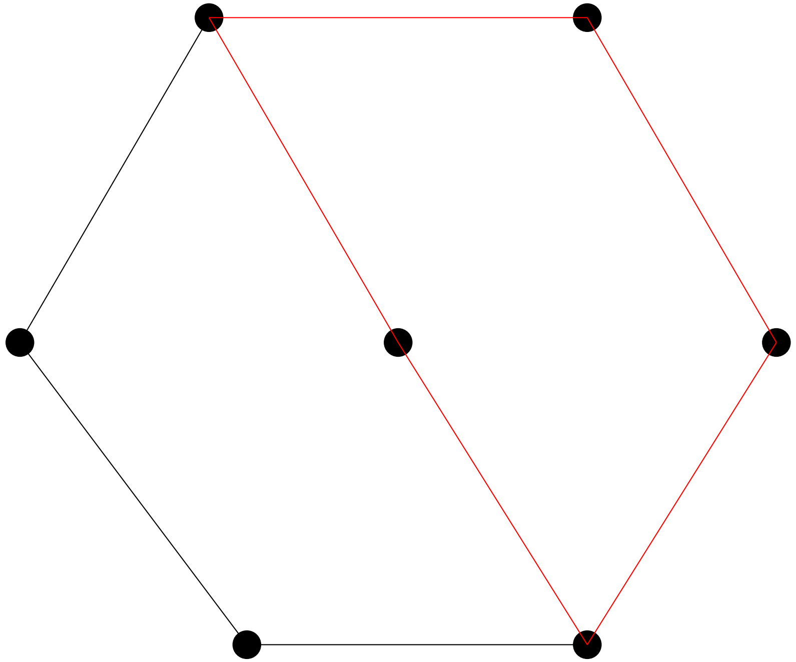

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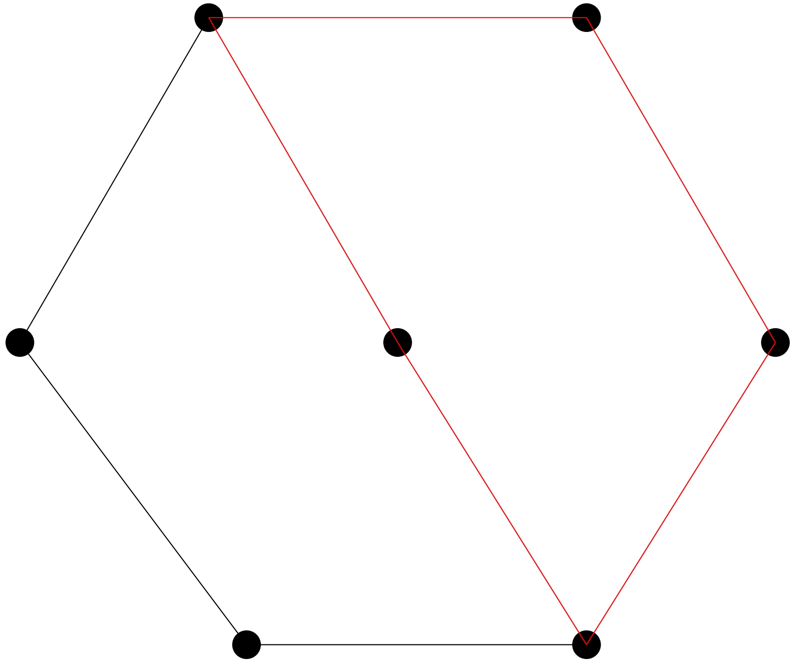

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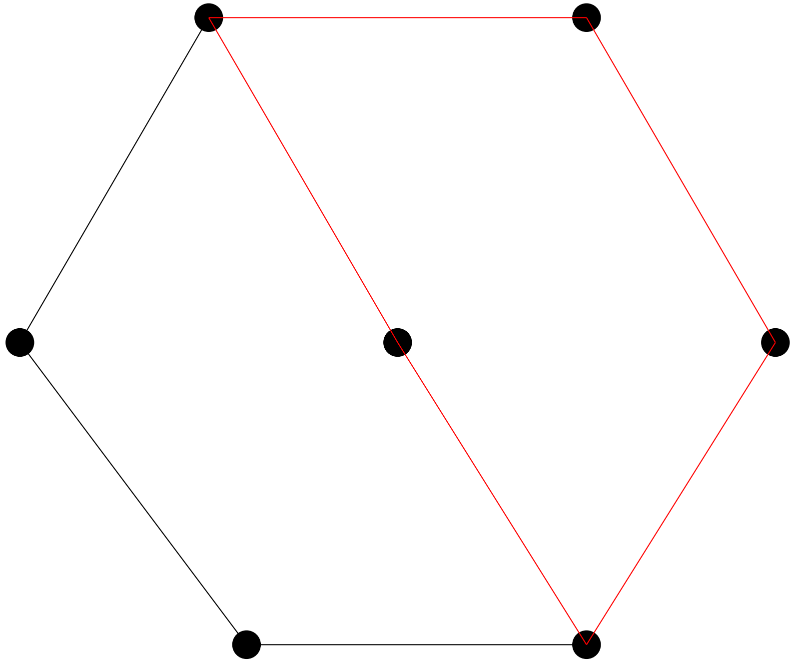

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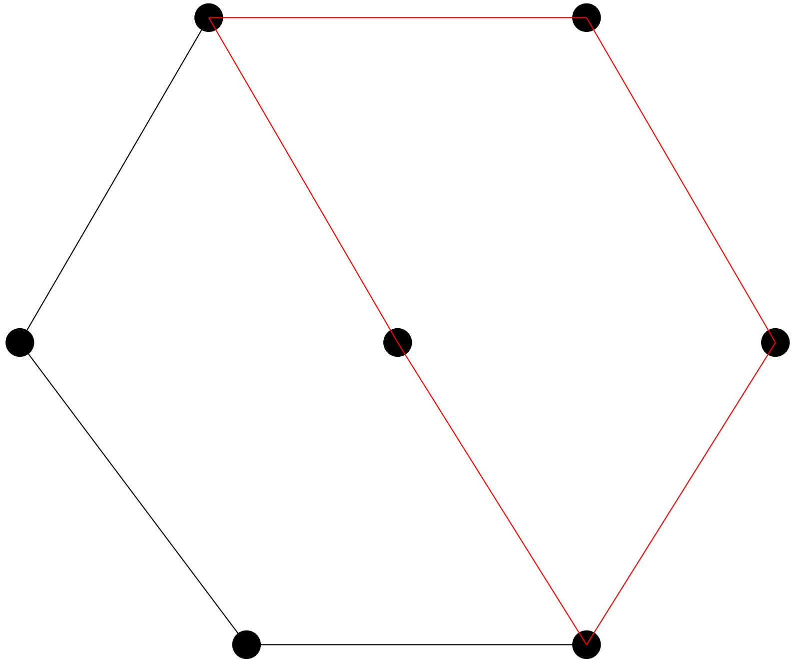

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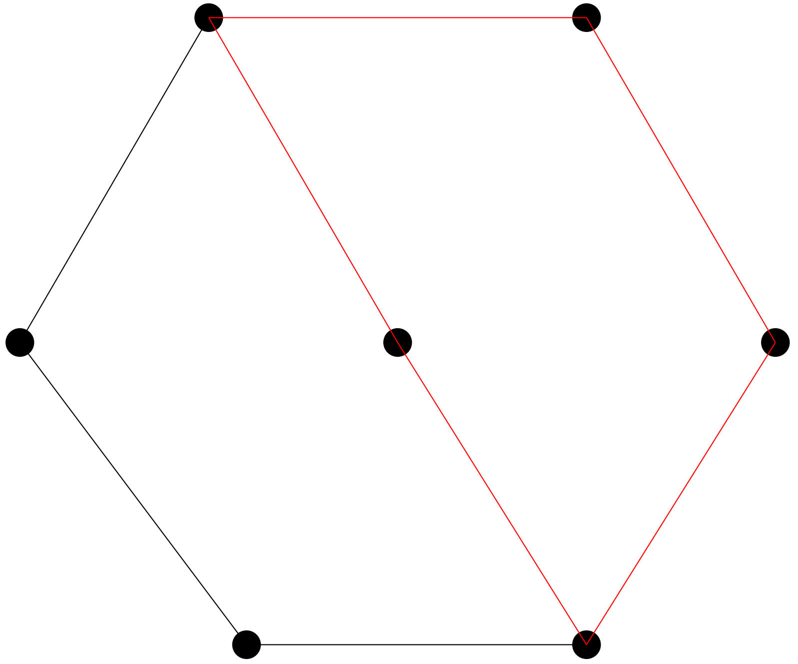

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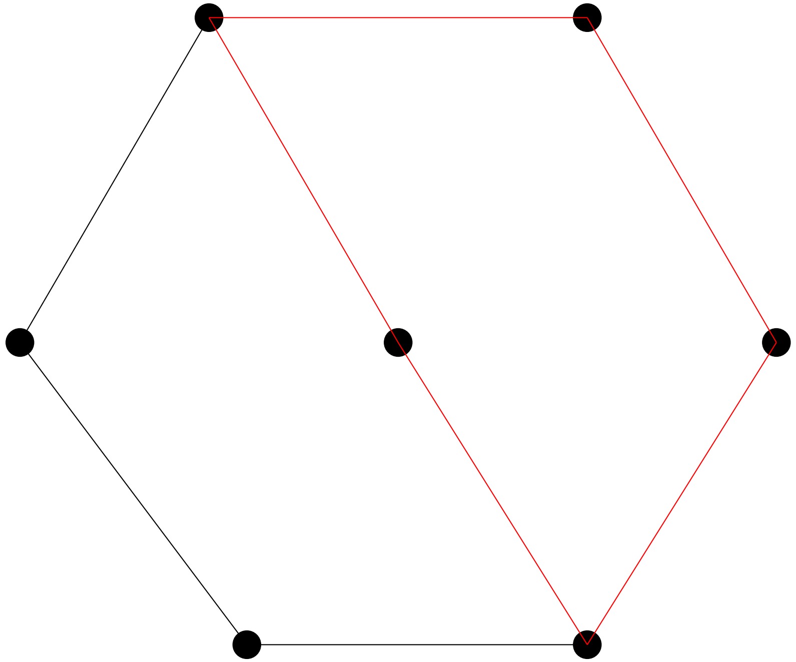
Definition (component)**.** A maximal connected subgraph is called a component



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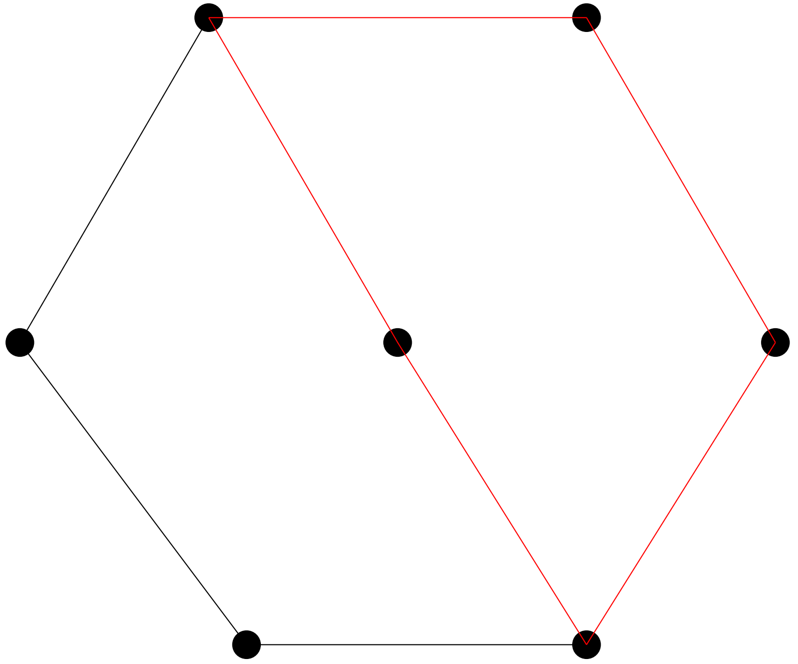
Definition. Graphs, G_1 and G_2 are called isomorphic if



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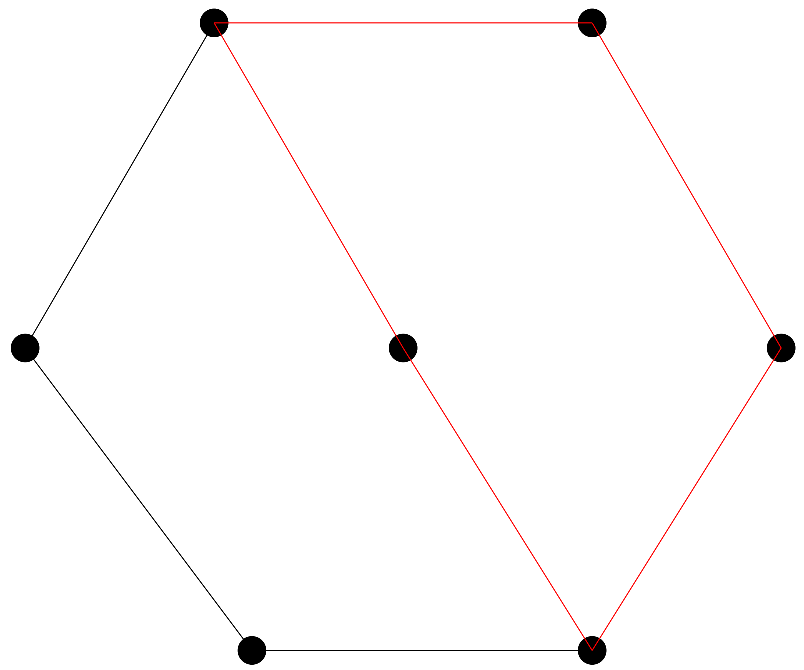

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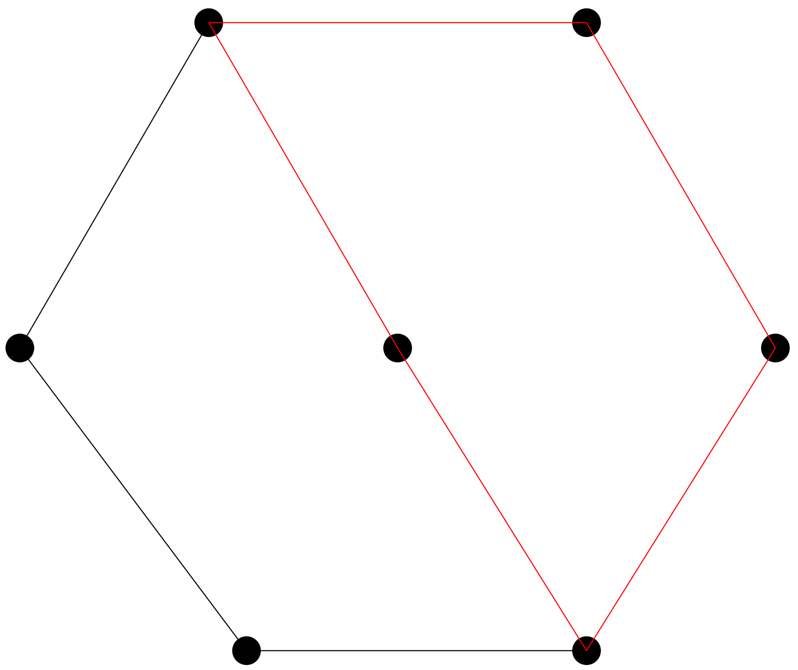

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