

Vize Soruları

1. Soru

$$\begin{aligned} 2x_1 - 5x_2 + x_3 &= 12 \\ -x_1 + 3x_2 - x_3 &= -8 \\ 3x_1 - 4x_2 + 2x_3 &= 16 \end{aligned}$$

LU yöntemi ile
çöz.

$$A = \begin{pmatrix} 2 & -5 & 1 \\ -1 & 3 & -1 \\ 3 & -4 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 12 \\ -8 \\ 16 \end{pmatrix}$$

$$A = L \cdot U \quad L \cdot Z = B \quad U \cdot X = Z$$

$$\begin{pmatrix} 2 & -5 & 1 \\ -1 & 3 & -1 \\ 3 & -4 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & -5 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 5 \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ \frac{3}{2} & 0 & 1 \end{pmatrix} \quad U = \begin{pmatrix} 2 & -5 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & 0 & 1 \end{pmatrix} \quad U = \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix}$$

$$L \cdot Z = B \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ \frac{3}{2} & 0 & 1 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} 12 \\ -8 \\ 16 \end{pmatrix}$$

$$z = \begin{pmatrix} 12 \\ -2 \\ 2 \end{pmatrix}$$

$$U \cdot X = Z \Rightarrow \begin{pmatrix} 2 & -5 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 12 \\ -2 \\ 4 \end{pmatrix}$$

$$X = \begin{pmatrix} 0 \\ -2 \\ 2 \end{pmatrix}$$

2. Soru

$$\begin{aligned} 4x_1 + x_2 + x_3 &= 1 \\ x_1 + 4x_2 + x_4 &= 2 \\ x_1 + 4x_3 + x_4 &= 0 \\ x^2 + x_3 + 4x_4 &= 1 \end{aligned}$$

Gauss-Seidel
ile çöz.

$$1 - x_2 - x_3 = x_1 = \frac{1}{4}$$

$$x_2 = \frac{2 - x_1 - x_4}{4} = \frac{7}{16}$$

$$x_3 = \frac{-x_4 - x_1}{4} = -\frac{1}{16}$$

$$1 - x_2 - x_3 = \frac{17 - \frac{49}{16} + \frac{1}{16}}{4} = \frac{17 - \frac{48}{16}}{4} = \frac{17 - 3}{4} = \frac{14}{4} = 3.5$$

$$1 - \frac{49}{16} + \frac{1}{16} = \frac{17 - \frac{48}{16}}{16} = \frac{223}{16} = 13.9375$$

3. Soru

$$\begin{aligned} x + y + z &= 2 \\ 2x + y + 3z &= 9 \\ x - 3y + z &= 10 \end{aligned}$$

Cramer yöntemi ile
çöz.

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 3 \\ 1 & -3 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 2 \\ 9 \\ 10 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 3 \\ 1 & -3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 3 \\ 1 & -3 & 1 \end{vmatrix} = 1 \cdot (1 \cdot 1 - (-3) \cdot 3) - 1 \cdot (1 \cdot 1 - 3 \cdot 1) + 1 \cdot (2 \cdot 1 - 1 \cdot (-3)) = 1 \cdot (1 + 9) - 1 \cdot (1 - 3) + 1 \cdot (2 + 3) = 10 - (-2) + 5 = 13$$

$$x_1 = \frac{|B|}{|A|} = \frac{2 \cdot 1 \cdot 1 + 9 \cdot 1 \cdot 1 + 10 \cdot 1 \cdot 1}{13} = \frac{2 + 9 + 10}{13} = \frac{21}{13}$$

$$x_1 = \frac{2 \cdot 1 \cdot 1 + 9 \cdot 1 \cdot 1 + 10 \cdot 1 \cdot 1}{13} = \frac{2 + 9 + 10}{13} = \frac{21}{13}$$

$$x_2 = \frac{2 \cdot 1 \cdot 1 + 9 \cdot 1 \cdot 1 + 10 \cdot 1 \cdot 1}{13} = \frac{2 + 9 + 10}{13} = \frac{21}{13}$$

$$x_1 = 1$$

$$x_2 = -2$$

$$x_3 = \frac{12}{4} = 3$$

$$x_3 = \frac{2 \cdot 1 \cdot 1 + 9 \cdot 1 \cdot 1 + 10 \cdot 1 \cdot 1}{13} = \frac{2 + 9 + 10}{13} = \frac{21}{13}$$

