Exercise sheet 1

1. Prove that a set map $f : X \to Y$ is injective if and only if it has a left inverse.
2. Prove that a set map $f : X \to Y$ is surjective if and only if it has a right inverse.
3. Prove that if $f : X \to Y$ is a homeomorphism, then f_* is an isomorphism.
4. Prove that if $r : X \to A$ is a retract, then r_* is surjective.
5. Consider the subset $A = S^1 \times x_0$ of $X = S^1 \times S^1$. Prove that A is a retract of X.
6. Show that for any $p \in \mathbb{R}^n$, there is a retract $r : \mathbb{R}^n \to \{p\}$
7. Show that $\mathbb{R}^n \setminus \{p\}$, where p is the origin, retracts onto S^{n-1}.
8. Prove that $\partial_n \circ \partial_{n+1} = 0$.
9. For any topological space, X, define the map $\epsilon : C_0(X) \to \mathbb{Z}$ by $\epsilon(\Sigma_i n_i \sigma_i) = \Sigma_i n_i$. Prove that:

a) ϵ is a homomorphism.
b) $\epsilon \circ \partial_1 = 0$. We can, therefore, define $\epsilon_* : H_0(X) \to \mathbb{Z}$
c) If X is path-connected, ϵ_* is an isomorphism.

To be updated...