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CODE FOR My MASTER THESIS WORK "Interior point method for the numerical simulation of the obstacle problem"

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```
function min_obstacle


clc; clear all;
% Geometry is imported from the workspace file
load  geometry_square.mat;
hs=inf;  % for structured mesh
[coordinates,edges,triangles]=initmesh(geom, 'Hmax', hs);
for i_refin=1:6
    [coordinates,edges,triangles]=refinemesh(geom,coordinates,edges,triangles);
end
kappa = 1.;
Theta = 0.5;                        % precision for the Newton iterations
tau = 0.9;
k = 0;
iter=[];                            % stores iteration number, corresponding kappa
N = size(coordinates,2);            % number of nodes in mesh
eps = 0.1/sqrt(N);                  % precision for the interior point method
if size(triangles,1) == 4
    triangles(4,:) = [] ;
end;
N_t = size(triangles,2);                     % returns the number of triangles in
BoundaryNodes=unique(edges(1:2,:));
Nodes = setdiff(1:N,BoundaryNodes);          % these are nodes in mesh excluding b
u = zeros(N,1);                              % starting values
C = 0.0000000000000001;
C1 = 0;
pic = 0;                                     % counter for pictures
% spparms('spumoni',1)
tic;                                           % timer on
psi_n = zeros(N,1);
for j=1:N
    psi_n(j) = f_psi(coordinates(:,j));
end
grad_u_grad_v = sparse(N,N);
f_phi = zeros(N,1);
jac=zeros(N_t,1);
for j = 1:N_t
    tri = triangles(:,j);
```

```
    jac(j)=det([1,1,1; coordinates(:,tri')]);
    grad_u_grad_v(tri,tri) = grad_u_grad_v(tri,tri)+Local_Stiff_1(coordinates(:,tr
    f_phi(tri)=f_phi(tri)+  [f(coordinates(:,tri(1)));f(coordinates(:,tri(2)));f(c
end
kappa_cond=0;
while kappa_cond==0                                     % here the predictor-correcto
    k=k+1;
    disp(['CORRECTOR STEP: ', 'k=',int2str(k),' \kappa=',num2str(kappa)]);
    conv=1;                                             % convergence criterion
    m=0;                                                % counter for Newton iterations
    disp('Newton iterations ...  ');
    while conv > Theta                                  % Newton iterations begin
        m=m+1;
        A = grad_u_grad_v;
        b = -grad_u_grad_v*u + f_phi;
        % Assembly
        for j = 1:N_t
            tri=triangles(:,j);
            bf=1./(u(tri)-psi_n(tri));
            A(tri,tri)  = A(tri,tri) + kappa * jac(j) * diag(bf.^2)/6;
            b(tri) = b(tri) + jac(j)  * kappa * bf/6 ;
        end
        corr = zeros(N,1);
        %corr(BoundaryNodes) = 0;
        %b = b - A * corr;
        %COMPUTATION OF THE CORRECTOR
        corr(Nodes)= A(Nodes,Nodes)\b(Nodes);
        vec = corr./(u-psi_n);
        Linf_vec = norm(vec,inf);
        L2_vec = sqrt(sum(vec(triangles).^2,1)*jac/6);
        if L2_vec>0
            C1 = Linf_vec/L2_vec;
        end
        C = max(C1,C);
        alpha = 1;
        while alpha*Linf_vec >0.99
            alpha = alpha/2;
        end;
        u = u+alpha*corr;
        conv=Linf_vec;
    end
    iter = [iter; [k, kappa, m]];
    clear corr;
    if kappa<=eps
        break
    end
    disp(['PREDICTOR STEP...']);
    A = grad_u_grad_v;
    b = -grad_u_grad_v*u + f_phi;
    % Assembly
    for j = 1:N_t
        tri=triangles(:,j);
        bf=(1./(u(tri)-psi_n(tri)));
        A(tri,tri) = A(tri,tri) + kappa * jac(j) * diag(bf.^2)/6;              %
    end
    pred = zeros(N,1);
    % COMPUTATION OF THE PREDICTOR
    pred(Nodes)= A(Nodes,Nodes)\b(Nodes);
    vecp = pred./(u-psi_n);
    Linf_vecp = norm(vecp,inf);
    rho = 0.99;
    while rho*Linf_vecp > 0.99
        rho = 0.9*rho;
        if rho < 0.001
            'small rho....'
```

```
            stop
        end
    end;
    % checking of the condition for rho
    Xi1=sum(vec(triangles).^4,1)*jac/6;
    Xi2=sum(vec(triangles).^4.*vecp(triangles),1)*jac/6;
    Xi3=sum(vec(triangles).^4.*vecp(triangles).^2,1)*jac/6;
    Xi4=sum(vec(triangles).^2.*vecp(triangles).*(1.+vecp(triangles)),1)*jac/6;
    Xi5=sum(vec(triangles).^2.*vecp(triangles).^2.*(1.+vecp(triangles)),1)*jac/6;
    Xi6=sum(vecp(triangles).^2.*(1.+vecp(triangles)).^2,1)*jac/6;
    Xi=Xi1+2*rho*Xi2+rho*rho*Xi3+2*rho*rho/(1-rho)*Xi4+2*rho^3/(1-rho)*Xi5+rho^4/(
    rho1=rho;
    Xi_min=Xi;
    while Xi > (Theta/C)^2
        if rho1<0.1
            rho1=0.7*rho1;
        else
            rho1=rho1-0.1;
        end;
        if rho1<=0.0001
            ' Xi^2 <= (Theta/C)^2 is not satisfied'
            break
        end
        Xi_min=Xi1+2*rho1*Xi2+rho1*rho1*Xi3+2*rho1*rho1/(1-rho1)*Xi4+2*rho1^3/(1-r
        if Xi_min<Xi
            rho=rho1;
            Xi=Xi_min;
        end
    end
    u=u+rho*pred;
    clear pred;
    kappa=(1-rho)*kappa;
end
toc;
clear f_phi;
c=kappa./(u-psi_n)-10;
pic=pic+1; show(pic,triangles,coordinates,u); title('Approximate solution u' )
show(pic+1,triangles, coordinates, c); title ('Lagrange multipliers'); view(0,0);
disp('------------------------------------------------');
disp('Iteration No |  kappa   | # of Newton iterations');
iter
return
```

Function which computes the local stiffness matrix

```
function A_elem = Local_Stiff_1(vertices,area)


G = [1,1,1;vertices] \ [0, 0; 1,0; 0,1];
A_elem = det([1,1,1;vertices]) * G * G'/2 ;
return
```

```
CORRECTOR STEP: k=1 \kappa=1
Newton iterations ...
PREDICTOR STEP...
CORRECTOR STEP: k=2 \kappa=0.609
Newton iterations ...
PREDICTOR STEP...
CORRECTOR STEP: k=3 \kappa=0.37088
```

```
Newton iterations ...
PREDICTOR STEP...
CORRECTOR STEP: k=4 \kappa=0.22587
Newton iterations ...
PREDICTOR STEP...
CORRECTOR STEP: k=5 \kappa=0.11497
Newton iterations ...
PREDICTOR STEP...
CORRECTOR STEP: k=6 \kappa=0.058518
Newton iterations ...
PREDICTOR STEP...
CORRECTOR STEP: k=7 \kappa=0.023934
Newton iterations ...
PREDICTOR STEP...
CORRECTOR STEP: k=8 \kappa=0.0073955
Newton iterations ...
PREDICTOR STEP...
CORRECTOR STEP: k=9 \kappa=0.0015457
Newton iterations ...
PREDICTOR STEP...
CORRECTOR STEP: k=10 \kappa=0.00016848
Newton iterations ...
Elapsed time is 23.809229 seconds.
-----------------------------------------------
Iteration No |  kappa   | # of Newton iterations

iter =

    1.0000    1.0000   13.0000
    2.0000    0.6090    1.0000
    3.0000    0.3709    1.0000
    4.0000    0.2259    1.0000
    5.0000    0.1150    1.0000
    6.0000    0.0585    1.0000
    7.0000    0.0239    1.0000
    8.0000    0.0074    1.0000
    9.0000    0.0015    1.0000
   10.0000    0.0002    1.0000
```

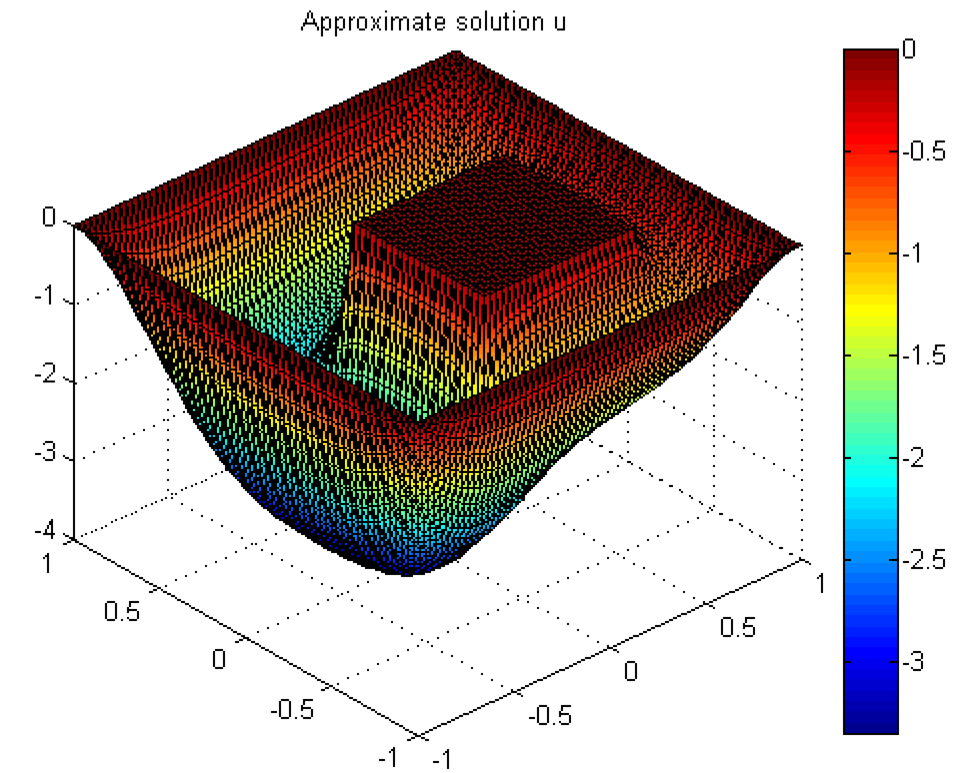
Function which describes the obstacle

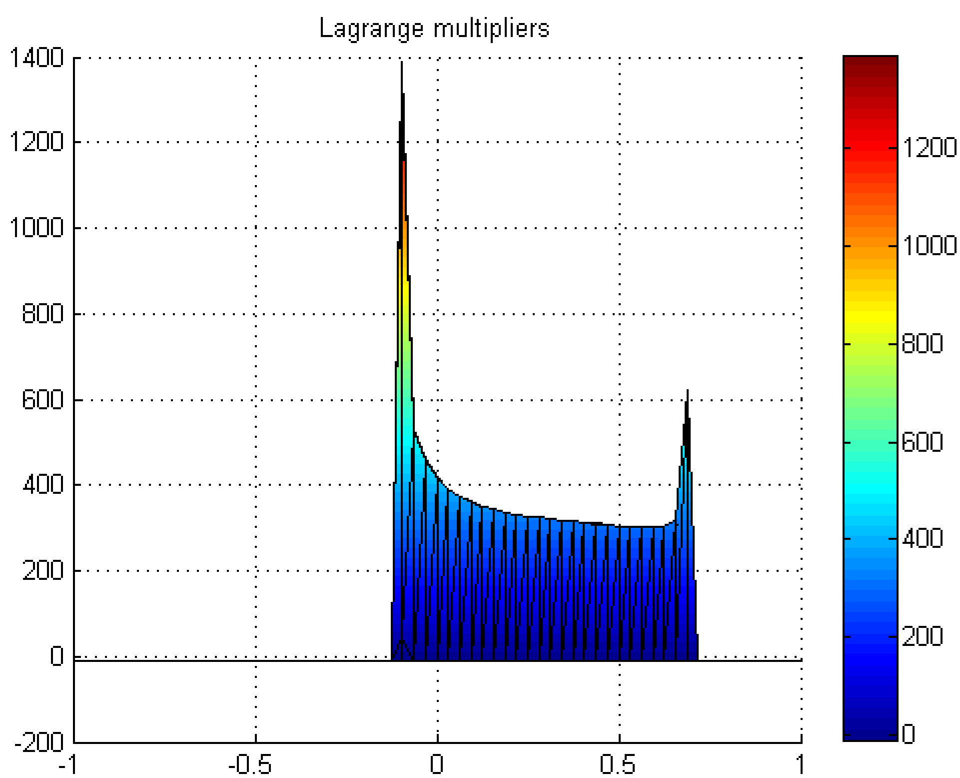
```
function Obstacle = f_psi(x);


% Obstacle =-1;                     %            horizontal plane
%----------------------------------------------------------------
% Obstacle = -0.2*x(1)-0.2*x(2)-0.5;          % oblique plane
% %--------------------------------------------------------------------
%  if (abs(x(1))<=0.5)
%      Obstacle = -2;
%  elseif (x(1)>-1) & (x(1)<-0.5)
%      Obstacle = 20*x(1)+8;
%  elseif (x(1)>0.5) & (x(1)<1)
%      Obstacle  = -20*x(1)+8;
%  else
%      Obstacle = -12;
%  end
% %--------------------------------------------------------------------
% if (abs(x(1)-0.2)<=0.3)
%     Obstacle = -2;
% else
```

```
%     Obstacle = -20;
% end
% -------------------------------------------------------------------
if (abs(x(1)-0.3)<=0.4) & (abs(x(2))<=0.4)             % piecewise linear
    Obstacle = -0.2;
elseif (x(1)<-0.1 & x(1)>-0.2 & abs(x(2))<=0.4)
    Obstacle = 100*x(1)+9.2;
elseif (x(1)<0.5 & x(1)>0.4 & abs(x(2))<=0.4)
    Obstacle = -100*x(1)-40.2;
elseif (abs(x(1)-0.3)<=0.4 & x(2)<.5 & x(2)>0.4)
    Obstacle = -100*x(2)-40.2;
elseif (abs(x(1)-0.3)<=0.4 & x(2)<-.4 & x(2)>-0.5)
    Obstacle = 100*x(2)+39.8;
else
    Obstacle = -10.2;
end
%----------------------------------------------------------------------
% if (x(1)-0.3)^2+x(2)^2<=0.25                    %     vertical cylinder
%        Obstacle= 1;
% else
%     Obstacle=-1;
% end;
%----------------------------------------------------------------------
% if (x(1)-0.3)^2+x(2)^2<=1                      %   cone, opened
%        Obstacle= -sqrt((x(1)-0.3)^2+x(2)^2)-2;
% else
%     Obstacle=-100;
% end;

%------------------------------------------------------------------
%  if (x(1)+0.3)^2+x(2)^2<=0.64                 %    sphere
%      Obstacle=sqrt(0.64-(x(1)+0.3)^2-x(2)^2)-2;
%  else
%      Obstacle=-2;
%  end
%
% R=0.7;
% if x(1)^2+x(2)^2<=R^2                    % sphere
%      Obstacle=sqrt(R^2-(x(1))^2-x(2)^2)-R-1;
% %  else
%      Obstacle=-100;
%  end
% ---------------------------------------------------------------------
%  if (abs(x(1)+0.3)<=0.8)                              %  horizontal cylinder
%       Obstacle = sqrt(0.64-(x(1)+0.3)^2)-1;
%   else
%       Obstacle = -1;
%   end
%-------------------------------------------------------------------------
%   Obstacle = sin(pi*(x(1)+1/4))-0.5;
return
```

Function for plotting of the surface on triangular mesh

```
function show(pic_no,triangles,coordinates,u)
figure(pic_no);
trisurf(triangles',coordinates(1,:),coordinates(2,:),full(u)','facecolor','interp'
colorbar; view([-42 40]);
%         zlim([-10,0]);
return
```


Approximate solution u
0
-1
-2
-3
-4
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1
0
-0.5
-1
-1.5
-2
-2.5
-3


Lagrange multipliers
1400
1200
1000
800
600
400
200
0
-200
-1
-0.5
0
0.5
1
1200
1000
800
600
400
200
0

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