Exercise sheet 6

1. Use the universal coefficient theorem to compute the cohomologies of S^n, the torus, and $\mathbb{RP}^2$, each with coeffients $\mathbb{Z}$, $\mathbb{Z}/2$, and $\mathbb{Q}$. Then redo them using the Mayer-Vietoris and, for S^n, using the long exact sequence of pairs.
2. Prove that the short exact sequence
$$0 \to Ext(H_{n-1}(X), G) \to H^n(X; G) \to Hom(H_n(X), G) \to 0$$
is natural and splits.
3. Prove that a degree d map $f : S^n \to S^n$ induces a map $f^* : H^n(S^n) \to H^n(S^n)$ which is multiplication by d.
4. Prove that following:
$$\delta(\phi \smile \tau) = \delta\phi \smile \tau + (-1)^m \phi \smile \delta\tau$$
5. Show that if $f : X \to Y$ is a continuous map then $f^*(x \smile y) = f^*(x) \smile f^*(y)$.
6. Show that we can define a relative version of the cup product:
$$\smile : H^m(X, A; R) \times H^n(X, B; R) \to H^{m+n}(X, A \cup B; R)$$
7. Compute the cohomology ring of
   (a) The projective plane.
   (b) The Klein bottle

from section 3.3 of Hatcher (latest online edition), questions, 2, 3, 5, 15, 8, and 9