Exercise sheet 6

1. Use the universal coefficient theorem to compute the cohomologies of S^n, the torus, and $\mathbb{RP}^2$, each with coeffients $\mathbb{Z}$, $\mathbb{Z}/2$, and $\mathbb{Q}$. Then redo them using the Mayer-Vietoris and, for S^n, using the long exact sequence of pairs.
2. Prove that the short exact sequence

   $$0 \to Ext(H_{n-1}(X), G) \to H^n(X; G) \to Hom(H_n(X), G) \to 0$$

   is natural and splits.