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Solution.
Denote, $[a]_n := \{kn + a \mid n \in \mathbb{Z}\}$
$[a]_n \cap [b]_n = \emptyset$, if $a \neq b$
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Example. Prove that there always exists a subset of a set of any n integers, whose sum is a multiple of n.

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The given set is $\{m_1, m_2, \ldots, m_n\}$
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There are $2^n - 1$ sums.
If the sums are not distinct, then we are done

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$|A| = n < n + 1 = |B|$, so f is not injective.
There is $p, q \in A$, so that $p \neq q$
$f(p) = f(q) \iff [p]_n = [q]_m \iff p, q \in [q]_n \iff$
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Pigeonhole principle: A and B are finite sets
If $f : A \to B$ and $\#(A) > \#(B)$, then f is not 1-1

Example. In a group of at least 13 people, at least two people share the same month of birth.

Example. Prove that from a set of $n+1$ integers, there must exist a pair whose difference is divisible by n.

Solution.
Denote, $[a]_n := \{kn + a \mid n \in \mathbb{Z}\}$
$[a]_n \cap [b]_n = \emptyset$, if $a \neq b$
$[0]_n \cup \ldots \cup [n-1]_n = \mathbb{Z}$
$p, q \in [a]_n \iff p - q$ divisible by n

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