Newton - Raphson

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Hata: $|x_{i+1} - x_i| \leq \varepsilon$ kök bu

$$\left| \frac{f''(x_0) \cdot f(x_0)}{f'(x_0)^2} \right| < 1 \quad \text{yakınsaklık koşulu}$$

!

Kiriş (Secant) Metodu

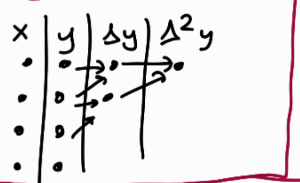
$$x_{i+1} = x_i - \frac{(x_i - x_{i-1})}{(y_i - y_{i-1})} \cdot y_i$$

Regula Falsi ?

$$x_0 = \frac{a \cdot f_b - b \cdot f_a}{f_b - f_a}$$

$|f(x_k)| \leq \varepsilon$ = kök

İleri Yönlü Sonlu Fark



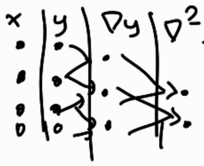
En küçük kareler

$$F_m(m,b) = -2\left(\sum_{i=1}^{n} x_i^2\right)m - 2\left(\sum_{i=1}^{n} x_i\right) + 2\left(\sum_{i=1}^{n} x_i y_i\right)$$

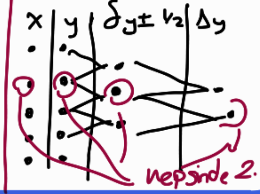
$$F_b(m,b) = -2\left(\sum_{i=1}^{n} x_i\right)m - 2\left(\sum_{i=1}^{n} 1\right)b + 2\left(\sum_{i=1}^{n} y_i\right)$$

$$F(m,b) = \sum_{i=1}^{n} (y_i - mx_i - b)^2 + \dots (y_n + m x_n + b)^2$$

Geri Yönlü Sonlu Fark



Merkezi Yönlü Sonlu Fark



Enterpolasyon

Polinom

$$P_n(x) = a_0 + a_1 x + a_2 x^2 + \dots a_n x^n$$

$$P_n(x_i) = f(x_i)$$

$$\begin{bmatrix} 1 & x_0 & x_0^2 & \dots & x_0^n \\ 1 & x_1 & x_1^2 & \dots & x_1^n \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_n & \dots & \dots & x_n^n \end{bmatrix}_{n \times n} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} f x_0 \\ f x_1 \\ f x_2 \\ \vdots \\ f x_n \end{bmatrix}$$

↓

$\text{Det} \neq 0$

Doğrusal

$$a_1 = \frac{f(x_i) - f(x_{i+1})}{x_i - x_{i+1}}$$

$a_0 = f(x_i) - a_0 \cdot x_i$ → m'ler eşittir

$$m = \frac{f x_{i+1} - f x_i}{x_{i+1} - x_i} \qquad y = y_i + \frac{\Delta y_i}{h}(x - x_i)$$

Kuadratik (3 noktası belli)

$$a = y_0 \qquad b = \frac{y_1 - y_0}{x_1 - x_0}$$

$$c = \frac{\left(\frac{y_2 - y_1}{x_2 - x_1} - \frac{y_1 - y_0}{x_1 - x_0} \right)}{x_2 - x_0}$$

$$P(x) = a + b(x - x_0) + c(x - x_0)(x - x_1)$$

Gregory-Newton

$$a_1 = f(x_1)$$

$$a_2 = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$a_3 = \frac{f(x_3) - a_1 - a_2(x_3 - x_1)}{(x_3 - x_1)(x_3 - x_2)}$$

$$P_x = a_1 + a_2(x - x_1) + a_3(x - x_1)(x - x_2)$$

....

İleri fark tablosuyla ($\hat{h}$ sabit) $x_{i+1} - x_i$

$$P_n(x) = y_0 + \frac{\Delta y_0}{1!h}(x - x_0) + \frac{\Delta^2 y_0}{2!h^2}(x - x_0)(x - x_1)$$

$$\dots \frac{\Delta^n y_0}{n!\,h^n}(x - x_0)(x - x_1)\dots(x - x_{n-1})$$

h sabit değilse

h = 1 yaparak hesapla

$$P(z) = x_0 + \Delta x_0 z + \frac{\Delta^2 x_0}{2!\,h}(z - z_0)(z - z_1 \dots)$$

$$P(z) = az^2 + bz + c$$

$x = P(z)$ old. için z yalnız bırak

$z - y$ tablosu yap

$$P(z) = y_0 + \frac{\Delta y_0}{1!h} \cdot (z - z_0) \dots$$

yalnız bıraktığın z'leri yerine koy

Lagrange

$$P(x) = \sum_{i=0}^{n} L_i(x) \cdot f(x_i)$$

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \left(\frac{x - x_j}{x_i - x_j} \right)$$

Sayısal İntegral

Simgesel Hesap ?

$$\int_{x_0}^{x_0 + rh} f(x)dx = [1 + E + E^2 \dots E^{r}]\, J f(x_0)$$

Trapez Kuralı (Yamuk)

$$\text{Alan} = \frac{h}{2}\left[y_0 + 2(y_1 + \dots y_{n-1}) + y_n \right]$$

trapz(x, f(x)) ← fark

h'ye göre vector

Simpson Kuralı

Trapeze göre daha doğru

a-b arası bilinen nokta var

$$\int_{x_i}^{x_{i+2}} f x dx = I \cong \frac{h}{3}\left[f(x_0) + 4 f x_1 + f x_2 \right]$$

Çift aralık

a-b " eşit aralıklı 2 nokta var

$$I \cong \frac{3h}{8}\left[f(x_0) + 3 f x_1 + 3 f x_2 + f x_3 \right]$$

$$y = \alpha x^2 + \beta x + \gamma$$

$$\int_a^b f x dx = \frac{b-a}{6}\left(f_a + 4\left(\frac{a+b}{2}\right) + f_b \right)$$

RIEMANN integrali

$$I = \int_{x_0}^{x_n} f(x)dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\left(\frac{x_n - x_0}{n} \right) \quad \Delta x$$

Newton-Cotes İntegrali

? bir sürü var ?