# On Out-of-distribution Detection with Energy-based Models Sven Elflein, Bertrand Charpentier, Daniel Zügner, Stephan Günnemann









## TL;DR

- EBMs do **not** strictly outperform Normalizing Flows across multiple training methods.
- Semantic features induced by supervision improves OOD detection in recent discriminative EBMs [2].
- Architectural modifications can also be used to improve OOD detection with EBMs.

# What is an energy-based model?

EBM defines a probability distribution over the data  $\mathbf{x} \in \mathbb{R}^D$  through the energy function  $E_{ heta}$  as

$$p_{\theta}(\mathbf{x}) = \frac{\exp(-E_{\theta}(\mathbf{x}))}{Z(\theta)} \tag{1}$$

where  $Z(\theta) = \int \exp(-E_{\theta}(\mathbf{x}))d\mathbf{x}$  is the normalizing constant.

## **Properties:**

- ✓ Flexible transformations
- Exact density evaluation
- ✓ Dimensionality reduction
- X No direct maximum likelihood training

## Why study EBMs for OOD detection?

- Recent research on generative models for OOD detection focuses on exact likelihood methods, e.g., Normalizing Flows
- → We compare behavior for EBMs vs. Normalizing Flows
- EBMs show good OOD detection capabilities [2], however, no detailed analysis has been done
- → We consider EBMs trained with different approaches
- $\rightarrow$  We investigate influences like supervision, dimensionality reduction, and architecture

# How to train an EBM?

We consider three approaches to train EBMs.

Sliced score matching (SSM) [4]. Efficient update formula based on random projection

$$\mathbb{E}_{p_{\mathbf{v}}} \mathbb{E}_{p(\mathbf{x})} \left[ \mathbf{v}^T \nabla_{\mathbf{x}} s_{\theta}(\mathbf{x}) \mathbf{v} + \frac{1}{2} \| s_{\theta}(\mathbf{x}) \|_2^2 \right]$$

where  $s_{\theta}(\mathbf{x}) = \nabla_{\mathbf{x}} p_{\theta}(\mathbf{x})$  and  $\mathbf{v} \sim p_{\mathbf{v}}$  is a simple distribution of random vectors.

Contrastive divergence (CD) [3]. Approximation of the gradient of the maximum likelihood objective by

$$\nabla_{\theta} p_{\theta}(\mathbf{x}) = \mathbb{E}_{p_{\theta}(\mathbf{x}')} \left[ \nabla_{\theta} E_{\theta}(\mathbf{x}') \right] - \nabla_{\theta} E_{\theta}(\mathbf{x})$$

**VERA** [1]. Learn the parameters  $\phi$  of a auxiliary distribution  $q_{\phi}$  as the optimum of

$$\log Z(\theta) = \max_{q_{\phi}} \mathbb{E}_{q_{\phi}(\mathbf{x})} \left[ f_{\theta}(\mathbf{x}) \right] + H(q_{\phi})$$

which can be plugged into Eq. (1) to obtain an alternative method for training EBMs with a variational approximation to estimate the entropy term  $H(q_{\phi})$ .

## SETUP

Natural and non-natural datasets. *Natural datasets*, e.g., images of other classes, require learning semantic features to differentiate. *Non-natural datasets*, e.g., noise, require detection farther from the training data manifold.

**OOD** detection eval. We compute the density  $p_{\theta}(\mathbf{x})$  and treat ID data as class 1 and OOD data as class 0 to compute AUC-PR.

## Experiments & Results

### Are EBMs better than Normalizing Flows?

EBMs do not consistently outperform Normalizing Flows across different training methods (Improvements: CD 11.9%, VERA 4.3%, SSM -4.3%)

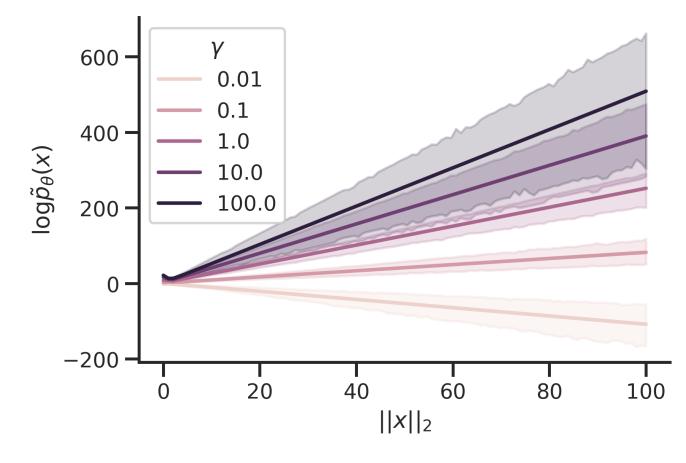
#### Does supervision improve OOD detection?

Use Joint Energy model (JEM) which incorporates supervision through cross-entropy objective.

Model	ID dataset	Natural	Non-natural
CD	CIFAR-10	-10.82	-9.11
	<b>FMNIST</b>	47.17	3.24
	Segment	1.85	0.89
	Sensorless		-0.02
	CIFAR-10	7.33	-27.94
SSM	<b>FMNIST</b>	50.61	-20.26
33IVI	Segment	25.89	-21.94
	Sensorless	-10.82 47.17 1.85 29.72 7.33 50.61	-40.73
	CIFAR-10	-1.16	-3.00
\/ED	<b>FMNIST</b>	33.66	-15.53
VERA	Segment	4.98	-0.57
	Sensorless 97.	97.93	0.07

% improvement in AUC-PR

- Supervision improves some results significantly on natural OOD datasets, but degrade on *non-natural* datasets.
- Investigation shows: Weighting parameter of cross-entropy loss  $\gamma$  affects the density estimates far from training data
  - → Non-natural data becomes harder to detect



### Sidestepping tuning of $\gamma$

Introduce supervision indirectly by training on embeddings obtained from a classification model.

Model	ID dataset	Natural	Non-natural
CD	CIFAR-10	48.60	3.37
	FMNIST	95.79	-13.52
SSM	CIFAR-10	53.84	-2.31
	FMNIST	58.40	59.59
VERA	CIFAR-10	50.16	16.97
	FMNIST	15.12	1.80

% improvement in AUC-PR

- OOD detection significantly improves on natural and in cases on non-natural datasets
- Shows that vanilla EBMs struggle to extract high-level, semantic features

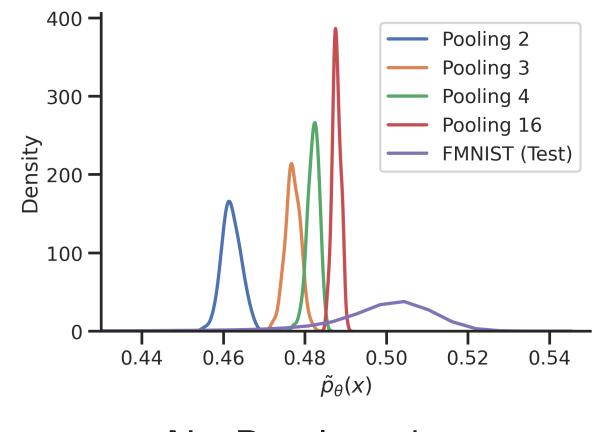
### Can we encourage semantic features?

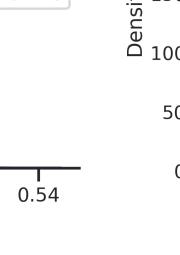
Introduce bottlenecks through  $1 \times 1$  convolutions into the architecture.

	Model	ID dataset	Natural	Non-natural
	CD	CIFAR-10 FMNIST	20.18 67.95	20.38 10.88
	SSM	CIFAR-10 FMNIST	14.76 1.75	33.34 -5.92
	VERA	CIFAR-10 FMNIST	19.66 26.84	33.22 32.94

% improvement in AUC-PR

- OOD detection improves consistently upon the baseline EBMs by learning higher-level features
- The difference in density assigned for low-level features compared to images increases significantly





0.35

No Bottleneck.

With Bottleneck.

# REFERENCES

- [1] W. Grathwohl, J. Kelly, M. Hashemi, M. Norouzi, K. Swersky, and D. Duvenaud. No MCMC for me: Amortized sampling for fast and stable training of energy-based models. arXiv:2010.04230 [cs], Oct. 2020.
- [2] W. Grathwohl, K.-C. Wang, J.-H. Jacobsen, D. Duvenaud, M. Norouzi, and K. Swersky. Your Classifier is Secretly an Energy Based Model and You Should Treat it Like One. arXiv:1912.03263 [cs, stat], Sept. 2020.
- [3] G. E. Hinton. Training Products of Experts by Minimizing Contrastive Divergence. Neural Computation, 14(8):1771–1800, Aug. 2002.
- [4] Y. Song, S. Garg, J. Shi, and S. Ermon. Sliced Score Matching: A Scalable Approach to Density and Score Estimation. arXiv:1905.07088 [cs, stat], June 2019.