Example. Number of ways to choose r distinct objects from a set of X of n objects.

Solution.

$$\mathcal{C}_r^n = \{ \qquad \qquad \qquad \qquad \qquad \qquad \}$$

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$$\binom{n}{r} := |\mathcal{C}_r^n| = \frac{n!}{(n-r)!r!}$$

Example. Find the number of positive divisors of 50.

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Solution.

$$50 = 5 \times 10$$

$\square$

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Solution.

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