Exercise sheet 6

Manifolds, MTH406

1. If 0 is a regular value of the a smooth function $f : \mathbb{R}^n \to \mathbb{R}$, then prove that $f^{-1}0$ is always an orientable manifold.

2. Prove that every Lie group is orientable.

3. Prove that if a manifold has trivial tangent bundle then it is orientable.

4. Prove that the total space, $T(M)$ of a tangent bundle, $\pi : T(M) \to M$, of a manifold M is always orientable, even if M is not orientable.

5. Prove that $F : M \to N$ is a smooth map between compact manifolds M and N, and ω a smooth n-form, where n is the dimension of N, then $\int_M F^*(\omega) = \pm \int_N \omega$

6. Compute $H^k(\mathbb{R})$ for each k.

7. Prove that an exact form on a compact manifold cannot be no-where vanishing (Hint: maxima and minuma). Using this, what can you deduce about $H^n(M)$ for an n-dimensional compact, connected, and orientable, manifold M?