

**Example.** Prove that any subset of cardinality n+1

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Solution.

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Denote the subset,  $A := \{a_1, a_2, ..., a_{n+1}\}$  $I := \{1, 2, \dots, n, n+1\}$  $B := \{\{1, 2\}, \{2, 4\}, \dots, \{n, 2n\}\}\$  $f: I \to B, \text{defined}, f(i) = [a_i]$ f cannot be injective, so  $f(i) = f(j) \iff [a_i] = \{a_i, a_i\} = [a_i] \in B$ i.e.  $a_i$  divides  $a_i$ 

Solution.

Denote the set,  $A := \{a_1, a_2, ..., a_6\}$  $a_1$  either knows or does not know  $\{b_1, b_2, b_3\} \subset \{a_2, \dots, a_6\}$  (pigeonhole principle)

Assume  $a_1$  knows  $\{b_1, b_2, b_3\}$ If any  $b_i, b_i \in \{b_1, b_2, b_3\}$  know each other,

then  $\{a_1, b_i, b_j\}$  form the triple.

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**Example.** Consider a graph with 6 vertices,

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**Example.** Consider a graph with 6 vertices, such that every pair vertices is joined by exactly one edge

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**Example.** Consider a graph with 6 vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue.

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**Example.** Consider a graph with 6 vertices, such that every pair vertices is joined by exactly one edge which can be coloured either red, or blue. Show that there are three vertices

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Solution. Same as above