Definition (Ramsay Number)**.**

Definition (Ramsay Number)**.** $R(p, q)$ the smallest number n

Definition (Ramsay Number)**.** $R(p, q)$ the smallest number n such that a graph with n vertices,

Definition (Ramsay Number)**.** $R(p, q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge,

Definition (Ramsay Number)**.** $R(p, q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue,

Definition (Ramsay Number)**.** $R(p, q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red,

Definition (Ramsay Number)**.** $R(p, q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Definition (Ramsay Number)**.** $R(p, q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example.

Definition (Ramsay Number)**.** $R(p,q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example. Consider a graph with

$$n = R(p-1,q) + R(p,q-1) - 1$$

vertices,

Definition (Ramsay Number)**.** $R(p,q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example. Consider a graph with

$$n = R(p-1,q) + R(p,q-1) - 1$$

vertices, such that every pair vertices is joined by exactly one edge

Definition (Ramsay Number)**.** $R(p,q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example. Consider a graph with

$$n = R(p-1,q) + R(p,q-1) - 1$$

vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue.

Definition (Ramsay Number)**.** $R(p, q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example. Consider a graph with

$$n = R(p-1, q) + R(p, q-1) - 1$$

vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue.
Assume $R(p-1, q)$ and $R(p, q-1)$ are both even.

Definition (Ramsay Number)**.** $R(p, q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example. Consider a graph with

$$n = R(p-1, q) + R(p, q-1) - 1$$

vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue. *Assume $R(p-1, q)$ and $R(p, q-1)$ are both even.* Show that there is a red p-clique or a blue q-clique.

Definition (Ramsay Number)**.** $R(p, q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example. Consider a graph with

$$n = R(p-1, q) + R(p, q-1) - 1$$

vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue.
Assume $R(p-1, q)$ *and* $R(p, q-1)$ *are both even.*
Show that there is a red p-clique or a blue q-clique.

Solution.

Definition (Ramsay Number)**.** $R(p, q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example. Consider a graph with

$$n = R(p-1, q) + R(p, q-1) - 1$$

vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue. *Assume* $R(p-1, q)$ *and* $R(p, q-1)$ *are both even.* Show that there is a red p-clique or a blue q-clique.

Solution.
Let the set of vertices be,

Definition (Ramsay Number)**.** $R(p,q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example. Consider a graph with

$$n = R(p-1,q) + R(p,q-1) - 1$$

vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue.
Assume $R(p-1,q)$ *and* $R(p,q-1)$ *are both even.*
Show that there is a red p-clique or a blue q-clique.

Solution.
Let the set of vertices be, $V :=$

Definition (Ramsay Number)**.** $R(p, q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example. Consider a graph with

$$n = R(p-1, q) + R(p, q-1) - 1$$

vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue.
Assume $R(p-1, q)$ and $R(p, q-1)$ are both even.
Show that there is a red p-clique or a blue q-clique.

Solution.
Let the set of vertices be, $V := \{v_1, v_2, \ldots, v_n\}$

Definition (Ramsay Number)**.** $R(p,q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example. Consider a graph with

$$n = R(p-1,q) + R(p,q-1) - 1$$

vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue.
Assume $R(p-1,q)$ and $R(p,q-1)$ are both even.
Show that there is a red p-clique or a blue q-clique.

Solution.
Let the set of vertices be, $V := \{v_1, v_2, \ldots, v_n\}$
v_1 is joined with $n-1$ vertices.

Definition (Ramsay Number)**.** $R(p,q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example. Consider a graph with

$$n = R(p-1,q) + R(p,q-1) - 1$$

vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue.
Assume $R(p-1,q)$ and $R(p,q-1)$ are both even.
Show that there is a red p-clique or a blue q-clique.

Solution.
Let the set of vertices be, $V := \{v_1, v_2, \ldots, v_n\}$
v_1 is joined with $n-1$ vertices.
Case 1: At least $R(p-1,q)$ edges are red

Definition (Ramsay Number)**.** $R(p,q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example. Consider a graph with

$$n = R(p-1,q) + R(p,q-1) - 1$$

vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue.
Assume $R(p-1,q)$ and $R(p,q-1)$ are both even.
Show that there is a red p-clique or a blue q-clique.

Solution.
Let the set of vertices be, $V := \{v_1, v_2, \ldots, v_n\}$
v_1 is joined with $n-1$ vertices.
Case 1: At least $R(p-1,q)$ edges are red
Let those vertices connected to v_1 via red edges be $\{v_{i_1}, v_{i_2}, \ldots, v_{i_m}\}$, where $m = R(p-1,q)$

Definition (Ramsay Number)**.** $R(p, q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example. Consider a graph with

$$n = R(p-1, q) + R(p, q-1) - 1$$

vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue.
Assume $R(p-1, q)$ and $R(p, q-1)$ are both even.
Show that there is a red p-clique or a blue q-clique.

Solution.
Let the set of vertices be, $V := \{v_1, v_2, \ldots, v_n\}$
v_1 is joined with $n-1$ vertices.
Case 1: At least $R(p-1, q)$ edges are red
Let those vertices connected to v_1 via red edges be $\{v_{i_1}, v_{i_2}, \ldots, v_{i_m}\}$, where $m = R(p-1, q)$

The sub-graph formed out of those vertices

Definition (Ramsay Number)**.** $R(p, q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example. Consider a graph with

$$n = R(p-1, q) + R(p, q-1) - 1$$

vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue.
Assume $R(p-1, q)$ and $R(p, q-1)$ are both even.
Show that there is a red p-clique or a blue q-clique.

Solution.
Let the set of vertices be, $V := \{v_1, v_2, \ldots, v_n\}$
v_1 is joined with $n-1$ vertices.
Case 1: At least $R(p-1, q)$ edges are red
Let those vertices connected to v_1 via red edges be $\{v_{i_1}, v_{i_2}, \ldots, v_{i_m}\}$, where $m = R(p-1, q)$
The sub-graph formed out of those vertices either has a q blue clique,

Definition (Ramsay Number)**.** $R(p, q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example. Consider a graph with

$$n = R(p-1, q) + R(p, q-1) - 1$$

vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue.
Assume $R(p-1, q)$ and $R(p, q-1)$ are both even.
Show that there is a red p-clique or a blue q-clique.

Solution.
Let the set of vertices be, $V := \{v_1, v_2, \ldots, v_n\}$
v_1 is joined with $n-1$ vertices.
Case 1: At least $R(p-1, q)$ edges are red
Let those vertices connected to v_1 via red edges be $\{v_{i_1}, v_{i_2}, \ldots, v_{i_m}\}$, where $m = R(p-1, q)$
The sub-graph formed out of those vertices either has a q blue clique, then we are done.

Definition (Ramsay Number)**.** $R(p, q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example. Consider a graph with

$$n = R(p-1, q) + R(p, q-1) - 1$$

vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue.
Assume $R(p-1, q)$ and $R(p, q-1)$ are both even.
Show that there is a red p-clique or a blue q-clique.

Solution.
Let the set of vertices be, $V := \{v_1, v_2, \ldots, v_n\}$
v_1 is joined with $n-1$ vertices.
Case 1: At least $R(p-1, q)$ edges are red
Let those vertices connected to v_1 via red edges be $\{v_{i_1}, v_{i_2}, \ldots, v_{i_m}\}$, where $m = R(p-1, q)$

The sub-graph formed out of those vertices either has a q blue clique, then we are done.
Or a $p-1$ red clique,

Definition (Ramsay Number)**.** $R(p, q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example. Consider a graph with

$$n = R(p-1, q) + R(p, q-1) - 1$$

vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue.
Assume $R(p-1, q)$ and $R(p, q-1)$ are both even.
Show that there is a red p-clique or a blue q-clique.

Solution.
Let the set of vertices be, $V := \{v_1, v_2, \ldots, v_n\}$
v_1 is joined with $n-1$ vertices.
Case 1: At least $R(p-1, q)$ edges are red
Let those vertices connected to v_1 via red edges be $\{v_{i_1}, v_{i_2}, \ldots, v_{i_m}\}$, where $m = R(p-1, q)$
The sub-graph formed out of those vertices either has a q blue clique, then we are done.
Or a $p-1$ red clique,
which combined with v_1, forms a p red clique.

Definition (Ramsay Number)**.** $R(p, q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example. Consider a graph with

$$n = R(p-1, q) + R(p, q-1) - 1$$

vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue.
Assume $R(p-1, q)$ and $R(p, q-1)$ are both even.
Show that there is a red p-clique or a blue q-clique.

Solution.
Let the set of vertices be, $V := \{v_1, v_2, \ldots, v_n\}$
v_1 is joined with $n-1$ vertices.
Case 1: At least $R(p-1, q)$ edges are red
Let those vertices connected to v_1 via red edges be $\{v_{i_1}, v_{i_2}, \ldots, v_{i_m}\}$, where $m = R(p-1, q)$

The sub-graph formed out of those vertices either has a q blue clique, then we are done.
Or a $p-1$ red clique,
which combined with v_1, forms a p red clique.
then we are done.

Definition (Ramsay Number)**.** $R(p, q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example. Consider a graph with

$$n = R(p-1, q) + R(p, q-1) - 1$$

vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue.
Assume $R(p-1, q)$ and $R(p, q-1)$ are both even.
Show that there is a red p-clique or a blue q-clique.

Solution.
Let the set of vertices be, $V := \{v_1, v_2, \ldots, v_n\}$
v_1 is joined with $n-1$ vertices.
Case 1: At least $R(p-1, q)$ edges are red
Let those vertices connected to v_1 via red edges be $\{v_{i_1}, v_{i_2}, \ldots, v_{i_m}\}$, where $m = R(p-1, q)$
The sub-graph formed out of those vertices either has a q blue clique, then we are done.
Or a $p-1$ red clique,
which combined with v_1, forms a p red clique.
then we are done.

Case 2: At least $R(p, q-1)$ edges are blue

Definition (Ramsay Number)**.** $R(p, q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example. Consider a graph with

$$n = R(p-1, q) + R(p, q-1) - 1$$

vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue.
Assume $R(p-1, q)$ and $R(p, q-1)$ are both even.
Show that there is a red p-clique or a blue q-clique.

Solution.
Let the set of vertices be, $V := \{v_1, v_2, \ldots, v_n\}$
v_1 is joined with $n-1$ vertices.
Case 1: At least $R(p-1, q)$ edges are red
Let those vertices connected to v_1 via red edges be $\{v_{i_1}, v_{i_2}, \ldots, v_{i_m}\}$, where $m = R(p-1, q)$
The sub-graph formed out of those vertices either has a q blue clique, then we are done.
Or a $p-1$ red clique,
which combined with v_1, forms a p red clique.
then we are done.

Case 2: At least $R(p, q-1)$ edges are blue
Simlar to case 1

Definition (Ramsay Number)**.** $R(p, q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example. Consider a graph with

$$n = R(p-1, q) + R(p, q-1) - 1$$

vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue.
Assume $R(p-1, q)$ and $R(p, q-1)$ are both even.
Show that there is a red p-clique or a blue q-clique.

Solution.
Let the set of vertices be, $V := \{v_1, v_2, \ldots, v_n\}$
v_1 is joined with $n-1$ vertices.
Case 1: At least $R(p-1, q)$ edges are red
Let those vertices connected to v_1 via red edges be $\{v_{i_1}, v_{i_2}, \ldots, v_{i_m}\}$, where $m = R(p-1, q)$
The sub-graph formed out of those vertices either has a q blue clique, then we are done.
Or a $p-1$ red clique,
which combined with v_1, forms a p red clique.
then we are done.

Case 2: At least $R(p, q-1)$ edges are blue
Simlar to case 1

Case 3: Exactly $R(p-1, q) - 1$ red edges,

Definition (Ramsay Number)**.** $R(p, q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example. Consider a graph with

$$n = R(p-1, q) + R(p, q-1) - 1$$

vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue.
Assume $R(p-1, q)$ and $R(p, q-1)$ are both even.
Show that there is a red p-clique or a blue q-clique.

Solution.
Let the set of vertices be, $V := \{v_1, v_2, \dots, v_n\}$
v_1 is joined with $n-1$ vertices.
Case 1: At least $R(p-1, q)$ edges are red
Let those vertices connected to v_1 via red edges be $\{v_{i_1}, v_{i_2}, \dots, v_{i_m}\}$, where $m = R(p-1, q)$
The sub-graph formed out of those vertices either has a q blue clique, then we are done.
Or a $p-1$ red clique,
which combined with v_1, forms a p red clique.
then we are done.

Case 2: At least $R(p, q-1)$ edges are blue
Simlar to case 1

Case 3: Exactly $R(p-1, q)-1$ red edges, **and** $R(p, q-1)-1$ blue edges

Definition (Ramsay Number)**.** $R(p, q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example. Consider a graph with

$$n = R(p-1, q) + R(p, q-1) - 1$$

vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue.
Assume $R(p-1, q)$ and $R(p, q-1)$ are both even.
Show that there is a red p-clique or a blue q-clique.

Solution.
Let the set of vertices be, $V := \{v_1, v_2, \ldots, v_n\}$
v_1 is joined with $n-1$ vertices.
Case 1: At least $R(p-1, q)$ edges are red
Let those vertices connected to v_1 via red edges be $\{v_{i_1}, v_{i_2}, \ldots, v_{i_m}\}$, where $m = R(p-1, q)$
The sub-graph formed out of those vertices
either has a q blue clique, then we are done.
Or a $p-1$ red clique,
which combined with v_1, forms a p red clique.
then we are done.

Case 2: At least $R(p, q-1)$ edges are blue
Simlar to case 1

Case 3: Exactly $R(p-1, q) - 1$ red edges, **and** $R(p, q-1) - 1$ blue edges adjacent to **each** vertex.

Definition (Ramsay Number)**.** $R(p,q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example. Consider a graph with

$$n = R(p-1,q) + R(p,q-1) - 1$$

vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue.
Assume $R(p-1,q)$ and $R(p,q-1)$ are both even.
Show that there is a red p-clique or a blue q-clique.

Solution.
Let the set of vertices be, $V := \{v_1, v_2, \ldots, v_n\}$
v_1 is joined with $n-1$ vertices.
Case 1: At least $R(p-1,q)$ edges are red
Let those vertices connected to v_1 via red edges be $\{v_{i_1}, v_{i_2}, \ldots, v_{i_m}\}$, where $m = R(p-1,q)$
The sub-graph formed out of those vertices
either has a q blue clique, then we are done.
Or a $p-1$ red clique,
which combined with v_1, forms a p red clique.
then we are done.

Case 2: At least $R(p,q-1)$ edges are blue
Simlar to case 1

Case 3: Exactly $R(p-1,q)-1$ red edges, **and** $R(p,q-1)-1$ blue edges adjacent to **each** vertex.

$$2 \times \text{number of red edges} = n \times (R(p-1,q)-1)$$

Definition (Ramsay Number)**.** $R(p, q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example. Consider a graph with

$$n = R(p-1, q) + R(p, q-1) - 1$$

vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue.
Assume $R(p-1, q)$ and $R(p, q-1)$ are both even.
Show that there is a red p-clique or a blue q-clique.

Solution.
Let the set of vertices be, $V := \{v_1, v_2, \ldots, v_n\}$
v_1 is joined with $n-1$ vertices.
Case 1: At least $R(p-1, q)$ edges are red
Let those vertices connected to v_1 via red edges be $\{v_{i_1}, v_{i_2}, \ldots, v_{i_m}\}$, where $m = R(p-1, q)$
The sub-graph formed out of those vertices either has a q blue clique, then we are done.
Or a $p-1$ red clique,
which combined with v_1, forms a p red clique.
then we are done.

Case 2: At least $R(p, q-1)$ edges are blue
Simlar to case 1

Case 3: Exactly $R(p-1, q) - 1$ red edges, **and** $R(p, q-1) - 1$ blue edges adjacent to **each** vertex.

$2 \times$ number of red edges $= n \times (R(p-1, q) - 1)$
But n is odd

Definition (Ramsay Number)**.** $R(p,q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example. Consider a graph with

$$n = R(p-1,q) + R(p,q-1) - 1$$

vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue.
Assume $R(p-1,q)$ and $R(p,q-1)$ are both even.
Show that there is a red p-clique or a blue q-clique.

Solution.
Let the set of vertices be, $V := \{v_1, v_2, \ldots, v_n\}$
v_1 is joined with $n-1$ vertices.
Case 1: At least $R(p-1,q)$ edges are red
Let those vertices connected to v_1 via red edges be $\{v_{i_1}, v_{i_2}, \ldots, v_{i_m}\}$, where $m = R(p-1,q)$
The sub-graph formed out of those vertices
either has a q blue clique, then we are done.
Or a $p-1$ red clique,
which combined with v_1, forms a p red clique.
then we are done.

Case 2: At least $R(p,q-1)$ edges are blue
Simlar to case 1

Case 3: Exactly $R(p-1,q)-1$ red edges, **and** $R(p,q-1)-1$ blue edges adjacent to **each** vertex.

$2 \times$ number of red edges $= n \times (R(p-1,q)-1)$
But n is odd and $R(p-1,q)-1$ is odd

Definition (Ramsay Number)**.** $R(p,q)$ the smallest number n such that a graph with n vertices, and each pair of vertices is joined by exactly one edge, and each coloured by either red or blue, will have either a p-clique which is entirely red, or a q-clique which is entirely blue.

Example. Consider a graph with

$$n = R(p-1,q) + R(p,q-1) - 1$$

vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue.
Assume $R(p-1,q)$ and $R(p,q-1)$ are both even.
Show that there is a red p-clique or a blue q-clique.

Solution.
Let the set of vertices be, $V := \{v_1, v_2, \ldots, v_n\}$
v_1 is joined with $n-1$ vertices.
Case 1: At least $R(p-1,q)$ edges are red
Let those vertices connected to v_1 via red edges be $\{v_{i_1}, v_{i_2}, \ldots, v_{i_m}\}$, where $m = R(p-1,q)$
The sub-graph formed out of those vertices
either has a q blue clique, then we are done.
Or a $p-1$ red clique,
which combined with v_1, forms a p red clique.
then we are done.

Case 2: At least $R(p,q-1)$ edges are blue
Simlar to case 1

Case 3: Exactly $R(p-1,q)-1$ red edges, **and** $R(p,q-1)-1$ blue edges adjacent to **each** vertex.

$2 \times$ number of red edges $= n \times (R(p-1,q)-1)$
But n is odd and $R(p-1,q)-1$ is odd $\square$

Inclusion and Exclusion principle

$|A_1 \cup A_2 \cup \ldots \cup A_n|$

Inclusion and Exclusion principle

$|A_1 \cup A_2 \cup \ldots \cup A_n| = \Sigma|A_i|$

Inclusion and Exclusion principle

$|A_1 \cup A_2 \cup \ldots \cup A_n| = \Sigma|A_i| -$

Inclusion and Exclusion principle

$|A_1 \cup A_2 \cup \ldots \cup A_n| = \Sigma|A_i| - \Sigma|A_i \cap A_j|$

Inclusion and Exclusion principle

$|A_1 \cup A_2 \cup \ldots \cup A_n| = \Sigma|A_i| - \Sigma|A_i \cap A_j| +$

Inclusion and Exclusion principle

$|A_1 \cup A_2 \cup \ldots \cup A_n| = \Sigma|A_i| - \Sigma|A_i \cap A_j| + \cdots$

Inclusion and Exclusion principle

$|A_1 \cup A_2 \cup \ldots \cup A_n| = \Sigma|A_i| - \Sigma|A_i \cap A_j| + \cdots + (-1)^n|A_1 \cap \ldots \cap A_n|$

Inclusion and Exclusion principle
$|A_1 \cup A_2 \cup \ldots \cup A_n| = \Sigma|A_i| - \Sigma|A_i \cap A_j| + \cdots + (-1)^n|A_1 \cap \ldots \cap A_n|$

$$|A_1 \cup A_2 \cup \ldots \cup A_n|$$

Inclusion and Exclusion principle

$|A_1 \cup A_2 \cup \ldots \cup A_n| = \Sigma|A_i| - \Sigma|A_i \cap A_j| + \cdots + (-1)^n|A_1 \cap \ldots \cap A_n|$

$$|A_1 \cup A_2 \cup \ldots \cup A_n| = |A_1 \cup \ldots \cup A_{n-1}| +$$

Inclusion and Exclusion principle
$|A_1 \cup A_2 \cup \ldots \cup A_n| = \Sigma|A_i| - \Sigma|A_i \cap A_j| + \cdots + (-1)^n|A_1 \cap \ldots \cap A_n|$

$$|A_1 \cup A_2 \cup \ldots \cup A_n| = |A_1 \cup \ldots \cup A_{n-1}| + |A_n| -$$

Inclusion and Exclusion principle
$|A_1 \cup A_2 \cup \ldots \cup A_n| = \Sigma|A_i| - \Sigma|A_i \cap A_j| + \cdots + (-1)^n|A_1 \cap \ldots \cap A_n|$

$$|A_1 \cup A_2 \cup \ldots \cup A_n| = |A_1 \cup \ldots \cup A_{n-1}| + |A_n| - |(A_1 \cup \ldots \cup A_{n-1}) \cap A_n|$$

Inclusion and Exclusion principle
$|A_1 \cup A_2 \cup \ldots \cup A_n| = \Sigma |A_i| - \Sigma |A_i \cap A_j| + \cdots + (-1)^n |A_1 \cap \ldots \cap A_n|$

$$\begin{aligned} |A_1 \cup A_2 \cup \ldots \cup A_n| &= |A_1 \cup \ldots \cup A_{n-1}| + |A_n| - |(A_1 \cup \ldots \cup A_{n-1}) \cap A_n| \\ &= |A_1 \cup \ldots \cup A_{n-1}| + \end{aligned}$$

Inclusion and Exclusion principle
$|A_1 \cup A_2 \cup \ldots \cup A_n| = \Sigma|A_i| - \Sigma|A_i \cap A_j| + \cdots + (-1)^n|A_1 \cap \ldots \cap A_n|$

$$\begin{aligned} |A_1 \cup A_2 \cup \ldots \cup A_n| &= |A_1 \cup \ldots \cup A_{n-1}| + |A_n| - |(A_1 \cup \ldots \cup A_{n-1}) \cap A_n| \\ &= |A_1 \cup \ldots \cup A_{n-1}| + |A_n| - \end{aligned}$$

Inclusion and Exclusion principle

$|A_1 \cup A_2 \cup \ldots \cup A_n| = \Sigma|A_i| - \Sigma|A_i \cap A_j| + \cdots + (-1)^n|A_1 \cap \ldots \cap A_n|$

$$\begin{aligned} |A_1 \cup A_2 \cup \ldots \cup A_n| &= |A_1 \cup \ldots \cup A_{n-1}| + |A_n| - |(A_1 \cup \ldots \cup A_{n-1}) \cap A_n| \\ &= |A_1 \cup \ldots \cup A_{n-1}| + |A_n| - |(A_1 \cap A_n) \cup \ldots \cup (A_{n-1} \cap A_n)| \end{aligned}$$

Inclusion and Exclusion principle
$|A_1 \cup A_2 \cup \ldots \cup A_n| = \Sigma|A_i| - \Sigma|A_i \cap A_j| + \cdots + (-1)^n|A_1 \cap \ldots \cap A_n|$

$$\begin{aligned}|A_1 \cup A_2 \cup \ldots \cup A_n| &= |A_1 \cup \ldots \cup A_{n-1}| + |A_n| - |(A_1 \cup \ldots \cup A_{n-1}) \cap A_n| \\ &= |A_1 \cup \ldots \cup A_{n-1}| + |A_n| - |(A_1 \cap A_n) \cup \ldots \cup (A_{n-1} \cap A_n)| \\ &= (\Sigma|A_i| -\end{aligned}$$

Inclusion and Exclusion principle
$|A_1 \cup A_2 \cup \ldots \cup A_n| = \Sigma|A_i| - \Sigma|A_i \cap A_j| + \cdots + (-1)^n|A_1 \cap \ldots \cap A_n|$

$$
\begin{aligned}
|A_1 \cup A_2 \cup \ldots \cup A_n| &= |A_1 \cup \ldots \cup A_{n-1}| + |A_n| - |(A_1 \cup \ldots \cup A_{n-1}) \cap A_n| \\
&= |A_1 \cup \ldots \cup A_{n-1}| + |A_n| - |(A_1 \cap A_n) \cup \ldots \cup (A_{n-1} \cap A_n)| \\
&= (\Sigma|A_i| - \Sigma|A_i \cap A_j| +
\end{aligned}
$$

Inclusion and Exclusion principle
$|A_1 \cup A_2 \cup \ldots \cup A_n| = \Sigma|A_i| - \Sigma|A_i \cap A_j| + \cdots + (-1)^n|A_1 \cap \ldots \cap A_n|$

$$\begin{aligned}
|A_1 \cup A_2 \cup \ldots \cup A_n| &= |A_1 \cup \ldots \cup A_{n-1}| + |A_n| - |(A_1 \cup \ldots \cup A_{n-1}) \cap A_n| \\
&= |A_1 \cup \ldots \cup A_{n-1}| + |A_n| - |(A_1 \cap A_n) \cup \ldots \cup (A_{n-1} \cap A_n)| \\
&= (\Sigma|A_i| - \Sigma|A_i \cap A_j| + \cdots + (-1)^{n-1}|A_1 \cap \ldots \cap A_{n-1}|)
\end{aligned}$$

Inclusion and Exclusion principle
$|A_1 \cup A_2 \cup \ldots \cup A_n| = \Sigma|A_i| - \Sigma|A_i \cap A_j| + \cdots + (-1)^n|A_1 \cap \ldots \cap A_n|$

$$
\begin{aligned}
|A_1 \cup A_2 \cup \ldots \cup A_n| &= |A_1 \cup \ldots \cup A_{n-1}| + |A_n| - |(A_1 \cup \ldots \cup A_{n-1}) \cap A_n| \\
&= |A_1 \cup \ldots \cup A_{n-1}| + |A_n| - |(A_1 \cap A_n) \cup \ldots \cup (A_{n-1} \cap A_n)| \\
&= (\Sigma|A_i| - \Sigma|A_i \cap A_j| + \cdots + (-1)^{n-1}|A_1 \cap \ldots \cap A_{n-1}|) \\
&\quad + |A_n| -
\end{aligned}
$$

Inclusion and Exclusion principle
$|A_1 \cup A_2 \cup \ldots \cup A_n| = \Sigma|A_i| - \Sigma|A_i \cap A_j| + \cdots + (-1)^n|A_1 \cap \ldots \cap A_n|$

$$\begin{aligned}
|A_1 \cup A_2 \cup \ldots \cup A_n| &= |A_1 \cup \ldots \cup A_{n-1}| + |A_n| - |(A_1 \cup \ldots \cup A_{n-1}) \cap A_n| \\
&= |A_1 \cup \ldots \cup A_{n-1}| + |A_n| - |(A_1 \cap A_n) \cup \ldots \cup (A_{n-1} \cap A_n)| \\
&= (\Sigma|A_i| - \Sigma|A_i \cap A_j| + \cdots + (-1)^{n-1}|A_1 \cap \ldots \cap A_{n-1}|) \\
&\quad + |A_n| - |(A_1 \cap A_n) \cup \ldots \cup (A_{n-1} \cap A_n)|
\end{aligned}$$

Inclusion and Exclusion principle
$|A_1 \cup A_2 \cup \ldots \cup A_n| = \Sigma|A_i| - \Sigma|A_i \cap A_j| + \cdots + (-1)^n|A_1 \cap \ldots \cap A_n|$

$$
\begin{aligned}
|A_1 \cup A_2 \cup \ldots \cup A_n| &= |A_1 \cup \ldots \cup A_{n-1}| + |A_n| - |(A_1 \cup \ldots \cup A_{n-1}) \cap A_n| \\
&= |A_1 \cup \ldots \cup A_{n-1}| + |A_n| - |(A_1 \cap A_n) \cup \ldots \cup (A_{n-1} \cap A_n)| \\
&= (\Sigma|A_i| - \Sigma|A_i \cap A_j| + \cdots + (-1)^{n-1}|A_1 \cap \ldots \cap A_{n-1}|) \\
&\quad + |A_n| - |(A_1 \cap A_n) \cup \ldots \cup (A_{n-1} \cap A_n)|
\end{aligned}
$$

Rest, exercise!

Example. Find the number of

Example. Find the number of positive integers

Example. Find the number of positive integers below
100

Example. Find the number of positive integers below 100 which are divisible by

Example. Find the number of positive integers below 100 which are divisible by 2,

Example. Find the number of positive integers below 100 which are divisible by 2, 5,

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.
A_2

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.
$A_2 := \{2, 4, \ldots, 100\}$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.
$A_2 := \{2, 4, \ldots, 100\}$
$|A_2| =$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.
$A_2 := \{2, 4, \ldots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor =$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.
$A_2 := \{2, 4, \ldots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor = 50$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.
$A_2 := \{2, 4, \ldots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor = 50$

A_5

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.
$A_2 := \{2, 4, \ldots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.

$A_2 := \{2, 4, \ldots, 100\}$

$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$

$|A_5| =$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.

$A_2 := \{2, 4, \ldots, 100\}$

$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$

$|A_5| = \lfloor 100/5 \rfloor =$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.
$A_2 := \{2, 4, \ldots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$
$|A_5| = \lfloor 100/5 \rfloor = 20$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.

$A_2 := \{2, 4, \dots, 100\}$

$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \dots, 100\}$

$|A_5| = \lfloor 100/5 \rfloor = 20$

A_7

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.

$A_2 := \{2, 4, \ldots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$
$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \ldots, 98\}$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.
$A_2 := \{2, 4, \ldots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$
$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \ldots, 98\}$
$|A_7| =$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.

$A_2 := \{2, 4, \ldots, 100\}$

$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$

$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \ldots, 98\}$

$|A_7| = \lfloor 100/7 \rfloor =$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.

$A_2 := \{2, 4, \dots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \dots, 100\}$
$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \dots, 98\}$
$|A_7| = \lfloor 100/7 \rfloor = 14$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.

$A_2 := \{2, 4, \ldots, 100\}$

$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$

$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \ldots, 98\}$

$|A_7| = \lfloor 100/7 \rfloor = 14$

$|A_2 \cap A_5| =$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.

$A_2 := \{2, 4, \ldots, 100\}$

$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$

$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \ldots, 98\}$

$|A_7| = \lfloor 100/7 \rfloor = 14$

$|A_2 \cap A_5| = |A_{10}| =$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.
$A_2 := \{2, 4, \ldots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$
$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \ldots, 98\}$
$|A_7| = \lfloor 100/7 \rfloor = 14$

$|A_2 \cap A_5| = |A_{10}| = \lfloor 100/10 \rfloor =$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.
$A_2 := \{2, 4, \ldots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$
$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \ldots, 98\}$
$|A_7| = \lfloor 100/7 \rfloor = 14$

$|A_2 \cap A_5| = |A_{10}| = \lfloor 100/10 \rfloor = 10$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.

$A_2 := \{2, 4, \ldots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$
$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \ldots, 98\}$
$|A_7| = \lfloor 100/7 \rfloor = 14$

$|A_2 \cap A_5| = |A_{10}| = \lfloor 100/10 \rfloor = 10$
$|A_5 \cap A_7| =$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.

$A_2 := \{2, 4, \ldots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$
$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \ldots, 98\}$
$|A_7| = \lfloor 100/7 \rfloor = 14$

$|A_2 \cap A_5| = |A_{10}| = \lfloor 100/10 \rfloor = 10$
$|A_5 \cap A_7| = |A_{35}| =$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.

$A_2 := \{2, 4, \ldots, 100\}$

$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$

$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \ldots, 98\}$

$|A_7| = \lfloor 100/7 \rfloor = 14$

$|A_2 \cap A_5| = |A_{10}| = \lfloor 100/10 \rfloor = 10$

$|A_5 \cap A_7| = |A_{35}| = \lfloor 100/35 \rfloor =$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.

$A_2 := \{2, 4, \ldots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$
$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \ldots, 98\}$
$|A_7| = \lfloor 100/7 \rfloor = 14$

$|A_2 \cap A_5| = |A_{10}| = \lfloor 100/10 \rfloor = 10$
$|A_5 \cap A_7| = |A_{35}| = \lfloor 100/35 \rfloor = 2$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.

$A_2 := \{2, 4, \ldots, 100\}$

$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$

$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \ldots, 98\}$

$|A_7| = \lfloor 100/7 \rfloor = 14$

$|A_2 \cap A_5| = |A_{10}| = \lfloor 100/10 \rfloor = 10$

$|A_5 \cap A_7| = |A_{35}| = \lfloor 100/35 \rfloor = 2$

$|A_7 \cap A_2| =$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.

$A_2 := \{2, 4, \ldots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$
$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \ldots, 98\}$
$|A_7| = \lfloor 100/7 \rfloor = 14$

$|A_2 \cap A_5| = |A_{10}| = \lfloor 100/10 \rfloor = 10$
$|A_5 \cap A_7| = |A_{35}| = \lfloor 100/35 \rfloor = 2$
$|A_7 \cap A_2| = |A_{14}| =$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.
$A_2 := \{2, 4, \ldots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$
$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \ldots, 98\}$
$|A_7| = \lfloor 100/7 \rfloor = 14$

$|A_2 \cap A_5| = |A_{10}| = \lfloor 100/10 \rfloor = 10$
$|A_5 \cap A_7| = |A_{35}| = \lfloor 100/35 \rfloor = 2$
$|A_7 \cap A_2| = |A_{14}| = \lfloor 100/14 \rfloor =$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.
$A_2 := \{2, 4, \ldots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$
$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \ldots, 98\}$
$|A_7| = \lfloor 100/7 \rfloor = 14$

$|A_2 \cap A_5| = |A_{10}| = \lfloor 100/10 \rfloor = 10$
$|A_5 \cap A_7| = |A_{35}| = \lfloor 100/35 \rfloor = 2$
$|A_7 \cap A_2| = |A_{14}| = \lfloor 100/14 \rfloor = 7$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.

$A_2 := \{2, 4, \dots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \dots, 100\}$
$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \dots, 98\}$
$|A_7| = \lfloor 100/7 \rfloor = 14$

$|A_2 \cap A_5| = |A_{10}| = \lfloor 100/10 \rfloor = 10$
$|A_5 \cap A_7| = |A_{35}| = \lfloor 100/35 \rfloor = 2$
$|A_7 \cap A_2| = |A_{14}| = \lfloor 100/14 \rfloor = 7$

$|A_2 \cap A_5 \cap A_7| =$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.

$A_2 := \{2, 4, \ldots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$
$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \ldots, 98\}$
$|A_7| = \lfloor 100/7 \rfloor = 14$

$|A_2 \cap A_5| = |A_{10}| = \lfloor 100/10 \rfloor = 10$
$|A_5 \cap A_7| = |A_{35}| = \lfloor 100/35 \rfloor = 2$
$|A_7 \cap A_2| = |A_{14}| = \lfloor 100/14 \rfloor = 7$

$|A_2 \cap A_5 \cap A_7| = |A_{70}| =$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.
$A_2 := \{2, 4, \ldots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$
$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \ldots, 98\}$
$|A_7| = \lfloor 100/7 \rfloor = 14$

$|A_2 \cap A_5| = |A_{10}| = \lfloor 100/10 \rfloor = 10$
$|A_5 \cap A_7| = |A_{35}| = \lfloor 100/35 \rfloor = 2$
$|A_7 \cap A_2| = |A_{14}| = \lfloor 100/14 \rfloor = 7$

$|A_2 \cap A_5 \cap A_7| = |A_{70}| = \lfloor 100/70 \rfloor =$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.

$A_2 := \{2, 4, \ldots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$
$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \ldots, 98\}$
$|A_7| = \lfloor 100/7 \rfloor = 14$

$|A_2 \cap A_5| = |A_{10}| = \lfloor 100/10 \rfloor = 10$
$|A_5 \cap A_7| = |A_{35}| = \lfloor 100/35 \rfloor = 2$
$|A_7 \cap A_2| = |A_{14}| = \lfloor 100/14 \rfloor = 7$

$|A_2 \cap A_5 \cap A_7| = |A_{70}| = \lfloor 100/70 \rfloor = 1$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

$$|A_2 \cup A_5 \cup A_7|$$

Solution.

$A_2 := \{2, 4, \ldots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$
$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \ldots, 98\}$
$|A_7| = \lfloor 100/7 \rfloor = 14$

$|A_2 \cap A_5| = |A_{10}| = \lfloor 100/10 \rfloor = 10$
$|A_5 \cap A_7| = |A_{35}| = \lfloor 100/35 \rfloor = 2$
$|A_7 \cap A_2| = |A_{14}| = \lfloor 100/14 \rfloor = 7$

$|A_2 \cap A_5 \cap A_7| = |A_{70}| = \lfloor 100/70 \rfloor = 1$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

$$|A_2 \cup A_5 \cup A_7| = (|A_2| + |A_5| + |A_7|)$$

Solution.

$A_2 := \{2, 4, \ldots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$
$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \ldots, 98\}$
$|A_7| = \lfloor 100/7 \rfloor = 14$

$|A_2 \cap A_5| = |A_{10}| = \lfloor 100/10 \rfloor = 10$
$|A_5 \cap A_7| = |A_{35}| = \lfloor 100/35 \rfloor = 2$
$|A_7 \cap A_2| = |A_{14}| = \lfloor 100/14 \rfloor = 7$

$|A_2 \cap A_5 \cap A_7| = |A_{70}| = \lfloor 100/70 \rfloor = 1$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.

$A_2 := \{2, 4, \ldots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$
$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \ldots, 98\}$
$|A_7| = \lfloor 100/7 \rfloor = 14$

$|A_2 \cap A_5| = |A_{10}| = \lfloor 100/10 \rfloor = 10$
$|A_5 \cap A_7| = |A_{35}| = \lfloor 100/35 \rfloor = 2$
$|A_7 \cap A_2| = |A_{14}| = \lfloor 100/14 \rfloor = 7$

$|A_2 \cap A_5 \cap A_7| = |A_{70}| = \lfloor 100/70 \rfloor = 1$

$$\begin{aligned}|A_2 \cup A_5 \cup A_7| = (|A_2| + |A_5| + |A_7|) \\ - (|A_2 \cap A_5| + |A_5 \cap A_7| + |A_7 \cap A_2|)\end{aligned}$$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.

$A_2 := \{2, 4, \ldots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$
$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \ldots, 98\}$
$|A_7| = \lfloor 100/7 \rfloor = 14$

$|A_2 \cap A_5| = |A_{10}| = \lfloor 100/10 \rfloor = 10$
$|A_5 \cap A_7| = |A_{35}| = \lfloor 100/35 \rfloor = 2$
$|A_7 \cap A_2| = |A_{14}| = \lfloor 100/14 \rfloor = 7$

$|A_2 \cap A_5 \cap A_7| = |A_{70}| = \lfloor 100/70 \rfloor = 1$

$$\begin{aligned}|A_2 \cup A_5 \cup A_7| = (|A_2| + |A_5| + |A_7|)& \\ - (|A_2 \cap A_5| + |A_5 \cap A_7| + |A_7 \cap A_2|)& \\ + |A_2 \cap A_5 \cap A_7|&\end{aligned}$$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.

$A_2 := \{2, 4, \ldots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$
$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \ldots, 98\}$
$|A_7| = \lfloor 100/7 \rfloor = 14$

$|A_2 \cap A_5| = |A_{10}| = \lfloor 100/10 \rfloor = 10$
$|A_5 \cap A_7| = |A_{35}| = \lfloor 100/35 \rfloor = 2$
$|A_7 \cap A_2| = |A_{14}| = \lfloor 100/14 \rfloor = 7$

$|A_2 \cap A_5 \cap A_7| = |A_{70}| = \lfloor 100/70 \rfloor = 1$

$$\begin{aligned}
|A_2 \cup A_5 \cup A_7| &= (|A_2| + |A_5| + |A_7|) \\
&\quad - (|A_2 \cap A_5| + |A_5 \cap A_7| + |A_7 \cap A_2|) \\
&\quad + |A_2 \cap A_5 \cap A_7| \\
&= (50 + 20 + 14)
\end{aligned}$$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.

$A_2 := \{2, 4, \ldots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$
$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \ldots, 98\}$
$|A_7| = \lfloor 100/7 \rfloor = 14$

$|A_2 \cap A_5| = |A_{10}| = \lfloor 100/10 \rfloor = 10$
$|A_5 \cap A_7| = |A_{35}| = \lfloor 100/35 \rfloor = 2$
$|A_7 \cap A_2| = |A_{14}| = \lfloor 100/14 \rfloor = 7$

$|A_2 \cap A_5 \cap A_7| = |A_{70}| = \lfloor 100/70 \rfloor = 1$

$$\begin{aligned}
|A_2 \cup A_5 \cup A_7| &= (|A_2| + |A_5| + |A_7|) \\
&\quad - (|A_2 \cap A_5| + |A_5 \cap A_7| + |A_7 \cap A_2|) \\
&\quad + |A_2 \cap A_5 \cap A_7| \\
&= (50 + 20 + 14) - (10 + 2 + 7)
\end{aligned}$$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.

$A_2 := \{2, 4, \ldots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$
$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \ldots, 98\}$
$|A_7| = \lfloor 100/7 \rfloor = 14$

$|A_2 \cap A_5| = |A_{10}| = \lfloor 100/10 \rfloor = 10$
$|A_5 \cap A_7| = |A_{35}| = \lfloor 100/35 \rfloor = 2$
$|A_7 \cap A_2| = |A_{14}| = \lfloor 100/14 \rfloor = 7$

$|A_2 \cap A_5 \cap A_7| = |A_{70}| = \lfloor 100/70 \rfloor = 1$

$$\begin{aligned}
|A_2 \cup A_5 \cup A_7| &= (|A_2| + |A_5| + |A_7|) \\
&\quad - (|A_2 \cap A_5| + |A_5 \cap A_7| + |A_7 \cap A_2|) \\
&\quad + |A_2 \cap A_5 \cap A_7| \\
&= (50 + 20 + 14) - (10 + 2 + 7) + 1
\end{aligned}$$

Example. Find the number of positive integers below 100 which are divisible by 2, 5, or 7

Solution.

$A_2 := \{2, 4, \ldots, 100\}$
$|A_2| = \lfloor 100/2 \rfloor = 50$

$A_5 := \{5, 10, \ldots, 100\}$
$|A_5| = \lfloor 100/5 \rfloor = 20$

$A_7 := \{7, 10, \ldots, 98\}$
$|A_7| = \lfloor 100/7 \rfloor = 14$

$|A_2 \cap A_5| = |A_{10}| = \lfloor 100/10 \rfloor = 10$
$|A_5 \cap A_7| = |A_{35}| = \lfloor 100/35 \rfloor = 2$
$|A_7 \cap A_2| = |A_{14}| = \lfloor 100/14 \rfloor = 7$

$|A_2 \cap A_5 \cap A_7| = |A_{70}| = \lfloor 100/70 \rfloor = 1$

$$\begin{aligned}
|A_2 \cup A_5 \cup A_7| &= (|A_2| + |A_5| + |A_7|) \\
&\quad - (|A_2 \cap A_5| + |A_5 \cap A_7| + |A_7 \cap A_2|) \\
&\quad + |A_2 \cap A_5 \cap A_7| \\
&= (50 + 20 + 14) - (10 + 2 + 7) + 1 \\
&= 66
\end{aligned}$$

□

Example. Consider the set $\{2^1-1, 2^2-1, \ldots 2^{n-1}-1\}$,

$$\begin{aligned}
|A_2 \cup A_5 \cup A_7| &= (|A_2| + |A_5| + |A_7|) \\
&\quad - (|A_2 \cap A_5| + |A_5 \cap A_7| + |A_7 \cap A_2|) \\
&\quad + |A_2 \cap A_5 \cap A_7| \\
&= (50 + 20 + 14) - (10 + 2 + 7) + 1 \\
&= 66
\end{aligned}$$

$\square$

Example. Consider the set $\{2^1-1, 2^2-1, \ldots 2^{n-1}-1\}$, where $n \geq 3$

$$
\begin{aligned}
|A_2 \cup A_5 \cup A_7| &= (|A_2| + |A_5| + |A_7|) \\
&\quad - (|A_2 \cap A_5| + |A_5 \cap A_7| + |A_7 \cap A_2|) \\
&\quad + |A_2 \cap A_5 \cap A_7| \\
&= (50 + 20 + 14) - (10 + 2 + 7) + 1 \\
&= 66
\end{aligned}
$$

$\square$

Example. Consider the set $\{2^1-1, 2^2-1, \ldots 2^{n-1}-1\}$, where $n \geq 3$ is odd.

$$
\begin{aligned}
|A_2 \cup A_5 \cup A_7| &= (|A_2| + |A_5| + |A_7|) \\
&\quad - (|A_2 \cap A_5| + |A_5 \cap A_7| + |A_7 \cap A_2|) \\
&\quad + |A_2 \cap A_5 \cap A_7| \\
&= (50 + 20 + 14) - (10 + 2 + 7) + 1 \\
&= 66
\end{aligned}
$$

$\square$

Example. Consider the set $\{2^1 - 1, 2^2 - 1, \ldots 2^{n-1} - 1\}$, where $n \geq 3$ is odd. Prove that there is a number from this set

$$\begin{aligned}
|A_2 \cup A_5 \cup A_7| &= (|A_2| + |A_5| + |A_7|) \\
&\quad - (|A_2 \cap A_5| + |A_5 \cap A_7| + |A_7 \cap A_2|) \\
&\quad + |A_2 \cap A_5 \cap A_7| \\
&= (50 + 20 + 14) - (10 + 2 + 7) + 1 \\
&= 66
\end{aligned}$$

$\square$

Example. Consider the set $\{2^1-1, 2^2-1, \ldots 2^{n-1}-1\}$, where $n \geq 3$ is odd. Prove that there is a number from this set divisible by n.

$$\begin{aligned}
|A_2 \cup A_5 \cup A_7| &= (|A_2| + |A_5| + |A_7|) \\
&\quad - (|A_2 \cap A_5| + |A_5 \cap A_7| + |A_7 \cap A_2|) \\
&\quad + |A_2 \cap A_5 \cap A_7| \\
&= (50 + 20 + 14) - (10 + 2 + 7) + 1 \\
&= 66
\end{aligned}$$

$\square$

Example. Consider the set $\{2^1-1, 2^2-1, \ldots 2^{n-1}-1\}$, where $n \geq 3$ is odd. Prove that there is a number from this set divisible by n.

Solution. Exercise to be discussed in tomorrow's lecture.

$$
\begin{aligned}
|A_2 \cup A_5 \cup A_7| &= (|A_2| + |A_5| + |A_7|) \\
&\quad - (|A_2 \cap A_5| + |A_5 \cap A_7| + |A_7 \cap A_2|) \\
&\quad + |A_2 \cap A_5 \cap A_7| \\
&= (50 + 20 + 14) - (10 + 2 + 7) + 1 \\
&= 66
\end{aligned}
$$

□