

MTH307

- Discrete Mathematics

MTH307

- Discrete Mathematics

Principles And Techniques In Combinatorics

by Chuan Chong Chen and Khee-meng Koh

MTH307

- Discrete Mathematics

Principles And Techniques In Combinatorics

by Chuan Chong Chen and Khee-meng Koh

Grading scheme:

Qizzes: 30%

Midsem: 30%

Final: 40%

The summation principal

The summation principal

$A_1,$

The summation principal

A_1, A_2

The summation principal

A_1, A_2 finite sets

The summation principal

A_1, A_2 finite sets

$$A_1 \cap A_2 = \emptyset$$

The summation principal

A_1, A_2 finite sets

$$A_1 \cap A_2 = \emptyset$$

The summation principal

A_1, A_2 finite sets

$A_1 \cap A_2 = \emptyset$ then

The summation principal

A_1, A_2 finite sets

$A_1 \cap A_2 = \emptyset$ then

$$A_1 \cup$$

The summation principal

A_1, A_2 finite sets

$A_1 \cap A_2 = \emptyset$ then

$$A_1 \cup A_2$$

The summation principal

A_1, A_2 finite sets

$A_1 \cap A_2 = \emptyset$ then

$$|A_1 \cup A_2|$$

The summation principal

A_1, A_2 finite sets

$A_1 \cap A_2 = \emptyset$ then

$$|A_1 \cup A_2| = |A_1| +$$

The summation principal

A_1, A_2 finite sets

$A_1 \cap A_2 = \emptyset$ then

$$|A_1 \cup A_2| = |A_1| + |A_2|$$

The summation principal

A_1, A_2 finite sets

$A_1 \cap A_2 = \emptyset$ then

$$|A_1 \cup A_2| = |A_1| + |A_2|$$

The summation principal

A_1, A_2 finite sets

$A_1 \cap A_2 = \emptyset$ then

$$|A_1 \cup A_2| = |A_1| + |A_2|$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

Example.

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

Example.

A

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

Example.

A

B

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

Example.

$|A|$

B

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

Example.

$$|A| = 10$$

B

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

Example.

$$|A| = 10$$

$$|B|$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

Example.

$$|A| = 10$$

$$|B| = 15$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$A \cap B$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B|$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = ?$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = ?$$

$$\text{because, } |A \cap B| + |A \setminus A \cap B| = |A|$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = ?$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = |A|$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = ?$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = 7$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = 7$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$$

$$|B \setminus A \cap B| = ??$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = 7$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$$

$$|B \setminus A \cap B| = ??$$

$$\text{because, } |A \cap B| + |B \setminus A \cap B| = |B|$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = 7$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$$

$$|B \setminus A \cap B| = ??$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |B \setminus A \cap B| = |B|$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = 7$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$$

$$|B \setminus A \cap B| = ??$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |B \setminus A \cap B| = \underbrace{|B|}_{15}$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = 7$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$$

$$|B \setminus A \cap B| = 12$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |B \setminus A \cap B| = \underbrace{|B|}_{15}$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

$$|A \cup B|$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = 7$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$$

$$|B \setminus A \cap B| = 12$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |B \setminus A \cap B| = \underbrace{|B|}_{15}$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

$$|A \cup B| = |A \setminus A \cap B|$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = 7$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$$

$$|B \setminus A \cap B| = 12$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |B \setminus A \cap B| = \underbrace{|B|}_{15}$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

$$|A \cup B| = |A \setminus A \cap B| + |B \setminus A \cap B|$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = 7$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$$

$$|B \setminus A \cap B| = 12$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |B \setminus A \cap B| = \underbrace{|B|}_{15}$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

$$|A \cup B| = |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B|$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = 7$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$$

$$|B \setminus A \cap B| = 12$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |B \setminus A \cap B| = \underbrace{|B|}_{15}$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

$$\begin{aligned} |A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= 7 \end{aligned}$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = 7$$

$$\text{because, } \underbrace{|A \cap B| + |A \setminus A \cap B|}_{3} = \underbrace{|A|}_{10}$$

$$|B \setminus A \cap B| = 12$$

$$\text{because, } \underbrace{|A \cap B| + |B \setminus A \cap B|}_{3} = \underbrace{|B|}_{15}$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

$$\begin{aligned} |A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= 7 + 12 \end{aligned}$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = 7$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$$

$$|B \setminus A \cap B| = 12$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |B \setminus A \cap B| = \underbrace{|B|}_{15}$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

$$\begin{aligned} |A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= 7 + 12 + 3 \end{aligned}$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = 7$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$$

$$|B \setminus A \cap B| = 12$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |B \setminus A \cap B| = \underbrace{|B|}_{15}$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

$$\begin{aligned}|A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= 7 + 12 + 3 \\ &= 22\end{aligned}$$

Generalizes to

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = 7$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$$

$$|B \setminus A \cap B| = 12$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |B \setminus A \cap B| = \underbrace{|B|}_{15}$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

$$\begin{aligned}|A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= 7 + 12 + 3 \\ &= 22\end{aligned}$$

Generalizes to

$$|A \cup B|$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = 7$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$$

$$|B \setminus A \cap B| = 12$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |B \setminus A \cap B| = \underbrace{|B|}_{15}$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

$$\begin{aligned}|A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= 7 + 12 + 3 \\ &= 22\end{aligned}$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = 7$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$$

$$|B \setminus A \cap B| = 12$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |B \setminus A \cap B| = \underbrace{|B|}_{15}$$

Generalizes to

$$|A \cup B| = |A \setminus A \cap B|$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

$$\begin{aligned} |A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= 7 + 12 + 3 \\ &= 22 \end{aligned}$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = 7$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$$

$$|B \setminus A \cap B| = 12$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |B \setminus A \cap B| = \underbrace{|B|}_{15}$$

Generalizes to

$$|A \cup B| = |A \setminus A \cap B| + |B \setminus A \cap B|$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

$$\begin{aligned} |A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= 7 + 12 + 3 \\ &= 22 \end{aligned}$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = 7$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$$

$$|B \setminus A \cap B| = 12$$

$$\text{because, } \underbrace{|A \cap B|}_{3} + |B \setminus A \cap B| = \underbrace{|B|}_{15}$$

Generalizes to

$$|A \cup B| = |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B|$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

$$\begin{aligned}|A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= 7 + 12 + 3 \\ &= 22\end{aligned}$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = 7$$

because, $|A \setminus A \cap B| = |A| - |A \cap B|$

$$|B \setminus A \cap B| = 12$$

because, $\underbrace{|A \cap B| + |B \setminus A \cap B|}_{3} = \underbrace{|B|}_{15}$

Generalizes to

$$\begin{aligned}|A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= (|A| - |A \cap B|) + |B| - |A \cap B| + |A \cap B|\end{aligned}$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

$$\begin{aligned}|A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= 7 + 12 + 3 \\ &= 22\end{aligned}$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = 7$$

because, $|A \setminus A \cap B| = |A| - |A \cap B|$

$$|B \setminus A \cap B| = 12$$

because, $|B \setminus A \cap B| = |B| - |A \cap B|$

Generalizes to

$$\begin{aligned}|A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= (|A| - |A \cap B|) + (|B| - |A \cap B|) + |A \cap B|\end{aligned}$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

$$\begin{aligned}|A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= 7 + 12 + 3 \\ &= 22\end{aligned}$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = 7$$

because, $|A \setminus A \cap B| = |A| - |A \cap B|$

$$|B \setminus A \cap B| = 12$$

because, $|B \setminus A \cap B| = |B| - |A \cap B|$

Generalizes to

$$\begin{aligned}|A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= (|A| - |A \cap B|) + (|B| - |A \cap B|) \\ &\quad + |A \cap B|\end{aligned}$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

$$\begin{aligned}|A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= 7 + 12 + 3 \\ &= 22\end{aligned}$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = 7$$

because, $|A \setminus A \cap B| = |A| - |A \cap B|$

$$|B \setminus A \cap B| = 12$$

because, $|B \setminus A \cap B| = |B| - |A \cap B|$

Generalizes to

$$\begin{aligned}|A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= |A| - |A \cap B| + |B| - |A \cap B| \\ &\quad + |A \cap B|\end{aligned}$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

$$\begin{aligned}|A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= 7 + 12 + 3 \\ &= 22\end{aligned}$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = 7$$

because, $|A \setminus A \cap B| = |A| - |A \cap B|$

$$|B \setminus A \cap B| = 12$$

because, $|B \setminus A \cap B| = |B| - |A \cap B|$

Generalizes to

$$\begin{aligned}|A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= |A| - \cancel{|A \cap B|} + |B| - \cancel{|A \cap B|} \\ &\quad + \cancel{|A \cap B|}\end{aligned}$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

$$\begin{aligned}|A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= 7 + 12 + 3 \\ &= 22\end{aligned}$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = 7$$

because, $|A \setminus A \cap B| = |A| - |A \cap B|$

$$|B \setminus A \cap B| = 12$$

because, $|B \setminus A \cap B| = |B| - |A \cap B|$

Generalizes to

$$\begin{aligned}|A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= |A| - \cancel{|A \cap B|} + |B| - \cancel{|A \cap B|} \\ &\quad + \cancel{|A \cap B|} \\ &= |A| + |B| - |A \cap B|\end{aligned}$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

$$\begin{aligned}|A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= 7 + 12 + 3 \\ &= 22\end{aligned}$$

Example.

$$|A| = 10$$

$$|B| = 15$$

$$|A \cap B| = 3$$

$$|A \cup B| = ??$$

$$|A \setminus A \cap B| = 7$$

because, $|A \setminus A \cap B| = |A| - |A \cap B|$

$$|B \setminus A \cap B| = 12$$

because, $|B \setminus A \cap B| = |B| - |A \cap B|$

Generalizes to

$$\begin{aligned}|A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= |A| - \cancel{|A \cap B|} + |B| - \cancel{|A \cap B|} \\ &\quad + \cancel{|A \cap B|} \\ &= |A| + |B| - |A \cap B|\end{aligned}$$

$$|A \cup B| = |A| + |B| - |A \cap B|$$

The summation principal

A_1, A_2, \dots, A_k finite sets

$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \dots \cup A_k| = |A_1| + |A_2| + \dots + |A_k|$$

The multiplication principal

The multiplication principal

A_1 ,

The multiplication principal

A_1, A_2

The multiplication principal

A_1, A_2 finite sets

The multiplication principal

A_1, A_2 finite sets

The multiplication principal

A_1, A_2 finite sets

$$A_1 \times$$

The multiplication principal

A_1, A_2 finite sets

$$A_1 \times A_2$$

The multiplication principal

A_1, A_2 finite sets

$$|A_1 \times A_2|$$

The multiplication principal

A_1, A_2 finite sets

$$|A_1 \times A_2| = |A_1| \times$$

The multiplication principal

A_1, A_2 finite sets

$$|A_1 \times A_2| = |A_1| \times |A_2|$$

The multiplication principal

A_1, A_2 finite sets

$$|A_1 \times A_2| = |A_1| \times |A_2|$$

The multiplication principal

A_1, A_2 finite sets

$$|A_1 \times A_2| = |A_1| \times |A_2|$$

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

Example. Number of ways

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

Example. Number of ways to arrange r distinct objects

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

Example. Number of ways to arrange r distinct objects out of n objects?

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

Example. Number of ways to arrange r distinct objects out of n objects?

X_r : total number of arrangements

Each arrangement,

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

Example. Number of ways to arrange r distinct objects out of n objects?

X_r : total number of arrangements

Each arrangement,

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

Example. Number of ways to arrange r distinct objects out of n objects?

X_r : total number of arrangements

Each arrangement,

(x_1, x_2, \dots, x_r)

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

Example. Number of ways to arrange r distinct objects out of n objects?

X_r : total number of arrangements

Each arrangement,

$$(x_1, x_2, \dots, x_r) \in$$

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

Example. Number of ways to arrange r distinct objects out of n objects?

X_r : total number of arrangements

Each arrangement,

$$(x_1, x_2, \dots, x_r) \in A \times (A \setminus x_1) \times \dots \times (A \setminus \{x_1, x_2, \dots, x_{r-1}\})$$

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

Example. Number of ways to arrange r distinct objects out of n objects?

X_r : total number of arrangements

Each arrangement,

$$(x_1, x_2, \dots, x_r) \in A \times (A \setminus x_1) \times \dots \times (A \setminus \{x_1, x_2, \dots, x_{r-1}\})$$

$$[n] := \{1, 2, \dots, n\}$$

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

Example. Number of ways to arrange r distinct objects out of n objects?

X_r : total number of arrangements

Each arrangement,

$$(x_1, x_2, \dots, x_r) \in A \times (A \setminus x_1) \times \dots \times (A \setminus \{x_1, x_2, \dots, x_{r-1}\})$$

$$[n] := \{1, 2, \dots, n\}$$

$$[n] \times [n-1] \times \dots \times [n-r+1] \rightarrow X_r$$

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

Example. Number of ways to arrange r distinct objects out of n objects?

X_r : total number of arrangements

Each arrangement,

$$(x_1, x_2, \dots, x_r) \in A \times (A \setminus x_1) \times \dots \times (A \setminus \{x_1, x_2, \dots, x_{r-1}\})$$

$$[n] := \{1, 2, \dots, n\}$$

$$[n] \times [n-1] \times \dots \times [n-r+1] \rightarrow X_r$$

$$(n_1, n_2, \dots, n_r) \rightarrow (x_{n_1}, x_{n_2}, \dots, x_{n_r})$$

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

A B C D E F G H

$(3, 2, 4) \rightarrow$

Example. Number of ways to arrange r distinct objects out of n objects?

X_r : total number of arrangements

Each arrangement,

$$(x_1, x_2, \dots, x_r) \in A \times (A \setminus x_1) \times \dots \times (A \setminus \{x_1, x_2, \dots, x_{r-1}\})$$

$$[n] := \{1, 2, \dots, n\}$$

$$[n] \times [n-1] \times \dots \times [n-r+1] \rightarrow X_r$$

$$(n_1, n_2, \dots, n_r) \rightarrow (x_{n_1}, x_{n_2}, \dots, x_{n_r})$$

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

A B D E F G H

$(3, 2, 4) \rightarrow C$

Example. Number of ways to arrange r distinct objects out of n objects?

X_r : total number of arrangements

Each arrangement,

$$(x_1, x_2, \dots, x_r) \in A \times (A \setminus x_1) \times \dots \times (A \setminus \{x_1, x_2, \dots, x_{r-1}\})$$

$$[n] := \{1, 2, \dots, n\}$$

$$[n] \times [n-1] \times \dots \times [n-r+1] \rightarrow X_r$$

$$(n_1, n_2, \dots, n_r) \rightarrow (x_{n_1}, x_{n_2}, \dots, x_{n_r})$$

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

A D E F G H
 $(3, 2, 4) \rightarrow$ C B

Example. Number of ways to arrange r distinct objects out of n objects?

X_r : total number of arrangements

Each arrangement,

$$(x_1, x_2, \dots, x_r) \in A \times (A \setminus x_1) \times \dots \times (A \setminus \{x_1, x_2, \dots, x_{r-1}\})$$

$$[n] := \{1, 2, \dots, n\}$$

$$[n] \times [n-1] \times \dots \times [n-r+1] \rightarrow X_r$$

$$(n_1, n_2, \dots, n_r) \rightarrow (x_{n_1}, x_{n_2}, \dots, x_{n_r})$$

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

A D E G H
 $(3, 2, 4) \rightarrow$ C B F

Example. Number of ways to arrange r distinct objects out of n objects?

X_r : total number of arrangements

Each arrangement,

$$(x_1, x_2, \dots, x_r) \in A \times (A \setminus x_1) \times \dots \times (A \setminus \{x_1, x_2, \dots, x_{r-1}\})$$

$$[n] := \{1, 2, \dots, n\}$$

$$[n] \times [n-1] \times \dots \times [n-r+1] \rightarrow X_r$$

$$(n_1, n_2, \dots, n_r) \rightarrow (x_{n_1}, x_{n_2}, \dots, x_{n_r})$$

Is a bijection

$$|X_r| =$$

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

A D E G H
 $(3, 2, 4) \rightarrow$ C B F

Example. Number of ways to arrange r distinct objects out of n objects?

X_r : total number of arrangements

Each arrangement,

$$(x_1, x_2, \dots, x_r) \in A \times (A \setminus x_1) \times \dots \times (A \setminus \{x_1, x_2, \dots, x_{r-1}\})$$

$$[n] := \{1, 2, \dots, n\}$$

$$[n] \times [n-1] \times \dots \times [n-r+1] \rightarrow X_r$$

$$(n_1, n_2, \dots, n_r) \rightarrow (x_{n_1}, x_{n_2}, \dots, x_{n_r})$$

Is a bijection

$$|X_r| = |[n]| \times |[n-1]| \times \dots \times |[n-r+1]|$$

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

Example. Number of ways to arrange r distinct objects out of n objects **in a circle**?

Example. Number of ways to arrange r distinct objects out of n objects?

X_r : total number of arrangements

Each arrangement,

$$(x_1, x_2, \dots, x_r) \in A \times (A \setminus x_1) \times \dots \times (A \setminus \{x_1, x_2, \dots, x_{r-1}\})$$

$$[n] := \{1, 2, \dots, n\}$$

$$[n] \times [n-1] \times \dots \times [n-r+1] \rightarrow X_r$$

$$(n_1, n_2, \dots, n_r) \rightarrow (x_{n_1}, x_{n_2}, \dots, x_{n_r})$$

Is a bijection

$$|X_r| = |[n]| \times |[n-1]| \times \dots \times |[n-r+1]| = \frac{n!}{(n-r)!}$$

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

Example. Number of ways to arrange r distinct objects out of n objects in a circle?

C_r : set of circular arrangements

Example. Number of ways to arrange r distinct objects out of n objects?

X_r : total number of arrangements

Each arrangement,

$$(x_1, x_2, \dots, x_r) \in A \times (A \setminus x_1) \times \dots \times (A \setminus \{x_1, x_2, \dots, x_{r-1}\})$$

$$[n] := \{1, 2, \dots, n\}$$

$$[n] \times [n-1] \times \dots \times [n-r+1] \rightarrow X_r$$

$$(n_1, n_2, \dots, n_r) \rightarrow (x_{n_1}, x_{n_2}, \dots, x_{n_r})$$

Is a bijection

$$|X_r| = |[n]| \times |[n-1]| \times \dots \times |[n-r+1]| = \frac{n!}{(n-r)!}$$

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

Example. Number of ways to arrange r distinct objects out of n objects in a circle?

C_r : set of circular arrangements

Define the relation,

Example. Number of ways to arrange r distinct objects out of n objects?

X_r : total number of arrangements

Each arrangement,

$$(x_1, x_2, \dots, x_r) \in A \times (A \setminus x_1) \times \dots \times (A \setminus \{x_1, x_2, \dots, x_{r-1}\})$$

$$[n] := \{1, 2, \dots, n\}$$

$$[n] \times [n-1] \times \dots \times [n-r+1] \rightarrow X_r$$

$$(n_1, n_2, \dots, n_r) \rightarrow (x_{n_1}, x_{n_2}, \dots, x_{n_r})$$

Is a bijection

$$|X_r| = |[n]| \times |[n-1]| \times \dots \times |[n-r+1]| = \frac{n!}{(n-r)!}$$

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

Example. Number of ways to arrange r distinct objects out of n objects?

X_r : total number of arrangements

Each arrangement,

$$(x_1, x_2, \dots, x_r) \in A \times (A \setminus x_1) \times \dots \times (A \setminus \{x_1, x_2, \dots, x_{r-1}\})$$

$$[n] := \{1, 2, \dots, n\}$$

$$[n] \times [n-1] \times \dots \times [n-r+1] \rightarrow X_r$$

$$(n_1, n_2, \dots, n_r) \rightarrow (x_{n_1}, x_{n_2}, \dots, x_{n_r})$$

Is a bijection

$$|X_r| = |[n]| \times |[n-1]| \times \dots \times |[n-r+1]| = \frac{n!}{(n-r)!}$$

Example. Number of ways to arrange r distinct objects out of n objects in a circle?

C_r : set of circular arrangements

Define the relation,

$$(x_1, x_2, \dots, x_r) \sim (x_r, x_1, \dots, x_{r-1})$$

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

Example. Number of ways to arrange r distinct objects out of n objects?

X_r : total number of arrangements

Each arrangement,

$$(x_1, x_2, \dots, x_r) \in A \times (A \setminus x_1) \times \dots \times (A \setminus \{x_1, x_2, \dots, x_{r-1}\})$$

$$[n] := \{1, 2, \dots, n\}$$

$$[n] \times [n-1] \times \dots \times [n-r+1] \rightarrow X_r$$

$$(n_1, n_2, \dots, n_r) \rightarrow (x_{n_1}, x_{n_2}, \dots, x_{n_r})$$

Is a bijection

$$|X_r| = |[n]| \times |[n-1]| \times \dots \times |[n-r+1]| = \frac{n!}{(n-r)!}$$

Example. Number of ways to arrange r distinct objects out of n objects in a circle?

C_r : set of circular arrangements

Define the relation,

$$(x_1, x_2, \dots, x_r) \sim (x_r, x_1, \dots, x_{r-1}) \sim \dots$$

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

Example. Number of ways to arrange r distinct objects out of n objects?

X_r : total number of arrangements

Each arrangement,

$$(x_1, x_2, \dots, x_r) \in A \times (A \setminus x_1) \times \dots \times (A \setminus \{x_1, x_2, \dots, x_{r-1}\})$$

$$[n] := \{1, 2, \dots, n\}$$

$$[n] \times [n-1] \times \dots \times [n-r+1] \rightarrow X_r$$

$$(n_1, n_2, \dots, n_r) \rightarrow (x_{n_1}, x_{n_2}, \dots, x_{n_r})$$

Is a bijection

$$|X_r| = |[n]| \times |[n-1]| \times \dots \times |[n-r+1]| = \frac{n!}{(n-r)!}$$

Example. Number of ways to arrange r distinct objects out of n objects in a circle?

C_r : set of circular arrangements

Define the relation,

$$(x_1, x_2, \dots, x_r) \sim (x_r, x_1, \dots, x_{r-1}) \sim \dots$$

\sim is an equivalence relation.

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

Example. Number of ways to arrange r distinct objects out of n objects?

X_r : total number of arrangements

Each arrangement,

$$(x_1, x_2, \dots, x_r) \in A \times (A \setminus x_1) \times \dots \times (A \setminus \{x_1, x_2, \dots, x_{r-1}\})$$

$$[n] := \{1, 2, \dots, n\}$$

$$[n] \times [n-1] \times \dots \times [n-r+1] \rightarrow X_r$$

$$(n_1, n_2, \dots, n_r) \rightarrow (x_{n_1}, x_{n_2}, \dots, x_{n_r})$$

Is a bijection

$$|X_r| = |[n]| \times |[n-1]| \times \dots \times |[n-r+1]| = \frac{n!}{(n-r)!}$$

Example. Number of ways to arrange r distinct objects out of n objects in a circle?

C_r : set of circular arrangements

Define the relation,

$$(x_1, x_2, \dots, x_r) \sim (x_r, x_1, \dots, x_{r-1}) \sim \dots$$

\sim is an equivalence relation.

Splits into equivalence classes

$$X_r := \sqcup A_k$$

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

Example. Number of ways to arrange r distinct objects out of n objects?

X_r : total number of arrangements

Each arrangement,

$$(x_1, x_2, \dots, x_r) \in A \times (A \setminus x_1) \times \dots \times (A \setminus \{x_1, x_2, \dots, x_{r-1}\})$$

$$[n] := \{1, 2, \dots, n\}$$

$$[n] \times [n-1] \times \dots \times [n-r+1] \rightarrow X_r$$

$$(n_1, n_2, \dots, n_r) \rightarrow (x_{n_1}, x_{n_2}, \dots, x_{n_r})$$

Is a bijection

$$|X_r| = |[n]| \times |[n-1]| \times \dots \times |[n-r+1]| = \frac{n!}{(n-r)!}$$

Example. Number of ways to arrange r distinct objects out of n objects in a circle?

C_r : set of circular arrangements

Define the relation,

$$(x_1, x_2, \dots, x_r) \sim (x_r, x_1, \dots, x_{r-1}) \sim \dots$$

\sim is an equivalence relation.

Splits into equivalence classes

$$X_r := \sqcup A_k$$

$$|X_r| := \sum_{i=1}^{|C_r|} |A_k| =$$

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

Example. Number of ways to arrange r distinct objects out of n objects?

X_r : total number of arrangements

Each arrangement,

$$(x_1, x_2, \dots, x_r) \in A \times (A \setminus x_1) \times \dots \times (A \setminus \{x_1, x_2, \dots, x_{r-1}\})$$

$$[n] := \{1, 2, \dots, n\}$$

$$[n] \times [n-1] \times \dots \times [n-r+1] \rightarrow X_r$$

$$(n_1, n_2, \dots, n_r) \rightarrow (x_{n_1}, x_{n_2}, \dots, x_{n_r})$$

Is a bijection

$$|X_r| = |[n]| \times |[n-1]| \times \dots \times |[n-r+1]| = \frac{n!}{(n-r)!}$$

Example. Number of ways to arrange r distinct objects out of n objects in a circle?

C_r : set of circular arrangements

Define the relation,

$$(x_1, x_2, \dots, x_r) \sim (x_r, x_1, \dots, x_{r-1}) \sim \dots$$

\sim is an equivalence relation.

Splits into equivalence classes

$$X_r := \sqcup A_k$$

$$|X_r| := \sum_{i=1}^{|C_r|} |A_k| = |C_r|r$$

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

Example. Number of ways to arrange r distinct objects out of n objects?

X_r : total number of arrangements

Each arrangement,

$$(x_1, x_2, \dots, x_r) \in A \times (A \setminus x_1) \times \dots \times (A \setminus \{x_1, x_2, \dots, x_{r-1}\})$$

$$[n] := \{1, 2, \dots, n\}$$

$$[n] \times [n-1] \times \dots \times [n-r+1] \rightarrow X_r$$

$$(n_1, n_2, \dots, n_r) \rightarrow (x_{n_1}, x_{n_2}, \dots, x_{n_r})$$

Is a bijection

$$|X_r| = |[n]| \times |[n-1]| \times \dots \times |[n-r+1]| = \frac{n!}{(n-r)!}$$

Example. Number of ways to arrange r distinct objects out of n objects in a circle?

C_r : set of circular arrangements

Define the relation,

$$(x_1, x_2, \dots, x_r) \sim (x_r, x_1, \dots, x_{r-1}) \sim \dots$$

\sim is an equivalence relation.

Splits into equivalence classes

$$X_r := \sqcup A_k$$

$$|X_r| := \sum_{i=1}^{|C_r|} |A_k| = |C_r|r$$

$$|C_r| =$$

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

Example. Number of ways to arrange r distinct objects out of n objects?

X_r : total number of arrangements

Each arrangement,

$$(x_1, x_2, \dots, x_r) \in A \times (A \setminus x_1) \times \dots \times (A \setminus \{x_1, x_2, \dots, x_{r-1}\})$$

$$[n] := \{1, 2, \dots, n\}$$

$$[n] \times [n-1] \times \dots \times [n-r+1] \rightarrow X_r$$

$$(n_1, n_2, \dots, n_r) \rightarrow (x_{n_1}, x_{n_2}, \dots, x_{n_r})$$

Is a bijection

$$|X_r| = |[n]| \times |[n-1]| \times \dots \times |[n-r+1]| = \frac{n!}{(n-r)!}$$

Example. Number of ways to arrange r distinct objects out of n objects in a circle?

C_r : set of circular arrangements

Define the relation,

$$(x_1, x_2, \dots, x_r) \sim (x_r, x_1, \dots, x_{r-1}) \sim \dots$$

\sim is an equivalence relation.

Splits into equivalence classes

$$X_r := \sqcup A_k$$

$$|X_r| := \sum_{i=1}^{|C_r|} |A_k| = |C_r| r$$

$$|C_r| = \frac{n!}{(n-r)! r}$$

The multiplication principal

A_1, A_2, \dots, A_k finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \dots \times |A_k|$$

Example. Number of ways to arrange r distinct objects out of n objects?

X_r : total number of arrangements

Each arrangement,

$$(x_1, x_2, \dots, x_r) \in A \times (A \setminus x_1) \times \dots \times (A \setminus \{x_1, x_2, \dots, x_{r-1}\})$$

$$[n] := \{1, 2, \dots, n\}$$

$$[n] \times [n-1] \times \dots \times [n-r+1] \rightarrow X_r$$

$$(n_1, n_2, \dots, n_r) \rightarrow (x_{n_1}, x_{n_2}, \dots, x_{n_r})$$

Is a bijection

$$|X_r| = |[n]| \times |[n-1]| \times \dots \times |[n-r+1]| = \frac{n!}{(n-r)!}$$

Example. Number of ways to arrange r distinct objects out of n objects in a circle?

C_r : set of circular arrangements

Define the relation,

$$(x_1, x_2, \dots, x_r) \sim (x_r, x_1, \dots, x_{r-1}) \sim \dots$$

\sim is an equivalence relation.

Splits into equivalence classes

$$X_r := \sqcup A_k$$

$$|X_r| := \sum_{i=1}^{|C_r|} |A_k| = |C_r| r$$

$$|C_r| = \frac{n!}{(n-r)! r}$$