

# Exercise sheet 7

Knots and Braids, MTH436

1. For a smooth manifold  $M$ , define the Euler Characteristic,  $\chi(M) := \sum_{k=0}^n (-1)^k H^k(M)$ . Prove that if  $M = U \cup V$ , where  $U$  and  $V$  are open subsets of  $M$ , then  $\chi(M) = \chi(U) + \chi(V) - \chi(U \cap V)$ .
2. Compute  $H_k(S^n)$  for each  $k \geq 0$  and  $n \geq 2$ .
3. Compute  $H_k(T)$  for each  $k \geq 0$  where  $T$  is the torus.