Theorem.

Theorem. *The number of partitions of* m

Theorem. *The number of partitions of m into a sum of non-zero integers*

Theorem. *The number of partitions of* m *into a sum of non-zero integers so that each term occurs at most twice*

Theorem. *The number of partitions of m into a sum of non-zero integers so that each term occurs at most twice is equal to the number of....*

Theorem. *The number of partitions of m into a sum of non-zero integers so that each term occurs at most twice is equal to the number of....*

Proof. $\square$

Theorem. *The number of partitions of m into a sum of non-zero integers so that each term occurs at most twice is equal to the number of....*

Proof. Number of partitions is the mth coefficient of

$\square$

Theorem. *The number of partitions of* m *into a sum of non-zero integers so that each term occurs at most twice is equal to the number of....*

Proof. Number of partitions is the mth coefficient of $(1 + x + x^2)$ $\square$

Theorem. *The number of partitions of m into a sum of non-zero integers so that each term occurs at most twice is equal to the number of....*

Proof. Number of partitions is the mth coefficient of $(1+x+x^2)(1+x^2+x^4)$ $\square$

Theorem. *The number of partitions of m into a sum of non-zero integers so that each term occurs at most twice is equal to the number of....*

Proof. Number of partitions is the mth coefficient of $(1+x+x^2)(1+x^2+x^4)(1+x^3+x^6)\dots$

$\square$

Theorem. *The number of partitions of m into a sum of non-zero integers so that each term occurs at most twice is equal to the number of....*

Proof. Number of partitions is the mth coefficient of
$(1+x+x^2)(1+x^2+x^4)(1+x^3+x^6)\dots$
$= \frac{1-x^3}{1-x}\frac{1-x^6}{1-x^2}\frac{1-x^9}{1-x^3}\cdots$

$\square$

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$= (1+x+x^2+\cdots)(1+x^2+x^4+\cdots)(1+x^4+x^8+\cdots)\cdots$ $\square$

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$=$ number of $\square$

Theorem. *The number of partitions of m into a sum of non-zero integers so that each term occurs at most twice is equal to the number of partitions with sizes* not *multiples of 3.*

Proof. Number of partitions is the mth coefficient of

$(1+x+x^2)(1+x^2+x^4)(1+x^3+x^6)\dots$

$= \frac{\cancel{1-x^3}}{1-x}\frac{1-x^6}{1-x^2}\frac{1-x^9}{\cancel{1-x^3}}\cdots$

$= \frac{1}{1-x}\frac{1}{1-x^2}\frac{1}{1-x^4}\cdots$

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$=$ number of partitions with sizes *not* multiples of 3 $\square$

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$=$ number of partitions with sizes *not* multiples of 3 $\square$

Theorem. *The number of partitions of m into a sum of non-zero integers so that each term occurs at most twice is equal to the number of partitions with sizes* not *multiples of 3.* **Theorem** (Euler)**.** .

Proof. Number of partitions is the mth coefficient of
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Theorem (Euler)**.** *The number of partitions of m into a sum of non-zero integers of distinct sizes, i.e. so that each term occurs at most once*

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Exercise.

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Exercise. Generalize the above theorems to paritions of sizes at most k.

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$\mathcal{P}_1$: # of partitions of n into k parts
$\mathcal{P}_2$: # of partitions of n into parts of at most size k

We will define a bijection,

$$f : \mathcal{P}_1 \to \mathcal{P}_2$$

$12 = 5 + 3 + 2 + 2$

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$*$

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$12 = 4 + 4 + .$

$\square$

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$$\begin{array}{lllll} * & * & * & * & * \\ * & * & * & & \\ * & * & & & \\ * & * & & & \end{array}$$

$$\begin{array}{llll} * & * & * & * \\ * & * & * & * \\ * & * & & \\ * & & & \\ * & & & \end{array}$$

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$* \ *$

$* \ * \ * \ *$

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$*$

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$$\begin{array}{ccccc} * & * & * & * & * \\ * & * & * & & \\ * & * & & & \\ * & * & & & \end{array}$$

$$\begin{array}{cccc} * & * & * & * \\ * & * & * & * \\ * & * & & \\ * & & & \\ * & & & \end{array}$$

$12 = 4 + 4 + 2 + 1 + 1$

.

$\square$

Exponential generating function

$$\begin{array}{llll} * & * & * & * \\ * & * & * & * \\ * & * & & \\ * & & & \\ * & & & \end{array}$$

$$12 = 4 + 4 + 2 + 1 + 1$$

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Exponential generating function

Definition.

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$*\ *\ *\ *$

$*\ *$

$*$

$*$

$$12 = 4 + 4 + 2 + 1 + 1$$

.

Exponential generating function

Definition (Exponential generating function)**.**

$$\begin{array}{llll} * & * & * & * \\ * & * & * & * \\ * & * & & \\ * & & & \\ * & & & \end{array}$$

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.

Exponential generating function

Definition (Exponential generating function)**.**

$$a_0 + a_1 x + a_2 \frac{x^2}{2!} + a_3 \frac{x^3}{3!} + \cdots$$

$$\begin{array}{llll} * & * & * & * \\ * & * & * & * \\ * & * & & \\ * & & & \\ * & & & \end{array}$$

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Exponential generating function

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$$a_0 + a_1 x + a_2 \frac{x^2}{2!} + a_3 \frac{x^3}{3!} + \cdots$$

Example.

$$\begin{array}{llll} * & * & * & * \\ * & * & * & * \\ * & * & & \\ * & & & \\ * & & & \end{array}$$

$$12 = 4 + 4 + 2 + 1 + 1$$

.

Exponential generating function

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Example. *Sequence*: $(1, 1, 1, \ldots,)$

$$\begin{array}{llll} * & * & * & * \\ * & * & * & * \\ * & * & & \\ * & & & \\ * & & & \end{array}$$

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$$\begin{array}{llll} * & * & * & * \\ * & * & * & * \\ * & * & & \\ * & & & \\ * & & & \end{array}$$

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$$\begin{array}{llll} * & * & * & * \\ * & * & * & * \\ * & * & & \\ * & & & \\ * & & & \end{array}$$

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$$\begin{array}{llll} * & * & * & * \\ * & * & * & * \\ * & * & & \\ * & & & \\ * & & & \end{array}$$

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Example.

$$\begin{array}{llll} * & * & * & * \\ * & * & * & * \\ * & * & & \\ * & & & \\ * & & & \end{array}$$

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Example. *Sequence*: $(P_0^n, P_1^n, P_2^n, \ldots, P_n^n)$

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$$P_0^n + P_1^n x + P_2^n \frac{x^2}{2!} + P_3^n \frac{x^3}{3!} + \cdots + P_n^n \frac{x^n}{n!} = (1+x)^n$$

$$\begin{array}{llll} * & * & * & * \\ * & * & * & * \\ * & * & & \\ * & & & \\ * & & & \end{array}$$

$$12 = 4 + 4 + 2 + 1 + 1$$

.

Exponential generating function

Definition (Exponential generating function).

$$a_0 + a_1 x + a_2 x^2 + \cdots = a_0 + (a_1 1!)\frac{x}{1!} + (a_2 2!)\frac{x^2}{2!} + \cdots$$

$$a_0 + a_1 x + a_2 \frac{x^2}{2!} + a_3 \frac{x^3}{3!} + \cdots$$

.

Example. *Sequence*: $(1, 1, 1, \ldots,)$
Generating function:

$$1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \cdots = e^x$$

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Generating function:

$$1 + 3x + 3^2\frac{x^2}{2!} + 3^3\frac{x^3}{3!} + \cdots = e^{3x}$$

Example. *Sequence*: $(P_0^n, P_1^n, P_2^n, \ldots, P_n^n)$
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Exponential generating function

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Example. *Sequence*: $(1, 3, 3^2, \ldots,)$
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Example. In how many ways can k balls chosen out of 2 red and 3 yellow be arranged?

.

Exponential generating function

Definition (Exponential generating function)**.**

$$a_0 + a_1 x + a_2 \frac{x^2}{2!} + a_3 \frac{x^3}{3!} + \cdots$$

Example. *Sequence*: $(1, 1, 1, \dots,)$
Generating function:

$$1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \cdots = e^x$$

Example. *Sequence*: $(1, 3, 3^2, \dots,)$
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$$1 + 3x + 3^2\frac{x^2}{2!} + 3^3\frac{x^3}{3!} + \cdots = e^{3x}$$

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Example. In how many ways can k balls chosen out of 2 red and 3 yellow be arranged?

Solution. .

Exponential generating function

Definition (Exponential generating function)**.**

$$a_0 + a_1 x + a_2 \frac{x^2}{2!} + a_3 \frac{x^3}{3!} + \cdots$$

Example. *Sequence*: $(1, 1, 1, \ldots,)$
Generating function:

$$1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \cdots = e^x$$

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$$1 + 3x + 3^2\frac{x^2}{2!} + 3^3\frac{x^3}{3!} + \cdots = e^{3x}$$

Example. *Sequence*: $(P_0^n, P_1^n, P_2^n, \ldots, P_n^n)$
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Example. In how many ways can k balls chosen out of 2 red and 3 yellow be arranged?

Solution.

$$(1 + \frac{x}{1!} + \frac{x^2}{2!})$$

.

Exponential generating function

Definition (Exponential generating function).

$$a_0 + a_1 x + a_2\frac{x^2}{2!} + a_3\frac{x^3}{3!} + \cdots$$

Example. *Sequence*: $(1, 1, 1, \ldots,)$
Generating function:

$$1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \cdots = e^x$$

Example. *Sequence*: $(1, 3, 3^2, \ldots,)$
Generating function:

$$1 + 3x + 3^2\frac{x^2}{2!} + 3^3\frac{x^3}{3!} + \cdots = e^{3x}$$

Example. *Sequence*: $(P^n_0, P^n_1, P^n_2, \ldots, P^n_n)$
Generating function:

$$P^n_0 + P^n_1 x + P^n_2\frac{x^2}{2!} + P^n_3\frac{x^3}{3!} + \cdots + P^n_n\frac{x^n}{n!} = (1+x)^n$$

Example. In how many ways can k balls chosen out of 2 red and 3 yellow be arranged?

Solution.

$$(1 + \frac{x}{1!} + \frac{x^2}{2!})(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!})$$

.

Exponential generating function

Definition (Exponential generating function)**.**

$$a_0 + a_1 x + a_2 \frac{x^2}{2!} + a_3 \frac{x^3}{3!} + \cdots$$

Example. *Sequence*: $(1, 1, 1, \ldots,)$
Generating function:

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Example. In how many ways can k balls chosen out of 2 red and 3 yellow be arranged?

Solution.

$$(1 + \frac{x}{1!} + \frac{x^2}{2!})(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!})$$

$k!$ times the kth coefficient

.

Exponential generating function

Definition (Exponential generating function)**.**

$$a_0 + a_1 x + a_2 \frac{x^2}{2!} + a_3 \frac{x^3}{3!} + \cdots$$

Example. *Sequence*: $(1, 1, 1, \ldots,)$
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Example. In how many ways can k balls chosen out of 2 red and 3 yellow be arranged?

Solution.

$$(1 + \frac{x}{1!} + \frac{x^2}{2!})(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!})$$

$k!$ times the kth coefficient

Example. In how many ways can k balls chosen out of infinitely many red and yellow ones be arranged?

.

Exponential generating function

Definition (Exponential generating function)**.**

$$a_0 + a_1 x + a_2 \frac{x^2}{2!} + a_3 \frac{x^3}{3!} + \cdots$$

Example. *Sequence*: $(1, 1, 1, \ldots,)$
Generating function:

$$1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \cdots = e^x$$

Example. *Sequence*: $(1, 3, 3^2, \ldots,)$
Generating function:

$$1 + 3x + 3^2\frac{x^2}{2!} + 3^3\frac{x^3}{3!} + \cdots = e^{3x}$$

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Example. In how many ways can k balls chosen out of 2 red and 3 yellow be arranged?

Solution.

$$(1 + \frac{x}{1!} + \frac{x^2}{2!})(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!})$$

$k!$ times the kth coefficient

Example. In how many ways can k balls chosen out of infinitely many red and yellow ones be arranged?

Solution. .

Exponential generating function

Definition (Exponential generating function)**.**

$$a_0 + a_1 x + a_2 \frac{x^2}{2!} + a_3 \frac{x^3}{3!} + \cdots$$

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Solution.

$$(1 + \frac{x}{1!} + \frac{x^2}{2!})(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!})$$

$k!$ times the kth coefficient

Example. In how many ways can k balls chosen out of infinitely many red and yellow ones be arranged?

Solution.

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Exponential generating function

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Example. *Sequence*: $(1, 3, 3^2, \dots,)$
Generating function:

$$1 + 3x + 3^2\frac{x^2}{2!} + 3^3\frac{x^3}{3!} + \cdots = e^{3x}$$

Example. *Sequence*: $(P_0^n, P_1^n, P_2^n, \dots, P_n^n)$
Generating function:

$$P_0^n + P_1^n x + P_2^n \frac{x^2}{2!} + P_3^n \frac{x^3}{3!} + \cdots + P_n^n \frac{x^n}{n!} = (1+x)^n$$

Example. In how many ways can k balls chosen out of 2 red and 3 yellow be arranged?

Solution.

$$(1 + \frac{x}{1!} + \frac{x^2}{2!})(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!})$$

$k!$ times the kth coefficient

Example. In how many ways can k balls chosen out of infinitely many red and yellow ones be arranged?

Solution.

$$(1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots)^2$$

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Exponential generating function

Definition (Exponential generating function)**.**

$$a_0 + a_1 x + a_2 \frac{x^2}{2!} + a_3 \frac{x^3}{3!} + \cdots$$

Example. *Sequence*: $(1, 1, 1, \ldots,)$
Generating function:

$$1 + 1x + 1\frac{x^2}{2!} + 1\frac{x^3}{3!} + \cdots = e^x$$

Example. *Sequence*: $(1, 3, 3^2, \ldots,)$
Generating function:

$$1 + 3x + 3^2\frac{x^2}{2!} + 3^3\frac{x^3}{3!} + \cdots = e^{3x}$$

Example. *Sequence*: $(P_0^n, P_1^n, P_2^n, \ldots, P_n^n)$
Generating function:

$$P_0^n + P_1^n x + P_2^n \frac{x^2}{2!} + P_3^n \frac{x^3}{3!} + \cdots + P_n^n \frac{x^n}{n!} = (1+x)^n$$

Example. In how many ways can k balls chosen out of 2 red and 3 yellow be arranged?

Solution.

$$(1 + \frac{x}{1!} + \frac{x^2}{2!})(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!})$$

$k!$ times the kth coefficient

Example. In how many ways can k balls chosen out of infinitely many red and yellow ones be arranged?

Solution.

$$(1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots)^2$$

$k!$ times the kth coefficient

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