

a)

x	1	2	3	4
y	1	3	4	3

a)

$$\begin{array}{c|c|c|c|c}
 x & y & \Delta y_0 & \Delta^2 y_0 & \Delta^3 y_0 \\
 \hline
 1 & 1 & & & \\
 2 & 3 & 2 & & \\
 3 & 4 & 1 & -1 & \\
 4 & 3 & -1 & -2 & -1
 \end{array}$$

b)

x	1	2	3	4
y	8	5	4	0

$$P(x) = y_0 + \frac{\Delta y_0}{h} (x-x_0) + \frac{\Delta^2 y_0}{2! h^2} (x-x_0)(x-x_1) + \frac{\Delta^3 y_0}{3! h^3} (x-x_0)(x-x_1)(x-x_2)$$

$$P(x) = 1 + \frac{2}{1} (x-1) + \frac{-1}{2! \cdot 1^2} (x-1)(x-2) + \frac{-1}{3! \cdot 1^3} (x-1)(x-2)(x-3)$$

$$P(x) = 1 + 2x - 2 - \frac{x^2}{2} + \frac{3x}{2} - 1 - \frac{x^3}{6} + x^2 + \frac{7x}{6} + 1$$

$$P(x) = -\frac{x^3}{6} + \frac{x^2}{2} + \frac{3x}{5} - 1$$

b)

$$\begin{array}{c|c|c|c|c}
 x & y & \Delta y_0 & \Delta^2 y_0 & \Delta^3 y_0 \\
 \hline
 1 & 8 & & & \\
 2 & 5 & -3 & & \\
 3 & 4 & -1 & 2 & \\
 4 & 0 & -4 & -3 & -5
 \end{array}$$

$$P(x) = 8 + \frac{-3}{1} (x-1) + \frac{2}{2! \cdot 1^2} (x-1)(x-2) + \frac{-5}{3! \cdot 1^3} (x-1)(x-2)(x-3)$$

$$P(x) = 8 - 3x + 3 + x^2 - 3x + 2 - \frac{5x^3}{6} + 5x^2 + \frac{35x}{6} + 5$$

$$P(x) = -\frac{5x^3}{6} + 6x^2 + \frac{29x}{6} + 10$$