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if $\mu_p = i_{p_*}(\mu_B)$ for some $\mu_B \in H_n(M, M \setminus B)$ for some $\mu_B \in B$, then for any $q \in B$, $\mu_q = i_{q_*}(\mu_B)$

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Therefore, $\mu_y \in U_{\mu_{B_1}}$. Similarly, $\mu_y \in U_{\mu_{B_2}}$.
$U_{\mu_B} \subset U_{\mu_{B_1}} \cap U_{\mu_{B_2}}$

So U_{μ_B} form a basis.

Each B is evenly covered:

$$\begin{aligned}
\pi^{-1}(B) &= \{\mu_p \mid p \in B\} \\
&= \{i_{p_*}(\mu_B) \mid p \in B\} \sqcup \{-i_{p_*}(\mu_B) \mid p \in B\} \\
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$\pi|_{U_{\mu_B}}$ bijection onto U_{μ_B}. $\square$

Theorem. *There is a 2 sheeted covering* $\pi : \widetilde{M} \to M$ *such that*

1. $\pi^{-1}(p) = \{\mu_p, -\mu_p\}$
2. $\widetilde{M}$ *is disconnected if and only if* M *is orientable.*

Proof. $\widetilde{M} := \{\mu_p \mid \mu_p \in H_n(M, M \setminus p)\}$
$\pi : \widetilde{M} \to M$, defined by $\pi(\mu_p) := p$.

Given $B \subset U \subset M$, where U is homeomorphic to $\mathbb{R}^n$ and B homeomorphic to an open ball in $\mathbb{R}^n$, and $\mu_B \in H_n(M, M \setminus B)$, define
$U_{\mu_B} := \{i_{p_*}(\mu_B) \mid p \in B\}$

If $\mu_x \in U_{\mu_{B_1}} \cap U_{\mu_{B_2}}$, then $x \in B \subset B_1 \cap B_2$.

If $\mu_y \in U_{\mu_B}$, then $\mu_y = i_{y_*}(\mu_B)$

$$\begin{tikzcd}
\mu_B \in H_n(M, M \setminus B) \arrow[r] & \mu_y \in H_n(M, M \setminus y) \\
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Maps basic open sets (i.e. U_{μ_B}) to basic open sets (i.e. B). $\square$

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Note: Given $\mu_p \in \widetilde{M}$, there is a B containing p

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The assignment follows the conpatibility condition. $\square$

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$\widetilde{M}$ disconnected $\implies$ M orientable

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Assignment $p \to \mu_p$ equivalent to $s : M \to \widetilde{M}$ such that $\pi \circ s = Id$.

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$\pi^{-1}(V) = \widetilde{V}_1 \sqcup \widetilde{V}_2$

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If $\widetilde{M}$ is connected, connect two points of a fibre by γ, so $\pi(\gamma)$ is a loop.

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So, $\widetilde{M}$ connected $\implies$ M not orientable.