# Stellar System Creator

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# CONTENTS

## CHAPTER ONE

# INTRODUCTION

The Solar System Creator is a python package that aims to ease the creation of realistic stellar systems in sci-fi settings. With minimal input, the user is able to create stars, planets, moons, asteroid regions and other celestial bodies, with accurate physical characteristics, declare their habitability, extract physical characteristics and visualize them.

# CHAPTER TWO

# CELESTIAL BODIES

Celestial bodies are different types of objects that constitute a stellar system. They can be very massive, or very light. They can be puffy or very dense. They can be bright or dim.

## 2.1 Star

A star is a celestial object that is massive enough to sustain nuclear fission in it's core. In this package we will only work with main sequence stars. Their *masses* vary between $0.08\,\mathrm{M_s}$ and $150\,\mathrm{M_s}$.

The more massive the star, the bigger it's *size*, the more *luminous* it is, and the smallest its *lifetime* will be. Ideal for *life* stars, are stars with *mass* between $0.6\,\mathrm{M_s}$ and $1.4\,\mathrm{M_s}$.

Stars can form *stellar binaries* and host *planets* and *asteroid belts*, either together or separately from their companion star (or both).

## 2.2 Planet

A planet is a celestial object that is *orbiting* around a *star* or a *binary stellar system* and is massive enough to be rounded due to its own gravity, but not massive enough to sustain nuclear fission in it's core.

In this package we will work with planets of a few distinct, but representative *composition types*, that fall under four categories: iron worlds, rocky worlds, water worlds, and ice/gas giants. Rocky and water worlds are ideal for life, either on the land, in the surface/underground ocean, or both.

Planets can host *satellites*, *trojans*, and *trojan satellites*, and orbit around *stars* or *stellar binaries*.

## 2.3 Asteroid Belt

An asteroid belt (or circumstellar disc) is is a torus, pancake or ring-shaped accumulation of matter composed of gas, dust, planetesimals, asteroids, or collision fragments in *orbit* around a *star*.

## 2.4 Satellite

A satellite is a celestial object that is *orbiting* around a *planet*. It's *composition* varies between a rocky world and water world. There is no a minimum *mass* requirement.

The more *massive* the parent *planet*, and the bigger the *distance* between child-parent, the more massive a satellite can be. If a satellite is massive enough, it can resemble a planet and can potentially sustain *life*.

## 2.5 Trojan

Trojans are multiple small celestial objects that are *orbiting* around a *star* while locked in the L4 or L5 *Lagrange point* of a *planet*. Their *composition* varies between a rocky world and water world. There is no a minimum *mass* requirement.

The more *massive* the parent *planet*, the higher the total *massive* trojan can be.

## 2.6 Trojan Satellite

A trojan satellite is a celestial object that is *orbiting* around a *star* while locked in the L4 or L5 *Lagrange point* of a *planet*. Its *composition* varies between a rocky world and water world. There is no a minimum *mass* requirement.

The more *massive* the parent *planet*, the more massive a trojan satellite can be. If a trojan satellite is massive enough, it can resemble a planet and can potentially sustain *life*.

## 2.7 Ring

An ring is is a torus, pancake or ring-shaped accumulation of matter composed of gas, dust, asteroids, or collision fragments in *orbit* around a *planet*.

It is not yet implemented in this package. Soon though!

CHAPTER

THREE

# CELESTIAL SYSTEMS

Celestial systems are systems that include one or more celestial bodies, and can be part of other celestial systems.

## 3.1 Binary System

A binary system is a system of two *celestial objects* that orbit around a common center.

In this package, we only implement stellar binary systems. Planetary binary systems are coming soon though!

There are two main types of binary systems:

1. P-type or close binaries. Their distance is small enough so that other bodies can orbit around both of them
2. S-type or wide binaries. Their distance is big enough so that other bodies can orbit each object individually.

In some cases a P-type system can also be an S-type system, meaning that objects orbit a) around both hosts, b) around one host.

Binary systems can also also be part of other binary systems. It is wise to not put too many binaries within binaries, since the orbits become highly unstable.

## 3.2 Stellar System

A stellar system consists of a parent/host *star* or a *stellar binary* and a small number of *planetary systems* and *asteroid belts* as children.

## 3.3 Planetary System

A planetary system consists of a parent/host *planet* or a *planetary binary* and a number of *satellites* and *rings* as children. They can also host *trojans* and *trojan satellites* on their *Lagrange positions* L4 and L5.

CHAPTER

FOUR

# QUANTITIES

Here, we will explore the various physical quantities found in this package.

## 4.1 Material

### 4.1.1 Mass

Mass is the quantity of mater in a physical body. In the context of this package, mass determines most of other physical characteristics, like *radius*, *luminosity*, *spin period* and *lifetime*.

Suggested (approximate) masses:

1. For moon-like satellites, less than 0.05 earth masses (Me)
2. For rocky planets: up to around 5 earth masses
3. For ice-giants: between 5 and 100 earth masses
4. For gas-giants: between 100 earth masses and 10 jupiter masses (Mj)
5. For long-lived, red stars: 0.081 and 0.5 solar masses (Ms)
6. For habitable stars: 0.6 to 1.4 solar masses
7. For short-live, big blue stars: 1.4 to 50 solar masses.

### 4.1.2 Density

Density ($\rho = \frac{M}{V}$) is the *mass* per unit *volume* of an a substance (or celestial object). Usual densities in the solar system are between 0.5 and 7 grams/cm^3.

### 4.1.3 Composition Type

The composition type of planets, planetoids, asteroids etc. is what the approximate composition of a celestial object will be. There are two types of iron worlds, two types of rocky worlds, four types of water worlds, one type of ice- and one of gas-giants.

1. Iron worlds (e.g. Mercury) are *small*, very *dense* and can sustain limited to no *tectonic activity*, which is essential to *life*.
2. Rocky worlds (e.g. Venus, Earth, Mars) are on average a bit *bigger*, *dense* and can sustain substantial *tectonic activity*.

3. Water worlds (e.g. Gliese 1214b) are of similar *size* or a bit bigger compared to rocky worlds, *less dense*, and depending on the amount of rock in their mantle, they may sustain *tectonic activity*.

4. Ice giants (e.g. Neptune, Uranus) are *bigger*, *puffy* and can not sustain any *tectonic activity*.

5. Gas giants (e.g. Saturn, Jupiter) are the *biggest* of planets, very *puffy* and can not sustain any *tectonic activity*.

I find that the *density* and *radius* of asteroids and moons that are not rocky, can be generally approximated by the four different water worlds, even though the *chemical composition* is not accurate by itself. Same for small gassy worlds (like Pluto).

### 4.1.4 Chemical Composition

Chemical composition is the ratio of different chemical compounds that constitute a substance. There are 3 main substances that are portraited in this package. That does not mean that there can not be other, it is just what the planetary *radius* models represent.

The main chemical compounds are iron (Fe), rock (MgSiO3), water (H2O), helium (He), Hydrogen (H2) and methane (CH4).

## 4.2 Geometric

### 4.2.1 Radius

Radius is the variable that defines the size of celestial objects. The radius determines the *circumference*, *surface area*, *volume* and *density*. among other characteristics. The suggested radius is determined by the *mass* of the object via various radius models. Use values $\pm 8$ % around the suggested value.

Models used:

1. For planetary models, see https://arxiv.org/pdf/0707.2895.pdf.

2. For hot gas-giant models, see https://arxiv.org/pdf/1804.03075.pdf.

3. For star models, see https://academic.oup.com/mnras/article/479/4/5491/5056185.

### 4.2.2 Circumference

The circumference is determined by the radius $C = 2\pi r$.

### 4.2.3 Surface Area

The surface area is determined by the radius $A = 4\pi r^2$.

### 4.2.4 Volume

The volume is determined by the radius $V = \frac{4\pi}{3}r^3$.

## 4.3 Rotational

### 4.3.1 Spin Period

The spin period is the amount of time it takes for a celestial body to rotate around itself compared to the distant stars.

Planetary spin period is determined by the *mass* and *radius* of the celestial body. The more massive the body, the faster it rotates. If there are satellites around the planet large enough (e.g. earth-moon), there is a substantial transfer of angular momentum between the two bodies, making the planet slow down (earth spin period would have been around 16 hr).

### 4.3.2 Day Period

The day period of a child body is determined by the *spin period* and the *orbital period* around the parent body.

### 4.3.3 Axial Tilt

The axial tilt of a child body, also known as obliquity, is the angle between an object's rotational axis and its orbital axis.

As of now, it is cosmetic and does not determine any other characteristics.

## 4.4 Life

### 4.4.1 Age

The suggested age of a star is set to be half of its *lifetime*. The suggested age of any other object is determined by it's parent age.

### 4.4.2 Lifetime

The lifetime of stars is determined by its mass and its luminosity ($T = \frac{M}{L} \cdot 10$ billion years).

The lifetime of each other body is determined by the lifetime of the parent minus a hundred million years, which is roughly the amount of time it takes for planets to form around stars (or satellites to be captured). It is by no means binding.

## 4.5 Surface

### 4.5.1 Emission

#### Albedo

(Bond) albedo $A$ is the measure of reflection of incident radiation of an object. A value of $A = 0$ means the object absorbs all incident radiation. A value of $A = 1$ means the object reflects all incident radiation.

Albedo values for planets in our solar system:

1. Mercury, Mars ~0.14.
2. Earth, Uranus, Neptune ~ 0.3.
3. Jupiter, Saturn ~ 0.5.
4. Venus ~ 0.75.

More information and example values on: https://en.wikipedia.org/wiki/Albedo.

#### Emissivity

Emissivity $\epsilon$ is the measure of how much of the overall radiation on the surface of an object is emitted outside. Most planets have an emissivity close to 1 (the maximum value). Emissivity of 0 (minimum value) means that no radiation is emitted.

#### Heat Distribution

Heat distribution $\beta$ is the measure of how well a planet distributes heat, and can take values from 0 to 1. It highly depends on the rotation speed of the planet.

The faster the rotation, the closer the number is to 1 (usually 1 for all non-tidally locked planets). The heat distribution is 0.5 for a planet that is *tidally locked* to its parent.

#### Normalized Greenhouse

The normalized greenhouse effect $g$ is a measure of the greenhouse effect in the atmosphere of a planet. It takes values between 0 and 1. The closer to 1, the higher the temperature of the planet will be. When you get close to 1 (e.g. 0.99), adding a new 9 at the end (e.g. 0.999) will dramatically increase the effect.

The normalized greenhouse effect on earth is approximately 0.34.

#### Incident Flux

Incident flux is the incoming radiation flux from all major sources. The total incident flux is added from the parent, the parent of the parent etc.

The incident flux $S$ from a single source of *luminosity* $L$ at effective distance :math:'r_{rm eff}' is given by the equation: $\frac{L}{r_{\rm eff}^2}$.

For a child orbiting at *mean distance* $a$ and *eccentricity* $e$, there are two types of effective distance we care about. The first is the one that is related to the average incoming flux and is given by: $r_F = a\left(1 - e^2\right)^{1/4}$.

The second is the one that is related to the average surface temperature and is given by: $r_T \approx a(1 + \frac{1}{8}e^2 + \frac{21}{512}e^4)$.

For more information: https://arxiv.org/pdf/1702.07314.pdf (eq. 3, 17, 19).

#### Temperature

The surface temperature of an object can be defined differently for objects that produce significant radiation (stars) and ones that are mostly heated by another stellar body (planets, satellites).

For a star of *luminosity* $L$ in solar luminosities and *radius* $R$ in solar radii, the surface temperature is given by the ideal body equation: $\frac{L}{R^2} \cdot 5778$ K.

For a planet of *bond albedo* $A$, *emissivity* $\epsilon$, *heat distribution* $\beta$, *normalized greenhouse* $g$ and *incident flux* $S$, the surface temperature is given by the equation: $\left(\frac{(1-A)S}{\beta\epsilon(1-g)}\right)^{1/4} \cdot 278.5$ K.

For more information of the planetary surface temperature: https://arxiv.org/pdf/1702.07314.pdf (eq. 6 and 16)

#### Luminosity

Luminosity is the measure of the emitted radiation from an object. It is determined by the *mass* of the object, and the *incident flux* if it is relatively higher than the emitted flux of radiation.

For star of *mass* $M$ in the main sequence, the luminosity is given by various mass-luminosity relationships as in https://en.wikipedia.org/wiki/Mass-luminosity_relation. Use values $\pm 8$ % around the suggested value.

For a planet of *surface temperature* $T$, *temperature-based incident flux* $S_T$, *albedo* $A$ and *surface area* $A_S$, the luminosity is determined by: $\sigma A_S T^4 + AS_T A_S/4$, where $\sigma$ is the Stefan-Boltzmann constant.

For more information on planetary luminosity:

1. https://www.acs.org/content/acs/en/climatescience/atmosphericwarming/singlelayermodel.html
2. https://en.wikipedia.org/wiki/Planetary_equilibrium_temperature

#### Peak Wavelength

The *surface temperature* $T$ of an object, if it is an (close to) ideal black body object can be associated with a profile of emitted wavelengths, with a single maximum. For our sun, the maximum of the emission spectrum is around 500 nm (green).

In general, the peak wavelength it is given by: $\lambda = \frac{2.898 \cdot 10^6 \text{ nm K}}{T}$.

### 4.5.2 Gravity

#### Surface Gravity

Surface gravity $g$ is the gravitational acceleration experienced at the surface of an object.

Earth's surface gravity is approximately 9.81 $\mathrm{m/s^2}$.

### Escape Velocity

Escape velocity $v_{\mathrm{esc}}$ is the initial speed a small object needs to escape the gravitational pull of the celestial object it is bound to.

Earth's escape velocity is approximately 11.19 km/s.

## 4.5.3 Internal Heating

Internal heating are a variety of processes with which an object generates heat.

### Tectonic Activity

The tectonic activity label is determined by amount of total internal heating $Q_{\mathrm{tot}}$. Since each internal heating process is not estimated very accurately, take the label as a suggestion, rather than an absolute truth, and optimize however you see fit. Plate tectonics are only present on sold planets, so any planets with substantial gaseous components will not have any plate tectonics.

The labels we use are:

1. 'Not applicable', if the composition is Icegiant or Gasgiant.
2. 'Unknown', if $Q_{\mathrm{tot}} = \mathrm{nan}$.
3. 'Stagnant', if $Q_{\mathrm{tot}} < 0.01\,\mathrm{W/m^2}$.
4. 'Low', if $Q_{\mathrm{tot}} < 0.04\,\mathrm{W/m^2}$.
5. 'Medium Low', if $Q_{\mathrm{tot}} < 0.07\,\mathrm{W/m^2}$.
6. 'Medium', if $Q_{\mathrm{tot}} < 0.15\,\mathrm{W/m^2}$.
7. 'Medium High', if $Q_{\mathrm{tot}} < 0.2\,\mathrm{W/m^2}$.
8. 'High', if $Q_{\mathrm{tot}} < 0.35\,\mathrm{W/m^2}$.
9. 'Extreme', if $Q_{\mathrm{tot}} \geq 0.35\,\mathrm{W/m^2}$.

### Primordial Heating

Primordial heating $Q_{\mathrm{prim}}$ of a planet is heating that originates from the initial formation heating. It is stored in the planet as an internal heating source that slowly (or quickly) escapes, depending on the planet materials.

For a planet of *mass* $M$, *surface area* $S$, *age* $T$, and initial heating $H_o$, the primordial heating is given by: $Q_{\mathrm{prim}} = H_o \mathrm{e}^{-\lambda T} M/S$, where $\lambda$ is a decay constant.

For this package we use a constant $H_o = 1.2 \cdot 10^{-11}$ Watts/kg and $\lambda = \frac{0.3391}{\text{billion years}}$.

More information on: https://www.sciencedirect.com/science/article/abs/pii/S003206331300161X Eq. 2.1 $\cdot M/S$

### Radiogenic Heating

Radiogenic heating $Q_\mathrm{rad}$ is the heating produced by slowly radiative isotopes in a planets mantle. Since we only care for the mantle, we take into account the planetary composition (we assume that only the rocky part of the planet is contributing to radiogenic heating). This model assumes the same percentages of radioactive isotopes as earth, although these may vary from planet to planet and stellar system to stellar system, depending on the age of the galaxy and local isotope abundances.

For a radiogenic isotope of number $k$ with heating production $H_k$, initial abundance $n_k(0)$, lifetime $\tau_k$, the heating heating produced at a certain time (*age*) $t$ is given by: $H_k(t) = H_k n_k(0)\mathrm{e}^{-t/\tau}$.

For a planet of *mass* $M$, *surface area* $S$, *age* $T$, and rocky mantle percentage $p_\mathrm{rocky}$ (see *chemical composition* and *composition type*), the total heating is given by: $Q_\mathrm{rad} = \sum_k \left(H_k(T)\right) p_\mathrm{rocky} M/S$.

More information on: https://www.sciencedirect.com/science/article/abs/pii/S0019103514004473?via%3Dihub#b0415%20. See table 1 for heat production and half-times, and table 2 for relative abundance.

### Tidal Heating

When an child object of *radius* $r$ rotates around a parent object of *mass* $M$ in an non-circular (eccentric) orbit, at a *mean distance* $a$ and *eccentricity* $e$, the shape of the child object can periodically change due to the variation of gravitational pull it feels from the parent object (it wobbles). This motion results in internal heating.

Tidal heating is proportional to: $Q_\mathrm{tidal} \propto M^{2.5} r^5 e^2 a^{-7.5}$

More information on:

1. https://academic.oup.com/mnras/article/391/1/237/1121115
2. https://www.liebertpub.com/doi/10.1089/ast.2015.1325 Eq.2
3. https://iopscience.iop.org/article/10.1088/0004-637X/789/1/30/pdf table 2, pg. 22

## 4.5.4 Induced Tide

Induced tides $h_\mathrm{tides}$ are the height differences that occur due to tidal forces on a planet's massive ocean's water level. The values provided are very crued and are only meant as a suggestion to the user. Tide height depends on many local parameters, as explained in artifexian's video.

For an object of *mass* $m$ and *radius* $r$ that interacts with a companion object of *mass* $m_c$ from a *mean distance* $a$ with orbital *eccentricity* $e$, will experience a maximum tide height of: $h_\mathrm{tides} = 3\frac{m_c}{m}\left(\frac{r}{a(1-e)}\right)^3 \frac{r}{2}$.

More information on the model used can be found on https://www.cambridge.org/resources/0521846560/7708_Tidal%20distortion.pdf (Eq. 20) where mean distance is replaced by the periapsis (which yields the maximum observed tides). A mistake must be noted: the example below eq. 20 needs $m_1$ to be the earth mass and $m_2$ the moon mass.

## 4.5.5 Angular Diameter

Angular diameter $\delta$ is the size of a celestial body in the sky. The angular diameter of the sun and the moon are similar on earth, and approximately ~ 0.5 degrees

The angular diameter of a body with *radius* $r$ at a distance $D$ is given by: $\delta = 2\arcsin(r/D)$

More information on: https://en.wikipedia.org/wiki/Angular_diameter

## 4.6 Orbital

An orbit is a curved trajectory of an object. This trajectory can either be circular, or elliptical.

### 4.6.1 Eccentricity

Eccentricity $e$ determines how elliptic the orbit of a child around a parent body is. $e = 0$ means that the orbit is circular, and $e = 1$ means that the orbit resembles a line (not a stable orbit).

The suggested eccentricity is $e = 0$ for single *star* systems. For *binary systems*, the suggested eccentricity depends on the eccentricity of the binary $e_c$, the *semi-major axis* $a_c$ of the child, the *mean distance* $a_b$ between the two binary system objects, and the secondary to total *mass* ratio $\mu$ of the binary. Small variations of the eccentricity (e.g. $\pm 0.05$) are suggested for a more realistic *orbit*.

For S-type *binary systems*, the suggested eccentricity is given by: $e_S = \frac{5}{4}\frac{a_c}{a_b}\frac{e_b}{1-e_b^2}$.

For P-type *binary systems*, the suggested eccentricity is given by: $e_P = \frac{5}{4}\frac{a_b}{a_c}(1-2\mu)\frac{4e_b+3e_b^3}{4+6e_b^2}$.

### 4.6.2 Semi-Major Axis

Semi-major axis $a$ is the mean distance between a child and a parent body.

When placing two objects in the same *celestial system* with semi-major axes $a_1$ and $a_2$, try to space them out in such a way that $a_2 = (1.4\,\text{to}\,2) \cdot a1$. Maintain this distance ratio for all new objects. Also, make sure to stay within the *minimum* and *maximum* limits since the program will not stop the user from exceeding the limits.

### 4.6.3 Semi-Minor Axis

Semi-minor axis $b$ is determined by the *semi-major axis* $a$ and the *eccentricity* $e$: $b = a\sqrt{1-e^2}$.

### 4.6.4 Apoapsis

Apoapsis ($a(1+e)$) is the furthest distance between a child and a parent body. It is determined by the *semi-major axis* $a$ and the *eccentricity* $e$.

### 4.6.5 Periapsis

Periapsis ($a(1-e)$) is the nearest distance between a child and a parent body. It is determined by the *semi-major axis* $a$ and the *eccentricity* $e$.

### 4.6.6 Lagrange Position

Lagrange positions/points (L#) are the equilibrium points between two *masses* (grandparent $m_1$ and parent $m_2$) that can potentially host an object of small mass $m_3$. There are 5 such points, three (L1, L2, L3) of which are unstable and two that are semi-stable (L4, L5). Points L4 and L5 can host from multiple asteroid-type bodies to a small planet, depending on $m_2$.

The rule of thumb is that the total *mass* of objects in these points must be smaller than $0.0018878 \cdot m_2$ or smaller than the Gascheu's limit. It must be noted that, since the three bodies are never just by themselves, there are other orbital instabilities introduced that further degrade the stability of the Lagrange points.

More information about the limits on:

1. https://hal.archives-ouvertes.fr/hal-00552502/document (For a discussion on Gascheu's limit)
2. https://www.aanda.org/articles/aa/abs/2007/07/aa6582-06/aa6582-06.html (For simple $0.0018878 \cdot m_2$ limit)

### 4.6.7 Contact

In this context, contact between two binary stars happens if the radius of at least one of the two stars resides outside the *Roche lobe* while the stars are in *periapsis*.

### 4.6.8 Roche Lobe

The Roche lobe is the region around a star in a binary system within which orbiting material is gravitationally bound to that star.

### 4.6.9 Orbital Period

Orbital period ($\sqrt{\frac{a^3}{M_{tot}}}$) is the time it takes for a child body to orbit around their parent. It is given by the *semi-major axis* $a$, and the total mass $M_{tot}$ of child and parent objects.

### 4.6.10 Orbital Velocity

Orbital velocity ($\sqrt{\frac{M_{tot}}{a}}$) is the mean speed at which the child body travels around the parent body. It is given by the *semi-major axis* $a$, and the total mass $M_{tot}$ of child and parent objects.

### 4.6.11 Orbit Type

Orbital type of a child object can be either prograde or retrograde - orbiting along or against the rotation of the parent.

### 4.6.12 Orbit Type Factor

Orbit type factor determines the *semi-major axis maximum limit*. It depends on the *orbit type* and is determined by the parent *eccentricity* $e_p$ and the child *eccentricity* $e_c$.

1. For a prograde orbit, it is given by $0.4895(1 - 1.0305e_p - 0.2738e_c)$.
2. For a retrograde orbit, it is given by $0.9309(1 - 1.0764e_p - 0.9812e_c)$.

### 4.6.13 Orbital Stability

Orbital stability demonstrates if the orbit of the child object around the parent object does not exceed any limits. More specifically, for a stable orbit we must have:

1. *Periapsis* > *roche limit*.
2. *Semi-major axis* > *p-type critical orbit*.
3. *Apoapsis* < *semi-major axis maximum limit*.
4. Optional: *Semi-major axis* > *parent inner orbit limit*.
5. For rock worlds: *Semi-major axis* > *parent outer rock formation limit*.

### 4.6.14 Inclination

Orbital inclination is the tilt of a child object's orbit around a celestial body. It varies between 0 and 180. Between 0 and 90, the orbit is prograde. Between 90 and 180, the orbit is retrograde.

However, as of now, it is cosmetic and does not determine any other characteristics, including the *orbit type*.

### 4.6.15 Argument of Periapsis

Check out https://en.wikipedia.org/wiki/Argument_of_periapsis.

The argument of periapsis, as of now, it is cosmetic and does not determine any other characteristics.

### 4.6.16 Longitude of the Ascending Node

Check out https://en.wikipedia.org/wiki/Longitude_of_the_ascending_node.

The longitude of the ascending node, as of now, it is cosmetic and does not determine any other characteristics.

### 4.6.17 Semi-Major Axis Minimum Limit

The semi-major axis minimum limit is the *roche limit* of the parent for the specific child *density*, or the *p-type critical limit* in binary systems.

### 4.6.18 Semi-Major Axis Maximum Limit

The semi-major axis maximum limit is the *hill_sphere* multiplied by the *orbit type factor*.

## 4.7 Children Orbit Limits

### 4.7.1 Tidal Locking Radius

The tidal locking radius is the furthest distance a child would be tidally locked to its parent. Tidal locking means that an orbiting object around a parent turns around itself at the same amount of time it turns around its parent. One side of the planet is always faces the parent, while the other side does not.

The tidal locking radius is larger the more *massive* the parent is, and the more *old* it is. Many parameters of the child object determine the tidal locking radius, but they are hard to estimate, so treat this this number as a suggestion rather than a hard limit.

More info on:

1. https://en.wikipedia.org/wiki/Tidal_locking#Timescale,
2. https://physics.stackexchange.com/questions/12541/tidal-lock-radius-in-habitable-zones
3. https://www.sciencedirect.com/science/article/abs/pii/S0019103583710109 (eq. 10 in CGS units)

### 4.7.2 Roche Limit

Roche limit is the minimum distance $d_{Roche}$ a child object of *mass* $m$, *radius* $r$ and *density* $\rho_m$ can orbit a parent object of *mass* $M$, *radius* $R$ and *density* $\rho_M$ before the child object breaks apart into pieces. An additional condition is that the child object must only be held together by its own gravity.

It is given the equation: $d_{Roche} = 2.44r\left(\frac{M}{m}\right)^{1/3}$ or $d_{Roche} = 2.44R\left(\frac{\rho_M}{\rho_m}\right)^{1/3}$.

### 4.7.3 Dense Roche Limit

Dense Roche limit is the *Roche limit* of a child of unknown *radius* and a parent object of *radius* $R$ *density* 10 times bigger than the child *density*.

It is approximately: $5.2568R$.

### 4.7.4 P-type binary Critical Orbit

In close binary systems (p-type binaries) of two stars with *masses* $m_1$ and $m_2$ with $m_1 > m_2$, *mean distance* $d$ and *eccentricity* $e$, the critical orbit limit is the *minimum* distance at which an orbit stays stable.

It is estimated by: $d(1.6 + 5.1e - 2.22e^2 + 4.12\mu - 5.09\mu^2 + 4.61e^2\mu^2 - 4.27e\mu)$, where $\mu = \frac{m_2}{m_1+m_2}$.

More info on: https://arxiv.org/pdf/2108.07815.pdf (eq. 3).

### 4.7.5 Inner Orbit Limit

The inner orbit limit of a parent for its children is the highest of *dense Roche limit* and *P-type critical orbit*. It is provided to assist the user determine the worst possible inner orbit limit.

### 4.7.6 Hill Sphere

Hill sphere is the maximum distance ($r_H$) an object (child) can affect even smaller body (grandchild), because of the presence of a bigger body around (parent). For example the moon is within the hill sphere of earth, and the hill sphere of earth is determined by earth's *mass* $m$ *semi-major axis* $a$, and *eccentricity* $e$, and the sun's *mass* $M$.

It is given by the equation: $r_H = a(1-e)\left(\frac{m}{3M}\right)^{1/3}$.

If we looking at a single object that is part of a binary system, a more accurate determination of the Hill sphere is given by the *Roche lobe*.

### 4.7.7 Roche Lobe

Roche lobe (or sphere) (not to be confused with *Roche limit*) is the region around a star in a binary system within which orbiting material is gravitationally bound to that star.

For an object of *mass* $m$ in a binary system, and a companion object of *mass* $m_c$, with a binary *mean distance* $d$ and *eccentricity* $e$, the roche lobe is approximated by: $d(1-e)\frac{0.49\mu^{2/3}}{0.6\mu^{2/3}+\ln\left(1+\mu^{1/3}\right)}$, where $\mu = \frac{m}{m_c}$.

### 4.7.8 S-type binary Critical Orbit

In wide binary systems (S-type binaries) of two objects with *masses* $m_1$ and $m_2$ with $m_1 > m_2$, *mean distance* $d$ and *eccentricity* $e$, the critical orbit limit is the *maximum* distance at which an orbit stays stable around a single object. This distance is different for each object and changes due to their mass difference.

For the primary star of *mass* $m_1$, $\mu$ is determined by the companions to the binary's mass ratio $\mu = \frac{m_2}{m_1+m_2}$.

For the secondary star of *mass* $m_2$, $\mu$ is determined by the companions to the binary's mass ratio $\mu = \frac{m_1}{m_1+m_2}$.

The critical distance is estimated by: $d(0.464 - 0.38\mu - 0.631e + 0.586\mu e + 0.15e^2 - 0.198\mu e^2)$.

More info on: https://arxiv.org/pdf/2108.07815.pdf (eq. 1).

### 4.7.9 Outer Orbit Limit

The outer orbit limit of a parent for its children is the lowest of *Hill Sphere* and *S-type critical orbit*. It is provided to assist the user determine the worst possible outer orbit limit.

### 4.7.10 Inner Rock Formation Limit

We use *Selsis insolation model* since it allows for easy, Solar system comparisons. Rock line is the distance at which iron and rock can form clusters, planetesimals and eventually planets. Since the rock line is determined by when rock and iron are more or less solid, I decided to use the boiling point of a fast rotating iron ball (*heating distribution* $\beta$ 1, *albedo* $A$ 0.15) @ $T = 2870$ K. for the optimistic inner rock line limit, giving a value of $\approx 0.087$ A.U for our sun.

Distance estimation: $\left(\frac{T}{T_{\text{eff}}}\right)^2 \sqrt{\frac{1-A}{\beta}}$, with $T_{\text{eff}} = 278.5$ K (https://arxiv.org/pdf/1702.07314.pdf eq. 6 with $L = 1$).

### 4.7.11 Outer Rock Formation Limit

We use *Selsis insolation model* since it allows for easy, Solar system comparisons. Rock line is the distance at which iron and rock can form clusters, planetesimals and eventually planets. Since the rock line is determined by when rock and iron are more or less solid, I decided to use the melting point of a slow rotating (*tidally locked*) rock ball (*heating distribution* $\beta$ 0.5, *albedo* $A$ 0.85) @ $T = 600$ K to find the equivalent solar system distance and multiply by 5/3.1 (similar to the *early solar system water frost line*) (lowest temperature from http://hyperphysics.phy-astr.gsu.edu/hbase/Geophys/meltrock.html) for the optimistic outer rock line limit, giving a value of $\approx 0.281$ A.U for our sun.

Distance estimation: $\frac{5}{3.1}\left(\frac{T}{T_{\text{eff}}}\right)^2 \sqrt{\frac{1-A}{\beta}}$, with $T_{\text{eff}} = 278.5$ K (https://arxiv.org/pdf/1702.07314.pdf eq. 6 with $L = 1$).

### 4.7.12 Inner Water Frost Limit

We use *Selsis insolation model* since it allows for easy, Solar system comparisons. As shown on the wikipedia page https://en.wikipedia.org/wiki/Frost_line_(astrophysics), there are different frost lines for different compounds. Water is important and seems to determine the line between gas planets and rocky planets.

It must be noted that gas giants can migrate to inner orbits after their creation, so it is not impossible to find one where venus or mercury would have been. It would however probably destabilize any other planets in its path.

The inner limit is taken from wiki's suggestion for big sized bodies at ~1.94 AU (~200 K).

### 4.7.13 Sol-Equivalent Water Frost Limit

We use *Selsis insolation model* since it allows for easy, Solar system comparisons. As shown on the wikipedia page https://en.wikipedia.org/wiki/Frost_line_(astrophysics), there are different frost lines for different compounds. Water is important and seems to determine the line between gas planets and rocky planets.

It must be noted that gas giants can migrate to inner orbits after their creation, so it is not impossible to find one where venus or mercury would have been. It would however probably destabilize any other planets in its path.

The Sol equivalent limit is from the average of the newest finds ~ 3.1 AU (~ 158.2 K).

### 4.7.14 Outer Water Frost Limit

We use *Selsis insolation model* since it allows for easy, Solar system comparisons. As shown on the wikipedia page https://en.wikipedia.org/wiki/Frost_line_(astrophysics), there are different frost lines for different compounds. Water is important and seems to determine the line between gas planets and rocky planets.

It must be noted that gas giants can migrate to inner orbits after their creation, so it is not impossible to find one where venus or mercury would have been. It would however probably destabilize any other planets in its path.

The outer limit is taken from wiki's mention on the early-days frost line at 5 AU (~124.5 K).

## 4.8 Insolation Models

Insolation or effective stellar flux is the effective flux that reaches a specific *orbital* distance, called threshold (or limit). Insolation changes with the *star's temperature*, as well as the environmental conditions of the target *habitable* world. We use insolation for a specific climate to normalize the *luminosity* of a *star's*, and try to estimate the threshold at which distance from the *star's* the aforementioned environmental conditions occur. By using extreme environmental conditions that could potentially support life, we can determine minima and maxima for *zones of habitability* around single- or multi-star *systems*.

There are different types of insolation models. In this package, we are using one that was designed by *Kopparapu*, and one that was designed by *Selsis*.

These models have multiple different thresholds, from which we only use a handful that are representative of the *habitability limits*. These are designated as *earth equivalent*, *conservative* or *relaxed* and *minimum* (inner) or *maximum* (outer).

### 4.8.1 Kopparapu

Kopparapu's insolation model is an elegant model that describes seven disparate thresholds of interest. It's main asset is the distinction of thresholds between different planetary masses for the runaway greenhouse effect.

The provided thresholds are:

1. Recent Venus: *inner*, *relaxed minimum limit*.
2. Runaway Greenhouse Effect, Subterran: *inner*.
3. Runaway Greenhouse Effect, Terran: *inner*, *conservative minimum limit*.
4. Runaway Greenhouse Effect, Superterran: *inner*.
5. Moist Greenhouse Effect: *inner*, *earth equivalent limit*.
6. Maximum Greenhouse Effect: *outer*, *conservative maximum limit*.
7. Early Mars: *outer*, *relaxed maximum limit*.

Sources: Kopparapu. et al. 2013 [1], 2014 [2], Wang and Cuntz 2019 [3]

1. https://iopscience.iop.org/article/10.1088/0004-637X/765/2/131
2. https://iopscience.iop.org/article/10.1088/2041-8205/787/2/L29
3. https://iopscience.iop.org/article/10.3847/1538-4357/ab0377 (overview of this and other models)

### 4.8.2 Selsis

Selsis' insolation model is a simple model that describes a multitude of disparate thresholds of interest. It's main assets are the distinction cloudy and non-cloudy greenhouse effect-based thresholds, the provision of very relaxed thresholds, and the ability to make once own thresholds. This last part is important in determining the *inner rock formation limit*, the *outer rock formation limit*, the *inner water frost limit*, the *sol-equivalent water frost limit*, and the *outer water frost limit*.

The provided thresholds are:

1. **Planet-based**
   a. Recent Venus: *inner*.
   b. Earth Equivalent': *inner*, *earth equivalent limit*.
   c. Early Mars: *outer*.
2. **0% Clouds**
   a. Runaway Greenhouse Effect, 0% Clouds: *inner*, *conservative minimum limit*.
   b. Start of water loss, 0% Clouds: *inner*.
   c. First C02 Condensation, 0% Clouds: *outer*.
   d. Maximum Greenhouse Effect, 0% Clouds: *outer*, *conservative maximum limit*.
3. **50% Clouds**
   a. Runaway Greenhouse Effect, 50% Clouds: *inner*.
   b. Start of water loss, 50% Clouds: *inner*.
   c. Maximum Greenhouse Effect, 50% Clouds: *outer*.
4. **100% Clouds**
   a. Runaway Greenhouse Effect, 100% Clouds: *inner*, *relaxed minimum limit*.
   b. Start of water loss, 100% Clouds: *inner*.
   c. Maximum Greenhouse Effect, 100% Clouds: *outer*, *relaxed maximum limit*.

Sources: Selsis. et al. 2007 [1], Wang and Cuntz 2019 [2] 1. https://www.aanda.org/articles/aa/pdf/2007/48 2. https://iopscience.iop.org/article/10.3847/1538-4357/ab0377 (overview of this and others models)

### 4.8.3 Relaxed Minimum Limit

Relaxed minimum limit is the smallest inner limit of an *insolation model*.

### 4.8.4 Relaxed Maximum Limit

Relaxed maximum limit is the furthest outer limit of an *insolation model*.

### 4.8.5 Conservative Minimum Limit

Conservative minimum limit is the furthest inner limit of an *insolation model*.

### 4.8.6 Conservative Maximum Limit

Conservative maximum limit is the smallest outer limit of an *insolation model*.

### 4.8.7 Earth-Equivalent Limit

Earth-equivalent limit is the limit that closely matches the distance of the earth from the sun of an *insolation model*.

## 4.9 Habitability

Habitability is a label given to stars or planets that satisfy certain criteria. These criteria are different for (*binary*) *stars*, *planets* and *satellites* and *trojan satellites*.

### 4.9.1 Habitable Zones

Habitable zone (HZ) is defined as an area between two orbital distances from a parent star or binary system that life can exist. The extremes of the zone are probably not as hospitable to complex organisms like ourselves, but you never know…

Depending on the system type, the habitable zone is estimated in different ways. They always depend on the *luminosity* and *temperature* of the star(s), and often on the orbital *eccentricity* of the potential habitable world. The single star habitable zone or *SSHZ* is the simplest type of HZ. For binary stars, there are three different types, the radiative habitable zone or *RHZ*, the permanent habitable zone or *PHZ*, the average habitable zone or *AHZ*.

#### Single Star Habitable Zone

The Single Star Habitable Zone (SSHZ) is the simplest type of habitable zone.

For a single *star* of *luminosity* $L$ and *temperature* $T$, the habitable thresholds $r_x$ (where $x = \text{inner}, \text{outer}$) with insolation $S_x(T)$ are estimated by: $r_x = \sqrt{\frac{L}{S_x(T)}}$.

#### Radiative Habitable Zone

The Radiative Habitable Zone (SSHZ) is the simplest type of habitable zone in a *binary system*. It does *not* take into account the forced and periodically changing *eccentricity* of the *planets* due to the asymmetry the two *stars* of different *mass* introduce to the *stellar system*.

Let us assume a *binary system* with *stars* $s = A, B$ of *luminosities* $L_s$ and *temperatures* $T_s$. They are separated by a mean *distance* $D$ and orbit around a common center with *eccentricity* $e$. We aim to calculate the *radiative* habitable thresholds with insolation $S_{s,x}(T_s)$ (where $x = \text{I, O}$). For convenience, we define the normalized luminosity: $\tilde{L}_{s,x} = \frac{L_s}{S_{s,x}(T_s)}$, the *SSHZ* $r_{s,x} = \sqrt{\tilde{L}_{s,x}}$, the double-star equivalent $r_{\text{AB,x}} = \sqrt{\tilde{L}_{A,x} + \tilde{L}_{B,x}}$, and the half mean distance $b = D/2$.

The S-type RHZ for *stars* $A$, namely $r_{A,x}^{\text{S-type}}$, thresholds are estimated by:

1. Inner: $r_{\mathrm{A,I}}^{\mathrm{S-type}} = r_{\mathrm{A,I}} \left(1 + \frac{\tilde{L}_{B,\mathrm{I}}}{(D - r_{\mathrm{A,I}})^2}\right)$.
2. Outer: $r_{\mathrm{A,O}}^{\mathrm{S-type}} = r_{\mathrm{A,O}} \left(1 + \frac{\tilde{L}_{B,\mathrm{O}}}{(D + r_{\mathrm{A,O}})^2}\right)$.

The P-type RHZ, namely $r_{\mathrm{AB,x}}^{\mathrm{P-type}}$, thresholds are estimated by:

1. Inner: $r_{\mathrm{AB,I}}^{\mathrm{P-type}} = \sqrt{\tilde{L}_{A,\mathrm{I}} \frac{r_{\mathrm{AB,I}} + b}{r_{\mathrm{AB,I}} - b} + \tilde{L}_{B,\mathrm{I}} \frac{r_{\mathrm{AB,I}} - b}{r_{\mathrm{AB,I}} + b} - b^2}$
2. Outer: $r_{\mathrm{AB,O}}^{\mathrm{P-type}} = \sqrt{\tilde{L}_{A,\mathrm{O}} \frac{r_{\mathrm{AB,O}} - b}{r_{\mathrm{AB,O}} + b} + \tilde{L}_{B,\mathrm{O}} \frac{r_{\mathrm{AB,O}} + b}{r_{\mathrm{AB,O}} - b} - b^2}$

#### Permanent Habitable Zone

The Permanent Habitable Zone (PHZ) is a complex type of habitable zone in a *binary system*. It takes into account the forced and periodically changing *eccentricity* of the *planets* due to the asymmetry the two *stars* of different *mass* introduce to the *stellar system*. It assumes that the climate of the potential habitable world is fast at immediately adjusting to the periodically changing *eccentricity*.

#### Average Habitable Zone

The Average Habitable Zone (AHZ) is a complex type of habitable zone in a *binary system*. It takes into account the forced and periodically changing *eccentricity* of the *planets* due to the asymmetry the two *stars* of different *mass* introduce to the *stellar system*. It assumes that the climate of the potential habitable world is slow at adjusting to the periodically changing *eccentricity*.

### 4.9.2 Single Star Habitability

For a single *star*, the habitability is determined by the following conditions:

1. There must be a viable *SSHZ* given by the *relaxed minimum limit* and the *relaxed maximum limit*.
2. The *relaxed maximum limit* must not be smaller than the *inner orbit limit*.
3. The *relaxed minimum limit* must not be larger than the *outer orbit limit*.
4. The *lifetime* or *age* must be bigger than 1 billion years. This minimum age limit is when life is expected to start developing (see here).

These are the most relaxed criteria, so keep in mind that habitability varies from object to object and the type of life that can survive on them.

### 4.9.3 Binary S-type Habitability

For a *star* in an S-type *binary system*, the habitability conditions are the same as the *single star habitability conditions* but the *SSHZ* is replaced with *AHZ*. For convenience, the conditions are listed here:

1. There must be a viable *AHZ* given by the *relaxed minimum limit* and the *relaxed maximum limit*.
2. The *relaxed maximum limit* must not be smaller than the *inner orbit limit*.
3. The *relaxed minimum limit* must not be larger than the *outer orbit limit*.
4. The *lifetime* or *age* must be bigger than 1 billion years. This minimum age limit is when life is expected to start developing (see here).

These are the most relaxed criteria, so keep in mind that habitability varies from object to object and the type of life that can survive on them.

### 4.9.4 Binary P-type Habitability

For a P-type *stellar binary*, the habitability is determined by the following conditions:

1. There must be a viable *P-Type AHZ* given by the *relaxed minimum limit* and the *relaxed maximum limit*.
2. The *relaxed maximum limit* must not be smaller than the *inner orbit limit*.
3. The *relaxed minimum limit* must not be larger than the *outer orbit limit*.
4. The *lifetime* or *age* of the primary *star* must be bigger than 1 billion years. This minimum age limit is when life is expected to start developing (see here).

These are the most relaxed criteria, so keep in mind that habitability varies from object to object and the type of life that can survive on them.

### 4.9.5 Planet Habitability

For a *planet*, the habitability is determined by the following conditions:

1. The *mass* must be less than $5\,\mathrm{M_e}$.
2. The *mass* must be more than $0.0268\,\mathrm{M_e}$ for water worlds and more than $0.1\,\mathrm{M_e}$ for other *composition types*.
3. The *radius* must be between $0.5\,\mathrm{R_e}$ and $1.5\,\mathrm{R_e}$.
4. The *planet's* parent must have a viable relaxed *HZ*.
5. The *semi-major axis* must be within the relaxed *HZ* of the parent.
6. The orbit must be *stable*.
7. The *tectonic activity* must be labeled as "Medium Low", "Medium" or "Medium High".

### 4.9.6 Satellite Habitability

For a *satellite*, the habitability conditions are the same as the *planetary habitability conditions* but the the reference to the *HZ* of the parent, must be changed to the grandparent. For convenience, the conditions are listed here:

1. The *mass* must be less than $5\,\mathrm{M_e}$.
2. The *mass* must be more than $0.0268\,\mathrm{M_e}$ for water worlds and more than $0.1\,\mathrm{M_e}$ for other *composition types*.
3. The *radius* must be between $0.5\,\mathrm{R_e}$ and $1.5\,\mathrm{R_e}$.
4. The *satellite's* grandparent must have a viable relaxed *HZ*.
5. The *semi-major axis* of the *satellite's* parent must be within the relaxed *HZ* of the *satellite's* grandparent.
6. The orbit must be *stable*.
7. The *tectonic activity* must be labeled as "Medium Low", "Medium" or "Medium High".

### 4.9.7 Trojan Satellite Habitability

For a *trojan satellite*, the habitability conditions are the same as the *planetary habitability conditions* but the the reference to the *HZ* of the parent, must be changed to the grandparent. For convenience, the conditions are listed here:

1. The *mass* must be less than $5\,\mathrm{M_e}$.
2. The *mass* must be more than $0.0268\,\mathrm{M_e}$ for water worlds and more than $0.1\,\mathrm{M_e}$ for other *composition types*.
3. The *radius* must be between $0.5\,\mathrm{R_e}$ and $1.5\,\mathrm{R_e}$.
4. The *trojan satellite's* grandparent must have a viable relaxed *HZ*.
5. The *semi-major axis* must be within the relaxed *HZ* of the grandparent.
6. The orbit of the parent must be *stable*.
7. The *tectonic activity* must be labeled as "Medium Low", "Medium" or "Medium High".