Exponential generating function

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Definition.

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$$a_0 + a_1 x + a_2 \frac{x^2}{2!} + a_3 \frac{x^3}{3!} + \cdots$$

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$$a_0 + a_1 x + a_2 x^2 + \cdots = a_0 + (a_1 1!)\frac{x}{1!} + (a_2 2!)\frac{x^2}{2!} + \cdots$$

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Solution.

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Example. Number of subsets of an n-element set.

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