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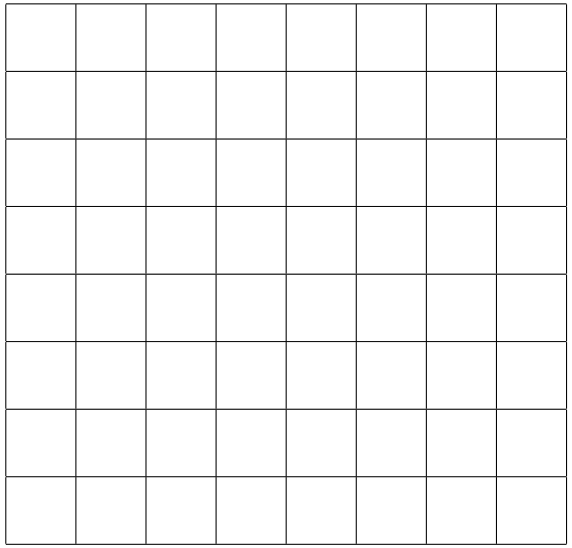

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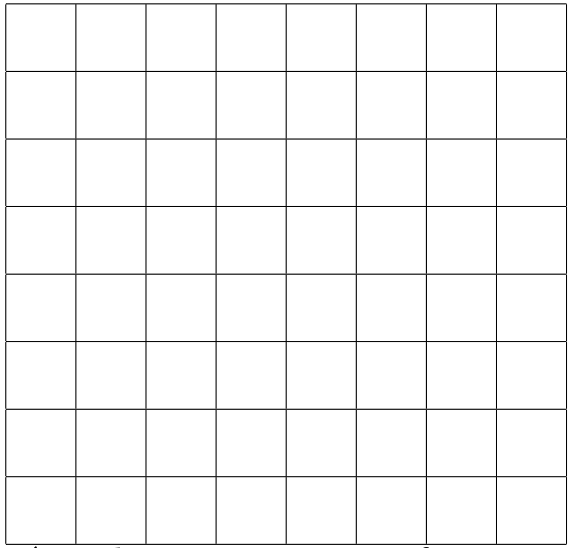

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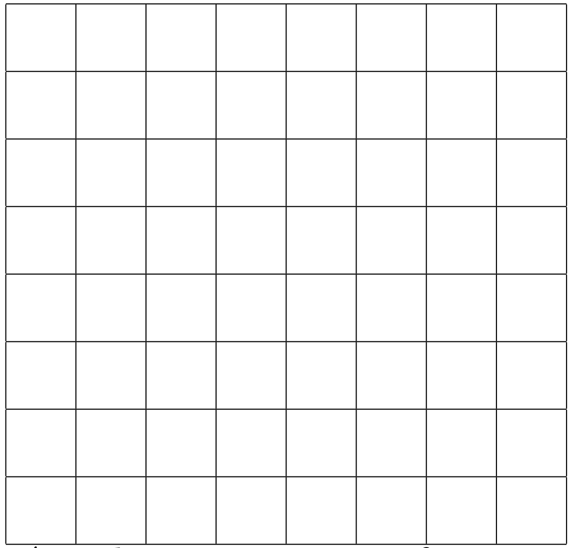

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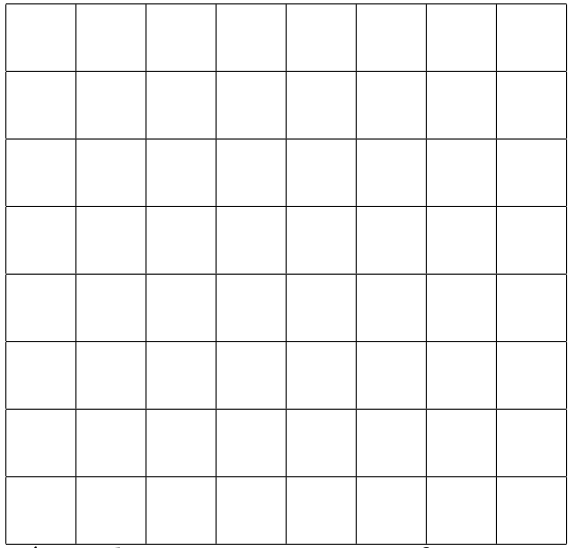

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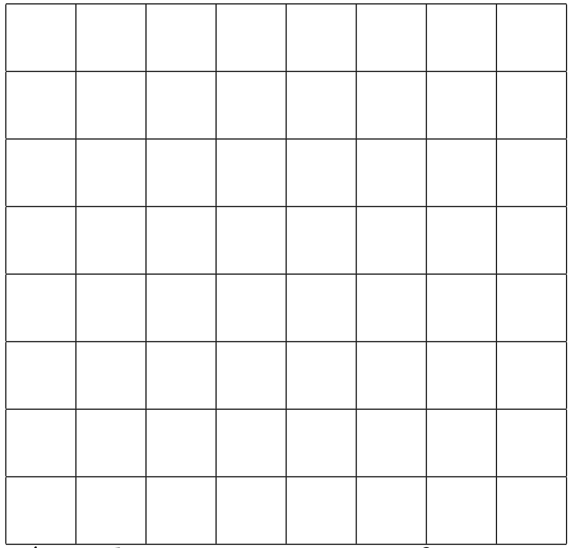

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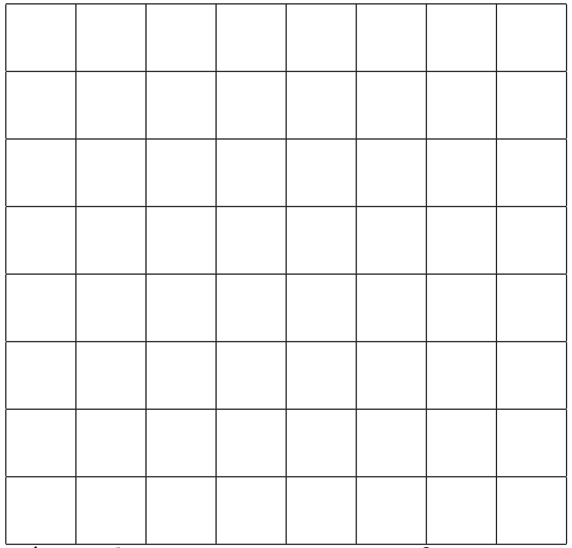

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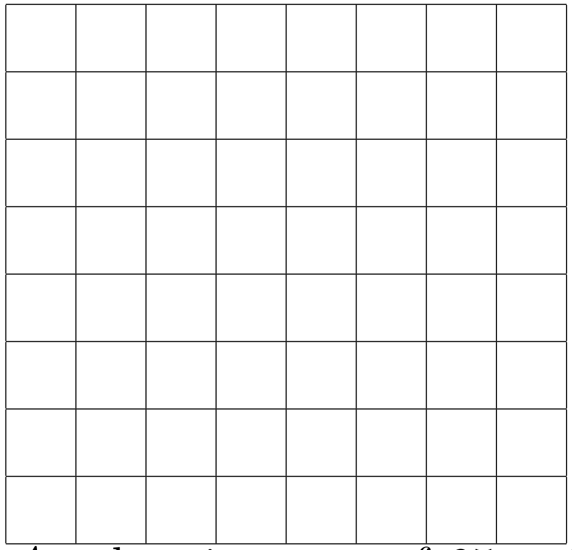

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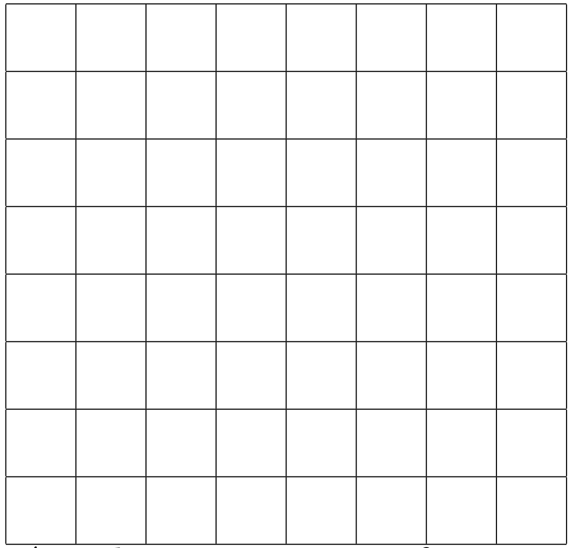

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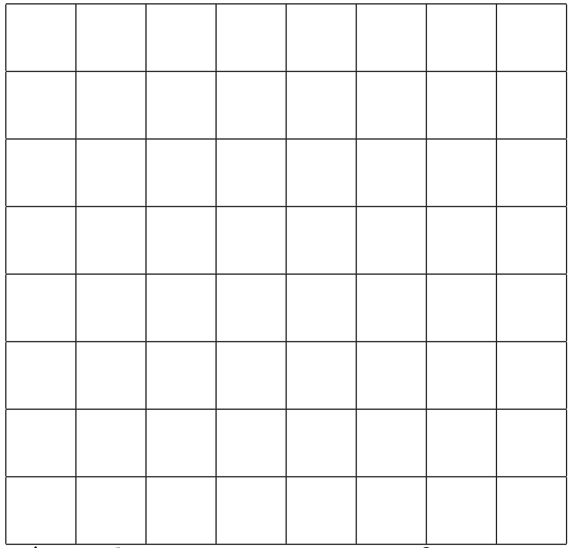

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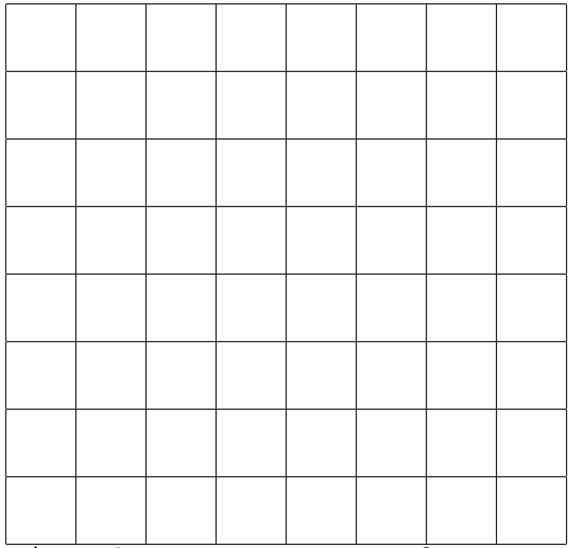

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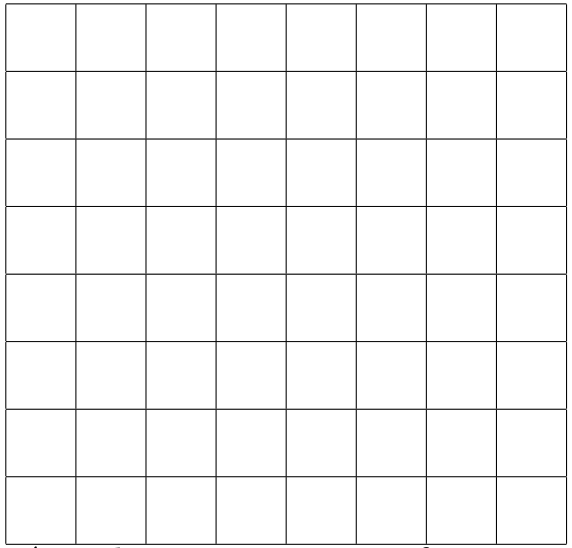

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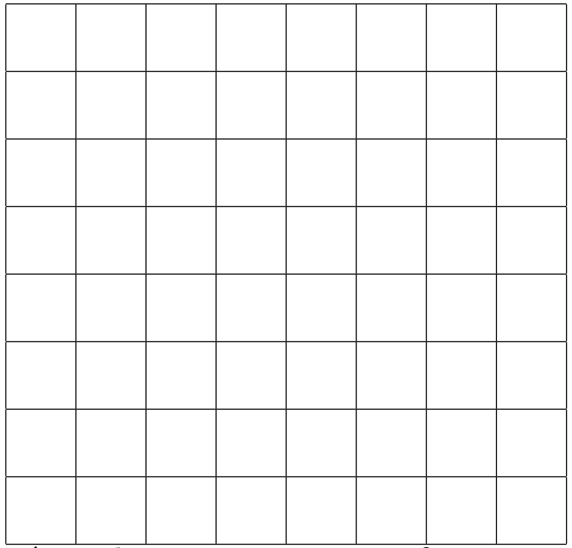

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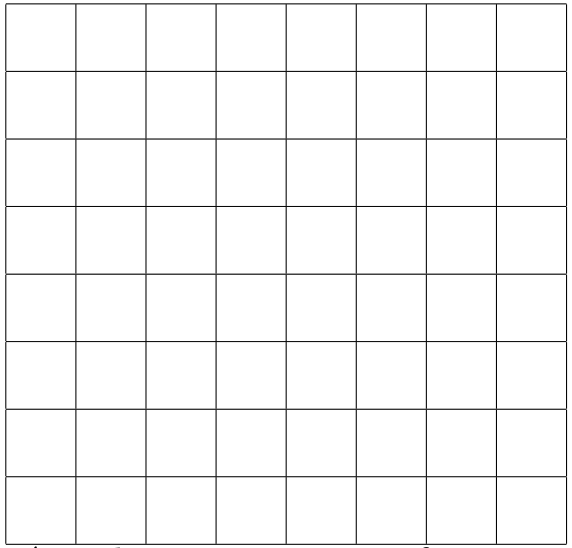

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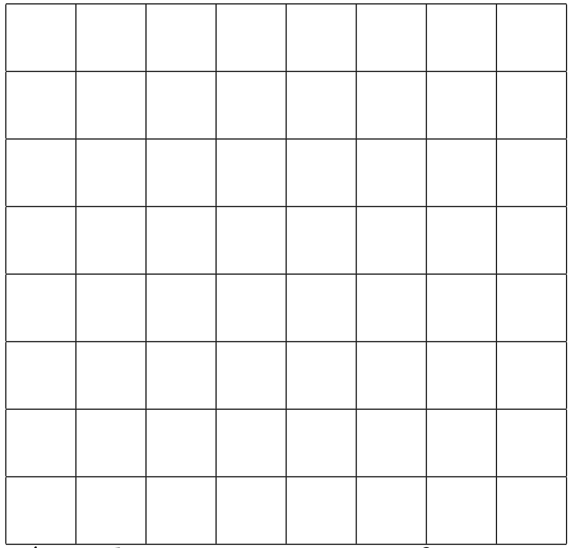

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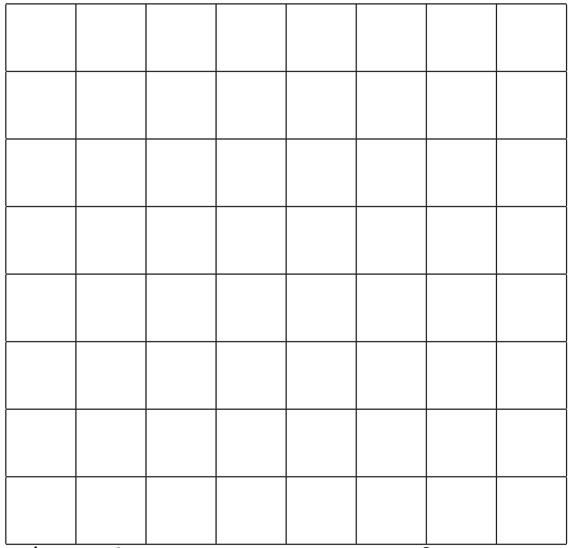

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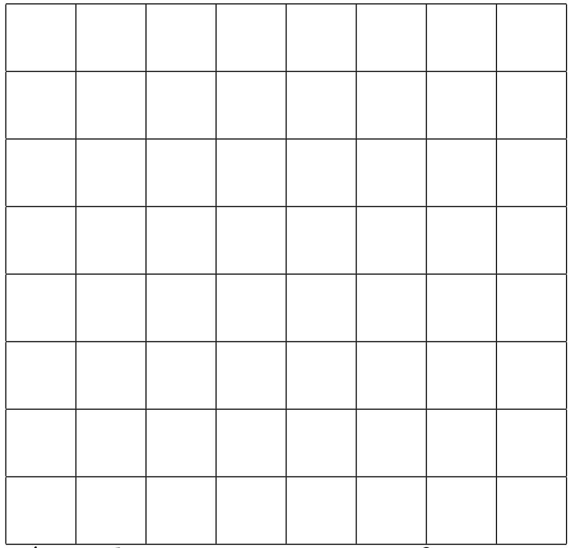

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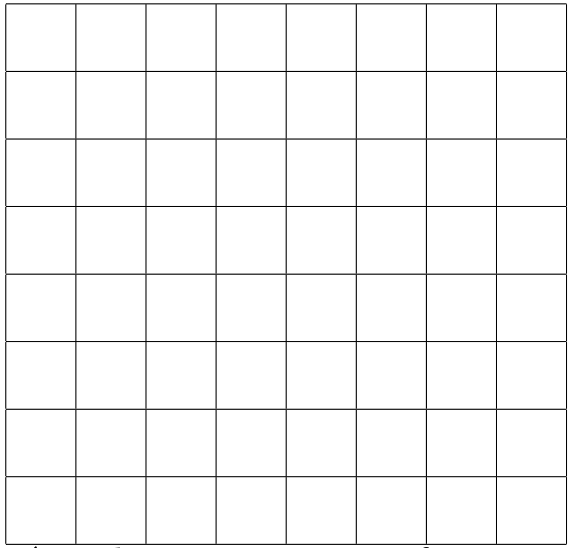

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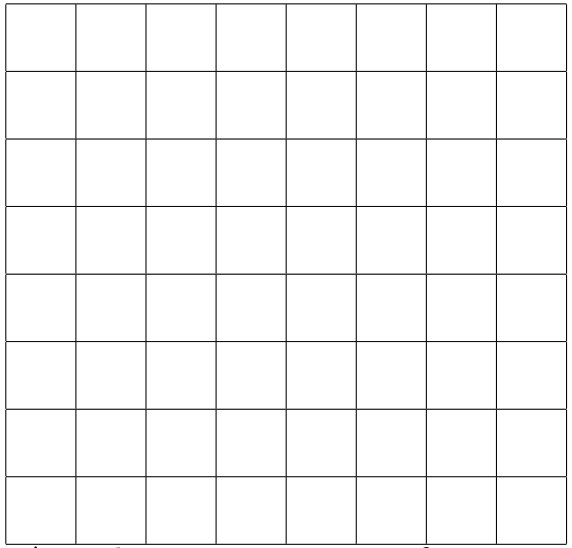

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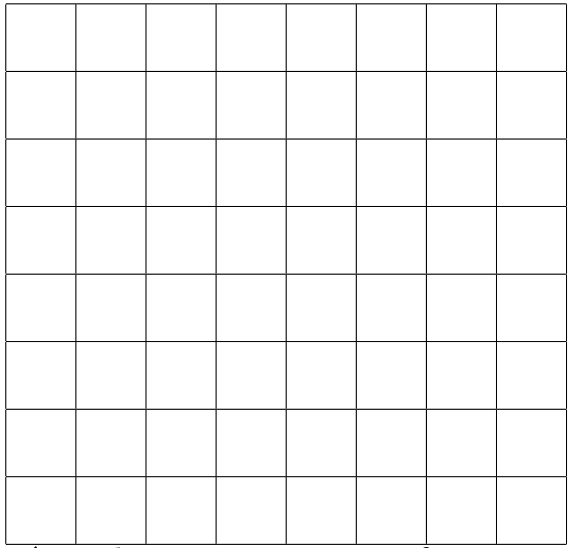

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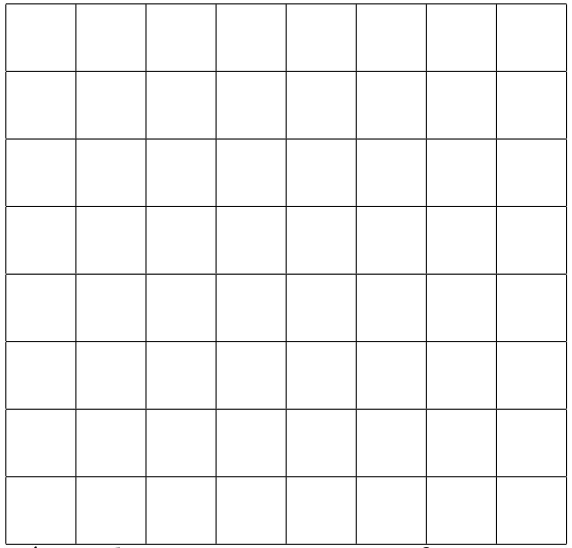

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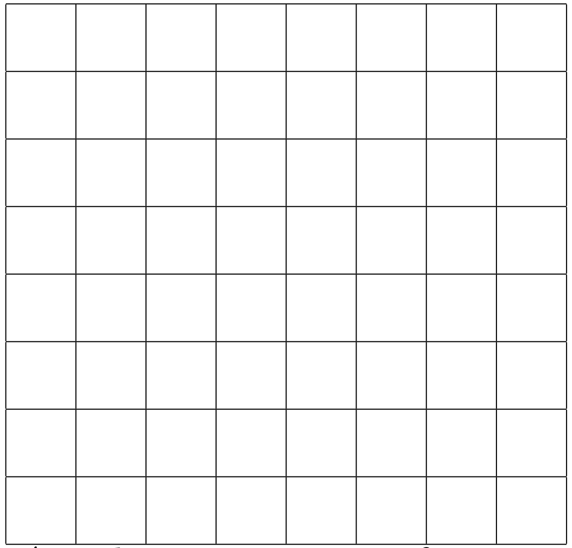

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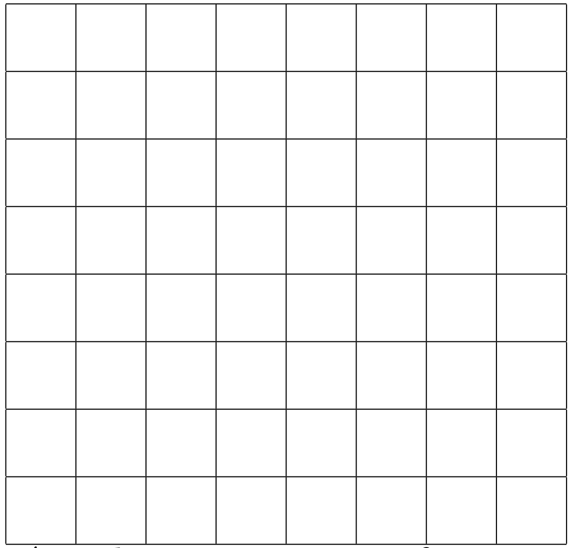

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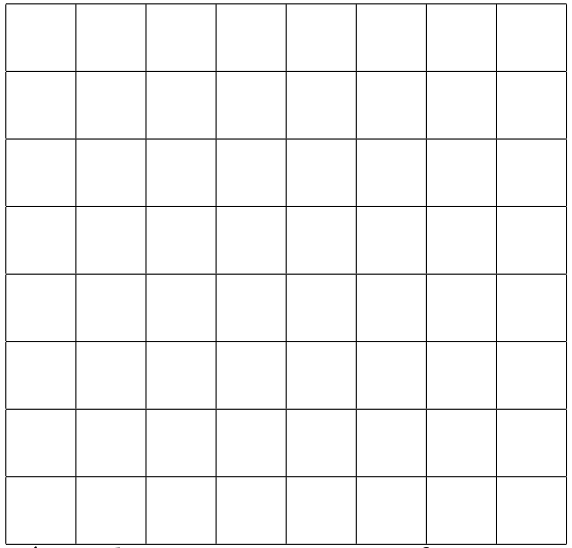

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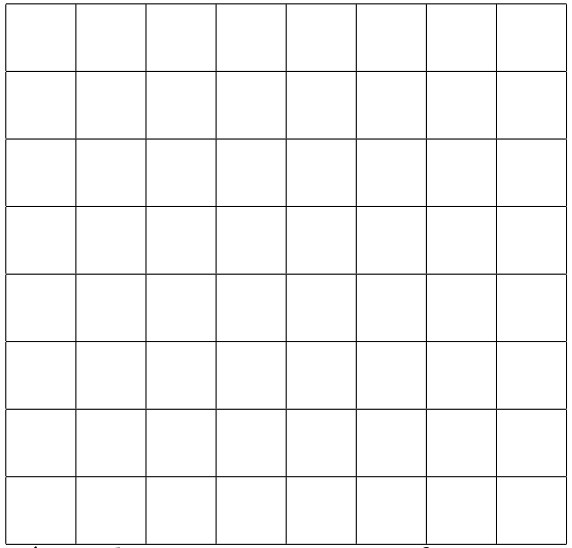

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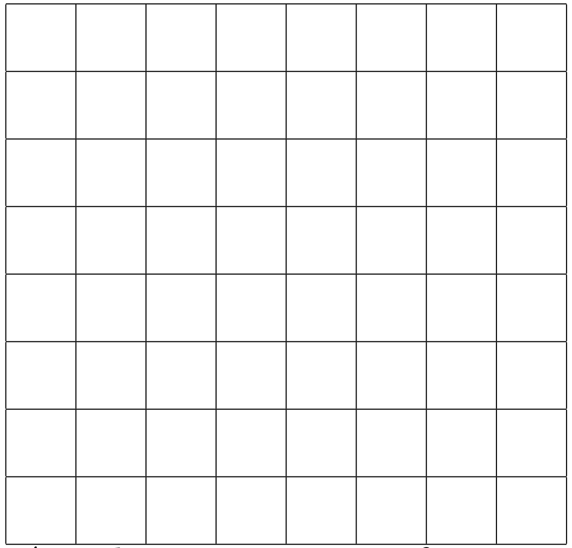

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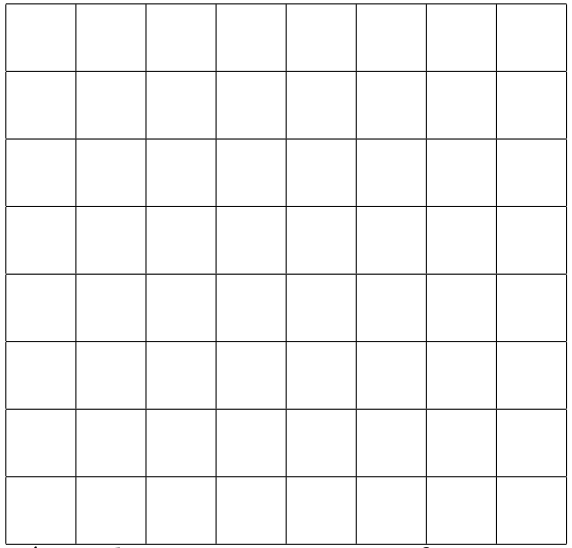

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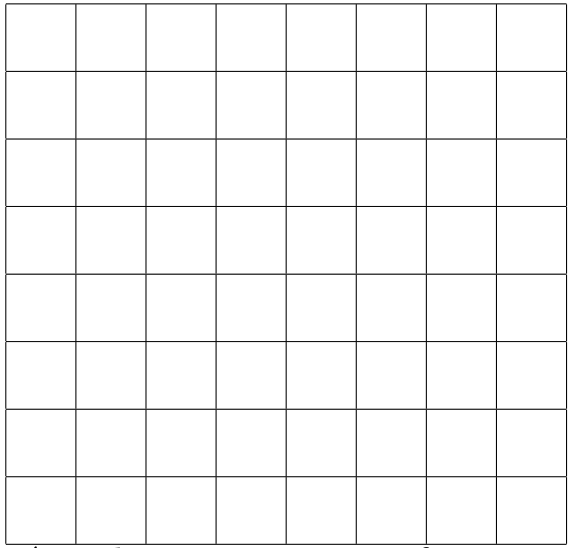

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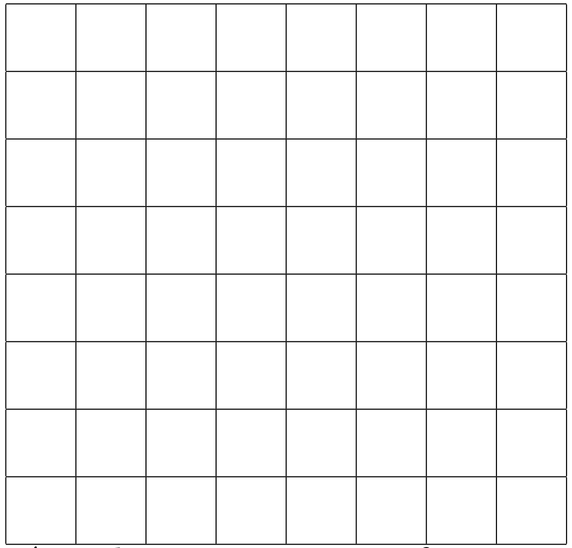

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Proof. $B = \{b_1, \}$ $\square$

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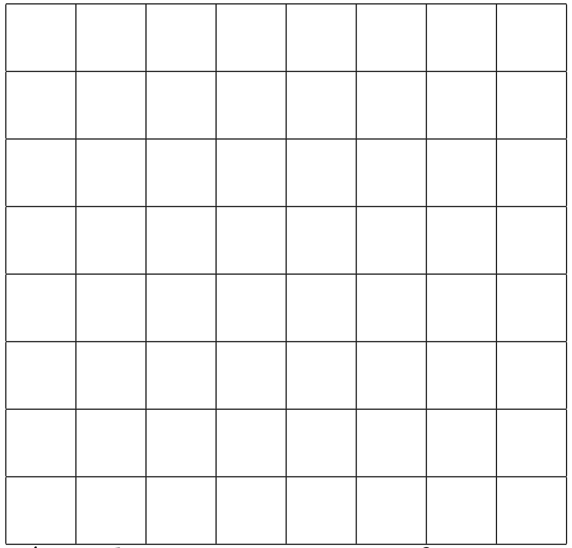

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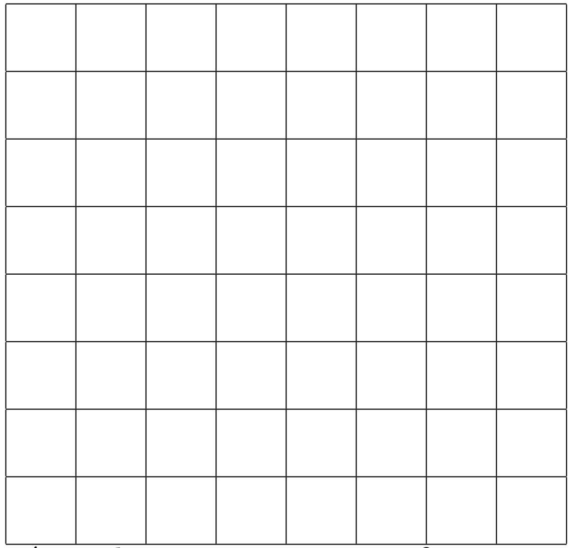

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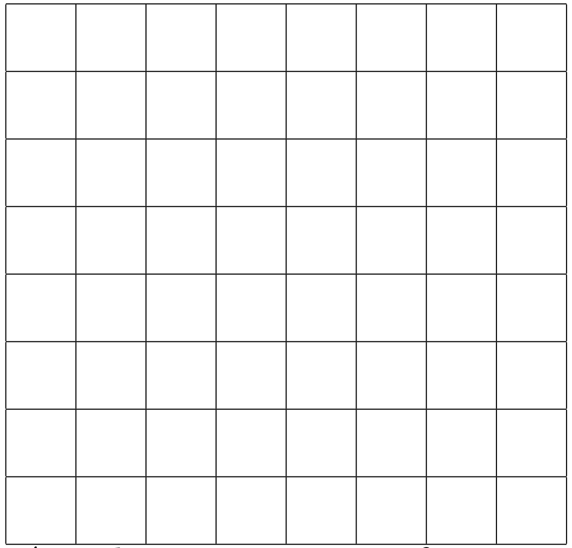

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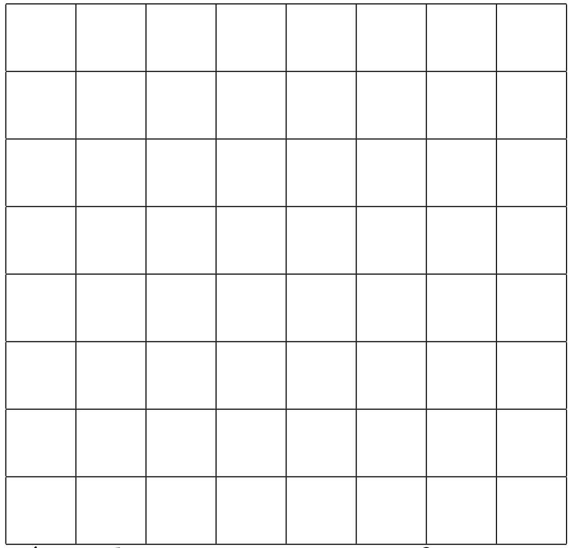
Solution. Divide the square into an 8×8 grid



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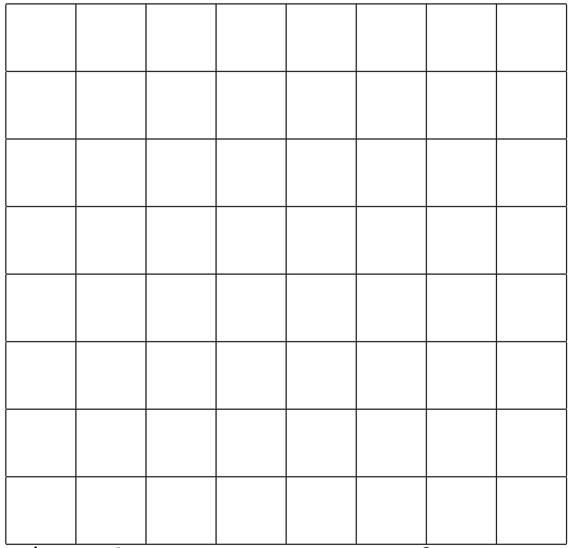

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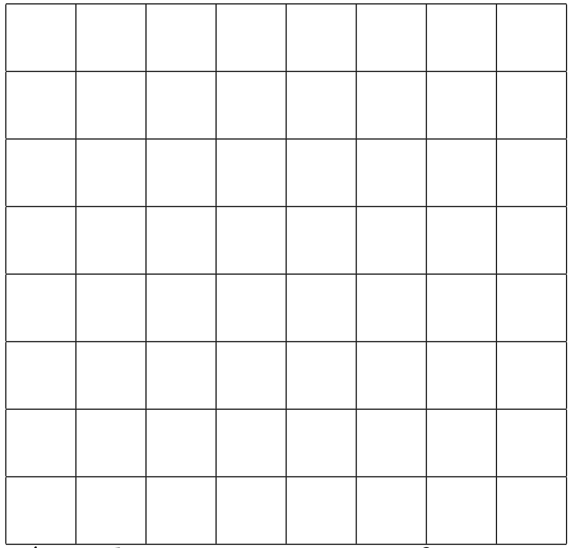

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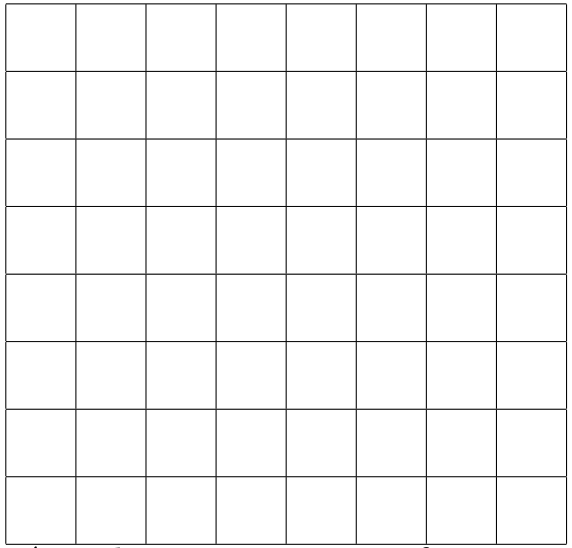

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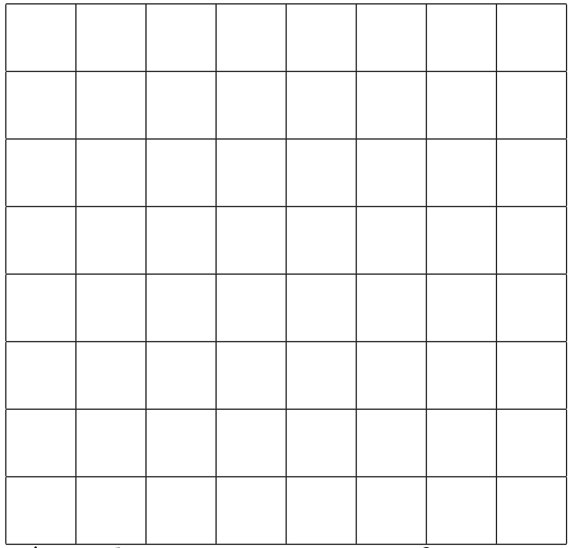

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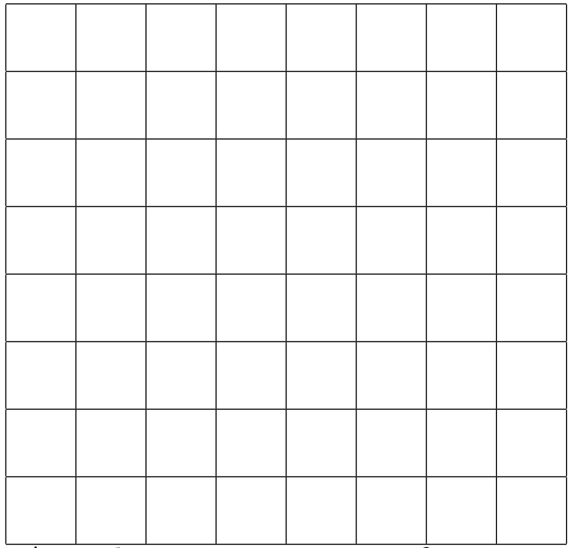

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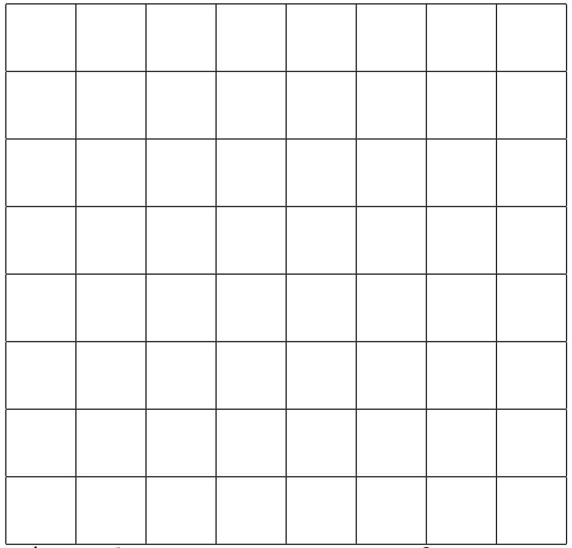

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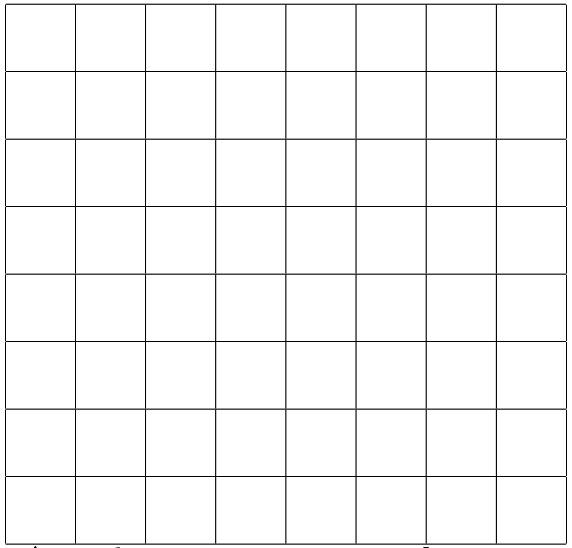

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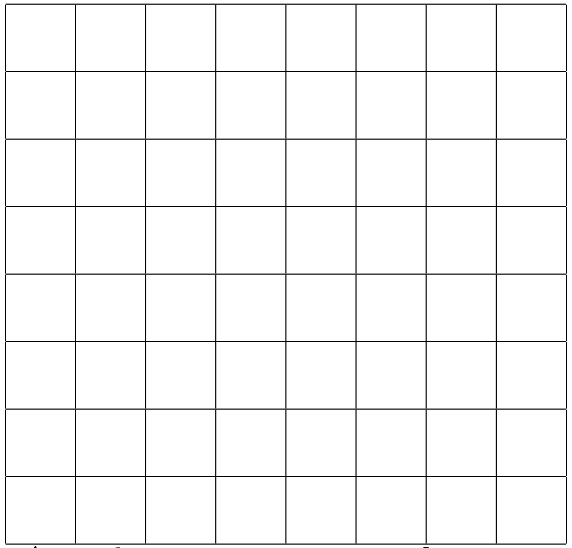

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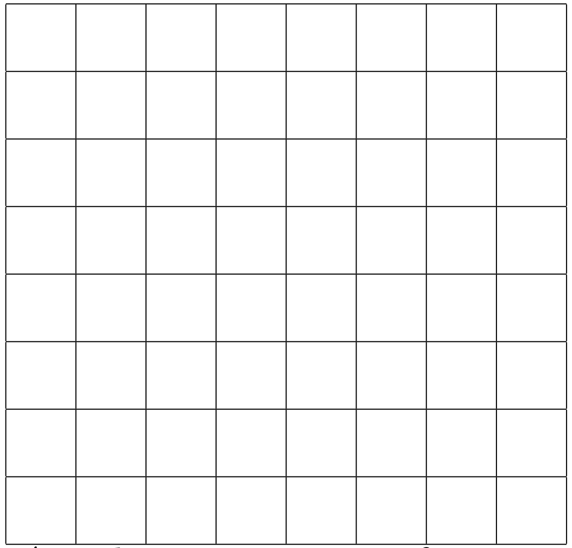

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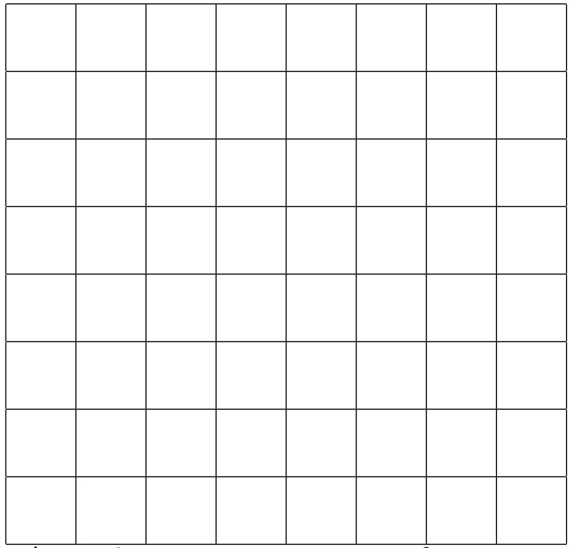

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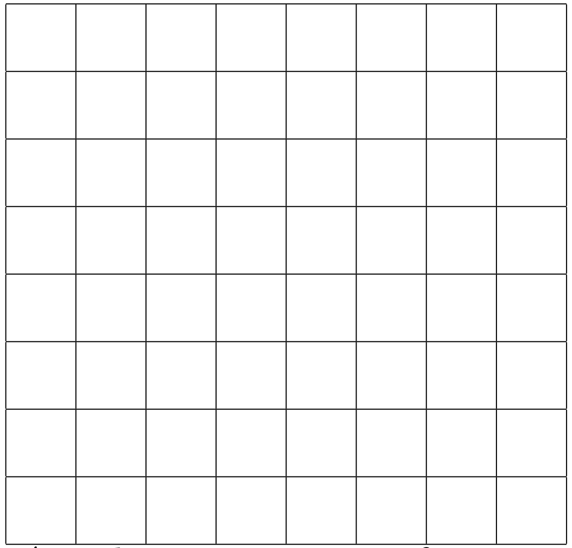

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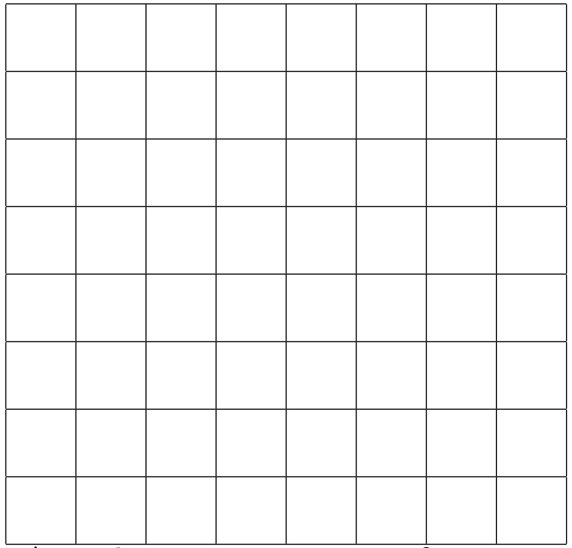

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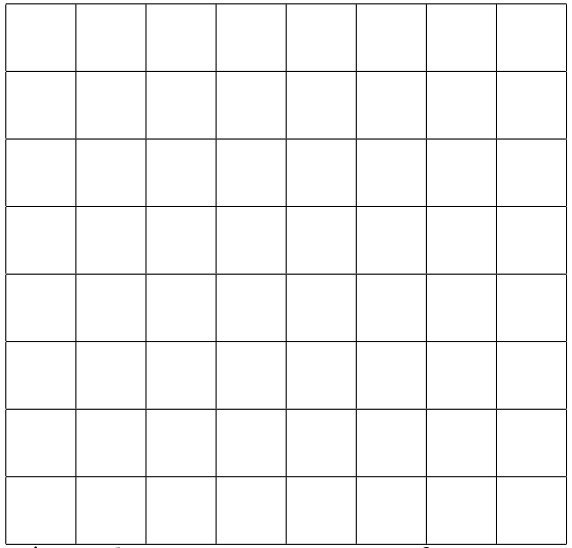

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