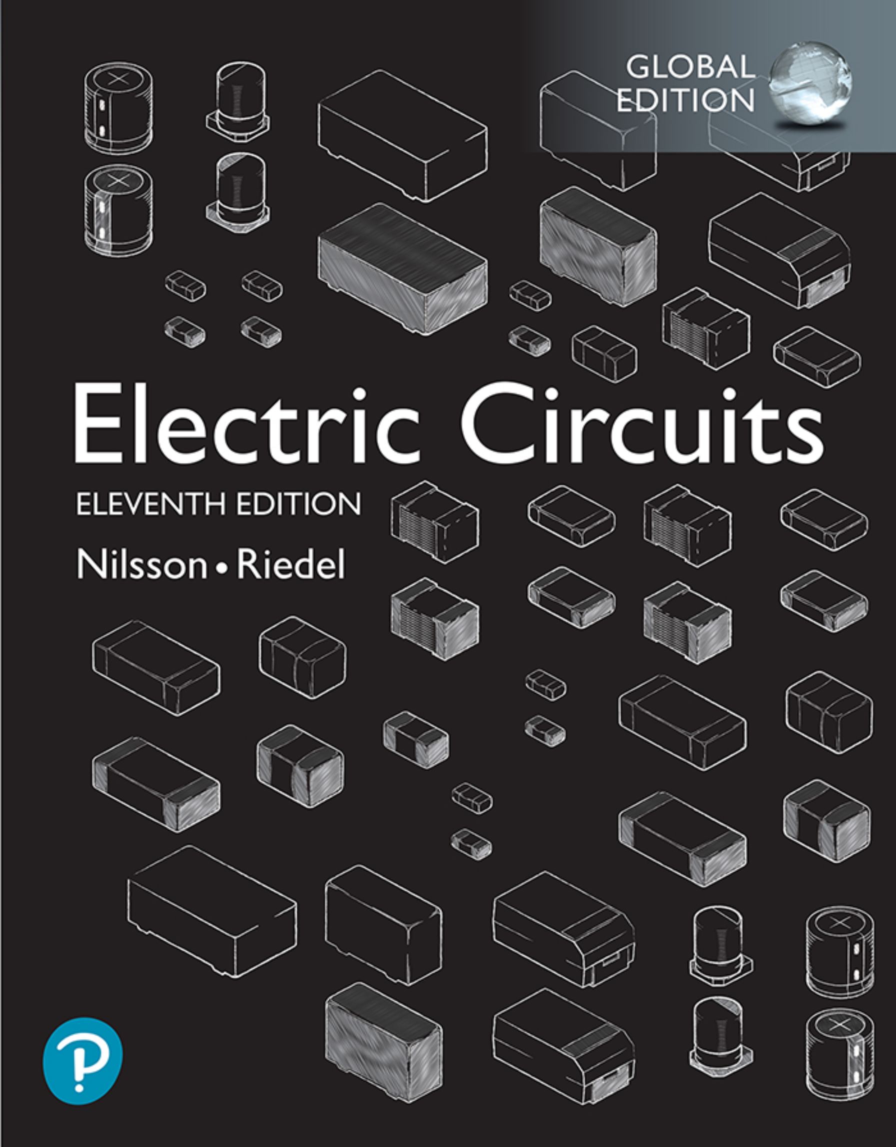

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TABLE 4.3 Steps in the Node-Voltage Method and the Mesh-Current Method

	Node-Voltage Method	Mesh-Current Method
Step 1 Identify nodes/meshes	Identify the essential nodes by circling them on the circuit diagram	Identify the meshes by drawing directed arrows inside each mesh
Step 2 Label node voltages/mesh currents Recognize special cases	Pick and label a reference node; then label the remaining essential node voltages • If a voltage source is the only component in a branch connecting the reference node and another essential node, label the essential node with the value of the voltage source • If a voltage source is the only component in a branch connecting two nonreference essential nodes, create a supernode that includes the voltage source and the two nodes on either side	Label each mesh current • If a current source is in a single mesh, label the mesh current with the value of the current source • If a current source is shared by two adjacent meshes, create a supermesh by combining the two adjacent meshes and temporarily eliminating the branch that contains the current source
Step 3 Write the equations	Write the following equations: • A KCL equation for any supernodes • A KCL equation for any remaining essential nodes where the voltage is unknown • A constraint equation for each dependent source that defines the controlling variable for the dependent source in terms of the node voltages • A constraint equation for each supernode that equates the difference between the two node voltages in the supernode to the voltage source in the supernode	Write the following equations: • A KVL equation for any supermeshes • A KVL equation for any remaining meshes where the current is unknown • A constraint equation for each dependent source that defines the controlling variable for the dependent source in terms of the mesh currents • A constraint equation for each supermesh that equates the difference between the two mesh currents in the supermesh to the current source eliminated to form the supermesh
Step 4 Solve the equations	Solve the equations to find the node voltages	Solve the equations to find the mesh currents
Step 5 Solve for other unknowns	Use the node voltage values to find any unknown voltages, currents, or powers	Use the mesh current values to find any unknown voltages, currents, or powers

ANALYZING A CIRCUIT WITH AN IDEAL OP AMP

1. Check for a negative feedback path. If it exists, assume the op amp operates in its linear region.
2. Write a KCL equation at the inverting input terminal.
3. Solve the KCL equation and use the solution to find the op amp's output voltage.
4. Compare the op amp's output voltage to the power supply voltages to determine if the op amp is operating in its linear region or if it is saturated.

GENERAL METHOD FOR NATURAL AND STEP RESPONSE OF *RL* AND *RC* CIRCUITS

1. Identify the variable $x(t)$, which is the inductor current for RL circuits and capacitor voltage for RC circuits.
2. Calculate the initial value X_0, by analyzing the circuit to find $x(t)$ for $t < 0$.
3. Calculate the time constant τ; for RL circuits $\tau = L/R$ and for RC circuits $\tau = RC$, where R is the equivalent resistance connected to the inductor or capacitor for $t \geq 0$.
4. Calculate the final value X_f, by analyzing the circuit to find $x(t)$ as $t \to \infty$; for the natural response, $X_f = 0$.
5. Write the equation for $x(t)$,
$x(t) = X_f + (X_0 - X_f)\,e^{-t/\tau}$, for $t \geq 0$.
6. Calculate other quantities of interest using $x(t)$.

NATURAL RESPONSE OF A PARALLEL *RLC* CIRCUIT

1. Determine the initial capacitor voltage (V_0) and inductor current (I_0) from the circuit.

2. Determine the values of α and ω_0 using the equations in Table 8.2.

3. If $\alpha^2 > \omega_0^2$, the response is overdamped and $v(t) = A_1e^{s_1t} + A_2e^{s_2t}, \quad t \geq 0$;
If $\alpha^2 < \omega_0^2$, the response is underdamped and $v(t) = B_1e^{-\alpha t}\cos\omega_d t + B_2e^{-\alpha t}\sin\omega_d t$, $t \geq 0$;
If $\alpha^2 = \omega_0^2$ the response is critically damped and $v(t) = D_1te^{-\alpha t} + D_2e^{-\alpha t}, \quad t \geq 0$.

4. If the response is overdamped, calculate s_1 and s_2 using the equations in Table 8.2;
If the response is underdamped, calculate ω_d using the equation in Table 8.2.

5. If the response is overdamped, calculate A_1 and A_2 by simultaneously solving the equations in Table 8.2;
If the response is underdamped, calculate B_1 and B_2 by simultaneously solving the equations in Table 8.2;
If the response is critically damped, calculate D_1 and D_2 by simultaneously solving the equations in Table 8.2.

6. Write the equation for $v(t)$ from Step 3 using the results from Steps 4 and 5; find any desired branch currents.

TABLE 8.2 Equations for analyzing the natural response of parallel *RLC* circuits

Characteristic equation	$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$
Neper, resonant, and damped frequencies	$\alpha = \frac{1}{2RC} \quad \omega_0 = \sqrt{\frac{1}{LC}} \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$
Roots of the characteristic equation	$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$
$\alpha^2 > \omega_0^2$: overdamped	$v(t) = A_1e^{s_1t} + A_2e^{s_2t}, \text{t} \geq 0$ $v(0^+) = A_1 + A_2 = V_0$ $\frac{dv(0^+)}{dt} = s_1A_1 + s_2A_2 = \frac{1}{C}\left(\frac{-V_0}{R} - I_0\right)$
$\alpha^2 < \omega_0^2$: underdamped	$v(t) = B_1e^{-\alpha t}\cos\omega_d t + B_2e^{-\alpha t}\sin\omega_d t, \text{t} \geq 0$ $v(0^+) = B_1 = V_0$ $\frac{dv(0^+)}{dt} = -\alpha B_1 + \omega_d B_2 = \frac{1}{C}\left(\frac{-V_0}{R} - I_0\right)$
$\alpha^2 = \omega_0^2$: critically damped	$v(t) = D_1te^{-\alpha t} + D_2e^{-\alpha t}, \text{t} \geq 0$ $v(0^+) = D_2 = V_0$ $\frac{dv(0^+)}{dt} = D_1 - \alpha D_2 = \frac{1}{C}\left(\frac{-V_0}{R} - I_0\right)$

(Note that the equations in the last three rows assume that the reference direction for the current in every component is in the direction of the reference voltage drop across that component.)

STEP RESPONSE OF A PARALLEL *RLC* CIRCUIT

1. Determine the initial capacitor voltage (V_0), the initial inductor current (I_0), and the final inductor current (I_f) from the circuit.

2. Determine the values of α and ω_0 using the equations in Table 8.3.

3. If $\alpha^2 > \omega_0^2$, the response is overdamped and $i_L(t) = I_f + A_1'e^{s_1t} + A_2'e^{s_2t}, \quad t \geq 0^+$;
If $\alpha^2 > \omega_0^2$ the response is underdamped and $i_L(t) = I_f + B_1'e^{-\alpha t}\cos\omega_d t + B_2'e^{-\alpha t}\sin\omega_d t, t \geq 0^+$;
If $\alpha^2 = \omega_0^2$, the response is critically damped and $i_L(t) = I_f + D_1'te^{-\alpha t} + D_2'e^{-\alpha t}, \quad t \geq 0^+$.

4. If the response is overdamped, calculate s_1 and s_2 using the equations in Table 8.3;
If the response is underdamped, calculate ω_d using the equation in Table 8.3.

5. If the response is overdamped, calculate A_1' and A_2' by simultaneously solving the equations in Table 8.3;
If the response is underdamped, calculate B_1' and B_2' by simultaneously solving the equations in Table 8.3;
If the response is critically damped, calculate D_1' and D_2' by simultaneously solving the equations in Table 8.3.

6. Write the equation for $i_L(t)$ from Step 3 using the results from Steps 4 and 5; find the inductor voltage and any desired branch currents.

TABLE 8.3 Equations for analyzing the step response of parallel *RLC* circuits

Characteristic equation	$s^2 + \frac{1}{RC}s + \frac{1}{LC} = \frac{I}{LC}$
Neper, resonant, and damped frequencies	$\alpha = \frac{1}{2RC} \quad \omega_0 = \sqrt{\frac{1}{LC}} \quad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$
Roots of the characteristic equation	$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}, \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$
$\alpha^2 > \omega_0^2$: overdamped	$i_L(t) = I_f + A_1'e^{s_1t} + A_2'e^{s_2t}, \quad t \geq 0$ $i_L(0^+) = I_f + A_1' + A_2' = I_0$ $\frac{di_L(0^+)}{dt} = s_1A_1' + s_2A_2' = \frac{V_0}{L}$
$\alpha^2 < \omega_0^2$: underdamped	$i_L(t) = I_f + B_1'e^{-\alpha t}\cos\omega_d t + B_2'e^{-\alpha t}\sin\omega_d t, \quad t \geq 0$ $i_L(0^+) = I_f + B_1' = I_0$ $\frac{di_L(0^+)}{dt} = -\alpha B_1' + \omega_d B_2' = \frac{V_0}{L}$
$\alpha^2 = \omega_0^2$: critically damped	$i_L(t) = I_f + D_1'te^{-\alpha t} + D_2'e^{-\alpha t}, \quad t \geq 0$ $i_L(0^+) = I_f + D_2' = I_0$ $\frac{di_L(0^+)}{dt} = D_1' - \alpha D_2' = \frac{V_0}{L}$

(Note that the equations in the last three rows assume that the reference direction for the current in every component is in the direction of the reference voltage drop across that component.)

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Courtesy of Anna Nilsson

In Memoriam

We remember our beloved author, James W. Nilsson, for his lasting legacy to the electrical and computer engineering field.

The first edition of *Electric Circuits* was published in 1983. As this book evolved over the years to better meet the needs of both students and their instructors, the underlying teaching methodologies Jim established remain relevant, even in the Eleventh Edition.

Jim earned his bachelor's degree at the University of Iowa (1948), and his master's degree (1952) and Ph.D. (1958) at Iowa State University. He joined the ISU faculty in 1948 and taught electrical engineering there for 39 years.

He became an IEEE fellow in 1990 and earned the prestigious IEEE Undergraduate Teaching Award in 1992.

For Anna

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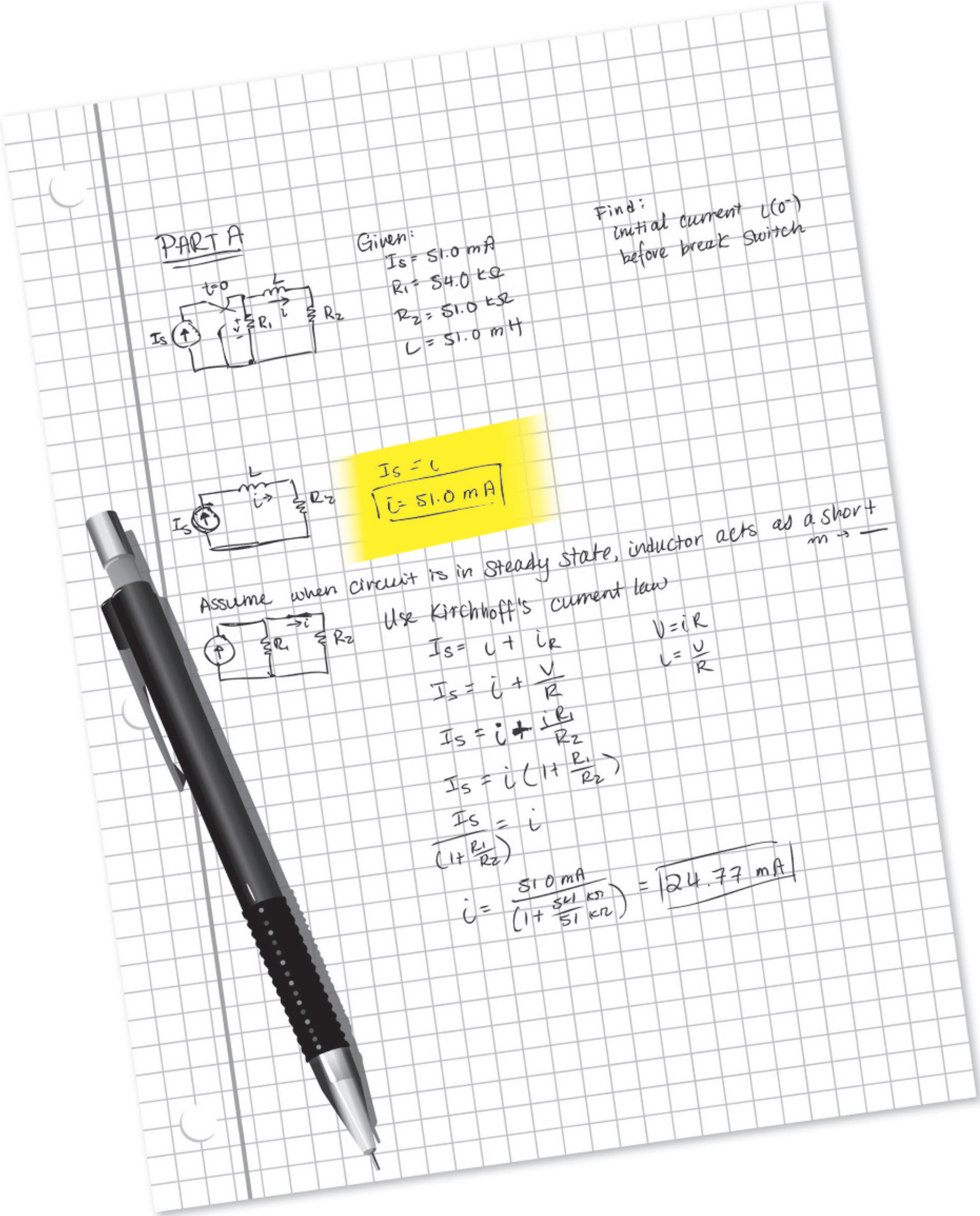

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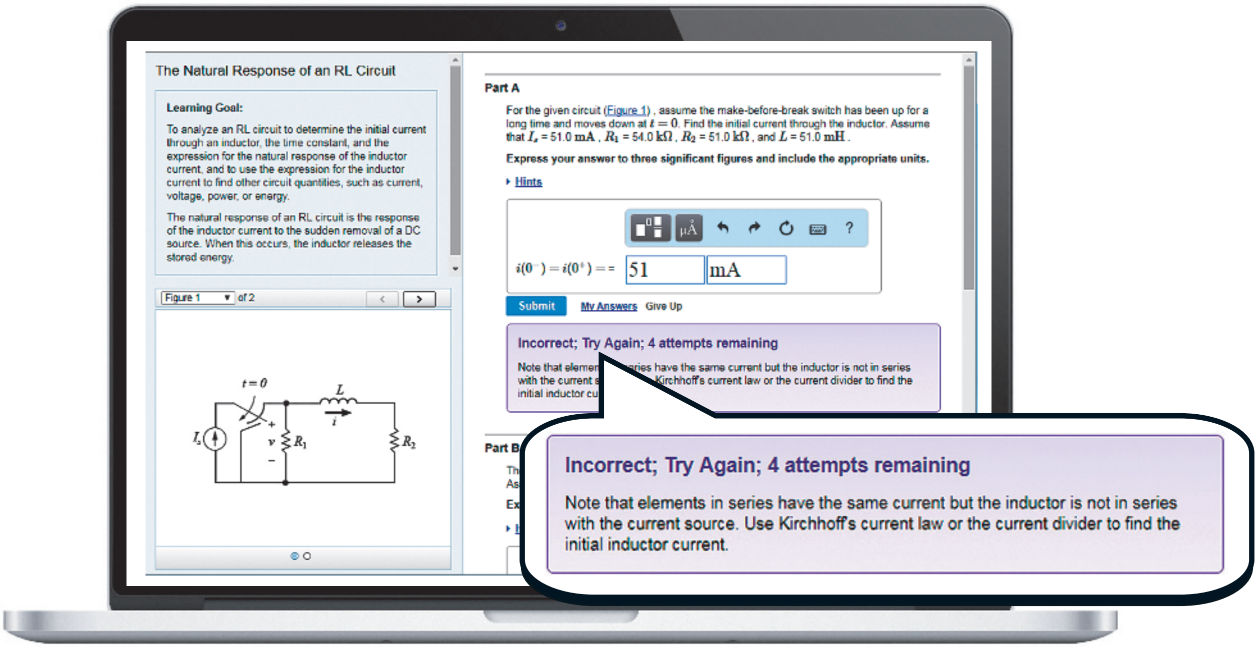

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Preface

The Eleventh Edition of *Electric Circuits* represents the most extensive revision to the text since the Fifth Edition, published in 1996. Every sentence, paragraph, subsection, and chapter has been examined to improve clarity, readability, and pedagogy. Yet the fundamental goals of the text are unchanged. These goals are:

- To build new concepts and ideas on concepts previously presented. This challenges students to see the explicit connections among the many circuit analysis tools and methods.
- To develop problem-solving skills that rely on a solid conceptual foundation. This challenges students to examine many different approaches to solving a problem before writing a single equation.
- To introduce realistic engineering experiences at every opportunity. This challenges students to develop the insights of a practicing engineer and exposes them to practice of engineering.

Why This Edition?

The Eleventh Edition of *Electric Circuits* incorporates the following new and revised elements:

- Analysis Methods – This new feature identifies the steps needed to apply a particular circuit analysis technique. Many students struggle just to get started when analyzing a circuit, and the analysis methods will reduce that struggle. Some of the analysis methods that are used most often can be found inside the book's covers for easy reference.
- Examples – Many students rely on examples when developing and refining their problem-solving skills. We identified many places in the text that needed additional examples, and as a result the number of examples has increased by nearly 35% to 200.
- End-of-chapter problems – Problem solving is fundamental to the study of circuit analysis. Having a wide variety of problems to assign and work is a key to success in any circuits course. Therefore, some existing end-of-chapter problems were revised, and some new end-of-chapter problems were added. Approximately 30% of the problems in the Eleventh Edition were rewritten.
- Fundamental equations and concepts – These important elements in the text were previously identified with margin notes. In this edition, the margin notes have been replaced by a second-color background, enlarged fonts, and a descriptive title for each fundamental equation and concept. In additional, many equation numbers have been eliminated to make it easier to distinguish fundamental equations from the many other equations in the text.
- Circuit simulation software – The PSpice® and Multisim® manuals have been revised to include screenshots from the most recent versions of these software simulation applications. Each manual presents the simulation material in the same order as the material is encountered in the text. These manuals include example simulations of circuits from the text. Icons identify end-of-chapter problems that are good candidates for simulation using either PSpice or Multisim.

- Solving simultaneous equations – Most circuit analysis techniques in this text eventually require you to solve two or more simultaneous linear algebraic equations. Appendix A has been extensively revised and includes examples of paper-and-pencil techniques, calculator techniques, and computer software techniques.
- Student Study Guides – Students who could benefit from additional examples and practice problems can use the Student Study Guides, which have been revised for the Eleventh Edition of the text. These guides have examples and problems covering the following material: balancing power, simple resistive circuits, node voltage method, mesh current method, Thévenin and Norton equivalents, op amp circuits, first-order circuits, second-order circuits, AC steady-state analysis, and Laplace transform circuit analysis.
- The Student Study Guides now include access to Video Solutions – complete, step-by-step solution walkthroughs to representative homework problems.
- Learning Catalytics – a "bring your own device" student engagement, assessment, and classroom intelligence system – is available with the Eleventh Edition. With Learning Catalytics you can:
  - Use open-ended questions to get into the minds of students to understand what they do or don't know, and adjust lectures accordingly.
  - Use a wide variety of question types to have students sketch a graph, annotate a circuit diagram, compose numeric or algebraic answers, and more.
  - Access rich analytics to understand student performance.
  - Use pre-built questions or add your own to make Learning Catalytics fit your course exactly.
- Pearson Mastering Engineering is an online tutorial and assessment program that provides students with personalized feedback and hints and instructors with diagnostics to track students' progress. With the Eleventh Edition, Mastering Engineering will offer new enhanced end-of-chapter problems with hints and feedback, Coaching Activities, and Adaptive Follow-Up assignments. Visit www.masteringengineering.com for more information.

Hallmark Features

Analysis Methods

Students encountering circuit analysis for the first time can benefit from step-by-step directions that lead them to a problem's solution. We have compiled these directions in a collection of analysis methods, and revised many of the examples in the text to employ these analysis methods.

Chapter Problems

Users of *Electric Circuits* have consistently rated the Chapter Problems as one of the book's most attractive features. In the Eleventh Edition, there are 1185 end-of-chapter problems with approximately 30% that have been revised from the previous edition. Problems are organized at the end of each chapter by section.

Practical Perspectives

The Eleventh Edition continues using Practical Perspectives to introduce the chapter. They provide real-world circuit examples, taken from real-world devices. Every chapter begins by describing a practical application of the material that follows. After presenting that material, the chapter revisits the Practical Perspective, performing a quantitative circuit analysis using the newly introduced chapter material. A special icon identifies end-of-chapter problems directly related to the Practical Perspective application. These problems provide additional opportunities for solving real-world problems using the chapter material.

Assessment Problems

Each chapter begins with a set of chapter objectives. At key points in the chapter, you are asked to stop and assess your mastery of a particular objective by solving one or more assessment problems. The answers to all of the assessment problems are given at the conclusion of each problem, so you can check your work. If you are able to solve the assessment problems for a given objective, you have mastered that objective. If you need more practice, several end-of-chapter problems that relate to the objective are suggested at the conclusion of the assessment problems.

Examples

Every chapter includes many examples that illustrate the concepts presented in the text in the form of a numeric example. There are now nearly 200 examples in this text, an increase of about 35% when compared to the previous edition. The examples illustrate the application of a particular concept, often employ an Analysis Method, and exemplify good problem-solving skills.

Fundamental Equations and Concepts

Throughout the text, you will see fundamental equations and concepts set apart from the main text. This is done to help you focus on some of the key principles in electric circuits and to help you navigate through the important topics.

Integration of Computer Tools

Computer tools can assist students in the learning process by providing a visual representation of a circuit's behavior, validating a calculated solution, reducing the computational burden of more complex circuits, and iterating toward a desired solution using parameter variation. This computational support is often invaluable in the design process. The Eleventh Edition supports PSpice and Multisim, both popular computer tools for circuit simulation and analysis. Chapter problems suited for exploration with PSpice and Multisim are marked accordingly.

Design Emphasis

The Eleventh Edition continues to support the emphasis on the design of circuits in many ways. First, many of the Practical Perspective discussions focus on the design aspects of the circuits. The accompanying Chapter Problems continue the discussion of the design issues in these practical examples. Second, design-oriented Chapter Problems have been labeled explicitly, enabling students and instructors to identify those problems with a design focus. Third, the identification of problems suited to exploration with PSpice or Multisim suggests design opportunities using these software tools. Fourth, some problems in nearly every chapter focus on the use of realistic component values in achieving a desired circuit design. Once such a problem has been analyzed, the student can proceed to a laboratory to build and test the circuit, comparing the analysis with the measured performance of the actual circuit.

Accuracy

All text and problems in the Eleventh Edition have undergone our strict hallmark accuracy checking process, to ensure the most error-free book possible.

Resources For Students

Mastering Engineering. Mastering Engineering provides tutorial homework problems designed to emulate the instructor's office hour environment, guiding students through engineering concepts with self-paced individualized coaching. These in-depth tutorial homework problems provide students with feedback specific to their errors and optional hints that break problems down into simpler steps. Visit www.masteringengineering.com for more information.

Learning Catalytics. Learning Catalytics is an interactive student response tool that encourages team-based learning by using student's smartphones, tablets, or laptops to engage them in interactive tasks and thinking. Visit www.learningcatalytics.com for more information.

Student Study Guides. These resources teach students techniques for solving problems presented in the text. Organized by concepts, these guides are a valuable problem-solving resource for all levels of students. The Student Study Guides now include access to Video Solutions, complete, step-by-step solution walkthroughs to representative homework problems.

Introduction to Multisim and Introduction to PSpice Manuals—Updated for the Eleventh Edition, these manuals are excellent resources for those wishing to integrate PSpice or Multisim into their classes.

Resources for Instructors

All instructor resources are available for download at the Instructor Resources Center. If you are in need of a login and password for this site, please contact your local Pearson representative.

Instructor Solutions Manual—Fully worked-out solutions to Assessment Problems and end-of-chapter problems.

PowerPoint lecture images—All figures from the text are available in PowerPoint for your lecture needs. An additional set of full lecture slides with embedded assessment questions are available upon request.

MasteringEngineering. This online tutorial and assessment program allows you to integrate dynamic homework with automated grading and personalized feedback. MasteringEngineering allows you to easily track the performance of your entire class on an assignment-by-assignment basis, or the detailed work of an individual student. For more information visit www.masteringengineering.com.

Learning Catalytics—This "bring your own device" student engagement, assessment and classroom intelligence system enables you to measure student learning during class, and adjust your lectures accordingly. A wide variety of question and answer types allows you to author your own questions, or you can use questions already authored into the system. For more information visit www.learningcatalytics.com or click on the Learning Catalytics link inside Mastering Engineering.

Prerequisites

In writing the first 12 chapters of the text, we have assumed that the reader has taken a course in elementary differential and integral calculus. We have also assumed that the reader has had an introductory physics course, at either the high school or university level, that introduces the concepts of energy, power, electric charge, electric current, electric potential, and electromagnetic fields. In writing the final six chapters, we have assumed the student has had, or is enrolled in, an introductory course in differential equations.

Course Options

The text has been designed for use in a one-semester, two-semester, or a three-quarter sequence.

- *Single-semester course:* After covering Chapters 1–4 and Chapters 6–10 (omitting Sections 7.7 and 8.5) the instructor can develop the desired emphasis by covering Chapter 5 (operational amplifiers), Chapter 11 (three-phase circuits), Chapters 13 and 14 (Laplace methods), or Chapter 18 (Two-Port Circuits).
- *Two-semester sequence:* Assuming three lectures per week, cover the first nine chapters during the first semester, leaving Chapters 10–18 for the second semester.
- *Academic quarter schedule:* Cover Chapters 1–6 in the first quarter, Chapters 7–12 in the second quarter, and Chapters 13–18 in the third quarter.

Note that the introduction to operational amplifier circuits in Chapter 5 can be omitted with minimal effect on the remaining material. If Chapter 5 is omitted, you should also omit Section 7.7, Section 8.5, Chapter 15, and those assessment problems and end-of-chapter problems that pertain to operational amplifiers.

There are several appendixes at the end of the book to help readers make effective use of their mathematical background. Appendix A presents several different methods for solving simultaneous linear equations; complex numbers are reviewed in Appendix B; Appendix C contains additional material on magnetically coupled coils and ideal transformers; Appendix D contains a brief discussion of the decibel; Appendix E is dedicated to Bode diagrams; Appendix F is devoted to an abbreviated table of trigonometric identities that are useful in circuit analysis; and an abbreviated table of useful integrals is given in Appendix G. Appendix H provides tables of common standard component values for resistors, inductors, and capacitors, to be used in solving many end-of-chapter problems. Selected Answers provides answers to selected end-of-chapter problems.

Acknowledgments

I will be forever grateful to Jim Nilsson for giving me the opportunity to collaborate with him on this textbook. I started by revising the PSpice supplement for the Third Edition, and became a co-author of the Fifth Edition. Jim was a patient and gracious mentor, and I learned so much from him about teaching and writing and hard work. It is a great honor to be associated with him through this textbook, and to impact the education of the thousands of students who use this text.

There were many hard-working people behind the scenes at our publisher who deserve my thanks and gratitude for their efforts on behalf of the Eleventh Edition. At Pearson, I would like to thank Norrin Dias, Erin Ault, Rose Kernan, and Scott Disanno for their continued support and encouragement, their professional demeanor, their willingness to lend an ear, and their months of long hours and no weekends. The author would also like to acknowledge the staff at Integra Software Solutions for their dedication and hard work in typesetting this text.

I am very grateful for the many instructors and students who have done formal reviews of the text or offered positive feedback and suggestions for improvement more informally. I am pleased to receive email from instructors and students who use the book, even when they are pointing out an error I failed to catch in the review process. I have been contacted by people who use our text from all over the world, and I thank all of you for taking the time to do so. I use as many of your suggestions as possible to continue to improve the content, the pedagogy, and the presentation in this text. I am privileged to have the opportunity to impact the educational experience of the many thousands of future engineers who will use this text.

Susan A. Riedel

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ELECTRIC CIRCUITS

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CHAPTER

1

Circuit Variables

CHAPTER CONTENTS

CHAPTER OBJECTIVES

1. Understand and be able to use SI units and the standard prefixes for powers of 10.
2. Know and be able to use the definitions of *voltage* and *current*.
3. Know and be able to use the definitions of *power* and *energy*.
4. Be able to use the passive sign convention to calculate the power for an ideal basic circuit element given its voltage and current.

Electrical engineering is an exciting and challenging profession for anyone who has a genuine interest in, and aptitude for, applied science and mathematics. Electrical engineers play a dominant role in developing systems that change the way people live and work. Satellite communication links, cell phones, computers, televisions, diagnostic and surgical medical equipment, robots, and aircraft represent systems that define a modern technological society. As an electrical engineer, you can participate in this ongoing technological revolution by improving and refining existing systems and by discovering and developing new systems to meet the needs of our ever-changing society.

This text introduces you to electrical engineering using the analysis and design of linear circuits. We begin by presenting an overview of electrical engineering, some ideas about an engineering point of view as it relates to circuit analysis, and a review of the International System of Units. We then describe generally what circuit analysis entails. Next, we introduce the concepts of voltage and current. We continue by discussing the ideal basic element and the need for a polarity reference system. We conclude the chapter by describing how current and voltage relate to power and energy.

Practical Perspective

Balancing Power

One of the most important skills you will develop is the ability to check your answers for the circuits you design and analyze using the tools developed in this text. A common method used to check for valid answers is to calculate the power in the circuit. The linear circuits we study have no net power, so the sum of the power associated with all circuit components must be zero. If the total power for the circuit is zero, we say that the power balances, but if the total power is not zero, we need to find the errors in our calculation.

As an example, we will consider a simple model for distributing electricity to a typical home. (Note that a more realistic model will be investigated in the Practical Perspective for Chapter 9.) The components labeled a and b represent the source of electrical power for the home. The components labeled c, d, and e represent the wires that carry the electrical current from the source to the devices in the home requiring electrical power. The components labeled f, g, and h represent lamps, televisions, hair dryers, refrigerators, and other devices that require power.

Once we have introduced the concepts of voltage, current, power, and energy, we will examine this circuit model in detail, and use a power balance to determine whether the results of analyzing this circuit are correct.

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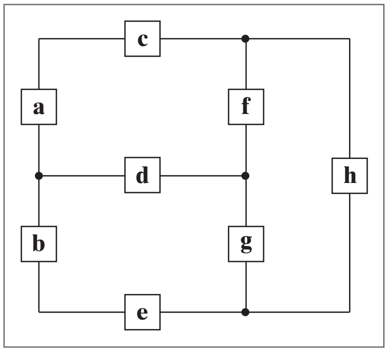

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1.1 Electrical Engineering: An Overview

The electrical engineering profession focuses on systems that produce, transmit, and measure electric signals. Electrical engineering combines the physicist's models of natural phenomena with the mathematician's tools for manipulating those models to produce systems that meet practical needs. Electrical systems pervade our lives; they are found in homes, schools, workplaces, and transportation vehicles everywhere. We begin by presenting a few examples from each of the five major classifications of electrical systems:

- communication systems
- computer systems
- control systems
- power systems
- signal-processing systems

Then we describe how electrical engineers analyze and design such systems.

Communication systems are electrical systems that generate, transmit, and distribute information. Well-known examples include television equipment, such as cameras, transmitters, receivers, and monitors; radio telescopes, used to explore the universe; satellite systems, which return images of other planets and our own; radar systems, used to coordinate plane flights; and telephone systems.

Figure 1.1 depicts the major components of a modern telephone system that supports mobile phones, landlines, and international calling. Inside a telephone, a microphone turns sound waves into electric signals. These signals are carried to local or mobile exchanges, where they are combined with the

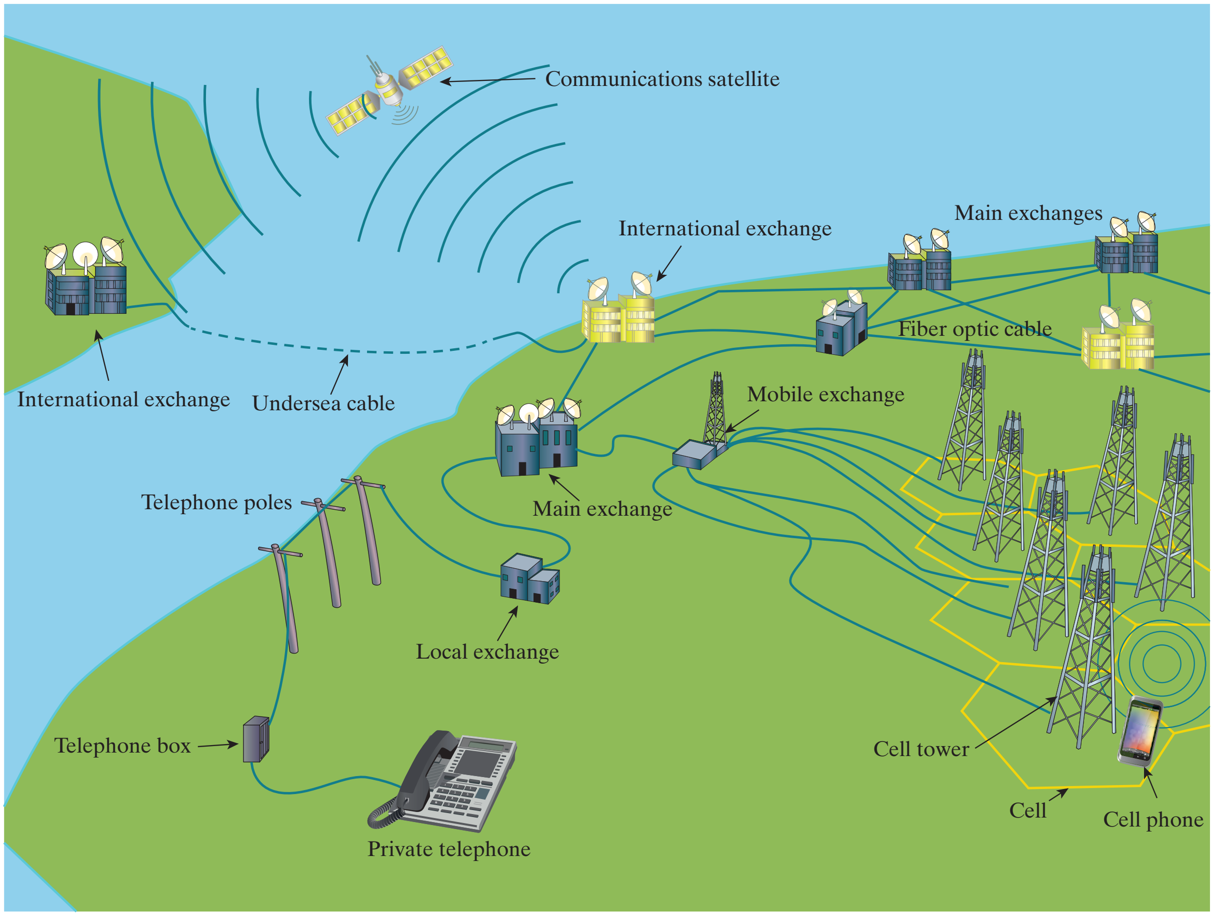

Figure 1.1 ▲ A telephone system.

signals from tens, hundreds, or thousands of other telephones. The form of the signals can be radio waves traveling through air, electrical signals traveling in underground coaxial cable, light pulses traveling in fiber-optic cable, or microwave signals that travel through space. The combined signals are broadcast from a transmission antenna to a receiving antenna. There the combined signals are separated at an exchange, and each is routed to the appropriate telephone, where an earphone acts as a speaker to convert the received electric signals back into sound waves. At each stage of the process, electric circuits operate on the signals. Imagine the challenge involved in designing, building, and operating each circuit in a way that guarantees that all of the hundreds of thousands of simultaneous calls have high-quality connections.

Computer systems use electric signals to process information ranging from word processing to mathematical computations. Systems range in size and power from simple calculators to personal computers to supercomputers that perform such complex tasks as processing weather data and modeling chemical interactions of complex organic molecules. These systems include networks of integrated circuits—miniature assemblies of hundreds, thousands, or millions of electrical components that often operate at speeds and power levels close to fundamental physical limits, including the speed of light and the thermodynamic laws.

Control systems use electric signals to regulate processes. Examples include the control of temperatures, pressures, and flow rates in an oil refinery; the fuel–air mixture in a fuel-injected automobile engine; mechanisms such as the motors, doors, and lights in elevators; and the locks in the Panama Canal. The autopilot and autolanding systems that help to fly and land airplanes are also familiar control systems.

Power systems generate and distribute electric power. Electric power, which is the foundation of our technology-based society, usually is generated in large quantities by nuclear, hydroelectric, solar, and thermal (coal-, oil-, or gas-fired) generators. Power is distributed by a grid of conductors that crisscross the country. A major challenge in designing and operating such a system is to provide sufficient redundancy and control so that failure of any piece of equipment does not leave a city, state, or region completely without power.

Signal-processing systems act on electric signals that represent information. They transform the signals and the information contained in them into a more suitable form. There are many different ways to process the signals and their information. For example, image-processing systems gather massive quantities of data from orbiting weather satellites, reduce the amount of data to a manageable level, and transform the remaining data into a video image for the evening news broadcast. A magnetic resonance imaging (MRI) scan is another example of an image-processing system. It takes signals generated by powerful magnetic fields and radio waves and transforms them into a detailed, three-dimensional image such as the one shown in Fig. 1.2, which can be used to diagnose disease and injury.

Considerable interaction takes place among the engineering disciplines involved in designing and operating these five classes of systems. Thus, communications engineers use digital computers to control the flow of information. Computers contain control systems, and control systems contain computers. Power systems require extensive communications systems to coordinate safely and reliably the operation of components, which may be spread across a continent. A signal-processing system may involve a communications link, a computer, and a control system.

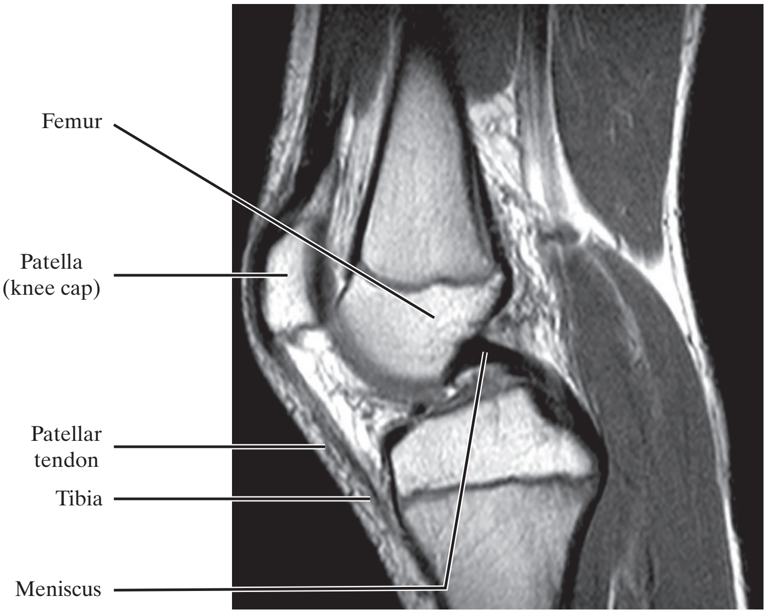

Figure 1.2 ▲ An MRI scan of an adult knee joint.

Neil Borden/Science Source/Getty Images

A good example of the interaction among systems is a commercial airplane, such as the one shown in Fig. 1.3. A sophisticated communications system enables the pilot and the air traffic controller to monitor the plane's location, permitting the air traffic controller to design a safe flight path for all of the nearby aircraft and enabling the pilot to keep the plane on its designated path. An onboard computer system manages engine functions, implements the navigation and flight control systems, and generates video information screens in the cockpit. A complex control system uses cockpit commands to adjust the position and speed of the airplane, producing the appropriate signals to the engines and the control surfaces (such as the wing flaps, ailerons, and rudder) to ensure the plane remains safely airborne and on the desired flight path. The plane must have its own power system to stay aloft and to provide and distribute the electric power needed to keep the cabin lights on, make the coffee, and activate the entertainment system. Signal-processing systems reduce the noise in air traffic communications and transform information about the plane's location into the more meaningful form of a video display in the cockpit. Engineering challenges abound in the design of each of these systems and their integration into a coherent whole. For example, these systems must operate in widely varying and unpredictable environmental conditions. Perhaps the most important engineering challenge is to guarantee that sufficient redundancy is incorporated in the designs, ensuring that passengers arrive safely and on time at their desired destinations.

Although electrical engineers may be interested primarily in one area, they must also be knowledgeable in other areas that interact with this area of interest. This interaction is part of what makes electrical engineering a challenging and exciting profession. The emphasis in engineering is on making things work, so an engineer is free to acquire and use any technique from any field that helps to get the job done.

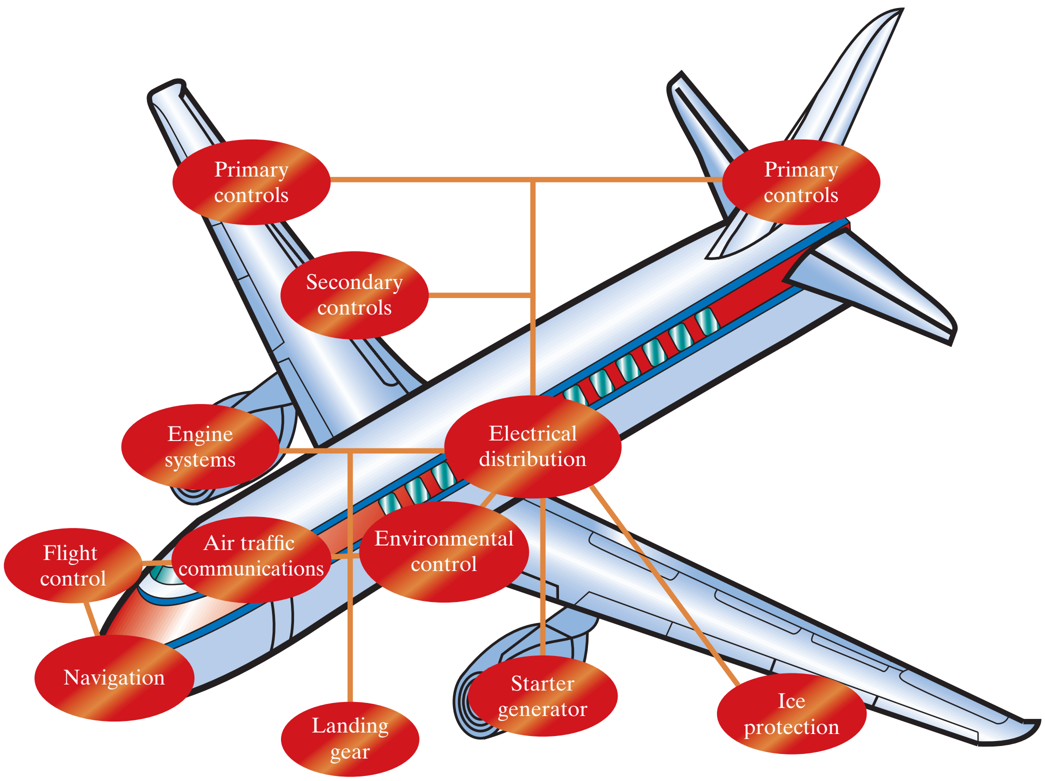

Figure 1.3 ▲ Interacting systems on a commercial aircraft.

Circuit Theory

An **electric circuit** is a mathematical model that approximates the behavior of an actual electrical system. Since electric circuits are found in every branch of electrical engineering, they provide an important foundation for learning how to design and operate systems such as those just described. The models, the mathematical techniques, and the language of circuit theory will form the intellectual framework for your future engineering endeavors.

Note that the term *electric circuit* is commonly used to refer to an actual electrical system as well as to the model that represents it. In this text, when we talk about an electric circuit, we always mean a model, unless otherwise stated. It is the modeling aspect of circuit theory that has broad applications across engineering disciplines.

Circuit theory is a special case of electromagnetic field theory: the study of static and moving electric charges. But applying generalized field theory to the study of electric signals is cumbersome and requires advanced mathematics. Consequently, a course in electromagnetic field theory is not a prerequisite to understanding the material in this book. We do, however, assume that you have had an introductory physics course in which electrical and magnetic phenomena were discussed.

Three basic assumptions permit us to use circuit theory, rather than electromagnetic field theory, to study a physical system represented by an electric circuit.

1. *Electrical effects happen instantaneously throughout a system.* We can make this assumption because we know that electric signals travel at or near the speed of light. Thus, if the system is physically small, electric signals move through it so quickly that we can consider them to affect every point in the system simultaneously. A system that is small enough so that we can make this assumption is called a **lumped-parameter system**.

2. *The net charge on every component in the system is always zero.* Thus, no component can collect a net excess of charge, although some components, as you will learn later, can hold equal but opposite separated charges.

3. *There is no magnetic coupling between the components in a system.* As we demonstrate later, magnetic coupling can occur *within* a component.

That's it; there are no other assumptions. Using circuit theory provides simple solutions (of sufficient accuracy) to problems that would become hopelessly complicated if we were to use electromagnetic field theory. These benefits are so great that engineers sometimes specifically design electrical systems to ensure that these assumptions are met. The importance of assumptions 2 and 3 becomes apparent after we introduce the basic circuit elements and the rules for analyzing interconnected elements.

Let's take a closer look at assumption 1. The question is, "How small does a physical system have to be to qualify as a lumped-parameter system?" To get a quantitative answer to this question, remember that electric signals propagate as waves. If the wavelength of the signal is large compared to the physical dimensions of the system, we have a lumped-parameter system. The wavelength λ is the velocity divided by the repetition rate, or **frequency**, of the signal; that is, $\lambda = c/f$. The frequency f is measured in hertz (Hz). For example, power systems in the United States operate at 60 Hz. If we use the speed of light ($c = 3 \times 10^8$ m/s) as the velocity of propagation, the wavelength is 5×10^6 m. If the power system of interest is physically smaller than this wavelength, we can represent it as a lumped-parameter system and use circuit theory to analyze its behavior.

How do we define *smaller*? A good rule is the *rule of 1/10th*: If the dimension of the system is less than 1/10th the dimension of the wavelength, you have a lumped-parameter system. Thus, as long as the physical dimension of the power system is less than 5×10^5 m (which is about 310 miles), we can treat it as a lumped-parameter system.

Now consider a communication system that sends and receives radio signals. The propagation frequency of radio signals is on the order of 10^9 Hz, so the wavelength is 0.3 m. Using the rule of 1/10th, a communication system qualifies as a lumped-parameter system if its dimension is less than 3 cm. Whenever any of the pertinent physical dimensions of a system under study approaches the wavelength of its signals, we must use electromagnetic field theory to analyze that system. Throughout this book we study circuits derived from lumped-parameter systems.

Problem Solving

As a practicing engineer, you will not be asked to solve problems that have already been solved. Whether you are improving the performance of an existing system or designing a new system, you will be working on unsolved problems. As a student, however, you will read and discuss problems with known solutions. Then, by solving related homework and exam problems on your own, you will begin to develop the skills needed to attack the unsolved problems you'll face as a practicing engineer.

Let's review several general problem-solving strategies. Many of these pertain to thinking about and organizing your solution strategy *before* proceeding with calculations.

1. *Identify what's given and what's to be found.* In problem solving, you need to know your destination before you can select a route for getting there. What is the problem asking you to solve or find? Sometimes the goal of the problem is obvious; other times you may need to paraphrase or make lists or tables of known and unknown information to see your objective.

   On one hand, the problem statement may contain extraneous information that you need to weed out before proceeding. On the other hand, it may offer incomplete information or more complexities than can be handled by the solution methods you know. In that case, you'll need to make assumptions to fill in the missing information or simplify the problem context. Be prepared to circle back and reconsider supposedly extraneous information and/or your assumptions if your calculations get bogged down or produce an answer that doesn't seem to make sense.

2. *Sketch a circuit diagram or other visual model.* Translating a verbal problem description into a visual model is often a useful step in the solution process. If a circuit diagram is already provided, you may need to add information to it, such as labels, values, or reference directions. You may also want to redraw the circuit in a simpler, but equivalent, form. Later in this text you will learn the methods for developing such simplified equivalent circuits.

3. *Think of several solution methods and decide on a way of choosing among them.* This course will help you build a collection of analytical tools, several of which may work on a given problem. But one method may produce fewer equations to be solved than another, or it may require only algebra instead of calculus to reach a solution. Such efficiencies, if you can anticipate them, can streamline your calculations considerably. Having an alternative method in mind also gives you a path to pursue if your first solution attempt bogs down.

4. *Calculate a solution.* Your planning up to this point should have helped you identify a good analytical method and the correct equations for the problem. Now comes the solution of those equations. Paper-and-pencil, calculator, and computer methods are all available for performing the actual calculations of circuit analysis. Efficiency and your instructor's preferences will dictate which tools you should use.

5. *Use your creativity.* If you suspect that your answer is off base or if the calculations seem to go on and on without moving you toward a solution, you should pause and consider alternatives. You may need to revisit your assumptions or select a different solution method. Or you may need to take a less conventional problem-solving approach, such as working backward from a solution. This text provides answers to all of the Assessment Problems and many of the Chapter Problems so that you may work backward when you get stuck. In the real world, you won't be given answers in advance, but you may have a desired problem outcome in mind from which you can work backward. Other creative approaches include allowing yourself to see parallels with other types of problems you've successfully solved, following your intuition or hunches about how to proceed, and simply setting the problem aside temporarily and coming back to it later.

6. *Test your solution.* Ask yourself whether the solution you've obtained makes sense. Does the magnitude of the answer seem reasonable? Is the solution physically realizable? Are the units correct? You may want to rework the problem using an alternative method to validate your original answer and help you develop your intuition about the most efficient solution methods for various kinds of problems. In the real world, safety-critical designs are always checked by several independent means. Getting into the habit of checking your answers will benefit you both as a student and as a practicing engineer.

These problem-solving steps cannot be used as a recipe to solve every problem in this or any other course. You may need to skip, change the order of, or elaborate on certain steps to solve a particular problem. Use these steps as a guideline to develop a problem-solving style that works for you.

1.2 The International System of Units

Engineers use quantitative measures to compare theoretical results to experimental results and compare competing engineering designs. Modern engineering is a multidisciplinary profession in which teams of engineers work together on projects, and they can communicate their results in a meaningful way only if they all use the same units of measure. The International System of Units (abbreviated SI) is used by all the major engineering societies and most engineers throughout the world; hence we use it in this book.

The SI units are based on seven *defined* quantities:

- length
- mass
- time
- electric current
- thermodynamic temperature
- amount of substance
- luminous intensity

These quantities, along with the basic unit and symbol for each, are listed in Table 1.1. Although not strictly SI units, the familiar time units

TABLE 1.1 The International System of Units (SI)

Quantity	Basic Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	degree kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

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of minute (60 s), hour (3600 s), and so on are often used in engineering calculations. In addition, defined quantities are combined to form **derived** units. Some quantities, such as force, energy, power, and electric charge, you already know through previous physics courses. Table 1.2 lists the derived units used in this book.

In many cases, the SI unit is either too small or too large to use conveniently. Standard prefixes corresponding to powers of 10, as listed in Table 1.3, are then applied to the basic unit. Engineers often use only the prefixes for powers divisible by 3; thus centi, deci, deka, and hecto are used rarely. Also, engineers often select the prefix that places the base number in the range between 1 and 1000. Suppose that a time calculation yields a result of 10^{-5} s, that is, 0.00001 s. Most engineers would describe this quantity as 10 μs, that is, 10×10^{-6} s, rather than as 0.01 ms or 10,000 ns.

Example 1.1 illustrates a method for converting from one set of units to another and also uses power-of-10 prefixes.

TABLE 1.2 Derived Units in SI

Quantity	Unit Name (Symbol)	Formula
Frequency	hertz (Hz)	s^{-1}
Force	newton (N)	$kg \cdot m/s^2$
Energy or work	joule (J)	$N \cdot m$
Power	watt (W)	J/s
Electric charge	coulomb (C)	$A \cdot s$
Electric potential	volt (V)	J/C
Electric resistance	ohm (Ω)	V/A
Electric conductance	siemens (S)	A/V
Electric capacitance	farad (F)	C/V
Magnetic flux	weber (Wb)	$V \cdot s$
Inductance	henry (H)	Wb/A

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TABLE 1.3 Standardized Prefixes to Signify Powers of 10

Prefix	Symbol	Power
atto	a	10^{-18}
femto	f	10^{-15}
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
centi	c	10^{-2}
deci	d	10^{-1}
deka	da	10
hecto	h	10^{2}
kilo	k	10^{3}
mega	M	10^{6}
giga	G	10^{9}
tera	T	10^{12}

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EXAMPLE 1.1 Using SI Units and Prefixes for Powers of 10

If a signal can travel in a cable at 80% of the speed of light, what length of cable, in inches, represents 1 ns?

Solution

First, note that $1\text{ ns} = 10^{-9}\text{ s}$. Also, recall that the speed of light $c = 3 \times 10^8\text{ m/s}$. Then, 80% of the speed of light is $0.8c = (0.8)(3 \times 10^8) = 2.4 \times 10^8\text{ m/s}$. Using a product of ratios, we can convert 80% of the speed of light from meters per second to inches per nanosecond. The result is the distance in inches traveled in 1 nanosecond:

$$\frac{2.4 \times 10^8\text{ meters}}{1\text{ second}} \cdot \frac{1\text{ second}}{10^9\text{ nanoseconds}} \cdot \frac{100\text{ centimeters}}{1\text{ meter}} \cdot \frac{1\text{ inch}}{2.54\text{ centimeters}}$$
$$= 9.45\text{ inches/nanosecond}.$$

Therefore, a signal traveling at 80% of the speed of light will cover 9.45 inches of cable in 1 nanosecond.

ASSESSMENT PROBLEMS

Objective 1—Understand and be able to use SI units and the standard prefixes for powers of 10

1.1 Assume a telephone signal travels through a cable at two-thirds the speed of light. How long does it take the signal to get from New York City to Miami if the distance is approximately 1100 miles?

Answer: 8.85 ms.

1.2 How many dollars per millisecond would the federal government have to collect to retire a deficit of \$100 billion in one year?

Answer: \$3.17/ms.

SELF-CHECK: Also try Chapter Problems 1.2, 1.3, and 1.6.

1.3 Circuit Analysis: An Overview

We look broadly at engineering design, specifically the design of electric circuits, before becoming involved in the details of circuit analysis. This overview provides you with a perspective on where circuit analysis fits within the whole of circuit design. Even though this book focuses on circuit analysis, we try to provide opportunities for circuit design where appropriate.

All engineering designs begin with a need (1), as shown in Fig. 1.4. This need may come from the desire to improve on an existing design, or it may be something brand new. A careful assessment of the need results in design specifications, which are measurable characteristics of a proposed design. Once a design is proposed, the design specifications (2) allow us to assess whether or not the design actually meets the need.

A concept (3) for the design comes next. The concept derives from a complete understanding of the design specifications coupled with an insight into the need, which comes from education and experience. The concept may be realized as a sketch, as a written description, or as some other form. Often the next step is to translate the concept into a mathematical model. A commonly used mathematical model for electrical systems is a circuit model (4).

The elements that make up the circuit model are called ideal circuit components. An **ideal circuit component** is a mathematical model of an actual electrical component, like a battery or a light bulb. The ideal circuit

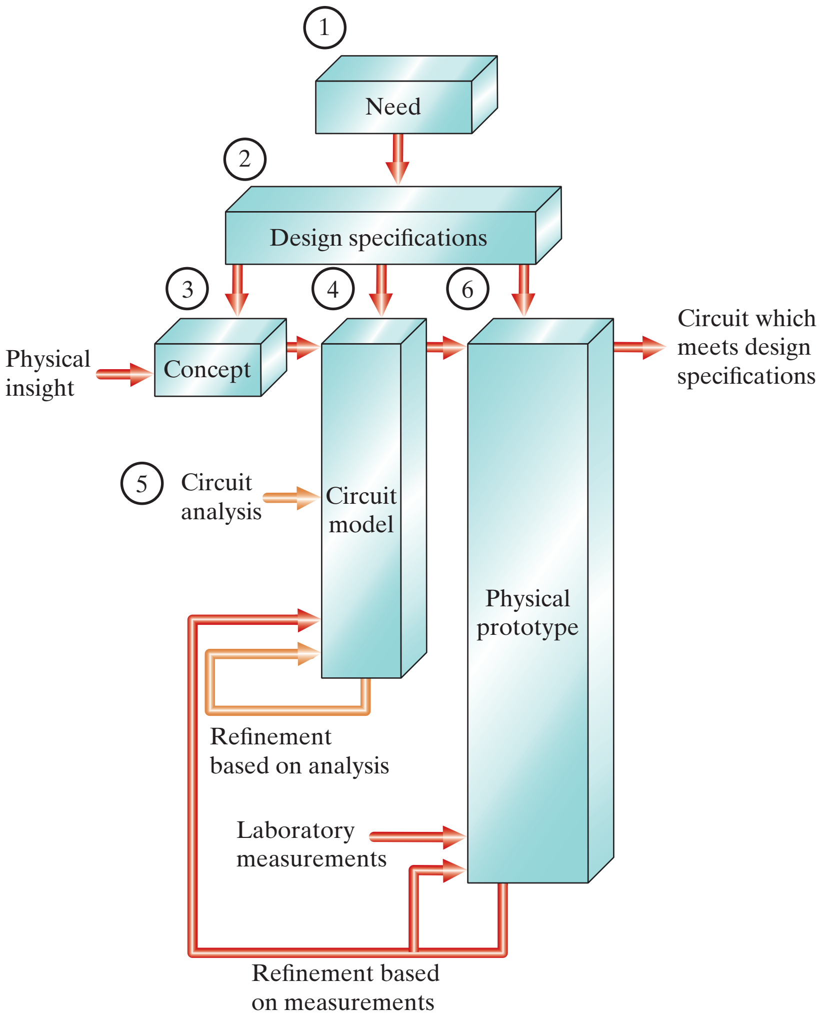


Figure 1.4 ▲ A conceptual model for electrical engineering design.

components used in a circuit model should represent the behavior of the actual electrical components to an acceptable degree of accuracy. The tools of circuit analysis 5, the focus of this book, are then applied to the circuit. **Circuit analysis** uses mathematical techniques to predict the behavior of the circuit model and its ideal circuit components. A comparison between the desired behavior, from the design specifications, and the predicted behavior, from circuit analysis, may lead to refinements in the circuit model and its ideal circuit elements. Once the desired and predicted behaviors are in agreement, a physical prototype 6 can be constructed.

The **physical prototype** is an actual electrical system, constructed from actual electrical components. Measurements determine the quantitative behavior of the physical system. This actual behavior is compared with the desired behavior from the design specifications and the predicted behavior from circuit analysis. The comparisons may result in refinements to the physical prototype, the circuit model, or both. This iterative process, in which models, components, and systems are continually refined, usually produces a design that accurately satisfies the design specifications and thus meets the need.

Circuit analysis clearly plays a very important role in the design process. Because circuit analysis is applied to circuit models, practicing engineers try to use mature circuit models so that the resulting designs will meet the design specifications in the first iteration. In this book, we use models that have been tested for at least 40 years; you can assume that they are mature. The ability to model actual electrical systems with ideal circuit elements makes circuit theory extremely useful to engineers.

Saying that the interconnection of ideal circuit elements can be used to quantitatively predict the behavior of a system implies that we can describe the interconnection with mathematical equations. For the mathematical equations to be useful, we must write them in terms of measurable quantities. In the case of circuits, these quantities are voltage and current, which we discuss in Section 1.4. The study of circuit analysis involves understanding the behavior of each ideal circuit element in terms of its voltage and current and understanding the constraints imposed on the voltage and current as a result of interconnecting the ideal elements.

1.4 Voltage and Current

The concept of electric charge is the basis for describing all electrical phenomena. Let's review some important characteristics of electric charge.

- Electric charge is bipolar, meaning that electrical effects are described in terms of positive and negative charges.
- Electric charge exists in discrete quantities, which are integer multiples of the electronic charge, 1.6022×10^{-19} C.
- Electrical effects are attributed to both the separation of charge and charges in motion.

In circuit theory, the separation of charge creates an electric force (voltage), and the motion of charge creates an electric fluid (current).

The concepts of voltage and current are useful from an engineering point of view because they can be expressed quantitatively. Whenever positive and negative charges are separated, energy is expended. **Voltage** is the energy per unit charge created by the separation. We express this ratio in differential form as

DEFINITION OF VOLTAGE

$$v = \frac{dw}{dq}, \tag{1.1}$$

where

v = the voltage in volts,

w = the energy in joules,

q = the charge in coulombs.

The electrical effects caused by charges in motion depend on the rate of charge flow. The rate of charge flow is known as the **electric current**, which is expressed as

DEFINITION OF CURRENT

$$i = \frac{dq}{dt}, \tag{1.2}$$

where

i = the current in amperes,

q = the charge in coulombs,

t = the time in seconds.

Equations 1.1 and 1.2 define the magnitude of voltage and current, respectively. The bipolar nature of electric charge requires that we assign polarity references to these variables. We will do so in Section 1.5.

Although current is made up of discrete moving electrons, we do not need to consider them individually because of the enormous number of them. Rather, we can think of electrons and their corresponding charge as one smoothly flowing entity. Thus, i is treated as a continuous variable.

One advantage of using *circuit models* is that we can model a component strictly in terms of the voltage and current at its terminals. Thus, two physically different components could have the same relationship between the terminal voltage and terminal current. If they do, for purposes of circuit analysis, they are identical. Once we know how a component behaves at its terminals, we can analyze its behavior in a circuit. However, when developing *component models*, we are interested in a component's internal behavior. We might want to know, for example, whether charge conduction is taking place because of free electrons moving through the crystal lattice structure of a metal or whether it is because of electrons moving within the covalent bonds of a semiconductor material. These concerns are beyond the realm of circuit theory, so in this book we use component models that have already been developed.

1.5 The Ideal Basic Circuit Element

An **ideal basic circuit element** has three attributes.

1. It has only two terminals, which are points of connection to other circuit components.
2. It is described mathematically in terms of current and/or voltage.
3. It cannot be subdivided into other elements.

Using the word *ideal* implies that a basic circuit element does not exist as a realizable physical component. Ideal elements can be connected in order to model actual devices and systems, as we discussed in Section 1.3. Using the word *basic* implies that the circuit element cannot be further reduced or subdivided into other elements. Thus, the basic circuit elements form the building blocks for constructing circuit models, but they themselves cannot be modeled with any other type of element.

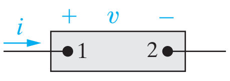


Figure 1.5 ▲ An ideal basic circuit element.

Figure 1.5 represents an ideal basic circuit element. The box is blank because we are making no commitment at this time as to the type of circuit element it is. In Fig. 1.5, the voltage across the terminals of the box is denoted by v, and the current in the circuit element is denoted by i. The plus and minus signs indicate the polarity reference for the voltage, and the arrow placed alongside the current indicates its reference direction. Table 1.4 interprets the voltage polarity and current direction, given positive or negative numerical values of v and i. Note that algebraically the notion of positive charge flowing in one direction is equivalent to the notion of negative charge flowing in the opposite direction.

Assigning the reference polarity for voltage and the reference direction for current is entirely arbitrary. However, once you have assigned the references, you must write all subsequent equations to agree with the chosen references. The most widely used sign convention applied to these references is called the **passive sign convention**, which we use throughout this book.

PASSIVE SIGN CONVENTION

Whenever the reference direction for the current in an element is in the direction of the reference voltage drop across the element (as in Fig. 1.5), use a positive sign in any expression that relates the voltage to the current. Otherwise, use a negative sign.

We apply this sign convention in all the analyses that follow. Our purpose for introducing it even before we have introduced the different types of basic circuit elements is to emphasize that selecting polarity references

TABLE 1.4 Interpretation of Reference Directions in Fig. 1.5

	Positive Value	Negative Value
v	voltage drop from terminal 1 to terminal 2	voltage rise from terminal 1 to terminal 2
	or	*or*
	voltage rise from terminal 2 to terminal 1	voltage drop from terminal 2 to terminal 1
i	positive charge flowing from terminal 1 to terminal 2	positive charge flowing from terminal 2 to terminal 1
	or	*or*
	negative charge flowing from terminal 2 to terminal 1	negative charge flowing from terminal 1 to terminal 2

is *not* a function either of the basic elements or the type of interconnections made with the basic elements. We apply and interpret the passive sign convention for power calculations in Section 1.6.

Example 1.2 illustrates one use of the equation defining current.

EXAMPLE 1.2 Relating Current and Charge

No charge exists at the upper terminal of the element in Fig. 1.5 for $t < 0$. At $t = 0$, a 5 A current begins to flow into the upper terminal.

a) Derive the expression for the charge accumulating at the upper terminal of the element for $t > 0$.

b) If the current is stopped after 10 seconds, how much charge has accumulated at the upper terminal?

Solution

a) From the definition of current given in Eq. 1.2, the expression for charge accumulation due to current flow is

$$q(t) = \int_0^t i(x)dx.$$

Therefore,

$$q(t) = \int_0^t 5dx = 5x\Big|_0^t = 5t - 5(0) = 5t \text{ C} \quad \text{for } t > 0.$$

b) The total charge that accumulates at the upper terminal in 10 seconds due to a 5 A current is $q(10) = 5(10) = 50$ C.

ASSESSMENT PROBLEMS

Objective 2—Know and be able to use the definitions of *voltage* and *current*

1.3 The current at the terminals of the element in Fig. 1.5 is

$$i = 0, \qquad t < 0;$$

$$i = 20e^{-5000t}\text{ A}, \qquad t \geq 0.$$

Calculate the total charge (in microcoulombs) entering the element at its upper terminal.

Answer: 4000 μC.

1.4 The expression for the charge entering the upper terminal of Fig. 1.5 is

$$q = \frac{1}{\alpha^2} - \left(\frac{t}{\alpha} + \frac{1}{\alpha^2}\right)e^{-\alpha t}\text{ C}.$$

Find the maximum value of the current entering the terminal if $\alpha = 0.03679\ \text{s}^{-1}$.

Answer: 10 A.

SELF-CHECK: Also try Chapter Problem 1.7.

1.6 Power and Energy

Power and energy calculations are important in circuit analysis. Although voltage and current are useful variables in the analysis and design of electrically based systems, the useful output of the system often is nonelectrical (e.g., sound emitted from a speaker or light from a light bulb), and this output is conveniently expressed in terms of power or energy. Also, all practical devices have limitations on the amount of power that they can handle. In the design process, therefore, voltage and current calculations by themselves are not sufficient to determine whether or not a design meets its specifications.

We now relate power and energy to voltage and current and at the same time use the power calculation to illustrate the passive sign convention. Recall from basic physics that power is the time rate of expending or absorbing energy. (A water pump rated 75 kW can deliver more liters per second than one rated 7.5 kW.) Mathematically, energy per unit time is expressed in the form of a derivative, or

DEFINITION OF POWER

$$p = \frac{dw}{dt}, \tag{1.3}$$

where

p = the power in watts,
w = the energy in joules,
t = the time in seconds.

Thus, 1 W is equivalent to 1 J/s.

The power associated with the flow of charge follows directly from the definition of voltage and current in Eqs. 1.1 and 1.2, or

$$p = \frac{dw}{dt} = \left(\frac{dw}{dq}\right)\left(\frac{dq}{dt}\right),$$

so

POWER EQUATION

$$p = vi, \tag{1.4}$$

where

p = the power in watts,
v = the voltage in volts,
i = the current in amperes.

Equation 1.4 shows that the **power** associated with a basic circuit element is the product of the current in the element and the voltage across the element. Therefore, power is a quantity associated with a circuit element, and we have to determine from our calculation whether power is being delivered to the circuit element or extracted from it. This information comes from correctly applying and interpreting the passive sign convention (Section 1.5).

If we use the passive sign convention, Eq. 1.4 is correct if the reference direction for the current is in the direction of the reference voltage drop across the terminals. Otherwise, Eq. 1.4 must be written with a minus sign. In other words, if the current reference is in the direction of a reference voltage rise across the terminals, the expression for the power is

$$p = -vi.$$

The algebraic sign of power is based on charge movement through voltage drops and rises. As positive charges move through a drop in voltage, they lose energy, and as they move through a rise in voltage, they gain energy. Figure 1.6 summarizes the relationship between the polarity references for voltage and current and the expression for power.

We can now state the rule for interpreting the algebraic sign of power:

INTERPRETING ALGEBRAIC SIGN OF POWER

- If the power is positive (that is, if $p > 0$), power is being delivered to the circuit element represented by the box.
- If the power is negative (that is, if $p < 0$), power is being extracted from the circuit element.

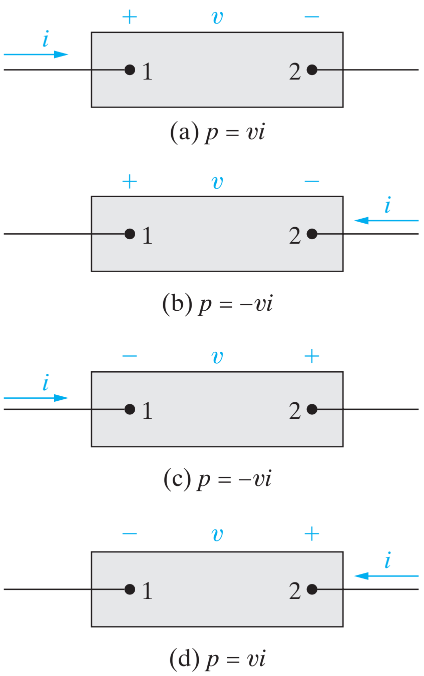


Figure 1.6 ▲ Polarity references and the expression for power.

Example 1.3 shows that the passive sign convention generates the correct sign for power regardless of the voltage polarity and current direction you choose.

EXAMPLE 1.3 Using the Passive Sign Convention

a) Suppose you have selected the polarity references shown in Fig. 1.6(b). Your calculations for the current and voltage yield the following numerical results:

$$i = 4\text{ A} \quad \text{and} \quad v = -10\text{ V}.$$

Calculate the power associated with the circuit element and determine whether it is absorbing or supplying power.

b) Your classmate is solving the same problem but has chosen the reference polarities shown in Fig. 1.6(c). Her calculations for the current and voltage show that

$$i = -4\text{ A} \quad \text{and} \quad v = 10\text{ V}.$$

What power does she calculate?

Solution

a) The power associated with the circuit element in Fig. 1.6(b) is

$$p = -(-10)(4) = 40\text{ W}.$$

Thus, the circuit element is absorbing 40 W.

b) Your classmate calculates that the power associated with the circuit element in Fig. 1.6(c) is

$$p = -(10)(-4) = 40\text{ W}.$$

Using the reference system in Fig. 1.6(c) gives the same conclusion as using the reference system in Fig. 1.6(b)—namely, that the circuit element is absorbing 40 W. In fact, any of the reference systems in Fig. 1.6 yields this same result.

Example 1.4 illustrates the relationship between voltage, current, power, and energy for an ideal basic circuit element and the use of the passive sign convention.

EXAMPLE 1.4 Relating Voltage, Current, Power, and Energy

Assume that the voltage at the terminals of the element in Fig. 1.5, whose current was defined in Assessment Problem 1.3, is

$$v = 0 \qquad t < 0;$$

$$v = 10e^{-5000t}\text{ kV}, \qquad t \geq 0.$$

a) Calculate the power supplied to the element at 1 ms.

b) Calculate the total energy (in joules) delivered to the circuit element.

Solution

a) Since the current is entering the + terminal of the voltage drop defined for the element in Fig. 1.5, we use a "+" sign in the power equation.

$$p = vi = (10{,}000e^{-5000t})(20e^{-5000t}) = 200{,}000e^{-10{,}000t}\text{ W}.$$

$$p(0.001) = 200{,}000e^{-10{,}000(0.001)} = 200{,}000e^{-10}$$

$$= 200{,}000(45.4 \times 10^{-6}) = 9.08\text{ W}.$$

b) From the definition of power given in Eq. 1.3, the expression for energy is

$$w(t) = \int_0^t p(x)dx.$$

To find the total energy delivered, integrate the expresssion for power from zero to infinity. Therefore,

$$w_{\text{total}} = \int_0^\infty 200{,}000e^{-10{,}000x}\,dx = \frac{200{,}000e^{-10{,}000x}}{-10{,}000}\bigg|_0^\infty$$

$$= -20e^{-\infty} - (-20e^{-0}) = 0 + 20 = 20\text{ J}.$$

Thus, the total energy supplied to the circuit element is 20 J.

ASSESSMENT PROBLEMS

Objective 3—Know and use the definitions of *power* and *energy*; Objective 4—Be able to use the passive sign convention

1.5 Assume that a 20 V voltage drop occurs across an element from terminal 2 to terminal 1 and that a current of 4 A enters terminal 2.

a) Specify the values of v and i for the polarity references shown in Fig. 1.6(a)–(d).
b) Calculate the power associated with the circuit element.
c) Is the circuit element absorbing or delivering power?

Answer: (a) Circuit 1.6(a): $v = -20$ V, $i = -4$ A;
circuit 1.6(b): $v = -20$ V, $i = 4$ A;
circuit 1.6(c): $v = 20$ V, $i = -4$ A;
circuit 1.6(d): $v = 20$ V, $i = 4$ A;
(b) 80 W;
(c) absorbing.

1.6 The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for $t < 0$. For $t \geq 0$, they are

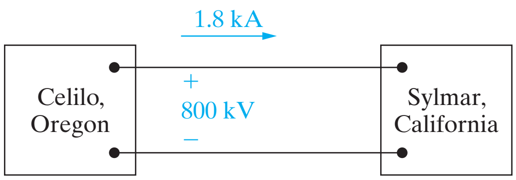
$$v = 80{,}000te^{-500t}\ \text{V}, \qquad t \geq 0;$$

$$i = 15te^{-500t}\ \text{A}, \qquad t \geq 0.$$

a) Find the time when the power delivered to the circuit element is maximum.
b) Find the maximum value of power.
c) Find the total energy delivered to the circuit element.

Answer: (a) 2 ms; (b) 649.6 mW; (c) 2.4 mJ.

1.7 A high-voltage direct-current (dc) transmission line between Celilo, Oregon, and Sylmar, California, is operating at 800 kV and carrying 1800 A, as shown. Calculate the power (in megawatts) at the Oregon end of the line and state the direction of power flow.



Answer: 1440 MW, Celilo to Sylmar

SELF-CHECK: Also try Chapter Problems 1.15, 1.18, and 1.25.

Practical Perspective

Balancing Power

A circuit model for distributing power to a typical home is shown in Fig. 1.7, with voltage polarities and current directions defined for all of the circuit components. Circuit analysis gives values for all of these voltages and currents, as summarized in Table 1.5. To determine whether or not the values given are correct, calculate the power associated with each component. Use the passive sign convention in the power calculations, as shown in the following.

$$p_a = v_a i_a = (120)(-10) = -1200\ \text{W} \qquad p_b = -v_b i_b = -(120)(9) = -1080\ \text{W}$$

$$p_c = v_c i_c = (10)(10) = 100\ \text{W} \qquad p_d = -v_d i_d = -(10)(1) = -10\ \text{W}$$

$$p_e = v_e i_e = (-10)(-9) = 90\ \text{W} \qquad p_f = -v_f i_f = -(-100)(5) = 500\ \text{W}$$

$$p_g = v_g i_g = (120)(4) = 480\ \text{W} \qquad p_h = v_h i_h = (-220)(-5) = 1100\ \text{W}$$

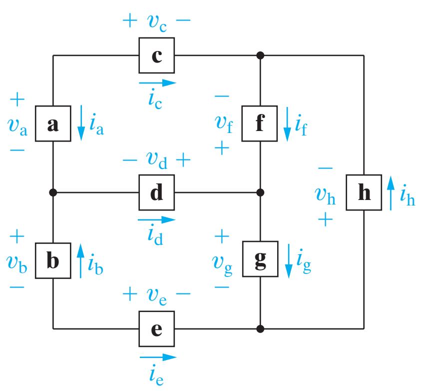


Figure 1.7 ▲ Circuit model for power distribution in a home, with voltages and currents defined.

The power calculations show that components a, b, and d are supplying power, since the power values are negative, while components c, e, f, g, and h are absorbing power. Now check to see if the power balances by finding the total power supplied and the total power absorbed.

$$p_{\text{supplied}} = p_a + p_b + p_d = -1200 - 1080 - 10 = -2290 \text{ W}$$

$$p_{\text{absorbed}} = p_c + p_e + p_f + p_g + p_h$$
$$= 100 + 90 + 500 + 480 + 1100 = 2270 \text{ W}$$

$$p_{\text{supplied}} + p_{\text{absorbed}} = -2290 + 2270 = -20 \text{ W}$$

Something is wrong—if the values for voltage and current in this circuit are correct, the total power should be zero! There is an error in the data, and we can find it from the calculated powers if the error exists in the sign of a single component. Note that if we divide the total power by 2, we get -10 W, which is the power calculated for component d. If the power for component d is $+10$ W, the total power would be 0. Circuit analysis techniques from upcoming chapters can be used to show that the current through component d should be -1 A, not $+1$ A as given in Table 1.5.

SELF-CHECK: Assess your understanding of the Practical Perspective by trying Chapter Problems 1.34 and 1.35.

TABLE 1.5 Voltage and Current Values for the Circuit in Fig. 1.7

Component	v(V)	i(A)
a	120	−10
b	120	9
c	10	10
d	10	1
e	−10	−9
f	−100	5
g	120	4
h	−220	−5

Summary

- The International System of Units (SI) enables engineers to communicate in a meaningful way about quantitative results. Table 1.1 summarizes the SI units; Table 1.2 presents some useful derived SI units. (See page 38.)
- A circuit model is a mathematical representation of an electrical system. Circuit analysis, used to predict the behavior of a circuit model, is based on the variables of voltage and current. (See page 40.)
  - **Voltage** is the energy per unit charge created by charge separation and has the SI unit of volt. (See page 41.)

    $$v = dw/dq$$

  - **Current** is the rate of charge flow and has the SI unit of ampere. (See page 41.)

    $$i = dq/dt$$

- The **ideal basic circuit element** is a two-terminal component that cannot be subdivided; it can be described mathematically in terms of its terminal voltage and current. (See page 42.)
- The **passive sign convention** uses a positive sign in the expression that relates the voltage and current at the terminals of an element when the reference direction for the current through the element is in the direction of the reference voltage drop across the element. (See page 42.)
- **Power** is energy per unit of time and is equal to the product of the terminal voltage and current; it has the SI unit of watt. (See page 44.)

  $$p = dw/dt = vi$$

  The algebraic sign of power is interpreted as follows:
  - If $p > 0$, power is being delivered to the circuit or circuit component.
  - If $p < 0$, power is being extracted from the circuit or circuit component. (See page 44.)

Problems

Section 1.2

1.1 There are approximately 520 million passenger vehicles registered in the United States. Assume that the battery in an average vehicle stores 480 watt-hours (Wh) of energy. Estimate (in gigawatt-hours) the total energy stored in US passenger vehicles.

1.2 Each frame of a movie file is played at a resolution of 960×640 picture elements (pixels). Each pixel requires 4 bytes of memory. Videos are displayed at the rate of 40 frames per second. If the size of this file is 64 gigabytes, find its length.

1.3 The 8 gigabyte (1 GB $= 2^{30}$ bytes) flash memory chip for an MP3 player is 10 mm by 20 mm by 2 mm. This memory chip holds 15,000 photos.

a) How many photos fit into a cube whose sides are 2 mm?

b) How many bytes of memory are stored in a cube whose sides are 100 μm?

1.4 The line described in Assessment Problem 1.7 is 900 mi in length. The line contains two conductors, each weighing 2526 lb per 1000 ft. How many kilograms of conductor are in the line?

1.5 A 40-inch monitor contains 4800×2160 picture elements, or pixels. Each pixel is represented in 32 bits of memory. A byte of memory is 8 bits.

a) How many megabytes (MB) of memory are required to store the information displayed on the monitor?

b) To display a video on the monitor, the image must be refreshed 30 times per second. How many terabytes (TB) of memory are required to store a 2 hr video?

c) For the video described in part (a), how fast must the image data in memory be moved to the monitor? Express your answer in gigabits per second (Gb/s).

1.6 Some species of bamboo can grow (250 mm/day). Assume individual cells in the plant are 10 μm long.

a) How long, on average, does it take a bamboo stalk to grow 1 cell length?

b) How many cell lengths are added in one week, on average?

Section 1.4

1.7 There is no charge at the upper terminal of the element in Fig. 1.5 for $t < 0$. At $t = 0$ a current of $125e^{-2500t}$ mA enters the upper terminal.

a) Derive the expression for the charge that accumulates at the upper terminal for $t > 0$.

b) Find the total charge that accumulates at the upper terminal.

c) If the current is stopped at $t = 0.5$ ms, how much charge has accumulated at the upper terminal?

1.8 The current entering the upper terminal of Fig. 1.5 is

$$i = 24 \cos 4000t \text{ A}$$

Assume the charge at the upper terminal is zero at the instant the current is passing through its maximum value. Find the expression for $q(t)$.

1.9 The current at the terminals of the element in Fig. 1.5 is

$$i = 0, \qquad t < 0;$$

$$i = 40te^{-500t} \text{ A}, \qquad t \geq 0.$$

a) Find the expression for the charge accumulating at the upper terminal.

b) Find the charge that has accumulated at $t = 1$ ms.

1.10 In electronic circuits it is not unusual to encounter currents in the microampere range. Assume a 35 μA current, due to the flow of electrons. What is the average number of electrons per second that flow past a fixed reference cross section that is perpendicular to the direction of flow?

1.11 How much energy is imparted to an electron as it flows through a 1.5 V battery from the positive to the negative terminals? Express your answer in joules.

Sections 1.5–1.6

1.12 The references for the voltage and current at the terminals of a circuit element are as shown in Fig. 1.6(d). The numerical values for v and i are -20 V and 5 A.

a) Calculate the power at the terminals and state whether the power is being absorbed or delivered by the element in the box.

b) Given that the current is due to electron flow, state whether the electrons are entering or leaving terminal 2.

c) Do the electrons gain or lose energy as they pass through the element in the box?

1.13 Repeat Problem 1.12 with a voltage of -60 V.

1.14 Two electric circuits, represented by boxes A and B, are connected as shown in Fig. P1.14. The reference direction for the current i in the interconnection and the reference polarity for the voltage v across the interconnection are as shown in the figure. For each of the following sets of numerical values, calculate the power in the interconnection and state whether the power is flowing from A to B or vice versa.

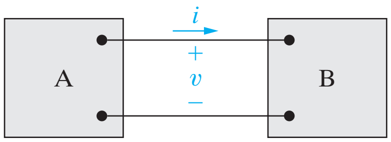
a) $i = 8$ A, $v = 40$ V

b) $i = -2$A, $v = -10$ V

c) $i = 2$A, $v = -50$ V

d) $i = -10$ A, $v = 20$ V

Figure P1.14

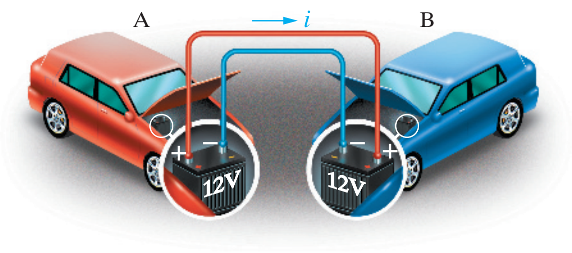


1.15 When a car has a dead battery, it can often be started by connecting the battery from another car across its terminals. The positive terminals are connected together as are the negative terminals. The connection is illustrated in Fig. P1.15. Assume the current i in Fig. P1.15 is measured and found to be 40 A.

a) Which car has the dead battery?

b) If this connection is maintained for 1.5 min, how much energy is transferred to the dead battery?

Figure P1.15



1.16 The manufacturer of a 1.5 V D flashlight battery says that the battery will deliver 9 mA for 40 continuous hours. During that time the voltage will drop from 1.5 V to 1.0 V. Assume the drop in voltage is linear with time. How much energy does the battery deliver in this 40 h interval?

1.17 One 12 V battery supplies 100 mA to a boom box. How much energy does the battery supply in 4 h?

1.18 The voltage and current at the terminals of the circuit elements in Fig 1.5 are zero for $t < 0$. For $t \geq 0$, they are

$$v = 3e^{-50t}\ \text{V},$$

$$i = 5e^{-50t}\ \text{mA}.$$

a) Calculate the power supplied to the element at 5 ms.

b) Calculate the total energy delivered to the circuit element.

1.19 The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for $t < 0$. For $t \geq 0$ they are
PSPICE
MULTISIM

$$v = 75 - 75e^{-1000t}\ \text{V},$$

$$i = 50e^{-1000t}\text{mA}.$$

a) Find the maximum value of the power delivered to the circuit.

b) Find the total energy delivered to the element.

1.20 The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for $t < 0$. For $t \geq 0$ they are

$$v = 50e^{-1600t} - 50e^{-400t}\ \text{V},$$

$$i = 5e^{-1600t} - 5e^{-400t}\ \text{mA}.$$

a) Find the power at $t = 625\ \mu$s.

b) How much energy is delivered to the circuit element between 0 and 625 μs?

c) Find the total energy delivered to the element.

1.21 The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for $t < 0$. For $t \geq 0$, they are
PSPICE
MULTISIM

$$v = (1600t + 1)e^{-800t}\ \text{V}, \qquad t \geq 0;$$

$$i = 50e^{-800t}\ \text{mA}, \qquad t \geq 0.$$

a) Find the time when the power delivered to the circuit element is maximum.

b) Find the maximum value of p in milliwatts.

c) Find the total energy delivered to the circuit element in microjoules.

1.22 The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for $t < 0$. For $t \geq 0$, they are

PSPICE
MULTISIM

$$v = (4000t + 3.2)e^{-1200t}\ \text{V},$$

$$i = (160t + 0.26)e^{-1200t}\ \text{A}.$$

a) At what instant of time is the maximum power delivered to the element?

b) Find the maximum power in watts.

c) Find the total energy delivered to the element in microjoules.

1.23 The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for $t < 0$ and $t > 50$ s. In the interval between 0 and 50 s, the expressions are

PSPICE
MULTISIM

$$v = t(1 - 0.030t)\ \text{V}, \quad 0 < t < 50\ \text{s};$$

$$i = 4 - 0.3t\ \text{A}, \quad 0 < t < 50\ \text{s}.$$

a) At what instant of time is the maximum power delivered to the element?

b) What is the power at the time found in part (a)?

c) At what instant of time is the power being extracted from the circuit element the maximum?

d) What is the power at the time found in part (c)?

e) Calculate the net energy delivered to the circuit at 0, 10, 20, 30, 40 and 50 s.

1.24 The voltage and current at the terminals of the circuit element in Fig. 1.5 are zero for $t < 0$. For $t \geq 0$, they are

PSPICE
MULTISIM

$$v = 500e^{-120t} \sin 250t\ \text{V},$$

$$i = 6e^{-150t} \sin 250t\ \text{A}.$$

a) Find the power absorbed by the element at $t = 20$ ms.

b) Find the total energy absorbed by the element.

1.25 The voltage and current at the terminals of the circuit element in Fig. 1.5 are

PSPICE
MULTISIM

$$v = 260 \cos 850\pi t\ \text{V}, \quad i = 9 \sin 850\pi t\ \text{A}.$$

a) Find the maximum value of the power being delivered to the element.

b) Find the maximum value of the power being extracted from the element.

c) Find the average value of p in the interval $0 \leq t \leq 3$ ms.

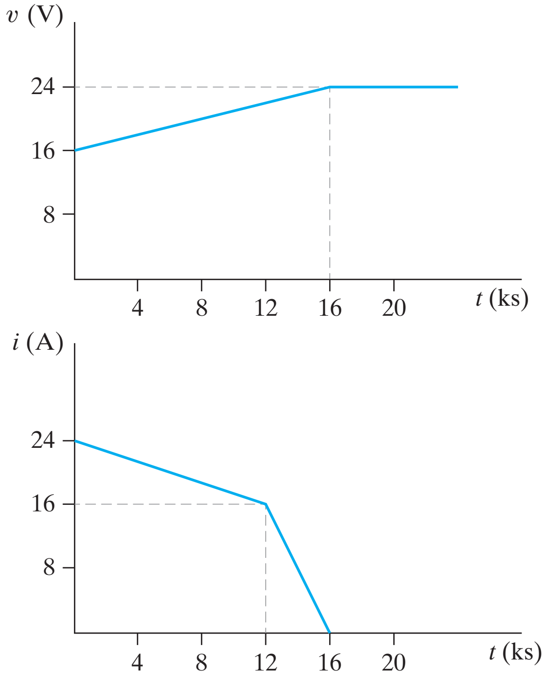
d) Find the average value of p in the interval $0 \leq t \leq 16.525$ ms.

1.26 The voltage and current at the terminals of an automobile battery during a charge cycle are shown in Fig. P1.26.

a) Calculate the total charge transferred to the battery.

b) Calculate the total energy transferred to the battery.

Figure P1.26

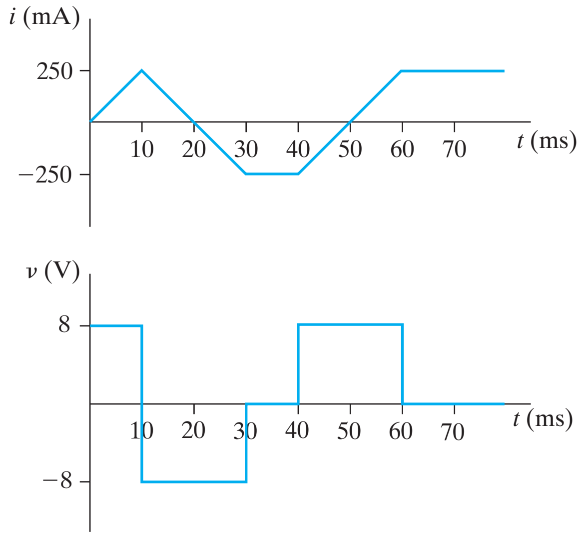


1.27 The voltage and current at the terminals of the circuit element in Fig. 1.5 are shown in Fig. P1.27.

a) Sketch the power versus t plot for $0 \leq t \leq 80$ ms.

b) Calculate the energy delivered to the circuit element at $t = 10$, 30, and 80 ms.

Figure P1.27



1.28 An industrial battery is charged over a period of several hours at a constant voltage of 120 V. Initially, the current is 20 mA and increases linearly to 30 mA in 10 ks. From 10 ks to 20 ks, the current is constant at 30 mA. From 20 ks to 30 ks the current decreases linearly to 10 mA. At 30 ks the power is disconnected from the battery.

a) Sketch the current from $t = 0$ to $t = 30$ ks.

b) Sketch the power delivered to the battery from $t = 0$ to $t = 30$ ks.

c) Using the sketch of the power, find the total energy delivered to the battery.

1.29 The numerical values for the currents and voltages in the circuit in Fig. P1.29 are given in Table P1.29. Find the total power developed in the circuit.

Figure P1.29

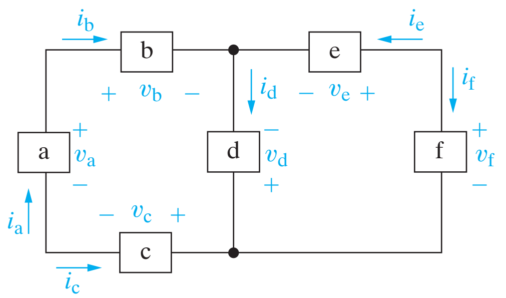


TABLE P1.29

Element	Voltage (V)	Current (mA)
a	40	−4
b	−24	−4
c	−16	4
d	−80	−1.5
e	40	2.5
f	120	−2.5

1.30 The numerical values of the voltages and currents in the interconnection seen in Fig. P1.30 are given in Table P1.30. Does the interconnection satisfy the power check?

Figure P1.30

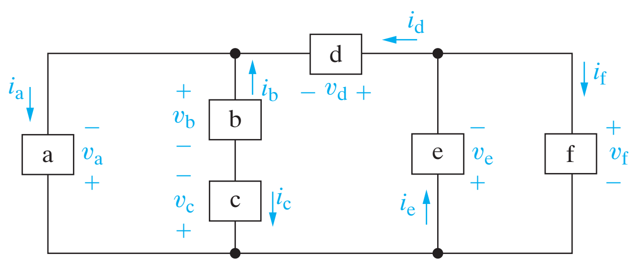


TABLE P1.30

Element	Voltage (kV)	Current (mA)
a	−3	−250
b	4	−400
c	1	400
d	1	150
e	−4	200
f	4	50

1.31 Assume you are an engineer in charge of a project and one of your subordinate engineers reports that the interconnection in Fig. P1.31 does not pass the power check. The data for the interconnection are given in Table P1.31.

a) Is the subordinate correct? Explain your answer.

b) If the subordinate is correct, can you find the error in the data?

Figure P1.31

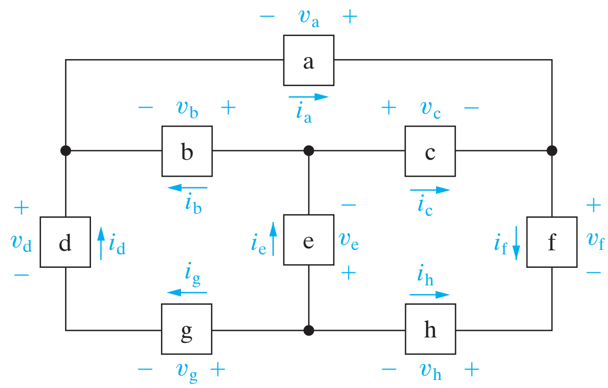


TABLE P1.31

Element	Voltage (V)	Current (A)
a	46.16	6.0
b	14.16	4.72
c	−32.0	−6.4
d	22.0	1.28
e	−33.6	−1.68
f	66.0	0.4
g	2.56	1.28
g	−0.4	0.4

1.32 The voltage and power values for each of the elements shown in Fig. P1.32 are given in Table P1.32.

a) Show that the interconnection of the elements satisfies the power check.

b) Find the value of the current through each of the elements using the values of power and voltage and the current directions shown in the figure.

1.33 The current and power for each of the interconnected elements in Fig. P1.33 is measured. The values are listed in Table P1.33.

a) Show that the interconnection satisfies the power check.

b) Identify the elements that absorb power.

c) Find the voltage for each of the elements in the interconnection, using the values of power and current and the voltage polarities shown in the figure.

Figure P1.32

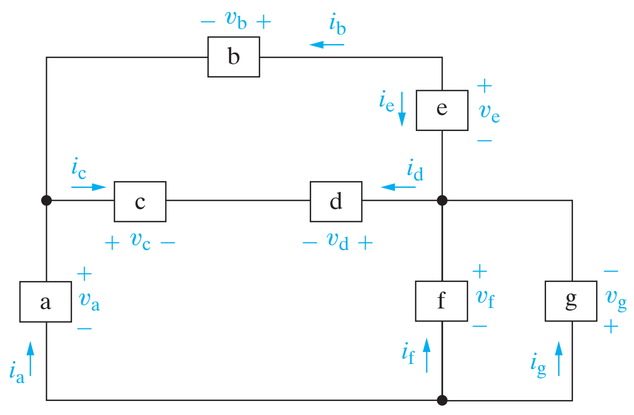


Figure P1.33

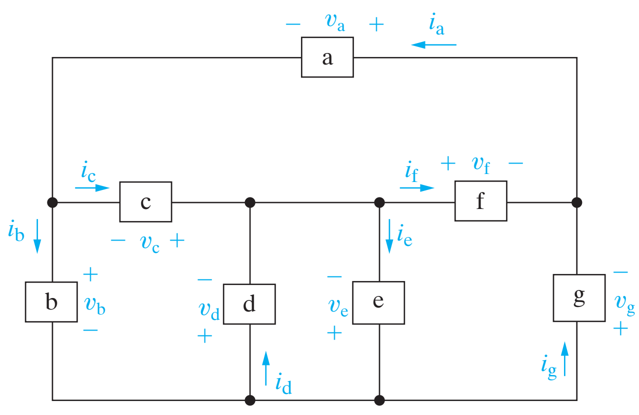


TABLE P1.32

Element	Power (kW)	Voltage (V)
a	0.6 supplied	400
b	0.05 supplied	−100
c	0.4 absorbed	200
d	0.6 supplied	300
e	0.1 absorbed	−200
f	2.0 absorbed	500
g	1.25 supplied	−500

TABLE P1.33

Element	Power (mW)	Current (mA)
a	175	25
b	375	75
c	150	−50
d	−320	40
e	160	20
f	120	−30
g	−660	55

1.34 PRACTICAL PERSPECTIVE Show that the power balances for the circuit shown in Fig. 1.7, using the voltage and current values given in Table 1.5, with the value of the current for component d changed to -1 A.

1.35 PRACTICAL PERSPECTIVE Suppose there is no power lost in the wires used to distribute power in a typical home.

a) Create a new model for the power distribution circuit by modifying the circuit shown in Fig 1.7. Use the same names, voltage polarities, and current directions for the components that remain in this modified model.

b) The following voltages and currents are calculated for the components:

$v_a = 120$ V	$i_a = -10$ A
$v_b = 120$ V	$i_b = 10$ A
$v_f = -120$ V	$i_f = 3$ A
$v_g = 120$ V	
$v_h = -240$ V	$i_h = -7$ A

If the power in this modified model balances, what is the value of the current in component g?

CHAPTER

2

Circuit Elements

CHAPTER CONTENTS

CHAPTER OBJECTIVES

1. Understand the symbols for and the behavior of the following ideal basic circuit elements: independent voltage and current sources, dependent voltage and current sources, and resistors.
2. Be able to state Ohm's law, Kirchhoff's current law, and Kirchhoff's voltage law, and be able to use these laws to analyze simple circuits.
3. Know how to calculate the power for each element in a simple circuit and be able to determine whether or not the power balances for the whole circuit.

There are five ideal basic circuit elements:

- voltage sources
- current sources
- resistors
- inductors
- capacitors

In this chapter, we discuss the characteristics of the first three circuit elements—voltage sources, current sources, and resistors. Although this may seem like a small number of elements, many practical systems can be modeled with just sources and resistors. They are also a useful starting point because of their relative simplicity; the mathematical relationships between voltage and current in sources and resistors are algebraic. Thus, you will be able to begin learning the basic techniques of circuit analysis with only algebraic manipulations.

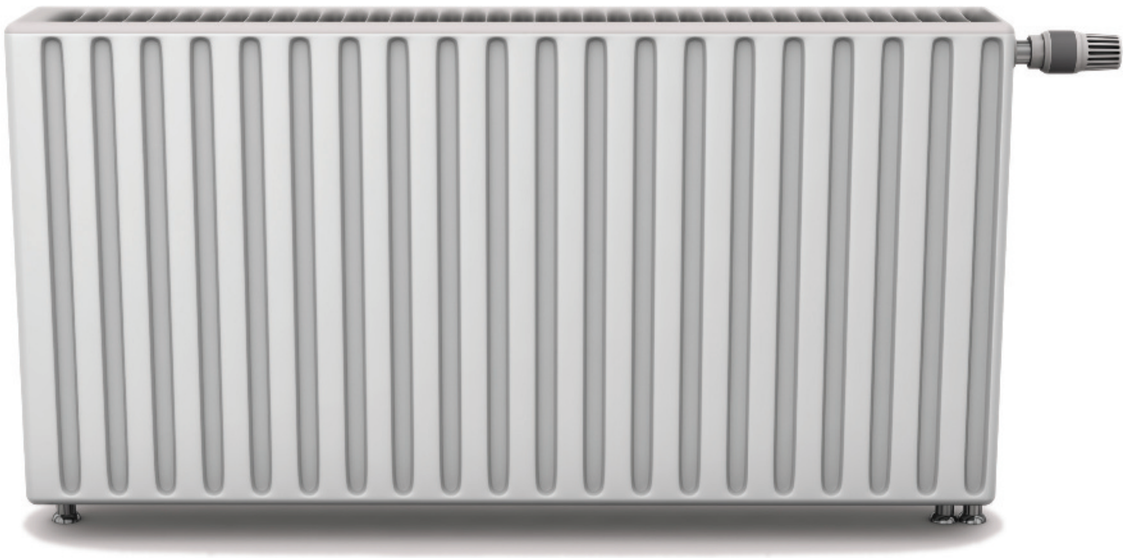
We will postpone introducing inductors and capacitors until Chapter 6, because their use requires that you solve integral and differential equations. However, the basic analytical techniques for solving circuits with inductors and capacitors are the same as those introduced in this chapter. So, by the time you need to begin manipulating more difficult equations, you should be very familiar with the methods of writing them.

Practical Perspective

Heating with Electric Radiators

You want to heat your small garage using a couple of electric radiators. The power and voltage requirements for each radiator are 1200 W, 240 V. But you are not sure how to wire the radiators to the power supplied to the garage. Should you use the wiring diagram on the left or the one on the right? Does it make any difference?

Once you have studied the material in this chapter, you will be able to answer these questions and determine how to heat the garage. The Practical Perspective at the end of this chapter guides you through the analysis of two circuits based on the two wiring diagrams shown below.



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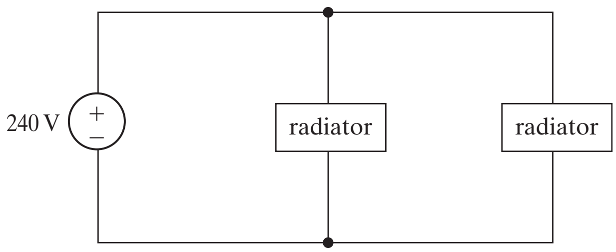
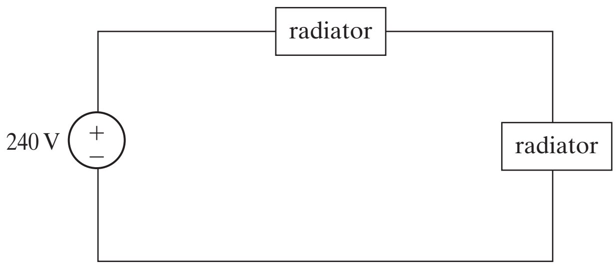




2.1 Voltage and Current Sources

An **electrical source** is a device capable of converting nonelectric energy to electric energy and vice versa. For example, a discharging battery converts chemical energy to electric energy, whereas a charging battery converts electric energy to chemical energy. A *dynamo* is a machine that converts mechanical energy to electric energy and vice versa. For operations in the mechanical-to-electric mode, it is called a *generator*. For transformations from electric to mechanical energy, it is called a *motor*. Electric sources either deliver or absorb electric power while maintaining either voltage or current. This behavior led to the creation of the ideal voltage source and the ideal current source as basic circuit elements.

- An **ideal voltage source** is a circuit element that maintains a prescribed voltage across its terminals regardless of the current flowing in those terminals.
- An **ideal current source** is a circuit element that maintains a prescribed current through its terminals regardless of the voltage across those terminals.

These circuit elements do not exist as practical devices—they are idealized models of actual voltage and current sources.

Using an ideal model for current and voltage sources constrains the mathematical descriptions of these components. For example, because an ideal voltage source provides a steady voltage even if the current in the element changes, it is impossible to specify the current in an ideal voltage source as a function of its voltage. Likewise, if the only information you have about an ideal current source is the value of current supplied, it is impossible to determine the voltage across that current source. We have sacrificed our ability to relate voltage and current in a practical source for the simplicity of using ideal sources in circuit analysis.

Ideal voltage and current sources can be further described as either *independent sources* or *dependent sources*.

- An **independent source** establishes a voltage or current in a circuit without relying on voltages or currents elsewhere in the circuit. The value of the voltage or current supplied is specified by the value of the independent source alone.
- A **dependent source**, in contrast, establishes a voltage or current whose value depends on the value of a voltage or current elsewhere in the circuit. You cannot specify the value of a dependent source unless you know the value of the voltage or current on which it depends.

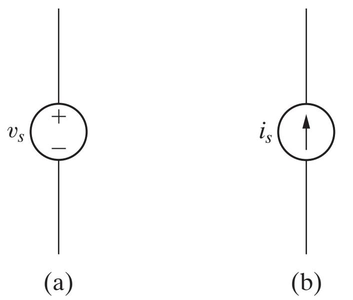


Figure 2.1 ▲ The circuit symbols for (a) an ideal independent voltage source and (b) an ideal independent current source.

The circuit symbols for the ideal independent sources are shown in Fig. 2.1. Note that a circle is used to represent an independent source. To completely specify an ideal independent voltage source in a circuit, you must include the value of the supplied voltage and the reference polarity, as shown in Fig. 2.1(a). Similarly, to completely specify an ideal independent current source, you must include the value of the supplied current and its reference direction, as shown in Fig. 2.1(b).

The circuit symbol for an ideal dependent source is a diamond, as shown in Fig. 2.2. There are four possible variations because both dependent current sources and dependent voltage sources can be controlled by either a voltage or a current elsewhere in the circuit. Dependent sources are sometimes called controlled sources.

To completely specify an ideal dependent voltage-controlled voltage source, you must identify the controlling voltage, the equation that

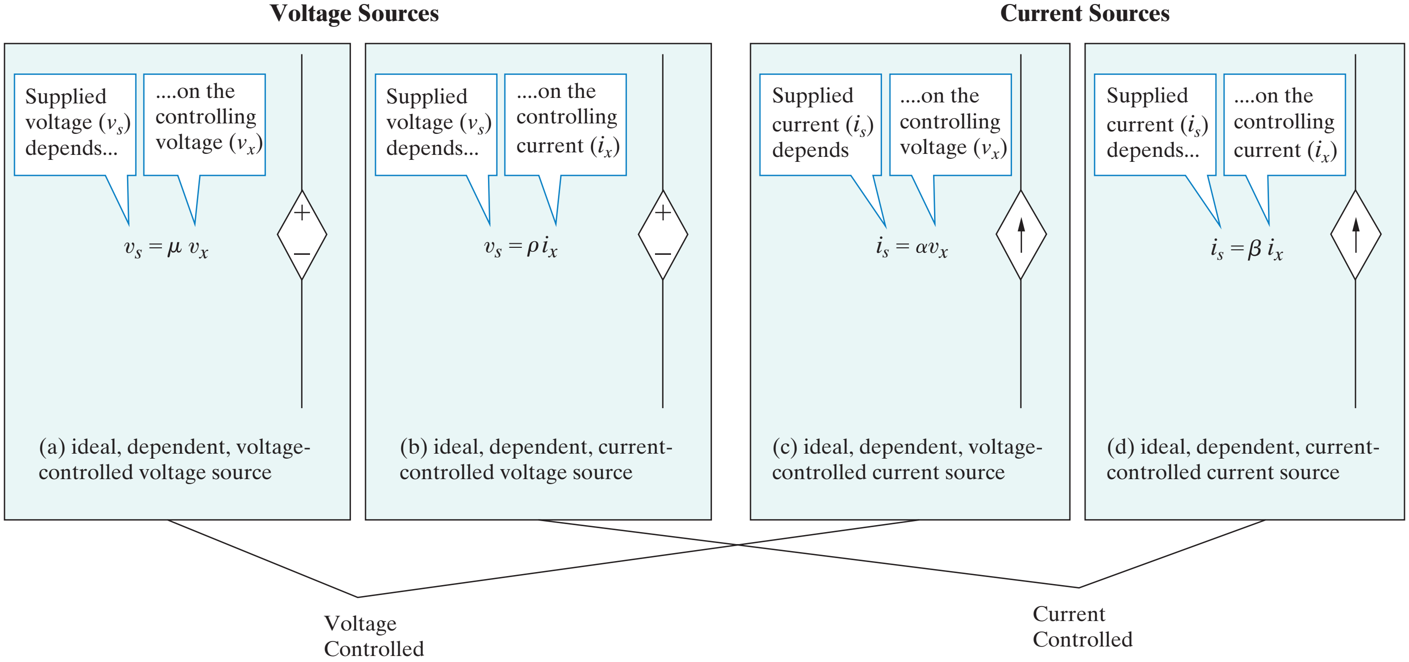


Figure 2.2 ▲ (a) (b) Circuit symbols for ideal dependent voltage sources and (c) (d) ideal dependent current sources.

permits you to compute the supplied voltage from the controlling voltage, and the reference polarity for the supplied voltage. For example, in Fig. 2.2(a), the controlling voltage is v_x, the equation that determines the supplied voltage v_s is

$$v_s = \mu v_x,$$

and the reference polarity for v_s is as indicated. Note that μ is a multiplying constant that is dimensionless.

Similar requirements exist for completely specifying the other ideal dependent sources. In Fig. 2.2(b), the controlling current is i_x, the equation for the supplied voltage v_s is

$$v_s = \rho i_x,$$

the reference polarity is as shown, and the multiplying constant ρ has the dimension volts per ampere. In Fig. 2.2(c), the controlling voltage is v_x, the equation for the supplied current i_s is

$$i_s = \alpha v_x,$$

the reference direction is as shown, and the multiplying constant α has the dimension amperes per volt. In Fig. 2.2(d), the controlling current is i_x, the equation for the supplied current i_s is

$$i_s = \beta i_x,$$

the reference direction is as shown, and the multiplying constant β is dimensionless.

Note that the ideal independent and dependent voltage and current sources generate either constant voltages or currents, that is, voltages and currents that are invariant with time. Constant sources are often called **dc sources**. The *dc* stands for *direct current*, a description that has a historical basis but can seem misleading now. Historically, a direct current was defined as a current produced by a constant voltage. Therefore, a constant voltage became known as a direct current, or dc, voltage. The use of *dc* for *constant* stuck, and the terms *dc current* and *dc voltage* are now universally accepted in science and engineering to mean constant current and constant voltage.

Finally, we note that ideal sources are examples of active circuit elements. An **active element** is one that models a device capable of generating electric energy. **Passive elements** model physical devices that cannot generate electric energy. Resistors, inductors, and capacitors are examples of passive circuit elements. Examples 2.1 and 2.2 illustrate how the characteristics of ideal independent and dependent sources limit the types of permissible interconnections of the sources.

EXAMPLE 2.1 Testing Interconnections of Ideal Sources

Use the definitions of the ideal independent voltage and current sources to determine which interconnections in Fig. 2.3 are permitted and which violate the constraints imposed by the ideal sources.

Solution

Connection (a) is permitted. Each source supplies voltage across the same pair of terminals, marked a and b. This requires that each source supply the same voltage with the same polarity, which they do.

Connection (b) is permitted. Each source supplies current through the same pair of terminals, marked a and b. This requires that each source supply the same current in the same direction, which they do.

Connection (c) is not permitted. Each source supplies voltage across the same pair of terminals, marked a and b. This requires that each source supply the same voltage with the same polarity, which they do not.

Connection (d) is not permitted. Each source supplies current through the same pair of terminals, marked a and b. This requires that each source supply the same current in the same direction, which they do not.

Connection (e) is permitted. The voltage source supplies voltage across the pair of terminals marked a and b. The current source supplies current through the same pair of terminals. Because an ideal voltage source supplies the same voltage regardless of the current, and an ideal current source supplies the same current regardless of the voltage, this connection is permitted.

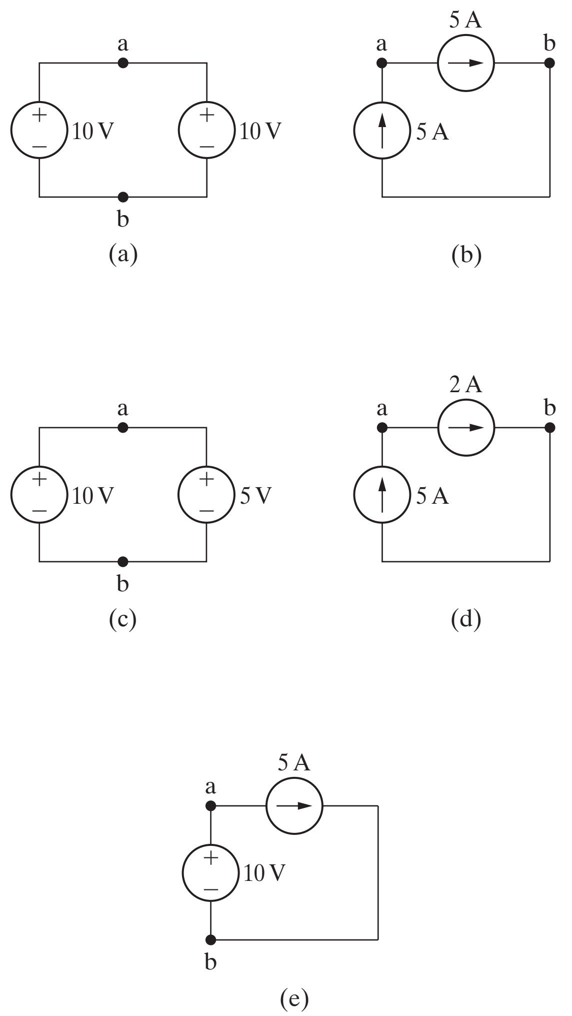


Figure 2.3 ▲ The circuits for Example 2.1.

EXAMPLE 2.2 Testing Interconnections of Ideal Independent and Dependent Sources

State which interconnections in Fig. 2.4 are permitted and which violate the constraints imposed by the ideal sources, using the definitions of the ideal independent and dependent sources.

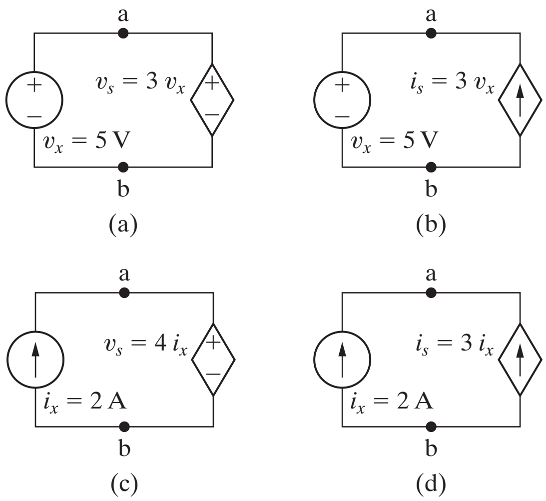


Figure 2.4 ▲ The circuits for Example 2.2.

Solution

Connection (a) is not permitted. Both the independent source and the dependent source supply voltage across the same pair of terminals, labeled a and b. This requires that each source supply the same voltage with the same polarity. The independent source supplies 5 V, but the dependent source supplies 15 V.

Connection (b) is permitted. The independent voltage source supplies voltage across the pair of terminals marked a and b. The dependent current source supplies current through the same pair of terminals. Because an ideal voltage source supplies the same voltage regardless of current, and an ideal current source supplies the same current regardless of voltage, this is a valid connection.

Connection (c) is permitted. The independent current source supplies current through the pair of terminals marked a and b. The dependent voltage source supplies voltage across the same pair of terminals. Because an ideal current source supplies the same current regardless of voltage, and an ideal voltage source supplies the same voltage regardless of current, this is a valid connection.

Connection (d) is not permitted. Both the independent source and the dependent source supply current through the same pair of terminals, labeled a and b. This requires that each source supply the same current in the same direction. The independent source supplies 2 A, but the dependent source supplies 6 A in the opposite direction.

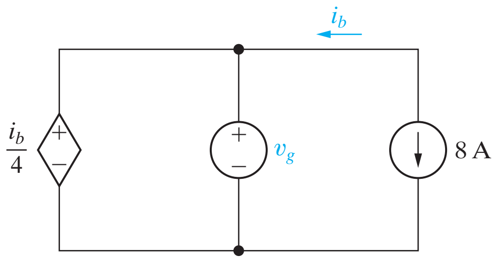
ASSESSMENT PROBLEMS

Objective 1—Understand ideal basic circuit elements

2.1 For the circuit shown,

a) What value of v_g is required in order for the interconnection to be valid?
b) For this value of v_g, find the power associated with the 8 A source.

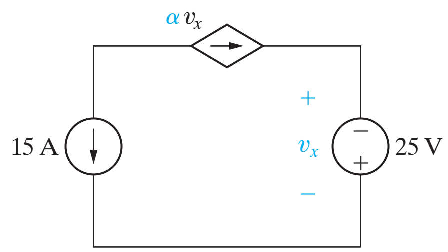
Answer: (a) −2 V;
(b) −16 W (16 W delivered).



2.2 For the circuit shown,

a) What value of α is required in order for the interconnection to be valid?
b) For the value of α calculated in part (a), find the power associated with the 25 V source.

Answer: (a) 0.6 A/V;
(b) 375 W (375 W absorbed).



SELF-CHECK: Also try Chapter Problems 2.6 and 2.7.

2.2 Electrical Resistance (Ohm's Law)

Figure 2.5 ▲ The circuit symbol for a resistor having a resistance R.

Resistance is the capacity of materials to impede the flow of current or, more specifically, the flow of electric charge. The circuit element modeling this behavior is the **resistor**. Figure 2.5 shows the resistor's circuit symbol, with R denoting the resistance value of the resistor.

To understand resistance, think about the electrons that make up electric current moving through, interacting with, and being resisted by the atomic structure of some material. The interactions convert some electric energy to thermal energy, dissipated as heat. Many useful electrical devices take advantage of resistance heating, including stoves, toasters, irons, and space heaters.

Most materials resist electric current; the amount of resistance depends on the material. Metals like copper and aluminum have small values of resistance, so they are often used as wires conducting electric current. When represented in a circuit diagram, copper or aluminum wiring isn't usually modeled as a resistor; the wire's resistance is so small compared to the resistance of other circuit elements that we can neglect the wiring resistance to simplify the diagram.

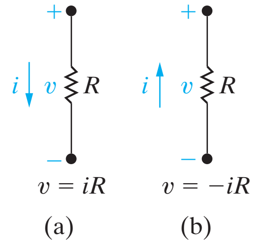


Figure 2.6 ▲ Two possible reference choices for the current and voltage at the terminals of a resistor and the resulting equations.

A resistor is an ideal basic circuit element, which is described mathematically using its voltage and current. The relationship between voltage and current for a resistor is known as **Ohm's law**, after Georg Simon Ohm, a German physicist who established its validity early in the nineteenth century. Consider the resistor shown in Fig. 2.6(a), where the current in the resistor is in the direction of the voltage drop across the resistor. For this resistor, Ohm's law is

OHM'S LAW

$$v = iR, \tag{2.1}$$

where

$$v = \text{the voltage in volts,}$$
$$i = \text{the current in amperes,}$$
$$R = \text{the resistance in ohms.}$$

For the resistor in Fig. 2.6(b), Ohm's law is

$$v = -iR, \tag{2.2}$$

where v, i, and R are again measured in volts, amperes, and ohms, respectively. We use the passive sign convention (Section 1.5) in determining the algebraic signs in Eqs. 2.1 and 2.2.

Figure 2.7 ▲ The circuit symbol for an 8 Ω resistor.

Resistance is measured in the SI unit ohms. The Greek letter Omega (Ω) is the standard symbol for an ohm. The circuit diagram symbol for an 8 Ω resistor is shown in Fig. 2.7.

Ohm's law expresses the voltage as a function of the current. However, expressing the current as a function of the voltage also is convenient. Thus, from Eq. 2.1,

$$i = \frac{v}{R}, \tag{2.3}$$

or, from Eq. 2.2,

$$i = -\frac{v}{R}. \tag{2.4}$$

The reciprocal of the resistance is referred to as **conductance**, is symbolized by the letter G, and is measured in siemens (S). Thus,

$$G = \frac{1}{R}. \tag{2.5}$$

An 8 Ω resistor has a conductance value of 0.125 S.

Ideal resistors model the behavior of physical devices. The word *ideal* reminds us that the resistor model makes several simplifying assumptions about the behavior of actual resistive devices. Assuming the resistance of the ideal resistor is constant, so that its value does not vary over time, is the most important of these simplifications. Most actual resistive devices have a time-varying resistance, often because the temperature of the device changes over time. The ideal resistor model represents a physical device whose resistance doesn't vary much from some constant value over the time period of interest in the circuit analysis. In this book, we assume that the simplifying assumptions about resistance devices are valid, and we thus use ideal resistors in circuit analysis.

We can calculate the power at the terminals of a resistor in several ways. The first approach is to use the defining equation (Section 1.6) to calculate the product of the terminal voltage and current. For the resistor shown in Fig. 2.6(a), we write

$$p = vi, \tag{2.6}$$

and for the resistor shown in Fig. 2.6(b), we write

$$p = -vi. \tag{2.7}$$

A second method expresses resistor power in terms of the current and the resistance. Substituting Eq. 2.1 into Eq. 2.6, we obtain

$$p = vi = (iR)i.$$

So

POWER IN A RESISTOR IN TERMS OF CURRENT

$$p = i^2R. \tag{2.8}$$

Likewise, substituting Eq. 2.2 into Eq. 2.7, we have

$$p = -vi = -(-iR)i = i^2 R. \tag{2.9}$$

Equations 2.8 and 2.9 are identical, demonstrating that regardless of voltage polarity and current direction, the power at the terminals of a resistor is positive. Therefore, resistors absorb power from the circuit.

A third method expresses resistor power in terms of the voltage and resistance. The expression is independent of the polarity references, so

POWER IN A RESISTOR IN TERMS OF VOLTAGE

$$p = \frac{v^2}{R}. \tag{2.10}$$

Sometimes a resistor's value will be expressed as a conductance rather than as a resistance. Using the relationship between resistance and conductance given in Eq. 2.5, we can also write Eqs. 2.9 and 2.10 in terms of the conductance, or

$$p = \frac{i^2}{G}, \tag{2.11}$$

$$p = v^2 G. \tag{2.12}$$

Equations 2.6–2.12 provide a variety of methods for calculating the power absorbed by a resistor. Each yields the same answer. In analyzing a circuit, look at the information provided and choose the power equation that uses that information directly.

Example 2.3 illustrates Ohm's law for a circuit with an ideal source and a resistor. Power calculations at the terminals of a resistor also are illustrated.

EXAMPLE 2.3 Calculating Voltage, Current, and Power for a Simple Resistive Circuit

In each circuit in Fig. 2.8, either the value of v or i is not known.

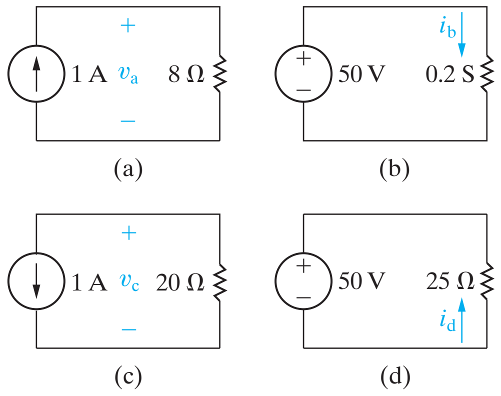


Figure 2.8 ▲ The circuits for Example 2.3.

a) Calculate the values of v and i.
b) Determine the power dissipated in each resistor.

Solution

a) The voltage v_a in Fig. 2.8(a) is a drop in the direction of the resistor current. The resistor voltage is the product of its current and its resistance, so,

$$v_a = (1)(8) = 8 \text{ V}.$$

The current i_b in the resistor with a conductance of 0.2 S in Fig. 2.8(b) is in the direction of the voltage drop across the resistor. The resistor current is the product of its voltage and its conductance, so

$$i_b = (50)(0.2) = 10 \text{ A}.$$

The voltage v_c in Fig. 2.8(c) is a rise in the direction of the resistor current. The resistor voltage is the product of its current and its resistance, so

$$v_c = -(1)(20) = -20 \text{ V}.$$

The current i_d in the 25 Ω resistor in Fig. 2.8(d) is in the direction of the voltage rise across the resistor. The resistor current is its voltage divided by its resistance, so

$$i_d = -\frac{50}{25} = -2 \text{ A}.$$

b) The power dissipated in each of the four resistors is

$$p_{8\Omega} = \frac{(8)^2}{8} = (1)^2(8) = 8 \text{ W}$$

(using Eq. 2.10 and Eq. 2.9);

$$p_{0.2S} = (50)^2(0.2) = 500 \text{ W}$$

(using Eq. 2.12);

$$p_{20\Omega} = \frac{(-20)^2}{20} = (1)^2(20) = 20 \text{ W}$$

(using Eq. 2.10 and Eq. 2.9);

$$p_{25\Omega} = \frac{(50)^2}{25} = (-2)^2(25) = 100 \text{ W}$$

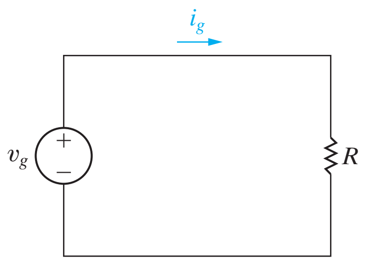
(using Eq. 2.10 and Eq. 2.9).

ASSESSMENT PROBLEMS

Objective 2—Be able to state and use Ohm's law

2.3 For the circuit shown,

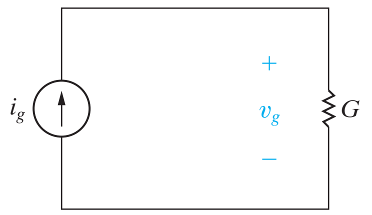
a) If $v_g = 1$ kV and $i_g = 5$ mA, find the value of R and the power absorbed by the resistor.
b) If $i_g = 75$ mA and the power delivered by the voltage source is 3 W, find v_g, R, and the power absorbed by the resistor.
c) If $R = 300$ Ω and the power absorbed by R is 480 mW, find i_g and v_g.



Answer: (a) 200 kΩ, 5 W;
(b) 40 V, 533.33 Ω, 3 W;
(c) 40 mA, 12 V.

2.4 For the circuit shown,

a) If $i_g = 0.5$ A and $G = 50$ mS, find v_g and the power delivered by the current source.
b) If $v_g = 15$ V and the power delivered to the conductor is 9 W, find the conductance G and the source current i_g.
c) If $G = 200$ μS and the power delivered to the conductance is 8 W, find i_g and v_g.



Answer: (a) 10 V, 5 W;
(b) 40 mS, 0.6 A;
(c) 40 mA, 200 V.

SELF-CHECK: Also try Chapter Problems 2.11 and 2.12.

2.3 Constructing a Circuit Model

Let's now move on to using ideal sources and resistors to construct circuit models of real-world systems. Developing a circuit model of a device or system is an important skill. Although this text emphasizes circuit-solving skills, as an electrical engineer you'll need other skills as well, one of the most important of which is modeling.

We develop circuit models in the next two examples. In Example 2.4, we construct a circuit model based on knowing how the system's components behave and how the components are interconnected. In Example 2.5, we create a circuit model by measuring the terminal behavior of a device.

EXAMPLE 2.4 Constructing a Circuit Model of a Flashlight

Construct a circuit model of a flashlight. Figure 2.9 shows a photograph of a widely available flashlight.

Figure 2.9 ▲ A flashlight can be viewed as an electrical system.
Thom Lang/Corbis/Getty Imagess

Solution

When a flashlight is regarded as an electrical system, the components of primary interest are the batteries, the lamp, the connector, the case, and the switch. Figure 2.10 shows these components. We now consider the circuit model for each component.

- A dry-cell battery maintains a reasonably constant terminal voltage if the current demand is not excessive. Thus, if the dry-cell battery is operating within its intended limits, we can model it with an ideal voltage source. The prescribed voltage is constant and equal to the sum of two dry-cell values.
- The ultimate output of the lamp is light energy, the result of heating the lamp's filament to a temperature high enough to cause radiation in the visible range. We can model the lamp with an ideal resistor. The resistor accounts for the amount of electric energy converted to thermal energy, but it does not predict how much of the thermal energy is converted to light energy. The resistor representing the lamp also predicts the steady current drain on the batteries, a characteristic of the system that also is of interest. In this model, R_l symbolizes the lamp resistance.
- The connector used in the flashlight serves a dual role. First, it provides an electrical conductive path between the dry cells and the case. Second, it is formed into a springy coil that applies mechanical pressure to the contact between the batteries and the lamp, maintaining contact between the two dry cells and between the dry cells and the lamp. Hence, in choosing the wire for the connector, we may find that its mechanical properties are more important than its electrical properties for the flashlight design. Electrically, we can model the connector with an ideal resistor with resistance R_1.
- The case also serves both a mechanical and an electrical purpose. Mechanically, it contains all the other components and provides a grip for the person using the flashlight. Electrically, it provides a connection between other elements in the flashlight. If the case is metal, it conducts current between the batteries and the lamp. If it is plastic, a metal strip inside the case connects the coiled connector to the switch. An ideal resistor with resistance R_c models the electrical connection provided by the case.

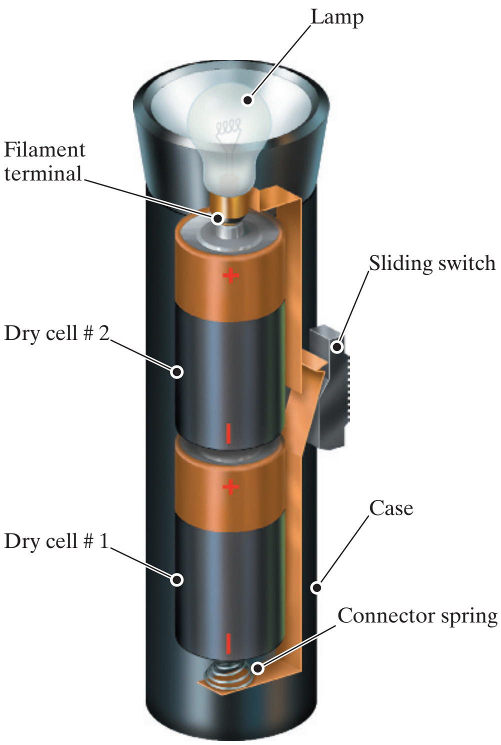


Figure 2.10 ▲ The arrangement of flashlight components.

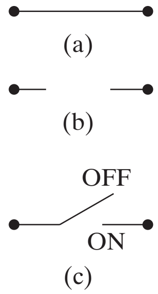


Figure 2.11 ▲ Circuit symbols. (a) Short circuit. (b) Open circuit. (c) Switch.

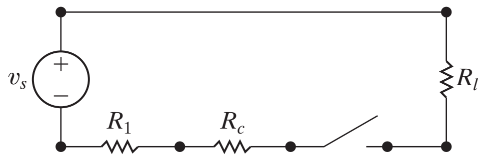


Figure 2.12 ▲ A circuit model for a flashlight.

- The switch has two electrical states: ON or OFF. An ideal switch in the ON state offers no resistance to the current, but it offers infinite resistance to current in the OFF state. These two states represent the limiting values of a resistor; that is, the ON state corresponds to a zero resistance, called a **short circuit** $(R = 0)$, and the OFF state corresponds to an infinite resistance called an **open circuit** $(R = \infty)$. Figures 2.11(a) and (b) depict a short circuit and an open circuit, respectively. The symbol shown in Fig. 2.11(c) represents a switch that can be either a short circuit or an open circuit, depending on the position of its contacts.

We now construct the circuit model of the flashlight shown in Fig. 2.10. Starting with the dry-cell batteries, the positive terminal of the first cell is connected to the negative terminal of the second cell. The positive terminal of the second cell is connected to one terminal of the lamp. The other terminal of the lamp makes contact with one side of the switch, and the other side of the switch is connected to the metal case. The metal spring connects the metal case to the negative terminal of the first dry cell. Note that the connected elements in Fig. 2.10 form a closed path or circuit. Figure 2.12 shows a circuit model for the flashlight.

Our flashlight example provides some general modeling guidelines.

1. **The *electrical* behavior of each physical component is of primary interest in a circuit model.** In the flashlight model, three very different physical components—a lamp, a coiled wire, and a metal case—are all represented by resistors because each circuit component resists the current flowing through the circuit.

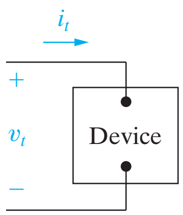
2. **Circuit models may need to account for undesired as well as desired electrical effects.** For example, the heat resulting from the lamp resistance produces the light, a desired effect. However, the heat resulting from the case and coil resistance represents an unwanted or *parasitic resistance*. It drains the dry cells and produces no useful output. Such parasitic effects must be considered, or the resulting model may not adequately represent the system.

3. **Modeling requires approximation.** We made several simplifying assumptions in developing the flashlight's circuit model. For example, we assumed an ideal switch, but in practical switches, contact resistance may be large enough to interfere with proper operation of the system. Our model does not predict this behavior. We also assumed that the coiled connector exerts enough pressure to eliminate any contact resistance between the dry cells. Our model does not predict the effect of inadequate pressure. Our use of an ideal voltage source ignores any internal dissipation of energy in the dry cells, which might be due to the parasitic heating just mentioned. We could account for this by adding an ideal resistor between the source and the lamp resistor. Our model assumes the internal loss to be negligible.

We used a basic understanding of the internal components of the flashlight to construct its circuit model. However, sometimes we know only the terminal behavior of a device and must use this information to construct the model. Example 2.5 presents such a modeling problem.

EXAMPLE 2.5 Constructing a Circuit Model Based on Terminal Measurements

The voltage and current are measured at the terminals of the device illustrated in Fig. 2.13(a), and the values of v_t and i_t are tabulated in Fig. 2.13(b). Construct a circuit model of the device inside the box.



v_t (V)	i_t (A)
−40	−10
−20	−5
0	0
20	5
40	10

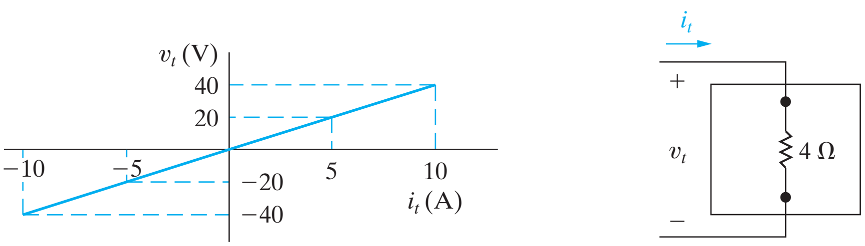
(a) (b)

Figure 2.13 ▲ The (a) device and (b) data for Example 2.5.

Solution

Plotting the voltage as a function of the current yields the graph shown in Fig. 2.14(a). The equation of the line in this figure is $v_t = 4i_t$, so the terminal voltage is directly proportional to the terminal current. Using Ohm's law, the device inside the box behaves like a 4 Ω resistor. Therefore, the circuit model for the device inside the box is a 4 Ω resistor, as seen in Fig. 2.14(b).

We come back to this technique of using terminal characteristics to construct a circuit model after introducing Kirchhoff's laws and circuit analysis.



(a) (b)

Figure 2.14 ▲ (a) The values of v_t versus i_t for the device in Fig. 2.13. (b) The circuit model for the device in Fig. 2.13.

SELF-CHECK: Assess your understanding of this example by trying Chapter Problems 2.14 and 2.16.

2.4 Kirchhoff's Laws

A circuit is **solved** when we determine the voltage across and the current in every element. While Ohm's law is an important tool for solving a circuit, it may not be enough to provide a complete solution. Generally, we need two additional algebraic relationships, known as Kirchhoff's laws, to solve most circuits.

Kirchhoff's Current Law

Let's try to solve the flashlight circuit from Example 2.4. We begin by redrawing the circuit as shown in Fig. 2.15, with the switch in the ON state. We have labeled the current and voltage variables associated with each resistor and the current associated with the voltage source, including reference polarities. For convenience, we use the same subscript for the voltage, current, and resistor labels. In Fig. 2.15, we also removed some of the terminal dots of Fig. 2.12 and have inserted nodes. Terminal dots are the start and end points of an individual circuit element. A **node** is a point where two or more circuit elements meet. In Fig. 2.15, the nodes are labeled a, b, c, and d. Node d connects the battery and the lamp and stretches all the way across the top of the diagram, though we label a single point for convenience. The dots on either side of the switch indicate its terminals, but only one is needed to represent a node, labeled node c.

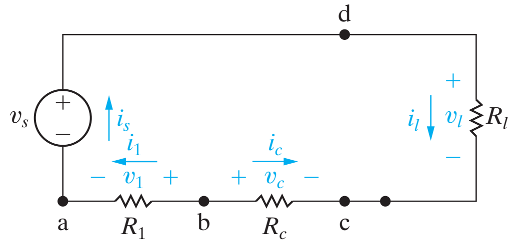


Figure 2.15 ▲ Circuit model of the flashlight with assigned voltage and current variables.

The circuit in Fig. 2.15 has seven unknowns: i_s, i_1, i_c, i_l, v_1, v_c, and v_l. Recall that $v_s = 3$ V, as it represents the sum of the terminal voltages of the two dry cells. To solve the flashlight circuit, we must find values for the seven unknown variables. From algebra, you know that to find n unknown quantities you must solve n simultaneous independent equations. Applying Ohm's law (Section 2.2) to each of the three resistors gives us three of the seven equations we need:

$$v_1 = i_1R_1, \tag{2.13}$$

$$v_c = i_cR_c, \tag{2.14}$$

$$v_l = i_lR_l. \tag{2.15}$$

What about the other four equations?

Connecting the circuit elements constrains the relationships among the terminal voltages and currents. These constraints are called Kirchhoff's laws, after Gustav Kirchhoff, who first stated them in a paper published in 1848. The two laws that state the constraints in mathematical form are known as Kirchhoff's current law and Kirchhoff's voltage law.

We can now state **Kirchhoff's current law (KCL)**:

KIRCHHOFF'S CURRENT LAW (KCL)

The algebraic sum of all the currents at any node in a circuit equals zero.

To use Kirchhoff's current law at a node, assign an algebraic sign corresponding to the current's reference direction for every current at the node. Assigning a positive sign to a current leaving the node requires assigning a negative sign to a current entering a node. Conversely, giving a negative sign to a current leaving a node requires giving a positive sign to a current entering a node.

We apply Kirchhoff's current law to the four nodes in the circuit shown in Fig. 2.15, using the convention that currents leaving a node are positive. The four equations are:

$$\text{node a} \qquad i_s - i_1 = 0, \tag{2.16}$$

$$\text{node b} \qquad i_1 + i_c = 0, \tag{2.17}$$

$$\text{node c} \qquad -i_c - i_l = 0, \tag{2.18}$$

$$\text{node d} \qquad i_l - i_s = 0. \tag{2.19}$$

But Eqs. 2.16–2.19 are not an independent set because any one of the four can be derived from the other three. In any circuit with n nodes, $n - 1$ independent equations can be derived from Kirchhoff's current law.[1] Let's disregard Eq. 2.19 so that we have six independent equations, namely, Eqs. 2.13–2.18. We need one more, which we can derive from Kirchhoff's voltage law.

Kirchhoff's Voltage Law

Before we can state Kirchhoff's voltage law, we must define a **closed path** or **loop**. Starting at an arbitrarily selected node, we trace a closed path in a circuit through selected basic circuit elements and return to the original node without passing through any intermediate node more than once. The circuit shown in Fig. 2.15 has only one closed path or loop. For example, choosing node a as the starting point and tracing the circuit clockwise, we form the closed path by moving through nodes d, c, b, and back to node a. We can now state **Kirchhoff's voltage law (KVL)**:

KIRCHHOFF'S VOLTAGE LAW (KVL)

The algebraic sum of all the voltages around any closed path in a circuit equals zero.

To use Kirchhoff's voltage law, assign an algebraic sign (reference direction) to each voltage in the loop. As we trace a closed path, a voltage will appear either as a rise or a drop in the tracing direction. Assigning a positive sign to a voltage rise requires assigning a negative sign to a voltage drop. Conversely, giving a negative sign to a voltage rise requires giving a positive sign to a voltage drop.

We now apply Kirchhoff's voltage law to the circuit shown in Fig. 2.15, tracing the closed path clockwise and assigning a positive algebraic sign to voltage drops. Starting at node d leads to the expression

$$v_l - v_c + v_1 - v_s = 0. \tag{2.20}$$

Now we have the seven independent equations needed to find the seven unknown circuit variables in Fig. 2.15.

Solving seven simultaneous equations for the simple flashlight lamp seems excessive. In the coming chapters, we present analytical techniques that solve a simple one-loop circuit like the one shown in Fig. 2.15 using a single equation. Before leaving the flashlight circuit, we observe two analysis details that are important for the techniques presented in subsequent chapters.

[1] We say more about this observation in Chapter 4.

1. If you know the current in a resistor, you also know the voltage across the resistor because current and voltage are directly related through Ohm's law. Thus, you can associate one unknown variable with each resistor, either the current or the voltage. For example, choose the current as the unknown variable. Once you solve for the unknown current in the resistor, you can find the voltage across the resistor. In general, if you know the current in a passive element, you can find the voltage across it, greatly reducing the number of simultaneous equations to solve. In the flashlight circuit, choosing the current as the unknown variable eliminates the voltages v_c, v_l, and v_1 as unknowns, and reduces the analytical task to solving four simultaneous equations rather than seven.

2. When only two elements connect to a node, if you know the current in one of the elements, you also know it in the second element by applying Kirchhoff's current law at the node. When just two elements connect at a single node, the elements are said to be **in series**, and you need to define only one unknown current for the two elements. Note that each node in the circuit shown in Fig. 2.15 connects only two elements, so you need to define only one unknown current. Equations 2.16–2.18 lead directly to

$$i_s = i_1 = -i_c = i_l,$$

which states that if you know any one of the element currents, you know them all. For example, choosing i_s as the unknown eliminates i_1, i_c, and i_l. The problem is reduced to determining one unknown, namely, i_s.

Examples 2.6 and 2.7 illustrate how to write circuit equations based on Kirchhoff's laws. Example 2.8 illustrates how to use Kirchhoff's laws and Ohm's law to find an unknown current. Example 2.9 expands on the technique presented in Example 2.5 for constructing a circuit model for a device whose terminal characteristics are known.

EXAMPLE 2.6 Using Kirchhoff's Current Law

Sum the currents at each node in the circuit shown in Fig. 2.16. Note that there is no connection dot (•) in the center of the diagram, where the 4 Ω branch crosses the branch containing the ideal current source i_a.

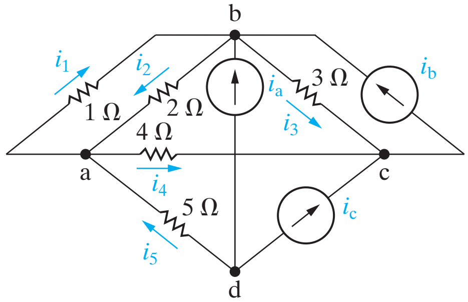


Figure 2.16 ▲ The circuit for Example 2.6.

Solution

In writing the equations, we use a positive sign for a current leaving a node. The four equations are

node a $\quad i_1 + i_4 - i_2 - i_5 = 0,$

node b $\quad i_2 + i_3 - i_1 - i_b - i_a = 0,$

node c $\quad i_b - i_3 - i_4 - i_c = 0,$

node d $\quad i_5 + i_a + i_c = 0.$

EXAMPLE 2.7 Using Kirchhoff's Voltage Law

Sum the voltages around each designated path in the circuit shown in Fig. 2.17.

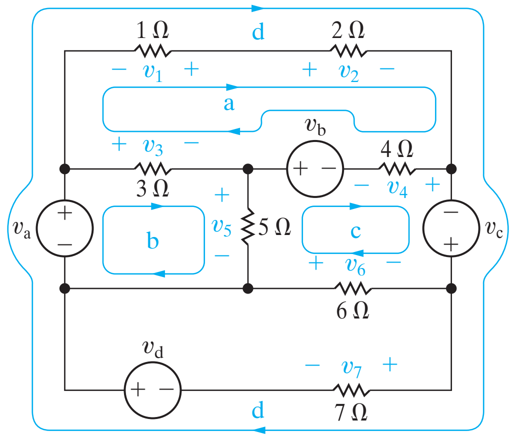


Figure 2.17 ▲ The circuit for Example 2.7.

Solution

In writing the equations, we use a positive sign for a voltage drop. The four equations are

path a $-v_1 + v_2 + v_4 - v_b - v_3 = 0,$

path b $-v_a + v_3 + v_5 = 0,$

path c $v_b - v_4 - v_c - v_6 - v_5 = 0,$

path d $-v_a - v_1 + v_2 - v_c + v_7 - v_d = 0.$

EXAMPLE 2.8 Applying Ohm's Law and Kirchhoff's Laws to Find an Unknown Current

a) Use Kirchhoff's laws and Ohm's law to find i_o in the circuit shown in Fig. 2.18.

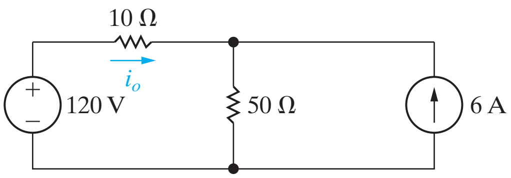


Figure 2.18 ▲ The circuit for Example 2.8.

b) Test the solution for i_o by verifying that the total power generated equals the total power dissipated.

Solution

a) We begin by redrawing the circuit and assigning an unknown current to the 50 Ω resistor and unknown voltages across the 10 Ω and 50 Ω resistors. Figure 2.19 shows the circuit. The nodes are labeled a, b, and c to aid the discussion.

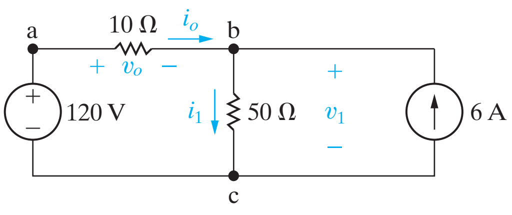


Figure 2.19 ▲ The circuit shown in Fig. 2.18, with the unknowns i_1, v_o, and v_1 defined.

Because i_o also is the current in the 120 V source, we have two unknown currents and therefore must derive two simultaneous equations involving i_o and i_1. One of the equations results from applying Kirchhoff's current law to either node b or c. Summing the currents at node b and assigning a positive sign to the currents leaving the node gives

$$i_1 - i_o - 6 = 0.$$

We obtain the second equation from Kirchhoff's voltage law in combination with Ohm's law. Noting from Ohm's law that $v_o = 10i_o$ and $v_1 = 50i_1$, we sum the voltages clockwise around the closed path c-a-b-c to obtain

$$-120 + 10i_o + 50i_1 = 0.$$

In writing this equation, we assigned a positive sign to voltage drops in the clockwise direction. Solving these two equations (see Appendix A) for i_o and i_1 yields

$$i_o = -3 \text{ A} \quad \text{and} \quad i_1 = 3 \text{ A}.$$

b) The power for the 50 Ω resistor is

$$p_{50\Omega} = (i_1)^2(50) = (3)^2(50) = 450 \text{ W}.$$

The power for the 10 Ω resistor is

$$p_{10\Omega} = (i_o)^2(10) = (-3)^2(10) = 90 \text{ W}.$$

The power for the 120 V source is

$$p_{120\text{V}} = -120 i_o = -120(-3) = 360 \text{ W}.$$

The power for the 6 A source is

$$p_{6\text{A}} = -v_1(6), \text{ and } v_1 = 50 i_1 = 50(3) = 150 \text{ V};$$

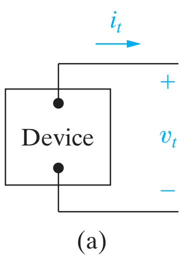
therefore

$$p_{6\text{A}} = -150(6) = -900 \text{ W}.$$

The 6 A source is delivering 900 W, and the 120 V source and the two resistors are absorbing power. The total power absorbed is $p_{6\text{A}} + p_{50\Omega} + p_{10\Omega} = 360 + 450 + 90 = 900$ W. Therefore, the solution verifies that the power delivered equals the power absorbed.

EXAMPLE 2.9 Constructing a Circuit Model Based on Terminal Measurements

We measured the terminal voltage and terminal current on the device shown in Fig. 2.20(a) and tabulated the values of v_t and i_t in Fig. 2.20(b).



v_t (V)	i_t (A)
30	0
15	3
0	6

(b)

Figure 2.20 ▲ (a) Device and (b) data for Example 2.9.

a) Construct a circuit model of the device inside the box.
b) Using this circuit model, predict the power this device will deliver to a 10 Ω resistor.

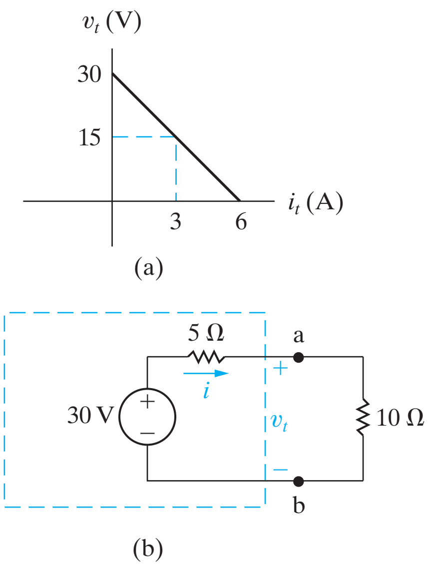


Figure 2.21 ▲ (a) The graph of v_t versus i_t for the device in Fig. 2.20(a). (b) The resulting circuit model for the device in Fig. 2.20(a), connected to a 10 Ω resistor.

Solution

a) Plotting the voltage as a function of the current yields the graph shown in Fig. 2.21(a). The equation of the line plotted is

$$v_t = 30 - 5 i_t.$$

What circuit model components produce this relationship between voltage and current? Kirchhoff's voltage law tells us that the voltage drops across two components in series add. From the equation, one of those components produces a 30 V drop regardless of the current, so this component's model is an ideal independent voltage source. The other component produces a positive voltage drop in the direction of the current i_t. Because the voltage drop is proportional to the current, Ohm's law tells us that this component's model is an ideal resistor with a value of 5 Ω. The resulting circuit model is depicted in the dashed box in Fig. 2.21(b).

b) Now we attach a 10 Ω resistor to the device in Fig. 2.21(b) to complete the circuit. Kirchhoff's current law tells us that the current in the 10 Ω resistor equals the current in the 5 Ω resistor. Using Kirchhoff's voltage law and Ohm's law, we can write the equation for the voltage drops around the circuit, starting at the voltage source and proceeding clockwise:

$$-30 + 5i + 10i = 0.$$

Solving for i, we get

$$i = 2\text{ A}.$$

This is the value of current flowing in the 10 Ω resistor, so compute the resistor's power using the equation $p = i^2R$:

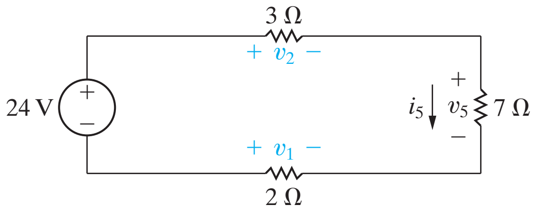
$$p_{10\Omega} = (2)^2(10) = 40\text{ W}.$$

ASSESSMENT PROBLEMS

Objective 3—Be able to state and use Ohm's law and Kirchhoff's current and voltage laws

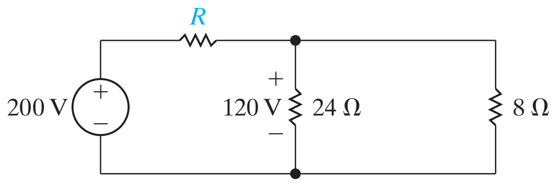
2.5 For the circuit shown, calculate (a) i_5; (b) v_1; (c) v_2; (d) v_5; and (e) the power delivered by the 24 V source.

Answer: (a) 2 A;
(b) −4 V;
(c) 6 V;
(d) 14 V;
(e) 48 W.



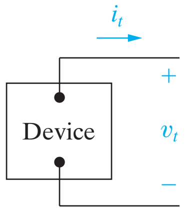
2.6 Use Ohm's law and Kirchhoff's laws to find the value of R in the circuit shown.

Answer: $R = 4\ \Omega$.



2.7 a) The terminal voltage and terminal current were measured on the device shown. The values of v_t and i_t are provided in the table. Using these values, create the straight-line plot of v_t versus i_t. Compute the equation of the line and use the equation to construct a circuit model for the device using an ideal voltage source and a resistor.
b) Use the model constructed in (a) to predict the power that the device will deliver to a 25 Ω resistor.

Answer: (a) A 25 V source in series with a 100 Ω resistor;
(b) 1 W.



(a)

v_t (V)	i_t (A)
25	0
15	0.1
5	0.2
0	0.25

(b)

2.8 Repeat Assessment Problem 2.7, but use the equation of the graphed line to construct a circuit model containing an ideal current source and a resistor.

Answer: (a) A 0.25 A current source connected between the terminals of a 100 Ω resistor;
(b) 1 W.

SELF-CHECK: Also try Chapter Problems 2.17, 2.18, 2.29, and 2.30.

2.5 Analyzing a Circuit Containing Dependent Sources

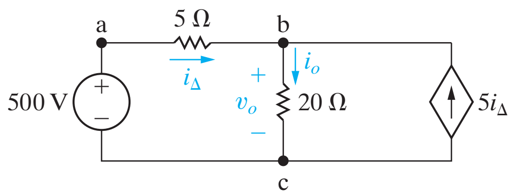


Figure 2.22 ▲ A circuit with a dependent source.

We conclude this introduction to elementary circuit analysis by considering circuits with dependent sources. One such circuit is shown in Fig. 2.22.

We want to use Kirchhoff's laws and Ohm's law to find v_o in this circuit. Before writing equations, it is good practice to examine the circuit diagram closely. This will help us identify the information that is known and the information we must calculate. It may also help us devise a strategy for solving the circuit using only a few calculations.

A look at the circuit in Fig. 2.22 reveals that:

- Once we know i_o, we can calculate v_o using Ohm's law.
- Once we know i_Δ, we also know the current supplied by the dependent source $5i_\Delta$.
- The current in the 500 V source is i_Δ, using Kirchhoff's current law at node a.

There are thus two unknown currents, i_Δ and i_o. We need to construct and solve two independent equations involving these two currents to produce a value for v_o. This is the approach used in Example 2.10.

EXAMPLE 2.10 Analyzing a Circuit with a Dependent Source

Find the voltage v_o for the circuit in Fig. 2.22.

Solution

The closed path consisting of the voltage source, the 5 Ω resistor, and the 20 Ω resistor contains the two unknown currents. Apply Kirchhoff's voltage law around this closed path, using Ohm's law to express the voltage across the resistors in terms of the currents in those resistors. Starting at node c and traversing the path clockwise gives:

$$-500 + 5i_\Delta + 20i_o = 0.$$

Now we need a second equation containing these two currents. We can't apply Kirchhoff's voltage law to the closed path formed by the 20 Ω resistor and the dependent current source because we don't know the value of the voltage across the dependent current source. For this same reason, we cannot apply Kirchhoff's voltage law to the closed path containing the voltage source, the 5 Ω resistor, and the dependent source.

We turn to Kirchhoff's current law to generate the second equation. Either node b or node c can be used to construct the second equation from Kirchhoff's current law, since we have already used node a to determine that the current in the voltage source and the 5 Ω resistor is the same. We select node b and produce the following equation, summing the currents leaving the node:

$$-i_\Delta + i_o - 5i_\Delta = 0.$$

Solve the KCL equation for i_o in terms of $i_\Delta (i_o = 6i_\Delta)$, and then substitute this expression for i_o into the KVL equation to give

$$500 = 5i_\Delta + 20(6i_\Delta) = 125i_\Delta.$$

Therefore,

$$i_\Delta = 500/125 = 4 \text{ A} \quad \text{and} \quad i_o = 6(4) = 24 \text{ A}.$$

Using i_o and Ohm's law for the 20 Ω resistor, we can solve for the voltage v_o:

$$v_o = 20i_o = 480 \text{ V}.$$

Think about a circuit analysis strategy before beginning to write equations because not every closed path yields a useful Kirchhoff's voltage law equation and not every node yields a useful Kirchhoff's current law equation. Think about the problem and select a fruitful approach and useful analysis tools to reduce the number and complexity of equations to be solved. Example 2.11 applies Ohm's law and Kirchhoff's laws to another circuit with a dependent source. Example 2.12 involves a much more complicated circuit, but with a careful choice of analysis tools, the analysis is relatively uncomplicated.

EXAMPLE 2.11 Applying Ohm's Law and Kirchhoff's Laws to Find an Unknown Voltage

a) Use Kirchhoff's laws and Ohm's law to find the voltage v_o as shown in Fig. 2.23.
b) Show that your solution is consistent with the requirement that the total power developed in the circuit equals the total power dissipated.

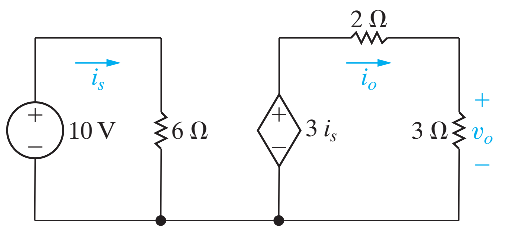


Figure 2.23 ▲ The circuit for Example 2.11.

Solution

a) A close look at the circuit in Fig. 2.23 reveals that:
   - There are two closed paths, the one on the left with the current i_s and the one on the right with the current i_o.
   - Once i_o is known, we can compute v_o using Ohm's law.

   We need two equations for the two currents. Because there are two closed paths and both have voltage sources, we can apply Kirchhoff's voltage law to each, using Ohm's law to express the voltage across the resistors in terms of the current in those resistors. The resulting equations are:

$$-10 + 6i_s = 0 \quad \text{and} \quad -3i_s + 2i_o + 3i_o = 0.$$

   Solving for the currents yields

$$i_s = 1.67 \text{ A} \quad \text{and} \quad i_o = 1 \text{ A}.$$

   Applying Ohm's law to the 3 Ω resistor gives the desired voltage:

$$v_o = 3i_o = 3 \text{ V}.$$

b) To compute the power delivered to the voltage sources, we use the power equation, $p = vi$, together with the passive sign convention. The power for the independent voltage source is

$$p = -10i_s = -10(1.67) = -16.7 \text{ W}.$$

   The power for the dependent voltage source is

$$p = -(3i_s)i_o = -(5)(1) = -5 \text{ W}.$$

   Both sources are supplying power, and the total power supplied is 21.7 W.

   To compute the power for the resistors, we use the power equation, $p = i^2R$. The power for the 6 Ω resistor is

$$p = (1.67)^2(6) = 16.7 \text{ W}.$$

   The power for the 2 Ω resistor is

$$p = (1)^2(2) = 2 \text{ W}.$$

   The power for the 3 Ω resistor is

$$p = (1)^2(3) = 3 \text{ W}.$$

   The resistors all absorb power, and the total power absorbed is 21.7 W, equal to the total power supplied by the sources.

EXAMPLE 2.12 Applying Ohm's Law and Kirchhoff's Law in an Amplifier Circuit

The circuit in Fig. 2.24 represents a common configuration encountered in the analysis and design of transistor amplifiers. Assume that the values of all the circuit elements—R_1, R_2, R_C, R_E, V_{CC}, and V_0—are known.

a) Develop the equations needed to determine the current in each element of this circuit.
b) From these equations, devise a formula for computing i_B in terms of the circuit element values.

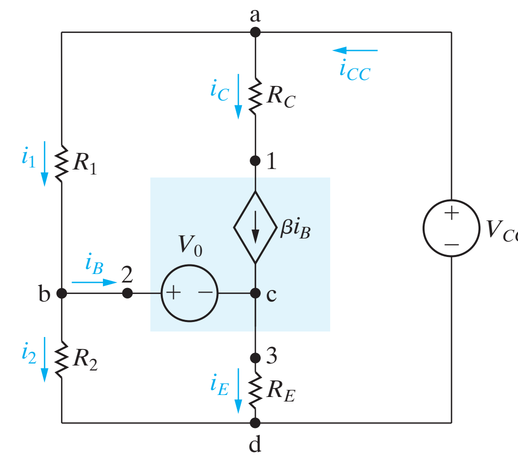


Figure 2.24 ▲ The circuit for Example 2.12.

Solution

Carefully examine the circuit to identify six unknown currents, designated i_1, i_2, i_B, i_C, i_E, and i_{CC}. In defining these six unknown currents, we observed that the resistor R_C is in series with the dependent current source βi_B, so these two components have the same current. We now must derive six independent equations involving these six unknowns.

a) We can derive three equations by applying Kirchhoff's current law to any three of the nodes a, b, c, and d. Let's use nodes a, b, and c and label the currents away from the nodes as positive:

$$(1)\quad i_1 + i_C - i_{CC} = 0,$$

$$(2)\quad i_B + i_2 - i_1 = 0,$$

$$(3)\quad i_E - i_B - i_C = 0.$$

A fourth equation results from imposing the constraint presented by the series connection of R_C and the dependent source:

$$(4)\quad i_C = \beta i_B.$$

We use Kirchhoff's voltage law to derive the remaining two equations. We must select two closed paths, one for each Kirchhoff's voltage law equation. The voltage across the dependent current source is unknown and cannot be determined from the source current βi_B, so select two closed paths that do not contain this dependent current source.

We choose the paths b-c-d-b and b-a-d-b, then use Ohm's law to express resistor voltage in terms of resistor current. Traverse the paths in the clockwise direction and specify voltage drops as positive to yield

$$(5)\quad V_0 + i_E R_E - i_2 R_2 = 0,$$

$$(6)\quad -i_1 R_1 + V_{CC} - i_2 R_2 = 0.$$

b) To get a single equation for i_B in terms of the known circuit variables, you can follow these steps:

- Solve Eq. (6) for i_1, and substitute this solution for i_1 into Eq. (2).
- Solve the transformed Eq. (2) for i_2, and substitute this solution for i_2 into Eq. (5).
- Solve the transformed Eq. (5) for i_E, and substitute this solution for i_E into Eq. (3). Use Eq. (4) to eliminate i_C in Eq. (3).
- Solve the transformed Eq. (3) for i_B, and rearrange the terms to yield

$$i_B = \frac{(V_{CC}R_2)/(R_1 + R_2) - V_0}{(R_1R_2)/(R_1 + R_2) + (1 + \beta)R_E}. \qquad (2.21)$$

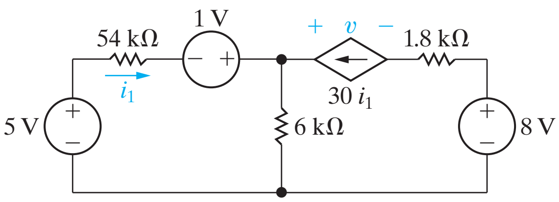
Problem 2.38 asks you to verify these steps. Note that once we know i_B, we can easily obtain the remaining currents.

ASSESSMENT PROBLEMS

Objective 4—Know how to calculate power for each element in a simple circuit

2.9 For the circuit shown, find (a) the current i_1 in microamperes, (b) the voltage v in volts, (c) the total power generated, and (d) the total power absorbed.

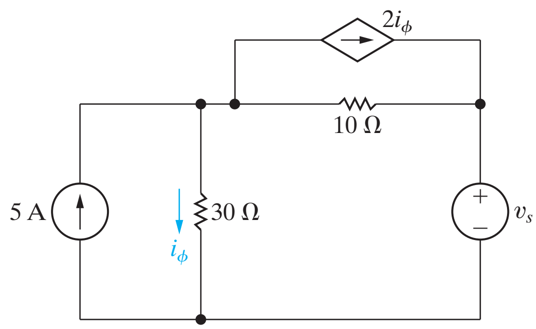
Answer: (a) 25 μA;
(b) −2 V;
(c) 6150 μW;
(d) 6150 μW.



2.10 The current i_ϕ in the circuit shown is 2 A. Calculate

a) v_s,
b) the power absorbed by the independent voltage source,
c) the power delivered by the independent current source,
d) the power delivered by the controlled current source,
e) the total power dissipated in the two resistors.

Answer: (a) 70 V;
(b) 210 W;
(c) 300 W;
(d) 40 W;
(e) 130 W.



SELF-CHECK: Also try Chapter Problems 2.33 and 2.34.

Practical Perspective

Heating with Electric Radiators

Let's determine which of the two wiring diagrams introduced at the beginning of this chapter should be used to wire the electric radiators to the power supplied to the garage. We begin with the diagram shown in Fig. 2.25. We can turn this into a circuit by modeling the radiators as resistors. The resulting circuit is shown in Fig. 2.26. Note that each radiator has the same resistance, R, and is labeled with a voltage and current value.

To find the unknown voltages and currents for the circuit in Fig. 2.26, begin by using Kirchhoff's voltage law to sum the voltage drops around the path on the circuit's left side:

$$-240 + v_1 = 0 \quad \Rightarrow \quad v_1 = 240\ \text{V}.$$

Now use Kirchhoff's voltage law to sum the voltage drops around the path on the circuit's right side:

$$-v_1 + v_2 = 0 \quad \Rightarrow \quad v_2 = v_1 = 240\ \text{V}.$$

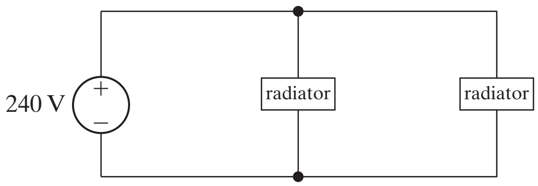


Figure 2.25 ▲ A wiring diagram for two radiators.

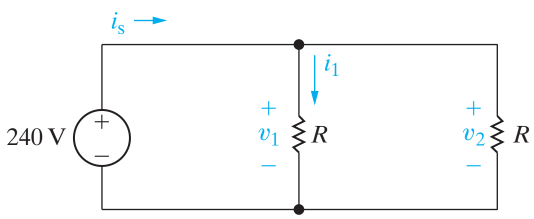


Figure 2.26 ▲ A circuit based on Fig. 2.25.

Remember that the power and voltage specifications for each radiator are 1200 W, 240 V. Therefore, the configuration shown in Fig. 2.25 satisfies the voltage specification, since each radiator would have a supplied voltage of 240 V.

Next, calculate the value of resistance R that correctly models each radiator. We want the power associated with each radiator to be 1200 W. Use the equation for resistor power that involves the resistance and the voltage:

$$p_1 = \frac{v_1^2}{R} = \frac{v_2^2}{R} = p_2 \qquad \Rightarrow \qquad R = \frac{v_1^2}{p_1} = \frac{240^2}{1200} = 48\ \Omega.$$

Each radiator can be modeled as a $48\ \Omega$ resistor, with a voltage drop of 240 V and power of 1200 W. The total power for two radiators is thus 2400 W.

Finally, calculate the power for the 240 V source. To do this, calculate the current in the voltage source, i_s, using Kirchhoff's current law to sum the currents leaving the top node in Fig. 2.26. Then use i_s to find the power for the voltage source.

$$-i_s + i_1 + i_2 = 0 \quad \Rightarrow \quad i_s = i_1 + i_2 = \frac{v_1}{R} + \frac{v_2}{R} = \frac{240}{48} + \frac{240}{48} = 10\ \text{A}.$$

$$p_s = -(240)(i_s) = -(240)(10) = -2400\ \text{W}.$$

Thus, the total power in the circuit is $-2400 + 2400 = 0$, and the power balances.

Now look at the other wiring diagram for the radiators, shown in Fig. 2.27. We know that the radiators can be modeled using $48\ \Omega$ resistors, which are used to turn the wiring diagram into the circuit in Fig. 2.28.

Start analyzing the circuit in Fig. 2.28 by using Kirchhoff's voltage law to sum the voltage drops around the closed path:

$$-240 + v_x + v_y = 0 \qquad \Rightarrow \qquad v_x + v_y = 240.$$

Next, use Kirchhoff's current law to sum to currents leaving the node labeled a:

$$-i_x + i_y = 0 \qquad \Rightarrow \qquad i_x = i_y = i.$$

The current in the two resistors is the same, and we can use that current in Ohm's law equations to replace the two unknown voltages in the Kirchhoff's voltage law equation:

$$48i + 48i = 240 = 96i \qquad \Rightarrow \qquad i = \frac{240}{96} = 2.5\ \text{A}.$$

Use the current in the two resistors to calculate the power for the two radiators.

$$p_x = p_y = Ri^2 = (48)(2.5)^2 = 300\ \text{W}.$$

Thus, if the radiators are wired as shown in Fig. 2.27, their total power is 600 W. This is insufficient to heat the garage.

Therefore, the way the radiators are wired has a big impact on the amount of heat that will be supplied. When they are wired using the diagram in Fig. 2.25, 2400 W of power will be available, but when they are wired using the diagram in Fig. 2.27, only 600 W of power will be available.

SELF-CHECK: Assess your understanding of the Practical Perspective by solving Chapter Problems 2.41–2.43.

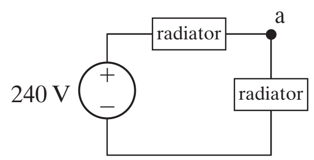


Figure 2.27 ▲ Another way to wire two radiators.

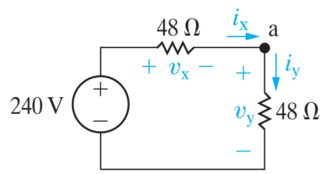


Figure 2.28 ▲ A circuit based on Fig. 2.27.

Summary

- The circuit elements introduced in this chapter are voltage sources, current sources, and resistors:
  - An **ideal voltage source** maintains a prescribed voltage regardless of the current in the source. An **ideal current source** maintains a prescribed current regardless of the voltage across the source. Voltage and current sources are either **independent**, that is, not influenced by any other current or voltage in the circuit, or **dependent**, that is, determined by some other current or voltage in the circuit. (See pages 56 and 57.)
  - A **resistor** constrains its voltage and current to be proportional to each other. The value of the proportional constant relating voltage and current in a resistor is called its **resistance** and is measured in ohms. (See page 60.)
- **Ohm's law** establishes the proportionality of voltage and current in a resistor. Specifically,

  $$v = iR$$

  if the current flow in the resistor is in the direction of the voltage drop across it, or

  $$v = -iR$$

  if the current flow in the resistor is in the direction of the voltage rise across it. (See page 60.)
- By combining the equation for power, $p = vi$, with Ohm's law, we can determine the power absorbed by a resistor:

  $$p = i^2R = v^2/R.$$

  (See pages 61–62.)
- Circuits have nodes and closed paths. A **node** is a point where two or more circuit elements join. When just two elements connect to form a node, they are said to be **in series**. A **closed path** is a loop traced through connecting elements, starting and ending at the same node and encountering intermediate nodes only once each. (See pages 67–68.)
- The voltages and currents of interconnected circuit elements obey Kirchhoff's laws:
  - **Kirchhoff's current law** states that the algebraic sum of all the currents at any node in a circuit equals zero. (See page 67.)
  - **Kirchhoff's voltage law** states that the algebraic sum of all the voltages around any closed path in a circuit equals zero. (See page 68.)
- A circuit is solved when the voltage across and the current in every element have been determined. By combining an understanding of independent and dependent sources, Ohm's law, and Kirchhoff's laws, we can solve many simple circuits.

Problems

Section 2.1

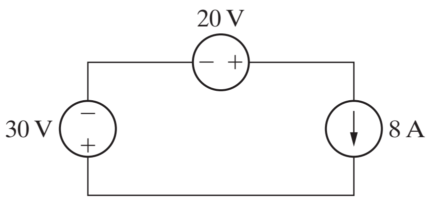
2.1 a) Is the interconnection of an ideal source in the circuit in Fig. P2.1 valid? Explain.

b) Identify which sources are developing power and which sources are absorbing power.

c) Verify that the total power developed in the circuit equals the total power absorbed.

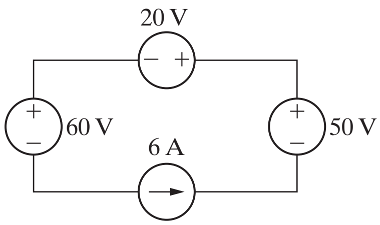
d) Repeat (a)–(c), reversing the polarity of the 30 V source.

Figure P2.1



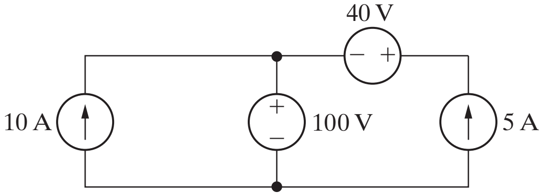
2.2 If the interconnection in Fig. P2.2 is valid, find the total power developed in the circuit. If the interconnection is not valid, explain why.

Figure P2.2



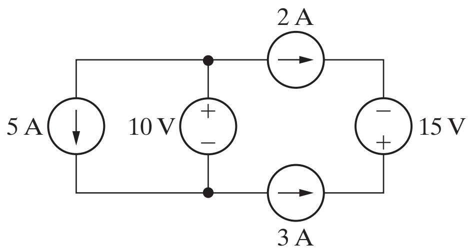
2.3 If the interconnection in Fig. P2.3 is valid, find the power developed by the current sources. If the interconnection is not valid, explain why.

Figure P2.3



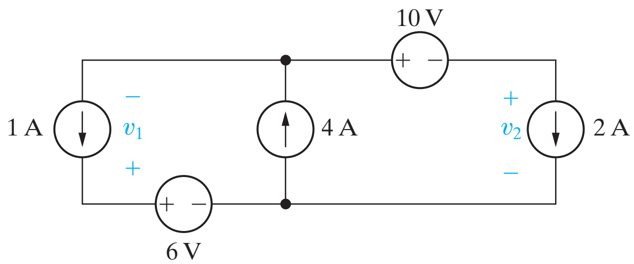
2.4 If the interconnection in Fig. P2.4 is valid, find the power developed by the voltage sources. If the interconnection is not valid, explain why.

Figure P2.4



2.5 The interconnection of ideal sources can lead to an indeterminate solution. With this thought in mind, explain why the solutions for v_1 and v_2 in the circuit in Fig. P2.5 are not unique.

Figure P2.5

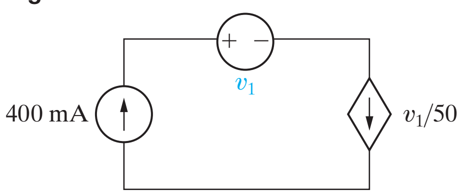


2.6 Consider the interconnection shown in Fig. P2.6.

a) What value of v_1 is required to make this a valid interconnection?

b) For this value of v_1, find the power associated with the voltage source.

Figure P2.6



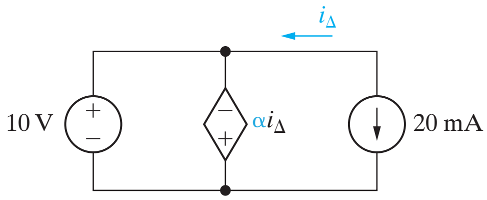
2.7 Consider the interconnection shown in Fig. P2.7.

a) What value of α is required to make this a valid interconnection?

b) For this value of α, find the power associated with the current source.

c) Is the current source supplying or absorbing power?

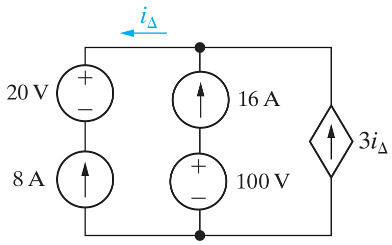
Figure P2.7



2.8 a) Is the interconnection in Fig. P2.8 valid? Explain.

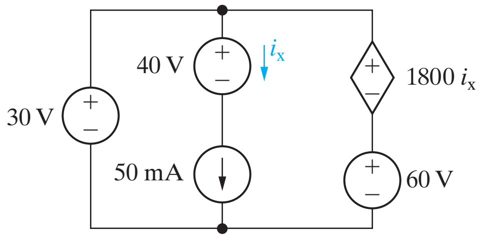
b) Can you find the total energy developed in the circuit? Explain.

Figure P2.8



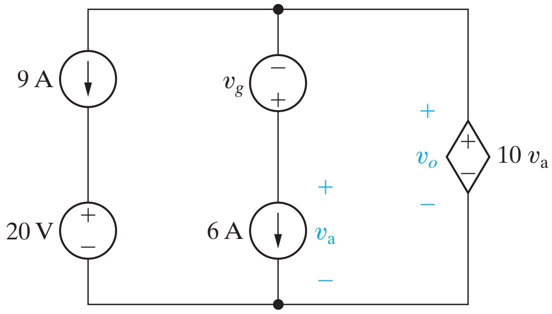
2.9 If the interconnection in Fig. P2.9 is valid, find the total power developed in the circuit. If the interconnection is not valid, explain why.

Figure P2.9



2.10 Find the total power developed in the circuit in Fig. P2.10 if $v_o = 10$ V.

Figure P2.10



Section 2.2–2.3

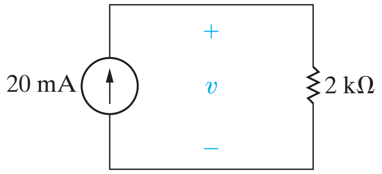
2.11 For the circuit shown in Fig. P2.11

a) Find v.

b) Find the power absorbed by the resistor.

c) Reverse the direction of the current source and repeat parts (a) and (b).

Figure P2.11



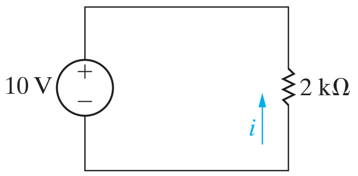
2.12 For the circuit shown in Fig. P2.12

a) Find i.

b) Find the power supplied by the voltage source.

c) Reverse the polarity of the voltage source and repeat parts (a) and (b).

Figure P2.12

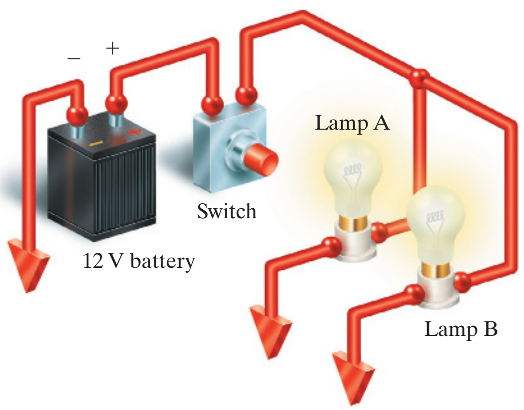


2.13 A pair of automotive headlamps is connected to a 12 V battery via the arrangement shown in Fig. P2.13. In the figure, the triangular symbol ▼ is used to indicate that the terminal is connected directly to the metal frame of the car.

a) Construct a circuit model using resistors and an independent voltage source.

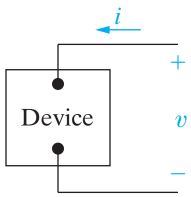
b) Identify the correspondence between the ideal circuit element and the symbol component that it represents.

Figure P2.13



2.14 The terminal voltage and terminal current were measured on the device shown in Fig. P2.14(a). The values of v and i are given in the table of Fig. P2.14(b). Use the values in the table to construct a circuit model for the device consisting of a single resistor from Appendix H.

Figure P2.14



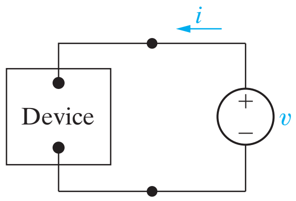
(a)

i (mA)	v (V)
−10	−120
−5	−60
5	60
10	120
15	180

(b)

2.15 A variety of voltage source values were applied to the device shown in Fig. P2.15(a). The power absorbed by the device for each value of voltage is recorded in the table given in Fig. P2.15(b). Use the values in the table to construct a circuit model for the device consisting of a single resistor from Appendix H.

Figure P2.15



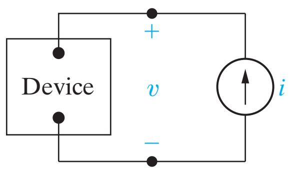
(a)

v (V)	p (mW)
−8	640
−4	160
4	160
8	640
12	1440
16	2560

(b)

2.16 A variety of current source values were applied to the device shown in Fig. P2.16(a). The power absorbed by the device for each value of current is recorded in the table given in Fig. P2.16(b). Use the values in the table to construct a circuit model for the device consisting of a single resistor from Appendix H.

Figure P2.16



(a)

i (mA)	p (mW)
0.5	12.5
1.0	50
1.5	112.5
2.0	200
2.5	312.5
3.0	450

(b)