Exercise sheet 7

Manifolds, MTH406

1. For a smooth manifold M, define the Euler Characteristic, $\chi(M) := \Sigma_{k=0}^{n}(-1)^k H^k(M)$. Prove that if $M = U \cup V$, where U and V are open subsets of M, than $\chi(M) = \chi(U) + \chi(V) - \chi(U \cap V)$.

2. Compute $H_k(S^n)$ for each $k \geq 0$ and $n \geq 2$.

3. Compute $H_k(T)$ for each $k \geq 0$