Quality Improvement

Chapter 6- Control Charts for Variables

PowerPoint presentation to accompany

Besterfield, Quality Improvement, 9e

Outline

- The Control Chart Techniques
- State of Introduction
- Control
- Specifications
- Process Capability
- Different Control Charts

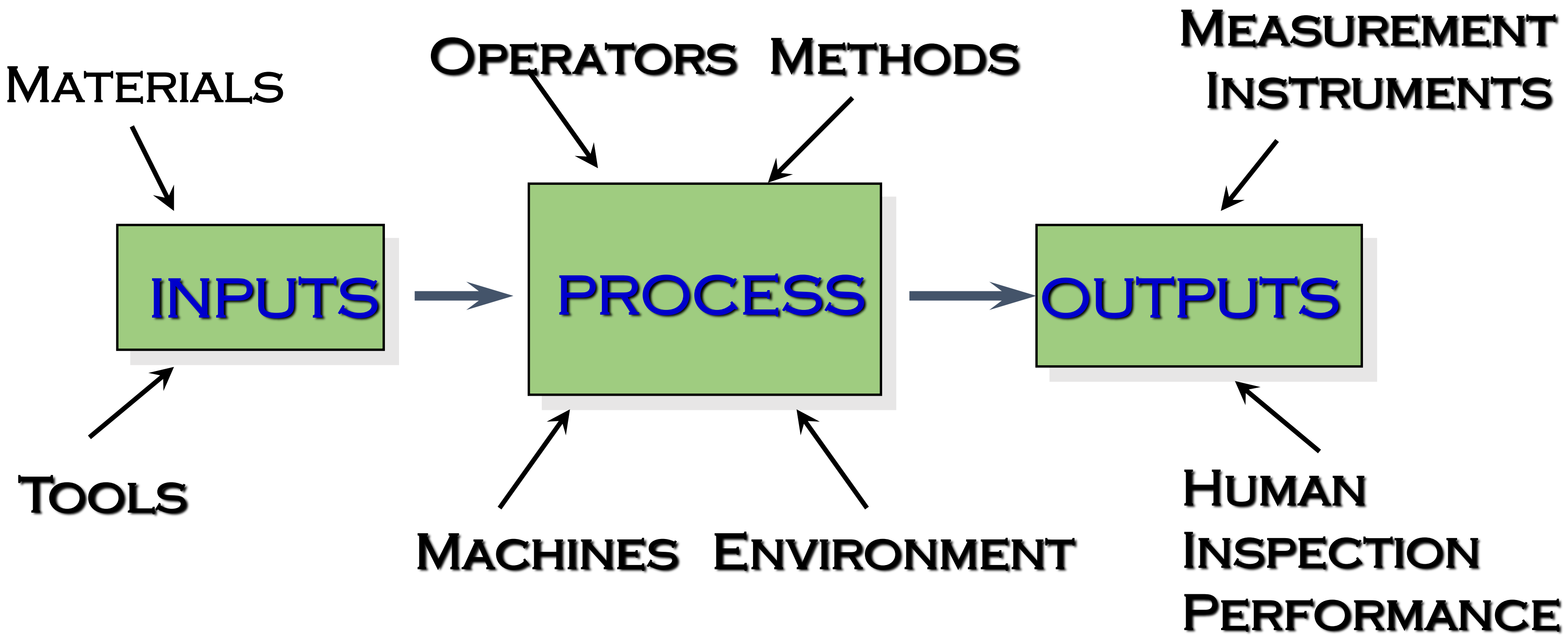
Variation

There are three categories of variation in piece part production:

1. Within-piece variation: Surface
2. Piece-to-piece variation: Among pieces produced at the same time
3. Time-to-time variation: Difference in product produced at different times of the day

Variation

Sources of Variation in production processes:



Variation

Sources of variation are:

1. Equipment:
    1. Toolwear
    2. Machine vibration
    3. Electrical fluctuations etc.
2. Material
    1. Tensile strength
    2. Ductility
    3. Thickness
    4. Porosity etc.

Variation

Sources of variation are:

3. Environment
    1. Temperature
    2. Light
    3. Radiation
    4. Humidity etc.
4. Operator
    1. Personal problem
    2. Physical problem etc.

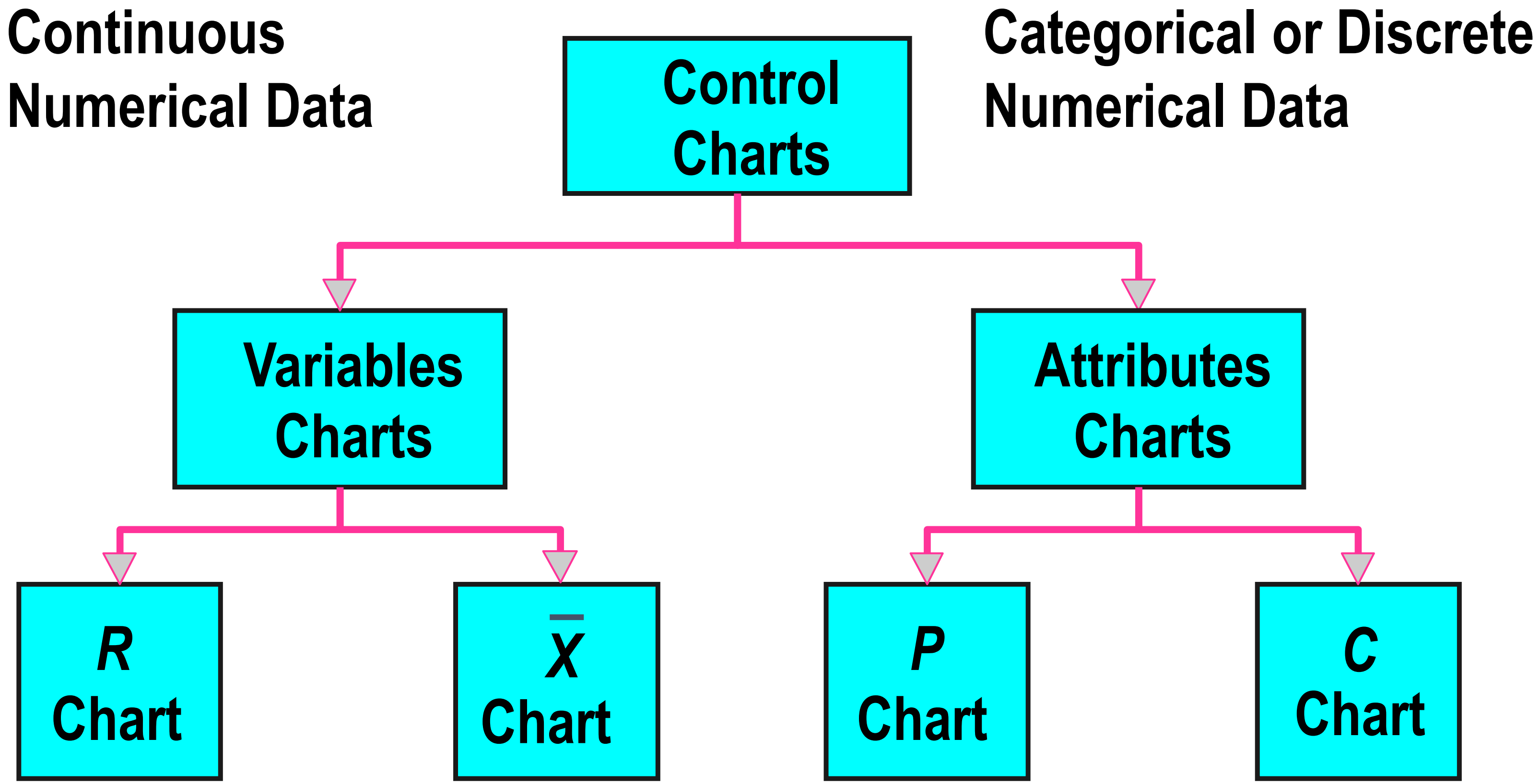
Variation

- There is also a reported variation which is due to the inspection activity.
- Variation due to inspection should be one tenth of the four other sources of variation.
- Variation may be due to chance causes (random causes) or assignable causes.
- When only chance causes are present, then the process is said to be in a state of statistical control. The process is stable and predictable.

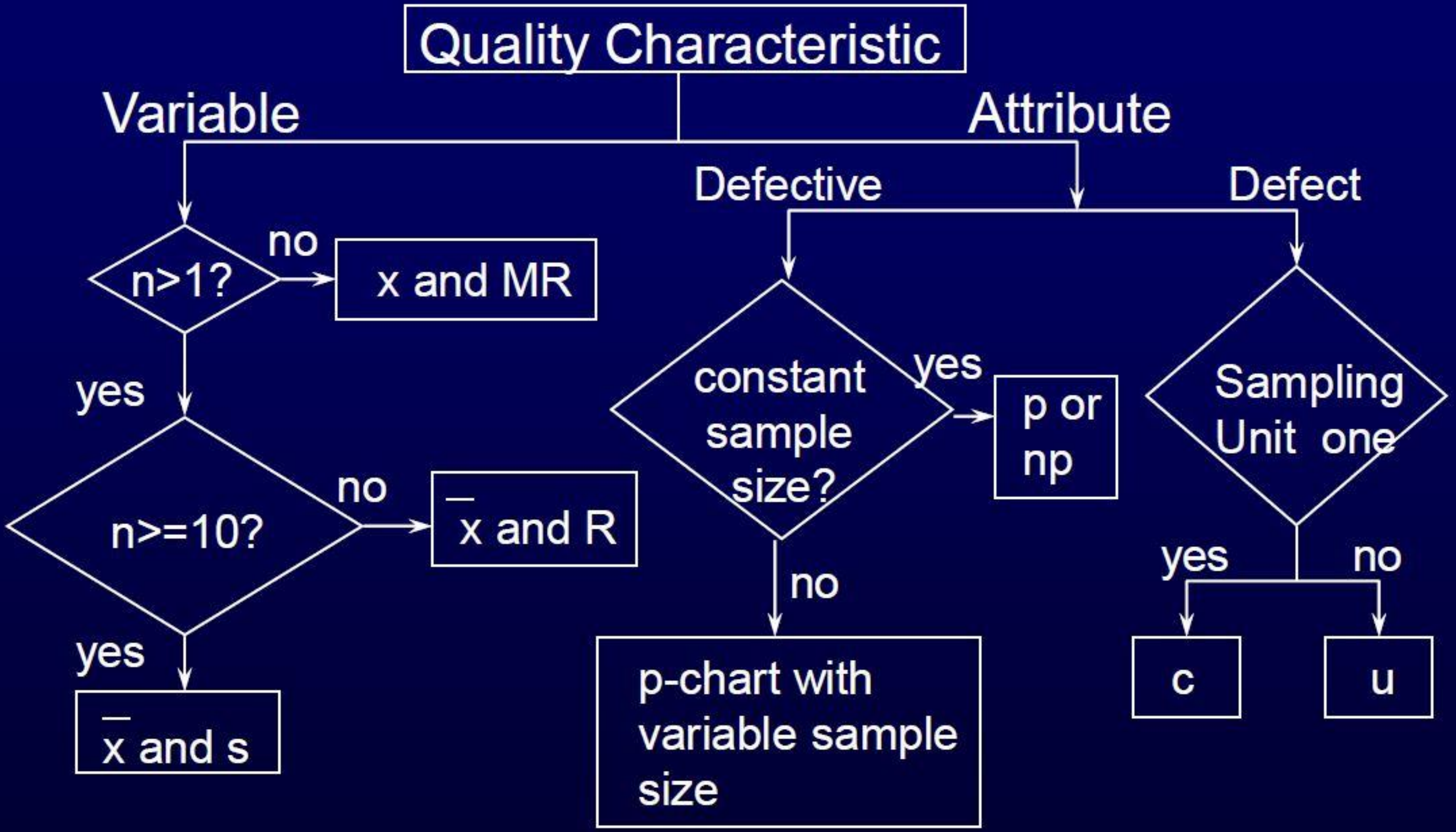
Control Charts

- Variable data
  - x-bar and R-charts
  - x-bar and s-charts
  - Charts for individuals (x-charts)
- Attribute data
  - For “defectives” (p-chart, np-chart)
  - For “defects” (c-chart, u-chart)

Control Charts



Control Chart Selection



Control Charts for Variables

The control chart for variables is a means of visualizing the variations that occur in the central tendency and the mean of a set of observations. It shows whether or not a process is in a stable state.

Control Charts

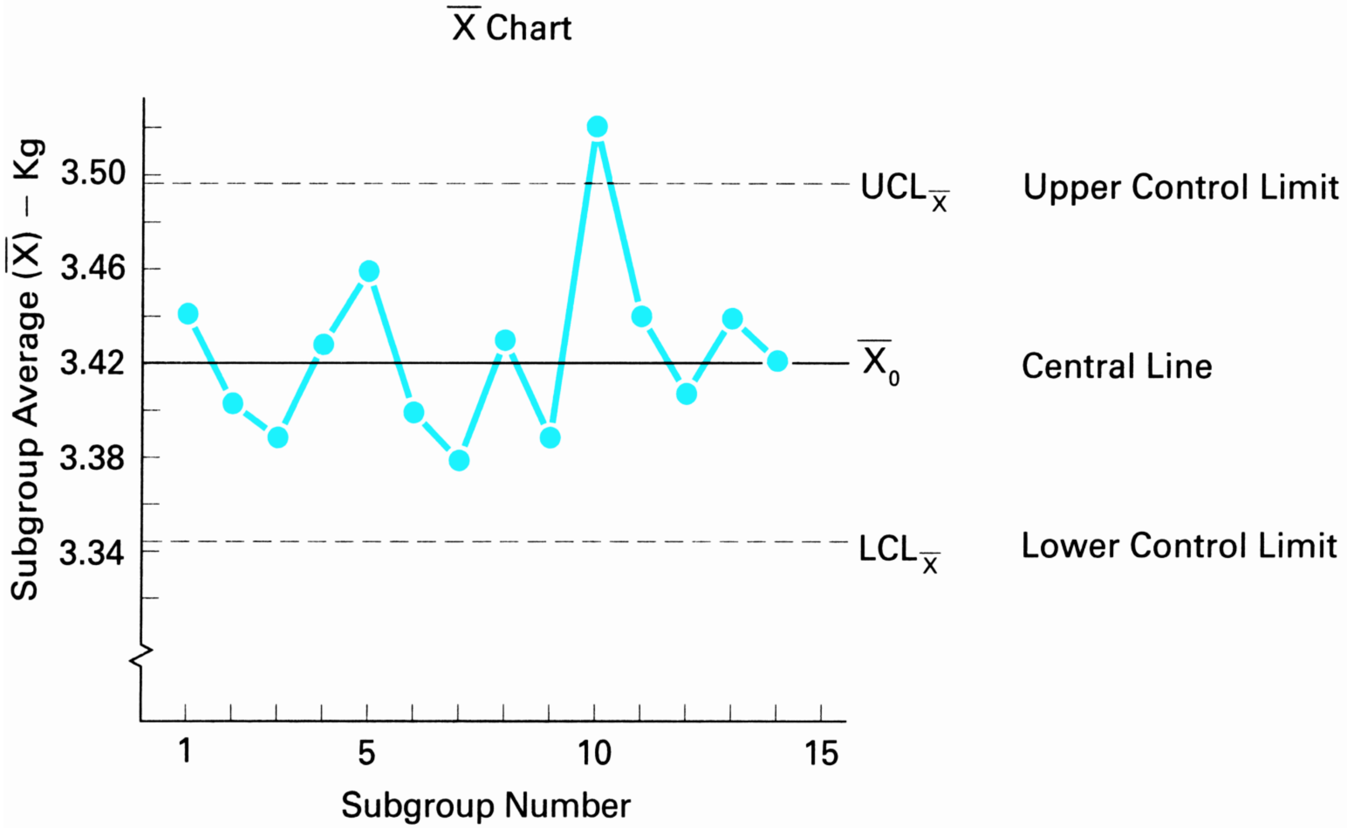


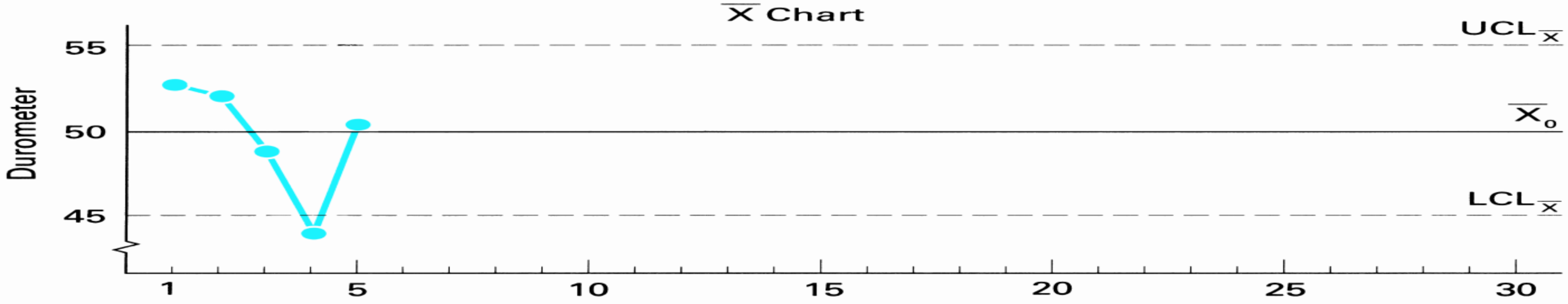
Figure 5-1 Example of a control chart

$\overline{X}$ AND R CHART

Work Center Number 365—2

Quality Characteristic Durometer Date 3/6

Time	8 30 AM	9 30 AM	10 40 AM	11 50 AM	1 30 PM									
Subgroup	1	2	3	4	5	6	7	8	9	10	11	12	13	14
X_1	55	51	48	45	53									
X_2	52	52	49	43	50									
X_3	51	57	50	45	48									
X_4	53	50	49	43	50									
Sum	211	210	196	176	201									
$\overline{X}$	52.8	52.5	49	44	50.2									
R	4	7	2	2	5									



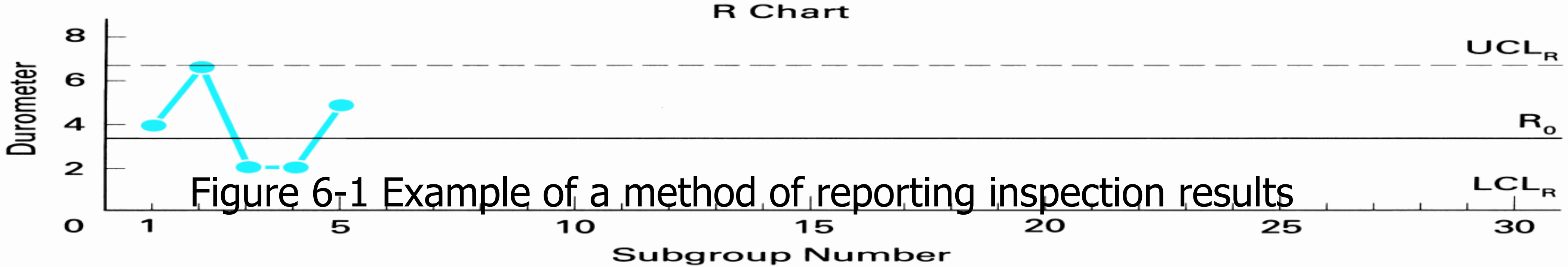


Figure 6-1 Example of a method of reporting inspection results

Variable Control Charts

The objectives of the variable control charts are:

1. For quality improvement
2. To determine the process capability
3. For decisions regarding product specifications
4. For current decisions on the production process
5. For current decisions on recently produced items

Control Chart Techniques

Procedure for establishing a pair of control charts for the average Xbar and the range R:

1. Select the quality characteristic
2. Choose the rational subgroup
3. Collect the data
4. Determine the trial center line and control limits
5. Establish the revised central line and control limits
6. Achieve the objective

Quality Characteristic

The Quality characteristic must be measurable.

It can expressed in terms of the seven basic units:

1. Length
2. Mass
3. Time
4. Electrical current
5. Temperature
6. Subatance
7. Luminosity

Rational Subgroup

A rational subgroup is one in which the variation within a group is due only to chance causes.

- **Within-subgroup variation** is used to determine the control limits.
- **Variation between subgroups** is used to evaluate long-term stability.

Rational Subgroup

There are two schemes for selecting the subgroup samples:

1. Select subgroup samples from product or service produced at one instant of time or as close to that instant as possible (instant time method)- more freq. used
2. Select from product or service produced over a period of time that is representative of all the products or services (period of time method)

Rational Subgroup

The first scheme will have a minimum variation within a subgroup.

The second scheme will have a minimum variation among subgroups.

The first scheme is the most commonly used since it provides a particular time reference for determining assignable causes.

Subgroup Size

- As the subgroup size increases, the control limits become closer to the central value, which make the control chart more sensitive to small variations in the process average
- As the subgroup size increases, the inspection cost per subgroup increases
- When destructive testing is used and the item is expensive, a small subgroup size is required

Subgroup Size

- From a statistical basis a distribution of subgroup averages are nearly normal for groups of 4 or more even when samples are taken from a non-normal distribution
- When a subgroup size of 10 or more is used, the s chart should be used instead of the R chart.
- See Table 6-1 for sample sizes

TABLE 6-1 Sample Sizes

Lot Size	Sample Size
91–150	10
151–280	15
281–400	20
401–500	25
501–1,200	35
1,201–3,200	50
3,201–10,000	75
10,001–35,000	100
35,001–150,000	150

Source: ANSI/ASQ Z1.9, Normal Inspection, Level II.

- The use of Table 6-1 , which was obtained from ANSI/ ASQ Z1.9, can be a valuable aid in making judgments on the amount of sampling required. If a process is expected to produce 4000 pieces per day, then 75 total inspections are suggested.
- Therefore, with a subgroup size of four, 19 subgroups would be a good starting point.

Data Collection

- Data collection can be accomplished using the type of figure shown in Figure 6-2.
- It can also be collected using the method in Table 6-2, i.e. Horizontal or vertical
- It is necessary to collect a minimum of 25 subgroups of data, fewergroups would not be proide sufficient data for central line and control limits.
- **A run chart** can be used to analyze the data in the development stage of a product or prior to a state of statistical control

$\overline{X}$ AND R CHART

Work Center Number 365—2
Quality Characteristic Durometer   Date 3/6

Time	8 30 AM	9 30 AM	10 40 AM	11 50 AM	1 30 PM									
Subgroup	1	2	3	4	5	6	7	8	9	10	11	12	13	14
X_1	55	51	48	45	53									
X_2	52	52	49	43	50									
X_3	51	57	50	45	48									
X_4	53	50	49	43	50									
Sum	211	210	196	176	201									
$\overline{X}$	52.8	52.5	49	44	50.2									
R	4	7	2	2	5									

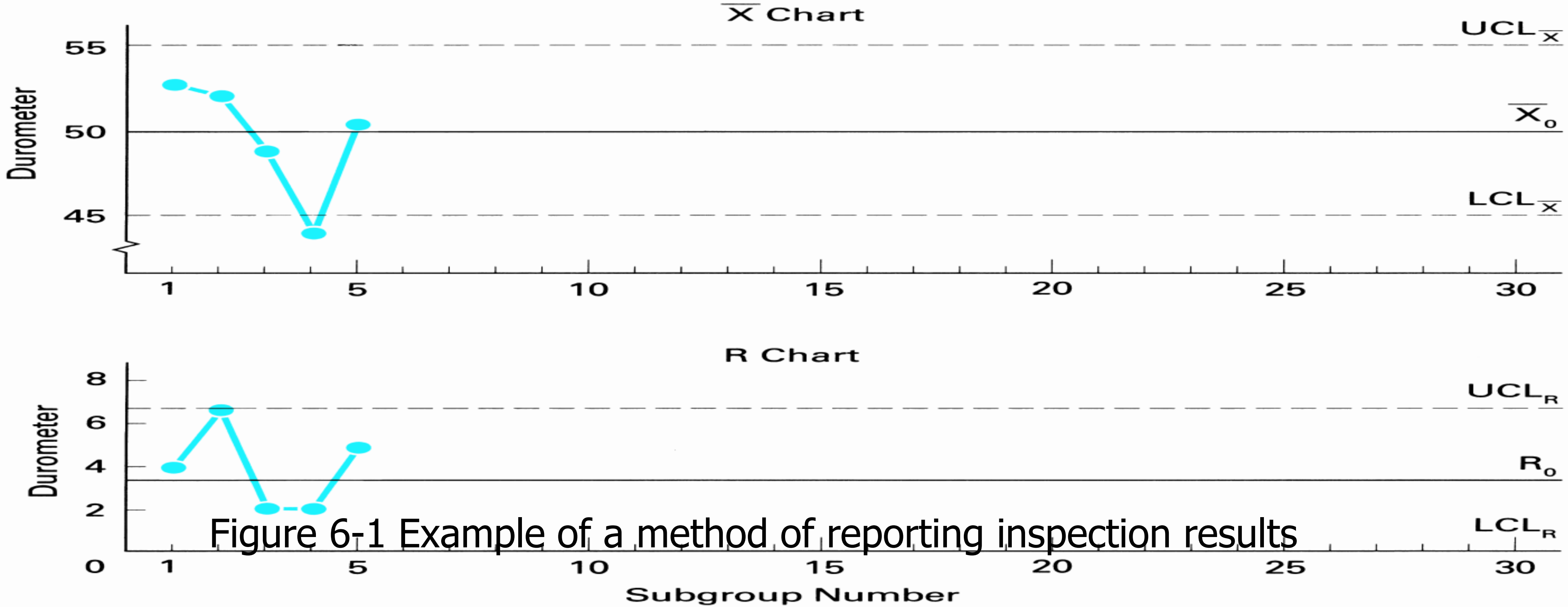


Figure 6-1 Example of a method of reporting inspection results

TABLE 6-2 Data on the Depth of the Keyway (mm)[a]

Subgroup Number	Date	Time	MEASUREMENTS X_1	X_2	X_3	X_4	Average $\overline{X}$	Range R	Comment
1	12/26	8:50	35	40	32	37	6.36	0.08	
2		11:30	46	37	36	41	6.40	0.10	
3		1:45	34	40	34	36	6.36	0.06	
4		3:45	69	64	68	59	6.65	0.10	New, temporary operator
5		4:20	38	34	44	40	6.39	0.10	
6	12/27	8:35	42	41	43	34	6.40	0.09	
7		9:00	44	41	41	46	6.43	0.05	
8		9:40	33	41	38	36	6.37	0.08	
9		1:30	48	44	47	45	6.46	0.04	
10		2:50	47	43	36	42	6.42	0.11	
11	12/28	8:30	38	41	39	38	6.39	0.03	
12		1:35	37	37	41	37	6.38	0.04	
13		2:25	40	38	47	35	6.40	0.12	
14		2:35	38	39	45	42	6.41	0.07	
15		3:55	50	42	43	45	6.45	0.08	
16	12/29	8:25	33	35	29	39	6.34	0.10	
17		9:25	41	40	29	34	6.36	0.12	
18		11:00	38	44	28	58	6.42	0.30	Damaged oil line
19		2:35	35	41	37	38	6.38	0.06	
20		3:15	56	55	45	48	6.51	0.11	Bad material
21	12/30	9:35	38	40	45	37	6.40	0.08	
22		10:20	39	42	35	40	6.39	0.07	
23		11:35	42	39	39	36	6.39	0.06	
24		2:00	43	36	35	38	6.38	0.08	
25		4:25	39	38	43	44	6.41	0.06	
Sum							160.25	2.19	

[a]For simplicity in recording, the individual measurements are coded from 6.00 mm.

Run Chart

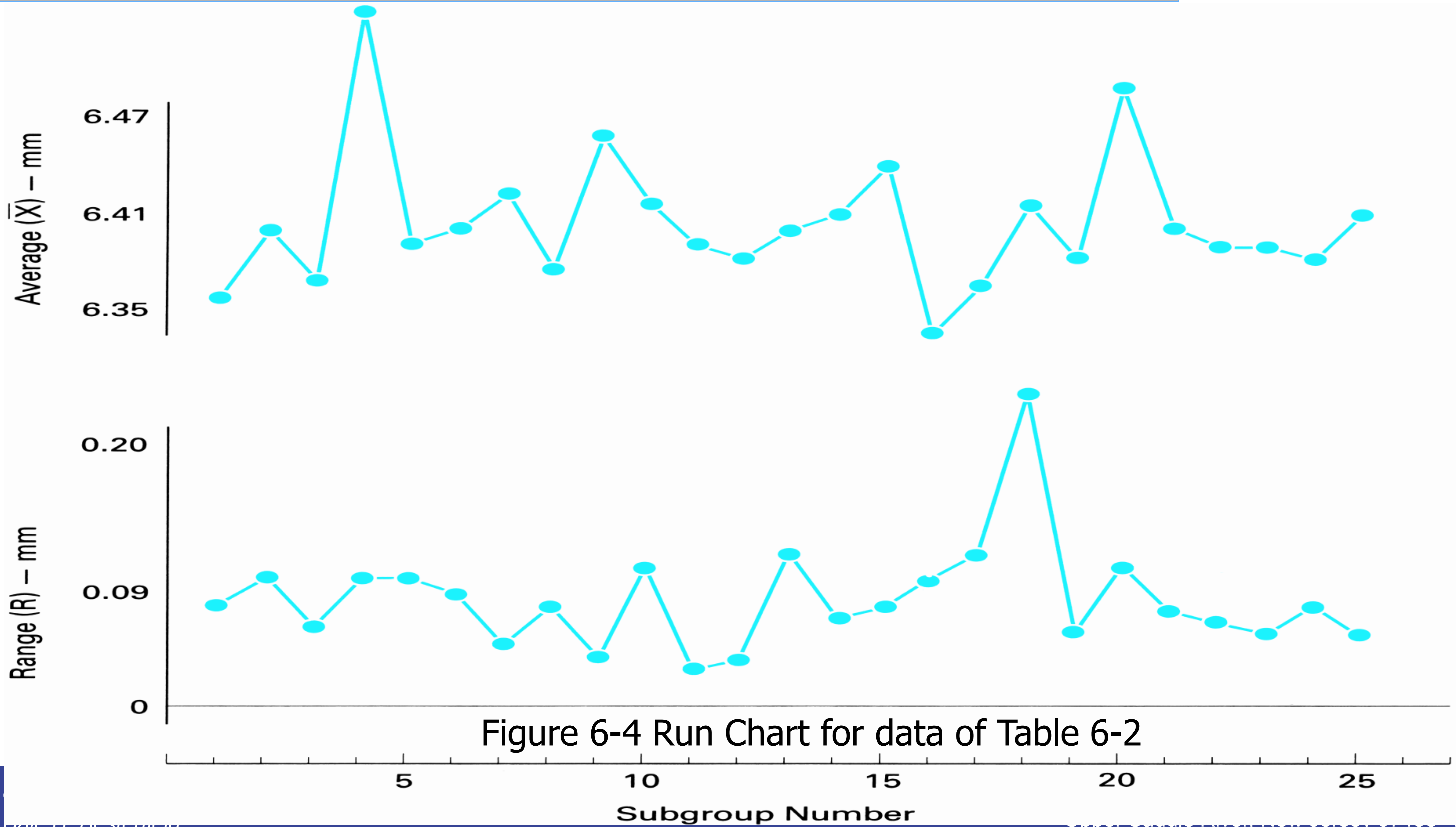


Figure 6-4 Run Chart for data of Table 6-2

Trial Central Lines

Central Lines are obtained using:

$$\overline{\overline{X}} = \frac{\sum_{i=1}^{g} \overline{X}_i}{g} \quad \text{and} \quad \overline{R} = \frac{\sum_{i=1}^{g} R_i}{g}$$

where $\overline{\overline{X}}$ = average of the subgroup averages (read "X double bar")

$\overline{X}_i$ = average of the ith subgroup

g = number of subgroups

$\overline{R}$ = average of the subgroup ranges

R_i = range of the ith subgroup

Trial control limits for the charts are established at ±3 standard deviations from the central value, as shown by the formulas

$$\mathrm{UCL}_{\overline{X}} = \overline{\overline{X}} + 3\sigma_{\overline{X}} \qquad \mathrm{UCL}_R = \overline{R} + 3\sigma_R$$

$$\mathrm{LCL}_{\overline{X}} = \overline{\overline{X}} - 3\sigma_{\overline{X}} \qquad \mathrm{LCL}_R = \overline{R} - 3\sigma_R$$

where UCL = upper control limit

LCL = lower control limit

$\sigma_{\overline{X}}$ = population standard deviation of the subgroup averages ($\overline{X}$'s)

σ_R = population standard deviation of the range

Trial Control Limits

In practice calculations are simplified by using the following equations where A_2, D_3 and D_4 are factors that vary with the subgroupsize and are found in Table B of the Appendix.

In practice, the calculations are simplified by using the product of the range ($\overline{R}$) and a factor (A_2) to replace the 3 standard deviations ($A_2\overline{R} = 3\sigma_{\overline{X}}$)[5] in the formulas for the $\overline{X}$ chart. For the R chart, the range $\overline{R}$ is used to estimate the standard deviation of the range (σ_R).[6] Therefore, the derived formulas are

$$\mathrm{UCL}_{\overline{X}} = \overline{\overline{X}} + A_2\overline{R} \qquad \mathrm{UCL}_R = D_4\overline{R}$$
$$\mathrm{LCL}_{\overline{X}} = \overline{\overline{X}} - A_2\overline{R} \qquad \mathrm{LCL}_R = D_3\overline{R}$$

[5]The derivation of $3\sigma_{\overline{X}} = A_2\overline{R}$ is based on the substitution of $\sigma_{\overline{X}} = \sigma/\sqrt{n}$ and an estimate of $\sigma = R/d_2$, where d_2 is a factor for the subgroup size.

$$3\sigma_x = \frac{3\sigma}{\sqrt{n}} = \frac{3}{d_2\sqrt{n}}\overline{R}; \quad \text{therefore, } A_2 = \frac{3}{d_2\sqrt{n}}$$

[6]The derivation of the simplified formula is based on the substitution of $d_3\sigma = \sigma R$ and $\sigma = \overline{R}/d_2$, which gives

$$\left(1 + \frac{3d_3}{d_2}\right)\overline{R} \quad \text{and} \quad \left(1 - \frac{3d_3}{d_2}\right)\overline{R}$$

for the control limits. Thus, D_4 and D_3 are set equal to the coefficients of $\overline{R}$.

TABLE B

TABLE B Factors for Computing Central Lines and 3σ Control Limits for $\bar{X}$, s, and R Charts

Observations in Sample, n	CHART FOR AVERAGES			CHART FOR STANDARD DEVIATIONS					CHART FOR RANGES					
	Factors for Control Limits			Factor for Central Line	Factors for Control Limits				Factor for Central Line	Factors for Control Limits				
	A	A_2	A_3	c_4	B_3	B_4	B_5	B_6	d_2	d_3	D_1	D_2	D_3	D_4
2	2.121	1.880	2.659	0.7979	0	3.267	0	2.606	1.128	0.853	0	3.686	0	3.267
3	1.732	1.023	1.954	0.8862	0	2.568	0	2.276	1.693	0.888	0	4.358	0	2.574
4	1.500	0.729	1.628	0.9213	0	2.266	0	2.088	2.059	0.880	0	4.698	0	2.282
5	1.342	0.577	1.427	0.9400	0	2.089	0	1.964	2.326	0.864	0	4.918	0	2.114
6	1.225	0.483	1.287	0.9515	0.030	1.970	0.029	1.874	2.534	0.848	0	5.078	0	2.004
7	1.134	0.419	1.182	0.9594	0.118	1.882	0.113	1.806	2.704	0.833	0.204	5.204	0.076	1.924
8	1.061	0.373	1.099	0.9650	0.185	1.815	0.179	1.751	2.847	0.820	0.388	5.306	0.136	1.864
9	1.000	0.337	1.032	0.9693	0.239	1.761	0.232	1.707	2.970	0.808	0.547	5.393	0.184	1.816
10	0.949	0.308	0.975	0.9727	0.284	1.716	0.276	1.669	3.078	0.797	0.687	5.469	0.223	1.777
11	0.905	0.285	0.927	0.9754	0.321	1.679	0.313	1.637	3.173	0.787	0.811	5.535	0.256	1.744
12	0.866	0.266	0.886	0.9776	0.354	1.646	0.346	1.610	3.258	0.778	0.922	5.594	0.283	1.717
13	0.832	0.249	0.850	0.9794	0.382	1.618	0.374	1.585	3.336	0.770	1.025	5.647	0.307	1.693
14	0.802	0.235	0.817	0.9810	0.406	1.594	0.399	1.563	3.407	0.763	1.118	5.696	0.328	1.672
15	0.775	0.223	0.789	0.9823	0.428	1.572	0.421	1.544	3.472	0.756	1.203	5.741	0.347	1.653
16	0.750	0.212	0.763	0.9835	0.448	1.552	0.440	1.526	3.532	0.750	1.282	5.782	0.363	1.637
17	0.728	0.203	0.739	0.9845	0.466	1.534	0.458	1.511	3.588	0.744	1.356	5.820	0.378	1.622
18	0.707	0.194	0.718	0.9854	0.482	1.518	0.475	1.496	3.640	0.739	1.424	5.856	0.391	1.608
19	0.688	0.187	0.698	0.9862	0.497	1.503	0.490	1.483	3.689	0.734	1.487	5.891	0.403	1.597
20	0.671	0.180	0.680	0.9869	0.510	1.490	0.504	1.470	3.735	0.729	1.549	5.921	0.415	1.585

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2		11:30	46	37	36	41	6.40	0.10	
3		1:45	34	40	34	36	6.36	0.06	
4		3:45	69	64	68	59	6.65	0.10	New, temporary
5		4:20	38	34	44	40	6.39	0.10	operator
6	12/27	8:35	42	41	43	34	6.40	0.09	
7		9:00	44	41	41	46	6.43	0.05	
8		9:40	33	41	38	36	6.37	0.08	
9		1:30	48	44	47	45	6.46	0.04	
10		2:50	47	43	36	42	6.42	0.11	
11	12/28	8:30	38	41	39	38	6.39	0.03	
12		1:35	37	37	41	37	6.38	0.04	
13		2:25	40	38	47	35	6.40	0.12	
14		2:35	38	39	45	42	6.41	0.07	
15		3:55	50	42	43	45	6.45	0.08	
16	12/29	8:25	33	35	29	39	6.34	0.10	
17		9:25	41	40	29	34	6.36	0.12	
18		11:00	38	44	28	58	6.42	0.30	Damaged oil line
19		2:35	35	41	37	38	6.38	0.06	
20		3:15	56	55	45	48	6.51	0.11	Bad material
21	12/30	9:35	38	40	45	37	6.40	0.08	
22		10:20	39	42	35	40	6.39	0.07	
23		11:35	42	39	39	36	6.39	0.06	
24		2:00	43	36	35	38	6.38	0.08	
25		4:25	39	38	43	44	6.41	0.06	
Sum							160.25	2.19	

[a]For simplicity in recording, the individual measurements are coded from 6.00 mm.

- Example 6.1

Example Problem 6-1

In order to illustrate the calculations necessary to obtain the trial control limits and the central line, the data in Table 6-2 concerning the depth of the shaft keyway will be used. From that table, $\sum \overline{X} = 160.25$, $\sum R = 2.19$, and $g = 25$; thus, the central lines are

$$\overline{\overline{X}} = \frac{\sum_{i=1}^{g} \overline{X}_i}{g} = \frac{160.25}{25} = 6.41 \text{ mm} \qquad \overline{R} = \frac{\sum_{i=1}^{g} R_i}{g} = \frac{2.19}{25} = 0.0876 \text{ mm}$$

From Table B in the Appendix, the values for the factors for a subgroup size (n) of 4 are $A_2 = 0.729$, $D_3 = 0$, and $D_4 = 2.282$. Trial control limits for the $\overline{X}$ chart are

$$\begin{aligned} UCL_{\overline{X}} &= \overline{\overline{X}} + A_2\overline{R} \\ &= 6.41 + (0.729)(0.0876) \\ &= 6.47 \text{ mm} \end{aligned}$$

$$\begin{aligned} LCL_{\overline{X}} &= \overline{\overline{X}} - A_2\overline{R} \\ &= 6.41 - (0.729)(0.0876) \\ &= 6.35 \text{ mm} \end{aligned}$$

Trial control limits for the R chart are

$$\mathrm{UCL}_R = D_4\overline{R} \qquad \mathrm{LCL}_R = D_3\overline{R}$$
$$= (2.282)(0.0876) \qquad = (0)(0.0876)$$
$$= 0.20 \text{ mm} \qquad = 0 \text{ mm}$$

Figure 6-5 shows the central lines and the trial control limits for the $\overline{X}$ and R charts for the preliminary data.

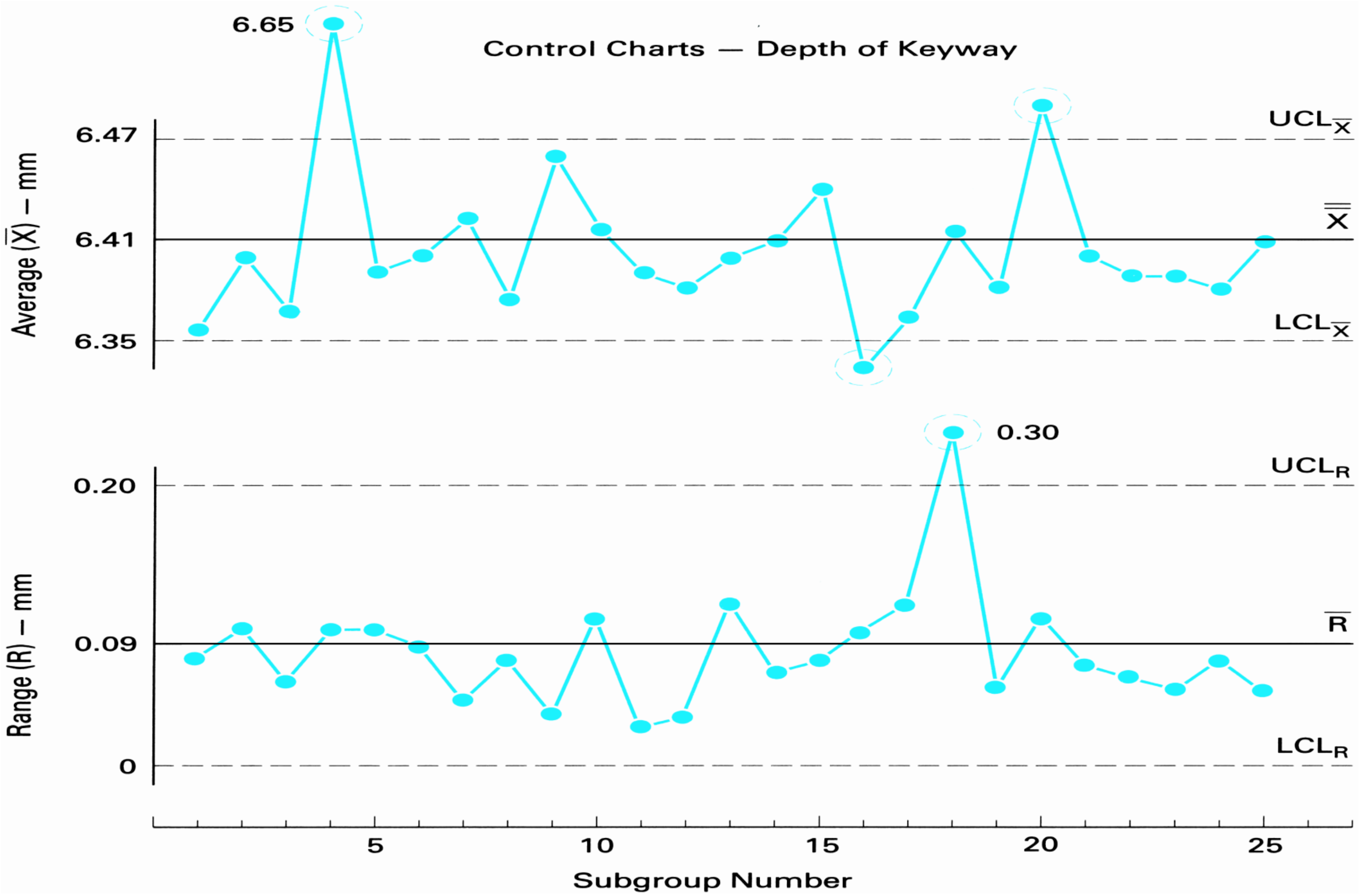


Figure 6-5 Xbar and R chart for preliminary data with trial control limits

Revised Central Line and Control Limits

- The first step is to post the preliminary data to the chart along with the control limits and central lines.
- The next step is to adopt standard values for the central lines, i.e., the best estimate of the standard values with the available data.
- If an analysis of the preliminary data shows **good control**, then X and R can be considered as representative of the process and these become the standard values, X_0 and R_0.

Revised Central Line and Control Limits

- Most processes are not in control when first analyzed.
- An analysis of Figure 6-5 shows that
  - there are out-of-control points on the X chart at subgroups 4, 16, and 20
  - and an out-of-control point on the R chart at subgroup 18
  - It also appears that there are a large number of points below the central line, which is no doubt due to the influence of the high points.

Revised Central Line and Control Limits

- The R chart is analyzed first to determine if it is stable.
- Because the out-of-control point at subgroup 18 on the R chart has an assignable cause (damaged oil line), it can be discarded from the data. The remaining plotted points indicate a stable process.

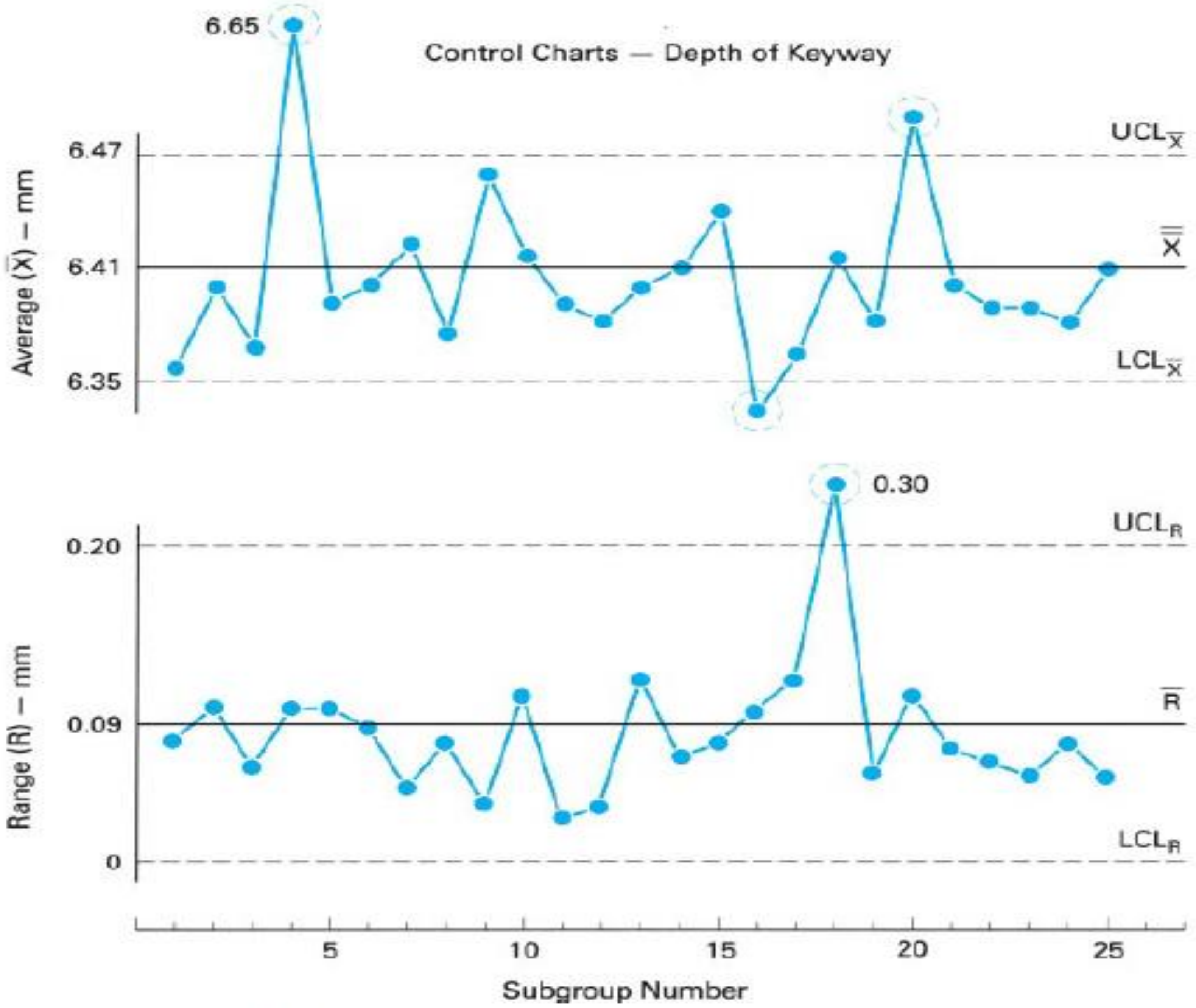


FIGURE 6-5 $\overline{X}$ and R Chart for Preliminary Data with Trial Control Limits

Revised Central Line and Control Limits

- The X-bar chart can now be analyzed. Subgroups 4 and 20 had an assignable cause, but the out-of-control condition for subgroup 16 did not.
- It is assumed that subgroup 16's out-of-control state is due to a chance cause and is part of the natural variation.
- Subgroups 4 and 20 for the $\bar{\bar{X}}$ chart and subgroup 18 for the R chart are not part of the natural variation and are discarded from the data, and new $\bar{\bar{X}}$ and the $\bar{R}$ values are computed with the remaining data.

Revised Central Line and Control Limits

- The calculations are simplified by using the following formulas:

$$\overline{\overline{X}}_{new} = \frac{\sum \overline{X} - \overline{X}_d}{g - g_d} \qquad \overline{R}_{new} = \frac{\sum R - R_d}{g - g_d}$$

where $\overline{X}_d$ = discarded subgroup averages

g_d = number of discarded subgroups

R_d = discarded subgroup ranges

Revised Central Line and Control Limits

- There are two techniques used to discard data.
- If either the X or the R value of a subgroup is out of control and has an assignable cause, both are discarded, or only the out-of-control value of a subgroup is discarded.
- In this lecture the latter technique is followed; thus, when an X value is discarded, its corresponding R value is **not** discarded and vice versa. A knowledge of the process may indicate which technique is most appropriate at any given time

- Where d_2 is a factor from Table B in the Appendix for estimating s_0 from R_0

Example Problem 6-2

Calculations for a new $\overline{X}$ are based on discarding the $\overline{X}$ values of 6.65 and 6.51 for subgroups 4 and 20, respectively. Calculations for a new $\overline{R}$ are based on discarding the R value of 0.30 for subgroup 18.

$$\overline{\overline{X}}_{\text{new}} = \frac{\Sigma\overline{X} - \overline{X}_d}{g - g_d} = \frac{160.25 - 6.65 - 6.51}{25 - 2} = 6.40 \text{ mm}$$

$$\overline{R}_{\text{new}} = \frac{\Sigma R - R_d}{g - g_d} = \frac{2.19 - 0.30}{25 - 1} = 0.079 \text{ mm}$$

These new values of $\overline{\overline{X}}$ and $\overline{R}$ are used to establish the standard values of $\overline{X}_0$, R_0, and σ_0. Thus,

$$\overline{X}_0 = \overline{\overline{X}}_{\text{new}} \qquad R_0 = \overline{R}_{\text{new}} \qquad \sigma_0 = \frac{R_0}{d_2}$$

Revised Control Limits

- These are revised control limits:
  - $UCL_{\overline{X}} = \overline{X}_0 + A\sigma_0 \qquad UCL_R = D_2\sigma_0$
  - $LCL_{\overline{X}} = \overline{X}_0 - A\sigma_0 \qquad LCL_R = D_1\sigma_0$
- Recall the previous limits were

$$\begin{aligned} \mathrm{UCL}_{\overline{X}} &= \overline{X} + 3\sigma_{\overline{X}} \qquad & \mathrm{UCL}_R &= \overline{R} + 3\sigma_R \\ \mathrm{LCL}_{\overline{X}} &= \overline{\overline{X}} - 3\sigma_{\overline{X}} \qquad & \mathrm{LCL}_R &= \overline{R} - 3\sigma_R \end{aligned}$$

$$\begin{aligned} \mathrm{UCL}_{\overline{X}} &= \overline{\overline{X}} + A_2\overline{R} \qquad & \mathrm{UCL}_R &= D_4\overline{R} \\ \mathrm{LCL}_{\overline{X}} &= \overline{\overline{X}} - A_2\overline{R} \qquad & \mathrm{LCL}_R &= D_3\overline{R} \end{aligned}$$

Standard Values

Example Problem 6-3

From Table B in the Appendix and for a subgroup size of 4, the factors are $A = 1.500$, $d_2 = 2.059$, $D_1 = 0$, and $D_2 = 4.698$. Calculations to determine $\overline{X}_0$, R_0, and σ_0 using the data previously given are

$$\overline{X}_0 = \overline{X}_{new} = 6.40 \text{ mm}$$

$$R_0 = \overline{R}_{new} = 0.079 \text{ (0.08 for the chart)}$$

$$\sigma_0 = \frac{R_0}{d_2} = \frac{0.079}{2.059} = 0.038 \text{ mm}$$

TABLE B Factors for Computing Central Lines and 3σ Control Limits for $\bar{X}$, s, and R Charts

TABLE B

Observations in Sample, n	CHART FOR AVERAGES			CHART FOR STANDARD DEVIATIONS					CHART FOR RANGES					
	Factors for Control Limits			Factor for Central Line	Factors for Control Limits				Factor for Central Line	Factors for Control Limits				
	A	A_2	A_3	c_4	B_3	B_4	B_5	B_6	d_2	d_3	D_1	D_2	D_3	D_4
2	2.121	1.880	2.659	0.7979	0	3.267	0	2.606	1.128	0.853	0	3.686	0	3.267
3	1.732	1.023	1.954	0.8862	0	2.568	0	2.276	1.693	0.888	0	4.358	0	2.574
4	1.500	0.729	1.628	0.9213	0	2.266	0	2.088	2.059	0.880	0	4.698	0	2.282
5	1.342	0.577	1.427	0.9400	0	2.089	0	1.964	2.326	0.864	0	4.918	0	2.114
6	1.225	0.483	1.287	0.9515	0.030	1.970	0.029	1.874	2.534	0.848	0	5.078	0	2.004
7	1.134	0.419	1.182	0.9594	0.118	1.882	0.113	1.806	2.704	0.833	0.204	5.204	0.076	1.924
8	1.061	0.373	1.099	0.9650	0.185	1.815	0.179	1.751	2.847	0.820	0.388	5.306	0.136	1.864
9	1.000	0.337	1.032	0.9693	0.239	1.761	0.232	1.707	2.970	0.808	0.547	5.393	0.184	1.816
10	0.949	0.308	0.975	0.9727	0.284	1.716	0.276	1.669	3.078	0.797	0.687	5.469	0.223	1.777
11	0.905	0.285	0.927	0.9754	0.321	1.679	0.313	1.637	3.173	0.787	0.811	5.535	0.256	1.744
12	0.866	0.266	0.886	0.9776	0.354	1.646	0.346	1.610	3.258	0.778	0.922	5.594	0.283	1.717
13	0.832	0.249	0.850	0.9794	0.382	1.618	0.374	1.585	3.336	0.770	1.025	5.647	0.307	1.693
14	0.802	0.235	0.817	0.9810	0.406	1.594	0.399	1.563	3.407	0.763	1.118	5.696	0.328	1.672
15	0.775	0.223	0.789	0.9823	0.428	1.572	0.421	1.544	3.472	0.756	1.203	5.741	0.347	1.653
16	0.750	0.212	0.763	0.9835	0.448	1.552	0.440	1.526	3.532	0.750	1.282	5.782	0.363	1.637
17	0.728	0.203	0.739	0.9845	0.466	1.534	0.458	1.511	3.588	0.744	1.356	5.820	0.378	1.622
18	0.707	0.194	0.718	0.9854	0.482	1.518	0.475	1.496	3.640	0.739	1.424	5.856	0.391	1.608
19	0.688	0.187	0.698	0.9862	0.497	1.503	0.490	1.483	3.689	0.734	1.487	5.891	0.403	1.597
20	0.671	0.180	0.680	0.9869	0.510	1.490	0.504	1.470	3.735	0.729	1.549	5.921	0.415	1.585

Thus, the control limits are

$$\begin{aligned}
UCL_{\bar{X}} &= \bar{X}_0 + A\sigma_0 \\
&= 6.40 + (1.500)(0.038) \\
&= 6.46 \text{ mm}
\end{aligned}$$

$$\begin{aligned}
LCL_{\bar{X}} &= \bar{X}_0 - A\sigma_0 \\
&= 6.40 - (1.500)(0.038) \\
&= 6.34 \text{ mm}
\end{aligned}$$

$$\begin{aligned}
UCL_R &= D_2\sigma_0 \\
&= (4.698)(0.038) \\
&= 0.18 \text{ mm}
\end{aligned}
\qquad
\begin{aligned}
LCL_R &= D_1\sigma_0 \\
&= (0)(0.038) \\
&= 0 \text{ mm}
\end{aligned}$$

Third, the parameter σ_0 is now available to obtain the initial estimate of the process capability, which is $6\sigma_0$. The true process capability is obtained in the next step (Achieve the Objective). Also, as mentioned in Chapter 5, a better estimate of the standard deviation is obtained from $\sigma_0 = R_0/d_2$ and the best estimate is obtained from $\sigma_0 = s_0/c_4$ from the $\overline{X}$, s control charts in the sample standard deviation control chart section.

Sixth, the process determines the central line and control limits. They are not established by design, manufacturing, marketing, or any other department, except for $\overline{X}_0$ when the process is adjustable.

Finally, when population values are known (μ and σ), the central lines and control limits may be calculated immediately, saving time and work. Thus $\overline{X}_0 = \mu, \sigma_0 = \sigma$, and $R_0 = d_2\sigma$, and the limits are obtained using the appropriate formulas. This situation would be extremely rare.

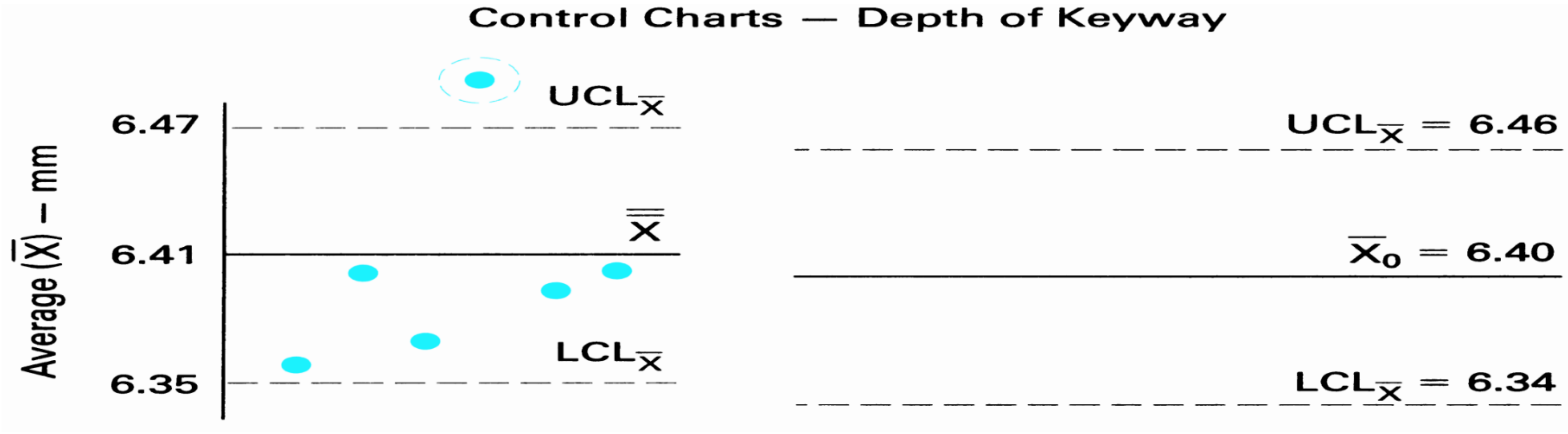


Figure 6-6 Trial control limits and revised control limits for Xbar and R charts

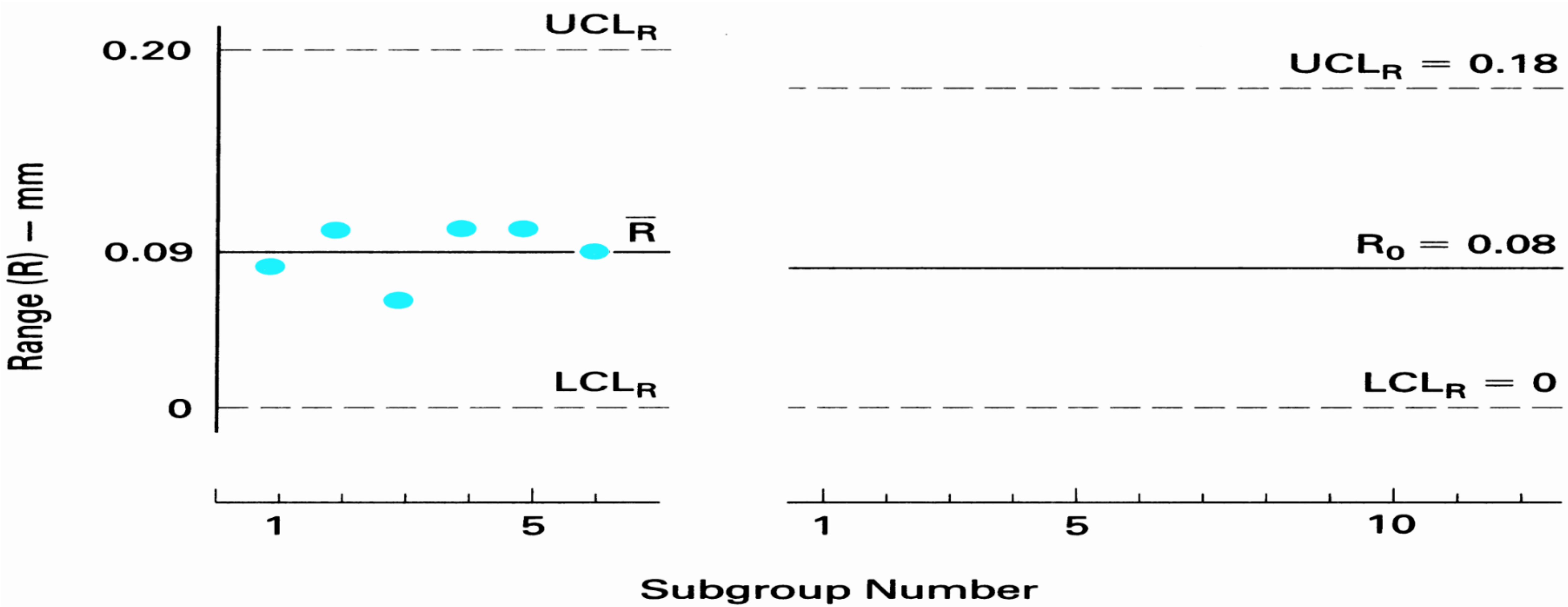


Figure 6-6 Trial control limits and revised control limits for Xbar and R charts

Achieving the Objective

Achieving the Objective

- When control charts are first introduced at a work center, an improvement in the process performance usually occurs.
- This initial improvement is especially noticeable when the process is dependent on the skill of the operator.
- Posting a quality control chart appears to be a psychological signal to the operator to improve performance.
- Most workers want to produce a quality product or service; therefore, when management shows an interest in the quality, the operatör responds.

Achieve the Objective

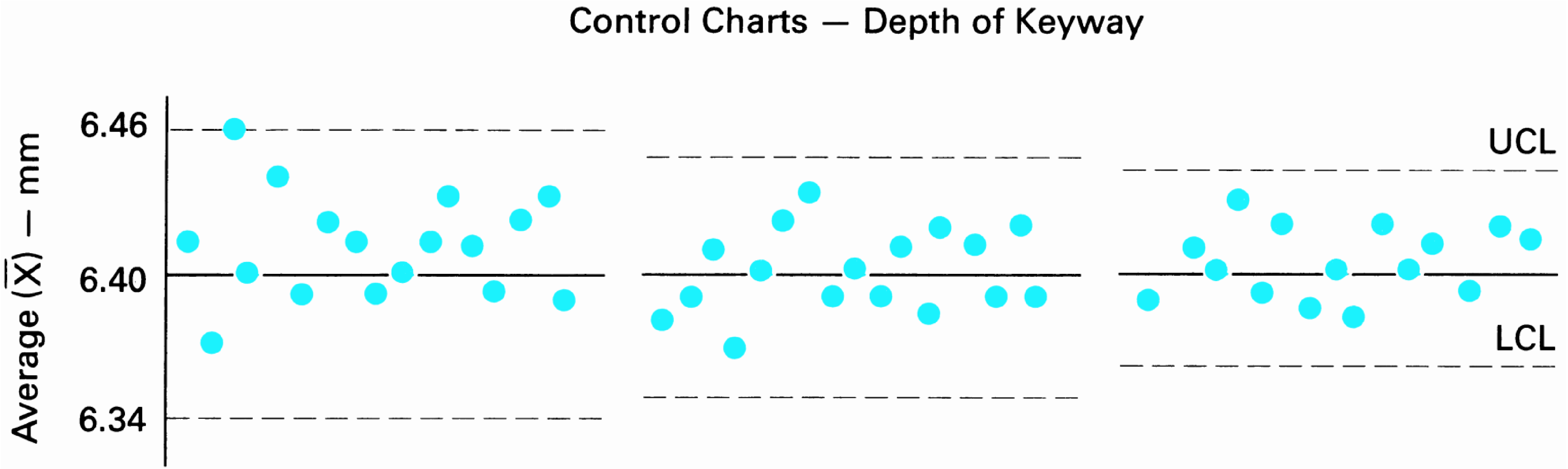


Figure 5-7 Continuing use of control charts, showing improved quality

Achieving the Objective

- Not all the improved performance in January was the result of operator effort. The first-line supervisor initiated a program of tool wear control, which was a contributing factor.
- At the end of January, new central lines and control limits were calculated using the data from subgroups obtained during the month.
- It is a good idea, especially when a chart is being initiated, to calculate standard values periodically to see if any changes have occurred.

Achieving the Objective

- Evaluation or testing of an idea requires 25 or more subgroups.
- The control chart will tell if the idea is good, poor, or has no effect on the process.
- Quality improvement occurs when the plotted points of the X chart converge on the central line, or when the plotted points of the R chart trend downward, or when both actions occur.
- If a poor idea is tested, then the reverse occurs. Of course, if the idea is neutral, it will have no affect on the plotted point pattern.

Achieve the Objective

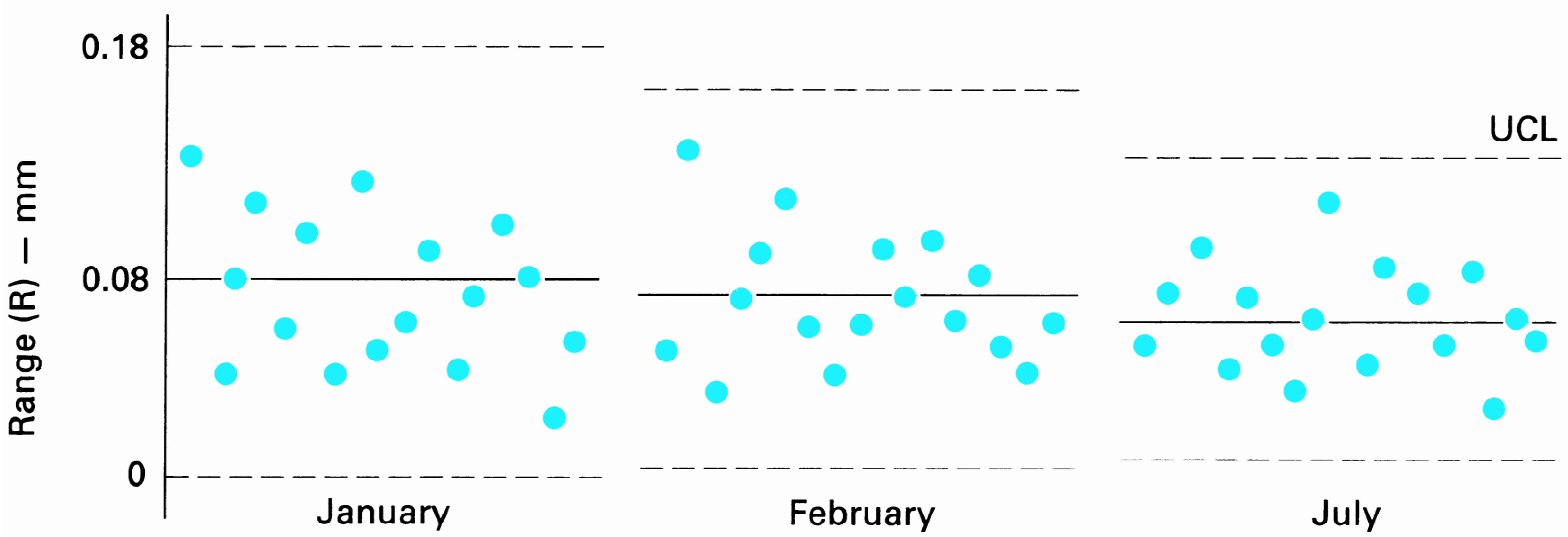


Figure 5-7 Continuing use of control charts, showing improved quality

The Sample Standard Deviation Control Chart

R chart vs *s*-chart

- Some organizations prefer the sample standard deviation, *s* , as the measure of the subgroup dispersion.
  - *R* chart is easier to compute and easier to explain.
  - *s* chart is calculated using all the data rather than just the high and the low value, hence *s* chart is more accurate than an *R* chart.
- When subgroup sizes are less than 10, both charts will graphically portray the same variation
- As subgroup sizes increase to 10 or more, extreme values have an undue influence on the *R* chart.
  - Therefore, at larger subgroup sizes, the *s* chart must be used.

$$\bar{s} = \frac{\sum_{i=1}^{g} \bar{s}_i}{g} \qquad \bar{\bar{X}} = \frac{\sum_{i=1}^{g} \bar{X}_i}{g}$$

$$\mathrm{UCL}_{\bar{X}} = \bar{X} + A_3\bar{s} \qquad \mathrm{UCL}_s = B_4\bar{s}$$

$$\mathrm{LCL}_{\bar{X}} = \bar{X} - A_3\bar{s} \qquad \mathrm{LCL}_s = B_3\bar{s}$$

where s_i = sample standard deviation of the subgroup values

$\bar{s}$ = average of the subgroup sample standard deviations

A_3, B_3, B_4 = factors found in Table B of the Appendix for obtaining the 3σ control limits for $\bar{X}$ and s charts from $\bar{s}$

Formulas for the computation of the revised control limits using the standard values of $\overline{X}_0$ and σ_0 are

$$\overline{X}_0 = \overline{\overline{X}}_{\text{new}} = \frac{\Sigma \overline{X} - \overline{X}_d}{g - g_d}$$

$$s_0 = \bar{s}_{\text{new}} = \frac{\Sigma s - s_d}{g - g_d} \qquad \sigma_0 = \frac{s_0}{c_4}$$

$$\text{UCL}_{\overline{X}} = \overline{X}_0 + A\sigma_0 \qquad \text{UCL}_s = B_6\sigma_0$$

$$\text{LCL}_{\overline{X}} = \overline{X}_0 - A\sigma_0 \qquad \text{LCL}_s = B_5\sigma_0$$

R Chart(trial/revized)

$$UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R} \qquad UCL_R = D_4\bar{R}$$

$$LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R} \qquad LCL_R = D_3\bar{R}$$

$$UCL_{\bar{X}} = \bar{X}_0 + A\sigma_0 \qquad UCL_R = D_2\sigma_0$$

$$LCL_{\bar{X}} = \bar{X}_0 - A\sigma_0 \qquad LCL_R = D_1\sigma_0$$

S-Chart(trial/revized)

$$UCL_{\bar{X}} = X + A_3\bar{s} \qquad UCL_s = B_4\bar{s}$$

$$LCL_{\bar{X}} = \bar{X} - A_3\bar{s} \qquad LCL_s = B_3\bar{s}$$

$$UCL_{\bar{X}} = \bar{X}_0 + A\sigma_0 \qquad UCL_s = B_6\sigma_0$$

$$LCL_{\bar{X}} = \bar{X}_0 - A\sigma_0 \qquad LCL_s = B_5\sigma_0$$

Calculation of s

The first step is to determine the standard deviation for each subgroup from the preliminary data. For subgroup 1, with values of 6.35, 6.40, 6.32, and 6.37, the standard deviation is

$$s = \sqrt{\frac{n \sum_{i=1}^{n} X_i^2 - \left(\sum_{i=1}^{n} X_i \right)^2}{n(n-1)}}$$

$$= \sqrt{\frac{4(6.35^2 + 6.40^2 + 6.32^2 + 6.37^2) - (6.35 + 6.40 + 6.32 + 6.37)^2}{4(4-1)}}$$

$$= 0.034 \text{ mm}$$

Subgroup Number	Date	Time	MEASUREMENTS X_1	X_2	X_3	X_4	Average $\overline{X}$	Sample Standard Deviation s	Comment
1	12/26	8:50	35	40	32	37	6.36	0.034	
2		11:30	46	37	36	41	6.40	0.045	
3		1:45	34	40	34	36	6.36	0.028	
4		3:45	69	64	68	59	6.65	0.045	New, temporary
5		4:20	38	34	44	40	6.39	0.042	operator
6	12/27	8:35	42	41	43	34	6.40	0.041	
7		9:00	44	41	41	46	6.43	0.024	
8		9:40	33	41	38	36	6.37	0.034	
9		1:30	48	44	47	45	6.46	0.018	
10		2:50	47	43	36	42	6.42	0.045	

Example Problem 6-4

Using the data of Table 6-3, determine the revised central line and control limits for $\overline{X}$ and s charts. The first step is to obtain $\bar{s}$ and $\overline{\overline{X}}$, which are computed from $\sum s$ and $\sum \overline{X}$, whose values are found in Table 6-3.

$$\bar{s} = \frac{\sum_{i=1}^{g} s_i}{g} = \frac{0.965}{25} = 0.039 \text{ mm} \qquad \overline{\overline{X}} = \frac{\sum_{i=1}^{g} \overline{X}_i}{g} = \frac{160.25}{25} = 6.41 \text{ mm}$$

From Table B the values of the factors—$A_3 = 1.628$, $B_3 = 0$, and $B_4 = 2.266$—are obtained, and the trial control limits are

$$\begin{aligned} UCL_{\bar{X}} &= \bar{\bar{X}} + A_3\bar{s} \\ &= 6.41 + (1.628)(0.039) \\ &= 6.47 \text{ mm} \end{aligned} \qquad \begin{aligned} LCL_{\bar{X}} &= \bar{\bar{X}} - A_3\bar{s} \\ &= 6.41 - (1.628)(0.039) \\ &= 6.35 \text{ mm} \end{aligned}$$

$$\begin{aligned} UCL_s &= B_4\bar{s} \\ &= (2.266)(0.039) \\ &= 0.088 \text{ mm} \end{aligned} \qquad \begin{aligned} LCL_s &= B_3\bar{s} \\ &= (0)(0.039) \\ &= 0 \text{ mm} \end{aligned}$$

Update in 05.04.2022

The next step is to plot the subgroup $\overline{X}$ and s on graph paper with the central lines and control limits. This step is show in Figure 6-8. Subgroups 4 and 20 are out of control on the $\overline{X}$ chart and, because they have assignable causes, they are discarded. Subgroup 18 is out of control on the s chart and, because it has an assignable cause, it is discarded. Computation to obtain the standard values of $\overline{X}_0$, s_0, and σ_0 are as follows:

Update in 05.04.2022

$$\overline{X}_0 = \overline{\overline{X}}_{new} = \frac{\sum \overline{X} - \overline{X}_d}{g - g_d} \qquad s_0 = \bar{s}_{new} = \frac{\sum s - s_d}{g - g_d}$$

$$= \frac{160.25 - 6.65 - 6.51}{25 - 2} \qquad = \frac{0.965 - 0.125}{25 - 1}$$

$$= 6.40 \text{ mm} \qquad = 0.035 \text{ mm}$$

$$\sigma_0 = \frac{s_0}{c_4} \qquad \text{from Table B, } c_4 = 0.9213$$

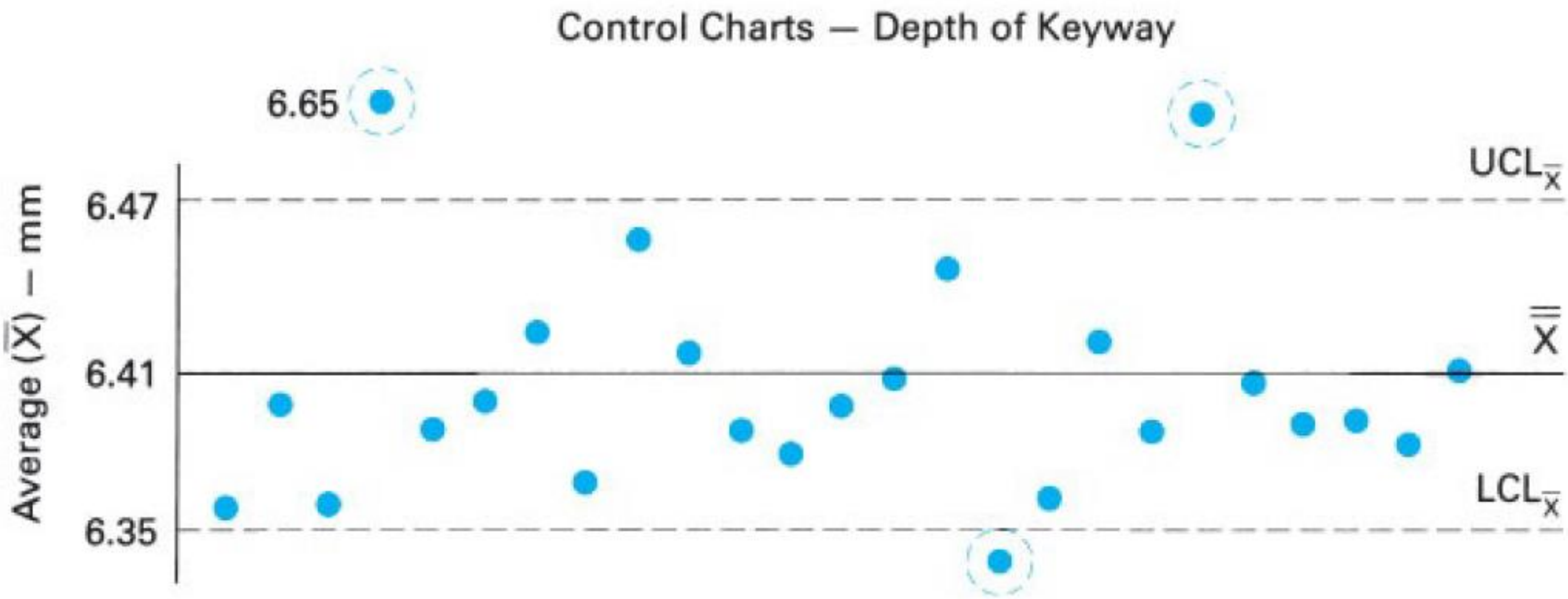
$$= \frac{0.035}{0.9213}$$

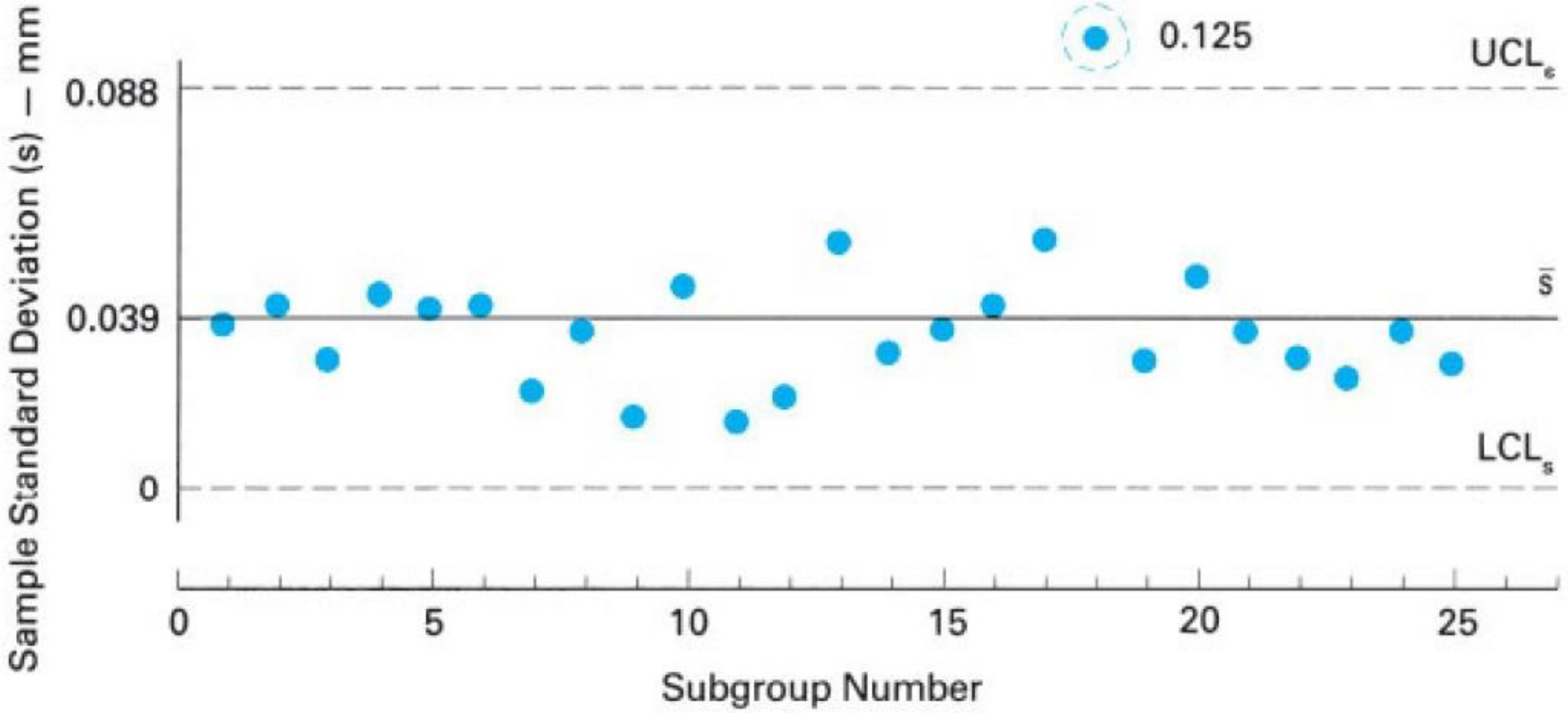
$$= 0.038 \text{ mm}$$

Update in 05.04.2022

- The reader should note that the standard deviation is the same as the value obtained from the range in the preceding section, i.e., recall that from Example 6.3 we have

$$\begin{aligned}
\overline{X}_0 &= \overline{X}_{new} = 6.40 \text{ mm} \\
R_0 = \overline{R}_{new} &= 0.079 \, (0.08 \text{ for the chart}) \\
\sigma_0 &= \frac{R_0}{d_2} \\
&= \frac{0.079}{2.059} \\
&= 0.038 \text{ mm}
\end{aligned}$$


Control Charts — Depth of Keyway
Average ($\bar{X}$) – mm
6.65
6.47
6.41
6.35
$UCL_{\bar{x}}$
$\bar{\bar{X}}$
$LCL_{\bar{x}}$


0.125
UCL$_s$
0.088
0.039
$\bar{s}$
LCL$_s$
0
Sample Standard Deviation (s) — mm
0
5
10
15
20
25
Subgroup Number