Exercise sheet 5

1. Define $\mathbb{RP}^n = (\mathbb{R}^{n+1} \setminus 0)/\sim$ where $(x_0, x_1, \ldots, x_n) \sim (y_0, y_1, \ldots, y_n)$ if and only if there is an $\alpha \in \mathbb{R}$ such that $(x_0, x_1, \ldots, x_n) = \alpha(y_0, y_1, \ldots, y_n)$. Show that it is a CW-complex for each n and compute its homology.

2. Define $\mathbb{CP}^n = (\mathbb{C}^{n+1} \setminus 0)/\sim$ where $(x_0, x_1, \ldots, x_n) \sim (y_0, y_1, \ldots, y_n)$ if and only if there is an $\alpha \in \mathbb{C}$ such that $(x_0, x_1, \ldots, x_n) = \alpha(y_0, y_1, \ldots, y_n)$. Show that it is a CW-complex for each n and compute its homology.

3. Compute $H_k(S^n; G)$ for any abelian group, G.

4. Prove that if $f : S^n \to S^n$ has degree d, then the induced map $f : H_k(S^n; G) \to H_k(S^n; G)$ is multiplication by n.