Exercise sheet 5

Manifolds, MTH406

1. Let ω_1 denote a k form and ω_2 denote an l-form on a smooth manifold. Prove that $\omega_1 \wedge \omega_2 = (-1)^k \omega_2 \wedge \omega_1$
2. Let ω_1 denote a k form and ω_2 denote an l-form on a smooth manifold. Prove that $\omega_1(\wedge \omega_2 \wedge \omega_3) = (\omega_1 \wedge \omega_2) \wedge \omega_3$
3. Let $\omega_1, \omega_2, \ldots, \omega_k$ denote 1-forms on M. Prove that $(\omega_1 \wedge \omega_2 \wedge \ldots \wedge \omega_k)_p(X_1, \ldots, X_k) = det(\omega_i(X_j))$ for any tangent vectors $X_1, X_2, \ldots, X_k$.
4. Prove that the exterior derivative, $d : \oplus A^k(R^n) \to \oplus A^k(R^n)$, defined by $df(X) = X(f)$ and $d(\Sigma a_I dx_1 \wedge \ldots dx^k) = \Sigma da_I \wedge x_1 \wedge \ldots dx^k$, satisfies $d \circ d = 0$
5. Prove that a 1-form ω is smooth if and only if $\omega(X) := \omega_p(X_p)$ is a smooth function for every smooth vector field X.
6. Consider a smooth map $F : M \to N$. Prove that $d(F^*(\omega) = F^*(d\omega)$