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The map, $i_{p_*}: H_n(M, M \setminus B) \to H_n(M, M \setminus p)$

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 $\pi: \widetilde{M} \to M \text{ such that }$

1.
$$\pi^{-1}(p) = \{\mu_p, -\mu_p\}$$

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$$H_n(M, M \setminus B) \longrightarrow H_n(M, M \setminus y)$$

$$\uparrow \qquad \qquad \downarrow$$

$$H_n(M, M \setminus B_i)$$

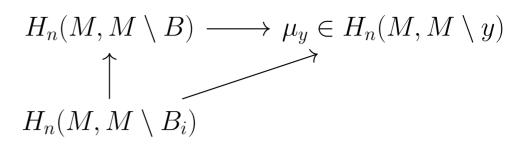
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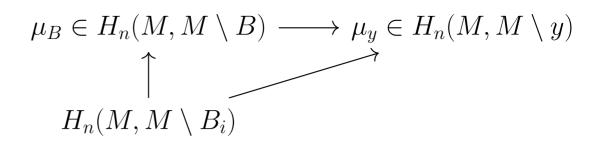
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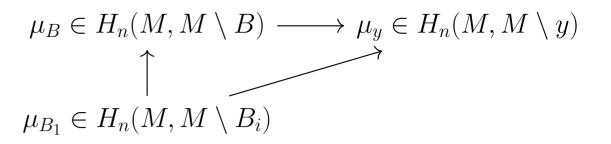
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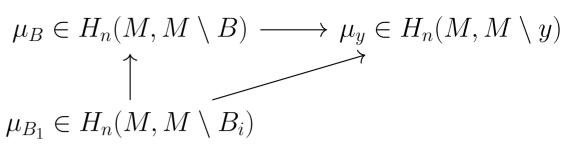
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Therefore,
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2. \widetilde{M} is disconnected if and only if M is orientable. Therefore, $\mu_y \in U_{\mu_{B_1}}$. Similarly, $\mu_y \in U_{\mu_{B_2}}$.

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$$\mu_{B} \in H_{n}(M, M \setminus B) \longrightarrow \mu_{y} \in H_{n}(M, M \setminus y)$$

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Therefore, $\mu_y \in U_{\mu_{B_1}}$. Similarly, $\mu_y \in U_{\mu_{B_2}}$. $U_{\mu_B} \subset U_{\mu_{B_1}} \cap U_{\mu_{B_2}}$

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Each B is evenly covered:

$$\pi^{-1}(B) = \{ \mu_p \mid p \in B \}$$

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 $\pi|_{U_{\mu_B}}$ bijection onto U_{μ_B} .

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$$\mu_{B} \in H_{n}(M, M \setminus B) \longrightarrow \mu_{y} \in H_{n}(M, M \setminus y)$$

$$\uparrow \qquad \qquad \downarrow \qquad \qquad \downarrow$$

$$\mu_{B_{1}} \in H_{n}(M, M \setminus B_{i})$$

Therefore, $\mu_y \in U_{\mu_{B_1}}$. Similarly, $\mu_y \in U_{\mu_{B_2}}$. $U_{\mu_B} \subset U_{\mu_{B_1}} \cap U_{\mu_{B_2}}$

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 $\pi|_{U_{\mu_B}}$ bijection onto U_{μ_B} .

Maps basic open sets (i.e. U_{μ_B}) to basic open sets (i.e. B).

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 $\mu_p \in \widetilde{M}$ also determines $\mu_p \in H_n(M, M \setminus p)$

Assign, $\pi_*^{-1}(\mu_p) \in H_n(\widetilde{M}, \widetilde{M} \setminus \mu_p)$ as local orientation at μ_p .

Note: Given $\mu_p \in \widetilde{M}$, there is a B containing p

 $H_n(\widetilde{M}, \widetilde{M} \setminus U_{\mu_B}) \xrightarrow{\pi_*} H_n(M, M \setminus B)$

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Proof. $\pi_*: H_n(\widetilde{M}, \widetilde{M} \setminus \mu_p) \to H_n(M, M \setminus p)$ is an isomorphism $H_{\widetilde{M}}(\widetilde{M}, \widetilde{M} \setminus \mu_p) \xrightarrow{\pi_*} H_{\widetilde{M}}(M, M \setminus p)$

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Lemma. A 2-sheeted cover of a connected space can have no more than two components.

Proof. $\pi: \widetilde{X} \to X$ is a 2-sheeted cover.

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Lemma. A 2-sheeted cover of a connected space M disconnected $\implies M$ orientable can have no more than two components. If it has two components, then each is homeomorphic to the ground space.

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So, M connected $\implies M$ not orientable.