MTH307
- Discrete Mathematics

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Principles And Techniques In Combinatorics
by Chuan Chong Chen and Khee-meng Koh

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Grading scheme:

Qizzes:	30%
Midsem:	30%
Final:	40%

The summation principal

The summation principal

$A_1,$

The summation principal

A_1, A_2

The summation principal

A_1, A_2 finite sets

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$A_1 \cap A_2 = \emptyset$

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$$|A_1 \cup A_2|$$

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$$|A_1 \cup A_2| = |A_1| +$$

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The summation principal

$A_1, A_2, \dots, A_k$ finite sets
$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \ldots \cup A_k| = |A_1| + |A_2| + \cdots + |A_k|$$

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Example.

The summation principal

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Example.
A

The summation principal

$A_1, A_2, \ldots, A_k$ finite sets
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Example.
A
B

The summation principal

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Example.

$|A|$

B

The summation principal

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$$|A_1 \cup A_2 \cup \ldots \cup A_k| = |A_1| + |A_2| + \cdots + |A_k|$$

Example.
$|A| = 10$
B

The summation principal

$A_1, A_2, \ldots, A_k$ finite sets
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Example.
$|A| = 10$
$|B|$

The summation principal

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Example.
$|A| = 10$
$|B| = 15$

The summation principal

$A_1, A_2, \ldots, A_k$ finite sets
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Example.
$|A| = 10$
$|B| = 15$
$A \cap B$

The summation principal

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Example.
$|A| = 10$
$|B| = 15$
$|A \cap B|$

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Example.
$|A| = 10$
$|B| = 15$
$|A \cap B| = 3$

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Example.
$|A| = 10$
$|B| = 15$
$|A \cap B| = 3$
$|A \cup B| =??$

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Example.
$|A| = 10$
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$|A \cap B| = 3$
$|A \cup B| = ??$

$|A \setminus A \cap B| = ?$

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Example.
$|A| = 10$
$|B| = 15$
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because, $|A \cap B| + |A \setminus A \cap B| = |A|$

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because, $\underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$

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Example.
$|A| = 10$
$|B| = 15$
$|A \cap B| = 3$
$|A \cup B| = ??$

$|A \setminus A \cap B| = 7$
because, $\underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$

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because, $\underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$

$|B \setminus A \cap B| = ??$
because, $|A \cap B| + |B \setminus A \cap B| = |B|$

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$|A \setminus A \cap B| = 7$
because, $\underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$

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$$|A \cup B| = |A \setminus A \cap B|$$

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$$|A \cup B| = |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B|$$
$$= 7$$

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$|A| = 10$
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$|A \setminus A \cap B| = 7$
because, $\underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$

$|B \setminus A \cap B| = 12$
because, $\underbrace{|A \cap B|}_{3} + |B \setminus A \cap B| = \underbrace{|B|}_{15}$

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$$|A \cup B| = |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B|$$
$$= 7 + 12$$

Example.
$|A| = 10$
$|B| = 15$
$|A \cap B| = 3$
$|A \cup B| = ??$

$|A \setminus A \cap B| = 7$
because, $\underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$

$|B \setminus A \cap B| = 12$
because, $\underbrace{|A \cap B|}_{3} + |B \setminus A \cap B| = \underbrace{|B|}_{15}$

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$$\begin{aligned} |A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= 7 + 12 + 3 \end{aligned}$$

Example.
$|A| = 10$
$|B| = 15$
$|A \cap B| = 3$
$|A \cup B| = ??$

$|A \setminus A \cap B| = 7$
because, $\underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$

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Example.
$|A| = 10$
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$|A \setminus A \cap B| = 7$
because, $\underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$

$|B \setminus A \cap B| = 12$
because, $\underbrace{|A \cap B|}_{3} + |B \setminus A \cap B| = \underbrace{|B|}_{15}$

$$\begin{aligned} |A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= 7 + 12 + 3 \\ &= 22 \end{aligned}$$

Generalizes to

The summation principal

$A_1, A_2, \ldots, A_k$ finite sets
$A_i \cap A_j = \emptyset$ for $i \neq j$ then

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Example.
$|A| = 10$
$|B| = 15$
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$|A \setminus A \cap B| = 7$
because, $\underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$

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because, $\underbrace{|A \cap B|}_{3} + |B \setminus A \cap B| = \underbrace{|B|}_{15}$

$$\begin{aligned} |A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= 7 + 12 + 3 \\ &= 22 \end{aligned}$$

Generalizes to

$$|A \cup B|$$

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$|B \setminus A \cap B| = 12$
because, $\underbrace{|A \cap B|}_{3} + |B \setminus A \cap B| = \underbrace{|B|}_{15}$

$$\begin{aligned} |A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= 7 + 12 + 3 \\ &= 22 \end{aligned}$$

Generalizes to

$$|A \cup B| = |A \setminus A \cap B|$$

The summation principal

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because, $\underbrace{|A \cap B|}_{3} + |B \setminus A \cap B| = \underbrace{|B|}_{15}$

$$\begin{aligned} |A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= 7 + 12 + 3 \\ &= 22 \end{aligned}$$

Generalizes to

$$|A \cup B| = |A \setminus A \cap B| + |B \setminus A \cap B|$$

The summation principal

$A_1, A_2, \ldots, A_k$ finite sets
$A_i \cap A_j = \emptyset$ for $i \neq j$ then

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Example.
$|A| = 10$
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because, $\underbrace{|A \cap B|}_{3} + |A \setminus A \cap B| = \underbrace{|A|}_{10}$

$|B \setminus A \cap B| = 12$
because, $\underbrace{|A \cap B|}_{3} + |B \setminus A \cap B| = \underbrace{|B|}_{15}$

$$\begin{aligned} |A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= 7 + 12 + 3 \\ &= 22 \end{aligned}$$

Generalizes to

$$|A \cup B| = |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B|$$

The summation principal

$A_1, A_2, \ldots, A_k$ finite sets
$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \ldots \cup A_k| = |A_1| + |A_2| + \cdots + |A_k|$$

Example.
$|A| = 10$
$|B| = 15$
$|A \cap B| = 3$
$|A \cup B| =??$

$|A \setminus A \cap B| = 7$
because, $$|A \setminus A \cap B| = |A| - |A \cap B|$$

$|B \setminus A \cap B| = 12$
because, $\underbrace{|A \cap B|}_{3} + |B \setminus A \cap B| = \underbrace{|B|}_{15}$

$$\begin{aligned} |A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= 7 + 12 + 3 \\ &= 22 \end{aligned}$$

Generalizes to

$$\begin{aligned} |A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= (|A| - |A \cap B|) \end{aligned}$$

The summation principal

$A_1, A_2, \ldots, A_k$ finite sets
$A_i \cap A_j = \emptyset$ for $i \neq j$ then

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Example.
$|A| = 10$
$|B| = 15$
$|A \cap B| = 3$
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$|A \setminus A \cap B| = 7$
because, $$|A \setminus A \cap B| = |A| - |A \cap B|$$

$|B \setminus A \cap B| = 12$
because, $$|B \setminus A \cap B| = |B| - |A \cap B|$$

$$\begin{aligned} |A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= 7 + 12 + 3 \\ &= 22 \end{aligned}$$

Generalizes to

$$\begin{aligned} |A \cup B| &= |A \setminus A \cap B| + |B \setminus A \cap B| + |A \cap B| \\ &= (|A| - |A \cap B|) + (|B| - |A \cap B|) \end{aligned}$$

The summation principal

$A_1, A_2, \ldots, A_k$ finite sets
$A_i \cap A_j = \emptyset$ for $i \neq j$ then

$$|A_1 \cup A_2 \cup \ldots \cup A_k| = |A_1| + |A_2| + \cdots + |A_k|$$

Example.
$|A| = 10$
$|B| = 15$
$|A \cap B| = 3$
$|A \cup B| =??$

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because, $$|A \setminus A \cap B| = |A| - |A \cap B|$$

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$$\boxed{|A \cup B| = |A| + |B| - |A \cap B|}$$

The summation principal

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The multiplication principal

The multiplication principal

$A_1,$

The multiplication principal

A_1, A_2

The multiplication principal

A_1, A_2 finite sets

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A_1, A_2 finite sets

The multiplication principal

A_1, A_2 finite sets

$A_1 \times$

The multiplication principal

A_1, A_2 finite sets

$$A_1 \times A_2$$

The multiplication principal

A_1, A_2 finite sets

$$|A_1 \times A_2|$$

The multiplication principal

A_1, A_2 finite sets

$$|A_1 \times A_2| = |A_1| \times$$

The multiplication principal

A_1, A_2 finite sets

$$|A_1 \times A_2| = |A_1| \times |A_2|$$

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The multiplication principal

$A_1, A_2, \ldots, A_k$ finite sets

$$|A_1 \times A_2 \times \ldots \times A_k| = |A_1| \times |A_2| \times \cdots \times |A_k|$$

The multiplication principal

$A_1, A_2, \ldots, A_k$ finite sets

$$|A_1 \times A_2 \times \ldots \times A_k| = |A_1| \times |A_2| \times \cdots \times |A_k|$$

Example. Number of ways

The multiplication principal

$A_1, A_2, \ldots, A_k$ finite sets

$$|A_1 \times A_2 \times \ldots \times A_k| = |A_1| \times |A_2| \times \cdots \times |A_k|$$

Example. Number of ways to arrange r distinct objects

The multiplication principal

$A_1, A_2, \ldots, A_k$ finite sets

$$|A_1 \times A_2 \times \ldots \times A_k| = |A_1| \times |A_2| \times \cdots \times |A_k|$$

Example. Number of ways to arrange r distinct objects out of n objects?

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$A_1, A_2, \ldots, A_k$ finite sets

$$|A_1 \times A_2 \times \ldots \times A_k| = |A_1| \times |A_2| \times \cdots \times |A_k|$$

Example. Number of ways to arrange r distinct objects out of n objects?
X_r: total number of arrangements
Each arrangement,

The multiplication principal

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Example. Number of ways to arrange r distinct objects out of n objects?
X_r: total number of arrangements
Each arrangement,

$(x_1, x_2, \ldots, x_r)$

The multiplication principal

$A_1, A_2, \ldots, A_k$ finite sets

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X_r: total number of arrangements
Each arrangement,

$(x_1, x_2, \ldots, x_r) \in$

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$$|A_1 \times A_2 \times \ldots \times A_k| = |A_1| \times |A_2| \times \cdots \times |A_k|$$

Example. Number of ways to arrange r distinct objects out of n objects?
X_r: total number of arrangements
Each arrangement,

$$(x_1, x_2, \ldots, x_r) \in A \times (A \backslash x_1) \times \cdots \times (A \backslash \{x_1, x_2, \ldots, x_{r-1}\})$$

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A B C D E F G H

$(3, 2, 4) \to$

Example. Number of ways to arrange r distinct objects out of n objects?

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Is a bijection

$$|X_r| =$$

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$$|X_r| = |[n]| \times |[n-1]| \times \ldots \times |[n-r+1]| = \frac{n!}{(n-r)!}$$

Example. Number of ways to arrange r distinct objects out of n objects in a circle?

The multiplication principal

$A_1, A_2, \dots, A_k$ finite sets

$$|A_1 \times A_2 \times \ldots \times A_k| = |A_1| \times |A_2| \times \cdots \times |A_k|$$

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Example. Number of ways to arrange r distinct objects out of n objects in a circle?

C_r : set of circular arrangements

The multiplication principal

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C_r : set of circular arrangements

Define the relation,

The multiplication principal

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Example. Number of ways to arrange r distinct objects out of n objects in a circle?

C_r : set of circular arrangements

Define the relation,

$$(x_1, x_2, \ldots, x_r) \sim (x_r, x_1, \ldots, x_{r-1})$$

The multiplication principal

$A_1, A_2, \ldots, A_k$ finite sets

$$|A_1 \times A_2 \times \ldots \times A_k| = |A_1| \times |A_2| \times \cdots \times |A_k|$$

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The multiplication principal

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$$[n] \times [n-1] \times \cdots \times [n-r+1] \to X_r$$
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Example. Number of ways to arrange r distinct objects out of n objects in a circle?
C_r : set of circular arrangements
Define the relation,

$$(x_1, x_2, \dots, x_r) \sim (x_r, x_1, \dots, x_{r-1}) \sim \dots$$

$\sim$ is an equivalence relation.

The multiplication principal

$A_1, A_2, \ldots, A_k$ finite sets

$$|A_1 \times A_2 \times \ldots \times A_k| = |A_1| \times |A_2| \times \cdots \times |A_k|$$

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Example. Number of ways to arrange r distinct objects out of n objects in a circle?
C_r : set of circular arrangements
Define the relation,

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$\sim$ is an equivalence relation.
Splits into equivalence classes

$$X_r := \sqcup A_k$$

The multiplication principal

$A_1, A_2, \dots, A_k$ finite sets

$$|A_1 \times A_2 \times \dots \times A_k| = |A_1| \times |A_2| \times \cdots \times |A_k|$$

Example. Number of ways to arrange r distinct objects out of n objects?
X_r: total number of arrangements
Each arrangement,

$$(x_1, x_2, \dots, x_r) \in A \times (A \backslash x_1) \times \cdots \times (A \backslash \{x_1, x_2, \dots, x_{r-1}\})$$

$$[n] := \{1, 2, \dots, n\}$$

$$[n] \times [n-1] \times \cdots \times [n-r+1] \to X_r$$
$$(n_1, n_2, \dots, n_r) \to (x_{n_1}, x_{n_2}, \dots, x_{n_r})$$

Is a bijection

$$|X_r| = |[n]| \times |[n-1]| \times \dots \times |[n-r+1]| = \frac{n!}{(n-r)!}$$

Example. Number of ways to arrange r distinct objects out of n objects in a circle?
C_r : set of circular arrangements
Define the relation,

$$(x_1, x_2, \dots, x_r) \sim (x_r, x_1, \dots, x_{r-1}) \sim \dots$$

$\sim$ is an equivalence relation.
Splits into equivalence classes

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