Definition. An *n-clique*

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Generalized Pigeonhole principle

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$|f^{-1}(b_i)| \geq k_i$ for some i.

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Example.

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$\{b_1, b_2, b_3\} \subset \{a_2, \ldots, a_6\}$ (pigeonhole principle)

Assume a_1 knows $\{b_1, b_2, b_3\}$

If any $b_i, b_j \in \{b_1, b_2, b_3\}$ know each other,

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Otherwise, none among $\{b_1, b_2, b_3\}$ know each other

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Example. In a group of 6 people, there is either a subset of 3 who know each other, or a subset of 3 who do not know each other.

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Example. Consider a graph with 6 vertices,

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Example. Consider a graph with 6 vertices, such that every pair vertices is joined by exactly one edge

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Example. Consider a graph with 6 vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue.

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then $\{a_1, b_i, b_j\}$ form the triple.

Otherwise, none among $\{b_1, b_2, b_3\}$ know each other

and form the triple $\square$

Example. Consider a graph with 6 vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue. Show that there are three vertices

Definition. An *n-clique* is an n-graph in which each pair from its n-vertices are joined by exactly one edge.

Generalized Pigeonhole principle

A and B finite sets

$f : A \to B := \{b_1, b_2, \ldots, b_n\}$.

$k_1, k_2, \cdots, k_n$ are positive integers

If $|A| \geq k_1 + k_2 + \cdots + k_n - (n-1)$, then

$|f^{-1}(b_i)| \geq k_i$ for some i.

Example. In a group of 6 people, there is either a subset of 3 who know each other, or a subset of 3 who do not know each other.

Solution.

Denote the set, $A := \{a_1, a_2, \ldots, a_6\}$

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Example. In a group of 6 people, there is either a subset of 3 who know each other, or a subset of 3 who do not know each other.

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Example. Consider a graph with 17 vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, blue, or green.

Example. In a group of 6 people, there is either a subset of 3 who know each other, or a subset of 3 who do not know each other.

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Solution.

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Solution.
Let the set of vertices be,

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Solution.
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Solution.
Let the set of vertices be, $V := \{v_1, v_2, \ldots, v_{17}\}$
v_1 is joined with 16 vertices.

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$16 = 3 \times 5 + 1$,

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At least $5 + 1 = 6$ edges containing v_1

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v_1 is joined with 16 vertices.
$16 = 3 \times 5 + 1$, so by pigeonhole principle,
At least $5 + 1 = 6$ edges containing v_1 are of the same colour C_1

Example. In a group of 6 people, there is either a subset of 3 who know each other, or a subset of 3 who do not know each other.

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Denote the set, $A := \{a_1, a_2, \ldots, a_6\}$
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$16 = 3 \times 5 + 1$, so by pigeonhole principle,
At least $5 + 1 = 6$ edges containing v_1 are of the same colour C_1
Let those edges be $\{v_{i_1}, v_{i_2}, \dots, v_{i_6}\}$

Example. In a group of 6 people, there is either a subset of 3 who know each other, or a subset of 3 who do not know each other.

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Denote the set, $A := \{a_1, a_2, \dots, a_6\}$
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Example. Consider a graph with 6 vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue. Show that there are three vertices so that the edges joining each pair of them are of the same colour.

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Let the set of vertices be, $V := \{v_1, v_2, \ldots, v_{17}\}$
v_1 is joined with 16 vertices.
$16 = 3 \times 5 + 1$, so by pigeonhole principle,
At least $5 + 1 = 6$ edges containing v_1 are of the same colour C_1
Let those edges be $\{v_{i_1}, v_{i_2}, \ldots, v_{i_6}\}$
If even one of them are joined by the colour C_1,

Example. In a group of 6 people, there is either a subset of 3 who know each other, or a subset of 3 who do not know each other.

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Denote the set, $A := \{a_1, a_2, \ldots, a_6\}$
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Assume a_1 knows $\{b_1, b_2, b_3\}$
If any $b_i, b_j \in \{b_1, b_2, b_3\}$ know each other,
then $\{a_1, b_i, b_j\}$ form the triple.
Otherwise, none among $\{b_1, b_2, b_3\}$ know each other and form the triple $\square$

Example. Consider a graph with 6 vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue. Show that there are three vertices so that the edges joining each pair of them are of the same colour.

Solution. Same as above $\square$

Example. Consider a graph with 17 vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, blue, or green. Show that there are three vertices so that the edges joining each pair of them are of the same colour.

Solution.
Let the set of vertices be, $V := \{v_1, v_2, \ldots, v_{17}\}$
v_1 is joined with 16 vertices.
$16 = 3 \times 5 + 1$, so by pigeonhole principle,
At least $5 + 1 = 6$ edges containing v_1 are of the same colour C_1
Let those edges be $\{v_{i_1}, v_{i_2}, \ldots, v_{i_6}\}$
If even one of them are joined by the colour C_1, we are done.

Example. In a group of 6 people, there is either a subset of 3 who know each other, or a subset of 3 who do not know each other.

Solution.
Denote the set, $A := \{a_1, a_2, \ldots, a_6\}$
a_1 either knows or does not know
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Definition (Ramsay Number)**.**

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Example. Consider a graph with $n = R(p-1,q) + R(p,q-1)$ vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue. Show that there is a red p-clique or a blue q-clique.

Solution.
Let the set of vertices be, $V := \{v_1, v_2, \ldots, v_n\}$
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Solution.
Let the set of vertices be, $V := \{v_1, v_2, \ldots, v_{17}\}$
v_1 is joined with 16 vertices.
$16 = 3 \times 5 + 1$, so by pigeonhole principle,
At least $5 + 1 = 6$ edges containing v_1 are of the same colour C_1
Let those edges be $\{v_{i_1}, v_{i_2}, \ldots, v_{i_6}\}$
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They either have a q blue clique, then we are done. Or a $p - 1$ red clique,

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They either have a q blue clique, then we are done. Or a $p-1$ red clique, combined with v_1, forms a p red clique. $\square$