Exercise sheet 1

Manifolds, MTH406

1. Prove that any open subset of a smooth manifold is smooth.
2. Prove that the product of smooth manifolds is smooth.
3. Prove that any chart $\phi : U \to \mathbb{R}^n$ on a smooth manifold is a smooth map.
4. Prove that if $F : M \to N$ and $G : N \to P$ are smooth maps between manifolds, then the composition $G \circ F$ is also smooth.
5. Prove that if X_p is a derivation at $p \in M$, then $X_p(c) = 0$ for any constant function c.