

## Exercise sheet 2

1. Prove that if  $f : X \rightarrow Y$  is continuous, then  $(f_{\#})_n : C_n(X) \rightarrow C_n(Y)$  satisfies,  $\partial_{n-1} \circ f_{\#} = f_{\#} \circ \partial_n$ . Use that to show that  $f_* : H_n(X) \rightarrow H_n(Y)$  defined as  $f_*([\sigma]) = [f \circ \sigma] = [f_{\#}(\sigma)]$  is well defined and also prove that  $(f \circ g)_* = f_* \circ g_*$  and  $Id_* = Id$ .
2. Prove that homotopy equivalence is an equivalence relation.
3. For,  $A \subset X$  if there is a homotopy  $F : X \times I \rightarrow Y$  such that  $F(x, 0) = x$ ,  $F(x, 1) \in A$ , and  $F(a, t) \in A$  for each  $a \in A$ , then  $A$  is called a deformation retract of  $X$ . Prove that if  $A$  is deformation retract of  $X$ , then  $A$  is homotopically equivalent to  $X$ .
4. Compute the homologies of  $\mathbb{R}^n$ ,  $B^n$  (the closed ball of dimension  $n$ ) and,  $D^n$  (the open ball of dimension  $n$ ).
5. Prove that chain homotopy is an equivalence relation.