Exercise sheet 4

Knots and Braids, MTH436

1. Consider a link L with n components.
   (a) Prove that $H_1(S^3 \setminus L) = \mathbb{Z}^n$
   (b) Prove that $H_2(S^3 \setminus L) = \mathbb{Z}^{n-1}$
2. Let K denote the trefoil knot.
   (a) Find a seifert surface for K.
   (b) Use that to compute $H_1(X_2)$ where X_2 denotes the 2-sheeted cover of $S^3 \setminus K$.
   (c) Compute the Seifert matrix corresponding with the Seifert surface and use that to find $H_(\tilde{X})$ as a $\mathbb{Z}[t, t^{-1}]$, where $\tilde{X}$ denotes the infinite cyclic cover.