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if $\mu_p = i_{p*}(\mu_B)$ for some $\mu_B \in H_n(M, M \setminus B)$ for some $\mu_B \in B$, then for any $q \in B$, $\mu_q = i_{q*}(\mu_B)$

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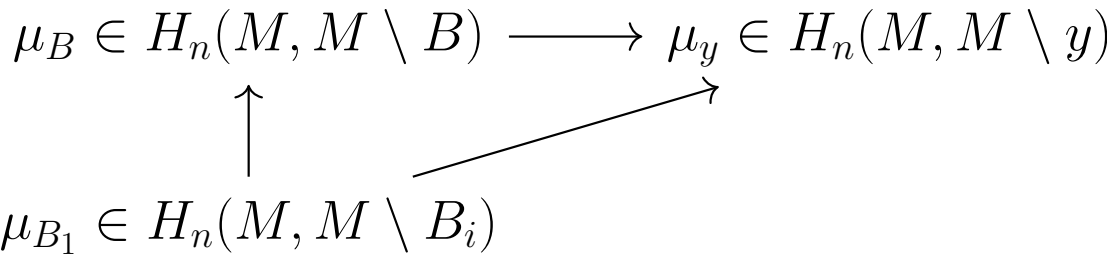
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$$\begin{aligned} \pi^{-1}(B) &= \{\mu_p \mid p \in B\} \\ &= \{i_{p*}(\mu_B) \mid p \in B\} \sqcup \{-i_{p*}(\mu_B) \mid p \in B\} \\ &= U_{\mu_B} \sqcup U_{-\mu_B} \end{aligned}$$

$\pi|_{U_{\mu_B}}$ bijection onto U_{μ_B} . □

Theorem. *There is a 2 sheeted covering $\pi : \widetilde{M} \rightarrow M$ such that*

1. $\pi^{-1}(p) = \{\mu_p, -\mu_p\}$

2. \widetilde{M} is disconnected if and only if M is orientable.

Proof. $\widetilde{M} := \{\mu_p \mid \mu_p \in H_n(M, M \setminus p)\}$
 $\pi : \widetilde{M} \rightarrow M$, defined by $\pi(\mu_p) := p$.

Given $B \subset U \subset M$, where U is homeomorphic to \mathbb{R}^n and B homeomorphic to an open ball in \mathbb{R}^n , and $\mu_B \in H_n(M, M \setminus B)$, define $U_{\mu_B} := \{i_{p*}(\mu_B) \mid p \in B\}$

If $\mu_x \in U_{\mu_{B_1}} \cap U_{\mu_{B_2}}$, then $x \in B \subset B_1 \cap B_2$.

If $\mu_y \in U_{\mu_B}$, then $\mu_y = i_{y*}(\mu_B)$

$$\begin{array}{ccc} \mu_B \in H_n(M, M \setminus B) & \longrightarrow & \mu_y \in H_n(M, M \setminus y) \\ \uparrow & \nearrow & \\ \mu_{B_1} \in H_n(M, M \setminus B_1) & & \end{array}$$

Therefore, $\mu_y \in U_{\mu_{B_1}}$. Similarly, $\mu_y \in U_{\mu_{B_2}}$.
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Maps basic open sets (i.e. U_{μ_B}) to basic open sets (i.e. B). □

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Checking compatibility:

Given $\mu_p \in \widetilde{M}$, there is a B containing p

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Lemma. *A 2-sheeted cover of a connected space \widetilde{M} disconnected $\implies M$ orientable can have no more than two components. If it has two components, then each is homeomorphic to the ground space.*

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Assignment $p \rightarrow \mu_p$ equivalent to $s : M \rightarrow \widetilde{M}$ such that $\pi \circ s = Id$.

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 $\pi^{-1}(V) = \widetilde{V}_1 \sqcup \widetilde{V}_2$

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If \widetilde{M} is connected, connect two points of a fibre by γ , so $\pi(\gamma)$ is a loop. Then $s(\pi(\gamma))$ is a lift of a loop contradicting uniqueness of a lift.

So, \widetilde{M} connected $\implies M$ not orientable.

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