Exercise sheet 2

1. Prove that if $f : X \to Y$ is continuous, then $(f_{\#})_n : C_n(X) \to C_n(Y)$ satisfies, $\partial_{n-1} \circ f_{\#} = f_{\#} \circ \partial_n$. Use that to show that $f_* : H_n(X) \to H_n(Y)$ defined as $f_*([\sigma]) = [f \circ \sigma] = [f_{\#}(\sigma)]$ is well defined and also prove that $(f \circ g)_* = f_* \circ g_*$ and $Id_* = Id$.
2. Prove that homotopy equivalence is an equivalence relation.
3. For, $A \subset X$ if there is a homotopy $F : X \times I \to Y$ such that $F(x, 0) = x$, $F(x, 1) \in A$, and $F(a, t) \in A$ for each $a \in A$, then A is called a deformation retract of X. Prove that if A is deformation retract of X, then A is homotopically eqiovalent to X.
4. Compute the homologies of $\mathbb{R}^n$, B^n (the closed ball of dimension n) and, D^n (the open ball of dimension n).
5. Prove that chain homotopy is an equivalence relation. $P(\sigma) := \Sigma_i (-1)^i F \circ (\sigma \times Id) \restriction_{[v_0, v_1, \ldots, v_i, w_i, \ldots, w_n]}$