Partitions of integers

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Examples

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Examples

3

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Examples

$3 = 1 + 1 + 1$

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$3 = 1 + 1 + 1 = 2 + 1$

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4

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Example. Find the number of partitions of m into integers

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Example. Find the number of partitions of m into integers with allowed sizes 1, 2, 3.

Partitions of 10:

$2 + 2 + 3 + 3 = 2 \times 2 + 2 \times 3$

$1 + 1 + 1 + 1 + 2 + 2 + 2 = 4 \times 1 + 3 \times 2$

$x^{10} = x^{m_1 \times 1 + m_2 \times 2 + m_3 \times 3} = (x^1)^{m_1}(x^2)^{m_2}(x^3)^{m_3}$

What about: $(1 + x + x^2 + x^3)^m$?

Not for partitions...identical balls in distinct boxes.

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Example. In how many ways can k balls chosen out of 2 red and 3 yellow be arranged?

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Example. *Sequence*: $(1, 3, 3^2, \ldots,)$
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$$1+3x+3^2\frac{x^2}{2!}+3^3\frac{x^2}{3!}+\cdots=e^{3x}$$

Example. *Sequence*: $(P_0^n, P_1^n, P_2^n, \ldots, P_n^n)$
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Example. In how many ways can k balls chosen out of 2 red and 3 yellow be arranged?

Solution.

$$(1+\frac{x}{1!}+\frac{x^2}{2!})(1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!})$$

Exponential generating function

Definition (Exponential generating function)**.**

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$k!$ times the kth coefficient

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Example. In how many ways can k balls chosen out of infinitely many red and yellow ones be arranged?

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